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RESEARCH MEMORANDUM

for the

Civil Aeronautics Authority

CALCULATIONS OF THE DYNAMIC STRESS OF SEVERAL

AIRPLANE WINGS IN VARIOUS GUSTS

By

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AIRPLANE WINGS IN VARIOUS GUSTS

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SUMMARY

A series of calculations of the dynamic response of airplane wings to gusts were made with the purpose of showing the relative response of a reference airplane, the DC-3 airplane, and of newer types of airplanes represented by the DC-4, DC-6, and L-49 airplanes. Additional calculations were made for the DC-6 airplane to show the effects of speed and altitude.

On the basis of the method of calculation used and the conditions selected for analysis, it is indicated that:

1. The newer airplanes show appreciably greater dynamic stress in gusts than does the reference airplane.
2. Increasing the forward speed or the operating altitude results in an increase of the dynamic stress ratio for the gust with a gradient distance of 10 chords.

INTRODUCTION

At the request of the Civil Aeronautics Authority, a series of calculations of the transient stresses induced by gusts in airplane wings have been made and analyzed. These calculations were made for the purpose of comparing the response of present-day large transport airplanes to that of an older transport airplane, which has proven itself to be satisfactory from the standpoint of gust experience. The results of the comparison are to be used in discussions of the advisability of including a transient stress requirement for gust loads in the civil air regulations of the International Civil Aviation Organization (ICAO) (reference 1, sec. 3.3.1.4.3).

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Sufficient time was not available to make a comprehensive survey of all the factors influencing the dynamic response of the chosen airplanes. The cases considered did, however, include investigation of the effects of gradient distance, forward speed, altitude, and two successive gusts on the dynamic response of the airplanes to gusts.

SYMBOLS

A	forcing function factor, pounds per second
A_f	fuselage forcing function factor, pounds per second
A_w	wing forcing function factor, pounds per second
Ate^{-bt}	forcing function assumed for conventional airplane
b	time constant, per second
H	gust-gradient distance, chords
K	spring constant, upward force on fuselage due to both wings when in assumed deflection shape, per unit tip deflection, pounds per foot
M_f	fuselage mass, slugs
M_w	wing mass, slugs
M_{we}	equivalent wing mass, slugs
U	maximum gust velocity, feet per second
V_L	maximum indicated airspeed in level flight, miles per hour (reference 2, sec. 04a.111)
V_{ne}	placard never-exceed-speed, miles per hour (reference 2, sec. 04a.743)
W	weight, pounds
λ	damping coefficient of airplane, pound-seconds per foot
λ_f	fuselage damping coefficient, pounds-seconds per foot
λ_w	wing damping coefficient, pound-seconds per foot

λ_{we}	equivalent wing damping coefficient, pound-seconds per foot
Δn_{rmax}	maximum acceleration increment the airplane would experience if it were rigid (g)
δ_{st}	static deflection, feet

METHOD

Calculations of the dynamic response of the airplane wings to gusts were made and the results put in the form of a ratio of the maximum deflection under dynamic conditions to the deflection computed under static conditions. The method of reference 3 was used in the calculations with the exception that the torsion of the wings was included as suggested in that reference as a fixed relation to wing bending. The torsion component was determined by computing the twist per g of load for the wing. In those cases where the twist was very small (DC-3 and DC-4 airplanes), the calculations were made by assuming it to be zero. The assumption that only the first mode of bending is important is the same as in reference 3. The gust is assumed to be uniform across the span of the wing.

The forcing function Ate^{-bt} may be considered to represent the effect of a gust having a velocity distribution in the direction of flight of the type shown in figure 1(a). The results of the calculations for the forcing function Ate^{-bt} were superimposed as indicated for the gust in figure 1(b) to represent the response to the triangular gust shape indicated in figure 1(c). The response to repeated gusts was obtained by superposition of the responses to gusts of the shape shown in figure 1(c). The gradient distance of the gust is assumed to be the distance traveled by the rigid airplane in order to reach the maximum value of the curve of acceleration increment.

CALCULATIONS AND RESULTS

The airplanes chosen by the Civil Aeronautics Authority for the calculations were the DC-3, DC-4, DC-6, and the L-49. The DC-3 airplane was designated as the reference airplane and the other airplanes were considered as representative of the present-day large transport airplanes. The conditions for which the calculations were made are given in tables I and II. All airplanes were assumed to be flying at high speed in level flight V_L . In the case of the DC-6 airplane, calculations were also made for flight at two other speeds and for flight at the high speed V_L at an altitude of 30,000 feet.

The basic constants used in the calculations were determined from manufacturers' data and are shown in Tables I and II. The equivalent masses and damping constants for the DC-4 and DC-6 airplanes were determined by using wing-deflection data determined by static test, and for the DC-3 and L-49 they were determined from computed deflection curves. The wing frequencies used were those determined by vibration tests in all cases. The mass distributions were determined as those that might be applicable at about a halfway point on a long flight.

The Glauert correction factor for the effects of compressibility on the slope of the lift curve was used only for the DC-6 airplane at the condition corresponding to a speed of 461 miles per hour, condition III. The correction was considered unnecessary for most of the other airplanes and conditions. Condition IV for the DC-6 airplane which represents a very high true speed at altitude was beyond the range of applicability of the simple correction factor so no correction was made.

The effect of wing torsion was adverse for the DC-6 airplane and favorable for the L-49 airplane, the spring and damping constants being reduced for the DC-6 airplane and increased for the L-49 airplane compared to the constants which would be determined if the bending alone were considered.

Time histories of wing-tip deflections for each condition, Tables I and II, were calculated for gusts varying from the shortest gradient distance to which the airplane would respond to a gust whose gradient distance was 20 chords. The shortest gradient distance considered applicable was taken as equal to the distance in which the rigid airplane would attain maximum load in penetrating a sharp-edge gust. The maximum wing deflection in each gust was divided by the static deflection determined by the formula

$$\delta_{st} = \frac{\Delta n_{r \max} \left(\frac{A_w W}{A} - 32.2 M_w \right)}{A}$$

in order to obtain the dynamic stress ratio for each condition and gust size.

The dynamic stress ratios have been plotted in figures 2, 3, and 4 as a function of the gust-gradient distance for each variable considered. Figure 2 shows the dynamic stress ratios for all airplanes flying at the high speed in level flight V_L at sea level. Figure 3 is for the DC-6 airplane flying at V_L , V_{ne} , and $V_{ne} + 100$ miles per hour in order to show the effect of speed as a function of gust sizes. Figure 4 is for the DC-6 airplane and shows the dynamic stress ratios which would be experienced by that airplane at the same indicated airspeed (V_L) flying at sea level and at an altitude of 30,000 feet.

In all cases, the curves are stopped at the shortest gradient distance to which the airplane would respond according to the criterion previously mentioned.

Calculations for the effect of repeated gusts were carried out according to the method used in reference 4 and made use of the results of the statistical analysis of the characteristics of repeated gusts in turbulent air reported in reference 5. The time histories of the results of the calculation for the gust-gradient distance of 10 chords were superimposed to represent the time history of the response of the wings to two gusts whose peaks were spaced 25 chords apart. The dynamic stress ratios were then determined as the ratio of the maximum deflection in the sequence to the static deflection determined from the first gust.

Since the repeated gust statistics of reference 5 show that the gust velocity of repeated gusts having the same frequency of occurrence in the atmosphere as a single gust of 30 feet per second is 25 feet per second, the dynamic stress ratios for the repeated gusts were reduced by the factor $25/30$. The reduction of the true dynamic stress ratios for the repeated gusts is made so that the resulting ratios may be multiplied directly by the static stress determined for the single gust of 30-foot-per-second gust velocity rather than by a static stress representing the reduced gust velocity of the repeated gust. Since the reduced dynamic stress ratios for the repeated gust and the true ratios for the single gust are multiplied by the same static stress, they are comparable measures of the stress in the airplane wings. The results for each airplane at the high speed in level flight at sea level are shown in Table III, together with the dynamic stress ratios for the single gust with a gradient distance of 10 chords.

Dynamic stress ratios for the ICAO design gust having a gradient distance of 100 feet (reference 1, sec. 3.3.1.4.2) are also included in table III. The values shown were determined by plotting a figure similar to figure 2 for flat-topped gusts of the type shown in figure 1(a) and by reading the dynamic stress ratio at the gradient distance in chords which corresponds to 100 feet of travel of the particular airplane.

DISCUSSION

Consideration of figures 2 to 4, together with table III, shows that the modern transport airplanes experience appreciable dynamic overstress as compared to the DC-3 airplane. Considerable spread is found between individual airplane types (fig. 2) and in some cases the general trend is not followed, notably for high speeds and high altitudes as shown in figures 3 and 4, respectively. Disregarding the exceptions, it is apparent that the results show that the dynamic structural response may be a factor in the design of modern transport airplanes.

Figure 2 shows that at the design high speed in level flight V_L , the DC-4, DC-6, and L-49 airplanes show a substantial increase in dynamic response as compared to the DC-3 airplane. For a triangular gust with a gradient distance of 10 chords, the increase in ratio varies from 0.07 to 0.30. Some of the spread noted is the result of including the effect of torsion. A special calculation for the DC-6 airplane, with torsion omitted, reduced the dynamic stress ratio by about 0.10 for a gradient distance of 10 chords. Since the L-49 airplane shows an opposite effect of torsion, the general effect of torsion is to spread the dynamic stress ratios, whereas its omission brings the curves closer together. It is apparent from brief consideration, therefore, that not only do the modern transport airplanes experience appreciable dynamic overstress but also that torsional effects are an important factor in the determination of dynamic stress.

Figure 3 shows, in general, that increased speed results in an appreciable increase in the dynamic stress ratio, but that at high speeds the gradient distance of the gust also has an important effect. At the gust-gradient distance of 10 chords, the increase in speed from V_L to V_{ne} gives an increase in dynamic stress ratio of about 0.12. The increase in speed beyond V_{ne} , however, results in only about an increase of 0.04 in the ratio. This small increase might partially be explained by the inclusion of the Glauert correction factor for compressibility in the calculations for the speed of $V_{ne} + 100$ miles per hour. The decided dropping of the curve at the shorter gradient distances for the highest speed probably results from the force caused by the sharp gust passing its maximum before the wing can respond fully.

The effect of altitude on the dynamic stress ratio, illustrated in figure 4, is very large, amounting to an increase of about 0.33 at the gradient distance of 10 chords. Analysis of the results indicates that the causes of the large increase are probably the increase in true speed obtained by keeping a constant equivalent airspeed equal to V_L and the reduction in the damping constant with altitude (tables I and II). The damping constant at altitude is only some 37 percent of the value it would be if the airplane were flying the true speed of 492 miles an hour at sea level. The drooping of the curve for altitude at the shorter gradient distances appears again to be the result of the high speed; that is, the sharp gust passes its maximum before the wing can respond fully.

Inspection of the values of dynamic stress ratio given in table III for the response to two standard gusts in succession but of opposite direction shows that the newer airplanes have substantially greater dynamic stress than the DC-3 airplane has under the same conditions. Comparing the results for the repeated gusts with those for the single gusts with gradient distances of 10 chords (table III) indicates that the general stress level for the repeated gusts is somewhat higher than for the single standard gusts.

The responses to the ICAO design gust with a gradient distance of 100 feet (table III) also show that the newer airplanes have much greater dynamic stress than does the DC-3 airplane, the values ranging from about 0.20 to 0.45 greater. Comparing the responses of the newer airplanes to the ICAO gust with those to the single gust with a gradient distance of 10 chords shows an increase in the value of dynamic stress ratio of about 0.15 for each airplane, whereas the increase for the DC-3 airplane is only about 0.01. The difference between the increases for the newer airplanes and that for the DC-3 airplane is a function of the different chord lengths of the airplane wings.

Inspection of the results presented in table III shows that, regardless of the comparison used, the stress levels for the newer airplanes are appreciably higher than that of the DC-3 airplane. On the basis of the analysis made, then, it appears that airplanes of the type represented by the DC-4, DC-6, and L-49 will show greater dynamic response to gusts than airplanes of the type represented by the DC-3 airplane. The analysis further points out that wing torsion may be an important consideration in the problem.

It is obvious that all the possible combinations of airplane conditions and gust conditions have not been investigated and that proper selection of the combinations could yield either greater or lesser values of dynamic stress ratio than those given in table III or in figures 2 to 4. Of the conditions investigated, however, the relative stress levels indicated for the repeated gust condition (table III) are felt to be the most realistic.

CONCLUDING REMARKS

On the basis of the calculation method used and the conditions selected for the analysis, it is indicated that:

1. The newer airplanes, represented by the DC-4, DC-6, and L-49 airplanes, show appreciably greater dynamic stress in gusts than does the reference airplane.
2. The effect of wing torsion may be an important factor in determining the dynamic stress level.

3. Increasing the forward speed or the operating altitude results in an increase of the dynamic stress ratio for the gust with a gradient distance of 10 chords.

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TABLE I

CHARACTERISTICS OF AIRPLANES CHOSEN FOR CALCULATIONS

Airplane	Condition	Weight W (lb)	Wing area (sq ft)	Wing loading (lb/sq ft)	Span (ft)	Mean aerodynamic chord (ft)	Natural wing frequency (cps)	Slope of lift curve (per radian)	Number of engines	True airspeed (mph)
DC-3	1	23,967	987	24.28	95	11.5	4.67	4.53	2	211 (v_L)
DC-6	1	84,680	1461	57.96	118.4	13.65	3.4	4.70	4	301 (v_L)
	2	84,680	1461	57.96	118.4	13.65	3.4	4.70	4	361 (v_{ne})
	3	84,680	1461	57.96	118.4	13.65	3.4	5.89	4	461 ($v_{ne} + 100$)
	4	84,680	1461	57.96	118.4	13.65	3.4	4.70	4	492 (v_L at 30,000 ft)
DC-4	1	61,650	1460	42.23	118.2	13.65	3.333	4.70	4	250 (v_L)
L-49	1	80,400	1650	48.73	123.0	14.65	3.583	4.67	4	271 (v_L)



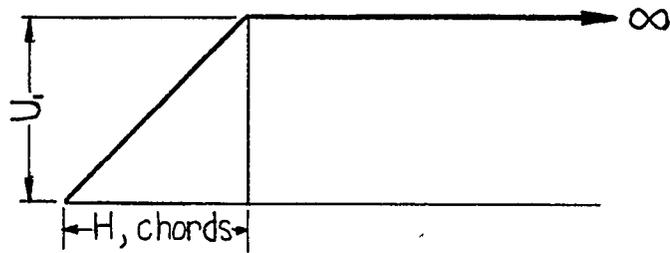
TABLE II
CHARACTERISTICS AND CALCULATION DATA FOR AIRPLANES IN TABLE I

Airplane	Condition	Gradient distance H (chords)	Weight W (lb)	M_{W_0} (slugs)	M_W (slugs)	M_F (slugs)	Damping factor λ (lb-sec/ft)	λ_{W_0} (lb-sec/ft)	λ_W (lb-sec/ft)	λ_F (lb-sec/ft)	Forcing function factor, A (lb/sec)
DC-3	1	0 10 20	23,967	20.59	407.48	336.84	1234.01	242.251	1090.48	143.53	1,015,000 207,153 65,500
DC-6	1	0 10 20	84,680	192.6	1501.11	1128.70	2700.41	553.390	2358.47	341.94	2,625,000 1,079,000 422,000
	2	0 10 20	84,680	192.6	1501.11	1128.70	3238.66	663.692	2828.56	410.10	3,230,000 1,312,000 504,000
	3	0 10 20	84,680	192.6	1501.11	1128.70	5186.70	1062.901	4529.93	656.77	4,280,000 1,554,000 567,000
	4	0 10 20	84,680	192.6	1501.11	1128.70	1650.50	338.234	1441.51	208.99	3,880,000 2,890,000 1,273,000
DC-4	1	0 10 20	61,650	204.0	1065	850	2245.74	480.736	1959.64	286.10	1,870,000 596,000 225,000
L-49	1	0 10 20	80,400	195.35	1516.2	981	2734.81	734.453	2330.39	405.08	2,300,000 808,551 299,382
Airplane	Condition	Gradient distance H (chords)	A_W (lb/sec)	A_F (lb/sec)	Spring constant K (lb/ft)	Time constant b (per sec)	True airspeed (mph)	Remarks			
DC-3	1	0 10 20	896943.26 183058.61 57881.56	118056.74 24094.39 7618.44	8022.61	7.82 1.59 0.503	211	Torsion negligible.			
DC-6	1	0 10 20	2292607.00 942370.98 368563.99	332392.00 136629.02 53436.00	33823.93	5.72 2.35 0.920	301	Torsion included.			
	2	0 10 20	2820999.32 1145867.21 440180.70	409000.68 166132.79 63819.30	32113.80	7.04 2.86 1.098	361	Torsion included.			
	3	0 10 20	3738042.44 1357223.82 495203.29	541957.56 196776.18 71796.71	26240.99	9.34 3.39 1.239	461	Torsion included Glauert compressibility correction to slope of lift curve.			
	4	0 10 20	3388692.67 2524052.02 1111805.59	491307.33 365947.98 161194.40	33826.78	6.29 4.69 2.065	492	Torsion included.			
DC-4	1	0 10 20	1631767.12 520071.23 196335.62	238232.88 75928.77 28664.38	39711.17	5.58 1.78 0.671	250	Torsion negligible.			
L-49	1	0 10 20	1959878.79 688983.46 255109.75	340121.21 119567.54 44272.25	45071.79	5.27 1.85 0.685	271	Torsion included.			

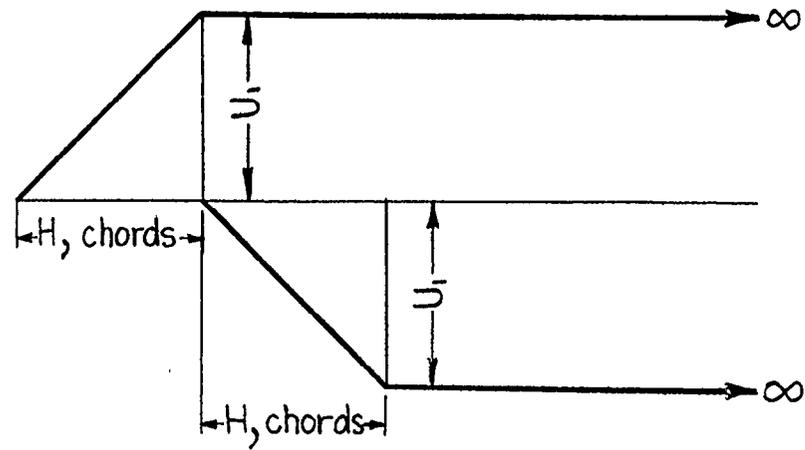
TABLE III
 COMPARISON OF RESPONSES OF AIRPLANES TO SINGLE
 AND REPEATED GUSTS AT V_L AT SEA LEVEL

Airplane	Standard repeated gusts, dynamic stress ratio	Standard single gust, dynamic stress ratio	ICAO single gust, dynamic stress ratio
DC-3	1.15	0.98	0.99
DC-4	1.41	1.19	1.33
DC-6	1.42	1.31	1.45
L-49	1.40	1.06	1.20

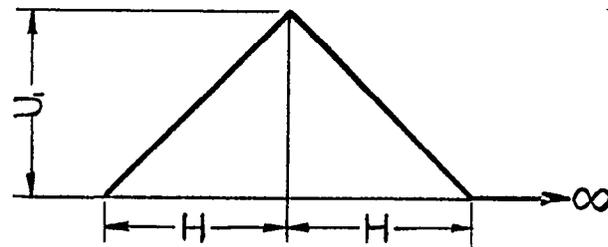




(a) Gust Represented by function
At e^{-bt}



(b) Superposition of gusts.



(c) Desired gust shape.

Figure 1.-Derivation of desired gust shape.



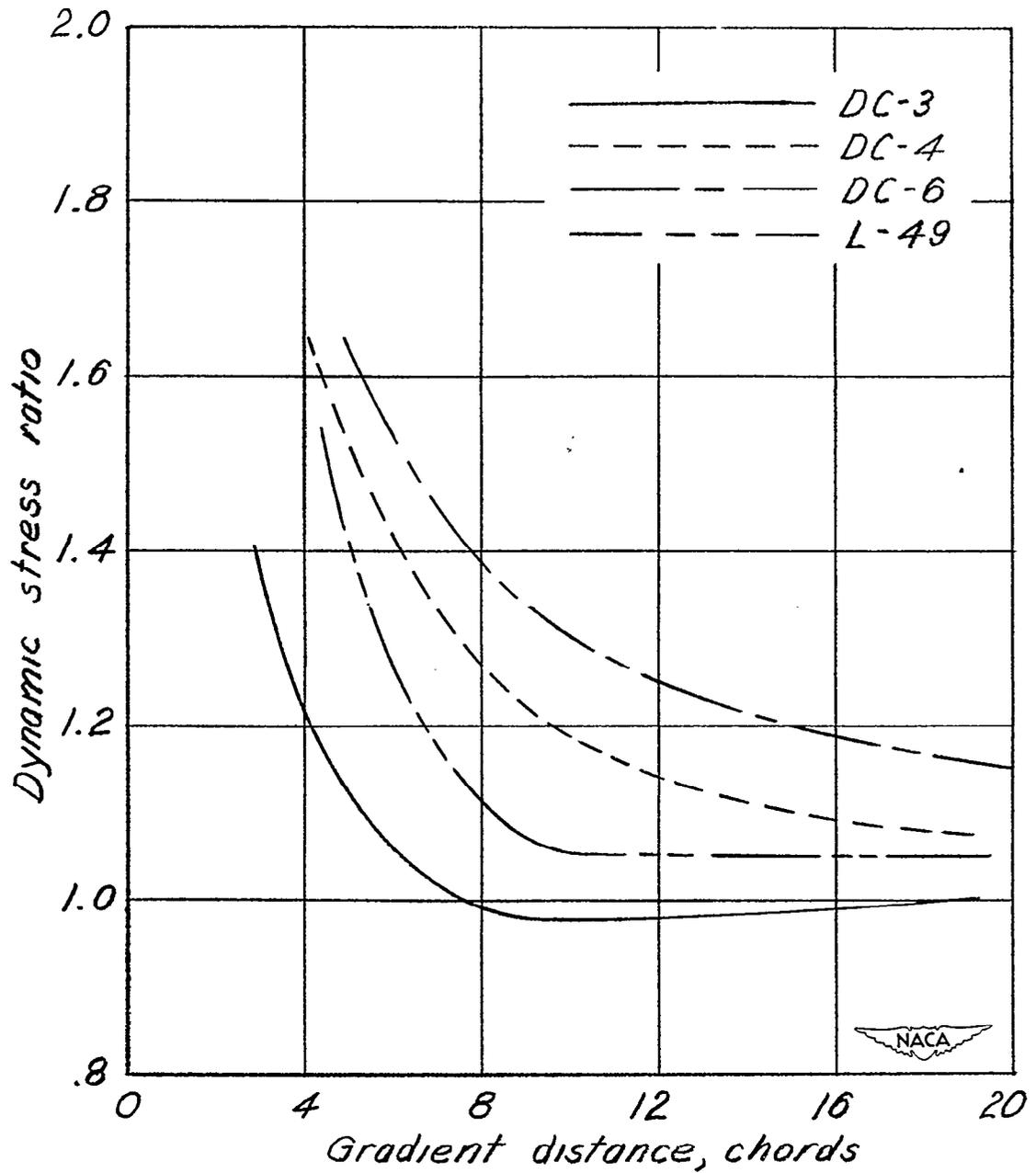


Figure 2.- Comparison of responses of airplanes at $V_{\frac{1}{2}}$ at sea level.

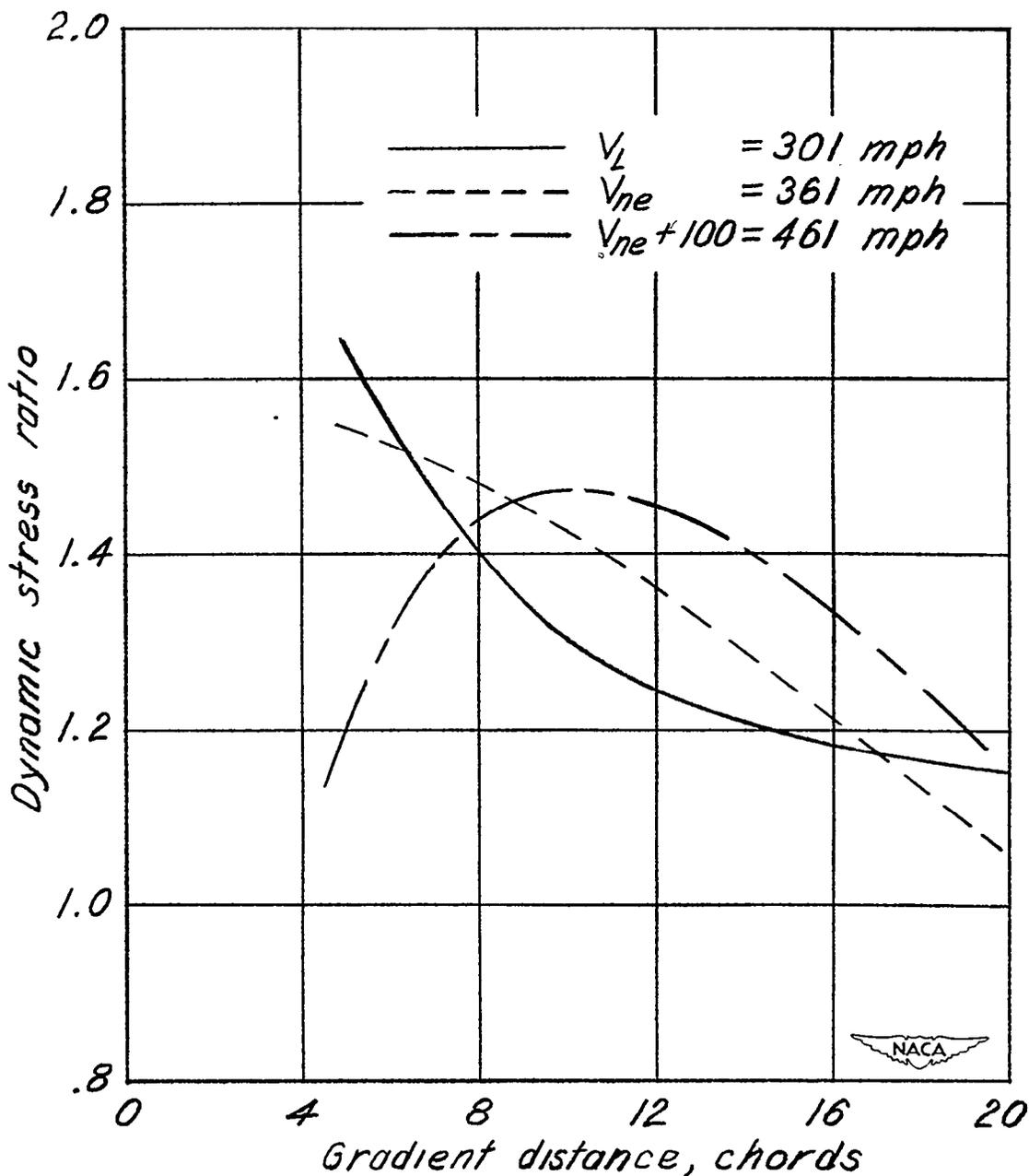


Figure 3.- Effect of forward speed on the dynamic stress ratio. DC-6 conditions I, II, and III.

