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RESEARCH MEMORANDUM

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INITIAL INCLINATION OF THE MIXING BOUNDARY SEPARATING AN EXHAUSTING SUPERSONIC JET FROM A SUPERSONIC AMBIENT STREAM

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NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

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INITIAL INCLINATION OF THE MIXING BOUNDARY SEPARATING AN EXHAUSTING SUPERSONIC JET FROM A SUPERSONIC AMBIENT STREAM

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SUMMARY

Calculations have been made of the initial inclination of the mixing boundary separating an exhausting supersonic jet from an external supersonic stream as a function of jet static-pressure ratio. The calculations are based on a ratio of specific heats of 7/5 for the external stream and 9/7 for the jet. The results are presented in curve form and cover, in terms of the conditions immediately ahead of the jet exit, the following range of variables: boattail and nozzle divergence angles from 0° to 20°, external-stream Mach numbers from 1.2 to 3.0, jet Mach numbers from 1.0 to 3.0, and jet static-pressure ratios within the approximate inviscid limits for supersonic flow, except at the higher jet and stream Mach numbers where the calculations were terminated somewhat short of these limits.

INTRODUCTION

When a propulsive jet exhausts supersonically into an ambient supersonic stream, a mixing boundary is created that separates the jet flow from the ambient stream. The initial inclination of this boundary immediately downstream of the exit is important to jet interference effects produced in the ambient stream and to the jet structure's contribution to this interference since the initial inclination establishes the base from which all other flow phenomena may be considered to develop. Once this initial inclination is known, the two-dimensional nature of the flow exactly at the exit permits all the initial flow phenomena and properties of the flow immediately downstream of the jet exit to be easily determined from tables and charts such as those given in references 1 to 4. A conservative estimate of the upper limit of the interference pressure and of the beginning of the interference pressure field produced in the ambient stream by the jet's presence is thus readily obtained, as are the basic values for a rigorous calculation of the interference field.

In reference 4, results of theoretical calculations are presented which show the effect upon the initial inclination of the mixing boundary and upon the initial inclination of the exit shock produced in the ambient stream of free-stream Mach number, jet Mach number, ratio of specific heats of the jet, and jet static-pressure ratio. The calculations of reference 4 correspond to a base annulus thickness of zero, no boattailing, and to a nozzle divergence angle of zero immediately ahead of the exit. (Jet flow and ambient flow are parallel immediately ahead of exit.) The purpose of the present calculations is to extend the results of reference 4 to a variety of combinations of boattail angle and nozzle divergence angle. The present results are based upon a ratio of specific heats of 7/5 for the free stream and 9/7 for the jet; the latter ratio was selected because it is generally representative of hot jets and because of the availability of tabulated values of some of the required flow properties for this ratio. The calculations cover ambient-stream Mach numbers of 1.2, 1.5, 2.0, 2.5, and 3.0 at the boattail surface immediately ahead of the exit, jet Mach numbers of 1.0, 1.5, 2.0, 2.5, and 3.0 at the nozzle surface immediately ahead of the exit, nozzle divergence angles and boattail angles ranging from 0° to 20° immediately ahead of the exit, and jet static-pressure ratios within the approximate inviscid limits for supersonic flow, except at the higher jet and stream Mach numbers where the calculations were terminated somewhat short of these limits.

SYMBOLS

β boattail angle immediately ahead of jet exit, deg

θ nozzle divergence angle immediately ahead of jet exit, deg

p static pressure

γ ratio of specific heats

M Mach number

δ_j initial inclination of mixing boundary (positive when it diverges with respect to jet axis), deg

Subscripts:

j value at nozzle surface immediately ahead of jet exit

∞ value at boattail surface immediately ahead of jet exit

ANALYSIS AND RESULTS

Analytical Considerations

Particular attention is drawn to the conditions of this analysis. Consider the segment of the general boattailed fuselage or nacelle shown in figure 1(a). The rigorous point of analysis covers the infinitesimal region immediately upstream of the jet exit and adjacent to the boattail and nozzle surfaces, and immediately downstream of the jet exit; the initial values must conform to these conditions in using the results. The point of analysis is magnified in figure 1(b) and indicates the initial ratio of specific heats, Mach number, and static pressure of the ambient flow and of the jet flow. Only positive values of β and θ will be considered, positive β denoting convergence of the boattail and positive θ denoting divergence of the nozzle in the conventional sense as indicated in the figure. The initial conditions imply, of course, that the ambient flow is inclined at the angle β and that the jet flow is inclined at the angle θ. The sign convention for the initial deflection of the jet or mixing boundary δ_j is also indicated in figure 1(b).

The value of the initial deflection δ_j is a function only of the initial properties introduced above. If β and θ are both zero, one may visualize that when $\frac{p_j}{p_\infty} > 1$ a shock originating at the jet exit is introduced into the ambient stream (excluding $M_\infty = 1$) and an expansion centered at the jet exit is introduced into the jet; the converse is true for $\frac{p_j}{p_\infty} < 1$ (excluding $M_j = 1$). Exactly at $\frac{p_j}{p_\infty} = 1$, the jet and ambient flow are free of these disturbances, and $\delta_j = 0$. However, if β and/or θ are not zero, then the value of δ_j at $\frac{p_j}{p_\infty} = 1$ is also not zero, and a disturbance is always introduced in the jet or in the ambient flow or in both. The type of disturbance phenomenon that originates at the jet exit and is introduced in the jet and/or in the ambient flow may be logically reasoned as follows.

In ambient flow:

$$\left.\begin{array}{ll} (\delta_j + \beta) > 0 & \text{Shock} \\ (\delta_j + \beta) = 0 & \text{None} \\ (\delta_j + \beta) < 0 & \text{Expansion} \end{array}\right\} \qquad (1)$$

In jet flow:

$$\left.\begin{array}{ll} (\delta_j - \theta) > 0 & \text{Expansion} \\ (\delta_j - \theta) = 0 & \text{None} \\ (\delta_j - \theta) < 0 & \text{Shock} \end{array}\right\} \qquad (2)$$

With increasing $\frac{p_j}{p_\infty}$ the general phenomena for finite β and/or θ vary in the following manner, the first given phenomenon pertaining to the jet and the second to the ambient stream: shock-expansion, shock-none, shock-shock, none-shock, and expansion-shock. (The special cases of M_j or $M_\infty = 1$ offer exceptions.)

The determination of the value of δ_j is based upon the requirement that the static pressures on both sides of the mixing boundary must be equal. For a given set of initial conditions there is no explicit solution for δ_j. In the present calculations values of flow deflection were assumed and applied to the jet or ambient stream as indicated in the preceding paragraph. Essentially, therefore, values of δ_j were assumed and values of $\frac{p_j}{p_\infty}$ were calculated for various initial values of M_j, M_∞, β, and θ by appropriate use of the shock and expansion equations and by rotation of axes (parallel to θ or to β).

Results

The results of the calculations are presented in figures 2 to 6. The curves for all values of M_j are shown together for a particular value of M_∞ and a particular combination of β and θ. The curves for a particular combination of β and θ can be simply converted to those applicable to the infinite number of combinations of β and θ that may be formed to give the same sum of $\beta + \theta$, provided both β and θ remain positive according to the original convention. For example, curves for $\beta = 10^\circ$ and $\theta = 0^\circ$ can be made applicable to $\beta = 8^\circ$ and $\theta = 2^\circ$, $\beta = 5^\circ$ and $\theta = 5^\circ$, $\beta = 0^\circ$ and $\theta = 10^\circ$, and so forth, because by maintaining the sum of $\beta + \theta$ constant, one is merely rotating the infinitesimally small fragment of the boattail and nozzle surface depicted in figure 1(b) about a point formed by the juncture of the boattail and nozzle surfaces, and is therefore not altering the other initial quantities.

Accordingly, the value of $\frac{p_j}{p_\infty}$ remains unchanged, and the value of δ_j changes by the amount that β (or θ) was changed. For example, if β were decreased 5°, δ_j would increase 5°. Equations (1) and (2) for determining the type of disturbance phenomenon are also applicable to these converted values of β, θ, and δ_j.

The present results for finite θ and β show a similarity to those of reference 4 for $\theta = 0^\circ$ and $\beta = 0^\circ$ in indicating a small effect of M_j and, therefore, by the reversibility of the problem, an equally small effect of M_∞. Although the present calculations were based on a base annulus of zero thickness, results shown in references 4 and 5 would appear to indicate that the results of the present calculations could be satisfactorily applied to configurations having thin base annuli.

The present calculations may also be used to obtain an idea of the order of magnitude of $\frac{p_j}{p_\infty}$ for which separation begins to take place on the boattail surface or on the nozzle surface. If the value of $(\delta_j + \beta) > 0$ and exceeds the critical turning angle for separation (see ref. 6) for the same initial conditions that exist on the boattail just ahead of the jet exit, then separation may be expected on the boattail surface. In like manner if the value of $(\delta_j - \theta) < 0$ and if $|\delta_j - \theta|$ exceeds the critical angle for separation (ref. 6) for the same initial conditions that exist on the nozzle just ahead of the jet exit, then separation may be expected on the nozzle surface.

CONCLUDING REMARKS

Calculations have been made of the initial inclination of the mixing boundary separating an exhausting supersonic jet from an external supersonic stream as a function of jet static-pressure ratio. The calculations are based on a ratio of specific heats of 7/5 for the external stream and 9/7 for the jet. The results are presented in curve form and cover, in terms of the conditions immediately ahead of the jet exit, the following range of variables: boattail and nozzle divergence angles from 0° to 20°, external-stream Mach numbers from 1.2 to 3.0, jet Mach numbers from 1.0 to 3.0, and jet static-pressure ratios within the approximate inviscid limits for supersonic flow, except at the higher jet and stream Mach numbers where the calculations were terminated somewhat short of these limits.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 29, 1955.

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3. Neice, Mary M.: Tables and Charts of Flow Parameters Across Oblique Shocks. NACA TN 1673, 1948.

4. Love, Eugene S., and Grigsby, Carl E.: Some Studies of Axisymmetric Free Jets Exhausting From Sonic and Supersonic Nozzles Into Still Air and Into Supersonic Streams. NACA RM L54L31, 1955.

5. Coletti, Donald E.: Measurements and Predictions of Flow Conditions on a Two-Dimensional Base Separating a Mach Number 3.36 Jet and a Mach Number 1.55 Outer Stream. NACA RM L54C08, 1954.

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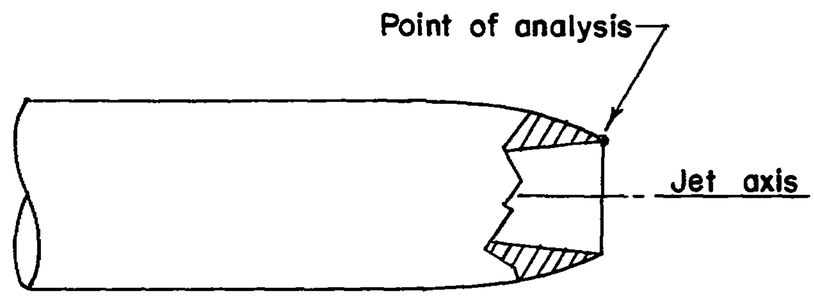
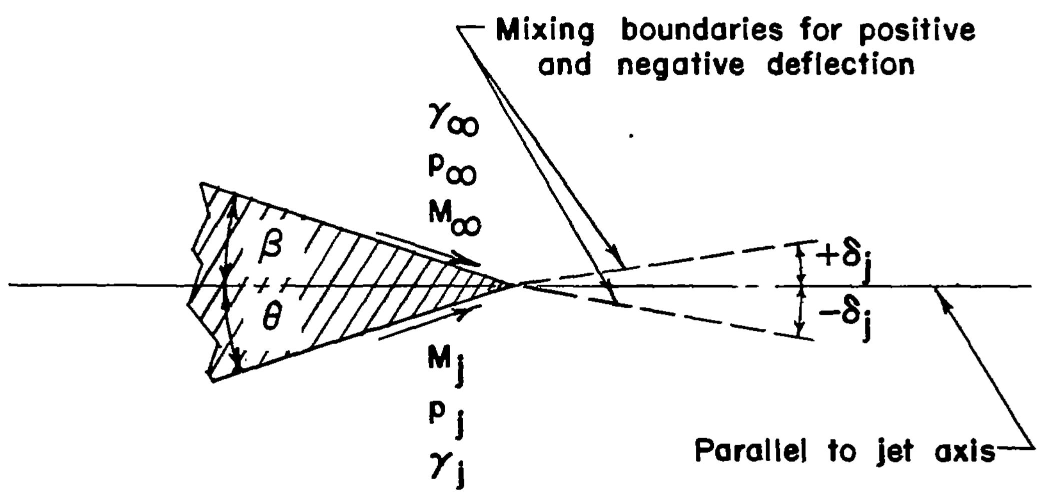


(a) Location of point of analysis.



(b) Magnification of point of analysis.

Figure 1.- Conditions of analysis used in present calculations.

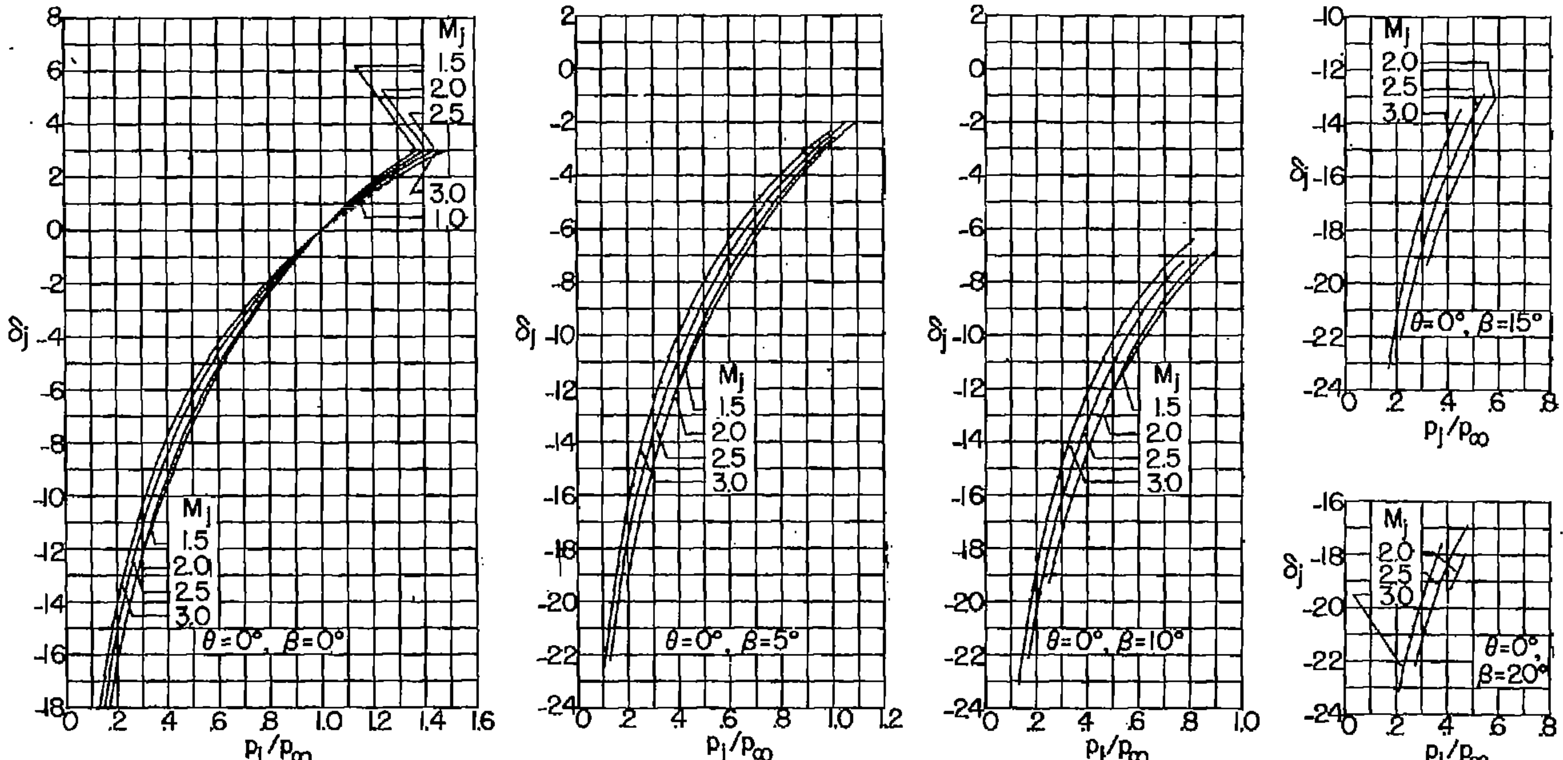


Figure 2.- The effects of jet static-pressure ratio upon the initial inclination of the mixing boundary at $M_\infty = 1.2$.

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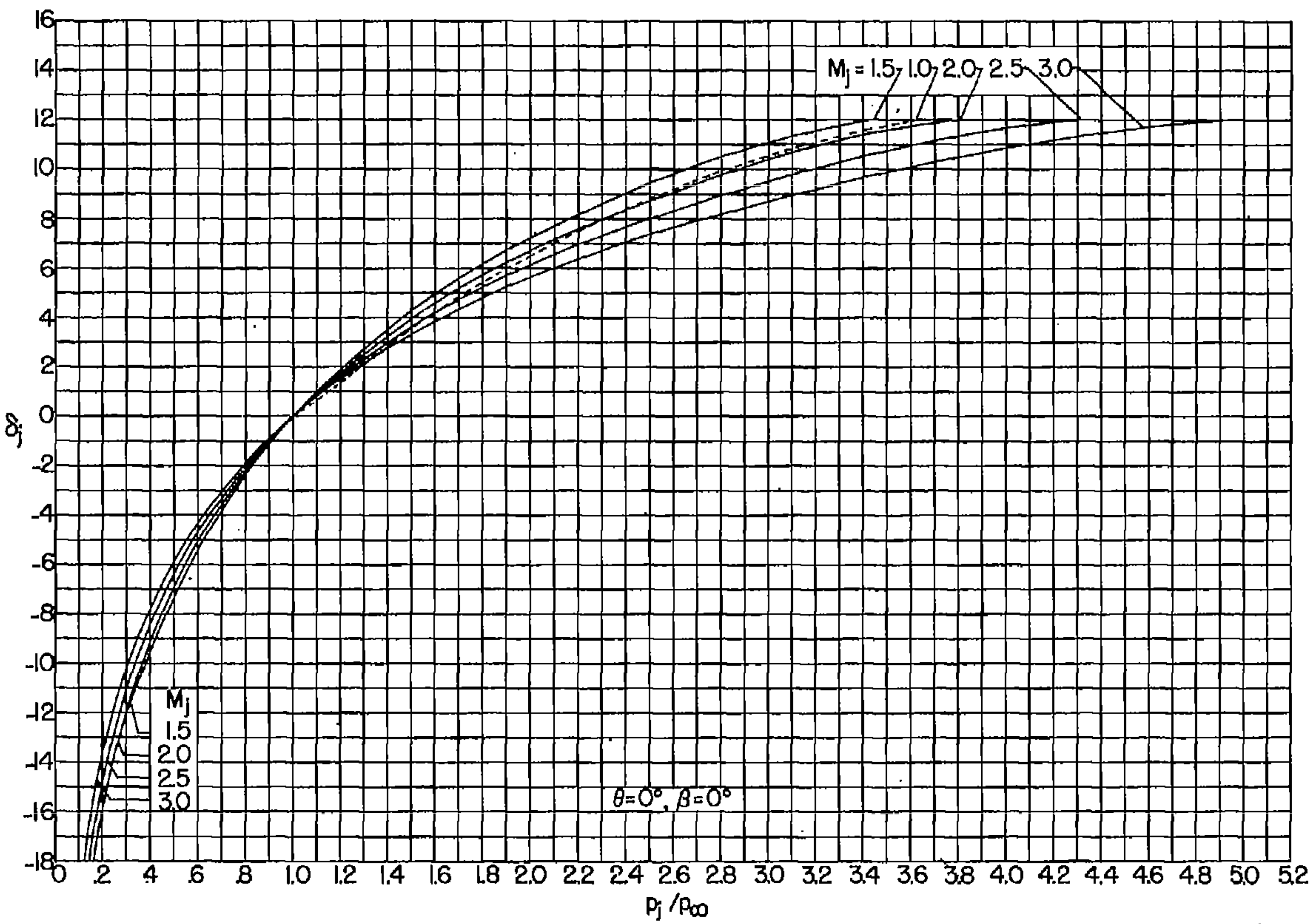

Figure 3.- The effects of jet static-pressure ratio upon the initial inclination of the mixing boundary at $M_\infty = 1.5$.

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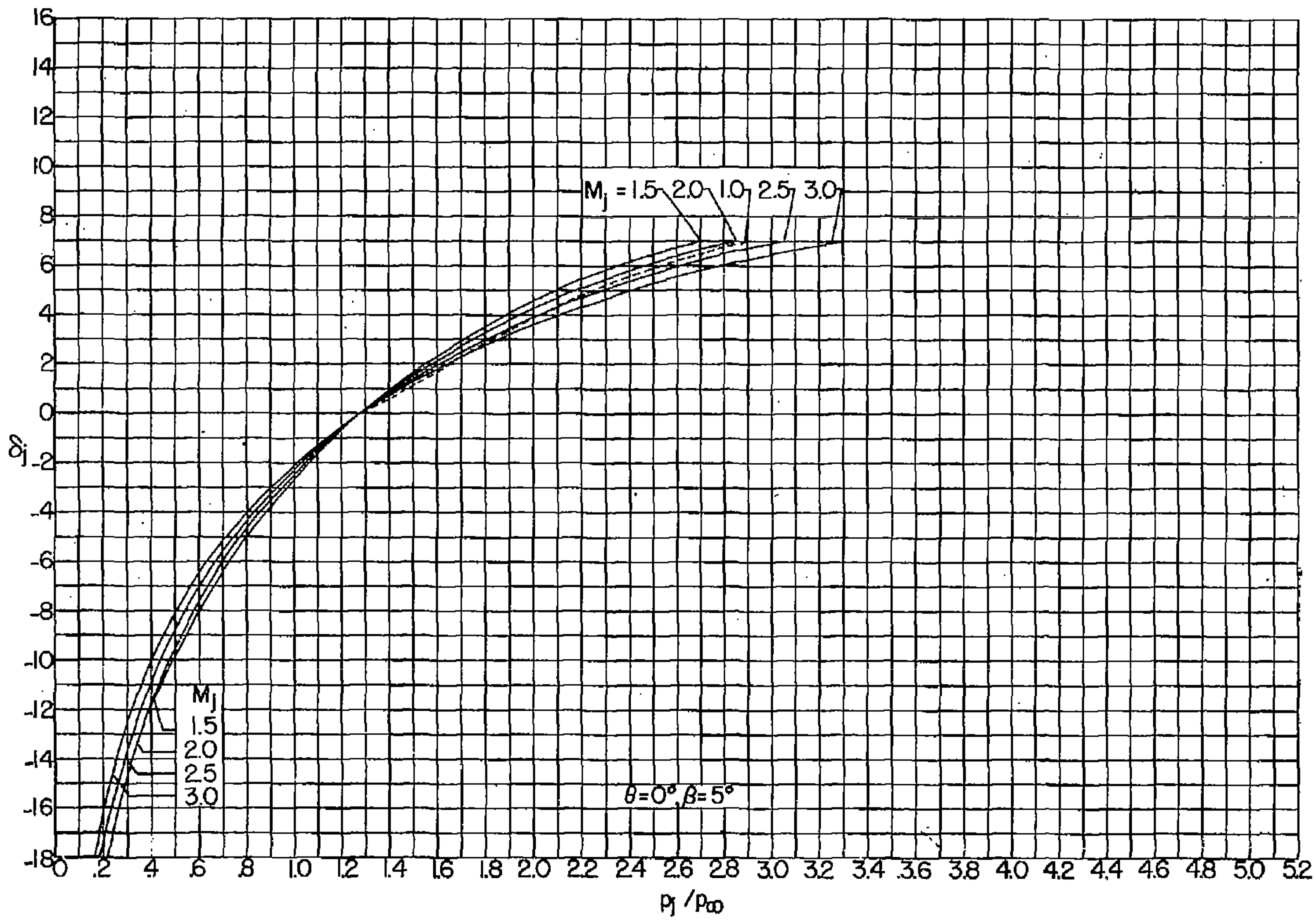

Figure 3.- Continued.

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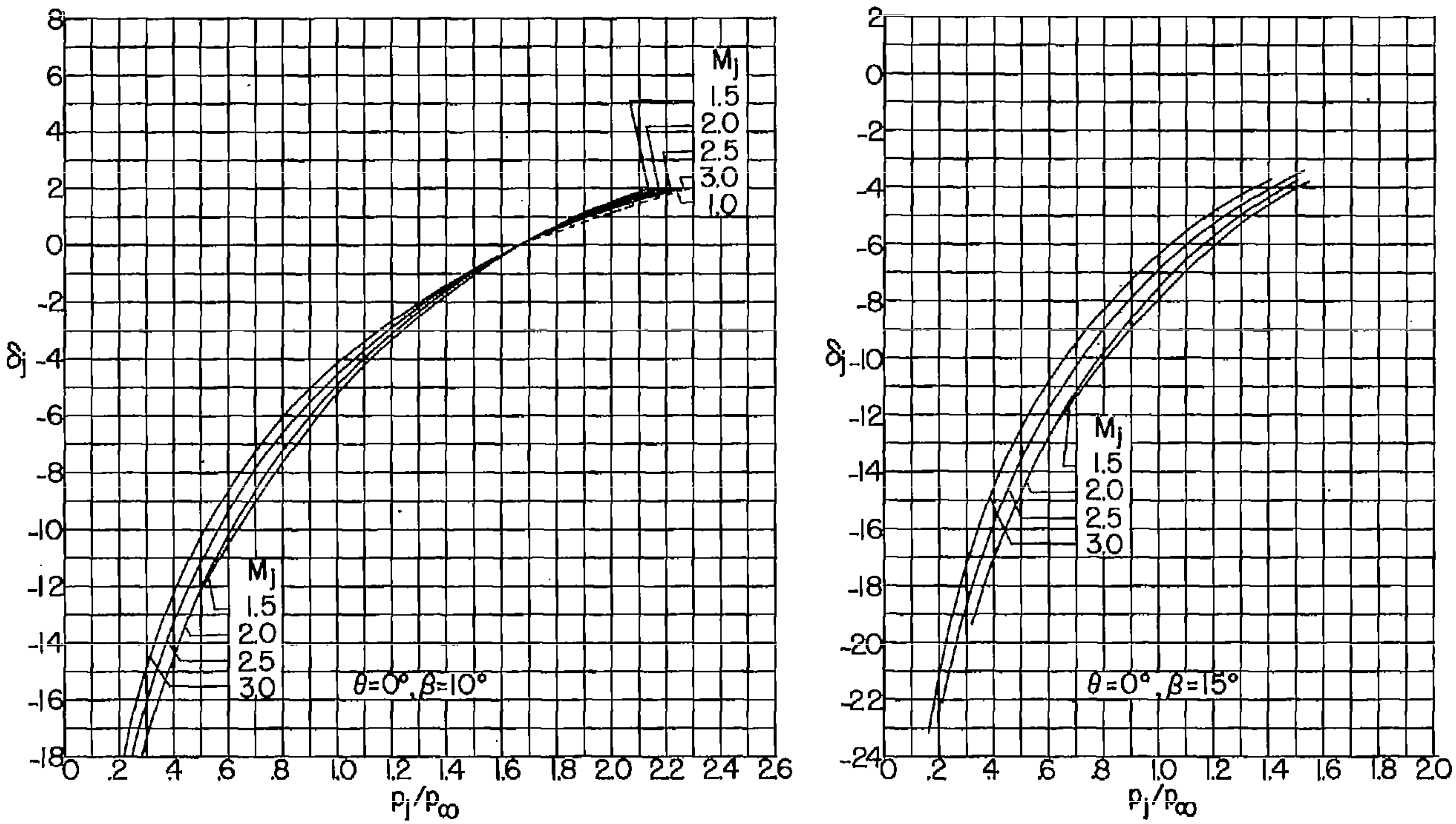

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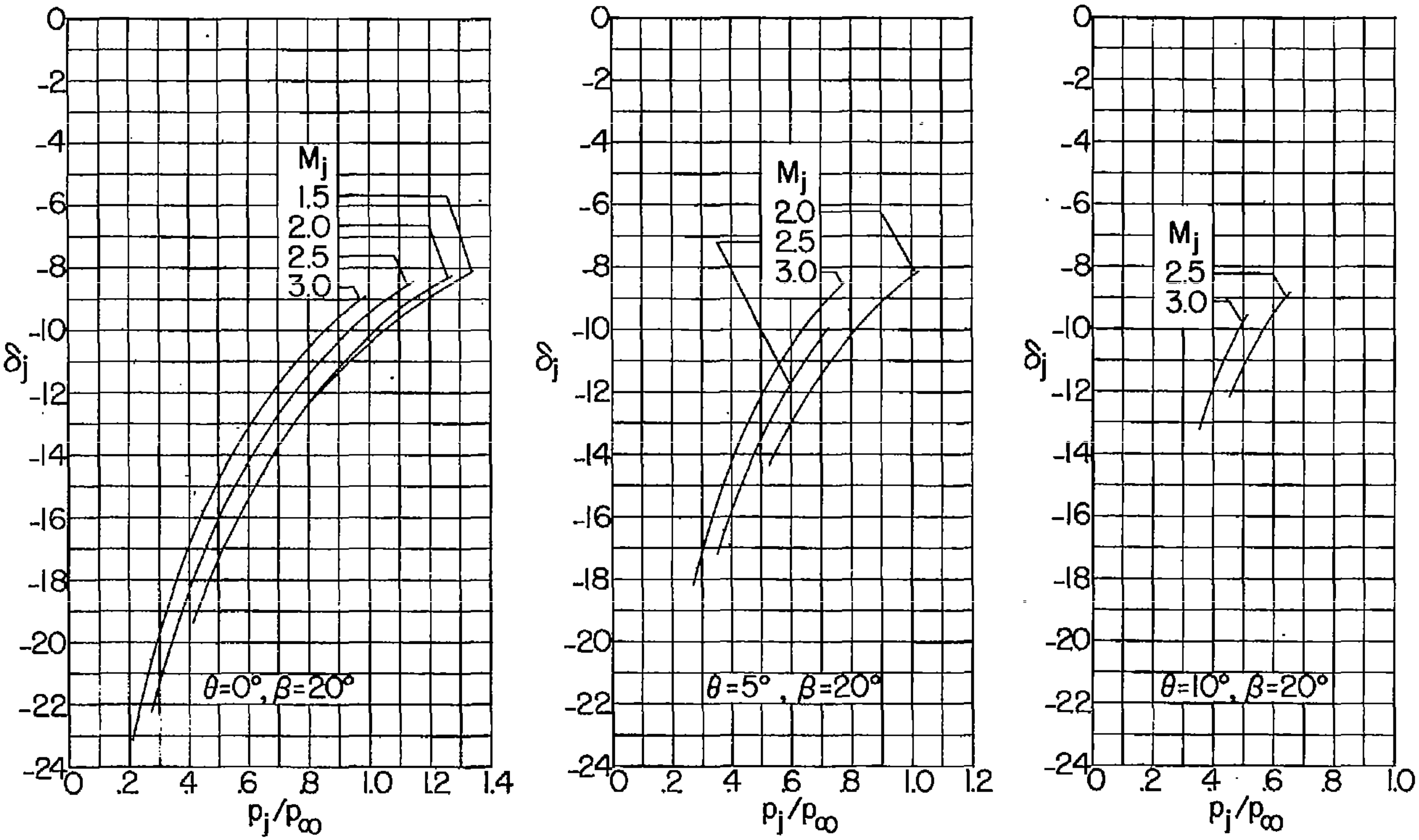

Figure 3.- Concluded.

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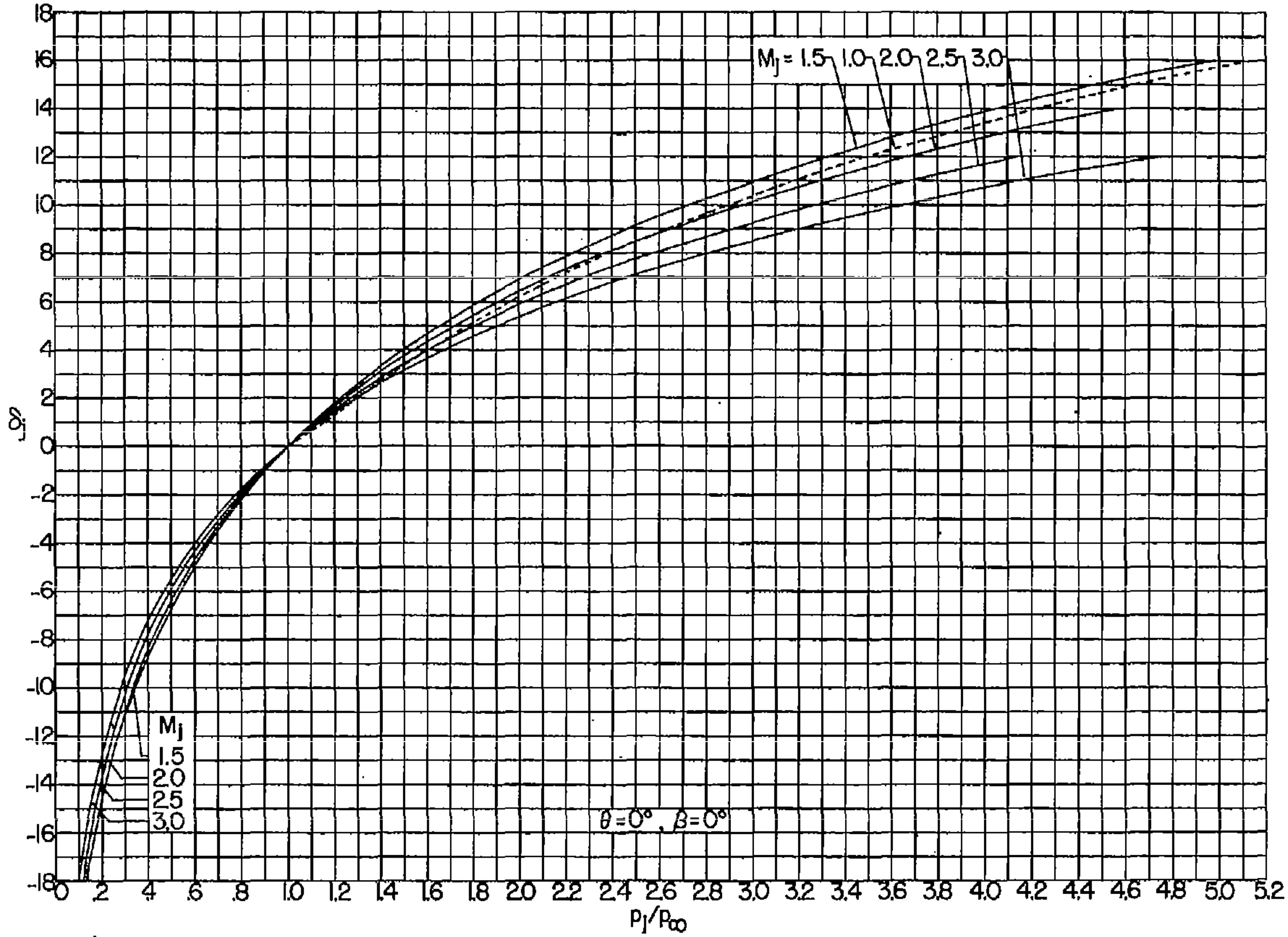

Figure 4.- The effects of jet static-pressure ratio upon the initial inclination of the mixing boundary at M_∞ = 2.0.

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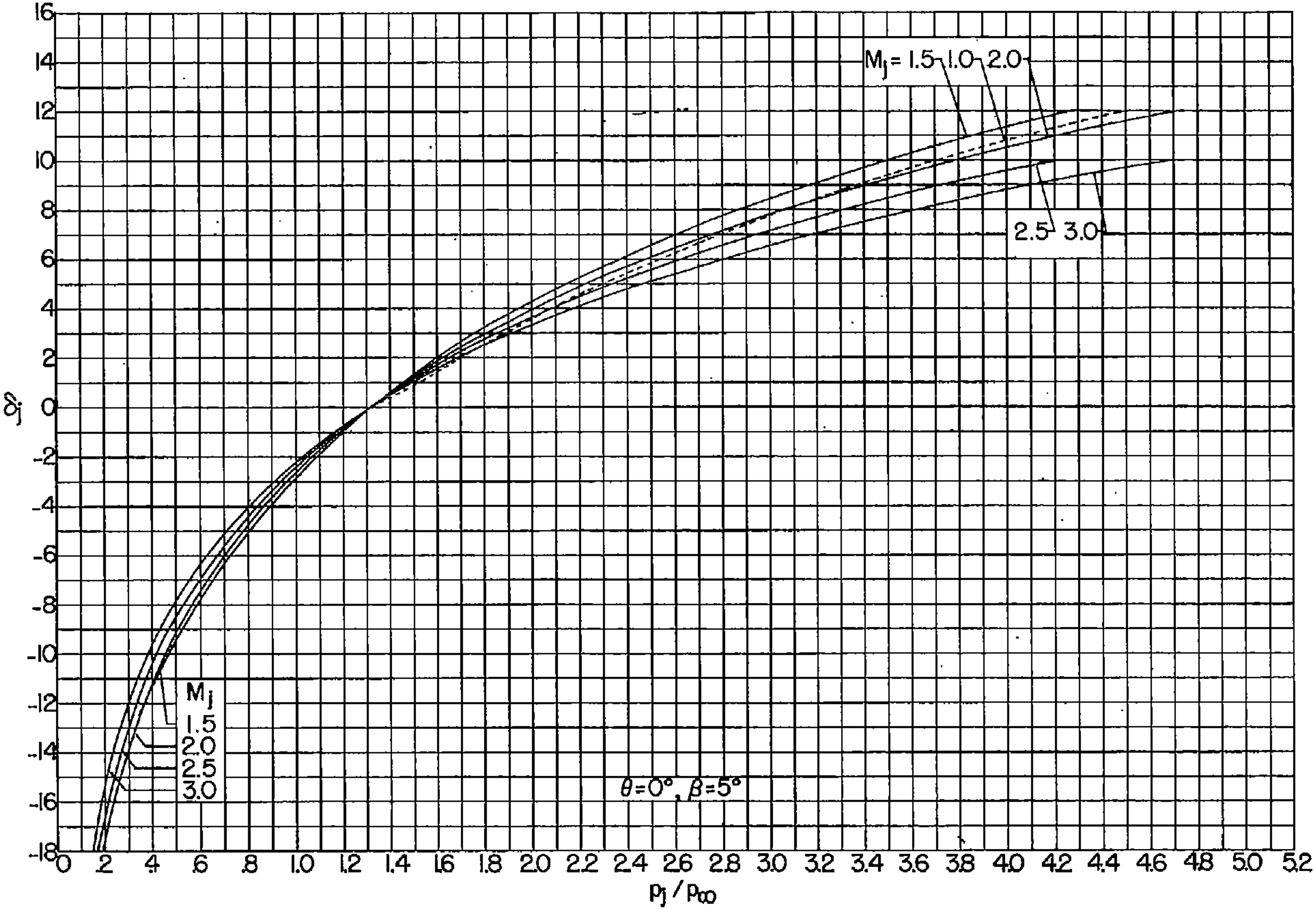

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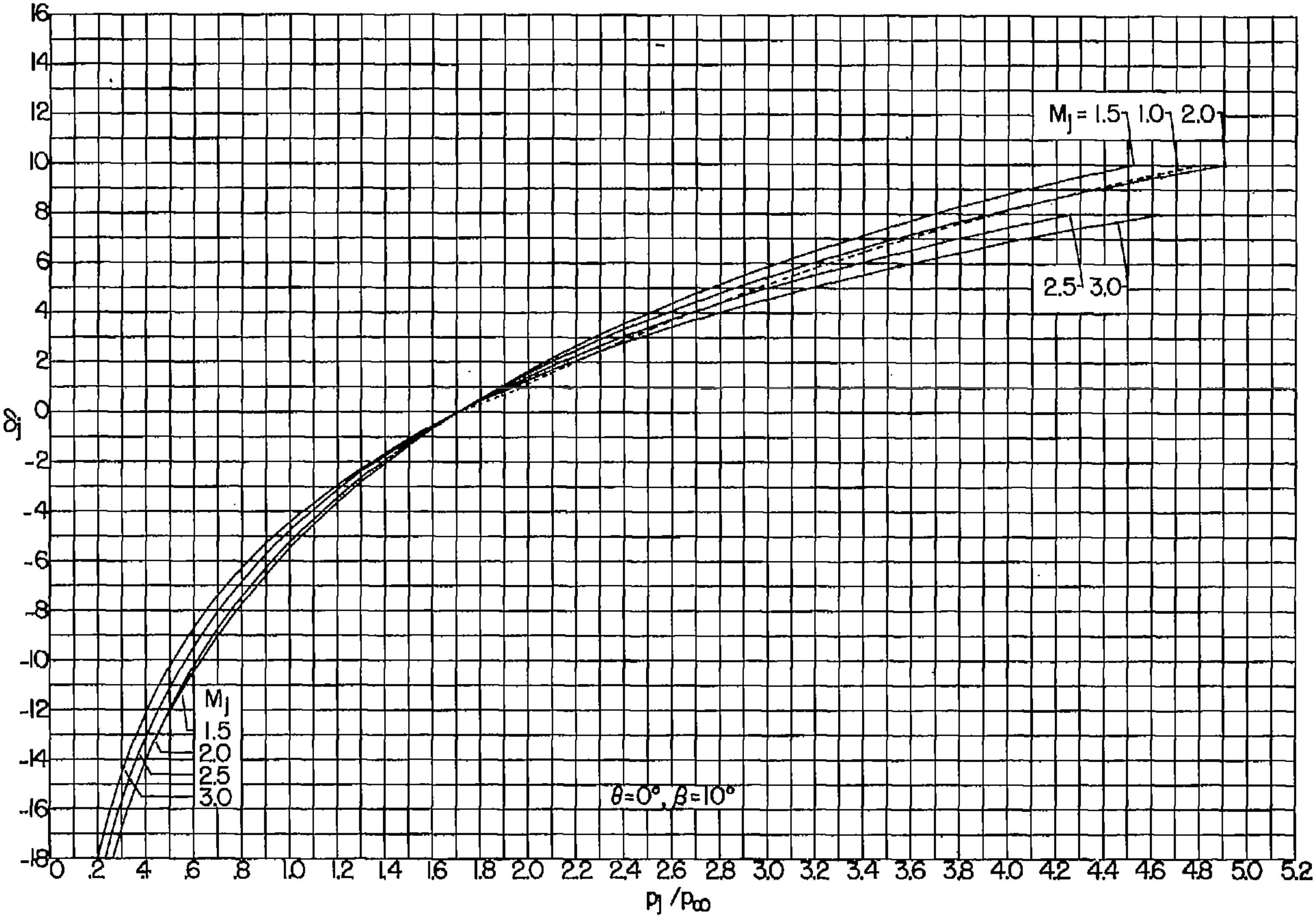

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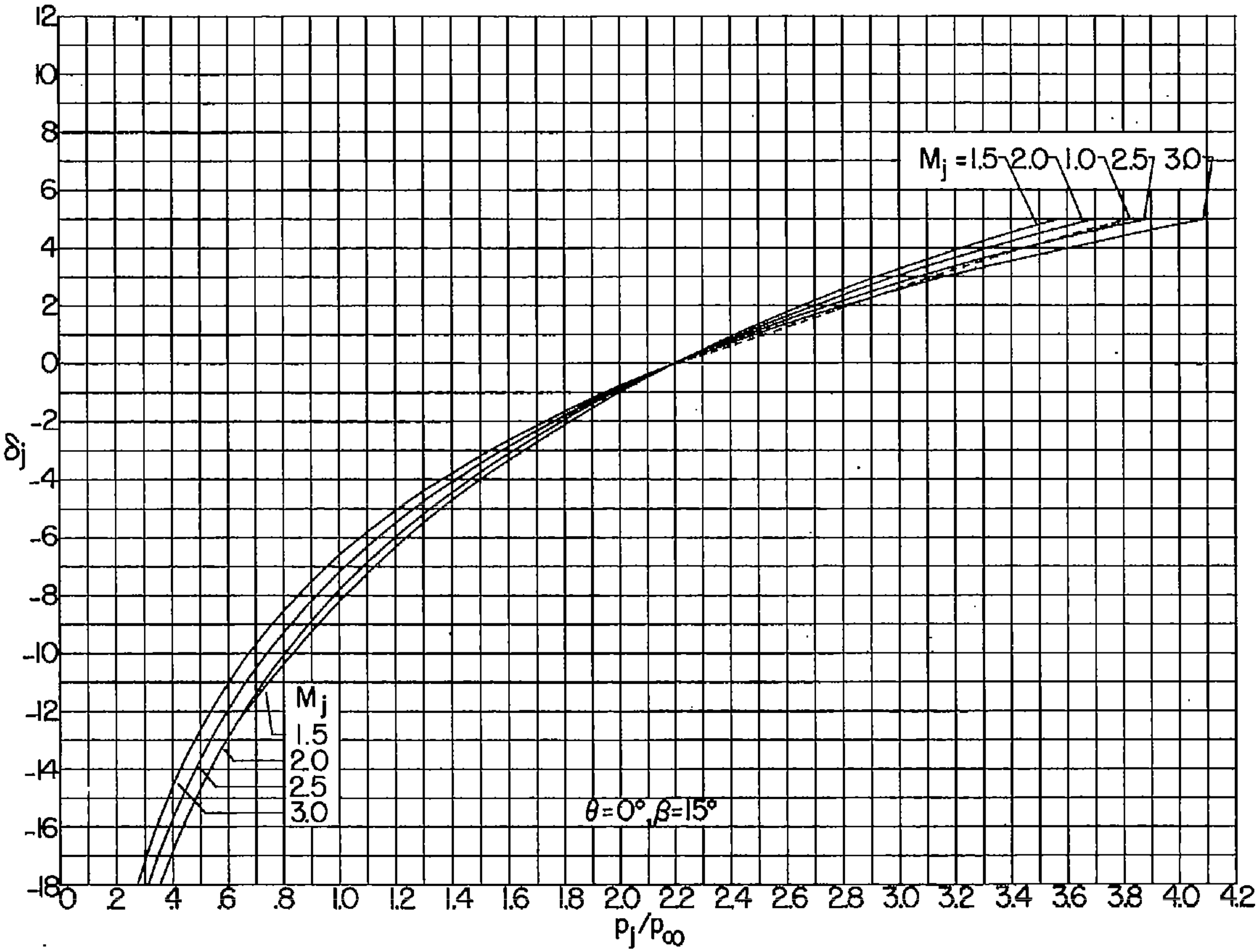

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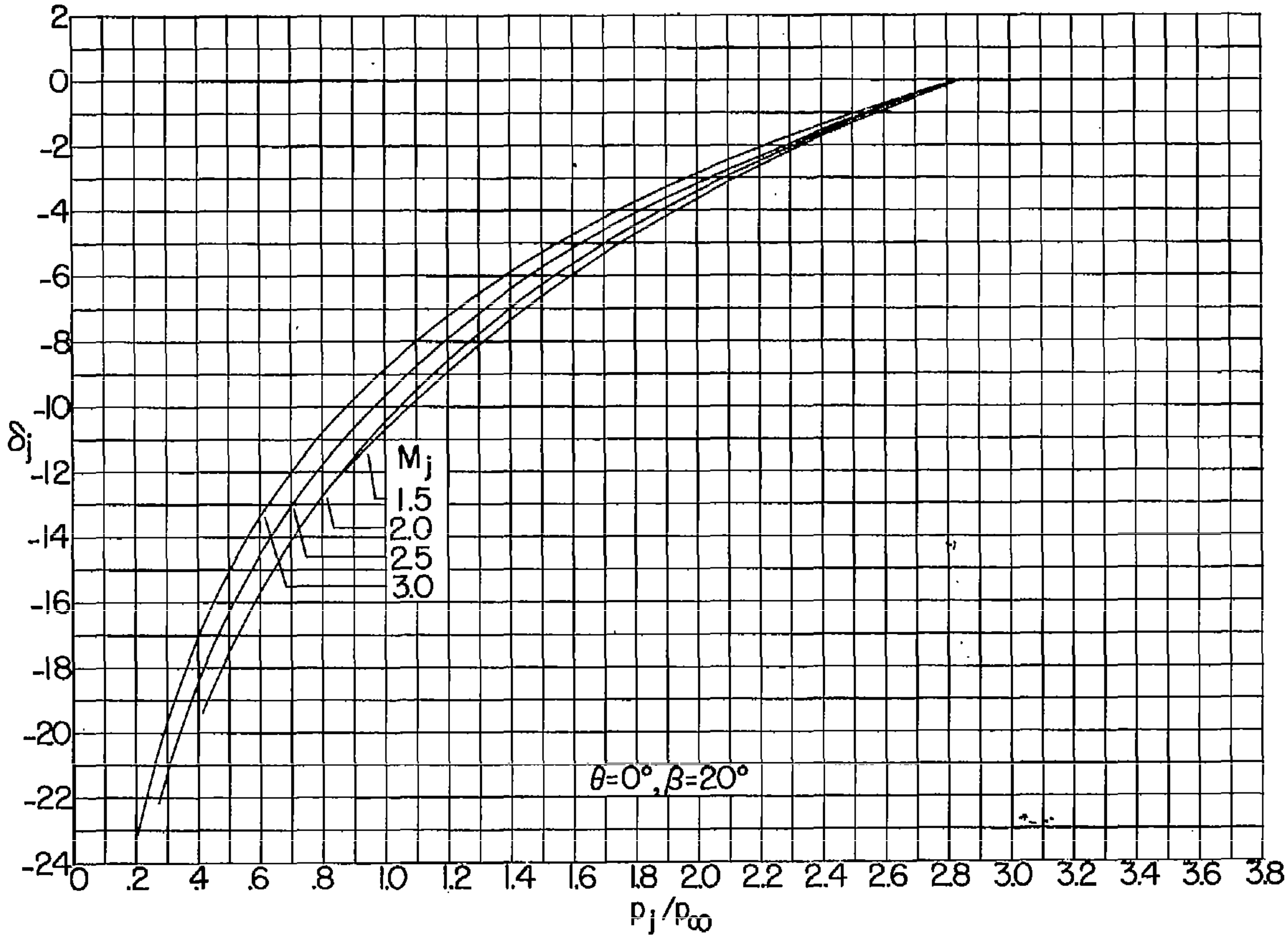

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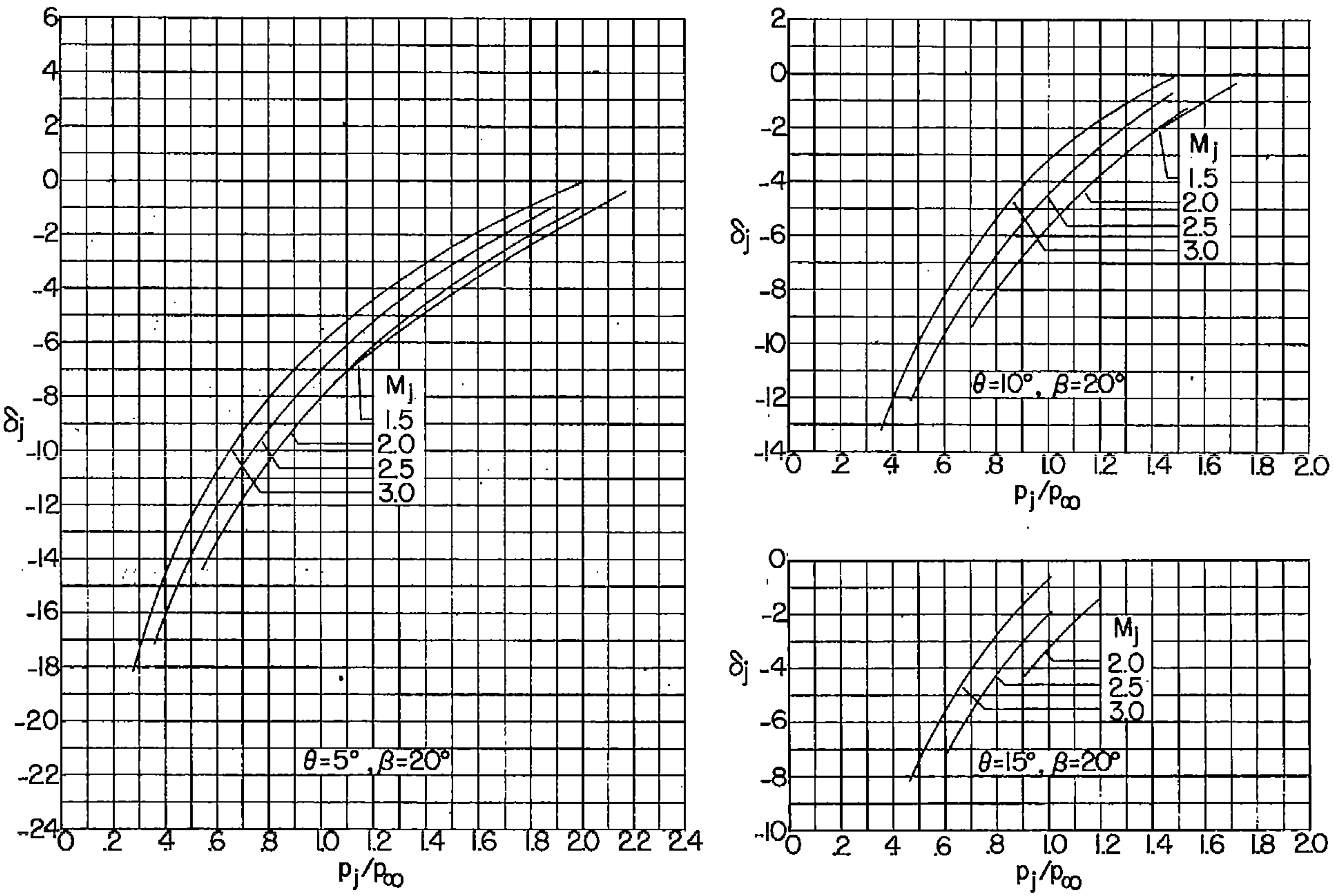

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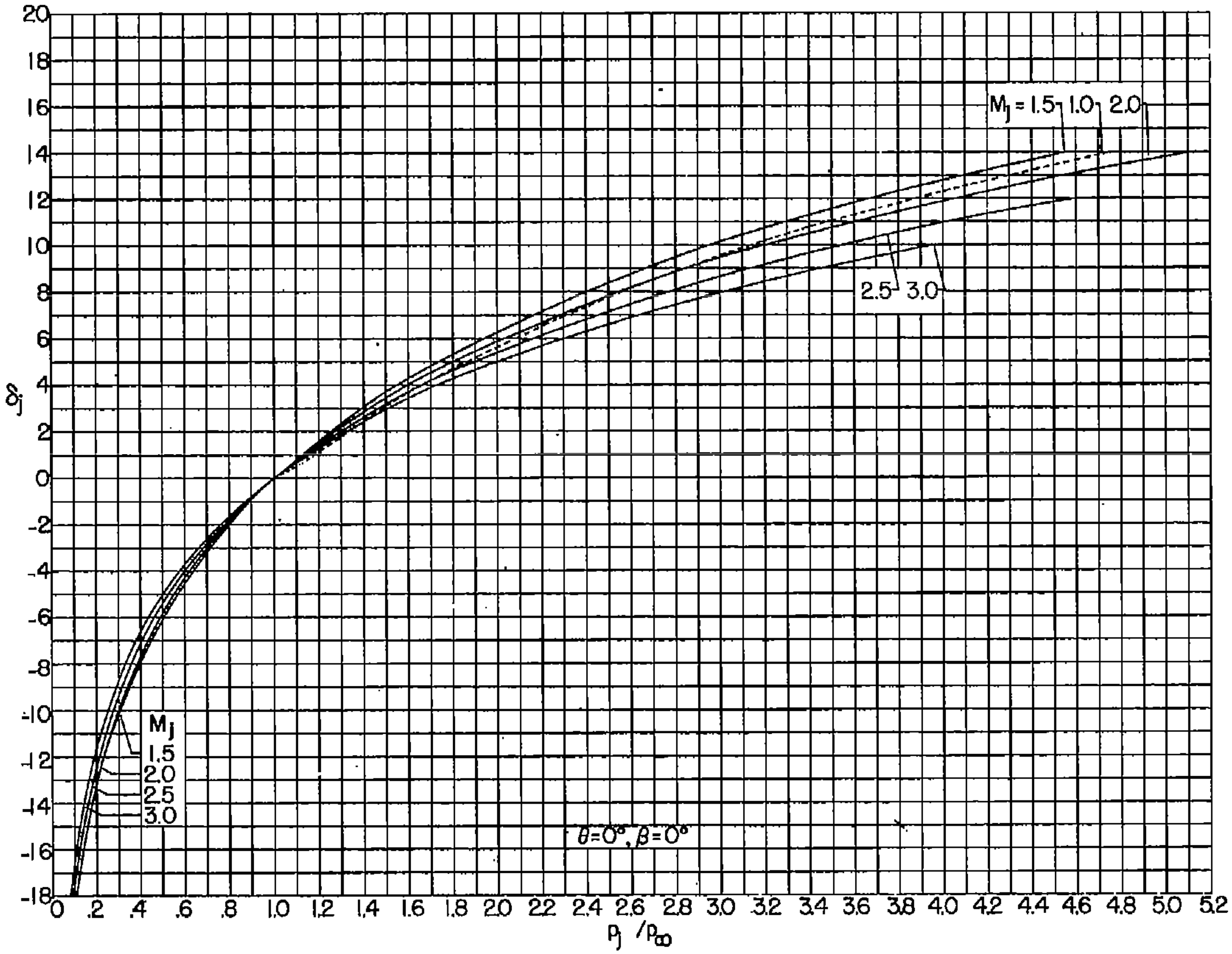
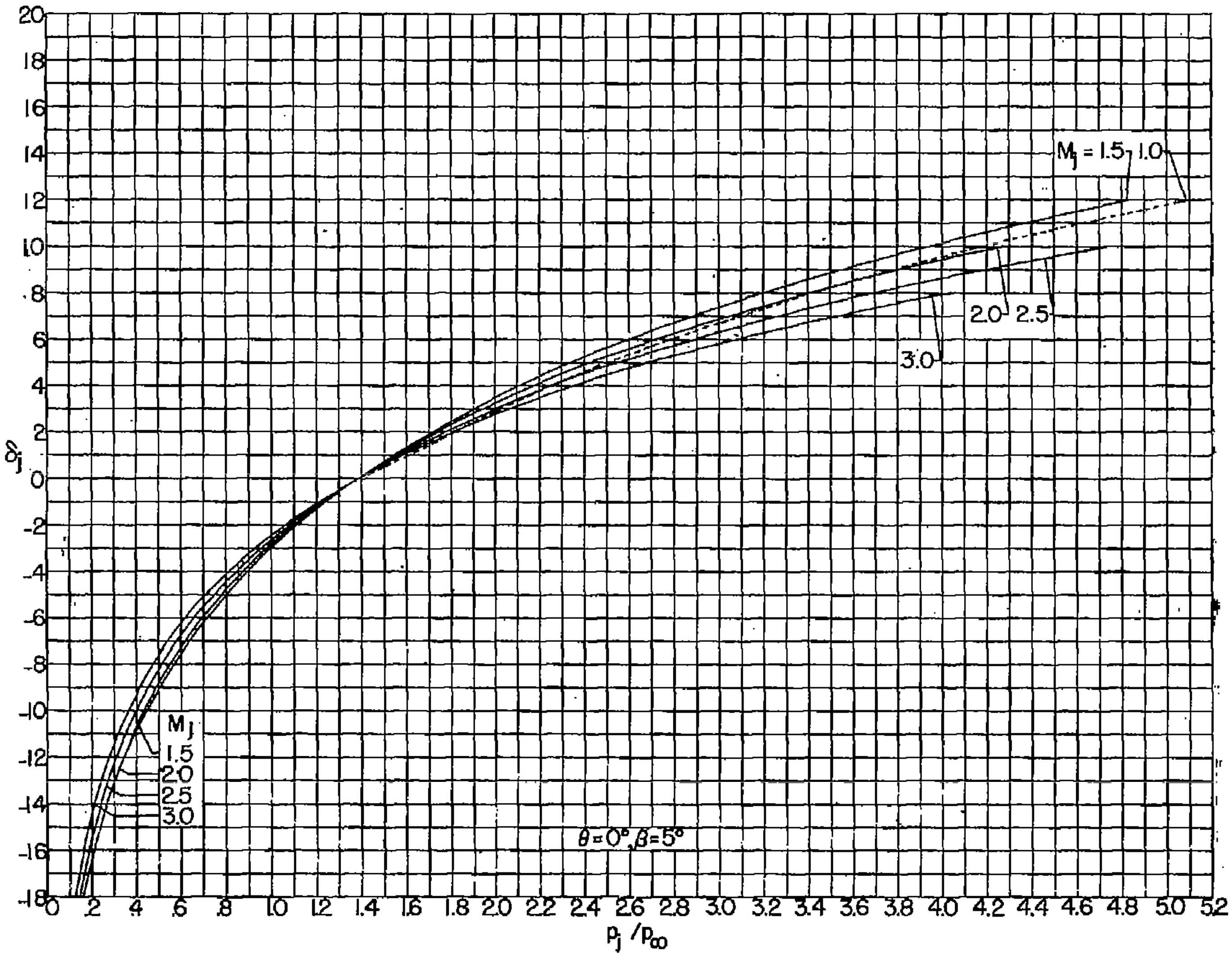


Figure 5.- The effects of jet static-pressure ratio upon the initial inclination of the mixing boundary at $M_\infty = 2.5$.



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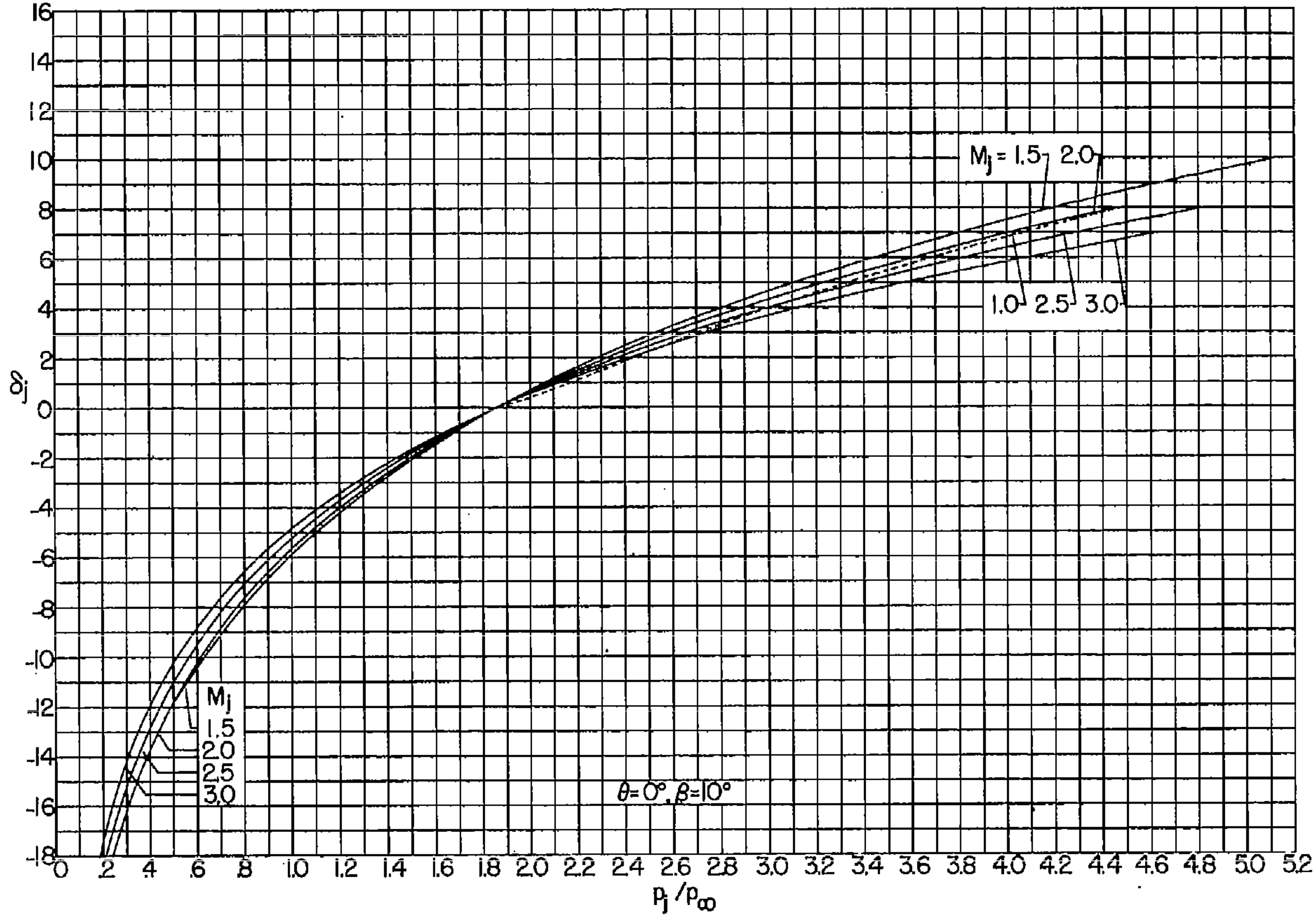

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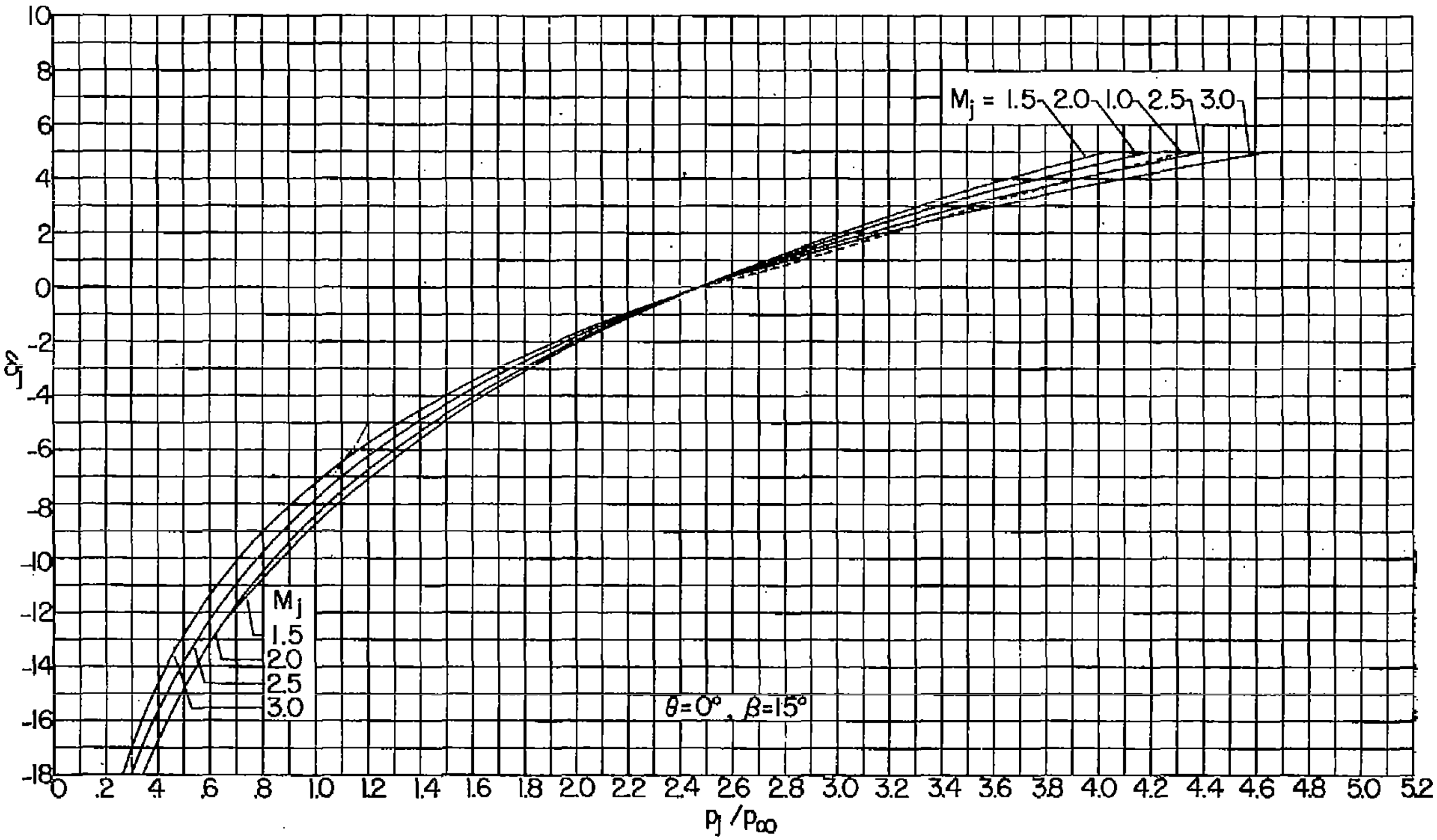

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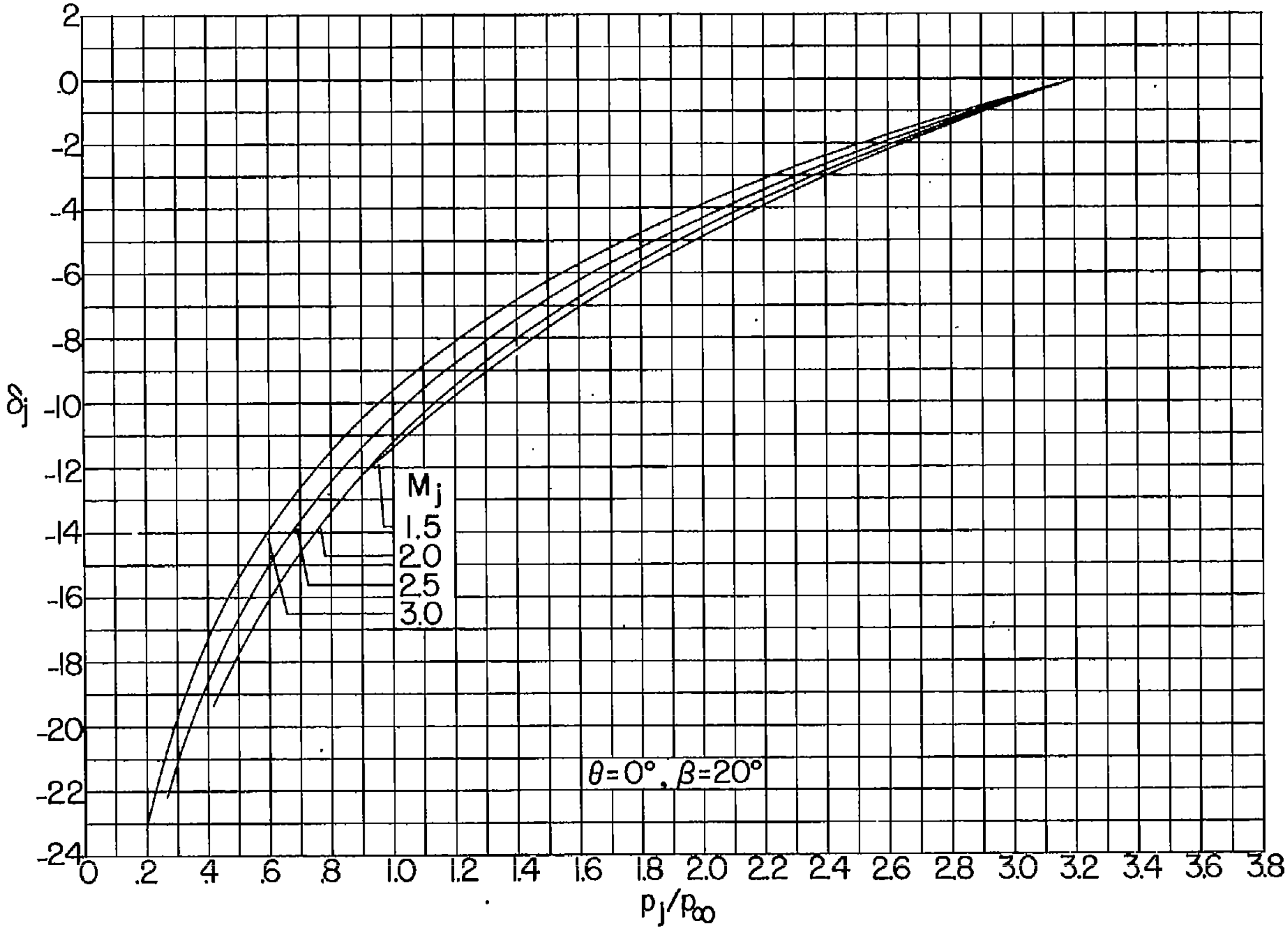

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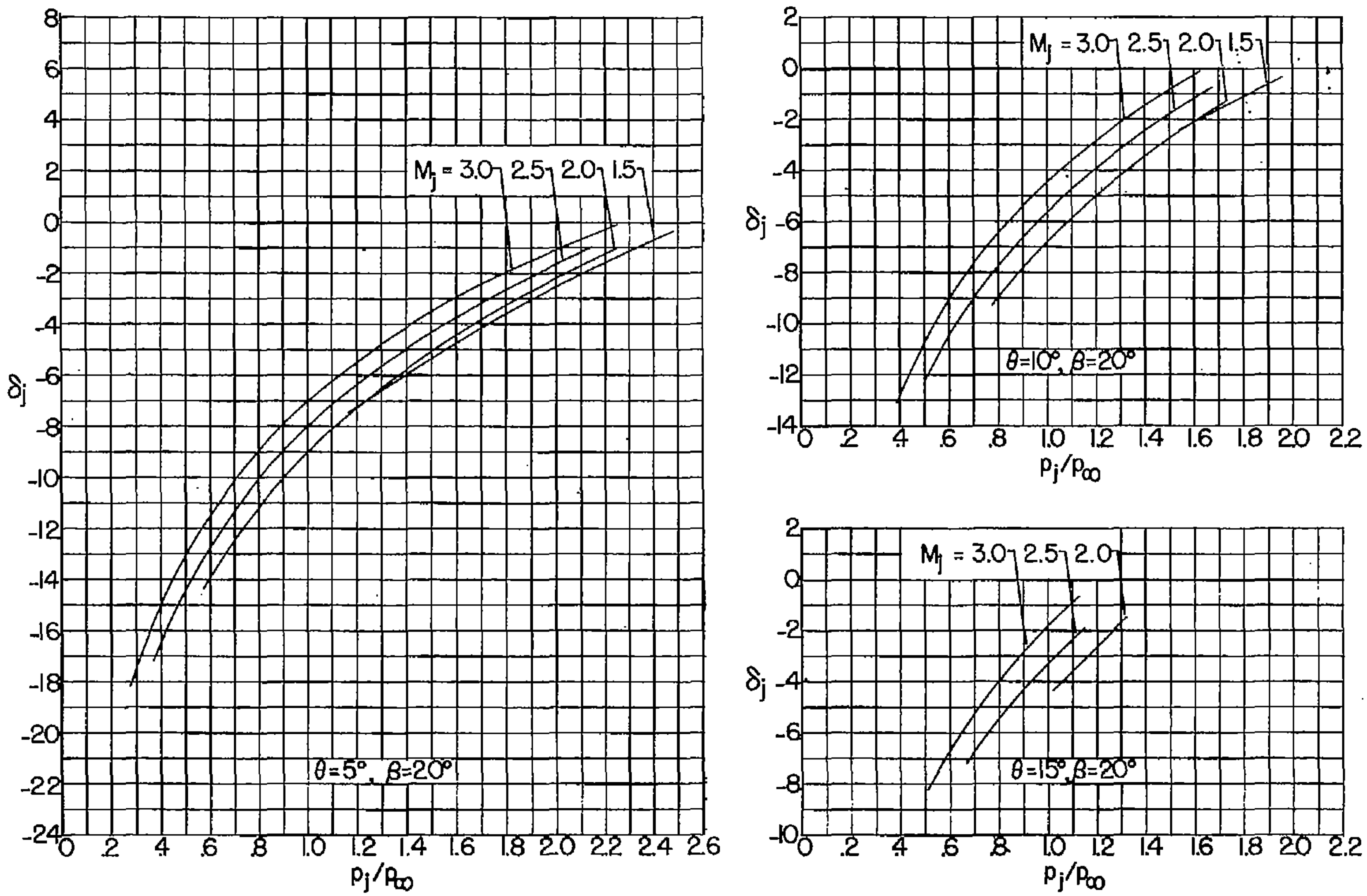

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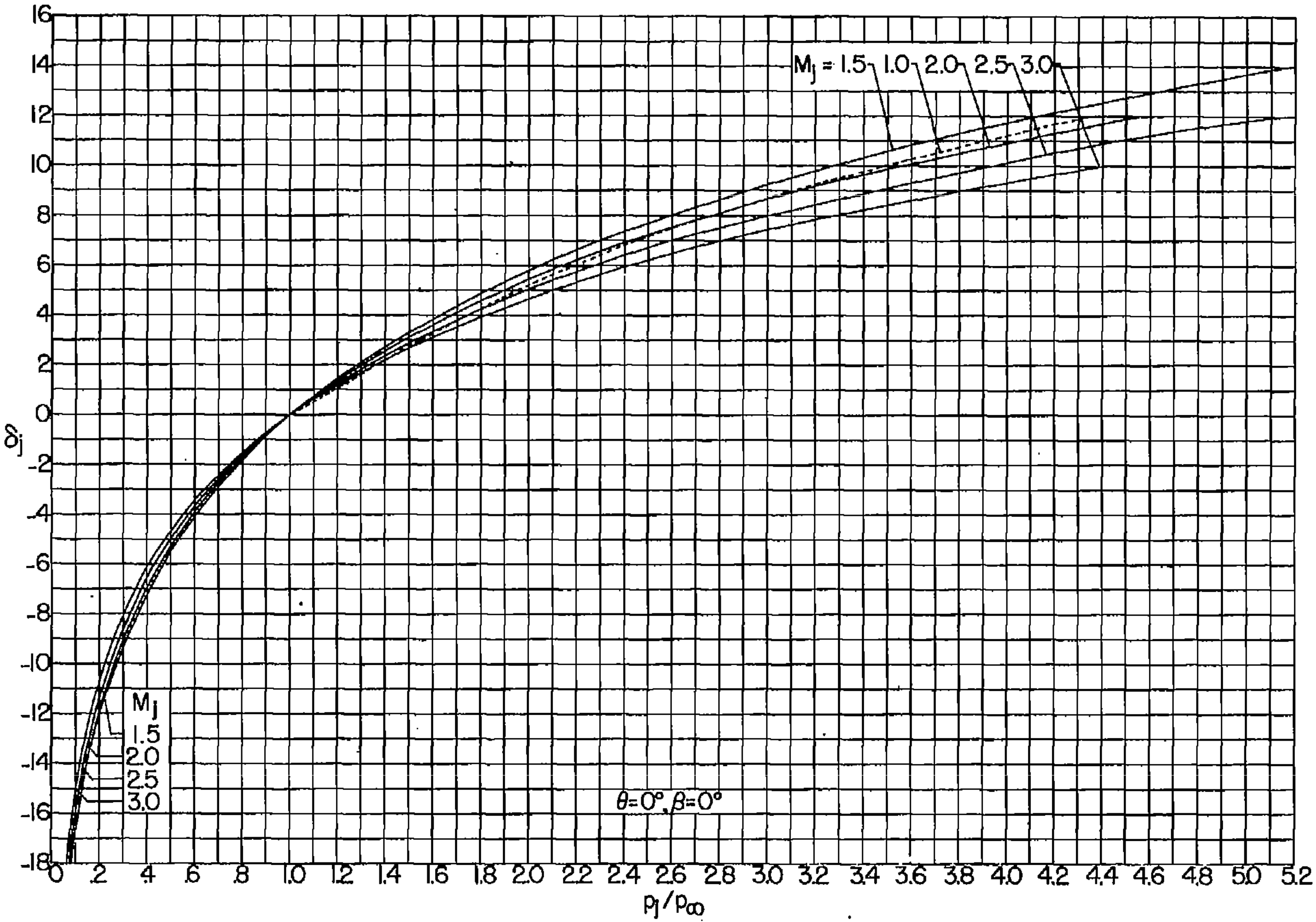

Figure 6.- The effects of jet static-pressure ratio upon the initial inclination of the mixing boundary at $M_\infty = 3.0$.

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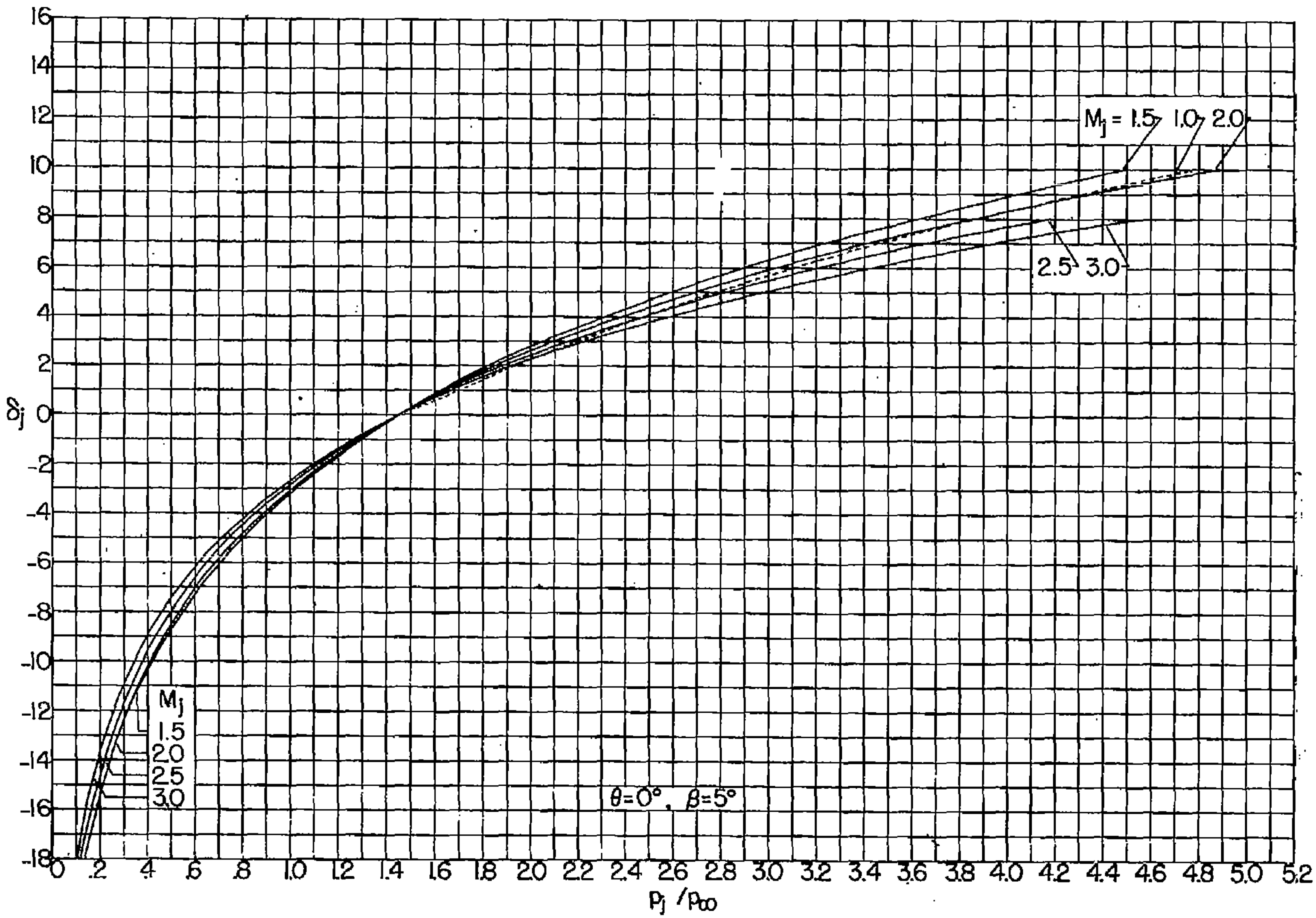

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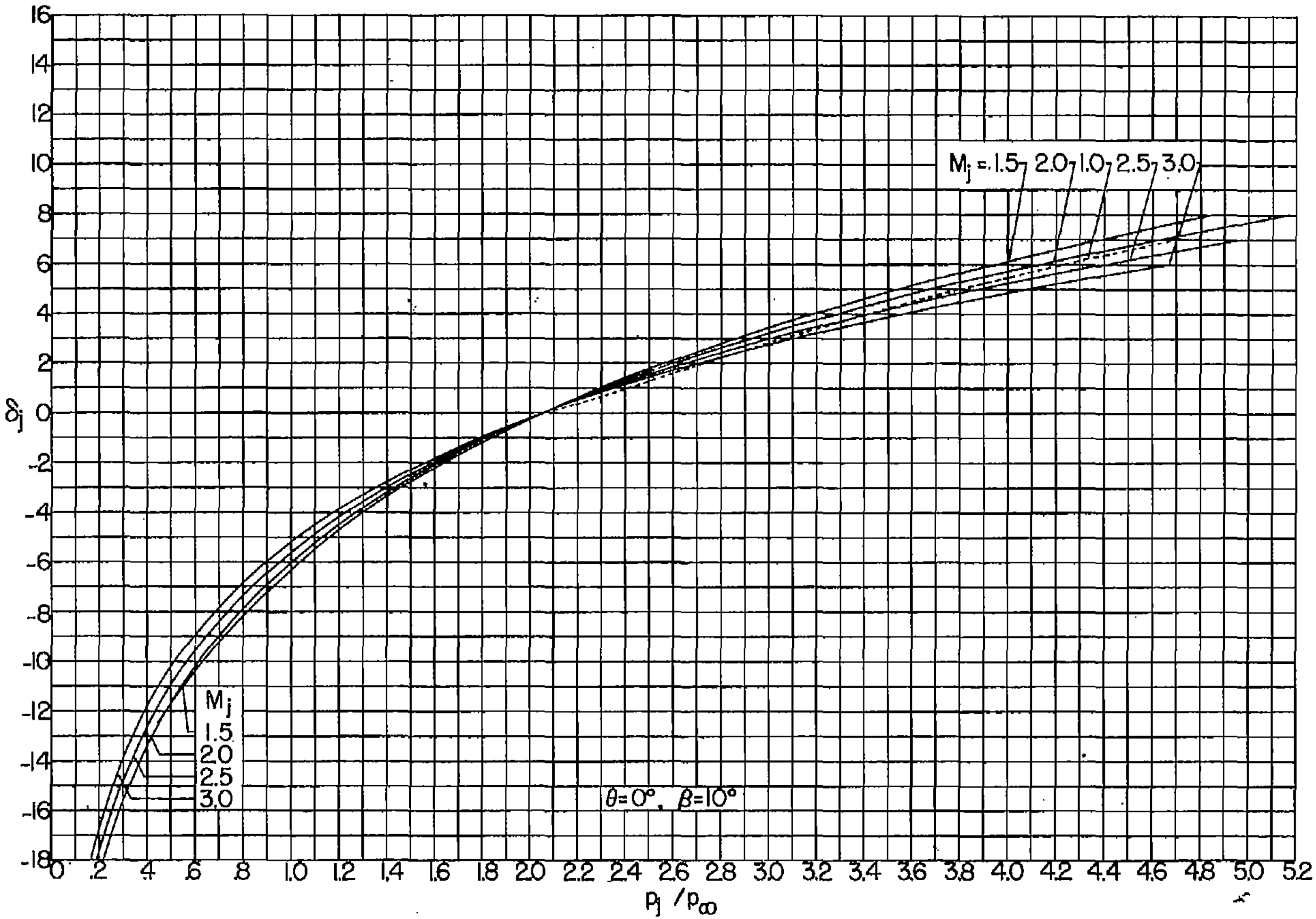

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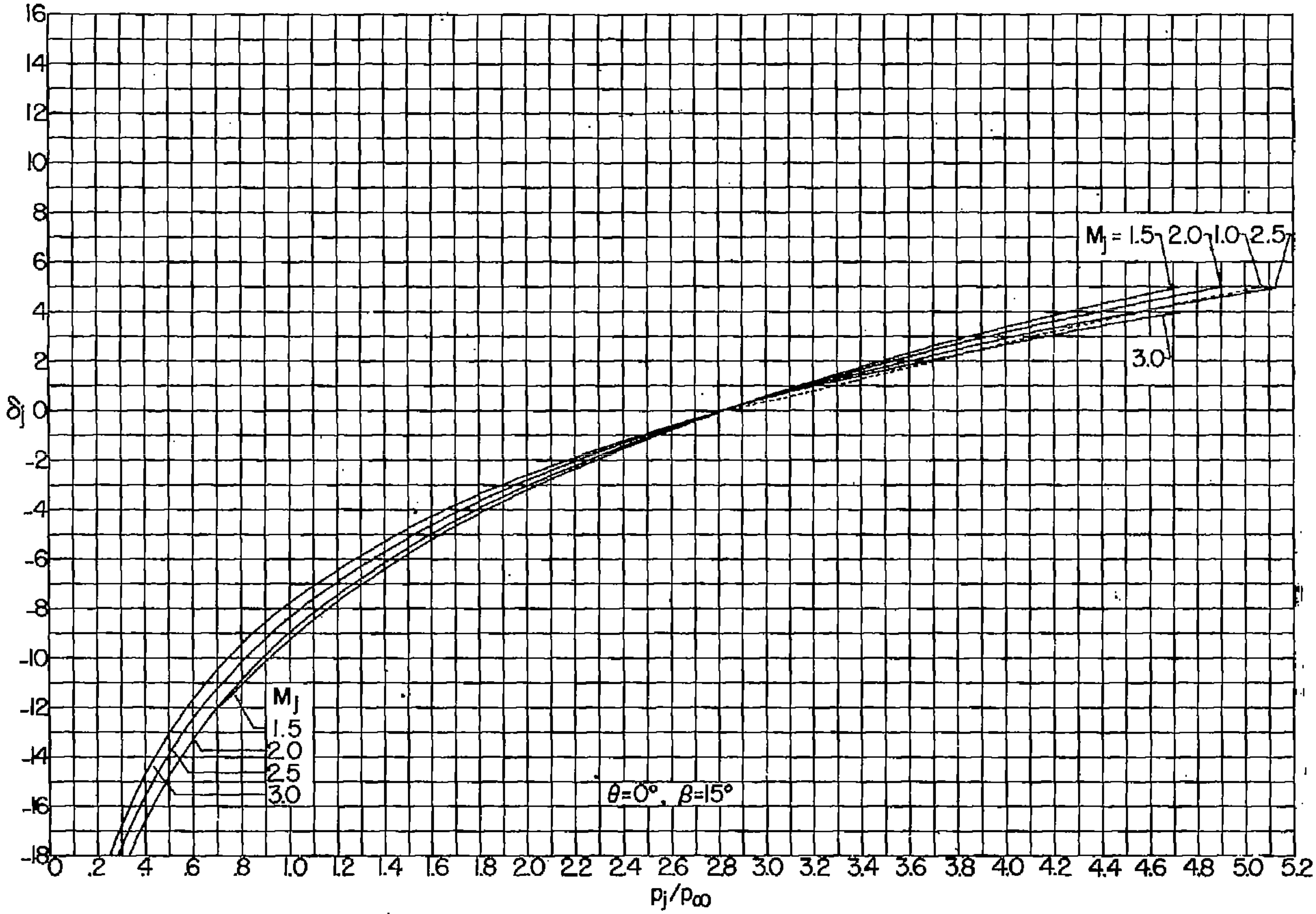

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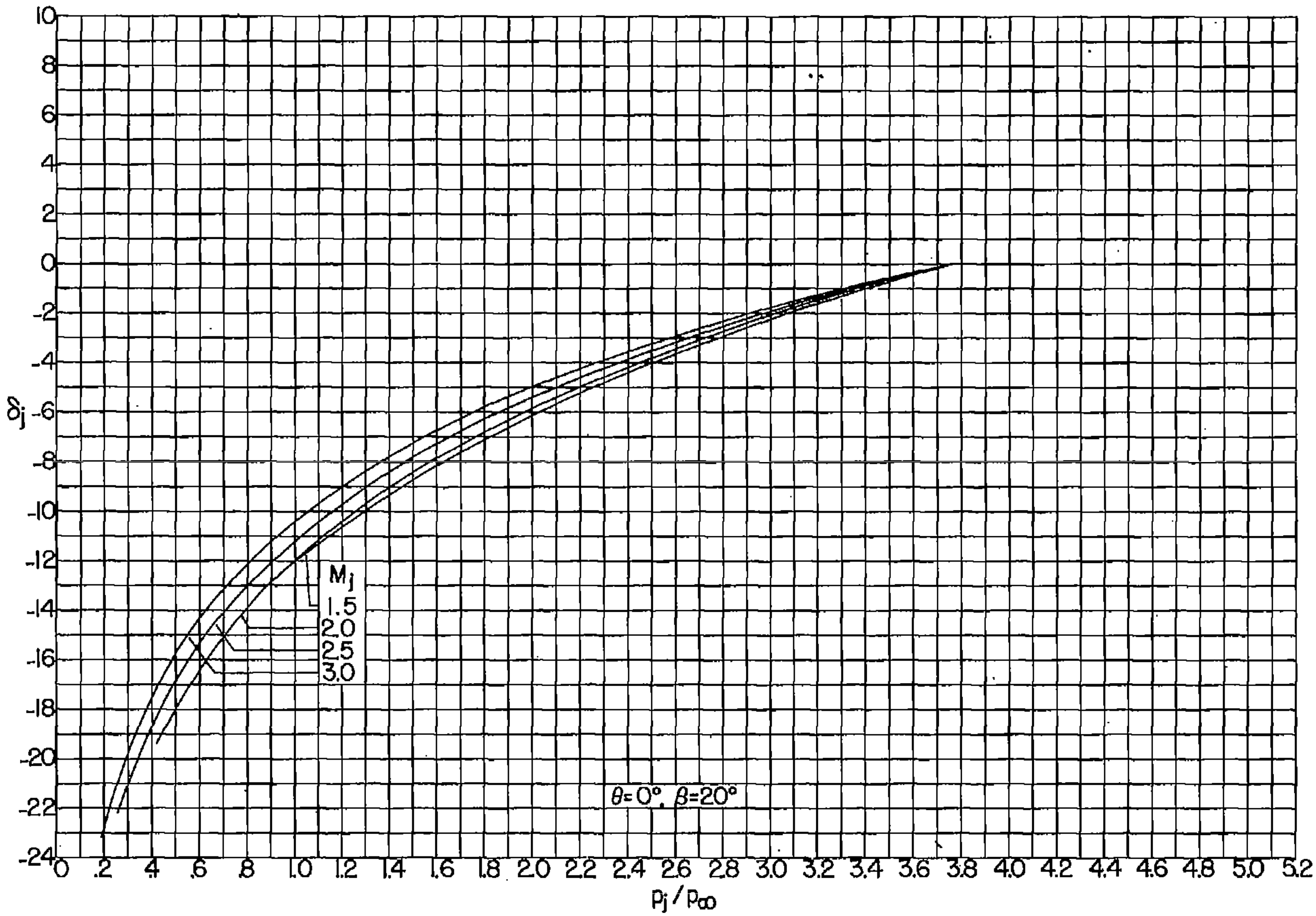

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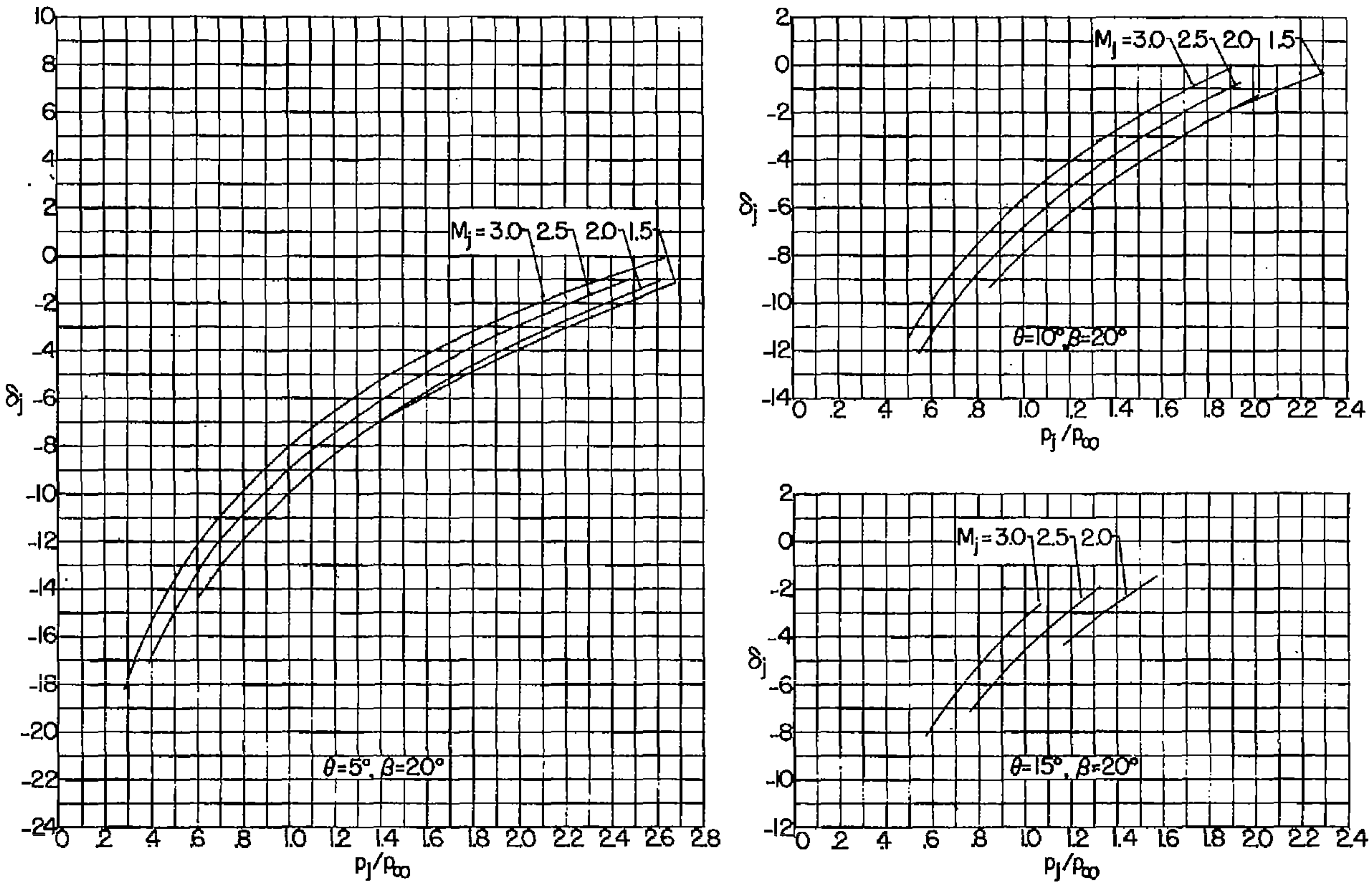

Figure 6.- Concluded.