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RESEARCH MEMORANDUM

ANALYTICAL INVESTIGATION OF MULTISTAGE-TURBINE EFFICIENCY CHARACTERISTICS IN TERMS OF WORK AND SPEED REQUIREMENTS

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
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SUMMARY

An analytical investigation of the interacting effects of turbine specific work output, rotor blade speed, stage number, and efficiency is presented. The correlating parameter used is a work-speed parameter, defined as the ratio of the mean-section rotor blade speed squared to the specific work output. Major assumptions made in the analysis include equal stage work and blade speed, no reheat effect, and a lower limit on rotor reaction of impulse at the mean section. The selected stator-exit flow angle was in the range of those encountered in such applications as turbopump and auxiliary drives.

The results of the analysis include both stage and over-all efficiency characteristics. For a given work-speed parameter, increasing stage number increased the efficiency, with an upper limit being incurred at a stage work-speed parameter of unity. Similarly, for a specified efficiency, increasing the stage number permitted a reduction in the over-all work-speed parameter. For a specified work-speed parameter and number of stages, the efficiency based on the static- to total-pressure ratio was lower than that based on the total-pressure ratio, the difference being greater as the stage number was reduced. This occurred because the leaving loss at the turbine exit became a greater percentage of the specific work output.

INTRODUCTION

The design of a turbine for a given application involves the selection of a considerable number of turbine variables. Included among these variables are specific work output, rotor blade speed, stage number, and efficiency. Because these four variables are interdependent aerodynamically, it is important that values selected for the design be compatible to ensure successful operation.

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This report presents an analytical investigation of this aerodynamic interdependence in terms of the effect of variations in the turbine specific work output, blade speed, and stage number on turbine efficiency. The fundamental parameter used is the "Parson's" characteristic number λ defined as the ratio of the square of the rotor mean-section blade speed to the specific work output. The method of this multistage-turbine analysis is similar to that used in the work of references 1 and 2, which concerned single- and two-stage-turbine efficiency characteristics.

Efficiency types considered in the analysis include those based on total-pressure and static- to total-pressure ratios across the turbine. Velocity diagrams include those of zero stage exit whirl and rotor impulse with each type covering a specified range of stage λ. Throughout the analysis equal stage work and rotor mean-section blade speed are assumed.

SYMBOLS

The following symbols are used in this report:

A/w	ratio of turbine blade surface area to turbine equivalent weight flow per unit annular area
B	ratio of turbine-exit to average axial component of kinetic energy
E	specific kinetic energy level, Btu/lb
g	acceleration due to gravity, 32.17 ft/sec^2
h	specific enthalpy, Btu/lb
J	mechanical equivalent of heat, 778.2 ft-lb/Btu
K,K_I	constants of proportionality
n	number of turbine stages
U	blade speed, ft/sec
V	absolute gas velocity, ft/sec
W	relative gas velocity, ft/sec
α	absolute gas angle as measured from axial direction, deg
η	total efficiency, based on total-pressure ratio across turbine

η_s static efficiency, based on static- to total-pressure ratio across turbine

λ work-speed parameter, $U_m^2/gJ\Delta h'$

Subscripts:

a first stage

av average

i intermediate stage

id ideal

l last stage

m mean section

S stage

s static

u tangential component

x axial component

0 turbine inlet

1 first-stage stator exit

2 first-stage rotor exit

3 intermediate-stage stator entrance

4 intermediate-stage stator exit

5 intermediate-stage rotor exit

7 last-stage stator exit

Superscripts:

' absolute total state

— over-all

METHOD OF ANALYSIS

As indicated in the INTRODUCTION, the fundamental parameter used to correlate turbine efficiency is the work-speed parameter λ. The over-all value of this parameter is defined as 4766

$$\overline{\lambda} = \frac{U_m^2}{gJ\Delta h'} \tag{1}$$

where U_m is the mean-section rotor blade speed and $\Delta h'$ is the over-all specific work output. A similar definition of the stage work-speed parameter λ_S can be written as

$$\lambda_S = \frac{U_{m,S}^2}{gJ\Delta h_S'} \tag{2}$$

In this analysis it is assumed that both the mean-section blade speed U_m and the specific work output of each turbine stage are the same; thus λ_S is the same for all stages. As a consequence, the relation between $\overline{\lambda}$ and λ_S is obtained as

$$\overline{\lambda} = \frac{\lambda_S}{n} \tag{3}$$

where n is the stage number.

Figure 1 presents a schematic diagram of a multistage turbine indicating the nomenclature used in this report. As indicated in the figure, three types of stages are considered: first, intermediate, and last, indicated by the subscripts a, i, and l, respectively. In the development of the efficiency equations, attention is focused first on the attainment of those for the three types of stages. Subsequently, the turbine over-all efficiency equations are developed in terms of the efficiency of the stages.

General Stage Efficiency Equations

First stage. - Equations for the first-stage efficiency based on the total-pressure and static- to total-pressure ratios are developed first. These efficiencies can also be used for the case of the single-stage turbine.

In reference 1 the development of equations to obtain the total efficiency of a single-stage turbine is presented. Combining equations (7) and (8) of this reference yields

$$\eta_a = \frac{\lambda_a}{\lambda_a + \dfrac{K_I\left(\dfrac{A}{w}\right)E}{\Delta V_{u,a}^2}} \tag{4}$$

where A/w represents the ratio of blade surface area to equivalent weight flow per unit annular area; E is the average of the specific kinetic energies entering and leaving the stator and rotor blade rows, where the velocities are relative to the blade rows; and K_I is a constant of proportionality.

In this development it is assumed that the loss coefficient (defined as the ratio of blade kinetic energy loss to average kinetic energy level) of the rotor is twice that of the stator. This assumption is more compatible with experimentally obtained coefficients than the assumption of equal coefficients made in reference 1.

As a result of this assumption, the expression for E is modified from that of equation (10) of reference 2 to yield

$$E = \frac{6\left(V_x^2\right)_{av} + V_{u,1}^2 + 2\left(W_{u,1}^2 + W_{u,2}^2\right)}{12\ g} \tag{5}$$

where

$$\left(V_x^2\right)_{av} = \frac{V_{x,0}^2 + 3V_{x,1}^2 + 2V_{x,2}^2}{6}$$

Let it be assumed that the expression $\left(V_x^2\right)_{av}$, which represents the average axial kinetic energy, can be approximated by the axial component of kinetic energy at the stator exit. This approximation is considered valid because (a) the stator-exit axial kinetic energy is the major contributor to the average, and (b) the difference between the stator-entrance and rotor-exit values and the average tends to be compensated for by the general trend of increasing axial velocity with stage axial station.

With this assumption, the stator-exit angle α can be introduced to obtain the axial component of kinetic energy in terms of the stator-exit whirl velocity as

$$\left(V_x^2\right)_{av} = V_{u,1}^2 \cot^2\alpha_1 \tag{6}$$

Substituting equations (5) and (6) into equation (4) yields

$$\eta_a = \frac{\lambda_a}{\lambda_a + K\left(\frac{A}{w}\right)\left\{\left(\frac{V_{u,1}}{\Delta V_{u,a}}\right)^2 (6 \cot^2\alpha_1 + 1) + 2\left[\left(\frac{W_{u,1}}{\Delta V_{u,a}}\right)^2 + \left(\frac{W_{u,2}}{\Delta V_{u,a}}\right)^2\right]\right\}} \tag{7}$$

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where

$$K = \frac{K_1}{12g}$$

Eliminating the relative whirl components by the equations

$$\frac{W_{u,1}}{\Delta V_{u,a}} = \frac{V_{u,1}}{\Delta V_{u,a}} - \lambda_a$$

$$\frac{W_{u,2}}{\Delta V_{u,a}} = \frac{V_{u,1}}{\Delta V_{u,a}} - \lambda_a - 1$$

finally yields

$$\eta_a = \frac{\lambda_a}{\lambda_a + K\left(\frac{A}{w}\right)\left\{\left(\frac{V_{u,1}}{\Delta V_{u,a}}\right)^2 (6 \cot^2\alpha_1 + 1) + 2\left[\left(\frac{V_{u,1}}{\Delta V_{u,a}} - \lambda_a\right)^2 + \left(\frac{V_{u,1}}{\Delta V_{u,a}} - \lambda_a - 1\right)^2\right]\right\}} \tag{8}$$

where variations in the velocity diagram types are obtained through variations in $\frac{V_{u,1}}{\Delta V_{u,a}}$.

The first-stage efficiency based on the static- to total-pressure ratio can be obtained using equation (23) of reference 1. This equation, slightly modified, is

$$\eta_{a,s} = \frac{\eta_a}{1 + \frac{\eta_a}{2}\left[\frac{1}{\lambda_a}\left(\frac{V_{u,1}}{\Delta V_{u,a}} - 1\right)^2 + \frac{B\left(V_x^2\right)_{av}}{gJ\Delta h'_a}\right]} \tag{9}$$

where B is introduced to consider the fact that the turbine-exit axial component of kinetic energy is greater, in general, than that of the average. Then altering the last expression in the demonimator of equation (9) yields

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$$\frac{B\left(V_x^2\right)_{av}}{gJ\Delta h_a'} = B\left(\frac{V_x}{V_{u,1}}\right)^2\left(\frac{V_{u,1}}{\Delta V_{u,a}}\right)^2\left(\frac{\Delta V_{u,a}^2}{gJ\Delta h_a'}\right)$$

$$= \frac{B\cot^2\alpha_1\left(\frac{V_{u,1}}{\Delta V_{u,a}}\right)^2}{\lambda_a}$$

As a result, equation (9) can be modified to

$$\eta_{a,s} = \frac{\eta_a}{1 + \frac{\eta_a}{2\lambda_a}\left[\left(\frac{V_{u,1}}{\Delta V_{u,a}} - 1\right)^2 + B\cot^2\alpha_1\left(\frac{V_{u,1}}{\Delta V_{u,a}}\right)^2\right]} \tag{10}$$

<u>Intermediate stage</u>. - The efficiency of an intermediate stage is based on the total-pressure ratio across the stage. However, the efficiency will be somewhat lower than that for the first stage, as the whirl into the stator must be included in attaining the kinetic energy of the stage. So the expression for E is altered from that of equation (5) to

$$E = \frac{6\left(V_x^2\right)_{av} + V_{u,3}^2 + V_{u,4}^2 + 2\left(W_{u,4}^2 + W_{u,5}^2\right)}{12g} \tag{11}$$

Since it is assumed that all the stages have the same rotor diagrams, the absolute whirl leaving any stage will be equal to the stator-entrance whirl of the subsequent stage. Therefore, the stator-entrance whirl $V_{u,3}$ can be obtained as

$$V_{u,3} = V_{u,4} - \Delta V_{u,i} \tag{12}$$

Substituting equation (11) into equation (4), continuing the development in the same manner as done for the first stage, and then including equation (12) gives the following equation for the intermediate-stage total efficiency

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$$\eta_i = \frac{\lambda_i}{\lambda_i + K\left(\frac{A}{w}\right)\left\{\left(\frac{V_{u,4}}{\Delta V_{u,i}}\right)^2 (6 \cot^2\alpha_4 + 1) + 2\left[\left(\frac{V_{u,4}}{\Delta V_{u,i}} - \lambda_i\right)^2 + \left(\frac{V_{u,4}}{\Delta V_{u,i}} - \lambda_i - 1\right)^2\right] + \left(1 - \frac{V_{u,4}}{\Delta V_{u,1}}\right)^2\right\}} \tag{13}$$

Last stage. - Equations need not be derived for the last-stage efficiency. The equation for total efficiency is the same as that just obtained for the intermediate stage; that is,

$$\eta_l = \eta_i$$

With η_l known, the last-stage efficiency based on the static- to total-pressure ratio across the stage can be computed from the equation

$$\eta_{l,s} = \frac{\eta_l}{1 + \frac{\eta_l}{2\lambda_l}\left[\left(\frac{V_{u,7}}{\Delta V_{u,l}} - 1\right)^2 + B \cot^2\alpha_7 \left(\frac{V_{u,7}}{\Delta V_{u,l}}\right)^2\right]} \tag{14}$$

This equation is the same in form as equation (10).

Velocity Diagram Consideration

Zero-exit-whirl case. - Zero stage exit whirl is used in this report over the range of λ_s from 0.5 to 1. The upper limit of 1 is selected because this value corresponds to the maximum stage efficiency (ref. 1). The lower limit occurs when impulse conditions exist across the rotor. The types of diagrams corresponding to these two limits are presented in figure 2.

Since there is no stage exit whirl in this case,

$$\frac{V_{u,1}}{\Delta V_{u,a}} = \frac{V_{u,4}}{\Delta V_{u,i}} = 1 \tag{15}$$

As a result, the pertinent efficiency equations (8), (10), (13), and (14) reduce to

$$\eta_a = \eta_i = \frac{\lambda_a}{\lambda_a + K\left(\frac{A}{w}\right)\left(6 \cot^2\alpha_1 + 3 - 4\lambda_a + 4\lambda_a^2\right)} \tag{16}$$

and

$$\eta_{a,s} = \eta_{l,s} = \frac{\eta_a}{1 + \frac{\eta_a}{2\lambda_a}\left(B \cot^2\alpha_1\right)} \tag{17}$$

<u>Rotor-impulse case</u>. - Over the range of λ_S from 0 to 0.5, impulse conditions are assumed across each rotor. Figure 2 illustrates this diagram type for values of λ_S of 0.25 and 0.5 (upper limit). This case is considered at length in reference 1, where the impulse condition defines equal relative velocities at the rotor entrance and exit. Under the assumption that the axial component of velocity at the rotor entrance and exit are also equal, the equation relating stator-exit whirl to the stage work-speed parameter is obtained as equation (27)(ref. 1). Rewritten here, it is

$$\frac{V_{u,1}}{\Delta V_{u,a}} = \frac{V_{u,4}}{\Delta V_{u,i}} = \lambda_S + \frac{1}{2} \tag{18}$$

Therefore, substituting equation (18) into the pertinent efficiency equations (8), (10), (13), and (14) yields

$$\eta_a = \frac{\lambda_a}{\lambda_a + K\left(\frac{A}{w}\right)\left[\left(\lambda_a + \frac{1}{2}\right)^2\left(6 \cot^2\alpha_1 + 1\right) + 1\right]} \tag{19}$$

$$\eta_{a,s} = \frac{\eta_a}{1 + \frac{\eta_a}{2\lambda_a}\left[\left(\frac{1}{2} - \lambda_a\right)^2 + B \cot^2\alpha_1\left(\lambda_a + \frac{1}{2}\right)^2\right]} \tag{20}$$

$$\eta_i = \frac{\lambda_i}{\lambda_i + K\left(\frac{A}{w}\right)\left[\left(\lambda_i + \frac{1}{2}\right)^2\left(6 \cot^2\alpha_4 + 1\right) + 1 + \left(\frac{1}{2} - \lambda_i\right)^2\right]} \tag{21}$$

$$\eta_{l,s} = \frac{\eta_l}{1 + \frac{\eta_l}{2\lambda_l}\left[\left(\frac{1}{2} - \lambda_l\right)^2 + B \cot^2\alpha_7\left(\lambda_l + \frac{1}{2}\right)^2\right]} \tag{22}$$

It might again be noted that in this analysis all the stage work-speed parameters are the same. That is,

$$\lambda_a = \lambda_i = \lambda_l = \lambda_S$$

Over-All Turbine Efficiency Equations

The previous section developes equations for obtaining stage efficiencies. This section concerns itself with the development of over-all turbine efficiency equations in terms of these stage efficiencies. In doing this, two assumptions are made:

(1) There is no reheat effect

(2) All stages other than the first are similar aerodynamically (only difference is that the first stage has zero stage inlet whirl).

Using these two assumptions, a general equation for the over-all total efficiency can be written as

$$\bar{\eta} = \frac{\overline{\Delta h'}}{\Delta h'_{id,a} + (n - 1)\Delta h'_{id,i}} \tag{23}$$

where $\Delta h'_{id,a}$ and $\Delta h'_{id,i}$ are the ideal specific work outputs corresponding to the total-pressure ratios across the stages. Then, using equation (2) and the definition of stage efficiency, equation (23) is reduced to

$$\bar{\eta} = \frac{n}{\frac{1}{\eta_a} + \frac{n - 1}{\eta_i}} \tag{24}$$

The equation for over-all efficiency based on the static- to total-pressure ratio across the turbine is slightly different because the kinetic energy leaving the last stage is considered as a loss. As a result,

$$\bar{\eta}_s = \frac{\overline{\Delta h'}}{\Delta h'_{id,a} + (n - 2)\Delta h'_{id,i} + \Delta h'_{id,s,l}} \tag{25}$$

Reduction of the equation then results in

$$\bar{\eta}_s = \frac{n}{\frac{1}{\eta_a} + \frac{n - 2}{\eta_i} + \frac{1}{\eta_{l,s}}} \tag{26}$$

DETERMINATION AND SELECTION OF CONSTANTS

Before the efficiency calculations can be made, values of $K\left(\frac{A}{W}\right)$, α_1, α_4, α_7, and B must be assigned. The product $K\left(\frac{A}{W}\right)$ was first obtained using the following loss values as being compatible with experimental results:

$$\lambda_a = 0.5$$

$$\alpha_1 = 60^\circ$$

$$\eta_a = 0.86$$

For these values a base value $K\left(\frac{A}{w}\right)$ was computed from equation (19) as

$$K\left(\frac{A}{w}\right)_{\alpha_1=60^\circ} = 0.0203$$

For the subject analysis, the stator-exit angles for all stages were selected to be equal and at a value of 75° as approaching a practical upper limit. This value of exit flow angle is also in the range of those encountered in such applications as turbopump and auxiliary drives.

As a result of the increase in angle, an increase in $K\left(\frac{A}{w}\right)$ was also made from its base value to include the effect of the reduction in equivalent weight flow per unit annular area. This increased value was obtained by the equation

$$K\left(\frac{A}{w}\right)_{\alpha_1=75^\circ} = K\left(\frac{A}{w}\right)_{\alpha_1=60^\circ} \frac{\cot 60^\circ}{\cot 75^\circ}$$

where the ratio of cotangents reflects the variation in axial component of velocity as the weight flow is reduced. As a result, for the calculations made,

$$K\left(\frac{A}{w}\right) = 0.0437$$

For most of the calculations B was selected as 2 to consider the turbine-exit axial kinetic energy being, in general, somewhat greater than that of the average. However, stage calculation results are also presented for $B = 1$ to illustrate the significance of variations in this parameter.

RESULTS OF ANALYSIS

Stage Efficiency Characteristics

Figure 3 presents the results of the stage efficiency calculations using equations (16), (17), and (19) to (22). Shown is stage efficiency as a function of stage work-speed parameter λ_S for $K\left(\frac{A}{w}\right) = 0.0437$, $\alpha_S = 75^{\circ}$, and two values of B, 1 and 2.

Inspection of figure 3 shows that the first-stage total efficiency η_a is either equal to or greater than that of the other stages η_i. This is due to the specified zero first-stage inlet whirl, which reduces the average kinetic energy level from that obtained when inlet whirl is incurred. The total efficiency is also high (0.80 to 0.86) in the range of λ_S from 0.4 to 1. However, as λ_S is reduced, the total efficiency drops off markedly. This occurs as a result of increased viscous losses due to the attainment of the required specific work output at the increased kinetic energy levels.

The stage static efficiency levels are only slightly less than those of total efficiency at high λ_S values. This difference in efficiency becomes much more significant, however, as λ_S is reduced and occurs as a result of the increased exit kinetic energy loss due to maintaining impulse conditions across the rotor.

A comparison of the calculation results obtained using the two values of B can also be made using figure 3. This effect is of secondary importance, affecting the static efficiency level by 3 to 4 percentage points at high λ_S values and by 1/2 to 1 point at low λ_S values. The higher value of B, 2, yields the lower efficiency. In view of this small effect, the over-all efficiency results are presented for only one value of B, selected as 2.

Over-All Efficiency Characteristics

The turbine over-all efficiency calculation results obtained using equations (3), (24), and (26) are presented in figures 4 and 5, where over-all total efficiency and over-all static efficiency are presented as functions of over-all work-speed parameter $\overline{\lambda}$ over a range of stage number from 1 to 16 for $K\left(\frac{A}{w}\right) = 0.0437$ and $\alpha_S = 75^{\circ}$.

Inspection of figure 4, where total efficiency characteristics are presented, shows the expected effect of staging. For a specified total efficiency, increasing the stage number permits a reduction in operating $\overline{\lambda}$ or, for a given blade speed, an increase in the specific work output. Similarly, for a given $\overline{\lambda}$, an increase in stage number results in an increase in the turbine total efficiency. There is, however, a limitation to this increase, as shown by the dotted line in the figure which shows limiting or maximum efficiency. For a given $\overline{\lambda}$, this limiting efficiency occurs at a stage number corresponding to $\lambda_S = 1$, which results in

$n = 1/\bar{\lambda}$ using equation (3). The value of this limiting total efficiency is approximately 0.87. Variation in this limiting efficiency would occur if different values of K and α_S were used.

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Inspection of figure 5, which presents the over-all static efficiency characteristics, shows similar results to those of figure 4. However, the level of efficiency is lower, especially as the stage number is reduced. This occurs because of the low static efficiency of the last stage due to the exit whirl loss. Furthermore, the limiting efficiency line is not only lower than the corresponding line in figure 4, but also changes in value as $\bar{\lambda}$ is varied. This occurs because the exit kinetic energy loss becomes a greater percentage of the specific work output as the number of stages is reduced.

CONCLUDING REMARKS

This report presents an analytical investigation of the interdependence of turbine efficiency, stage number, blade speed, and specific work output. The results are presented in terms of the effect of variations in over-all work-speed parameter on efficiency based on both total-pressure and static- to total-pressure ratio across the turbine for a range of turbine stage number. Consideration of effects such as those presented herein is very important in the selection of a turbine configuration for a given application, not only to ensure satisfactory operation, but also to permit the selection to be optimum from a design standpoint.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, November 25, 1957

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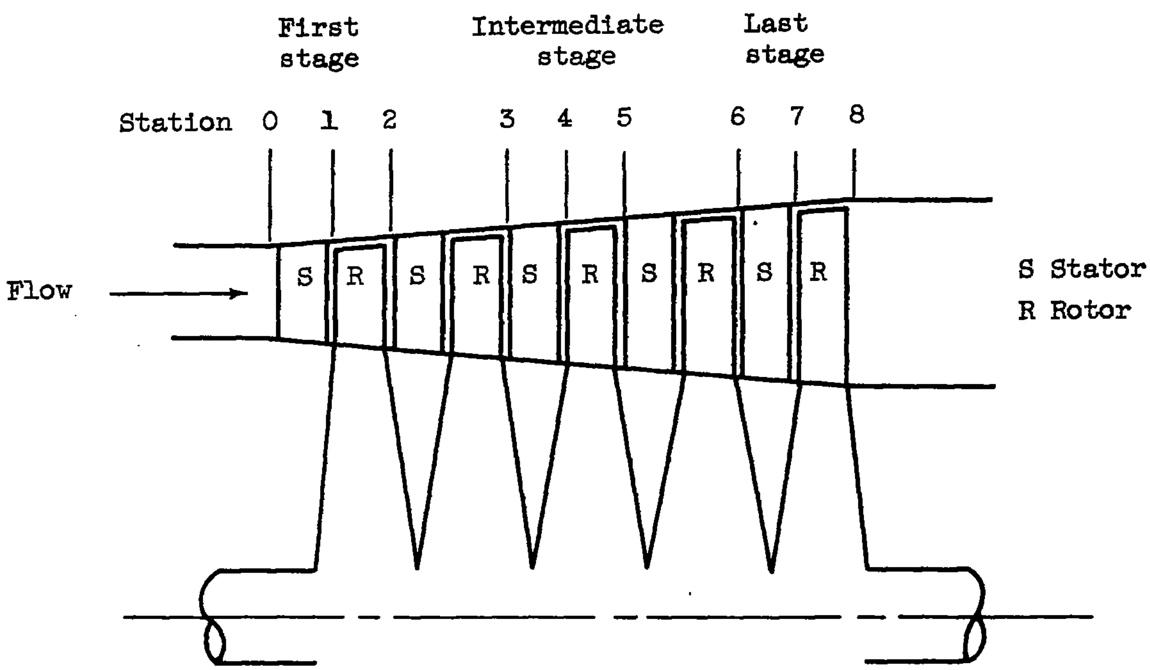
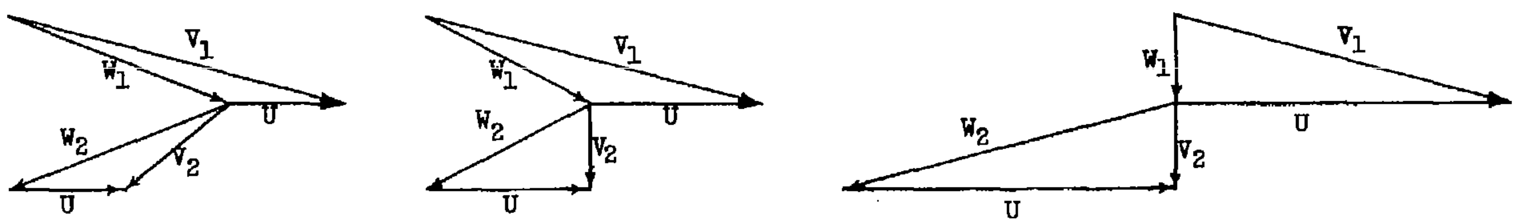


Figure 1. - Diagrammatic sketch of multistage turbine with station nomenclature.



(a) Rotor impulse. Work-speed parameter, 0.25.

(b) Rotor impulse, zero exit whirl. Work-speed parameter, 0.5.

(c) Zero exit whirl. Work-speed parameter, 1.

Figure 2. - Types of velocity diagrams considered in analysis.

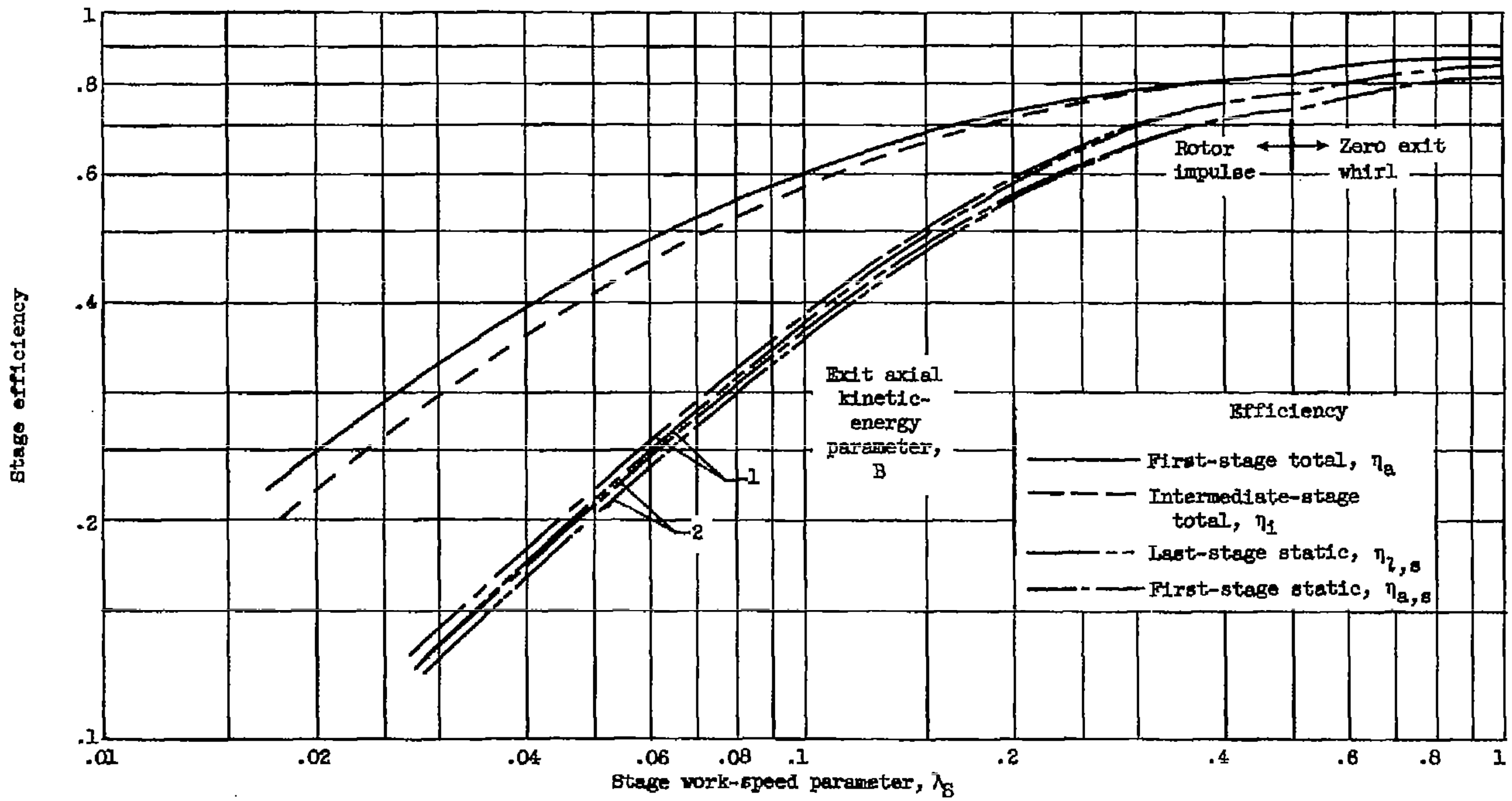


Figure 3. - Stage efficiency as function of stage work-speed parameter. $K\left(\frac{A}{w}\right)$, 0.0437; stator-exit flow angle, α_S, 75°.

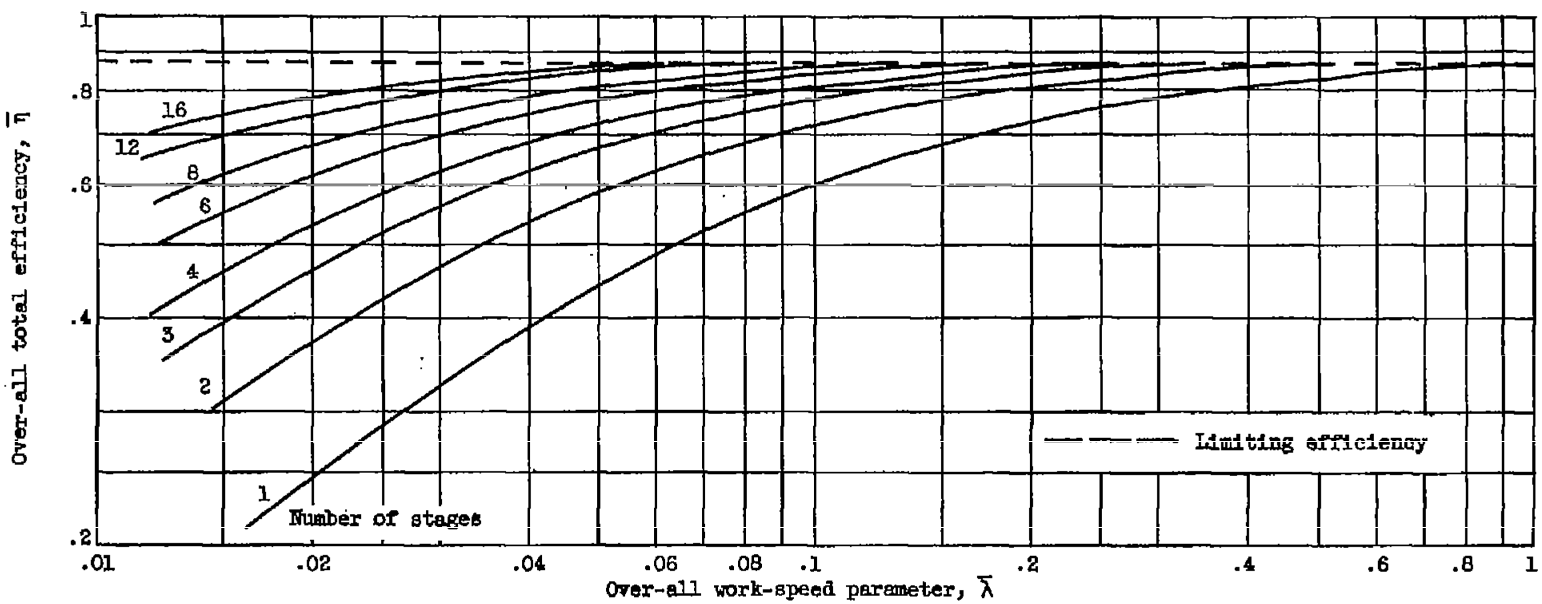


Figure 4. - Over-all total efficiency characteristics. $K\left(\frac{A}{W}\right)$, 0.0437; stator-exit flow angle, α_B, 75°.

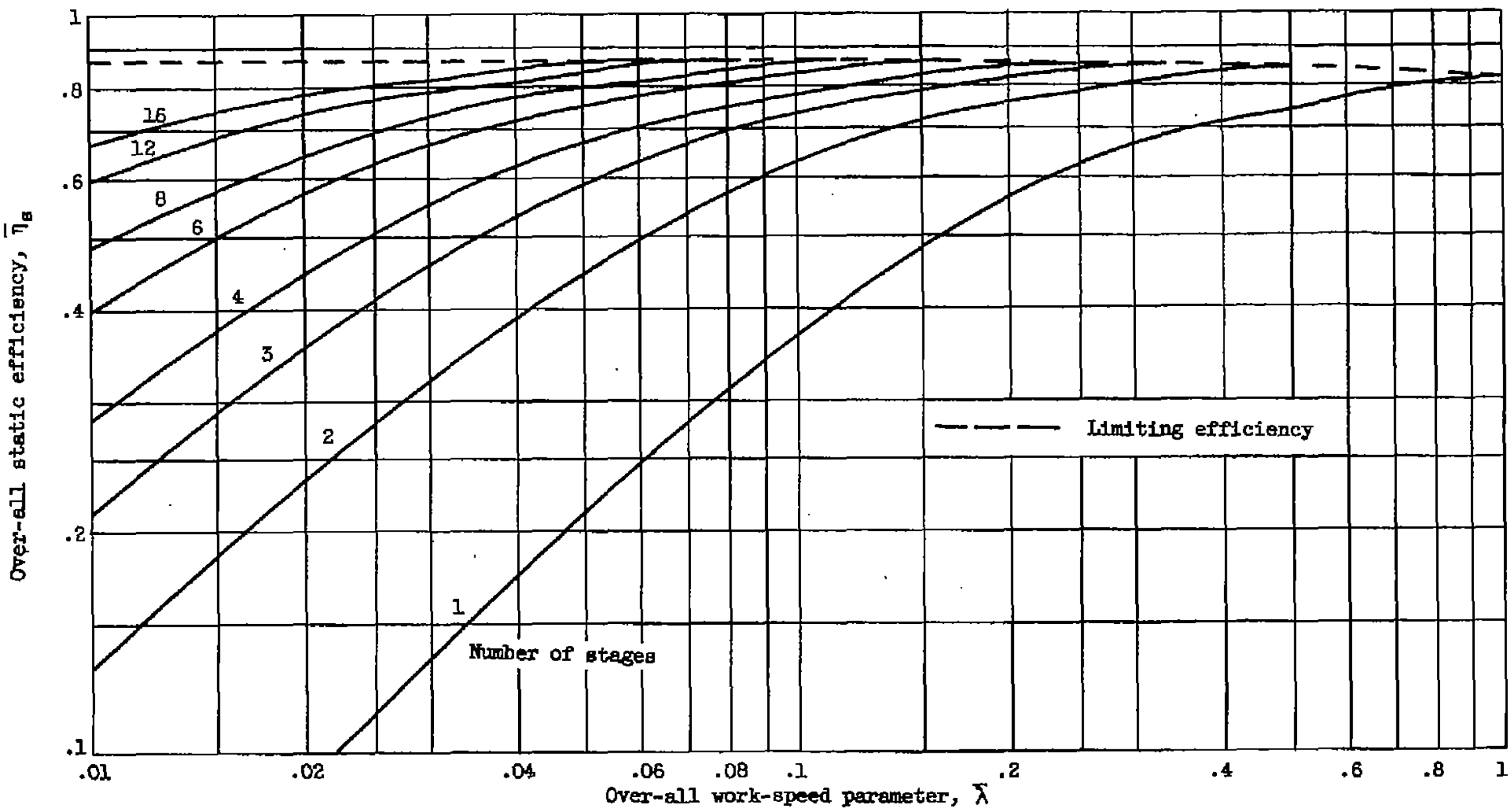


Figure 5. - Over-all static efficiency characteristics. Exit axial kinetic-energy ratio, B, 2; $K\left(\frac{A}{W}\right)$, 0.0437; stator-exit flow angle, α_S, 75°.