NACA

RESEARCH MEMORANDUM

A STUDY OF THE DYNAMIC STABILITY OF THE BELL X-1 RESEARCH AIRPLANE

By Edward C. Polhamus

Langley Aeronautical Laboratory
Langley Air Force Base, Va.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WASHINGTON

January 10, 1950

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

A STUDY OF THE DYNAMIC STABILITY OF THE BELL X-1 RESEARCH AIRPLANE

By Edward C. Polhamus

SUMMARY

The period and damping of the lateral oscillation of the Bell X-1 research airplane have been calculated for a range of Mach numbers and wing loadings for an altitude of 30,000 feet. The calculations included an investigation of the effect of various parameters. The calculations indicate that according to current flying-quality requirements the damping of the lateral oscillation is unsatisfactory for practically all conditions investigated. Flight results indicate even poorer damping of the lateral oscillation. However, there is strong evidence that the yawing moment due to rolling velocity is probably very much less than the theoretical values and may even be zero. If a zero value of the yawing moment due to rolling velocity is assumed and combined with only a 20-percent reduction in the value of the yawing moment due to yawing velocity, the calculations for the damping of the lateral oscillation are found to be in good agreement with the flight results.

INTRODUCTION

Reference 1 presents charts from which estimates of the dynamic-lateral-stability characteristics of aircraft with light wing loadings can be made. Subsequent studies (references 2 and 3), however, have indicated that for aircraft with relatively heavy wing loadings and flying at high altitudes the number of important variables involved makes it impractical to construct generalized charts even when the stability derivatives involved are accurately known. Therefore, each configuration must be studied separately and in detail.

The purpose of the present investigation was to calculate the period and damping of the lateral oscillation of the Air Force-NACA sponsored Bell X-1 transonic research airplane (see fig. 1) at Mach numbers ranging from 0.4 to 1.4 for several flight conditions. Inasmuch as there is considerable uncertainty in regard to the values of some of the stability derivatives of this aircraft, the effect of deviations from the assumed values on the period and damping of the lateral oscillation has also been investigated.

COEFFICIENTS AND SYMBOLS

C_L	trim lift coefficient ($W \cos \gamma / qS$)
C_l	rolling-moment coefficient (Rolling moment/qSb)
C_n	yawing-moment coefficient (Yawing moment/qSb)
C_y	lateral-force coefficient (Lateral force/qS)
q	dynamic pressure, pounds per square foot ($\rho V^2/2$)
ρ	mass density of air, slugs per cubic foot
V	airplane velocity, feet per second
S	wing area, square feet
b	wing span, feet
W	weight of airplane, pounds
m	mass of airplane, slugs (W/g)
g	acceleration of gravity, feet per second per second
μ	relative-density factor ($m/\rho Sb$)
k_{X_0}	radius of gyration in roll about principal longitudinal axis, feet
k_{Z_0}	radius of gyration in yaw about principal normal axis, feet
β	angle of sideslip, radians
α	angle of attack of airplane reference axis, degrees (see fig. 2)
η	angle of attack of principal longitudinal axis of airplane, degrees (see fig. 2)

ϵ	angle between reference axis and principal axis, degrees (see fig. 2)
γ	angle of flight path to horizontal, degrees (see fig. 2)
r	yawing angular velocity, radians per second
p	rolling angular velocity, radians per second
M	Mach number (V/a)
a	local speed of sound, feet per second

$$C_{l_\beta} = \frac{\partial C_l}{\partial \beta}$$

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta}$$

$$C_{Y_\beta} = \frac{\partial C_Y}{\partial \beta}$$

$$C_{l_p} = \frac{\partial C_l}{\partial \frac{pb}{2V}}$$

$$C_{n_p} = \frac{\partial C_n}{\partial \frac{pb}{2V}}$$

$$C_{Y_p} = \frac{\partial C_Y}{\partial \frac{pb}{2V}}$$

$$C_{l_r} = \frac{\partial C_l}{\partial \frac{rb}{2V}}$$

ϵ	angle between reference axis and principal axis, degrees (see fig. 2)
γ	angle of flight path to horizontal, degrees (see fig. 2)
r	yawing angular velocity, radians per second
p	rolling angular velocity, radians per second
M	Mach number (V/a)
a	local speed of sound, feet per second

$$C_{l_\beta} = \frac{\partial C_l}{\partial \beta}$$

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta}$$

$$C_{Y_\beta} = \frac{\partial C_Y}{\partial \beta}$$

$$C_{l_p} = \frac{\partial C_l}{\partial \frac{pb}{2V}}$$

$$C_{n_p} = \frac{\partial C_n}{\partial \frac{pb}{2V}}$$

$$C_{Y_p} = \frac{\partial C_Y}{\partial \frac{pb}{2V}}$$

$$C_{l_r} = \frac{\partial C_l}{\partial \frac{rb}{2V}}$$

$$C_{n_r} = \frac{\partial C_n}{\partial \frac{rb}{2V}}$$

$$C_{Y_r} = \frac{\partial C_Y}{\partial \frac{rb}{2V}}$$

P period of the lateral oscillation, seconds

$T_{1/2}$ time for lateral oscillation to reduce to half amplitude, seconds

$C_{1/2}$ cycles for lateral oscillations to reduce to half amplitude

CALCULATIONS

All calculations in this investigation were made by the methods of reference 3. The calculations included the effect of Mach number and wing loading on the period and damping of the lateral oscillation and the effect of changes in the magnitude of some of the parameters on the period and damping.

The values of the mass and the aerodynamic parameters used for the basic investigation are presented in table I. The low-speed static-stability derivatives C_{n_β}, C_{l_β}, and C_{Y_β} were obtained from unpublished wind-tunnel data. These parameters were corrected for subsonic compressibility effects by assuming that the wing-fuselage contribution was unaffected by compressibility while the tail contribution to these parameters could be corrected by the three-dimensional Prandtl-Glauert transformation (reference 4). The values of the damping-in-roll parameter C_{l_p} through the Mach number range were obtained from reference 5. The incompressible values of the wing contribution to the rotary derivatives C_{l_r}, C_{n_r}, and C_{n_p} were obtained from reference 6 and were corrected for compressibility by the method of reference 7. The vertical tail contributions to the rotary derivatives were estimated by the method of reference 8 using values of the tail contribution to C_{Y_β} which were corrected for compressibility.

[redacted]

The aerodynamic characteristics for the supersonic case were estimated with the aid of references 9 and 10.

RESULTS AND DISCUSSION

Basic conditions.- The period and damping of the lateral oscillation for several wing loadings through the Mach number range from 0.40 to 0.75 at an altitude of 30,000 feet are presented in figure 3. The results indicate that increasing the Mach number caused a decrease in the period while increasing the wing loading caused a slight increase in the period. The effect of Mach number and wing loading on the damping is not very consistent but, in general, it can be said that at the higher Mach numbers an increase in the wing loading increases the time required to damp the oscillation to one-half of its original amplitude. Also presented in figure 3 is a comparison of the damping characteristics with the requirements for satisfactory damping of the lateral oscillation as set forth in reference 11. On the basis of these requirements, it can be seen that the damping of the oscillation would be unsatisfactory for Mach numbers above about 0.50.

The results of the calculations made at a supersonic Mach number of 1.4 are presented in figure 4 as charts of the variation of the period and the time and cycles to damp to one-half amplitude with wing loading. The results indicate, as was the case in the subsonic range, that although the wing loading has little effect on the period of the oscillation increasing the wing loading from 54 to 85 pounds per square foot increases $T_{1/2}$ from 1.2 seconds to 4.6 seconds and $C_{1/2}$ from 1.6 to 4.8. Also presented in figure 4 is a comparison of the damping characteristics with the requirements set forth in reference 11 for periods less than 2 seconds. The results indicate that according to this criterion the damping of the oscillation is unsatisfactory at a Mach number of 1.4 for wing loadings above about 65.

Effect of various parameters.- Inasmuch as no wind-tunnel data are available for the rotary derivatives of this airplane and since little is known about the effects of compressibility on these parameters, some additional calculations were made to determine the effect of changes in these parameters on the period and damping of the oscillation. Condition 10 (table I) was chosen for the basic condition and the results of variations from the basic condition are presented in figure 5. The results indicate that changes in the rotary derivatives as much as ±50 percent have a negligible effect on the period of the lateral oscillation. However, the effect on the time for the oscillation to damp to one-half of the original amplitude is more pronounced and for this configuration the damping-in-yaw parameter C_{n_r} is the most important of the rotary derivatives.

The effect of the inclination of the principal axis with reference to the fuselage reference axis and the flight-path angle were also investigated and the results are presented in figure 5. It should be mentioned that changes in the inclination of the principal axis with respect to the fuselage reference line are important only because at a given angle of attack they change the inclination of the principal axis with respect to the flight path. Neither of these parameters has any appreciable effect on the period of the oscillation in the range investigated. A change in the flight-path angle from level flight to a 45° climb caused a slight decrease in the time to damp to one-half amplitude while a change in the inclination of the principal axis from 0° to 3° down at the nose caused an increase in the time to damp of a little over 1 second.

Comparison with flight results.- A time history of the stick-fixed lateral oscillation of the actual airplane is presented in figure 6. This record was obtained in gliding flight at an altitude of 32,000 feet, a Mach number of 0.75, and a wing loading of about 85 pounds per square foot. (See reference 12.) The flight tests indicate a period of the lateral oscillation of about 1.8 seconds which is in fair agreement with the calculated period of about 1.5 seconds. (See fig. 3.) Although the calculations indicate, according to current flying-qualities requirements, that the damping of the oscillation is unsatisfactory, they do not indicate it to be as poor as the flight results show. According to the flight results, the time required to damp the oscillation to one-half of its original amplitude is about 6.5 seconds while the calculations give about 2.7 seconds.

Inasmuch as the calculations did not check the flight results, an attempt was made to determine the cause of this discrepancy. As was pointed out in the previous section, less is probably known about the rotary derivatives of this airplane than about any of the other parameters necessary for the calculation of the period and damping of the lateral oscillation. Figure 5 indicates that decreases in C_{n_r} or C_{n_p} or both from the assumed values might account for the discrepancy in the damping. Reference 13 and some recent unpublished results obtained by both the rolling-flow and forced-rotation methods indicate that the tail contribution to C_{n_p} is much less than that indicated by the theory of reference 8 and, in some cases, it has been found to be zero. It is also quite possible that separation and wake effects might, at least in the small-angle range, cause a considerable decrease in C_{n_r}. The results of additional calculations investigating the individual and combined effects of losses in C_{n_r} and C_{n_p} are presented in the following table using condition 15 (table I) as the base:

Condition	C_{n_r}	C_{n_p}	P	$T_{1/2}$
15 (table I)	−0.276	0.094	1.54	2.71
15 − A	−.276	0	1.49	3.75
15 − B	−.276	−.015	1.48	3.99
15 − C	−.210	0	1.49	5.12
15 − D	−.138	0	1.49	8.47
15 − E	−.170	0	1.49	6.56
Flight results	------	-----	1.8	6.5

The results indicate that even a zero tail contribution to C_{n_p}, which results in a negative value for the total, (condition 15 − B) does not, by itself, account for the discrepancy in the damping. However, when a zero value of C_{n_p} was combined with various values of C_{n_r} it was found that a reduction in C_{n_r} of only about 20 percent (condition 15 − E) was required to bring the calculations in agreement with the flight results.

CONCLUSIONS

Based on calculations of the lateral oscillation of the Bell X-1 transonic research airplane, the following conclusions have been reached:

1. Theoretical computations indicate that, according to current flying-quality requirements, the damping of the lateral oscillation would be considered unsatisfactory for Mach numbers from about 0.50 to 0.75, (the highest subsonic case investigated) for all wing loadings and for wing loadings above about 65 for a Mach number of 1.4 (the only supersonic case investigated).

2. Flight results indicate even poorer damping of the lateral oscillation. However, there is strong evidence that the yawing moment due to rolling velocity is probably very much less than the theoretical values and may even be zero. If a zero value of the yawing moment due to rolling velocity is assumed and combined with only a 20-percent reduction in the value of the yawing moment due to yawing velocity, the

calculations for the damping of the lateral oscillation are found to be in good agreement with the flight results.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va.

REFERENCES

1. Zimmerman, Charles H.: An Analysis of Lateral Stability in Power-Off Flight with Charts for Use in Design. NACA Rep. 589, 1937.

2. Sternfield, Leonard: Effect of Product of Inertia on Lateral Stability. NACA TN 1193, 1947.

3. Sternfield, Leonard: Some Considerations of the Lateral Stability of High-Speed Aircraft. NACA TN 1282, 1947.

4. Göthert, B.: Plane and Three-Dimensional Flow at High Subsonic Speeds. NACA TM 1105, 1946.

5. Bird, John D.: Some Theoretical Low-Speed Span Loading Characteristics of Swept Wings in Roll and Sideslip. NACA TN 1839, 1949.

6. Toll, Thomas A., and Queijo, M. J.: Approximate Relations and Charts for Low-Speed Stability Derivatives of Swept Wings. NACA TN 1581, 1948.

7. Fisher, Lewis R.: Approximate Corrections for the Effects of Compressibility on the Subsonic Stability Derivatives of Swept Wings. NACA TN 1854, 1949.

8. Bamber, Millard J.: Effect of Some Present-Day Airplane Design Trends on Requirements for Lateral Stability. NACA TN 814, 1941.

9. Harmon, Sidney M.: Stability Derivatives of Thin Rectangular Wings at Supersonic Speeds. Wing Diagonals Ahead of Tip Mach Lines. NACA TN 1706, 1948.

10. Lagerstrom, P. A., Wall, D., and Graham, M. D.: Formulas in Three-Dimensional Wing Theory. Rep. No. SM-11901, Douglas Aircraft Co., Inc., July 8, 1946.

11. Anon.: Flying Qualities of Piloted Airplanes. U. S. Air Force Specification No. 1815-B, June 1, 1948.

12. Williams, Walter, C., Forsyth, Charles M., and Brown, Beverly P.: General Handling-Qualities Results Obtained during Acceptance Flight Tests of the Bell XS-1 Airplane. NACA RM L8A09, 1948.

13. Bird, John D., Lichtenstein, Jacob H., and Jaquet, Byron M.: Investigation of the Influence of Fuselage and Tail Surfaces on Low-Speed Static Stability and Rolling Characteristics of a Swept-Wing Model. NACA RM L7H15, 1947.

[redacted]

TABLE I

PARAMETERS USED IN CALCULATIONS OF PERIOD AND DAMPING OF

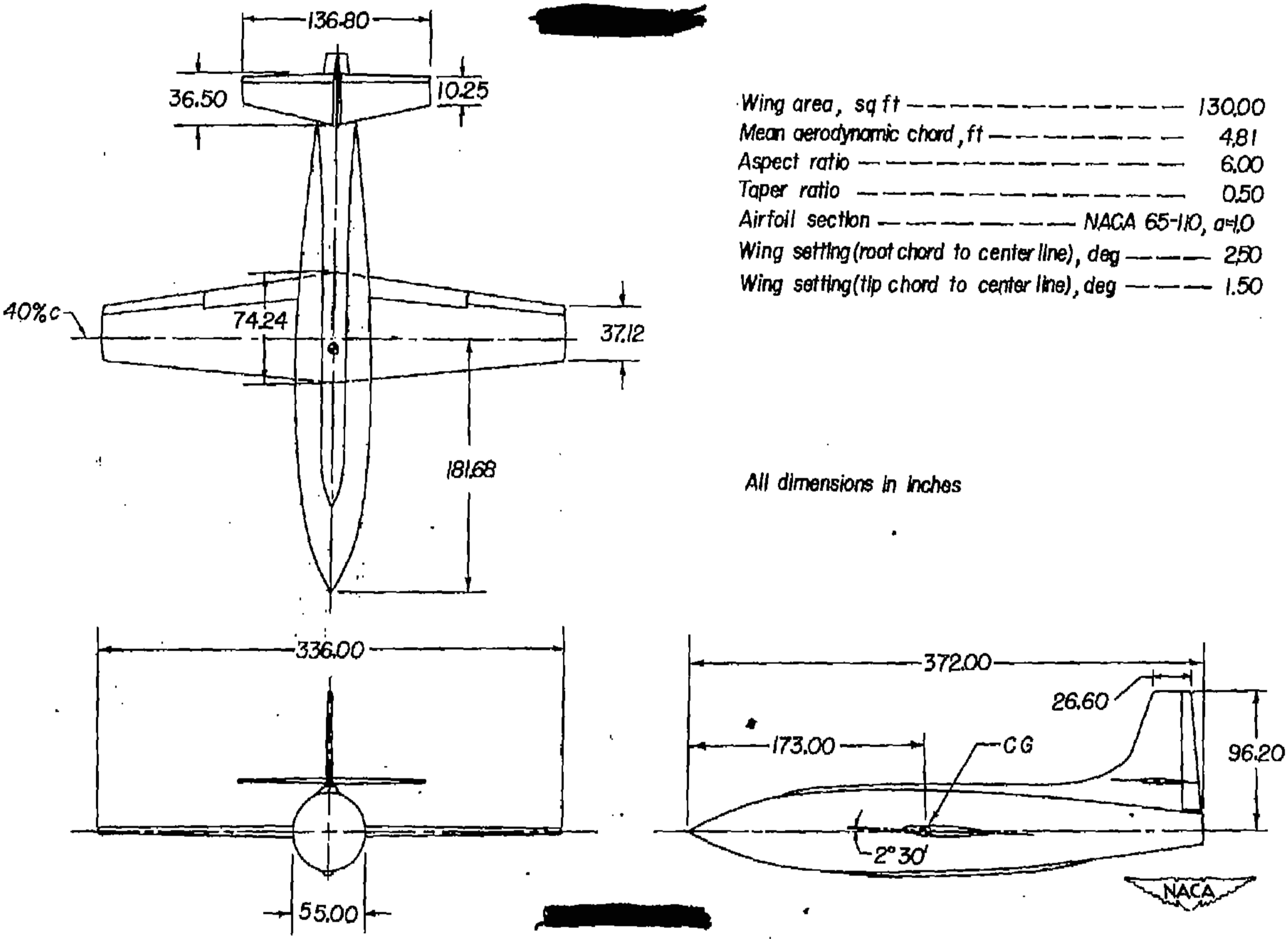
LATERAL OSCILLATION OF BELL X-1 RESEARCH AIRPLANE

[$\epsilon = 2^{\circ}$; $C_{Y_r} = 0$; attitude = 30,000 ft]

Condition	M	W/S	μ	$\left(\frac{k_{X_0}}{b}\right)^2$	$\left(\frac{k_{Z_0}}{b}\right)^2$	η	C_L	C_{n_β}	C_{l_β}	C_{Y_β}	C_{l_p}	C_{l_r}	C_{n_p}	C_{n_r}	C_{Y_p}
1	0.4	54	67.50	0.0078	0.0433	4.4	0.765	0.205	-0.099	-0.930	-0.425	0.288	0.053	-0.305	0
2	.5	54	67.50	.0078	.0433	1.2	.490	.211	-.084	-.884	-.435	.222	.065	-.281	0
3	.6	54	67.50	.0078	.0433	-.6	.341	.200	-.091	-.838	-.446	.199	.085	-.262	0
4	.7	54	67.50	.0078	.0433	-1.7	.250	.205	-.105	-.866	-.465	.190	.099	-.271	0
5	.75	54	67.50	.0078	.0433	-2.0	.218	.205	-.110	-.878	-.474	.187	.105	-.274	0
6	1.40	54	67.50	.0078	.0433	-3.0	.062	.179	-.121	-.670	-.500	.104	.113	-.248	.014
7	.5	65	81.25	.0067	.0419	2.2	.590	.212	-.084	-.884	-.435	.257	.069	-.294	0
8	.6	65	81.25	.0067	.0419	.1	.410	.200	-.088	-.838	-.446	.218	.080	-.263	0
9	.7	65	81.25	.0067	.0419	-1.2	.301	.205	-.101	-.866	-.465	.202	.094	-.272	0
10	.75	65	81.25	.0067	.0419	-1.6	.262	.205	-.107	-.878	-.474	.200	.101	-.275	0
11	1.40	65	81.25	.0067	.0419	-2.7	.074	.179	-.121	-.670	-.500	.104	.115	-.248	.018
12	.5	85	106.3	.0051	.0409	4.1	.771	.212	-.095	-.884	-.435	.300	.053	-.312	0
13	.6	85	106.3	.0051	.0409	1.5	.547	.221	-.086	-.838	-.446	.257	.074	-.279	0
14	.7	85	106.3	.0051	.0409	-.3	.394	.218	-.096	-.866	-.465	.228	.086	-.273	0
15	.75	85	106.3	.0051	.0409	-.9	.343	.217	-.101	-.878	-.474	.224	.094	-.276	0
16	1.40	85	106.3	.0051	.0409	-2.5	.097	.179	-.121	-.670	-.500	.104	.117	-.248	.021

[redacted]

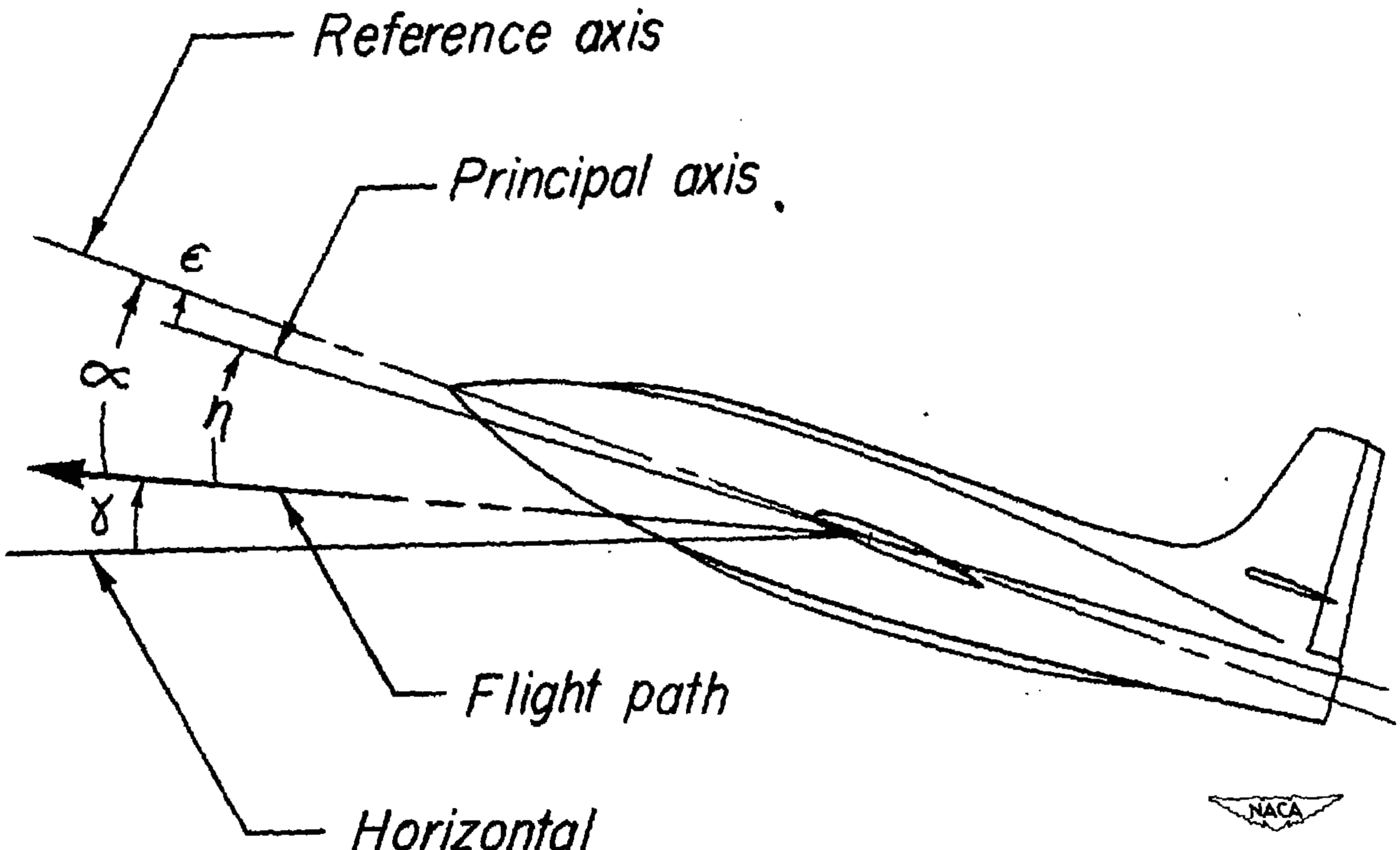
[redacted]



[redacted]

Figure 1.- Drawing of Bell X-1 transonic research airplane.

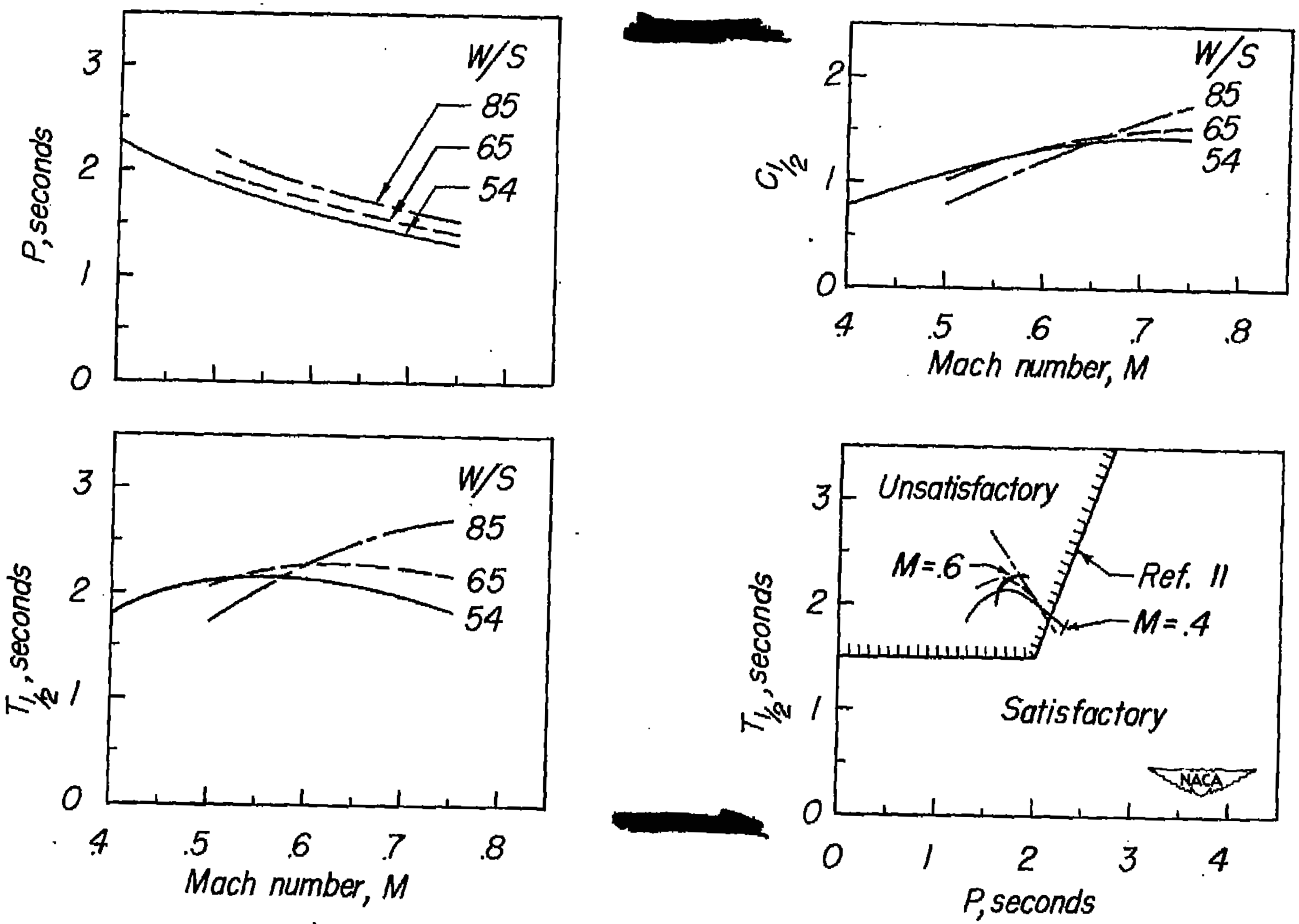
[redacted]



[redacted]

Figure 2.- Angular relationships in flight. Arrows indicate positive direction of angles.

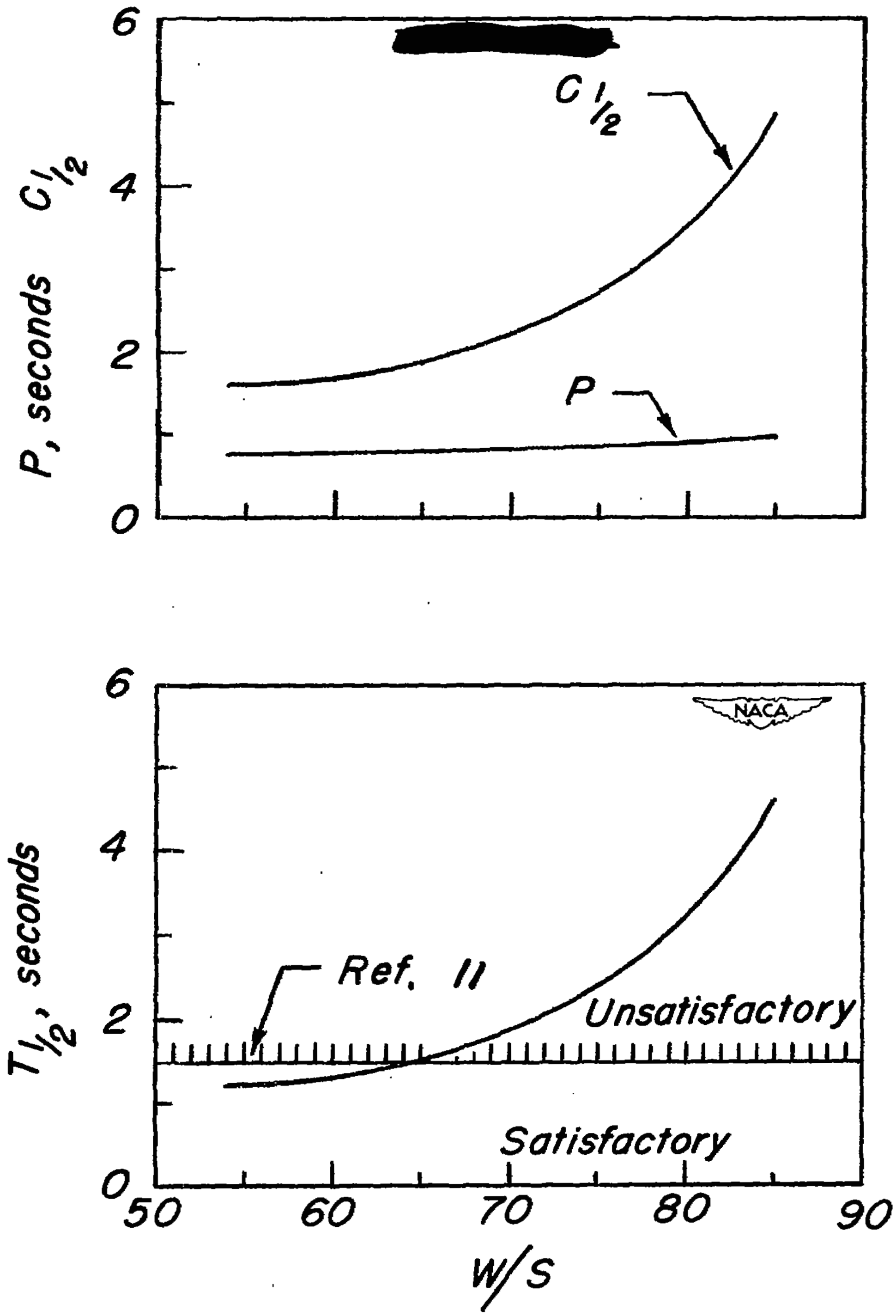
[redacted]



[redacted]

Figure 3.- Effect of Mach number and wing loading on the period and damping of the lateral oscillation. Altitude = 30,000 feet.

[redacted]



[redacted]

Figure 4.- Effect of wing loading on the period and damping of the lateral oscillation. M = 1.5; altitude = 30,000 feet.

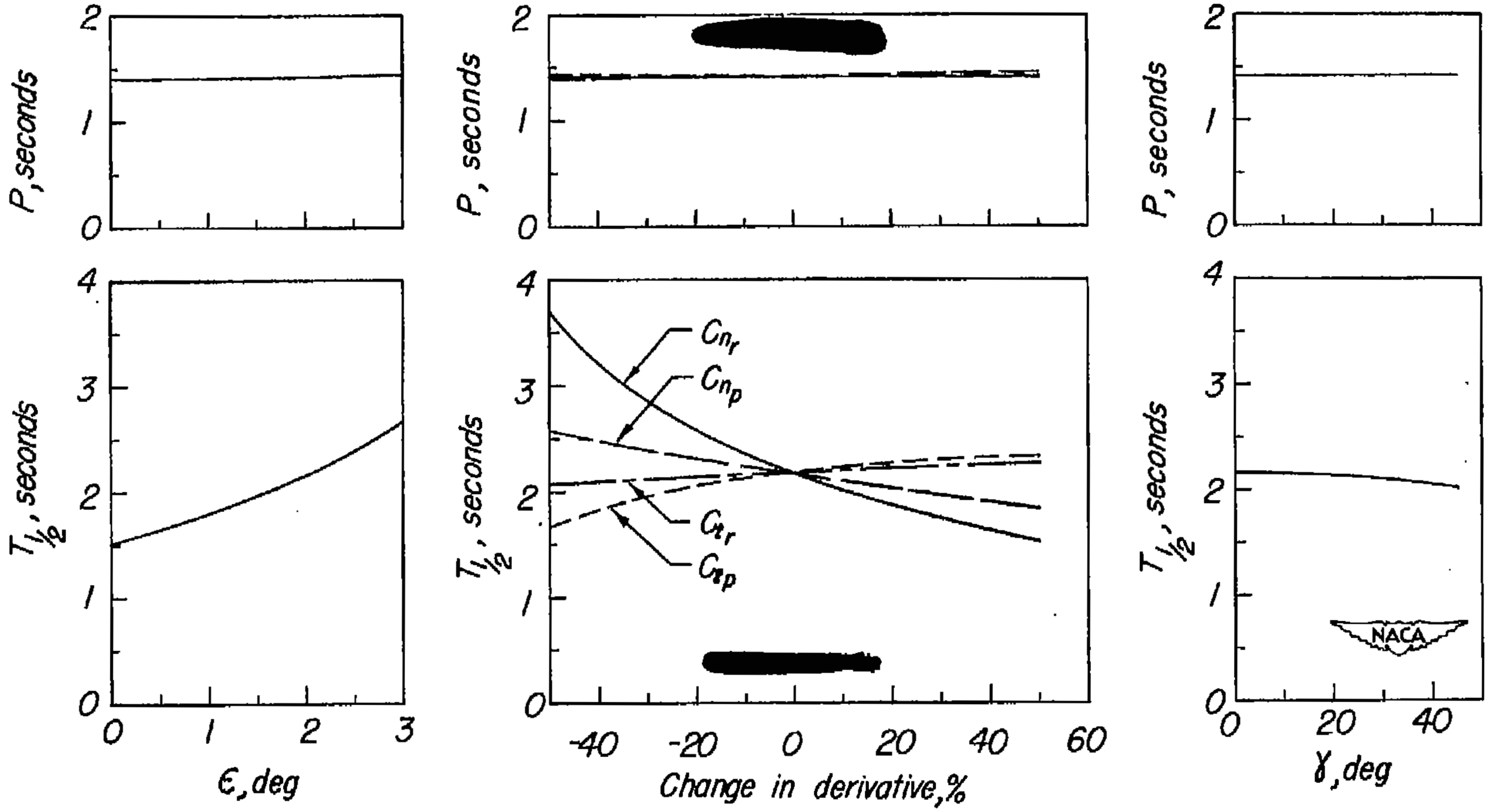
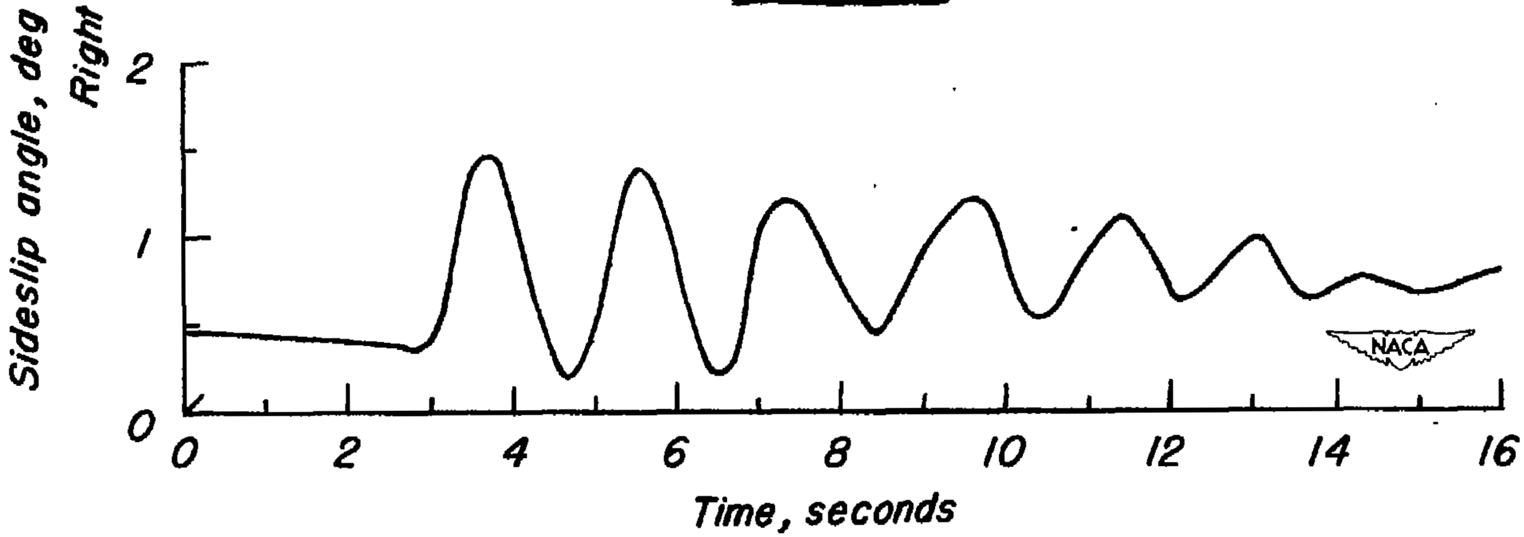


Figure 5.- Effect of the rotary derivatives, the inclination of the principal axis and the flight path angle on the period and damping of the lateral oscillation. M = 0.75; altitude = 30,000 feet; $\frac{W}{S} = 65$.

[redacted]



[redacted]

Figure 6.- Time history of the stick-fixed lateral oscillation at a Mach number of about 0.75, an altitude of 32,000 feet and a wing loading of about 85 (data from reference 12).