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# RESEARCH MEMORANDUM

CALCULATED LIFT DISTRIBUTIONS OF A CONSOLIDATED VULTEE

B-36 AND TWO BOEING B-47 AIRPLANES

COUPLED AT THE WING TIPS

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## NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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SUMMARY

Calculated lift distributions and associated aerodynamic parameters are presented for a Consolidated Vultee B-36 and two Boeing B-47 airplanes coupled at the wing tips for different angles of attack relative to each other and with different deflections of the ailerons of the B-47 airplane. For the purpose of calculation, the lift was assumed to be carried by a series of horseshoe vortices centered on the quarter-chord line, and the induced flow angle was equated to the angle of attack at the three-quarter-chord line.

INTRODUCTION

Configurations consisting of several airplanes flying wing tip to wing tip are sometimes considered for the purpose of improving range or economy. In this paper calculated aerodynamic loadings and some related aerodynamic parameters are presented for such a configuration of recent interest, namely a Consolidated Vultee B-36 and two Boeing B-47 airplanes coupled at the wing tips. (See fig. 1.)

For the purpose of calculation, the lift was assumed to be carried by rectangular horseshoe vortices centered at the quarter-chord line. The strengths of these vortices, and, hence, the local lift on the wing were determined by the condition that the induced flow angle at the three-quarter-chord line be equal to the angle of attack at that line at several points along the span.

Lift-distribution curves and equations for the lift, induced drag, bending moment, and rolling moment are presented for the airplanes flying alone and in combination. The aerodynamic parameters for the combination

are expressed as increments to be added to the known data for the uncoupled airplanes in order to minimize errors inherent in the analysis of this paper.

The manner in which this information may be used in a performance analysis is described briefly, but no attempt has been made to make such an analysis.

#### SYMBOLS

A	aspect ratio
$\alpha$	angle of attack measured from zero (section) lift attitude of the tip of the wing of the B-36 airplane, degrees
$\alpha_{\delta}$	effective angle of attack due to aileron deflection, degrees
b	wing span, inches
$M_b$	aerodynamic bending moment (about axes parallel to the plane of symmetry), inch-pounds
c	wing chord, inches
$\bar{c}$	average wing chord, inches
$c_r$	root chord, inches
$c_t$	tip chord, inches
$c_{MAC}$	mean aerodynamic chord, inches
$D_i$	induced drag, pounds
$\delta$	aileron deflection of B-47 airplane (positive downward for outboard aileron when flying in combination and for right aileron when flying alone), degrees
$\epsilon$	angle of attack of B-47 wing tip relative to B-36 wing tip, degrees
F	concentrated normal force at the juncture (positive upward on the B-36 airplane), pounds

g	acceleration due to gravity, inches per second <sup>2</sup>
l	section lift per unit span, pounds per inch
$\lambda$	wing taper ratio (Tip chord/Root chord)
A	angle of sweep at the quarter-chord line
S	wing area, square feet
$C_L$	wing lift coefficient
$c_l$	section lift coefficient
$C_{D_i}$	induced drag coefficient
$I_z$	mass moment of inertia of B-47 airplane about its center line, slug-inch <sup>2</sup>
L	lift, pounds
M	pitching moment about intersection of quarter-chord line and plane of symmetry, inch-pounds
n	load factor
q	dynamic pressure, pounds per square foot
$L'$	aerodynamic rolling moment (positive if it tends to turn the right wing up on the right B-47 airplane), inch-pounds
W	gross weight of airplane, pounds
y	lateral distance from the center line, inches
$y^*$	dimensionless lateral distance from the center line $\left(\frac{y}{b/2}\right)$

Subscripts:

1	pertaining to the B-36 airplane
2	pertaining to the B-47 airplane
c	coupled condition
T	pertaining to the combination of one B-36 and two B-47 airplanes

rw right wing  
lw left wing  
A, B, C, D corresponding to the angle-of-attack distributions shown in figure 1

## ANALYSIS

### Method of Calculations

The concept of representing the lifting action of a wing by a continuous concentrated vortex at the quarter-chord line and equating the induced flow angle at the three-quarter-chord line to the angle of attack at that line has been found (in reference 1, for instance) to give reliable results for the spanwise lift distribution and associated aerodynamic parameters for a wide range of plan forms. Use of a pattern of discrete horseshoe vortices instead of a continuous vortex has been found to give equally good results (reference 2). However, the commonly used methods based on these two approximations to a true lifting surface employ mathematical techniques which are inapplicable to the lift distributions associated with plan forms of the type under consideration. Nor do these methods take into account a number of points along the span sufficient to describe such plan forms. For the calculations of the present paper a horseshoe-vortex pattern similar to that of reference 2 was used (see fig. 2), without resorting to the mathematical techniques of reference 2, however.

The downwash velocity due to each of the 21 horseshoe vortices was calculated at each of the 10 downwash points shown in figure 2. For symmetrical lift distributions each vortex on one semispan (of the combined configuration) has the same strength as the vortex at the same station on the other semispan, so that only 11 of the vortex strengths need be calculated. By summing the downwash velocities due to each of the 21 vortices at each of the 10 downwash points and equating the sum to the angles of attack at those 10 points, a set of 10 simultaneous equations for the 11 unknown vortex strengths may be obtained.

In order to solve this set of equations the strength of each corrector vortex (see fig. 2) was assumed to be given in terms of that of the two adjacent vortices by an extrapolation formula based on the assumption that the lift distribution near the tip may be approximated by a series of the type  $Ax^{1/2} + Bx^{3/2}$ ,  $x$  being the distance inboard of the tip. This extrapolation formula results in a strength of the corrector vortex which is equal to 0.995 of that of the vortex adjacent to it

minus 0.271 of that of the vortex once removed from it. Substitution of this relation into the 10 simultaneous equations serves to eliminate the unknown strength of the corrector vortex. The equations can then be solved for the unknown strengths of the other 10 vortices.

As a result of the fairly large lateral spacing of these vortices, the lift distributions calculated in the foregoing manner tend to be smoother than the actual distribution is likely to be. To avoid this effect an impractically large number of vortices would have to be used. However, the discontinuity in chord at the juncture has only a small and largely localized effect on the lift distribution; furthermore, it is possible that the juncture will be faired in an actual installation. The discontinuities in angle of attack due to aileron deflection have been taken into account in the following manner. Lift distributions were calculated for full-span ailerons and for ailerons of 60 percent span; the resulting curves were faired with the aid of knowledge of lift distributions of uncoupled airplanes due to aileron deflections. A lift distribution corresponding to the actual aileron (about 48 percent span) was then extrapolated from the other two distributions. The lift distribution obtained in this manner is likely to be less reliable than the lift distributions due to the other angle-of-attack conditions, but the results may be useful for aileron deflections which are small compared with the other angles of attack.

The lift distributions were integrated to obtain total lifts, bending moments, and rolling moments. The induced drags were obtained, on the basis of Munk's stagger theorem, by calculating the downwash induced on an unswept wing by the given lift distributions and integrating the product of the local lift and induced downwash angle (in radians) over the span.

In representing the lifting action of the wings by a vortex pattern, the assumptions are made that the flow is incompressible and inviscid and that the airfoil sections are thin. The net effect of viscosity (that is, of the boundary layer) and of airfoil thickness is to decrease the lift compared with that calculated. The effect of compressibility is to increase the lift. At the Mach numbers at which the airplane configuration under consideration is likely to cruise, these effects tend to cancel at low angles of attack, so that no allowance has been made for either. At high angles of attack, however, local regions on the wings tend to stall, particularly near the wing tips of the B-47 airplanes and at the wing juncture, which tends to act somewhat like the root of a sweptforward wing and to stall at relatively low angles of attack. When local regions are stalled the lift distributions cannot be predicted by the theoretical method used in this paper.

### Scope of Calculations

The configuration under consideration is shown in figure 1(a) and some of its geometric parameters are presented in table 1. The four fundamental angle-of-attack conditions for which spanwise lift distributions have been calculated are shown in figure 1(b). Case A represents a uniform angle of attack on both airplanes, case B an angular displacement of the B-47 airplanes relative to the B-36 airplane, case C the washout (linear twist distribution) of the B-36 airplane, and case D a uniform effective angle of attack due to aileron deflection on the B-47 airplanes.

### RESULTS OF THE CALCULATIONS

All angles of attack that occur in this section are in degrees. In the case of the B-47 airplane there is no ambiguity in referring to the wing angle of attack, since the wing of the B-47 airplane considered in the present calculations has neither twist nor camber. The wing of the B-36 airplane, however, has both camber and twist. (The sum of the aerodynamic and structural twist was considered to be equivalent to 2° of washout in the present calculations.) For definiteness the angle of attack of the B-36 airplane referred to in this section is that of the B-36 wing tip measured from the angle at which the wing-tip section would have zero lift in two-dimensional flow.

The effective angle of attack due to aileron deflection  $\alpha_\delta$  can be obtained for any small aileron deflection  $\delta$  by multiplying  $\delta$  (in degrees) by the two-dimensional value of  $\alpha_\delta \equiv \frac{\partial C_l}{\partial \delta} \frac{\partial C_l}{\partial \alpha}$  for the airfoil-aileron combination of the B-47 airplane.

### Lift Distributions

The lift distributions corresponding to the angle-of-attack distributions shown in figure 1 are shown in figure 3. They are plotted in the dimensional form  $(cc_l)$  rather than in the frequently used nondimensional form  $(cc_l/\bar{c})$  in order to avoid confusion between the average chords of two airplanes. The local lift per inch of span corresponding to an arbitrary angle-of-attack distribution may then be found by superposition for any point along the span of either airplane as:

$$l = \frac{q}{8251} \left[ \alpha (cc_l)_A + \epsilon (cc_l)_B + 2(cc_l)_C + \alpha_\delta \delta (cc_l)_D \right] \quad (1)$$

The lift distributions for several angles of attack are also presented in dimensionless form in figure 4; the corresponding lift distributions for the two airplanes flying alone are shown for comparison. It appears that as a result of flying the airplanes in combination the lift on the B-36 airplane is increased about 10 percent and that on the B-47 airplane, about 20 percent. As would be expected, the largest increase is incurred near the juncture of the two airplanes.

#### Lifts, Moments, and Induced Drags

Curves of the lift and induced drag coefficients for the airplane combination are presented in figure 5. The coefficients have been made dimensionless with the combined wing area. These curves are for zero aileron deflection. The total lifts and induced drags for the entire combination due to all angle-of-attack conditions considered are given by the expressions:

$$L_{T_c} = q \left[ 713\alpha + 280\epsilon - 18(\alpha\delta) + 530 \right] \quad (2)$$

$$D_{i_{T_c}} = q \left[ 0.97\alpha^2 + 0.58\alpha\epsilon + 0.17\alpha(\alpha\delta) + 1.98\alpha + 0.56\epsilon^2 - 0.02\epsilon(\alpha\delta) - 0.54\epsilon + 0.42(\alpha\delta)^2 + 0.17\alpha\delta + 2.08 \right] \quad (3)$$

The lifts and induced drags for the two airplanes flying alone as calculated in a manner similar to that employed for the combination are given in figure 6, the coefficients being made dimensionless with the individual wings areas. Expressions for the lift and induced drag of the individual airplanes are:

$$\left. \begin{aligned} L_1 &= q(427\alpha + 507) \\ L_2 &= q[104(\alpha + \epsilon)] \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} D_{i_1} &= q(1.12\alpha^2 + 2.86\alpha + 2.35) \\ D_{i_2} &= q \left[ 0.26(\alpha + \epsilon)^2 + 0.16(\alpha\delta)^2 \right] \end{aligned} \right\} \quad (5)$$

where, for the sake of consistency, the symbol  $(\alpha + \epsilon)$  is used to designate the angle of attack of the B-47 airplane.

The differences

$$\Delta L_T = L_T - L_1 - 2L_2 \quad (6)$$

$$\Delta D_{i_T} = D_{i_1} + 2D_{i_2} - D_{i_T} \quad (7)$$

represent (for any angle-of-attack condition) the increase in lift and the saving in induced drag obtained by coupling the airplanes. If the presence of one airplane does not affect the profile and parasite drag of the others, and if the leakage or other parasite drag at the juncture is negligible compared with the total drag, then the difference in induced drag is the difference in total drag (for the same angle-of-attack condition, coupled as against uncoupled). The differences defined by equations (6) and (7) may be expressed as:

$$\Delta L_T = q(78\alpha + 72\epsilon - 18\alpha_\delta\delta + 23) \quad (8)$$

$$\Delta D_{i_T} = q \left[ 0.67\alpha^2 + 0.47\alpha\epsilon - 0.17\alpha(\alpha_\delta\delta) + 0.88\alpha - 0.04\epsilon^2 + 0.02\epsilon(\alpha_\delta\delta) + 0.54\epsilon - 0.10(\alpha_\delta\delta)^2 - 0.17\alpha_\delta\delta + 0.27 \right] \quad (9)$$

In order to facilitate the calculation of the forces required to balance the B-47 airplanes in roll, expressions for the rolling moments and lift on the individual airplanes in the presence of each other are required:

$$\left. \begin{aligned} L_{1c} &= q \left[ 458\alpha + 39\epsilon - 15(\alpha_\delta\delta) + 515 \right] \\ L_{2c} &= q \left[ 128\alpha + 120\epsilon - 2(\alpha_\delta\delta) + 8 \right] \end{aligned} \right\} \quad (10)$$

$$L'_{2c} = q \left[ -4,700\alpha - 2,800\epsilon + 21,500(\alpha_\delta\delta) - 2,000 \right] \quad (11)$$

The rolling moment is that for the B-47 airplane on the right of the B-36 airplane.

The bending moments of the aerodynamic wing loads about the center lines of the individual airplanes may be of interest for structural calculations and have a bearing on the calculations of the static longitudinal stability of the configuration. They are

$$(M_b)_{1_c} = q \left[ 137,700\alpha + 18,900\varepsilon - 7,700(\alpha_\delta\delta) + 118,500 \right] \quad (12)$$

$$(M_b)_{2_{c_{rw}}} = q \left[ 18,400\alpha + 17,900\varepsilon + 10,200(\alpha_\delta\delta) + 600 \right] \quad (13)$$

$$(M_b)_{2_{c_{lw}}} = q \left[ 23,100\alpha + 20,700\varepsilon - 11,300(\alpha_\delta\delta) + 2,600 \right] \quad (14)$$

The rolling moment given in equation (11) is the net of the bending moments given in equations (13) and (14). The calculated bending moments and the rolling moment of the B-47 airplane for the airplanes flying alone are:

$$(M_b)_1 = q(119,900\alpha + 112,100) \quad (15)$$

$$\left. \begin{aligned} (M_b)_{2_{rw}} &= q \left[ 16,000(\alpha + \varepsilon) + 8,600\alpha_\delta\delta \right] \\ (M_b)_{2_{lw}} &= q \left[ 16,000(\alpha + \varepsilon) - 8,600\alpha_\delta\delta \right] \end{aligned} \right\} \quad (16)$$

$$(L')_2 = q(17,200\alpha_\delta\delta) \quad (17)$$

The differences between equations (10) to (14) and equations (4) and (15) to (17) may be regarded as increments due to coupling, which may be written as:

$$(\Delta L)_1 = q(31\alpha + 39\varepsilon - 15\alpha_\delta\delta + 8) \quad (18)$$

$$(\Delta L)_2 = q(24\alpha + 16\varepsilon - 2\alpha_\delta\delta + 8) \quad (19)$$

$$(\Delta M_b)_1 = q(17,800\alpha + 18,900\varepsilon - 7,700\alpha_\delta\delta + 6,400) \quad (20)$$

$$(\Delta M_b)_{2_{rw}} = q(2400\alpha + 1900\varepsilon + 1600\alpha_\delta\delta + 600) \quad (21)$$

$$(\Delta M_b)_{2_{lw}} = q(7100\alpha + 4700\varepsilon - 2700\alpha_\delta\delta + 2600) \quad (22)$$

$$(\Delta L')_2 = q(4700\alpha - 2800\varepsilon + 4300\alpha_\delta\delta - 2000) \quad (23)$$

The wing aerodynamic center is the center of pressure of the aerodynamic loading due to a uniform angle of attack. If the chordwise center of pressure of the angle-of-attack loading at each station is assumed to be at the quarter-chord point, the aerodynamic center of the B-36 airplane when coupled is 2 percent of the mean aerodynamic chord of that airplane rearward of its position when uncoupled. Similarly, the aerodynamic center of the B-47 airplane is 5 percent of the mean aerodynamic chord of the B-47 airplane rearward of its position when uncoupled. However, this information cannot be used directly for estimating the stability of the individual airplanes in the configuration or the configuration as a whole, since the concentrated normal forces at the junctures exert pitching moments on the airplanes.

#### APPLICATION OF THE RESULTS

The results presented in the preceding section may be used in calculation of the performance and stability of the airplane combination. Since the manner in which they are used is, perhaps, not obvious, some considerations which are basic to such calculations are presented in the following sections.

#### Calculation of the Aerodynamic Forces

In order to calculate the lifts and moments required for static equilibrium as well as the drag incurred at any attitude, the lifts and drags of both airplanes (including fuselages, empennage, and external stores) must be known as functions of the individual angles of attack. The rolling moment of the B-47 airplane due to aileron deflection must also be known as a function of aileron deflection. This information may

be written in the form of equations similar to equations (4), (5), (15), and (17), for instance. (The constants in the equations which pertain to the entire airplanes differ from the constants in equations (4), (5), (15), (17), and so on, because one set of equations applies to the theoretical values of the forces on the airplanes due to the wings alone, whereas the other set of equations applies to the actual forces due to the wings, fuselage, and tail surfaces.) The increments given by equations (8), (9), (18), and so on, are then added to these equations with the use of the defining equations (6) and (7) in the case of equations (8) and (9), so that

$$L_{T_c} = L_1 + 2L_2 + \Delta L \quad (24)$$

$$D_{T_c} = D_1 + 2D_2 - \Delta D_i + 2D_p \quad (25)$$

where  $D_p$  is an estimated value of the leakage or other parasite drag (if any) at each juncture, expressed in terms of the angles  $\alpha$  and  $\epsilon$ , if possible.

#### Equilibrium Conditions

The total normal forces, pitching moments, and rolling moments on the individual airplanes and on the combination must be zero for steady flight. The condition of zero rolling moment is automatically satisfied in symmetrical flight of an uncoupled airplane. In the presence of the B-36, however, the left wing of the B-47 airplane tends to carry more lift than the right wing, so that there is a tendency for a net rolling moment to act on the B-47 airplane. In addition to this aerodynamic rolling moment a concentrated normal force exists at the juncture, in general, which also exerts a rolling moment on the B-47 airplane. There are two ways in which the net rolling moment may be reduced to zero; either the relative angle of attack of the airplanes  $\epsilon$  is adjusted to a suitable value or the ailerons of the B-47 airplane are displaced a certain amount. In general, both of these methods are likely to be used simultaneously.

From a mathematical point of view, at any given condition of airplane weights, centers of gravity, and dynamic pressure there are four unknown quantities ( $\alpha$ ,  $\epsilon$ ,  $\alpha_0\delta$ , and the elevator deflection) which must be determined. If the lift and drag due to elevator deflection are negligible compared with the total airplane lift and drag, the condition of zero pitching moment serves to determine the elevator deflection and need not be considered further. (If there are appreciable changes in lift and drag due to elevator deflection, the elevator deflection must be considered a fourth unknown variable and the condition of zero pitching

moment considered simultaneously with the other conditions in a manner similar to that discussed in the following paragraphs.) There are then two conditions of equilibrium (the total lift must equal the total weight, and the net rolling moment on the B-47 must be zero) for the three unknown quantities  $\alpha$ ,  $\epsilon$ , and  $\alpha_\delta\delta$ . A third condition is required to solve for the values of the three parameters. The nature of the third condition is discussed in a subsequent section.

The conditions that in steady flight the total lift must equal the combined gross weight of the three airplanes and that the rolling moment on the B-47 airplane must be zero may be written as

$$L_{T_c} = W_1 + 2W_2 \quad (26)$$

$$(L')_{2_c} = -F \frac{b_2}{2} \quad (27)$$

where  $F$  is the concentrated normal force at the juncture, positive if upward on the B-36 airplane, and  $(L')_{2_c}$ , the aerodynamic rolling moment. In steady flight the force  $F$  may be expressed as

$$F = L_{2_c} - W_2 \quad (28)$$

which may be substituted in equation (27) to yield

$$(L')_{2_c} = -\frac{b_2}{2}(L_{2_c} - W_2) \quad (29)$$

Equations (26) and (29) are linear in the three parameters  $\alpha$ ,  $\epsilon$ , and  $\alpha_\delta\delta$  and, consequently, may be solved for  $\epsilon$  and  $\alpha_\delta\delta$  in terms of  $\alpha$  in the form

$$\left. \begin{aligned} \epsilon &= K_1 + K_2\alpha \\ \alpha_\delta\delta &= K_3 + K_4\alpha \end{aligned} \right\} \quad (30)$$

where the constants  $K_1$ ,  $K_2$ ,  $K_3$ , and  $K_4$  depend on  $q$ ,  $W_1$ , and  $W_2$ .

### Drag of the Airplane Combination

The value of the angle of attack may be determined from several conditions. The condition of primary interest is that of minimum drag. The value of the angle of attack which minimizes the drag may be obtained by substituting the expressions for  $\epsilon$  and  $\alpha\delta$  from equation (30) into the expression for the total drag of the combination. The resulting expression has the form

$$D_{Tc} = K_5\alpha^2 + K_6\alpha + K_7$$

and is a minimum if

$$\alpha = -\frac{K_6}{2K_5}$$

In order to calculate the tip force incurred in this condition, for structural reasons the values of  $\epsilon$  and  $\alpha\delta$  may be calculated from equations (30) and, together with  $\alpha$ , substituted in the expression for  $L_2$ . The value of the force  $F$  is then obtained from equation (28).

Another condition of interest may be that of zero tip force. The value of  $\alpha$  for this condition may be obtained by substituting the expressions for  $\epsilon$  and  $\alpha\delta$  into the expression for  $L_2$  and equating that expression to  $W_2$ . The resulting expression contains only  $\alpha$  and known constants. The drag can then be calculated from the general expression for drag by substituting the value of  $\alpha$  obtained from this expression and the corresponding values of  $\epsilon$  and  $\alpha\delta$  obtained from equation (30) into that general expression.

A third condition of possible interest is that of zero aileron deflection. The value of  $\alpha$  for that case is

$$\alpha = -\frac{K_3}{K_4}$$

as a result of equation (30). With this value of  $\alpha$ , with the value of  $\epsilon$  calculated from the first of equations (30), and with  $\alpha\delta = 0$  the drag for this condition can be calculated from the general equation for the drag; the tip force, if desired for structural reasons, can be calculated by obtaining the value of  $L_2$  and subtracting  $W_2$  from it.

## Stability

Calculation of the stability of a combination of coupled airplanes is a difficult problem since many of the simplifications possible for uncoupled airplanes, such as a separate analysis of longitudinal and of lateral stability or a separate calculation of static and dynamic stability, are not possible or may lead to insignificant results in the case of the combination. A further complication of an analysis of the stability of the combination results from the fact that the concentrated force at the wing tip must be taken into account.

Some of the results presented in this paper may be used in the calculation of some of the aerodynamic parameters which enter into a stability analysis. For instance, calculation of the airplane lifts and rolling moments due to several angle-of-attack conditions has been described in a preceding section. The yawing moments due to these angle-of-attack conditions can be calculated from the products of the induced angles of attack (which have been calculated as part of the calculations of the induced drag but are not themselves given in this paper) and the local lifts. The yawing moment due to sideslip and the side forces are not likely to be affected by coupling.

The aerodynamic pitching moments of the individual uncoupled airplanes can be corrected for coupling effects by subtracting from them the products of the increments in bending moment and of the tangent of the angle of sweepback. This correction applies to the pitching moments about the intersections of the quarter-chord lines and the planes of symmetry. In order to apply this correction, the pitching moments of the uncoupled airplanes must be referred to these points; the corrected moments can then be transferred back to any point of interest by using the lifts on the coupled airplanes. A further important correction to the pitching moments arises from the fact that the downwash angles are affected by coupling. The spanwise load distributions of figure 3 provide the basic data from which the downwash angles can be calculated.

The concentrated tip force given by equation (27) applies only to cases in which static equilibrium exists. For the purposes of stability calculations, the tip force must be determined from conditions of dynamic equilibrium and its vertical component and yawing moment and pitching moment added to the aerodynamic lifts and moments.

## DISCUSSION

In order to aid in the evaluation of the results obtained in this paper, some of the previously stated assumptions on which the calculations

are based are summarized in this section and some additional considerations that have a bearing on the application of the results are discussed as well.

In the calculations of this paper the juncture has been assumed to be well faired. A gap between the wing tips would have a pronounced effect on the lift distribution near the tip and, if sufficiently large, on the lift distribution over the entire span of the configuration.

The horseshoe-vortex method as used in this paper is known to yield reliable lift distributions for relatively simple plan forms, for which its results may be compared with those of other methods. For a complicated plan form and for complicated angle-of-attack distribution, such as those considered in this paper, the accuracy of the lift distribution calculated by means of a relatively coarse vortex-grid is open to question. The fact that both compressibility and boundary-layer effects on lift are disregarded in the method of this paper is likely to have little effect on the results, but at the probable cruising speed of the airplane combination these effects are likely to cancel approximately. Fuselage effects have been disregarded in the calculations, as is usually done in calculations for uncoupled airplanes.

There is insufficient experience with such plan forms to permit a quantitative estimate of the accuracy. However, the lift distributions are logical and have been checked by several approximate theoretical methods. Most of the lift distributions (except near the juncture and at the fuselages), lifts, bending moments, and rolling moments are estimated to be accurate within a few percent; the induced drags are likely to be less accurate, and all results which pertain to the aileron-deflected case are likely to be less accurate than the others. These statements pertain only to low angles of attack; once any part of the wing combination has stalled, the results of this paper become inapplicable. This fact should be kept in mind, since the angle-of-attack combination for least total drag, as determined by the preceding equations, may well involve angles of attack of the B-47 airplanes sufficient to stall parts of their wings, sweptback wings being particularly likely to stall at relatively small angles of attack.

The lifts, induced drags, bending moments, and rolling moments have been expressed as increments due to coupling. The reason for so doing is that the use of known results for the uncoupled airplanes, together with relatively small increments due to coupling, tends to eliminate or diminish the errors inherent in the calculations of this paper. Also, use of uncoupled wing data permits the inclusion of tail and fuselage forces and, to some extent, of fuselage-wing interference effects in the final coupled-airplane forces.

The basic premise of this procedure is that fuselage effects are not affected by coupling, and that tail effects can be corrected by taking the change in downwash at the tail due to the change in wing lift into account. However, coupling the airplanes may result in a leakage or other parasite drag, which is not taken into account in the calculations of this paper, unless the juncture is well faired for all relative angles of attack and preferably of yaw as well. If there is such a drag and if its magnitude can be estimated, its effect on the total airplane drag can easily be included in the calculation of the total drag, as in equation (25), for instance.

For calculating the shifts in the aerodynamic center due to coupling and for estimating the static longitudinal stability characteristics of the airplane combination, the chordwise location of the local aerodynamic centers is required. They may be assumed to be at the quarter-chord points, but this assumption is not warranted except for wings of high aspect ratios, and even for such wings it is not valid near the root and the tips. However, the effect of these regions on the pitching and rolling moments is small, particularly inasmuch as the effect of the root and that of the tip tend to cancel. Also, by using increments applied to known data for the uncoupled airplanes any remaining effects of local aerodynamic centers off the quarter-chord line tend to be minimized.

No aeroelastic effects have been taken into account in the calculations of this paper. At or below the cruising speed of the airplane combination aeroelastic effects (in the ordinary sense) are likely to be unimportant. However, a concentrated load at the coupled wing tips may give rise to wing deflections of sufficient magnitude to cause a pronounced effect on the lift distribution and, hence, on the other parameters calculated in this paper. This effect will depend on the magnitude of the concentrated tip load and on the stiffness of the airplane wings. If this load is small, as it has to be from purely structural considerations, its effect on the lift distribution is also relatively small.

#### CONCLUDING REMARKS

Lift distributions and some associated aerodynamic parameters have been calculated for a Consolidated Vultee B-36 and two Boeing B-47 airplanes coupled at the wing tips. The results of the calculations have been expressed as increments to be added to the known data for the

uncoupled airplanes in order to minimize errors inherent in the calculations. The manner in which the results can be used in calculations of the performance has been discussed, but no such calculations have been made.

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#### REFERENCES

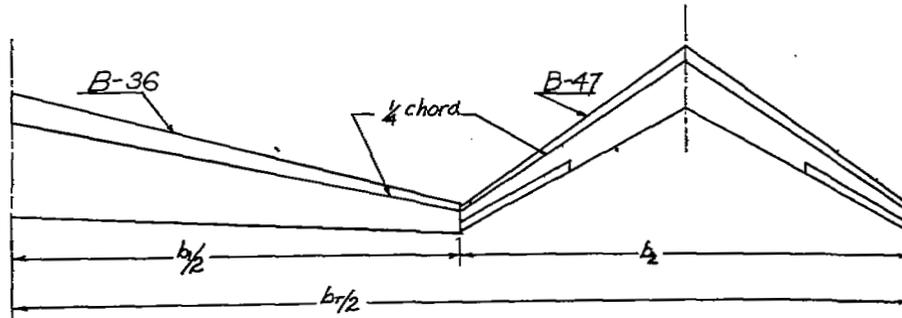
1. Van Dorn, Nicholas H., and DeYoung, John: A Comparison of Three Theoretical Methods of Calculating Span Load Distribution on Swept Wings. NACA TN 1476, 1947.
2. Falkner, V. M.: The Calculation of Aerodynamic Loading on Surfaces of Any Shape. R. & M. No. 1910, British A.R.C., 1943.

TABLE 1

## GEOMETRY OF B-36 AND B-47 AIRPLANES

Airplane	b (ft)	S (ft <sup>2</sup> )	A	c <sub>r</sub> (in.)	c <sub>t</sub> (in.)	$\bar{c}$ (in.)	c <sub>MAC</sub> (in.)	$\lambda$	$\Delta c/l$ (deg)	Twist (deg)
B-36	230	4772	11.1	400	100	250	280	0.25	12.5	2, washout
B-47	116	1428	9.43	208	87.4	147.7	156	.42	35.0	0
B-36 and B-47	462	7628	27.982	---	---	---	---	---	---	---


 NACA

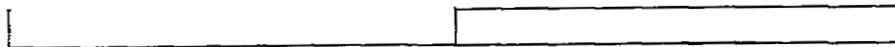


(a) Plan-form layout.

Case A,  
Uniform angle  
of attack



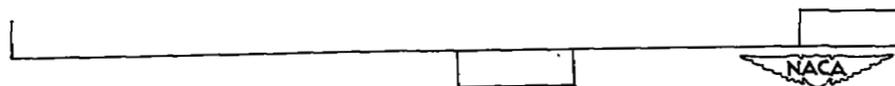
Case B,  
Unit angular  
displacement  
of B-47



Case C,  
Unit washout  
of B-36



Case D,  
Aileron  
displacement  
of B-47



(b) Angle-of-attack distributions.

Figure 1.- Wing plan form and angle-of-attack distribution.

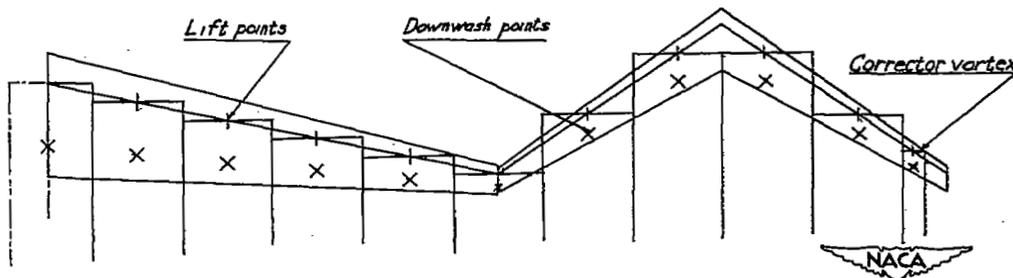


Figure 2.- Location of vortices on plan form.

B-36

B-47

20

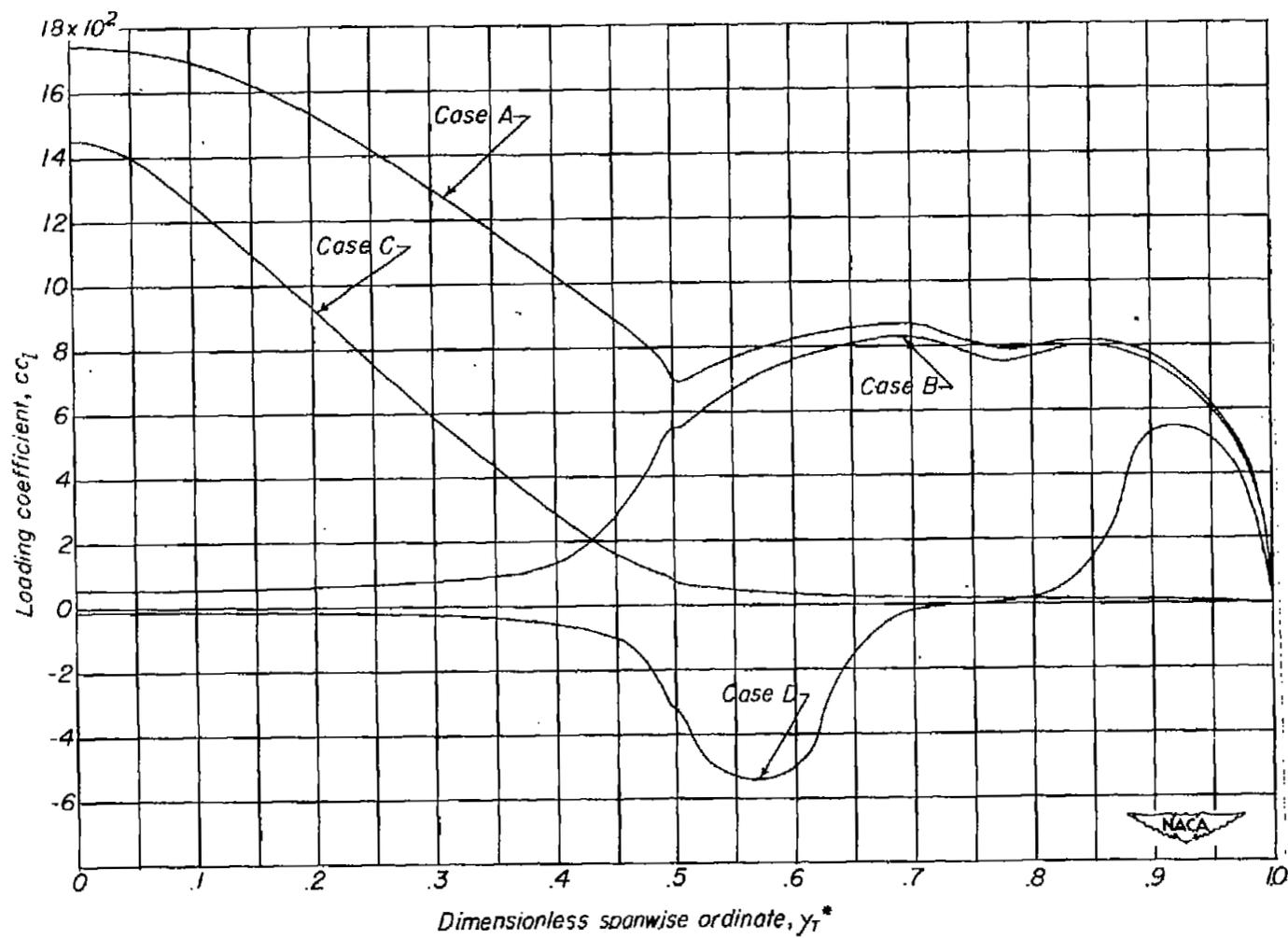


Figure 3.- Lift distributions for  $\alpha$ ,  $\epsilon$ ,  $\alpha_8\delta$ , and washout of 1 radian.

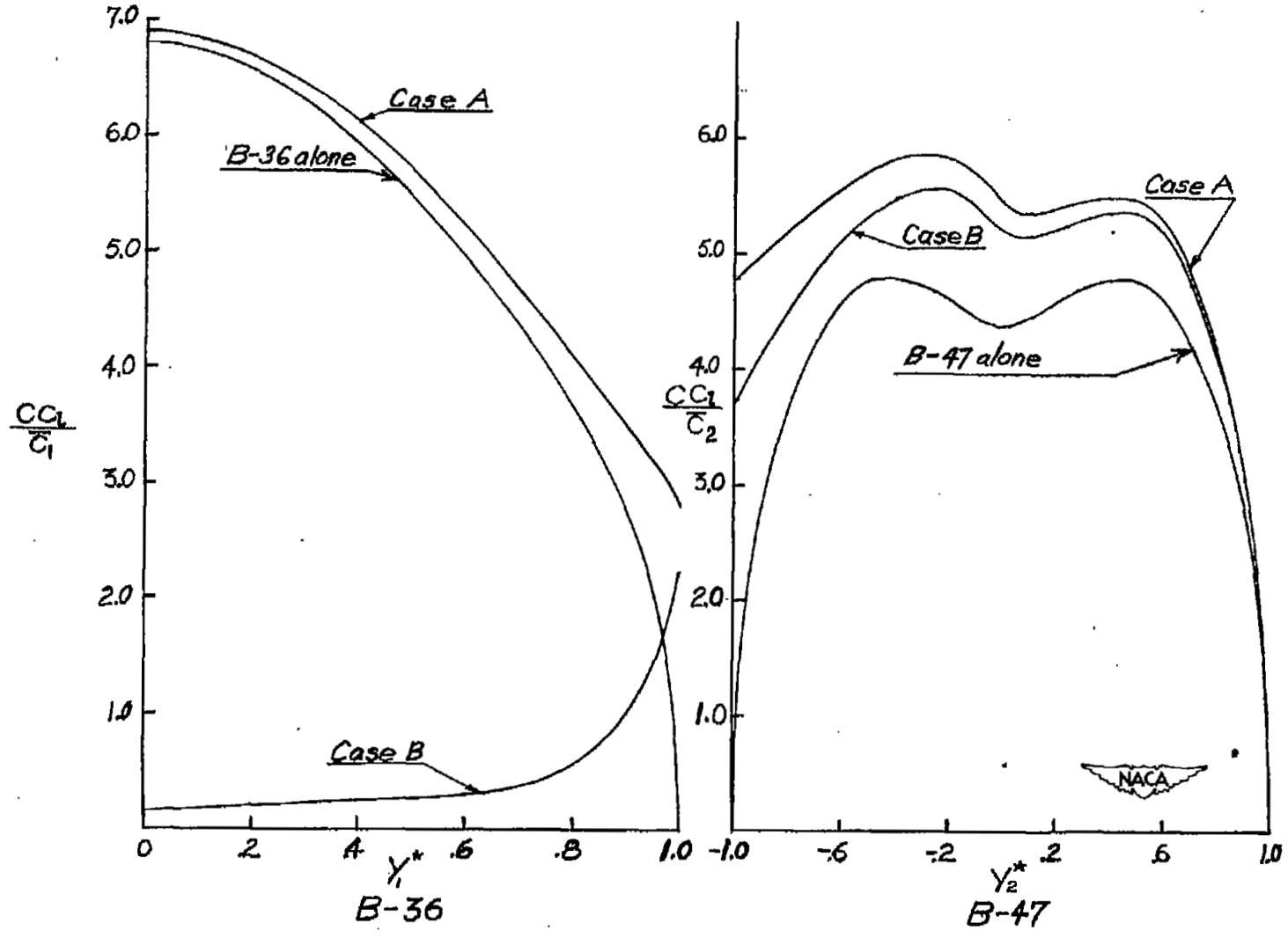


Figure 4.- Individual lift distributions for  $\alpha$  and  $\epsilon$  of 1 radian.

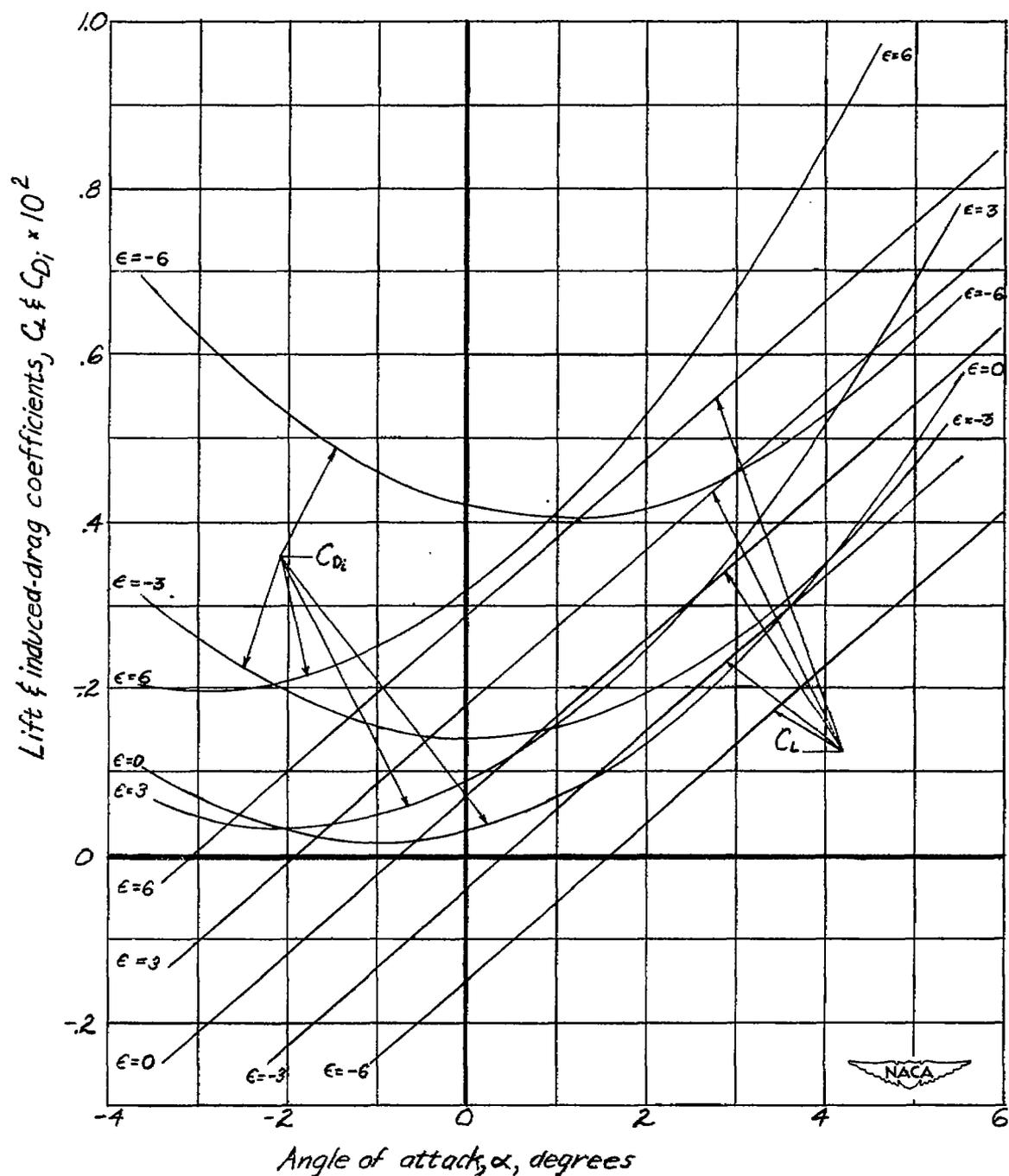


Figure 5.- Lift and induced-drag curves for the combination at various relative angles of attack  $\epsilon$ .

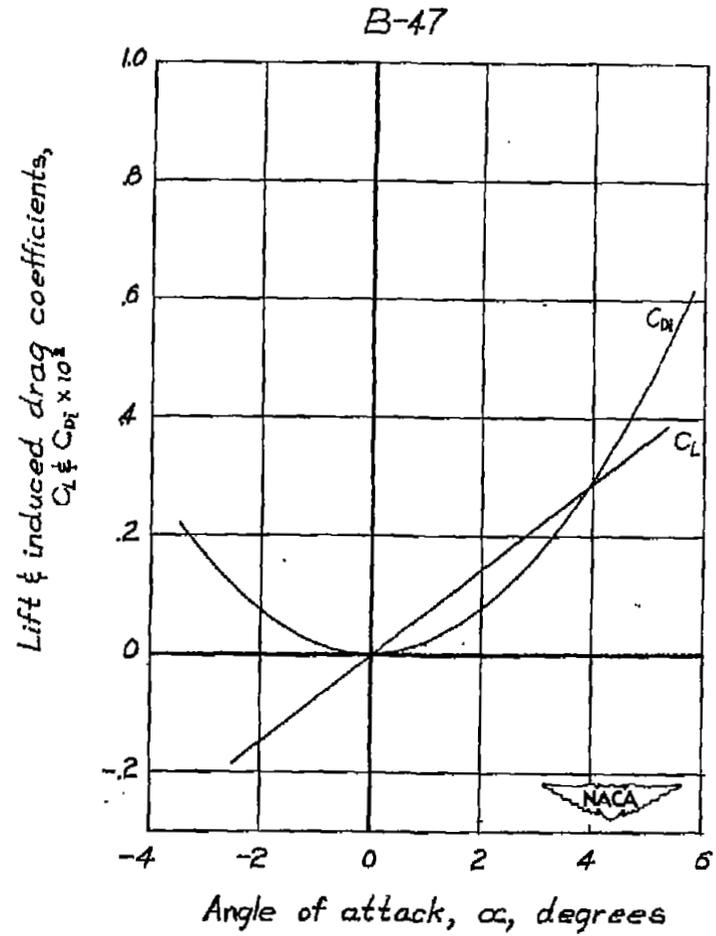
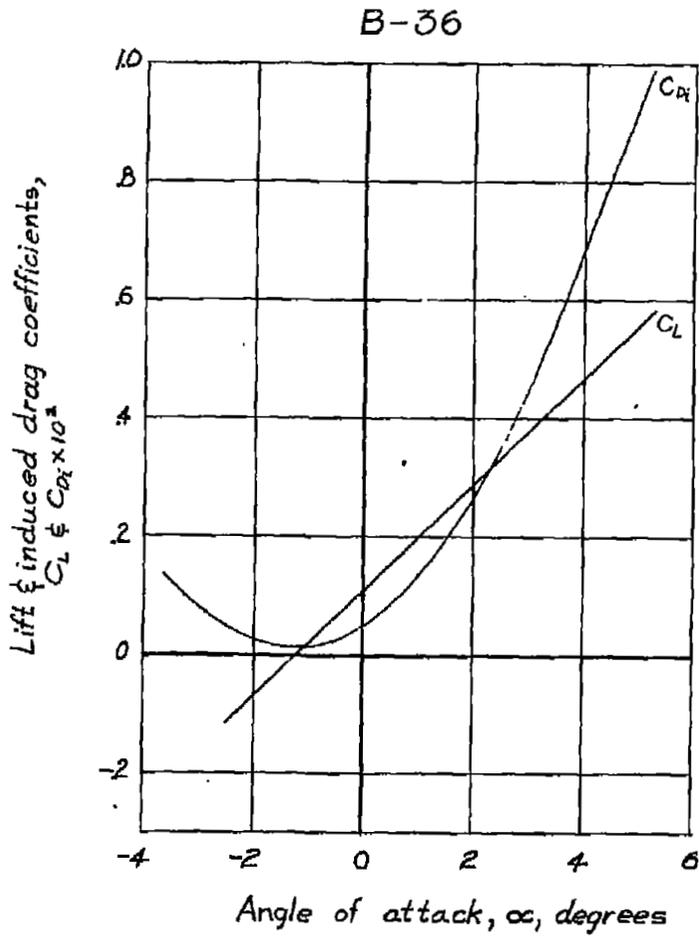


Figure 6.- Lift and drag curves for the B-36 and B-47 airplanes alone.  
( $\alpha$  referred to tip of B-36.)