

# NACA

## RESEARCH MEMORANDUM

THEORETICAL ANALYSIS OF THE ROLLING MOTIONS OF AIRCRAFT USING  
A FLICKER-TYPE AUTOMATIC ROLL STABILIZATION SYSTEM HAVING  
A DISPLACEMENT-PLUS-RATE RESPONSE

By

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CLASSIFIED DOCUMENT

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NATIONAL ADVISORY COMMITTEE  
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ERRATUM

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An error has been found in the labeling of the curves of figure 3.  
A copy of the corrected figure is attached.



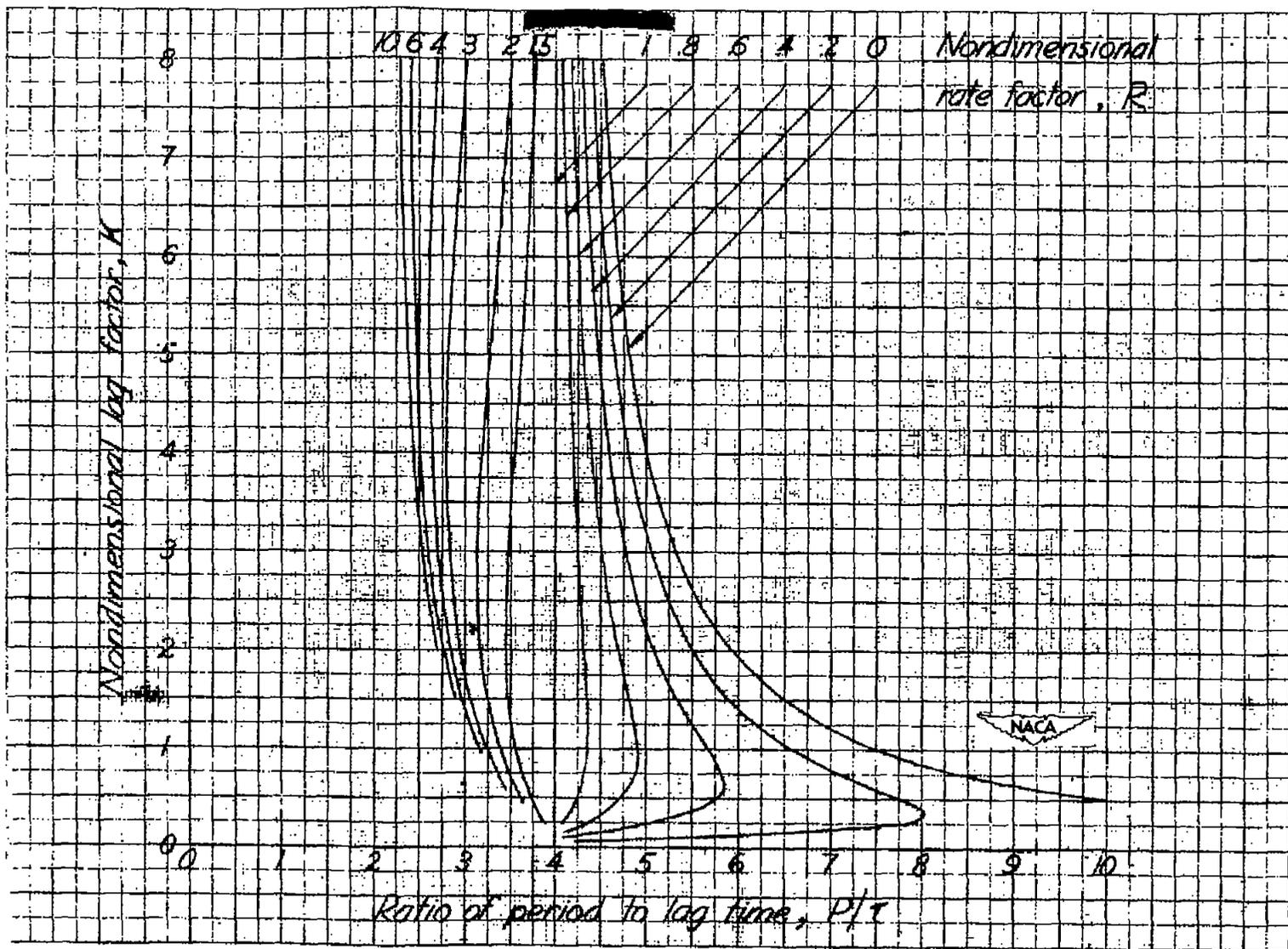


Figure 3.- Chart for determining the steady-state period.



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## SUMMARY

A general analysis is presented which allows the amplitude and period of the steady rolling oscillations of aircraft using the displacement-plus-rate response, flicker-type automatic roll stabilization system to be determined. It is shown that the inertia, damping, and control characteristics of the aircraft in roll and the lag time and rate factor of the automatic system are sufficient to define these rolling oscillations, and charts are presented from which the amplitude and period of the resultant steady oscillations may be found. The addition of the rate-sensitive element to the displacement response, flicker-type system reduces the amplitude and increases the frequency of the steady-state oscillations. Because of the inherent residual oscillations, this system may not be considered ideal for many stabilization problems; however, this flicker-type system appears to offer a simple solution to those applications where steady rolling oscillations may not be objectionable. Current trends in pilotless-aircraft designs indicate that the subject system can provide roll stabilization with small amplitude residual oscillations. Close agreement is shown to exist between the theoretical results and roll-simulator tests.

## INTRODUCTION

The stabilization of aircraft in roll by means of automatic control is one of the important problems existing in the field of pilotless-aircraft research. This paper, like reference 1, has as its purpose the study of a particular type of automatic control system and its adaptability to providing roll stabilization in terms of the conventional aircraft parameters.

The automatic roll stabilization system considered herein is the displacement-plus-rate response, flicker-type automatic pilot. In this system the sense of the control moment is dependent on both the angular displacement and the rate of displacement from a given reference. The control

moment is considered to be constantly applied in either one direction or the other at all times; hence, the name flicker-type system. It is recognized that the continuous application of the control moment may not be ideal for many stabilization problems; however, such a system may prove an economic and simple solution to those applications where residual oscillations of small magnitude are not objectionable.

The aircraft and the automatic control system parameters which affect the rolling motions of an aircraft are found by considering the aircraft as a single-degree-of-freedom system in roll controlled by the automatic pilot. Charts are presented from which the amplitude and period of the resulting steady-state rolling oscillations may be found. Roll-simulator tests employing a subject automatic roll stabilization system are included to show the agreement between this theoretical approach and the experimental results.

#### SYMBOLS

$\phi$	angle of bank, radians
$P$	angular rolling velocity, radians per second ( $d\phi/dt$ )
$I_x$	moment of inertia about longitudinal axis of aircraft, slug-feet <sup>2</sup>
$L$	rolling moment in general and control moment in particular, foot-pounds
$L_p$	rate of change of rolling moment with angular rolling velocity, foot-pounds per radian per second ( $\partial L/\partial p$ )
$\delta$	combined differential deflection of ailerons, radians
$L_\delta$	rate of change of rolling moment with aileron deflection, foot-pounds per radian ( $\partial L/\partial \delta$ )
$\delta L_\delta$	control moment, foot-pounds ( $L$ )
$t$	time, seconds
$\Delta t$	time increments measured from translated time origins, seconds
$\tau$	time lag in operation of the servomotor, seconds
$a$	damping-to-inertia ratio, per second ( $ L_p/I_x $ )

K	nondimensional lag factor ( $\alpha r$ )
$\Lambda$	proportional rate factor of subsidiary feedback loop, radians per radian per second (see appendix, equation (8))
R	nondimensional rate factor ( $\alpha \Lambda$ )
$\eta$	angular output of the differentiator, radians ( $-\Lambda p$ )
$P_{\max}$	maximum angular rolling velocity, radians per second
C	fractions of $P_{\max}$
$\epsilon$	servomotor operating error, radians (see appendix, equation (6))
$\phi_1$	reference bank angle, radians (considered as zero herein)
A	amplitude of steady-state rolling oscillations (one-half total displacement), radians
$A_{R=0}$	amplitude of steady-state rolling oscillations when $R = 0$ , (that is, for the displacement response, flicker-type system), radians
B	nondimensional amplitude factor ( $LI_x/L_p^2$ )
P	period of the steady-state rolling oscillations, seconds
ln	natural logarithm
e	base of natural logarithm (2.7183)

Numerical subscripts refer to time limits (see appendix).

#### DESCRIPTION OF THE AUTOMATIC STABILIZATION SYSTEM

In general, the displacement-plus-rate response, flicker-type automatic roll stabilization system is shown as a block diagram in figure 1 and operates as follows: The detection of an error  $\epsilon$  causes the servomotor to operate the controls to apply a constant rolling moment to the aircraft. The sense of this rolling moment  $L$  is dependent upon the sign of  $\epsilon$ . A differentiator whose output is an angle proportional to the rolling velocity of the aircraft forms a subsidiary or parallel feedback loop. The angular displacement of the aircraft in roll is also detected through another feedback loop.

The two summing devices determine the difference between actual angle of bank, the angular output of the differentiator, and the reference,  $\phi_i$ , thus forming the error  $e$  causing the servomotor to function. Since the error controlling the system is a function of both the roll displacement and the rate of roll displacement and the applied control moment is a constant to the right or left, the term displacement-plus-rate response, flicker-type system is applied. Reference 2 reports the successful use of such a system on a current research pilotless aircraft.

Since the subject system is essentially nonlinear, attention is called to references 3, 4, and 5 which include additional methods of analysis for nonlinear systems.

#### METHOD OF ANALYSIS

In this analysis of the rolling motions of an aircraft stabilized in roll by the displacement-plus-rate response, flicker-type automatic control system, several assumptions are made:

(1) There is a constant time lag in the operation of the servomotor, that is, the application of the constant rolling moment to the aircraft occurs at a finite time after the error signal is applied to the servomotor.

(2) The reversal of the control moment after the time lag is assumed to be instantaneous.

(3) The aircraft response to the constant rolling moment is found from the single-degree-of-freedom, rolling-moment equation. In this analysis this rolling-moment equation is written in terms of actual moments and not in coefficient form. Hence, the damping-in-roll and the aerodynamic roll-control effectiveness must be specified by definite conditions of velocity and dynamic pressure.

(4) The case of out-of-trim moments producing roll has been omitted.

The mathematical procedure used in this analysis is closely related to that of reference 1 and is presented in this paper as the appendix. The analysis is briefly summarized in four steps:

(1) The conditions that cause a change in the sign of the error are formulated.

(2) The angle-of-bank variation during a complete cycle of a rolling oscillation is written in terms of the general aircraft and system parameters which define the motion.

(3) The conditions required for the uniform oscillations at steady state are found.

(4) The general expressions which define the amplitude and period of the steady-state oscillations are found.

Although the case of out-of-trim moments producing roll is not considered herein, it is believed that a method similar to that used in reference 1 can be applied to the displacement-plus-rate response, flicker-type system.

### PRESENTATION OF RESULTS

The displacement-plus-rate response, flicker-type roll stabilization system is characterized by the tendency of the automatically controlled aircraft to oscillate at a definite amplitude and frequency. These resultant oscillations are defined as steady-state oscillations. The motions of the aircraft in approaching steady state following a disturbance are not considered herein; however, a method similar to that used in reference 1 for the displacement response, flicker-type system or a graphical method may be employed for this transient state.

In this section only the equations that were used for plotting the included curves are presented, their derivations being in the appendix. All of the symbols used here and in the appendix have been presented in a previous section.

### Required Parameters

The analysis shows that five primary parameters are required to define the steady-state rolling oscillations of the aircraft. These are:

- (1) The rolling inertia of the aircraft,  $I_x$
- (2) The damping in roll of the aircraft,  $L_p$
- (3) The control moment causing the aircraft to roll,  $L$
- (4) The constant time lag in the operation of the servomotor,  $\tau$
- (5) The proportional rate factor of the subsidiary feedback loop,  $\Lambda$ .

All of these parameters must be known or estimated before the amplitude and period of the steady-state oscillations can be determined.

The results of the analysis are presented in terms of three factors which include all of the parameters mentioned. These factors are:

- (1) The nondimensional lag factor  $K$  which is defined as

$$K \equiv \left| \frac{L_p}{I_x} \right| \tau \equiv a\tau$$

- (2) The nondimensional amplitude factor  $B$  which is defined as

$$B \equiv \frac{LI_x}{L_p^2}$$

- (3) The nondimensional rate factor  $R$  which is defined as

$$R \equiv \left| \frac{L_p}{I_x} \right| \Lambda \equiv a\Lambda$$

Each of these factors is used in finding the amplitude and period at steady state.

#### Steady-State Amplitude

The amplitude of the steady-state rolling oscillations is defined as one-half the total angle-of-bank displacement. The total displacement was found from the expressions for the maximum angles of bank for each half cycle of a steady-state rolling oscillation. The amplitude  $A$  of the steady-state oscillations is given as

$$A = B \left\{ K + C_0(1 - R) + \ln \left[ \frac{1}{2 - (1 - C_0)e^{-K}} \right] \right\} \quad (1)$$

where  $C_0$  is the fraction of the maximum rate of roll that exists at the time the sign of the error changes, that is, when the servomotor is signaled to reverse.

It has been shown in the appendix that the steady-state value of  $C_0$  is a function of  $K$  and  $R$  and that once  $K$  and  $R$  are defined,  $C_0$  is

unique. At steady state, therefore, the bracketed term of equation (1) is completely determined. Since the steady-state amplitude varies directly with the amplitude factor, figure 2 is given as the solution of equation (1) and presents the variation of the ratio of A/B with K for various values of R. (The value of A computed from this fig. is in degrees.) Figure 2(a) is presented for small values of K against A/B for various values of R. Figure 2(b) is given for larger values of K against R for various values of A/B.

### Steady-State Period

The period of the steady-state oscillations is defined as the time required to complete one cycle. The equation for the period P at steady state is given as

$$P = 2\tau \left\{ 1 - \frac{1}{K} \ln \left[ \frac{1 - C_0}{2 - (1 - C_0)e^{-K}} \right] \right\} \quad (2)$$

Although the nondimensional rate factor R is not explicit in equation (2), it is necessarily implied since the steady-state value of  $C_0$  is defined by both K and R. Figure 3 is the graphical interpretation of equation (2) for the steady state and is a plot of the ratio  $P/\tau$  against K for various values of R. At the values of K and R for the particular system, the value of  $P/\tau$  read from figure 3 multiplied by the lag time will give the period of the steady-state roll oscillations.

Attention is called to the fact that the period does not vary directly with the lag time although the curve is presented in terms of the ratio of the period to the lag time. This fact is true because the value of K is also a function of  $\tau$ . For example, at a particular R, the doubling of  $\tau$  does not exactly double the period since the value of  $P/\tau$  is changed due to the resultant doubling of K.

### DISCUSSION OF RESULTS

To show more clearly the effect of the rate-of-roll component on the operation of the complete system, a comparison can be made between the system with and without the rate-sensitive element, that is, between the displacement response and the displacement-plus-rate response, flicker-type systems.

Referring to figure 1, the purpose of the rate of sensitive element, the differentiator, is to cause the error  $\epsilon$ , which energizes the servomotor,

to be a function of the rolling velocity as well as the roll displacement. Without the differentiator the servomotor is signaled to reverse when the bank angle  $\phi$  equals zero. (The reference bank angle  $\phi_1$  is considered zero herein.) It is evident that the amplitude of the rolling oscillations could be reduced if the signal for reversal of control moments could occur as the aircraft is approaching the zero reference position. This effective leading signal is obtained by using the differentiator whose output  $\eta$  is proportional to the rolling velocity, since the signal reversal will then occur when the bank angle  $\phi$  is equal to the differentiator output  $\eta$ .

The reduction in amplitude of the steady-state rolling oscillations due to the addition of the rate-sensitive element is shown in figure 4. The ordinate is the ratio of the amplitude with the rate factor  $R$  to the amplitude when  $R = 0$ , that is, the ratio of the amplitude of the displacement-plus-rate response, flicker system to the displacement response, flicker system. The abscissa is the nondimensional rate factor  $R$ , and the curves are presented for various values of the nondimensional lag factor  $K$ .

In addition to showing the beneficial effects of the rate component on the amplitude, the expected benefits of a reduction of servomotor operating lag are also revealed. For displacement-response systems where the operating lag cannot easily be further reduced, the addition of the rate-sensitive element appears to be the obvious step to take in the improvement of amplitude characteristics of flicker-type systems as described herein and in reference 1.

An accompanying effect of the addition of the rate component to the system is a corresponding reduction in the period of the steady-state oscillations. For the range of values of  $K$  shown on figure 4, the reduction in period results in the frequency of the oscillations with the rate component being approximately three times the frequency without this element for the highest values of  $R$  considered. In practice this feature may be undesirable if the response characteristics of an element of the system are frequency variant. An example of such an application is the use of a rate gyroscope as the rate-sensitive element. In order that the response of the rate-gyroscope system be reasonably linear with respect to the rolling velocity, it must operate well below its natural frequency. Consideration should be given, therefore, to this and any other characteristics that may be influenced by the increase in the frequency of oscillation due to increased values of  $R$ . This effect of the rate factor  $R$  on the period can also be noted on figure 3. For any value of the nondimensional lag factor  $K$ , the period for any  $R$  is always less than the value for  $R = 0$ , the displacement response case.

From the preceding discussion it is recognized that the adjustment of the rate-sensitive element can, within reasonable limits, afford an exceptional amount of control over the amplitude of the rolling oscillations experienced in the automatically stabilized aircraft.

## APPLICATION OF RESULTS

Table I has been prepared in an effort to indicate whether or not the topic automatic roll stabilization system might be of value in current problems. The aircraft shown are not identical with any particular pilotless aircraft designs, although current trends are revealed in the magnitude of the characteristics presented. Also shown in table I is a comparison of the amplitude and period for the displacement response (reference 1) and the displacement-plus-rate response, flicker-type systems for the same aircraft. The value of the proportional rate factor  $\Lambda$  was chosen very arbitrarily and is not intended to indicate a most satisfactory or optimum value. It is actually the value found for the system used in the roll-simulator tests discussed later in this report. It is noted in the examples given that the rate element causes a reduction in the steady-state amplitude to as much as one-thirtieth of that found for the displacement response, flicker system. The accompanying increase in the frequency of the steady-state oscillations is shown by the reduction in the period. For aircraft 2 and 4 where the displacement-response system gives very large amplitude oscillations, the system with rate decreases the amplitude to values of a more tolerable magnitude. The effects of lag are shown for aircraft 3.

Although no special trends as to the effects of the other parameters can be determined from this table, the method presented herein allows a rapid determination of the effects of changes in the various factors with velocity, altitude, Mach number, and so forth.

## Roll-Simulator Tests

In order to substantiate these theoretical results, experimental tests were run using the roll simulator developed by the Instrument Research Division of the Langley Aeronautical Laboratory. This instrument is a single-degree-of-freedom system which is controlled by an actual automatic system mounted on a moving table. The control torque-to-inertia ratio and aircraft damping-to-inertia ratio are electromechanically simulated.

A gyroscopic unit comprising both displacement and rate gyroscopes and an electrically operated servomotor were used as the displacement-plus-rate response, flicker-type automatic stabilization system. This system is schematically represented in figure 5. The rider arm is attached to the outer gymbal of the displacement gyroscope and remains oriented, providing the zero space reference. The commutator drum is attached to the mounting case of the displacement gyroscope through the rate gyroscope linkage and rotates as the aircraft displacement is changed. The rate gyroscope is mounted to be sensitive to rolling velocity and is spring-restrained about its precessional axis. The commutator drum is also rotated through an angle

proportional to the angular deflection of the rate gyroscope and thus proportional to the rolling velocity by a suitable linkage. The contact of the rider arm with either of the segments of the commutator drum energizes the servomotor, causing the control moment to be reversed and maintained until a control reversal is again signaled.

The results of two simulator tests are presented in table II. These results are considered to be in good agreement with the theoretical results because of the following restriction placed on these tests: The rate gyroscope used had stops which limited its deflection; therefore, for the present theory to apply, only those conditions which enabled the system to function without the rate gyroscope hitting or remaining on the stops had to be employed. In each case the damping-to-inertia ratio required was out of the linear range of simulation for the instrument.

#### CONCLUDING REMARKS

A general analysis of the steady rolling oscillations of aircraft using a displacement-plus-rate response, flicker-type automatic roll stabilization system has been presented. This system may not be considered satisfactory for many stabilization problems because of the inherent steady oscillations; however, this flicker-type system appears promising where steady-state oscillations may not be objectional. Current trends in pilotless-aircraft designs indicate that the topic system can provide roll stabilization with small amplitude residual oscillations. It has been shown that the addition of a rate-sensitive element to the displacement response, flicker-type system of reference 1 is an effective method of improving the amplitude characteristics. Accompanying the reduction of the steady-state amplitude due to the rate component is an increase in the frequency of the rolling oscillations. Charts have been presented for the determination of the amplitude and period of the resultant steady-state oscillations, and these theoretical considerations have been shown to be in close agreement with roll-simulator tests.

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## APPENDIX

## DERIVATION OF EQUATIONS

## Basic Equations of Motion

The basic differential equation of motion in this analysis is the rolling-moment equation for one degree of freedom:

$$I_x \frac{d^2\phi}{dt^2} - L_p \frac{d\phi}{dt} = L \quad (1)$$

The right-hand member of equation (1) is the constant control moment. The solution of this linear differential equation yields the relationships

$$\begin{aligned} \phi = & \frac{LI_x}{L_p^2} \left[ e^{(L_p/I_x)t} - \left( \frac{L_p}{I_x} \right) t - 1 \right] \\ & + p(0) \frac{I_x}{L_p} \left[ e^{(L_p/I_x)t} - 1 \right] + \phi(0) \end{aligned} \quad (2)$$

$$p = \frac{L}{L_p} \left[ e^{(L_p/I_x)t} - 1 \right] + p(0) e^{(L_p/I_x)t} \quad (3)$$

where  $\phi(0)$  and  $p(0)$  are, respectively, the angle of bank and rolling velocity at the time origin considered.

Equations (2) and (3) are modified in form for use in this paper. It is evident that the variations expressed by these equations are those which follow the initial conditions defined when  $t = 0$ . Herein, the notation  $\Delta t$  is used in place of  $t$  and indicates time increments measured from a defined origin where  $\Delta t = 0$ , thus allowing the equations to be translated along the time axis when proper consideration is given to initial conditions, that is, whenever  $\Delta t$  is defined as zero. A second change is made through the use of the substitution of

$$a \equiv \left| \frac{L_p}{I_x} \right|$$

where  $a$  is termed the damping-to-inertia ratio and the vertical bars indicate the absolute magnitude. The final modification is the expression of  $p(0)$  as a fraction of the maximum rolling velocity. From equation (3)

$$p_{\max} = -\frac{L}{L_p}$$

and

$$p(0) \equiv C p_{\max}$$

where  $C$  is the fraction of  $p_{\max}$  at the time origin ( $\Delta t = 0$ ) considered.

Making these changes, the basic equations of motion are

$$\phi = \frac{L I_x}{L_p^2} \left[ (1 - C) e^{-a \Delta t} + a \Delta t - (1 - C) \right] + \phi(0) \quad (4)$$

$$p = -\frac{L}{L_p} \left[ 1 - (1 - C) e^{-a \Delta t} \right] \quad (5)$$

#### Signal Reversal

The time of signal reversal of the system is defined as the time at which the sign of the error changes, the error sign change causing the servomotor to reverse. Actual application of the reversed control moment occurs  $\tau$  seconds after signal reversal.

From the block diagram shown in figure 1, it is seen that the error may be written as

$$\epsilon = \phi_1 - (\phi - \eta)$$

where  $\phi_1$  is a reference bank angle and is taken as zero herein,  $\phi$  is the actual bank angle, and  $\eta$  is the output of the differentiator in the subsidiary feedback loop. Thus, the action of the servomotor can be written as

$$\begin{aligned}
 L &= |L| \text{ sign } \epsilon \\
 &= |L| \text{ sign } [- (\phi - \eta)]
 \end{aligned}
 \tag{6}$$

It is recognized that the conditions at signal reversal are therefore defined by  $\epsilon = 0$  or

$$\phi - \eta = 0 \tag{7}$$

The purpose of the differentiator is to provide an effect of lead into the automatic control system; that is, the signal reversal is to occur while the aircraft is approaching the zero reference position due to the rate of rolling instead of at the zero reference, as in the case in the displacement response, flicker-type system. The output  $\eta$  of the differentiator is written as

$$\eta = -\Lambda P \tag{8}$$

where the negative sign is required if the action of the subsidiary feedback loop is to cause the system to lead. The factor  $\Lambda$  is the rate proportionality factor with the units radians per radian per second. Substituting into equation (7) gives

$$\phi + \Lambda P = 0 \tag{9}$$

as the conditions which must exist at signal reversal.

Since the maximum value of the rolling velocity can be defined, the maximum value of  $\eta$  can be represented as

$$\eta_{\max} = \Lambda |P_{\max}|$$

The following notation is convenient for use herein:

$$\eta = -C\eta_{\max} \tag{10}$$

where  $C$  is defined as a fraction of the maximum rate of roll and has the sign of the rolling velocity at the instant considered.

These conditions at signal reversal and the basic equations of motion can be used to describe the angle-of-bank variation during a cycle of the roll oscillations.

#### First Half Cycle of Roll Oscillation

A general description of a rolling oscillation is begun by considering the aircraft banked to the left with the constant right rolling moment rolling the aircraft to the right. (See fig. 6.) The condition of signal reversal is also assumed ( $\phi_0 = \eta_0$ ) and is given by  $\Delta t = 0$ ,  $\phi(0) = \phi_0$ , and  $C = C_0$ , the numerical subscript referring to point 0, figure 6. From equation (10).

$$\eta_0 = -C_0 \eta_{\max} \quad (11)$$

From equations (4), (5), and (10) the angle of bank, rolling velocity, and differentiator output variations during the interval between points 1 and 2 (fig. 6) are written as follows:

$$\phi_{01} = \frac{LI_x}{L_p^2} \left[ (1 - C_0)e^{-a \Delta t} + a \Delta t - (1 - C_0) \right] + \phi_0 \quad (12)$$

$$P_{01} = -\frac{L}{L_p^2} \left[ 1 - (1 - C_0)e^{-a \Delta t} \right] \quad (13)$$

$$\eta_{01} = -\left[ 1 - (1 - C_0)e^{-a \Delta t} \right] \eta_{\max} \quad (14)$$

Since conditions of signal reversal were assumed at point 0, the actual reversal of the control moment occurs after a lag period of  $\tau$  seconds, or at point 1 (fig. 6). Substituting  $\Delta t = \tau$  into equations (12), (13), and (14), and defining the nondimensional lag factor  $K \equiv a\tau$ , the conditions at control reversal are found to be

$$\phi_1 = \frac{LI_x}{L_p^2} \left[ (1 - C_0)e^{-K} + K - (1 - C_0) \right] + \phi_0 \quad (15)$$

$$P_1 = C_1 P_{\max} \quad (16)$$

$$\eta_1 = -C_1 \eta_{\max} \quad (17)$$

where  $C_1$  is positive since the roll is to be right and is given by

$$C_1 = \left[ 1 - (1 - C_0)e^{-K} \right] \quad (18)$$

The motion following the reversal of the control moment, expressed as  $\phi_{12}$ , is found by the superposition of the incremental changes in bank induced by the reversal of the control moment. The control moment considered herein is an aerodynamic control moment expressed by

$$L = \delta L_{\delta}$$

where  $\delta$  is the combined maximum differential aileron deflection (measured from zero deflection) used as a flicker control and  $L_{\delta}$  is the control-effectiveness parameter. Since the flicker-type control causes a change from maximum control deflection in one direction to maximum in the other, the incremental changes in bank induced by control reversal are found by considering the control moment as twice that given by the previous equation. Time increments in this case are to be from control reversal, point 1, with the initial conditions at this point considered.

$$\begin{aligned} \phi_{12} = B & \left[ (1 - C_1)e^{-a \Delta t} + a \Delta t - (1 - C_1) \right] + \phi_1 \\ & - 2B(e^{-a \Delta t} + a \Delta t - 1) \end{aligned} \quad (19)$$

where  $\phi_1$  is given by equation (15) and  $B$  is termed the amplitude factor and is defined as

$$B \equiv \frac{L I_x}{L_p^2}$$

Substituting equations (18) for  $C_1$  and (15) for  $\phi_1$ , the angle-of-bank variation following control reversal takes the form

$$\phi_{12} = \phi_0 + B \left\{ K + (1 - C_0)(e^{-K} - 1) - a \Delta t + (e^{-a \Delta t} - 1) [(1 - C_0)e^{-K} - 2] \right\} \quad (20)$$

It is now assumed that point 2 (fig. 6) represents the time of the next signal reversal. This condition will be satisfied when

$$\phi_2 = \eta_2 \quad (21)$$

Differentiating (20) and substituting  $t_{12}$  for the time increment in reaching point 2, the fraction  $C_2$  of the maximum rolling velocity existing there is found to be

$$C_2 = 2e^{-at_{12}} - (1 - C_0)e^{-(K+at_{12})} - 1 \quad (22)$$

The following angles are also required:

$$\phi_2 = \phi_0 + B \left\{ K + (1 - C_0)(e^{-K} - 1) - at_{12} + (e^{-at_{12}} - 1) [(1 - C_0)e^{-K} - 2] \right\} \quad (23)$$

$$\eta_2 = -C_2 \eta_{\max} \quad (24)$$

It is evident at this point that the remainder of the cycle could be defined in a similar way; however, this is not required in the evaluation of steady-state conditions.

#### Conditions at Steady State

At steady state, with no out-of-trim moments producing roll, the oscillations are symmetrical about the zero angle of bank reference position. The

angles of bank and rolling velocities existing at signal reversals must therefore be the negatives of one another. The following conditions therefore exist at steady state:

$$C_2 = -C_0$$

$$\phi_2 = -\phi_0$$

$$\eta_2 = -\eta_0$$

From equation (22) it is possible to solve for the time to reach  $C_2$ .

$$t_{12} = -\frac{1}{a} \ln \left[ \frac{1 - C_0}{2 - (1 - C_0)e^{-K}} \right] \quad (25)$$

Substituting equation (25) into (23) gives

$$\phi_2 - \phi_0 = B \left\{ K + 2C_0 + \ln \left[ \frac{1 - C_0}{2 - (1 - C_0)e^{-K}} \right] \right\} \quad (26)$$

The left-hand member of equation (26) is equal to  $-2\phi_0$  because of the steady-state conditions and may be written as

$$\begin{aligned} -2\phi_0 &= -2\eta_0 \\ &= -2(-AC_0 R_{max}) \\ &= 2AC_0 \left| \frac{L}{L_p} \right| \end{aligned}$$

Substituting this into equation (26) and using the expression for the amplitude factor B gives

$$2C_o \Lambda \left| \frac{L}{L_p} \right| = \left| \frac{L}{L_p} \right| \left| \frac{1}{\frac{L_p}{I_x}} \right| \left\{ K + 2C_o + \ln \left[ \frac{1 - C_o}{2 - (1 - C_o)e^{-K}} \right] \right\}$$

or

$$2C_o R = K + 2C_o + \ln \left[ \frac{1 - C_o}{2 - (1 - C_o)e^{-K}} \right] \quad (27)$$

where R is termed the nondimensional rate factor and is defined by

$$R \equiv \left| \frac{L_p}{I_x} \right| \Lambda \equiv a\Lambda \quad (28)$$

Equation (27) is called the conditional equation for a half cycle. It is the relationship that must exist between the factors at steady state. Since K and R are defined by aircraft and system parameters, the  $C_o$  existing at steady state is determined.

#### Steady-State Amplitude Equation

The amplitude of the oscillation is defined as one-half the total amplitude displacement. Since the oscillations are symmetrical about the reference axis, it is only necessary to find the maximum angle of bank. The time to reach zero rate of roll was found from the differentiation of equation (20). This time was substituted into equation (20), and the amplitude was found to be given by

$$A = B \left\{ K + C_o(1 - R) + \ln \left[ \frac{1}{2 - (1 - C_o)e^{-K}} \right] \right\} \quad (29)$$

At steady state the bracketed term is defined for a given K and R, and the amplitude may be presented as A/B as a function of K and R.

## Steady-State Period Equation

The period of the oscillation is defined as the time required to complete one cycle. It is evident then that the period  $P$  is

$$P = 2(\tau + t_{12})$$

Using equation (25) for  $t_{12}$ , the period is written finally as

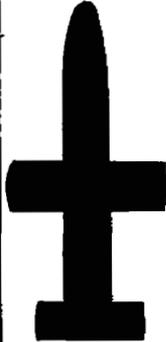
$$P = 2\tau \left\{ 1 - \frac{1}{K} \ln \left[ \frac{1 - C_0}{2 - (1 - C_0)e^{-K}} \right] \right\} \quad (30)$$

Although the nondimensional rate factor  $R$  is not explicit in equation (30), it is necessarily implied since, at steady state,  $C_0$  is related to both  $K$  and  $R$ .

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2. Pitkin, Marvin, Gardner, William N., and Curfman, Howard J., Jr.: Results of Preliminary Flight Investigation of Aerodynamic Characteristics of the NACA Two-Stage Supersonic Research Model RM-1 Stabilized in Roll at Transonic and Supersonic Velocities. NACA RM No. L6J23, 1946.
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4. Minorsky, N.: Introduction to Non-Linear Mechanics. Part I - Topological Methods of Non-Linear Mechanics. Rep. 534, David Taylor Model Basin, Navy Dept., Dec. 1944.
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TABLE I

					
Aircraft	1	2	3	4	
Weight, lb	600	150	500	1200	
Diameter, in.	10	8	20	17	
Length/Diameter	23	15	6	12	
Aspect Ratio	1.5	2.5	4.6	4.0	
Mach Number	1.7	1.5	1.5	0.85	
L, ft-lb	218	347	930	270	
L <sub>p</sub> , ft-lb/rad/sec	-50.5	-6.58	-420	-17.5	
I <sub>x</sub> , slug-ft <sup>2</sup>	2.1	0.3	7.8	14.8	
τ, sec	0.025	0.025	0.025	0.050	0.025
$K = \tau \left  \frac{L_p}{I_x} \right $	0.60	0.55	1.35	2.70	0.029
Λ, rad/rad/sec	0.122	0.122	0.122	0.122	0.122
$R = \Lambda \left  \frac{L_p}{I_x} \right $	2.93	2.67	6.59	6.59	0.144
Amplitude, deg	1.01	15.6	1.01	2.63	1.3
Period, sec	0.085	0.086	0.074	0.072	0.188
Amplitude, R = 0, deg	7.75	95	3.71	7.0	32.5
Period, R = 0, sec	0.232	0.24	0.168	0.27	0.75

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TABLE II

Roll Simulator Tests

Test	1	2
$a =  L_p/I_x $	11.06	10.93
$ L/I_x $	78.0	77.0
$B =  L/I_x  / a^2$	0.639	0.645
$\tau$ , sec	0.0312	0.0282
$\Lambda$ , rad/rad/sec	0.122	0.122
$K = a\tau$	0.345	0.308
$R = a\Lambda$	1.35	1.33
Amplitude, deg (measured)	2.10	1.91
Period, sec (measured)	0.186	0.189
Amplitude, deg (calculated)	2.11	1.80
Period, sec (calculated)	0.118	0.109

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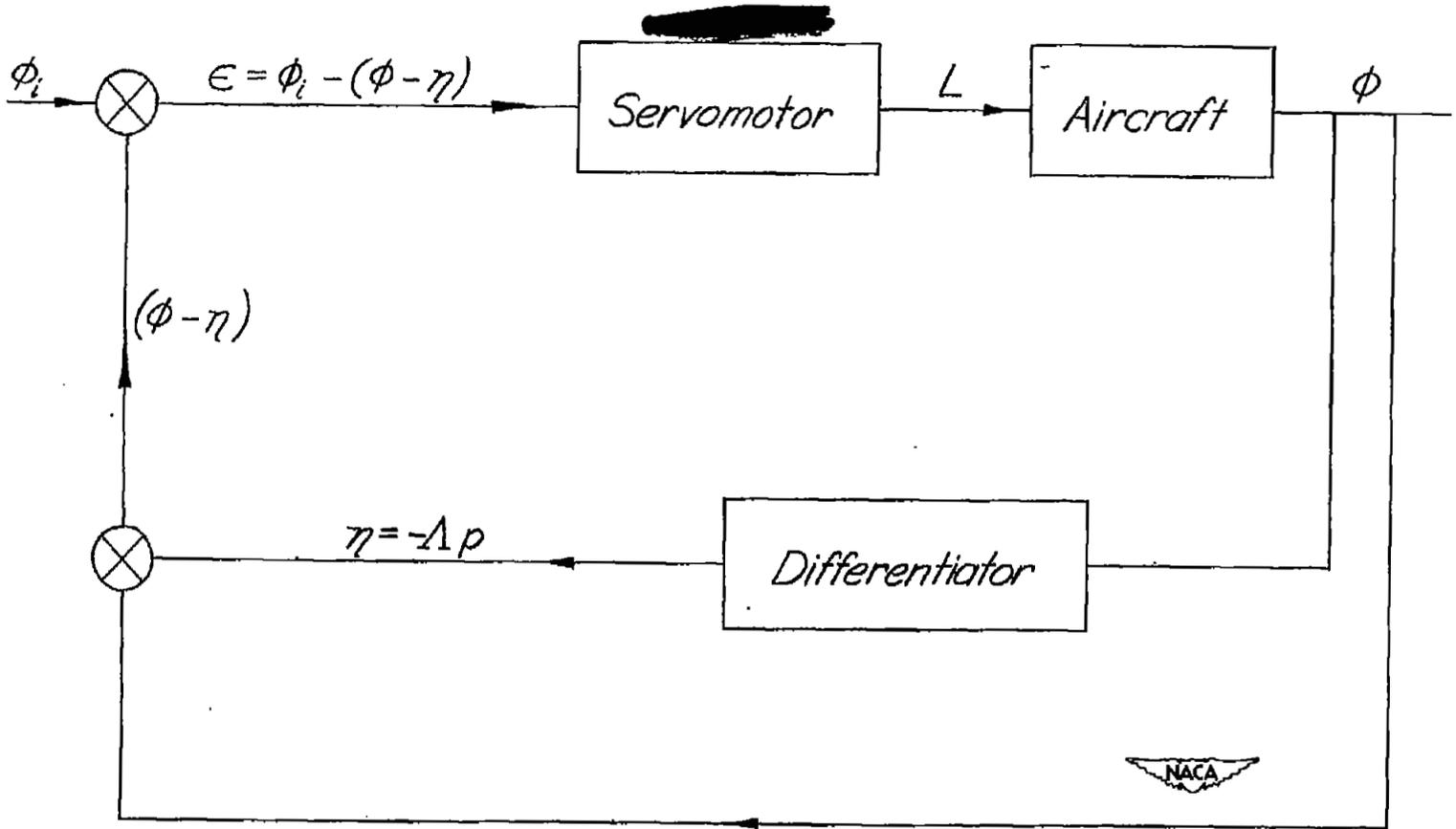
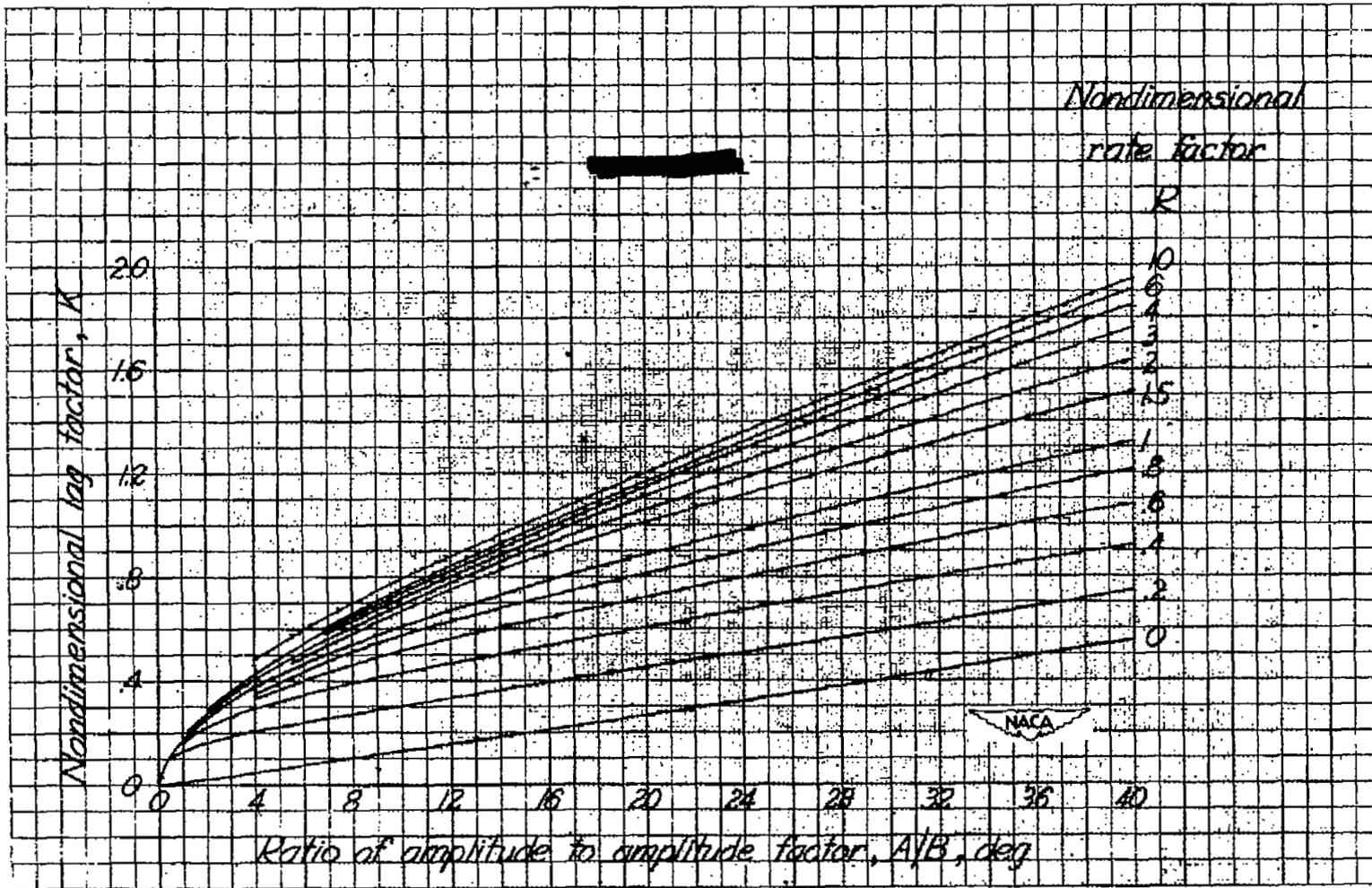
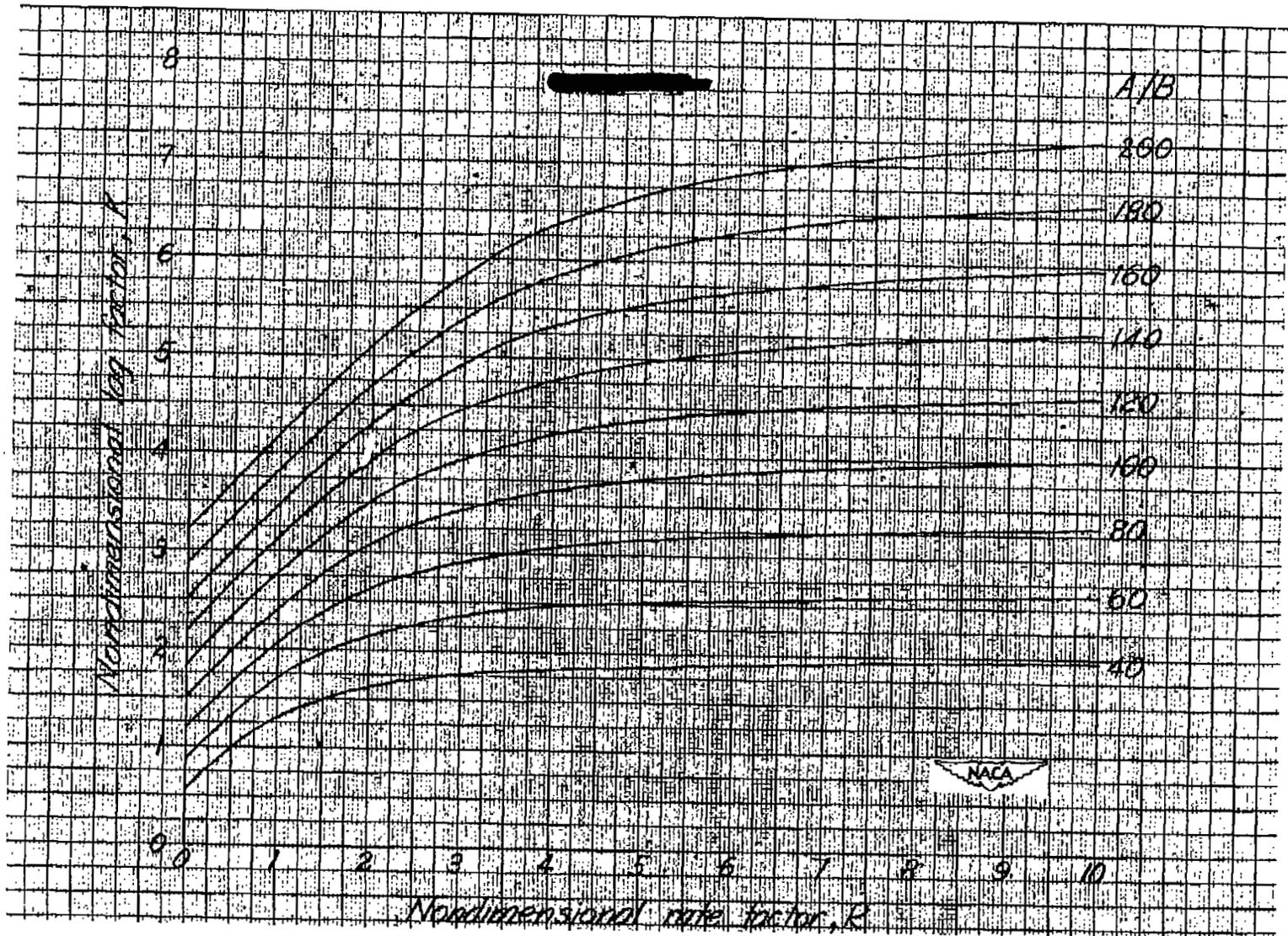


Figure 1.— Block diagram of the displacement-plus-rate response, flicker-type automatic roll stabilization system.



(a) For small values of  $K$ .

Figure 2.- Chart for determining the steady-state amplitude.



(b) For large values of  $K$ .

Figure 2.- Concluded.

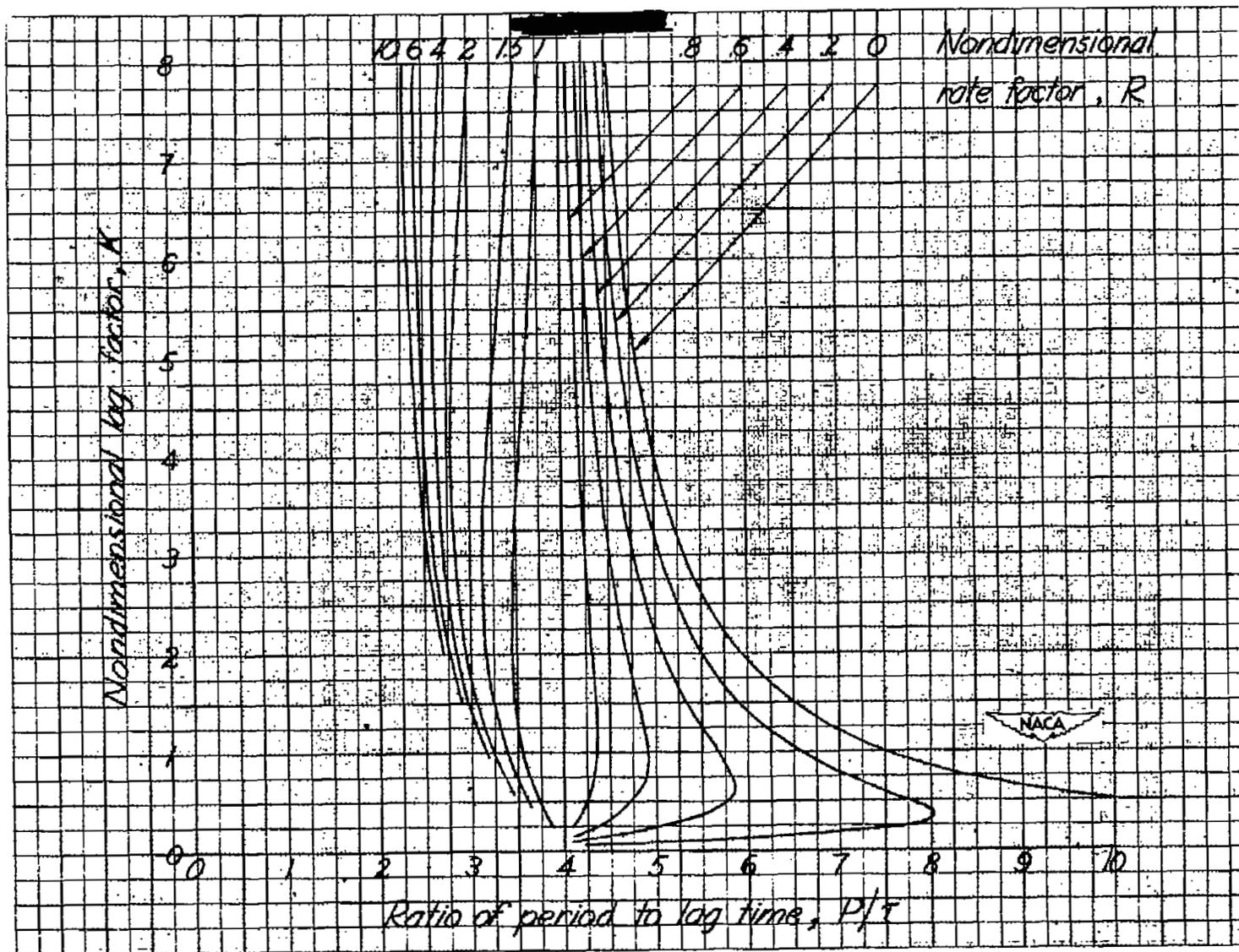


Figure 3.-- Chart for determining the steady-state period.

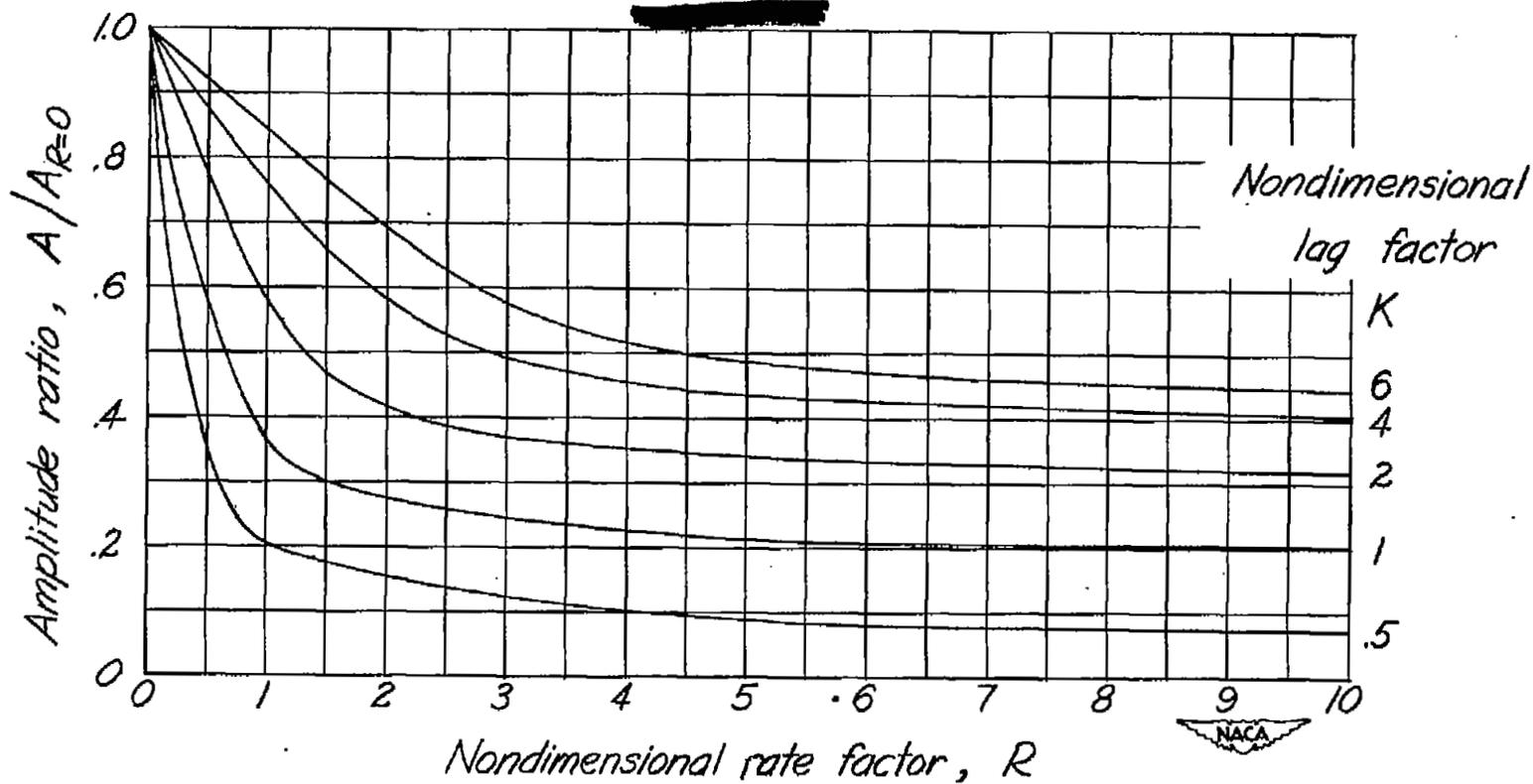


Figure 4.— Curves showing the effect of the rate component on the steady-state amplitude.

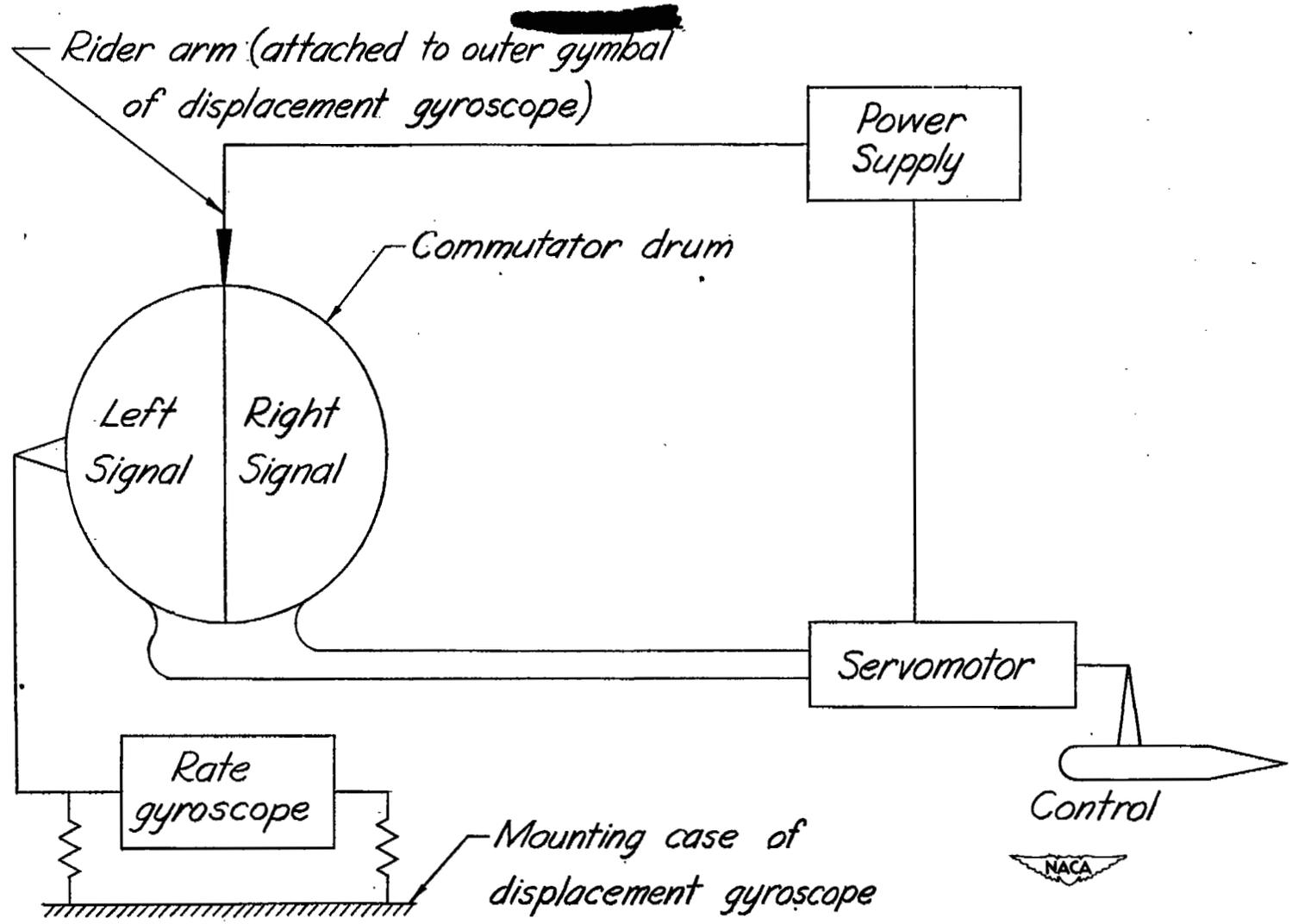


Figure 5.- Schematic diagram of the displacement-plus-rate response, flicker-type system used in the roll simulator tests.

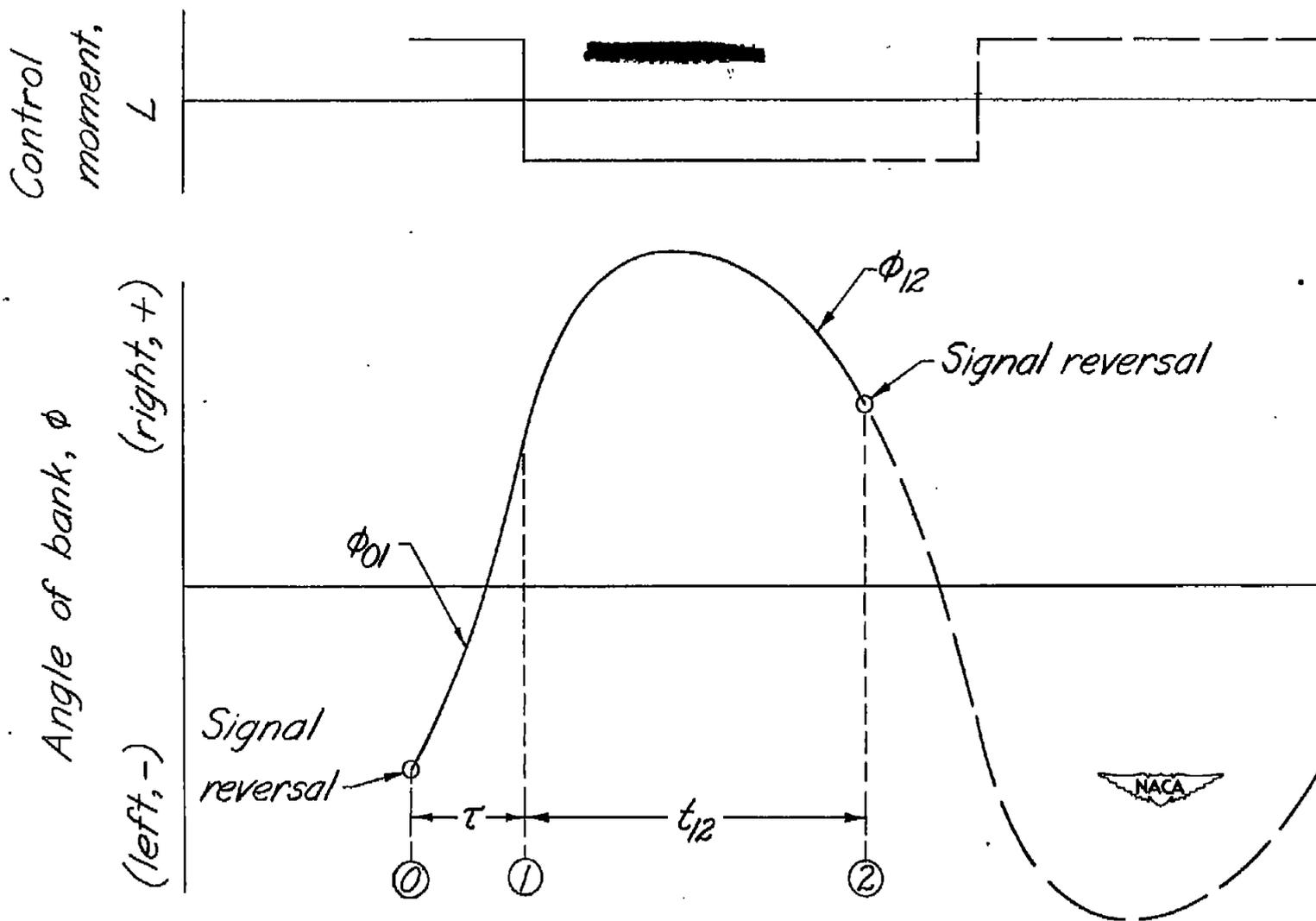


Figure 6.- Mathematical nomenclature for one-half cycle of roll.

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