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TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 816

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By Louis Breguet

Journées Techniques Internationales de l'Aéronautique
November 23-27, 1936

Washington
January 1937

1.7.3

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To begin with, I shall explain what a gyroplane is.

The gyroplane belongs to the helicopter family which, as the name implies, has wings in the form of propellers.

In fact, a helicopter consists of large propellers with, substantially, vertical axes set in motion by an engine; the reaction of the air on the revolving blades produces an upward lift in excess of the weight of the entire apparatus which, as a result, can ascend in the air without forward speed.

It will be remembered that, in order to obtain sustentation without speed, a great many methods have been conceived intended to furnish the lifting wings with a proper movement with respect to the body.

The first inspiration was found in nature itself, that "incomparable model," and has actually led to the design of airplanes with flapping wings whose possibility of realization cannot be denied.

But, even as man has, in the remote past, invented the wheel to replace the alternative movement of natural locomotion by a rotary motion, so the rotation of lifting blades should appear in mind as a more mechanical process than flapping: Whence the idea to make these wings revolve in continuous motion around a central axis, each wing describing a circle - the whole system constituting a sort of individual whirling arms of which the center, fixed in the body, may be kept stationary.

The idea of sustentation of flying machines by propellers is quite old. Long before Jules Verne wrote his

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*"Le Gyroplane - Sa Technique et ses Possibilités." From Journées Techniques Internationales de l'Aéronautique, November 23-27, 1936. Published by Chambre Syndicale des Industries Aéronautiques.

"Robur le Conquérant," many inventors had thought of helicopters; one of the best known studies is that by Ponton d'Hamécourt. More recently, Colonel Charles Renard treated the problem comprehensively in his now celebrated Communications to the Academy of Sciences. The first, entitled "On the Possibility of Sustentation in the Air of a Flying Machine of the Helicopter Type by Employing the Explosion Engines in Their Actual State of Lightness," dates from November 23, 1903. Then on December 7, of the same year, he presented his second note entitled "On the Quality of Lifting Propellers" which, on November 7, 1904, was followed by another, entitled "A New Method of Constructing Aerial Propellers."

I was impressed at that time by the works of Colonel Renard, one of whose students I had the honor to be, and I have taken up again the problems treated by him by superposing on the motion of rotation, alone considered then, a motion of translation. In effect, a gyroplane is a helicopter designed to move diagonally in the air at a speed as high as possible.

This translation causes the speed of rotation to combine with that of advance in every point of the blade. As the angle formed by these speeds changes while each blade makes a complete revolution and the speed of rotation becomes additive for half a revolution to the speed of translation, some precautions must be taken to keep the forces from becoming excessive at certain moments so as to prevent rupture of the blades or throwing the apparatus out of balance.

In my first gyroplane patent I provided for the use of flexible blades with automatic incidence control. Then in 1908, I patented a differential linkage of opposite blades for the purpose of balancing the loads by incidence variations, the incidence of the advancing blade decreasing and that of the retreating blade increasing.

I also made provision in my gyroplane No. 3, for the mechanism described by Colonel Renard in his communication of 1904, and which consisted of hinging the blades to the hub. Due to this fact, the blades - being subject on the one hand to the centrifugal force, constant for a given speed of rotation and, on the other hand, to changing aerodynamic reactions resulting from the translation - were able to orientate themselves at any instant, according to the resultant forces.

During the period of one revolution the blades undulate then and flap in alternate motion, each at its own count, with a phase displacement in ratio to the air loads and an amplitude which can be regulated by an automatic incidence change in function of flapping. When the blades advance in the direction of translation of the body which they support, they are lifted up at the same time as they move at an angle with respect to the motion of rotation of the hub. The inverse process takes place during the half-revolution during which the blades retreat. In this way the alternating loads to which the rotating wings are subjected in their combined movement of translation and gyration, as well as the couple necessary for their engagement, are regulated.

The essential advantage of helicopters and gyroplanes lies, as we have seen, in their power of sustentation without forward speed. Thus a helicopter can take off and land vertically without speed, whereas the modern airplane with high specific wing loading cannot take off or land unless it has a speed of the order of 100 kilometers (62.14 miles) per hour. As a corollary, it requires large landing fields, leveled off and well kept. The airplane cannot, in effect, fly below a certain speed without grave danger of instability, spoken of in aviation circles as "dangers of pancaking."

To get away from the constraint of vast airports is something that interests both military and civil aviation. For the military airplane this release is chiefly important in time of war, when it may not only be difficult to find suitable areas near the front but also to keep them in good shape. The landing field is apt to be a target of bombing raids which leave it unfit for further airplane use.

Granted that the gyroplane can rise vertically from any clear piece of ground: It must then be able to fly at suitable speeds without excessive power input. With this in mind, I was particularly interested in ascertaining the possible efficiency of this method of translation obtained simply by a suitable forward tilt of the blade shaft and the extent to which this efficiency and speed obtained are comparable with those of modern airplanes.

Before launching into this problem, I want to answer a question which has so often been posed to me: What is the difference between a gyroplane and a helicopter?

Etymologically, gyroplane means "an apparatus which moves by turning," and this name was coined during a conversation I had in 1905 with the late Professor Charles Richet. A gyroplane has no propulsive propeller, since its rotating wings driven by the engines are sufficient both for propulsion and for sustentation.

An autogiro, such as that of Mr. de la Cierva, the eminent Spanish engineer, is an apparatus whose wings rotate in autorotation. In autogiros, in effect, the revolving blades are not controlled by the engine but mounted free on the central shaft. The engine drives, as in the airplane, one or more regular propellers; it is the relative wind, due to the translation provided by these propellers, that sets the revolving blades in autorotation - the plane of the blades, of necessity, being tilted with respect to the plane of rotation.

In brief, the autogiro is actually an airplane whose wings are free to rotate about a central axis, as a windmill set nearly horizontal; in revolving, these wings materialize, in some way, according to wind-tunnel tests, a lifting disk, and the machine behaves as if it had a fixed wing, but of considerably larger area, equal to the swept-disk area of the blades. It is, by virtue of this enlarged area, that the autogiro can fly at low speed.

In the autogiro the plane of the blades is tilted toward the rear and is drag-producing - the drag being overcome by the propeller thrust; while in the gyroplane the plane of the blades tilts forward in order to assure propulsion.

The gyroplane - quite apart from the faculty of vertical flight, which the autogiro with free wings does not possess - offers additional advantages, particularly in regard to the over-all efficiency, which is enhanced by the absence of the propulsive propeller. Propulsion and sustentation by the same rotating wing system, allows much higher forward speeds, and it has been proved that the propulsive efficiency is then practically equal to unity.

My first gyroplane with flexible wings was built during 1905-1906, at Douai, and made its first free flight in 1907, with one man aboard. This achievement - the first of its kind - formed the subject of a report presented to the Academy of Sciences by Mr. Lipmann (reference 1).

Before building this gyroplane, I had made a great number of systematic experiments on a large wind-tunnel balance. The first results of these tests were equally presented in a communication at the Fourth Aeronautical Congress, held at Nancy, in September 1909.

The conclusions at which I arrived from the study of the best airfoils and especially from the introduction of a new concept, that of the solidity ratio or ratio of blade area to swept-disk area, had already been very encouraging.

For a given lifted weight P, with a propeller radius D, and a power W, I had obtained a lifting quality $q = \frac{P^{3/2}}{DW}$, which was distinctly superior to that indicated by Colonel Renard, whose propellers had an excessive relative width, especially toward the tip.

Moreover, it seemed to me that the translation should improve this quality which would, up to certain speeds, compensate the power necessary for translation.

I wrote, in fact, in 1909: "The trouble met with on surfaces working successively on the same air column should lead us to think that, for a lifting propeller in diagonal motion, the supporting column of air being constantly renewed, the inconvenience of the surfaces between them should, due to this fact, be notably less great than when at rest.

"I have, indeed, checked this fact but without being able to put it in figures. On a day of average and intermittent wind, I have observed that at every gust the lifting force developed by my gyroplane No. 1, increased quite freely.

"I also noted another fact: While testing my second gyroplane, which was a combination of helicopter and airplane, the center of thrust of the propellers - which, at rest, coincided with the axis of rotation - was, during flight, shifted quite freely forward, the shift of the c.g. amounting, probably, to as much as 50 cm (19.67 in.); the propeller diameter being 8 m (26.25 ft.), and the forward speed of the order of 10 m/s (32.808 ft./sec.).

I have reproduced the sketch and photograph of the balance which I constructed for my experiments along with the graph on direct-lift propellers, and a picture of my 1907 gyroplane (figs. 1, 2, 3, 4).

Notwithstanding these results and the very encouraging trials of my machine, I was due to abandon the solution of this important problem because of lack of funds.

Then, too, while devoting myself to these researches, Santos-Dumont, Voisin, Blériot, and Esnault-Pelterie had made successful flights in regular airplanes. And so I decided to build an airplane but on the basis of the results of my own experiments.

The wings of my airplane were therefore conceived as scaled-up versions of my gyroplane blades; they had one spar and flexible ribs.

Further, my studies on propeller efficiency enabled me to see how to adapt them best to an airplane, and the flights of my first airplane, in 1910, revealed a particularly interesting efficiency. This is how I came to abandon the subject of gyroplanes until some years after the war.

It is now five years since Cierva presented his curious machine which he called "autogiro," in France, and which actually surprised me with its stability in flight. The blades were joined to the hub by articulations such as I had employed in 1908.

I might add that mounting the blades freely to the hub suppresses the gyroscopic couples, which may affect the stability of the machine as a theoretical study of the problem will prove. This practical proof justified me in thinking that gyroplanes should also have the same stability.

At that particular time, I had designed a new machine which was to be built by one of my coworkers, Mr. Dorand - a son of Colonel Dorand. In this machine the blades were again mounted in articulations to the hub and could, in addition, revolve around their own axis, thus making it possible to control the incidence. The incidence was automatically changeable by an eccentric lever; lower when the blade rises, higher when the blade drops.

The differential incidence control was realized by a plate mounted on ball bearings, which the pilot could control either for changing the incidence in any meridian or for changing the whole system affecting the pitch. The direction was assured by a differential control of the pitch of two systems of coaxial blades revolving in opposite direction. This arrangement had the advantage of assuring direction even when hovering.

The last gyroplane I constructed was, in fact, only a laboratory model. Its lines, as seen in figures 5 and 6, were not refined, and its drag was quite high. The sole purpose was to aid my experiments on blade-control mechanism and maneuverability.

Concurrently, I launched into a theoretical study of translation - a study which was to confirm the tests made at the Eiffel laboratory and as published in 1927 in the Bulletin of the S.T.Aé. These tests were made by Mr. Lapresle on rigid propellers with fairly large solidity and a wide range of incidence variations. These experiments, carried out in systematic order, confirmed in startling manner everything I had suspected, and were of inestimable value to me.

I have established in this respect, various general formulas, and requested my collaborator, Mr. Devillers, to help me put them in mathematical form. They appear, at first glance, quite complicated, which is but natural. But they are in full accord with both the Eiffel tests and my own past and recent experiments.

I shall commence by indicating several simple principles concerning the velocity distribution over the blades of a lifting propeller of diameter D, revolving at n revolutions per second, and animated by a horizontal movement of translation at speed V.

The calculation, compared with the test data, has shown me that the aerodynamic action of the air on the blades depends practically only on the velocity components in a plane at right angles to the blade span. In other words, the radial velocities or velocities of sideslip have no substantial effect on the lift and power coefficients - this assumption being, moreover, unfavorable.

Other scientists or technicians who have treated this problem, arrived at the same conclusion (reference 2).

At any one instant there is thus introduced into the velocity distribution, the component of the speed of translation V along the normal to the span of each blade, such as, for instance, V_1 for the blade A, and V_2 for blade B (fig. 7).

1. Consider blade A advancing in the direction of translation by rotating about axis O; the effective resultant velocity at the tip then is the sum $V_3 = U_A + V_1$ of the speed of rotation $U_A = \pi n D$ and of the component V_1 perpendicular to the span of the speed of translation V.

The extremity of the resultant speed U' at any one point M of the blade, is therefore found on the straight line EF to be deduced from the straight line OU_A, the place of the extremities of the speeds of rotation by a translation V_1 in the direction of the advance.

The line EF meets the axis OA of the blade at O' which is the point of zero velocity or the instantaneous center of rotation.

The triangles O'OE and OAU_A forthwith give:

$$\frac{OO'}{OA} = \frac{OE}{U_A}, \quad \frac{OO'}{\frac{D}{2}} = \frac{V_1}{\pi n D}, \quad OO' = \frac{V_1}{2\pi n}$$

Let H represent a point on the perpendicular to the direction V of the translation and in such a manner that OO' is the projection of OH.

The triangles OO'H and OVE are similar as their respective sides are perpendicular

$$\frac{OO'}{OE} = \frac{OO'}{V_1} = \frac{OH}{V}, \quad OH = \frac{V}{V_1}\, OO' = \frac{V}{2\pi n} = \text{const.}$$

The angle OO'H being straight when the blade A effects its rotation, the instantaneous center O' is shifted on the circle I, passing through O and the diameter $d = OH = \frac{V}{2\pi n}$, perpendicular to the direction of translation, the direction of OH being deduced from that of the translation by a 90-degree rotation in the sense of the rotation n.

The distribution of the aerodynamic velocities is the same as if, at each instant, the blade turned about the instantaneous center O' at the angular velocity $2\pi n$, which it has about its axis O.

In fact, by virtue of the verification of this general principle, it is seen that the resultant velocity U' in M is, by definition, $V_1 + 2\pi n$ OM; i.e., after replacing V_1 by $2\pi n$ OO':

$$U' = 2\pi n\ (OO' + OM) = 2\pi n\ O'M$$

The velocity U' is fully the same as if, at every instant, the rotation took place at n revolutions per second about point O', which is always the point where axis OA of the blade and circle I meet. So, as long as point O' is outside of the blade area - that is, so long as the blade does not sweep the inside of circle I, the velocities U' are all in the same direction.

Thus it is for the rotation of 180°, which the blade advances, while rotating, in the sense of the translation.

2\. Consider, then, a blade B (fig. 7), whose tip speed $U_B = \pi n D$ is in the direction opposite to the effective component V_2 of the translatory speed V.

The straight line E'F' representing the velocity distribution, is again deduced from the straight line OU_B, which represents the distribution of the rotational speeds by a translation V_2, but which is now in the inverse sense of U_B. The resultant velocity cancels out, in the instantaneous center O", the intersection point of blade axis and circle I.

It is seen that, for every part of the blade within circle I, the sections are attacked on their trailing edge. The circle I, the place of the instantaneous centers of rotation, defines by its inside area the region which I have called the reversed-velocity region. Within this region the blade drag is always activating as concerns the engine torque, while the lift is negative, the blades being attacked at their back. The distribution of the resultant velocities over the blade is again the same as if it rotated about the instantaneous center O" at the rotational speed $2\pi n$, which the propeller possesses about its central axis O.

This theory of the gyroplane, as outlined above, is based on the fact that it is possible to effect the summation of the elementary actions of the air on the rotating blades, considered as wings of an airplane having a certain aspect ratio λ and a minimum drag coefficient c_{x_0}. The problem then reduces to finding the fictitious aspect ratio λ to be applied to this blade.

Obviously, this λ depends on the blade number N, the ratio h_0 of blade area to swept-disk area, which I have called "solidity ratio," on the parameter of translation $\gamma = V/nD$, and lastly, on a residual aspect ratio, to which a fictitious residual solidity ratio h_r corresponds.

It will be remembered that the geometrical aspect ratio λg of a surface s_1 is the ratio E^2/s_1 between the square of the span and the surface - that is to say, $\lambda g = \frac{R^2}{s_1} = \frac{D^2}{4 s_1}$ for a blade of surface s_1. But, on considering it as a propeller with N blades, by definition $N s_1 = h_0 \frac{\pi D^2}{4}$, it gives for the geometrical aspect ratio of a blade:

$$\lambda g = \frac{1}{\pi \frac{h_0}{N}}.$$

It is known that the interference of the blades, operating because of their rotation in their mutual downflow, is manifested by a rise in induced velocities normal to the plane of rotation, and proceeds, as concerns the induced drag $c_{x_i} = \frac{c_z^2}{\pi\lambda}$, a function of c_z, as if the geometric aspect ratio λg was lowered and replaced by a fictitious aspect ratio λ so much smaller as the interference is more pronounced. It was this which decided me, in the first place, for operation at a fixed point (static thrust) to multiply h_0 by $N + 1$ which, for $N = 2$, gave a fictitious aspect ratio three times smaller than the geometric aspect ratio λg.

Then I had to introduce the residual aspect ratio λ_r which I express in terms of a fictitious solidity ratio h_r, the introduction of which simplifies the mathematical representation, and so that $\lambda_r = \frac{1}{2\pi h_r}$ at a fixed point.

The wind-tunnel tests warranted the use of $\lambda_r = 35$ for an isolated wing in translation, and $\lambda_r = 10.5$ for the wings in rotation, such as those of a wing system rotating at a fixed point, the latter value corresponding to $h_r = 0.015$. Thus the formula for the fictitious aspect ratio of a helicopter blade at a fixed point, reads as follows:

$$\lambda_o = \frac{1}{\pi\left(h_o \frac{N+1}{N} + 2h_r\right)}$$

In effect, h_r may be dependent on the blade number, but this formula is intended to be applied to gyroplanes having at least four, and no more than 8, blades, and it is sufficiently approximate for the study under consideration.

The coefficient h_r represents an altogether new notion in aerodynamics and signifies that, for blades which are infinitely extended, a residual aspect ratio corresponding to an interference limit, should be considered.

In the Eiffel wind-tunnel tests on a four-blade propeller yielding $h_o = 0.28$, the geometric aspect ratio of a blade being $\lambda g = 4.5$, we observed at a fixed point, results corresponding to a fictitious aspect ratio of $\lambda_o = 0.9$; that is, a marked decrease with respect to λg, and explaining the quite mediocre results obtained experimentally.

It is only by adopting a fictitious aspect ratio comprising the residual term, that use can be made of the induced parabola of Prandtl's theory for each blade section. Otherwise, it is impossible to find even the sense and magnitude of the experimentally observed results. This was confirmed in my experiments of 1907 on the dynamometric balance - according to which the variation of the solidity ratio h_o results in the lifting quality passing through a maximum for a value of h_o proportional to h_r; or else, when h_r is neglected, it increases indefinitely in proportion as the blades become smaller. The solid curves in the chart (fig. 8) represent the results of my tests of 1907, and the dashed curves the theoretical result corresponding to $h_r = 0.015$ for blades extending as far as the hub. The discrepancy between the experimental and the theoretical curves is due to the fact that the blades of my propellers in 1907, did not reach to the hub.

I estimate that the method of conducting the calculations is more exact that that frequently resorted to for autogiro rotor blades; i.e., computing the interference on the basis of induced vertical velocity uniformly distributed over the swept-disk·area, this velocity being determined by comparing the disk constituted by this area to an airplane wing. It is, in effect, difficult to acknowledge such a distribution - much too advantageous in translation - of the vertical flow of the air for large propellers revolving considerably slower than the propulsive propellers - at a speed of from 2 to 4 revolutions per second, for example, and where the blades during half of a revolution are inactive while sweeping the reversed-velocity region.

I effected the calculations on the basis of a mean and uniform lift coefficient, but proceeded from an experimental polar when, in the reversed-velocity region, the sections are attacked at their trailing edge.

Without automatic incidence adaptation, this would change periodically because of the tilting of the axis of the propellers, but the vertical flapping motions of the blades permitted by the articulations play, on that account, the part of a regulator.

To compute the lift and the power input, I then effected the integrations of the air action along the blades by replacing for each section the square U'^2 of the resultant aerodynamic velocity by its mean value derived from the integration in the period. The integrations were made separately for the exterior and the interior of the reversed-velocity region. For the interior, I assumed $c_x' = 2c_x$ and $c_z' = -0.5\ c_z$, c_x and c_z being the lift and drag coefficients on the active parts of the blades.

I also computed the resistance offered to the rotational speed generated by the blades in their plane of rotation, with consideration for the unsymmetry of the air loads set up when the propeller is in translation. Adding the drag of the body to that of the accessories gives the total drag.

This drag necessitates an angle of forward propulsive inclination of the axis of the propellers and its effect is included in the term for the power input W to keep the propellers rotating.

I confined myself to the case where the reversed-velocity region remains within the swept-disk area, whence my formulas are valid up to $\frac{V}{nD} = \pi$, which seemed to me to be sufficient. In this manner I have obtained (reference 3) for blades substantially rectangular in plan form, the following formulas (in meter, kilogram, second units).

Gyroplane Formulas

$h_o = \frac{s}{\frac{\pi D^2}{4}}$, effective solidity ratio for total blade area s.

N, number of blades.

h_r, residual solidity ratio (0.015 for my actual gyroplanes).

V, forward speed.

n, revolutions per second of the coaxial propellers.

D, propeller radius.

$\gamma = \frac{V}{nD}$, parameter of translation.

σV^2, parasite drag at zero altitude.

λ, fictitious aspect ratio of the blades.

$c_{zf} = \sqrt{\pi \lambda c_{x_o}}$, lift coefficient corresponding to the fineness ratio of an element, for a minimum drag coefficient c_{x_o}.

$c_z = \mu c_{zf}$, lift coefficient of an element, assumed constant for all active parts of the blades.

$c_x = (1+\mu^2)\, c_{x_o}$, drag coefficient of an element.

P, total weight, equal to the lift in horizontal flight.

W, power input at propeller shaft.

C, sum of engine torques applied at propellers.

δ, relative air density at contemplated altitude of flight.

I. Fictitious aspect ratio:

$$\lambda = \frac{1}{\pi\left[\frac{h_o}{N} + h_r + \frac{h_o + h_r}{1 + 1.28\ \gamma}\right]} \tag{1}$$

II. Lift coefficient:

$$\alpha_z = \frac{P}{\delta n^2\ D^4} =$$

$$0.162\ \mu h_o \sqrt{\frac{c_{x_o}}{\frac{h_o}{N} + h_r + \frac{h_o+h_r}{1+1.28\gamma}}}\ (1+0.15\gamma^2-0.01\gamma^3) \tag{2}$$

III. Power coefficient:

$$\beta = \frac{W}{\delta n^3 D^5} = 0.383\ (1+\mu^2)\ c_{x_o}\ h_o\ (1+0.3\gamma^2+0.006\gamma^4) + \frac{\sigma}{D^2}\ \gamma^3 \tag{3}$$

IV. Angle of propulsive inclination θ:

$$\tan\theta =$$

$$\frac{1+\mu^2}{\mu}\sqrt{c_{x_o}\left(\frac{h_o}{N} + h_r + \frac{h_o+h_r}{1+1.28\gamma}\right)}\ \frac{0.475\gamma+0.02\gamma^3+6.17\ \frac{\sigma}{c_{x_o} h_o (1+\mu^2)D^2}\ \gamma^2}{1+0.006\gamma^3} \tag{4}$$

V. Lifting quality:

$$q = \frac{P^{3/2}}{DW} = \delta^{1/2}\ \frac{\alpha_z{}^{3/2}}{\beta} \tag{5}$$

VI. Apparent relative drag:

$$\tan\Phi = \frac{W}{PV} = \frac{\beta}{\alpha_z\ \gamma} = \tan\Phi_a + \delta\ \frac{\sigma}{P}\ V^2 \tag{6}$$

$\tan\Phi_a$ being the relative drag of the wing system alone; that is, for $\sigma = 0$, so that

$$\tan\Phi_a =$$

$$2.36\ \frac{1+\mu^2}{\mu}\sqrt{c_{x_o}\left(\frac{h_o}{N} + h_r + \frac{h_o+h_r}{1+1.28\gamma}\right)}\ \frac{1+0.3\gamma^2+0.006\gamma^4}{\gamma(1+0.15\gamma^2-0.01\gamma^3)} \tag{6a}$$

VII. Propeller torque:

$$C = \frac{\beta}{\alpha_z} \frac{DP}{2\pi} \tag{7}$$

VIII. Lift referred to speed V:

$$P = \delta \frac{\alpha_z}{\gamma^2} D^2 V^2 \tag{8}$$

IX. Power referred to speed V:

$$W = \delta \frac{\beta}{\gamma^3} D^2 V^3 \tag{9}$$

X. Polar versus swept-disk area S:

$$C_x = \frac{64}{\pi} \frac{\beta}{\gamma^3} \tag{10}$$

$$W = \frac{\delta}{16} C_x SV^3 \tag{11}$$

$$C_z = \frac{64}{\pi} \frac{\alpha_z}{\gamma^2} \tag{12}$$

$$P = \frac{\delta}{16} C_z SV^2 \tag{13}$$

XI. Semicubic induced parabola asymptotic to the polar:

$$C_x = \frac{C_z^{3/2}}{2} \tag{14}$$

corresponding to the quality at fixed point $q = 0.443\ \delta^{1/2}$ deduced from the Froude theory.

It follows from formula (1), which allows for the translation, that the blade interference decreases very quickly in function of the translation parameter γ, this phenomenon being analytically expressed by the rise in fictitious aspect ratio λ interposed in the induced parabola of a blade (fig. 9). This λ is minimum at static thrust $(\gamma = 0)$ and then takes the aforementioned value:

$$\lambda_o = \frac{1}{\pi \left[h_o \frac{N + 1}{N} + 2h_r \right]}$$

In forward motion, when the propeller makes a complete revolution, it advances by V/n, thus sweeps the total area:

$$S' = \frac{\pi D^2}{4} + \frac{VD}{n} = \frac{\pi D^2}{4}\left(1 + \frac{4}{\pi}\frac{V}{nD}\right)$$

The ratio of the actual blade area $s = h_o \frac{\pi D^2}{4}$ to this area S' is:

$$\frac{s}{S'} = \frac{h_o}{1 + \frac{4}{\pi}\frac{V}{nD}} = \frac{h_o}{1 + 1.28\ \gamma}$$

Formula (1) shows that, on condition of increasing h_o of the residual solidity ratio h_r, it is precisely this characteristic ratio which intervenes to cause, through its decrease, the increase of λ in function of the translation.

Lastly, if γ becomes very great, the limit of the fictitious aspect ratio is reached at:

$$\lambda m = \frac{1}{\pi\left(\frac{h_o}{N} + h_r\right)}$$

which is identical to the geometric aspect ratio λg of the blade except for the added residual solidity ratio h_r.

Figure 9 shows for gyroplanes with 4 or 6 blades, the rapid increase of λ with the translation parameter γ, the fictitious aspect ratio becoming substantially 2.5 times greater when passing from $\gamma = 0$ (static thrust) to $\gamma = 3$.

In the expression (2) of the lift coefficient α_z the composition of the velocities gives the parenthesis $(1 + 0.15\ \gamma^2 - 0.01\ \gamma^3)$ the fairly small subtractive term $0.01\ \gamma^3$ arising from the passage of the blades into the reversed-velocity region.

It is surprising to note that up to the limit of validity $\gamma = \pi$ of my formulas, the reversed-velocity region remains within the swept-disk area; the passage of the blades into this circle lowers the aerodynamic qualities of a propeller in translation very little.

Intuitively, it is seen that - the aerodynamic reactions being proportional to the square of the resultant velocity - the blade which recedes with respect to translation is, by reason of the smallness of the existing resultant velocity, bound to be practically inactive over its whole area lying within the reversed-velocity region.

The power coefficient β in formula (3) assumes, at each instant, the propulsive equilibrium realized in horizontal flight. The power absorbed by the drag of the body and of the accessories is, according to (3), derived from the integrations:

$$\Delta W = \delta\sigma \frac{n^3 D^5}{D^2} \gamma^3$$

or, substituting V/nD for γ and simplifying:

$$\Delta W = \delta\sigma V^3$$

This power is equal to that of traction, with an efficiency equal to unity, whatever the translation parameter γ may be.

This conclusion is exact only when, as I have done, the quantities of the second order are neglected with respect to the angle of propulsive inclination θ, $\cos\theta$ having been compared to unity and $\sin\theta$ to θ during my calculations.

The chart (fig. 10) illustrates the application of my formulas to propellers tested during 1925-27 in the Eiffel wind tunnel - propellers with excessive solidity and very drag-producing hub, μ reaching as high as 3.9.

Chart 11 shows the evolution of the lift coefficients α_z and the power coefficients β against $\gamma = V/nD$ for two gyroplanes. The one of considerable parasite drag and having four blades, is substantially the same as the experimental aircraft I have tested; the other, fitted with six blades, represents a very refined gyroplane of the future.

Figure 12 shows the angle of propulsive inclination θ, insuring propulsion in horizontal flight independent of the relative air density for the two types of gyroplane.

Figure 13 gives the apparent relative drag changes

$\tan \Phi$ against $\gamma = V/nD$, independent of the altitude, and the lifting quality q at sea level for the tested gyroplane against that of the future. It will be seen that q passes through so much higher a maximum as the parasite drags are lower; this maximum, reached for a value of V/nD, decreases as these drags increase. By the same argument, the relative drags $\tan \Phi$ pass through so much lower a minimum and reach so much higher a value V/nD as the parasite drags are smaller. For aerodynamically clean machines, as the future ones will be, this minimum ranges around 0.11 for a value of V/nD approaching 2.5, and it is surprising to note that over a very large region the relative drag remains practically constant and equal to its minimum.

This is an advantage not possessed by the airplane and enables a gyroplane in cruising flight to increase its speed while conserving its power in proportion to its lighter weight with fuel consumption.

Another remarkable feature is, that at the regime of minimum $\tan \Phi$, the angle of propulsive inclination θ remains practically constant and equal to a slope of around 10°, as a glance at figures 12 and 13 reveals.

The graph 14 shows $\tan \Phi_a$, $\tan \theta$ and the lifting quality q plotted against $\gamma = V/nD$ for a gyroplane with zero parasite drag $(\sigma = 0)$ at sea-level altitude (corresponding to wing system rotating only).

Quality q increases to a maximum of 0.64 on approaching $\frac{V}{nD} = 2$, then drops a little to reach 0.615 at $\frac{V}{nD} = 3$ The apparent relative drag, $\tan \Phi_a$, decreases constantly as far as $\frac{V}{nD} = 3$, where it reaches substantially its minimum of 0.069.

The slope $\tan \theta$, corresponding to the wing system alone, is inferior to $\tan \Phi_a$ as far as $\frac{V}{nD} = 3$, these two quantities then becoming equal.

Now, for any gyroplane, let $R = Ra + Rn$ be the total drag balanced by the angle of propulsive inclination θ: Ra being the drag due to the revolving blades, and Rn the drag due to body, hub, and accessories.

The condition of propulsion in longitudinal flight gives, obviously, $\tan\theta = R/P$. But, as the apparent over-all relative drag is $\tan\Phi = W/PV$, the substitution of $R/\tan\theta$ for the weight P in the formula for $\tan\theta$ gives:

$$\tan\Phi = \frac{W}{RV}\tan\theta$$

According to the charts for θ and $\tan\Phi$, it seems that up to the minimum of $\tan\Phi$, $\tan\Phi$ being greater than $\tan\theta$, the power input W is greater than RV and, beyond, W may be inferior to RV.

This paradoxical result follows from evaluating R with respect to V/nD rather than V. In the regime of minimum $\tan\Phi$, W/RP is very close to unity for a clean gyroplane, and approaches 0.5 when the gyroplane has a high drag, such as that analyzed in this study.

Finally, it may be noted that in view of formula (6), the power equation of a gyroplane can be put in the following form:

$$\frac{W}{P} = V\tan\Phi_a + \delta\frac{\sigma}{P}V^3 \qquad (15)$$

wherein the parasite drags do not interfere except in their relation to the total weight of the gyroplane.

For a very clean apparatus, we may put $\frac{\sigma}{P} = \frac{1}{350000}$ to $\frac{1}{400000}$. Formula (15) shows that, for a gyroplane of given parasite drag, weight, and horsepower, the highest speed V is obtained when $\tan\Phi_a$ is minimum, or at values V/nD much higher than considered here, i.e., $\frac{V}{nD} = \pi$.

The most favorable value for μ is unity, as is readily apparent from formula (6a), although $\tan\Phi_a$ increases slowly with μ so long as this coefficient does not exceed 1.5.

Assume, for example, that $\frac{\sigma}{P} = \frac{1}{400000}$, $\delta = 0.74$ (3,000 meters = 9,842 ft.), and that it is possible to adapt the propellers for a value of $V/nD = 2.3$ to 2.6 or substantially, $\tan\Phi_a = 0.072$. Then the preceding formula enables us to compute the horsepower per kilogram of total weight or the total weight per horsepower with respect to the maximum speed at this altitude. The result is:

Altitude of Flight, 3,000 m (9,842 ft.)

Maximum speed km/h	350	400	450	500	550	600	650	700
Horsepower per kilogram	0.116	.140	.169	.200	.235	.276	.320	.370
Total weight in kilograms per horsepower	8.65	7.13	5.92	5	4.25	3.62	3.12	2.70

km/h x 0.62137 = mi./hr. kg x 2.20462 = lb.

Now we attempt to find the speed of translation V, up to which the resultant velocity at the tip of an advancing blade does not exceed the velocity of sound. For a speed of translation V and a tip speed πnD of the blades, the resultant maximum aerodynamic velocity at the tip of the advancing blade is:

$$U' = V + \pi nD = V\left(1 + \frac{\pi nD}{V}\right) = V\left(1 + \frac{\pi}{\gamma}\right) \qquad (16)$$

It is seen that for a given speed V, U' will be so much lower as the translation parameter γ itself is greater. So, to prevent U' from reaching some velocity and thereby vitiating the aerodynamic qualities of the blades, it is advantageous in this respect that γ should approach π. With $\gamma = \pi$, the velocity $U' = 2V$ reaches that of sound; that is, 330 m/s for a forward speed of V = 165 m/s, or 595 km/h.

Chart 15 compares $\tan \Phi$ for a gyroplane of the future and an aerodynamically clean airplane corresponding to $c_{x_0} = 0.018$ and a 130 kg/m² loading against the speed at 3,000 m. The over-all relative drag of the airplane is equal to its relative aerodynamic drag divided by the propeller efficiency η, which has been fixed at 0.77. It is seen that the gyroplane prevails over the airplane as soon as the speed exceeds 380 km/h, and likewise, at speeds below 130 km/h, unattainable by the airplane which assumedly has been fitted with the best high-lift devices.

m/s x 3.28083 = ft./sec. kg/m x 0.204818 = lb./sq.ft.

In proof of the foregoing, the diagram (fig. 16) shows, plotted against the speed at 3,000 meters, the power absorption for the airplane and for the gyroplane, and for the latter the development of quality q at this altitude, q varying in inverse ratio of the horsepower.

Between 130 and 380 kilometers per hour, the airplane needs less power to fly than a gyroplane, but the gyroplane can make 500 kilometers per hour with only 2,900 horsepower, whereas the airplane, notwithstanding its high fineness ratio, needs 4,700 horsepower.

Chart 17 represents, in function of $\gamma = \frac{V}{nD}$, the changes in speed of advance, speed of propeller rotation, power absorption, and of the total propeller torque for horizontal flight at 3,000 meters - that is, for the entire speed range of horizontal flight, from hovering to maximum forward speed.

The surprising fact is, that contrary to what occurs with the ordinary propeller, the number of revolutions per second of the propellers decreases consistently as the speed V increases, which is evident as a result of the correlative increase in lift coefficient α_z.

Thus the tip speed πnD of the blades decreases in proportion to the increase in forward speed V, so that the sum $V + \pi nD$ may be almost considered as being a constant. This explains why, with this particular gyroplane, the tip speed at static thrust is 260 meters per second, and at 480 kilometers per hour, the resultant speed $V + \pi nD$ will only be 274 meters per second; i.e., only 5 percent higher and well below that of the velocity of sound.

This variation in the number of revolutions can, obviously, be mitigated by modifying the blade incidence, but there is a possibility that it will be necessary to provide a speed change for the gyroplane of the future with its high forward speeds.

According to figures 16 and 17, the gyroplane absorbs the same power at static thrust as at 450 kilometers per hour, which indicates quite clearly that this type of aircraft affords in some fashion, gratuitously, a translation at already very high speed. The power input is minimum for $\frac{V}{nD} = 0.9$, corresponding to a speed V of 225 kilometers per hour, while the propeller torque itself is minimum at a slightly lower speed, such as $V/nD = 0.6$ and $V = 150$ kilometers per hour.

Chart 18 shows the changes is coefficient β/α_z of the propeller torque against V/nD for the investigated and the future gyroplane. As for the airplane, the speed of minimum torque is that of the ceiling, and that is also the most advantageous for flight with one or more engines cut out.

Charts 19 and 20 reveal - plotted against V/nD - the slope of the lift and power curves α_z/γ^2 and β/γ^3, respectively, which follow when horsepower and lift are referred to speed of advance V, as for the conventional airplane, rather than to the number of revolutions n.

Lastly, chart 21 gives the polars versus swept-disk area conformable to formulas (10) and (12) for the tested gyroplane and for that of the future. The coefficient C_x is defined by the power equation (11) and coefficient C_z by the lift equation (13).

Drag $\tan \Phi$ and lifting quality q are given in terms of C_x and C_z by the formulas:

$$\tan \Phi = \frac{W}{PV} = \frac{C_x}{C_z} \tag{17}$$

$$q = \frac{P^{3/2}}{DW} = \frac{\sqrt{\pi}}{8} \delta^{1/2} \frac{C_z{}^{3/2}}{C_x} \tag{18}$$

When γ tends toward zero, i.e., upon approaching static sustentation, C_x and C_z increase indefinitely, and the polar has an infinitely rising branch; $\tan \Phi$ then increases indefinitely, the asymptotic direction being the axis of C_x. The quality at zero altitude then tends toward a limit q_o, making the polar asymptotic to the semicubic induced parabola:

$$C_x = \frac{\sqrt{\pi}}{8 q_o} C_z{}^{3/2} \tag{19}$$

Froude's theory affords a satisfactory approximation of the quqlity q_o at static thrust and without ground interference. It supposes the induced speed to be uniformly distributed over the swept-disk area, the value u on passing into this area, and $2u$ after passage. It finally affords the power input Pu and the quality at zero altitude $q_0 = \frac{\sqrt{\pi}}{4} = 0.443$, with the corresponding semi-cubic induced parabola previously cited and

$$C_x = \frac{1}{2} C_z^{3/2} \tag{20}$$

I have indicated in the foregoing that, in order to move at sufficient speeds, it was indispensable both from the point of view of design and of the stability, to hinge the rotating blades to the hub, and gave the reasons why this is justified. When the blades are rigid - and this is important - and the parameter of translation is quite high, the momentous variations in the lifting force exerted on a blade during rotation, produce periodic bending stresses which are not admissible unless the structure is very heavy. Besides, it undoubtedly engenders critical vibrations. The calculation of which I have given the results, are predicated on the assumption, from the aerodynamic point of view, that the blades are rigid and consequently make no allowance for the flapping action permitted by the articulations, and whose analysis is a very difficult problem.

Suffice it to say that this flapping, even for high values of Υ, has practically no detrimental effect on $\tan \Phi$. I shall demonstrate, moreover, the necessity, from the aerodynamic point of view, for allowing the blades 2° of freedom about the two perpendicular axes - one in the meridian plane, the other in a parallel plane in order to recover the powers brought into play.

It is said that when a wing in uniform translation is actuated by a vertical, sustained, periodic flapping motion, it is possible to effect a decrease and even a nullification of the drag by combining the oscillation of the aerodynamic resultant with the incidence variations (reference 4).

The diminution of the power necessary for the advance is found in the power consumed for upholding the flapping motion, with a propulsive efficiency solely a function of the effective aspect ratio of the wing. The efficiency is improved when the wing oscillates about an axis parallel to the span so as to attenuate the incidence variations by tending toward unity if the aerodynamic incidence were kept constant. In this extreme case the influence of the flapping motion will be zero and, likewise also, the power necessary to sustain this motion.

On the gyroplane the flapping motions are free, being caused by the variations of the resultant aerodynamic velocity. The blades, doubly hinged, are free to oscillate in a meridian, and in a palallel plane. Although the propeller is tilted, I designate the former with vertical flapping; the other, with horizontal flapping. When a blade advances in the sense of the speed of translation, it is raised with a certain phase difference by assuming, in this manner, at one of its points, a speed v, which combines with the aerodynamic speed U'. The resultant aerodynamic velocity, without its magnitude being substantially changed, then inclines upward at an angle $\epsilon = v/U'$.

The result is that the drag coefficient in the plane of rotation is increased by the component $\epsilon\ c_z$ of the lift coefficient; at the same time, the incidence is, of course, decreased by ϵ. But I have made the calculations on the basis of a mean lift coefficient, taking into account the natural and controlled incidence variations. In comparison with these figures, according to the foregoing, it will be seen that the drag in the plane of rotation is increased when the blade advances in the sense of the translation.

The inverse process takes place outside of the reversed-velocity region, when the blade recedes, but, as the resultant aerodynamic velocity is then much lower, there is no compensation. In addition, the drag within the reversed-velocity region - the lift being negative - is increased. The amplitude of this flapping is a function of the intensity of the restoring forces formed by the centrifugal force and the blade weight. When the blade faces in the inverse sense of the speed of advance, it is substantially perpendicular to the axis of rotation, and even slightly tilted downward owing to its own inertia. With fixed pitch the greatest elongation is obtained in about the most forward position, and the highest speed of climb in the meridian perpendicular to the translation.

In practice, according to our patented device, the amplitude of these vertical flapping motions is limited by the automatic pitch decrease, with the aid of an eccentric lever, in direct ratio to the rise. The maximum speed and elongation are thus reached sooner. I shall confine myself, on this subject, to the following little-known fundamental phenomena which underlie the theory of flapping motion.

1) Every vertical flapping motion develops - due to the fact that it superposes itself on the rotation of the propellers - combined centrifugal forces, perpendicular to the meridian plane of this flapping, which tend to make the blade advance when it is raised and retreat when it is lowered.

Every vertical flapping motion is therefore, necessarily, accompanied by a horizontal flapping motion of lower amplitude, these two flapping motions being not in phase.

2) The increase in power necessary for the rotation due to the drag increase in the plane of rotation, is compensated - at efficiency approaching flapping - by the power supplied in vertical flapping by the displacement of the lift.

3) This recovery is effected through the energy, in the horizontal flapping motion, of the combined centrifugal forces which, in this fashion, play the role of transformers of energy. As these combined centrifugal forces are due to vertical flapping, it is readily seen that the recovery of energy is contingent upon the combined flapping motions, vertical and horizontal: whence follows the justification of the principle of double articulation; no fraction of the considerable energy employed in the vertical flapping motion can be transformed and recovered except by permitting the horizontal flapping to be effected freely.

I shall give the mathematical demonstration of these fundamental properties.

The motive force of a blade being its rotation about its own axis at uniform speed ω, the vertical flapping constitutes a relative motion and gives rise to complementary accelerations.

Let β be the upward inclination of the blade in the plane of rotation, $v = r \frac{d\beta}{dt}$ the speed of rise of an element dm of the mass of the blade situated at distance r, v forming, with the axis of rotation, the angle β. The elementary combined centrifugal force of mass dm is perpendicular to v and to the axis of rotation, hence to the meridian of the blade and has, by virtue of the Coriolis theorem, the value:

$$dF_c = 2\omega\, v\, \beta\, dm = 2\omega\, r\, dm\, \beta \frac{d\beta}{dt} \tag{21}$$

With M, the total mass of the blade

r_g, distance of its center of gravity from the axis of rotation, we have:

$$\Sigma\, r\, dm = M\, r_g$$

The resultant combined centrifugal force then has, after integrating for the whole blade, the magnitude:

$$F_c = 2\omega\, r_g\, M\, \beta \frac{d\beta}{dt} \tag{22}$$

It is seen that this force F_c has the same magnitude as if the total mass were concentrated in the center of gravity, although this is not to be interpreted as being applied at that point.

If $H = M\,\omega^2\, r_g$ is the centrifugal force to which the blade is subject in its rotation about its axis, we may write:

$$F_c = 2H \frac{\beta}{\omega} \frac{d\beta}{dt} \tag{23}$$

which shows that this force can become relatively very important.

As the blade rises, this force - directed in the sense of motion due to rotation ω - is active. Contrariwise, it is resistant when the blade is lowered, and zero when the blade is perpendicular to the axis of rotation $(\beta = 0)$, or when its inclination is maximum or minimum $\left(\frac{d\beta}{dt} = 0\right)$. By integrating along the blade at a given instant, the resultant couple in relation to the articulation parallel to the axis of rotation, due to the forces dF_c, has the value:

$$J = \Sigma\, r\, dF_c = 2\omega\, \beta \frac{d\beta}{dt} \Sigma\, r^2\, dm = 2I\, \omega\, \beta \frac{d\beta}{dt} \tag{24}$$

where $I = \Sigma\, r^2\, dm = M\, \rho^2$ is the moment of inertia of the blade in ratio to the articulation, and ρ the corresponding radius of gyration. Thus it is seen that the force F_c is applied at a distance a from the axis, so that $a\, F_c = \Phi$. From formulas (22) and (24) follows:

$$a = \frac{I}{Mr_g} = \frac{M\rho^2}{Mr_g} = \frac{\rho^2}{r_g} \tag{25}$$

But if ρ_g is the radius of gyration relative to the center of gravity, $\rho^2 = \rho_g{}^2 + r_g{}^2$, hence:

$$a = r_g + \frac{\rho_g{}^2}{r_g} \tag{26}$$

the well-known formula defining the center of shock with respect to the axis which, in consequence, is the point of application of force F_c farther away from the axis than the center of gravity.

The combined centrifugal couple J thus defined, is periodic and of particular value; it contributes directly to the conservation of the horizontal flapping motion.

Then let ψ be the elongation of horizontal flapping at time interval t, positive when in direction of motion due to rotation ω, all flapping motions having as common period that $T = \frac{2\pi}{\omega}$ of a propeller revolution.

I shall demonstrate this important theorem in the following manner: The recoverable vertical flapping energy at each propeller revolution is precisely the work of the combined centrifugal couple J in the horizontal flapping motion. The work of couple J in the period is evidently the sum of the work $\underline{T}$ and $\underline{T'}$ of this couple in the rotation at uniform angular velocity ω on one hand, and in the horizontal flapping superposed on this motion, on the other.

The differential of the work $\underline{T}$ is

$$d\underline{T} = J\, \omega\, dt = 2I\, \omega^2\, \beta\, d\beta = I\, \omega^2\, d\beta^2 \tag{27}$$

after substituting $d\beta^2$ for $2\beta\, d\beta$.

As elongation β, and consequently its square, also assume the same values at the end of an interval equal to the period, it is seen that the work $\underline{T}$ within the period is zero.

The couple J can therefore furnish work only in the horizontal flapping motion, and the value of this work in period T is:

$$\underline{T'} = \int_0^T J \frac{d\psi}{dt} dt = 2I \omega \int_0^T \beta \frac{d\beta}{dt} \frac{d\psi}{dt} dt \qquad (28)$$

Now it remains to be proved that this work is precisely equal to that of the aerodynamic lift during vertical flapping. With this in view, I shall write the equation for vertical flapping.

The blade rotates at speed $\omega + \frac{d\psi}{dt}$, so that the centrifugal returning moment due to an element of mass dm is:

$$dC_i = r^2 \, dm \left(\omega + \frac{d\psi}{dt}\right)^2 \beta \qquad (29)$$

Disregarding $\left(\frac{d\psi}{dt}\right)^2$ before ω^2 and $2\omega \frac{d\psi}{dt}$ and designating the moment of inertia of a blade with respect to one articulation with I - (it is practically the same thing, whether considering one or the other of the two articulations) - we have:

$$C_i = I\beta \left(\omega^2 + 2\omega \frac{d\psi}{dt}\right) \qquad (30)$$

With C_a as constant couple due to the lift, and C_p as constant couple due to the weight of the blade, the differential equation of the vertical flapping motion reads as follows:

$$I \frac{d^2\beta}{dt^2} = C_a - C_p - C_i \qquad (31)$$

That is, by replacing C_i by its value:

$$I \left(\frac{d^2\beta}{dt^2} + \omega^2 \beta + 2\omega \beta \frac{d\psi}{dt}\right) = C_a - C_p \qquad (32)$$

This equation is absolutely general, whatever the laws of incidence variations affecting C_a may be. In this vertical flapping the elementary work of the aerodynamic lift decreased by that of the weight of the blade is:

$$dT_1 = (C_a - C_p)\ d\beta = I\left(d\beta\ \frac{d^2\beta}{dt^2} + \omega^2\ \beta\ d\beta + 2\omega\ \beta\ d\beta\ \frac{d\beta}{dt}\right) \tag{33}$$

but

$$d\beta\ \frac{d^2\beta}{dt^2} = d\beta\ \frac{d}{dt}\ \frac{d\beta}{dt} = \frac{d\beta}{dt}\ d\ \frac{d\beta}{dt} = \frac{1}{2}\ d\left(\frac{d\beta}{dt}\right)^2, \quad \beta\ d\beta = \frac{1}{2}\ d\beta^2$$

Hence, we can write:

$$dT_1 = I\left[\frac{1}{2}\ d\left(\frac{d\beta}{dt}\right)^2 + \frac{\omega^2}{2}\ d\beta^2 + 2\omega\ \beta\ d\beta\ \frac{d\psi}{dt}\right] \tag{34}$$

At the beginning and end of the period, β and $\frac{d\beta}{dt}$, and likewise, their squares, assume the same values; hence, the first two terms yield zero work in this period.

The work recoverable in the duration of a period, reduces to

$$\underline{T_1} = 2I\ \omega \int_0^T \beta\ \frac{d\beta}{dt}\ \frac{d\psi}{dt}\ dt \tag{35}$$

which is precisely the value of the work $\underline{T'}$ of couple J in horizontal flapping. And, since the work of the blade weight is obviously zero in the period, work $\underline{T_1}$ represents exactly that which is recovered from the work of the aerodynamic lift.

It is therefore readily seen that the recovery of energy hinges on the combination of the two simultaneously dephased flapping motions, horizontal and vertical, and owing to the intervention of the combined centrifugal forces. Without the freedom of horizontal flapping, no recovery of energy is possible.

This brings us to the design of the gyroplane of the future (figs. 22, 23, 24), which weighs from 15 to 17 tons, has three-blade rotors of 25-meter diameter, and a solidity ratio $h_0 = 0.07$ or $34\ m^2$ area.

I have always assumed $\mu = 1.5$, the mean lift coefficient c_z of the blade elements being 1.5 times that which corresponds to their fineness ratio.

The engines, four in number, housed in one compartment of the aircraft, should develop, at 3,000 meters, a total maximum power of around 3,600 horsepower. The gyroplane should be able to fly at 3,000 meters, with only 2,000 horsepower, at a forward speed of 250 kilometers per hour (155 miles per hour), whereas with 2,400 horsepower, the speed is to be 400 kilometers per hour (248 miles per hour). The drag of the body and of the accessories corresponds to that of a 0.56 m^2 thin flat plate.

I have compared, as seen, the possibilities of such a gyroplane with those of an airplane of the same tonnage, both in horizontal flight with full load, at 3,000 meters.

The weight balance for a design of the same quality is in favor of the gyroplane, whose rotating wing system - not being subjected to any appreciable bending moment - is definitely much lighter than the fixed wings of an airplane. One may figure the gain in dead weight at 10 percent of the total weight. The airplane, to counteract this added weight, would have to be equipped with less powerful engines, which in turn would lower its top speed.

Now, in regard to cruising flight, the gradual reduction in weight due to fuel consumption must be borne in mind. Then, by judiciously combining the altitude increase with that of $\gamma = V/nD$, it is possible to realize the condition of flight with constant horsepower, while remaining within the limits between which the over-all fineness $\tan \Phi$ changes little - in fact, remains practically constant over fairly large speed ranges, as I have already indicated.

Under these conditions, the formula $V = \frac{W}{P \tan \Phi}$ shows that the speed increases continuously in inverse ratio of the total weight without the altitude reached at the end of the trip becoming excessive.

If the fuel consumption amounts to 36 percent of the total weight, which is equivalent to stages of 4,600 kilometers in still air, the speed of 400 kilometers may even be raised to 625 kilometers per hour, which corresponds to a mean speed of 500 kilometers.

Such a result is impossible to achieve with the airplane considered here, because $\tan \Phi$ increases much faster with the speed than it does for the gyroplane and, to raise the speed, it would have to reach heights where the power of its engines could not be maintained. In fact, it does not seem possible, with the very best airplanes, actually to envisage a mean speed of over 400 kilometers per hour, at the time, at 8,000 or 10,000 meters altitude.

Objections may be raised to my assumed 130 kg/m² wing loading of the airplane. But these are figures actually in use, and I have chosen for a gyroplane a somewhat large diameter, carrying at full load only 440 kg/m² blade loading, so as to provide a margin of excess sustentation at take-off in order to be able, with engines cut out, to descend in a glide, like an autogiro, the wings - with a loading of only 31 kg/m² - being in autorotation with respect to swept-disk area; and lastly, to be able to fly with one of the four engines stopped.

If an airplane of 200 kg/m² loading could be realized, it would necessarily have to be launched by catapult. The reduction in wing structure involves, probably $c_{x_0} = 0.021$. That being so, the gyroplane which I have considered should not have a higher over-all fineness than the airplane at 3,000 meters, except at speeds above 420 kilometers per hour instead of 380 kilometers per hour, as before.

Quite apart from the advantages of speed and lightness of design, the gyroplane has other particular qualities not possessed by any other type of aircraft - and important enough to justify the studies undertaken, even if the maximum speed should not exceed that of our conventional airplanes. These are:

1. Practically no response to aerial eddies, the flexibility of the articulated revolving rotors forming a particularly efficacious aerodynamic suspension.

2. The absence of stalling, since stationary sustentation is possible, and the facility in case of engine failure, to descend in the manner of airplanes with low wing loading, like the autogiro descends.

3. Possibility of joining several engines to the central shaft and installation in a comfortable engine compartment, affording uninterrupted inspection and ease of accessibility, with liberty of cutting out the engines at will.

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km/h x 0.62137 = mi./hr.

With the high reduction gear ratio (10 to 20) between engine and propellers, a simple worm (endless screw) could be used and which actually has been developed and is of sufficient efficiency.

4. Possibility of vertical ascent on ground or water. The gyroplanes will be more or less amphibians. One can even visualize refueling being effected with much less difficulty than with seaplanes.

5. Small over-all dimensions for storage, since the articulated blades are easily folded.

6. Inappreciable military qualities, since a gyroplane can take observations in cases where absence of motion is particularly desired; small gyroplanes seem to be made for artillery spotting.

7. As regards naval aviation, gyroplanes of from 2-4 tons could replace the actually used deckplane, along with the bulky and heavy catapults, to good advantage. Airplane carriers will, undoubtedly, no longer be necessary.

Deck-landing gyroplanes, with their small bulk, once the blades are folded, can be used in much larger numbers on every battleship.

Such tempting results are, quite obviously, not obtainable before overcoming certain difficulties besetting every new development, and which are beyond the scope of this report. But I do feel that it is fitting to make known at this time the conclusions to which my investigations have enabled me to arrive.

Certain readers - even among technicians most familiar with aeronautical problems - may be surprised, but I am firmly convinced that when they have studied my formulas and reflected on the posed problem, their final conclusions will be similar to mine.

I may have been led to assume, in my examples, qualities beyond reach in the near future, but even so, the judgment and nature of my conclusions are, I believe, incontestable.

I hope I have been able to make you share my personal opinion, that the gyroplane problem should not be given up but, on the contrary, attacked in all seriousness. Success will so much more quickly crown the efforts still necessary, as these efforts are more unanimous, better understood, more encouraged and coordinates, and it is hoped that France will again take first place in this new stage of progress in aerial navigation.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

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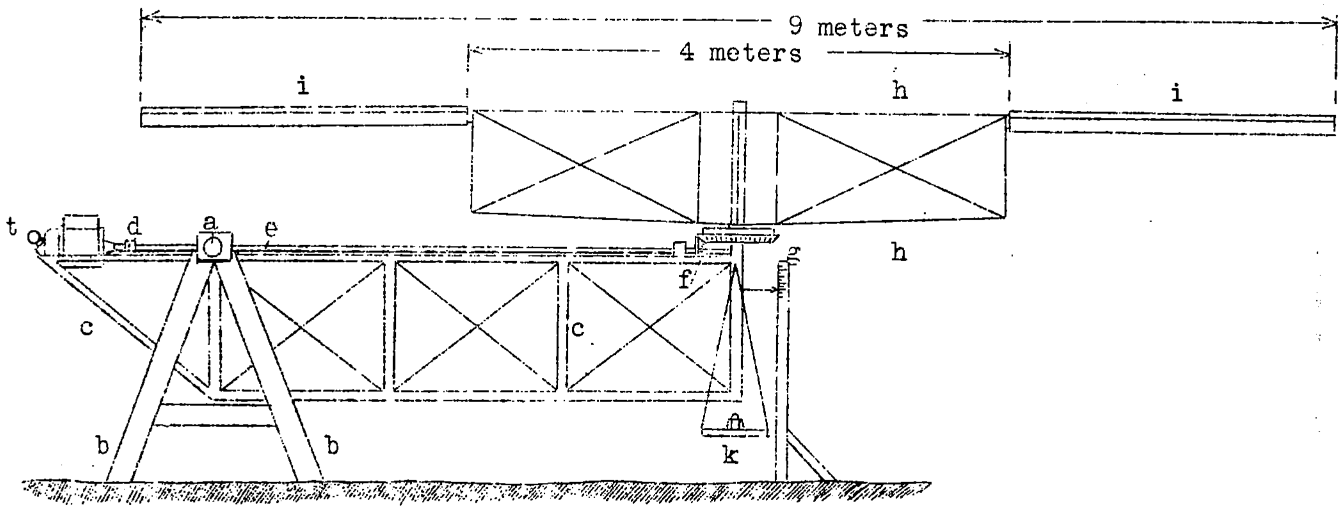


Figure 1. The dynamometric balance of 1907.

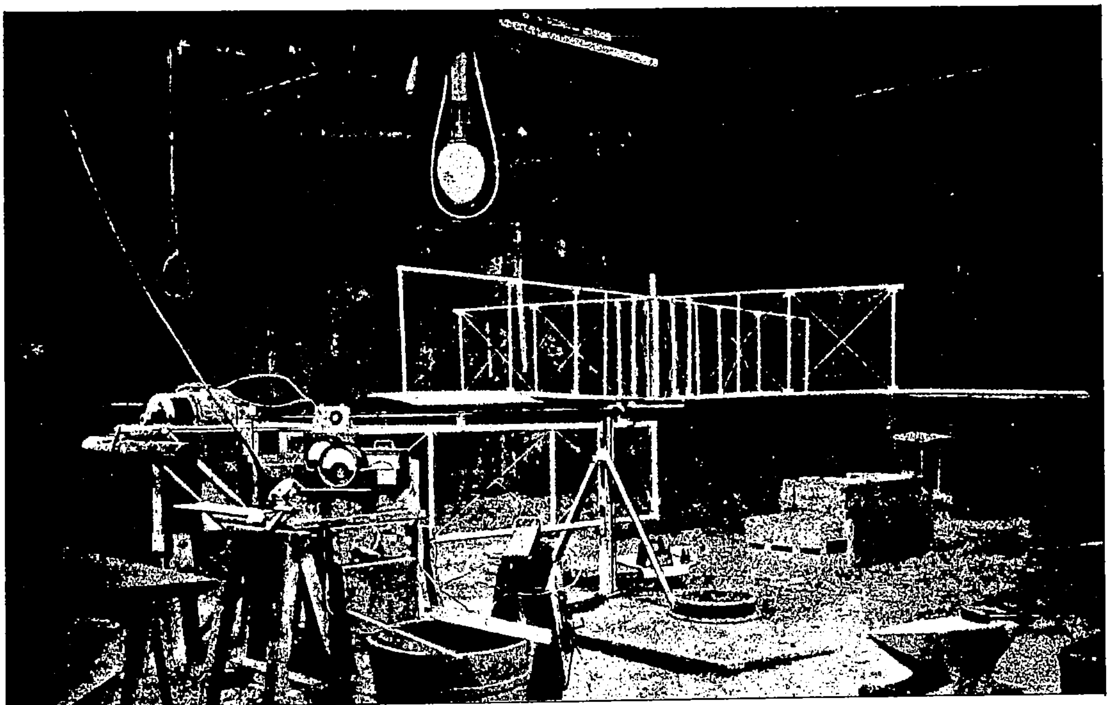


Figure 2 The dynamometric balance of 1907

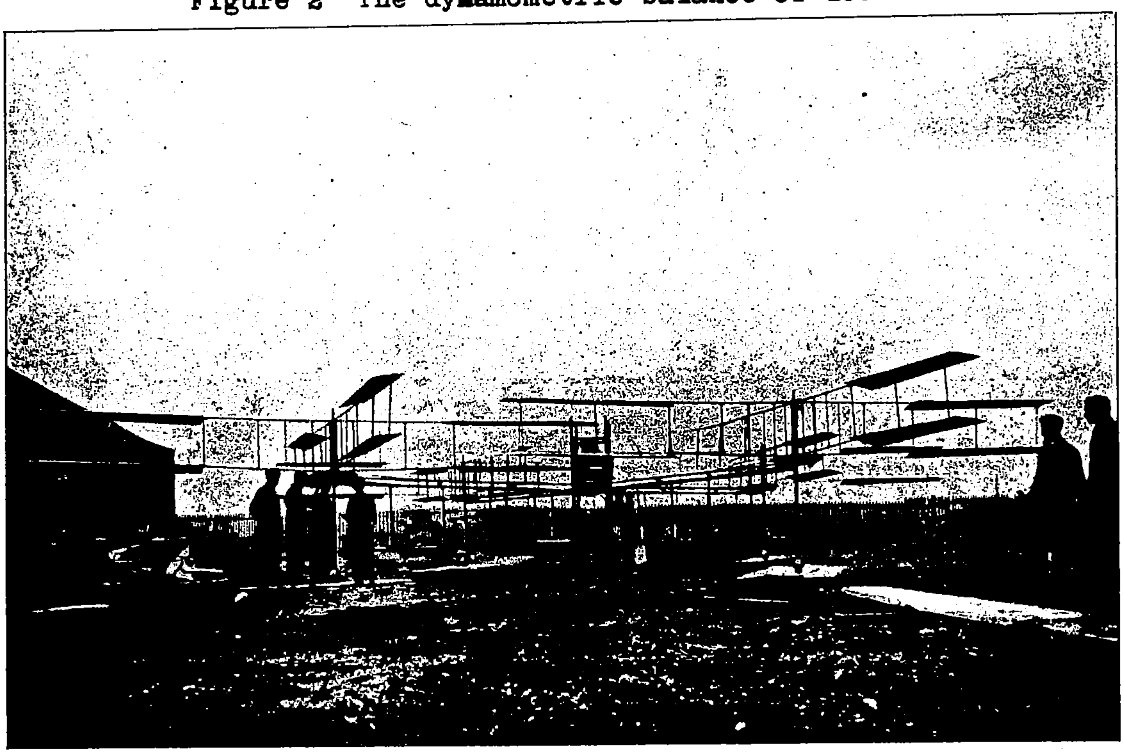


Figure 4 The gyroplane of 1907

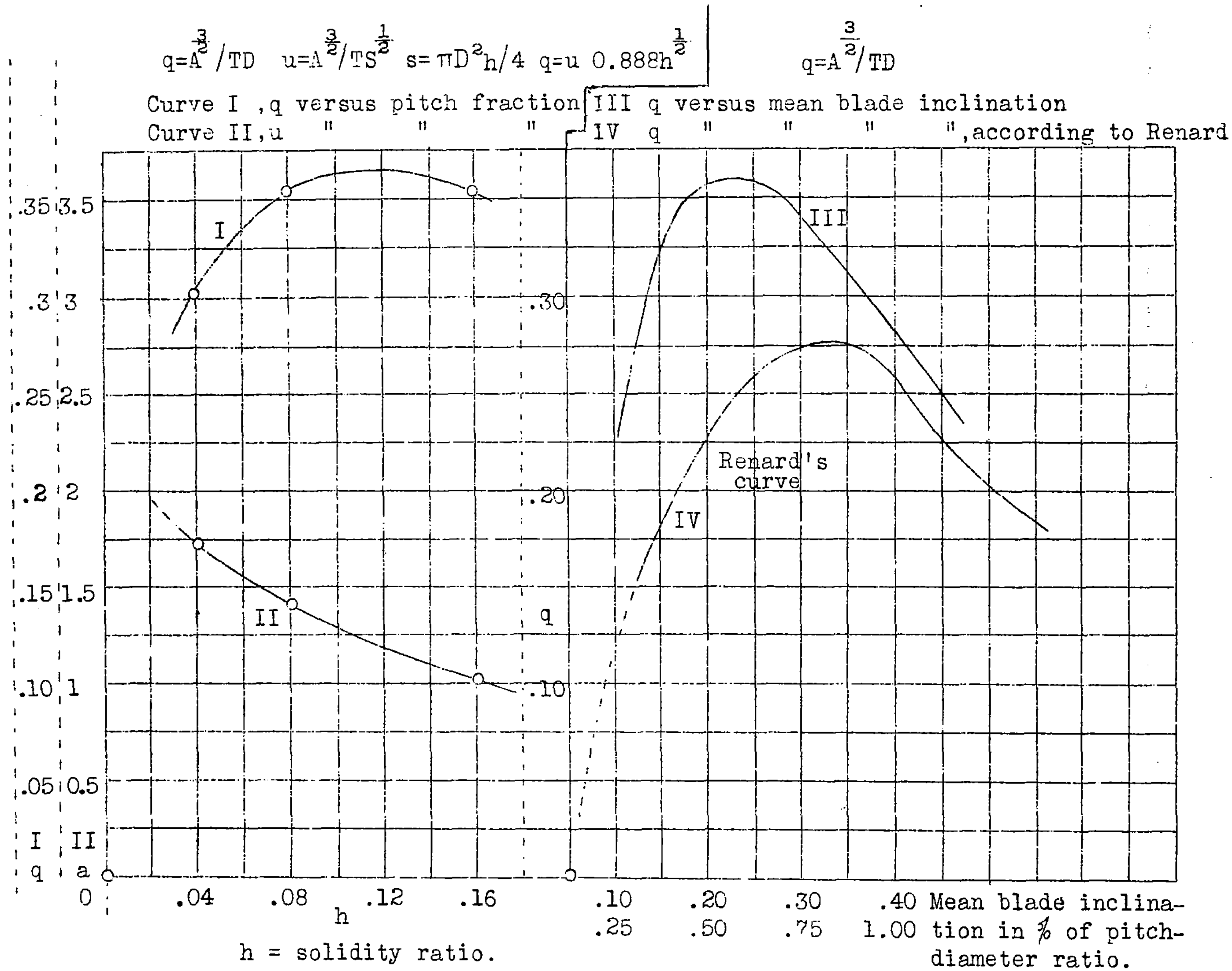


Figure 3.- Test data obtained in 1907 with the balance, Fig.2.

Figure 5.- 1935 gyroplane in test flight.

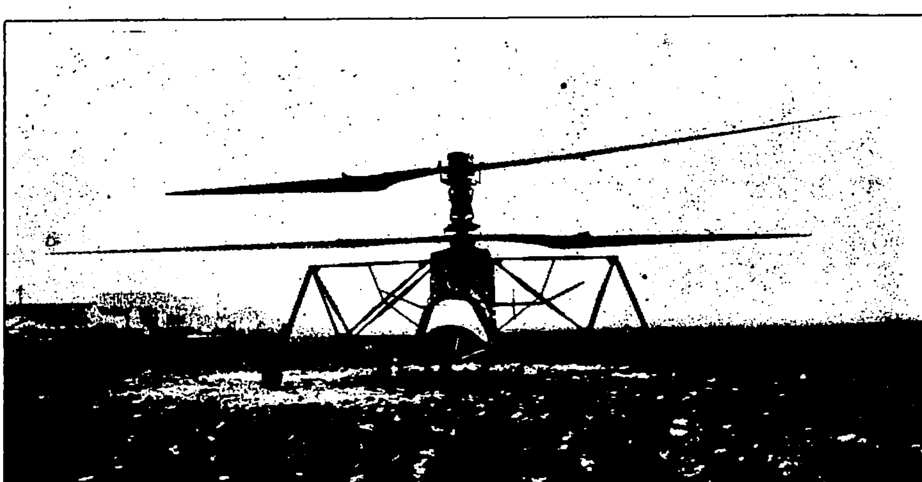


Figure 6.- 1935 experimental gyroplane.

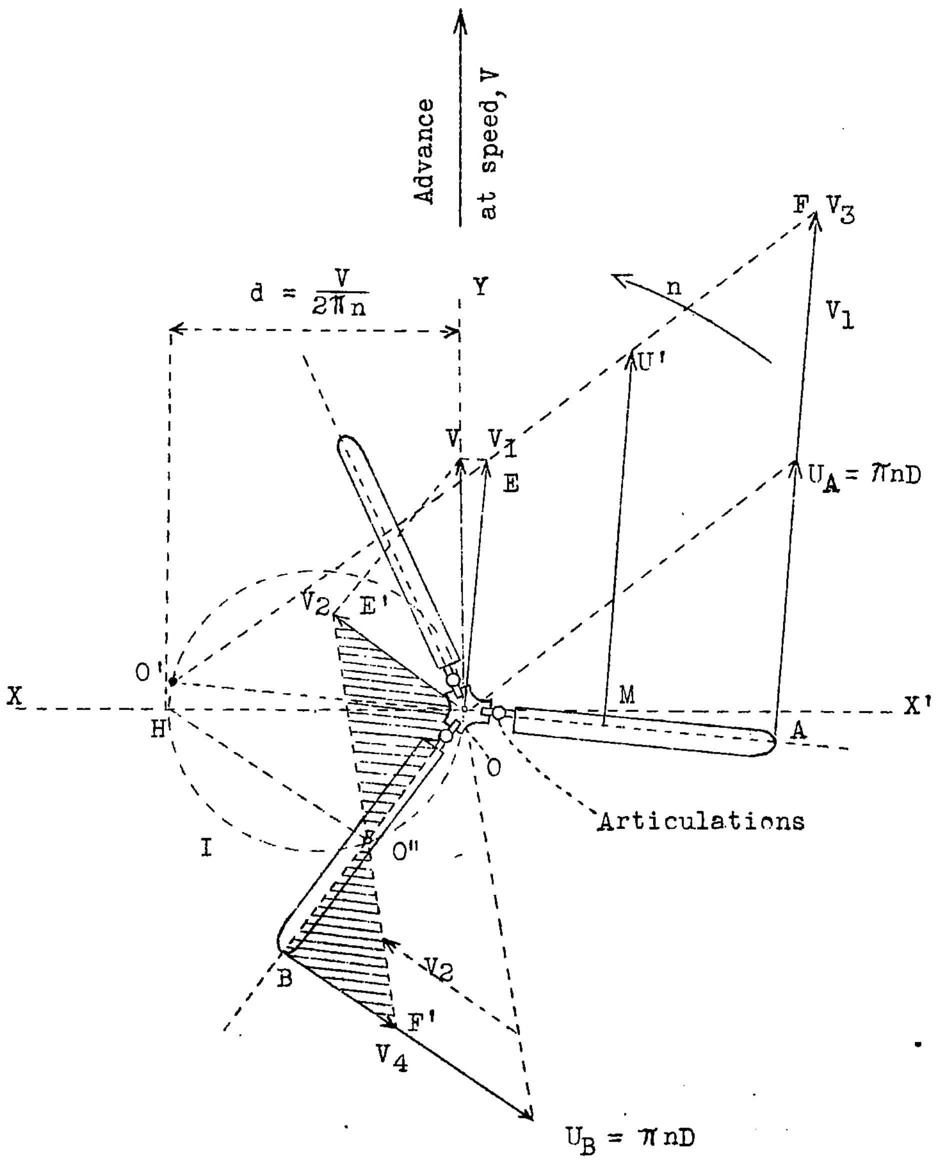


Figure 7.- Distribution of speed and reversed-velocity region for a propeller revolving at rate n and advancing at speed V.

$u = \dfrac{P^{\frac{3}{2}}}{s^{\frac{1}{2}} W}$, $q = 0.888 u \sqrt{h_o}$ I experimental curve

II theoretical curves for $\mu = \sqrt{3}$:

$$u^4 = \frac{0.000142}{c_{x_o}\left[h_o(1+1/N)+0.03\right]^3} \qquad (c_{x_o} = 0.015)$$

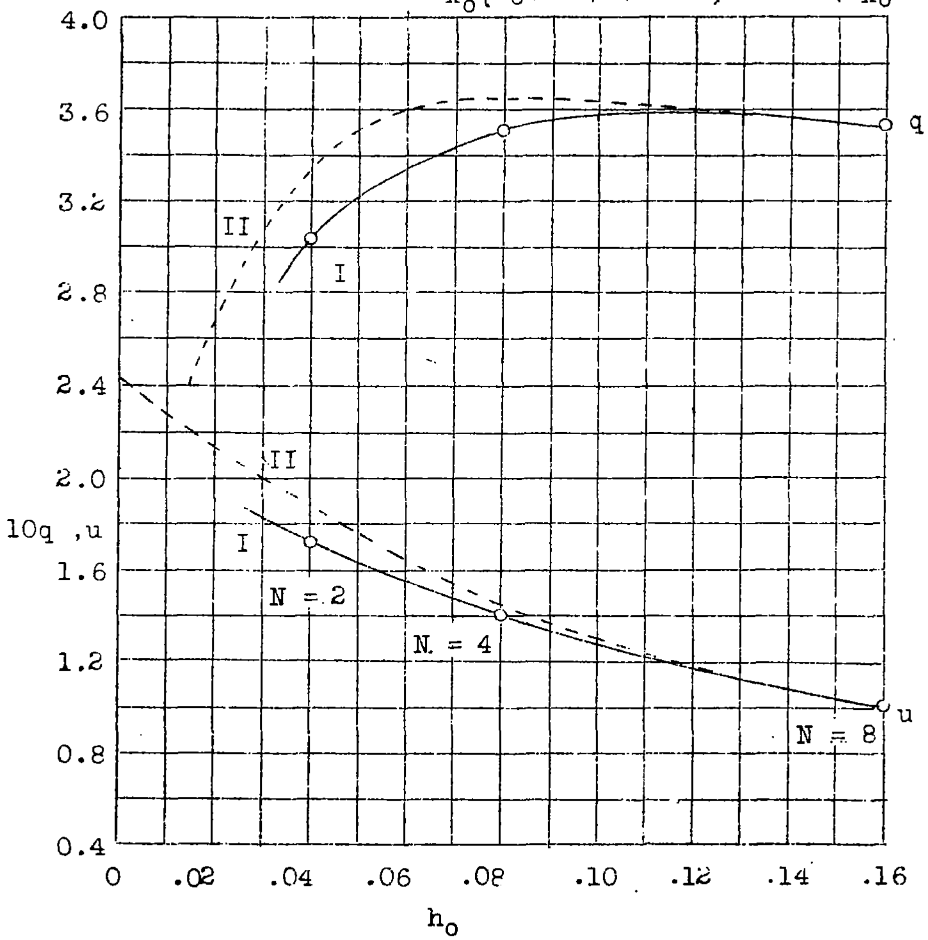


Figure 8.- Lifting quality q and inherent quality u of blades of total area s plotted against solidity ratio $h_o \dfrac{s}{\pi D^2/4}$ (near ground $\delta = 1$).

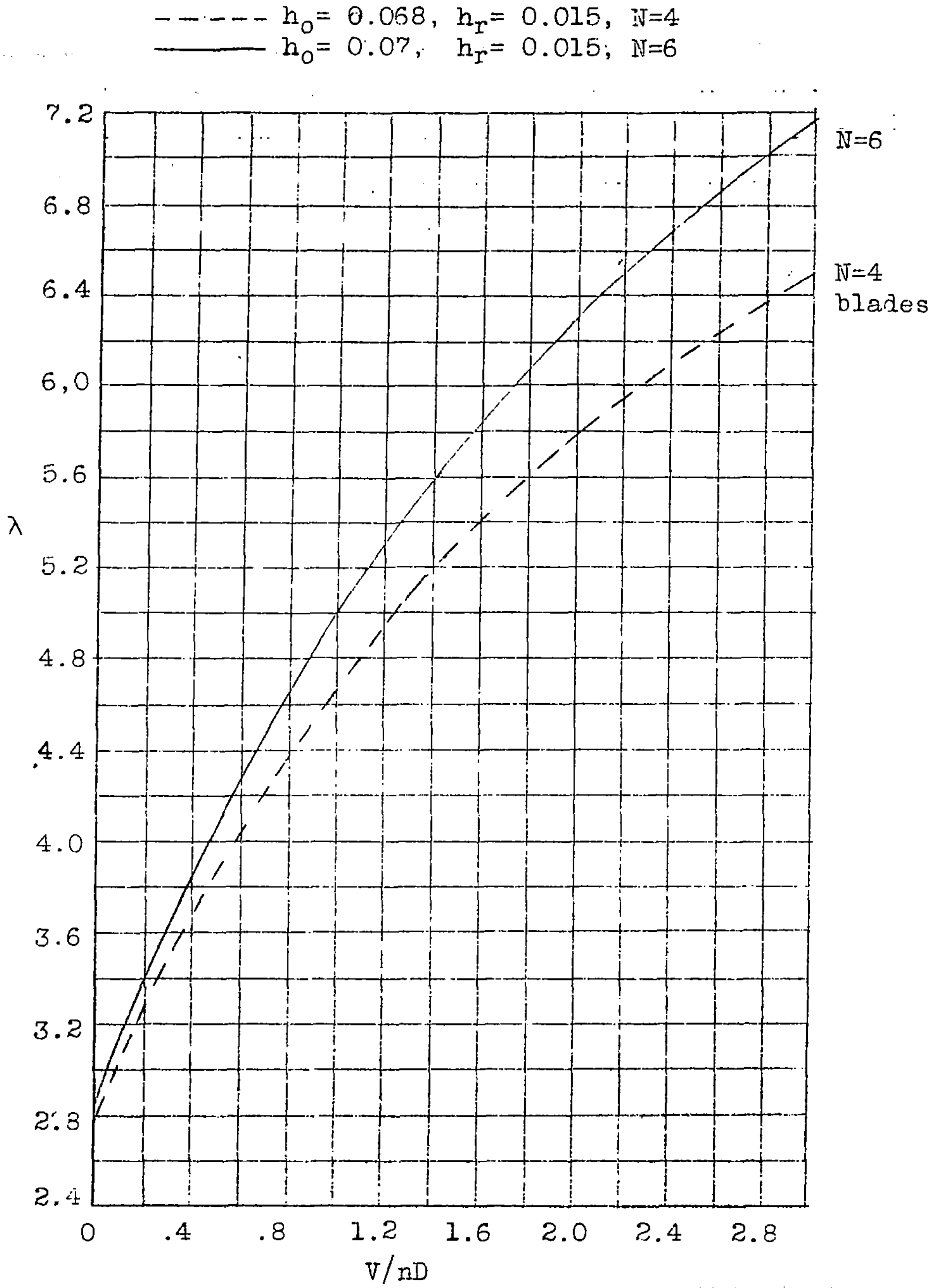


Figure 9.- Gyroplane.- Effective aspect ratio λ of a blade.

Propeller flatwise in the wind - tested in Eiffel wind tunnel.

Relative pitch: 0.55, $h_o = 0.14$ (2 blades)

$h_o = 0.28$ (4 blades)

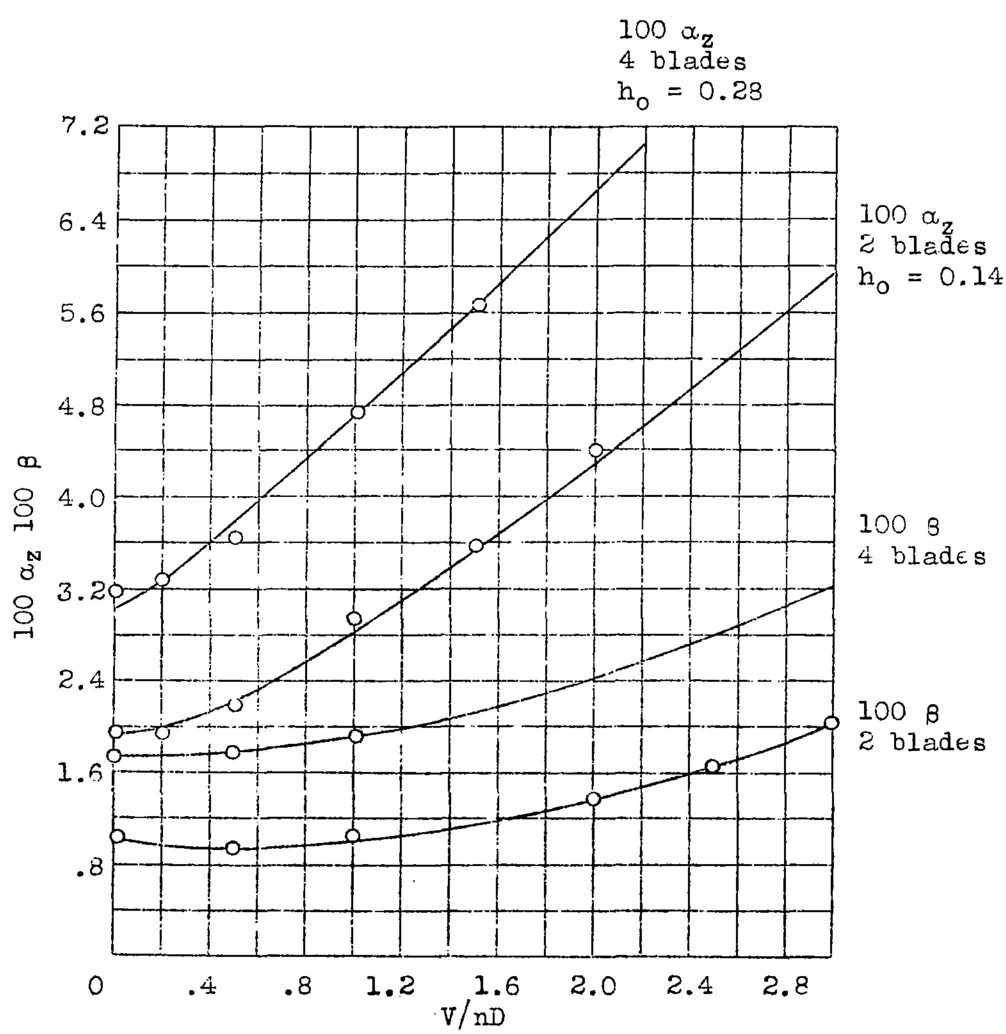


Figure 10.- Theoretical curves for lift coefficients and horsepower. The dots represent tests.

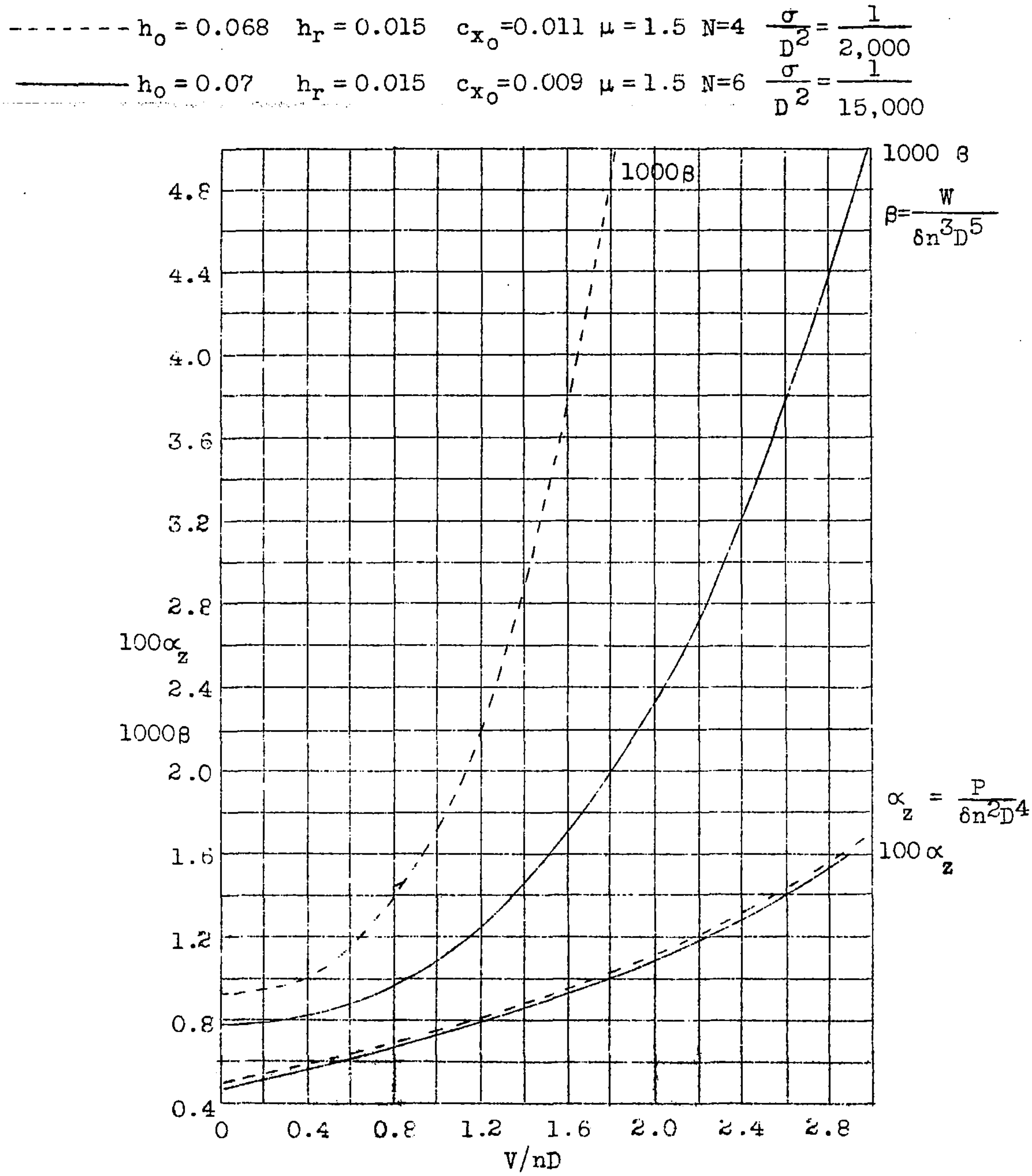


Figure 11.- Gyroplane. Lift and power coefficients.

I, $h_o = 0.068$, $h_r = 0.015$, $c_{x_o} = 0.011$,

$\mu = 1.5$, $N = 4$ blades, $\sigma/D^2 = 1/2000$

II, $h_o = 0.07$, $h_r = 0.015$, $c_{x_o} = 0.009$,

$\mu = 1.5$, $N = 6$ blades, $\sigma/D^2 = 1/15000$

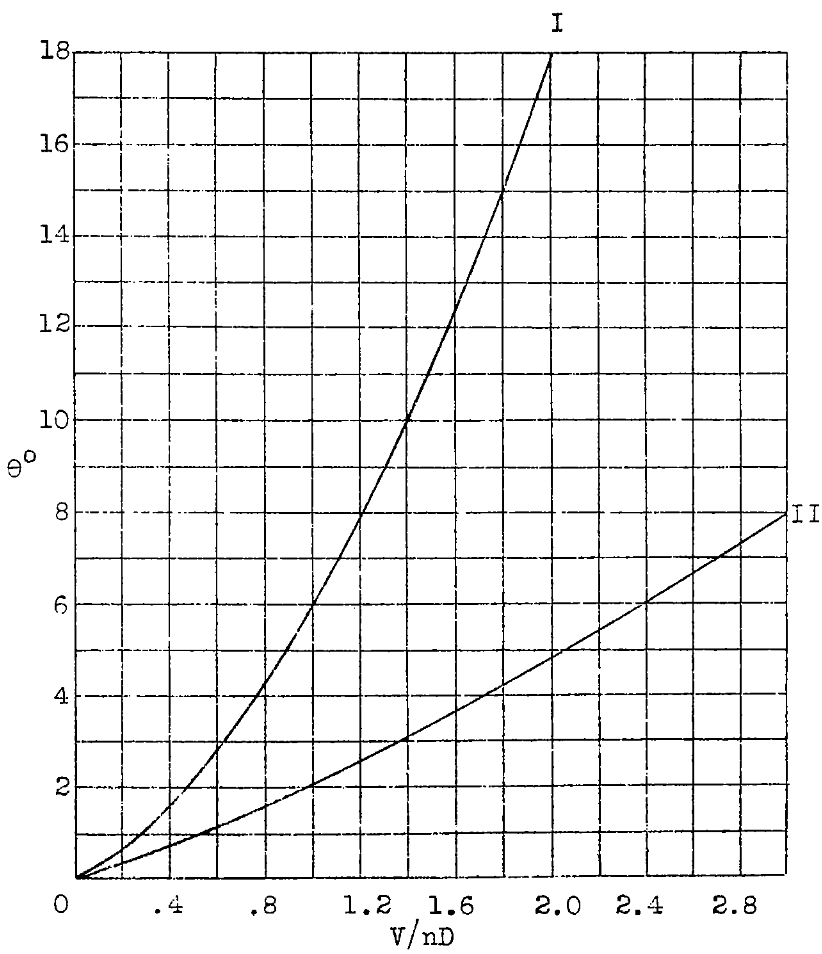


Figure 12.- Angle of propulsive inclination θ in horizontal flight.

	h_o	h_r	c_{x_o}	μ	N	$\frac{\sigma}{D^2}$
I	0.068	0.015	0.011	1.5	4	1/2,000
II	0.07	0.015	0.009	1.5	6	1/15,000

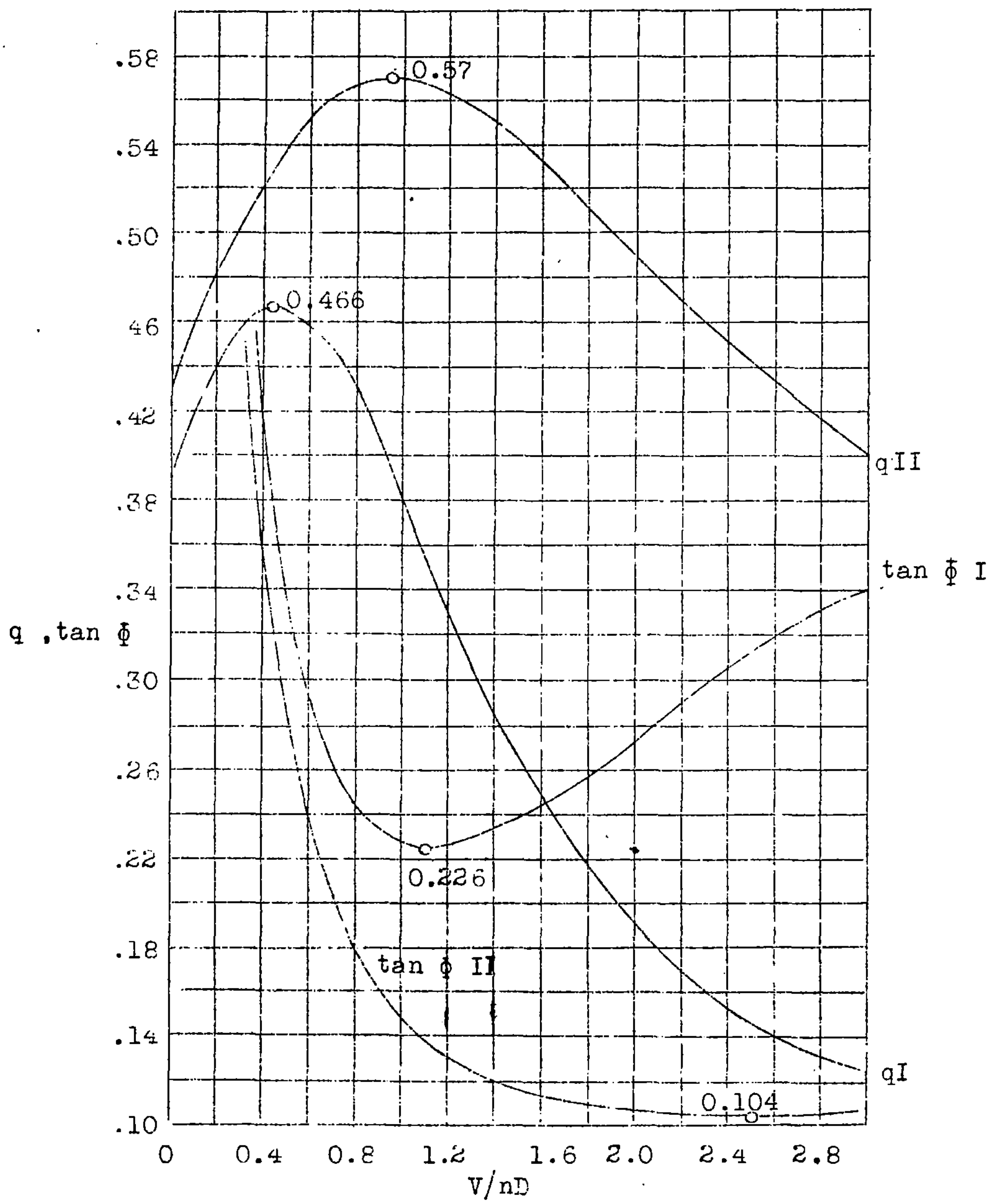


Figure 13.- Quality q and apparent relative drag tan ϕ.

$\sigma = 0$, $h_o = 0.07$, $h_r = 0.015$, $c_{x_o} = 0.009$,

$\mu = 1.5$, $N = 6$, $\frac{\sigma}{D^2} = 0$

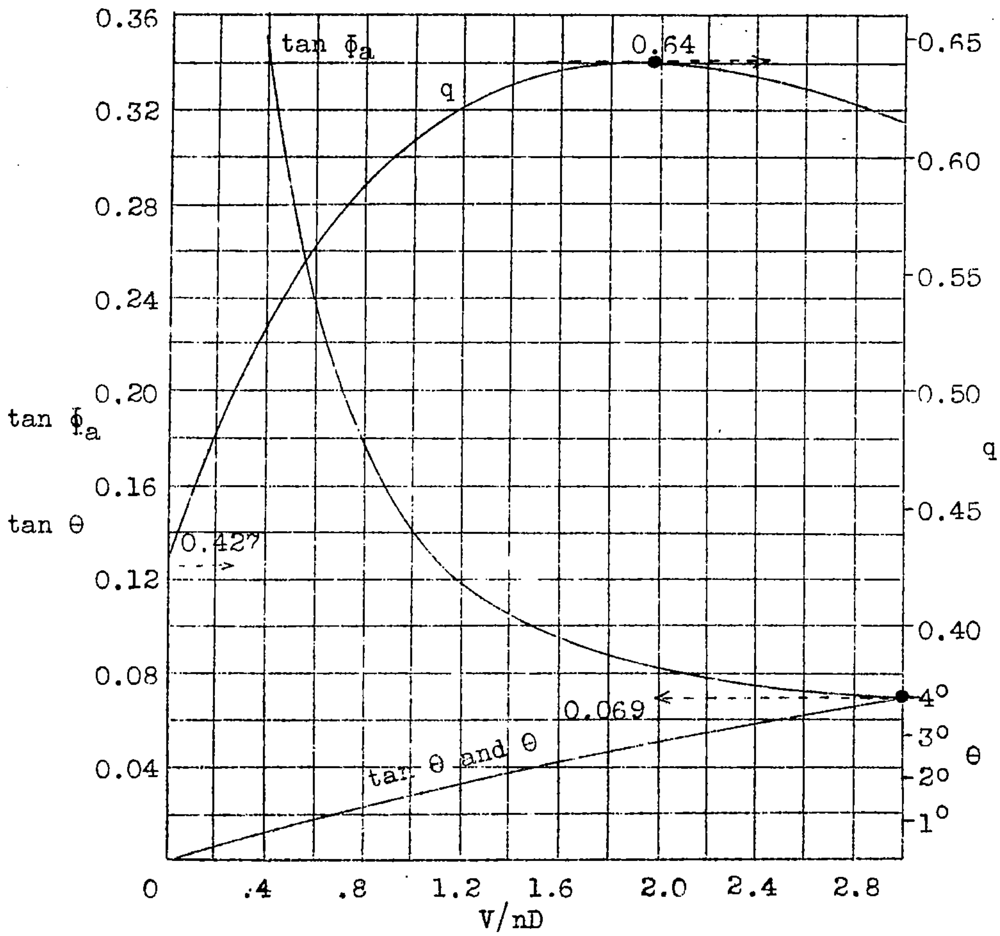


Figure 14.- Gyroplane without parasite drag(rotors revolving only). Overall fineness tan Φ_a , quality q at zero height and angle tan Θ of rotor tilt.

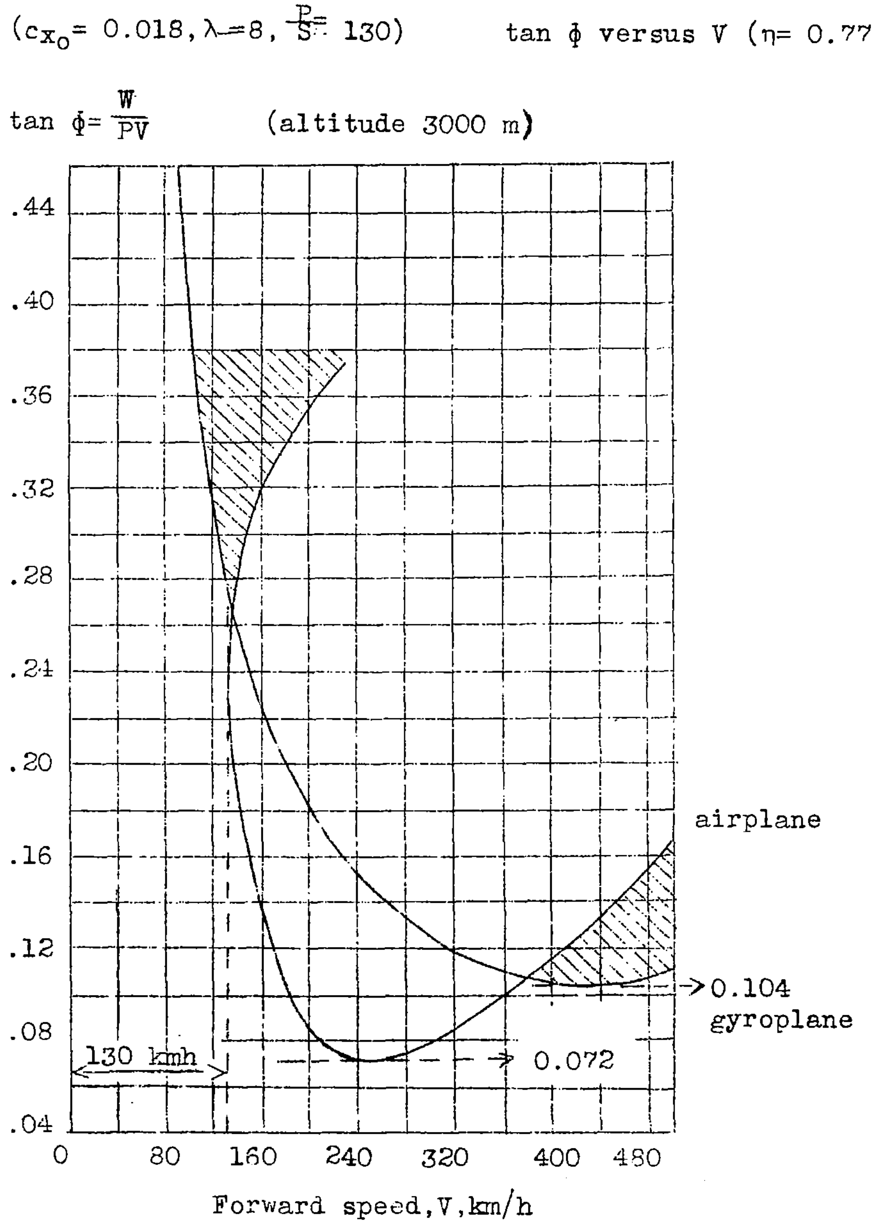


Figure 15.- Gyroplane of the future (σ/D^2= 1/15000) and airplane of exceptional aerodynamic qualities.

$$q = \frac{P^{\frac{3}{2}}}{DW} = \sqrt{\delta}\ \frac{\alpha_z^{\frac{3}{2}}}{\beta}$$

Gyroplane of the future $\left(\frac{\sigma}{D^2} = \frac{1}{15,000}\right.$, D=25m, P=15 tons)

Airplane of same weight (c_{x_0}=0.018 , λ =8 , $\frac{P}{S}$=130, η=0.77)

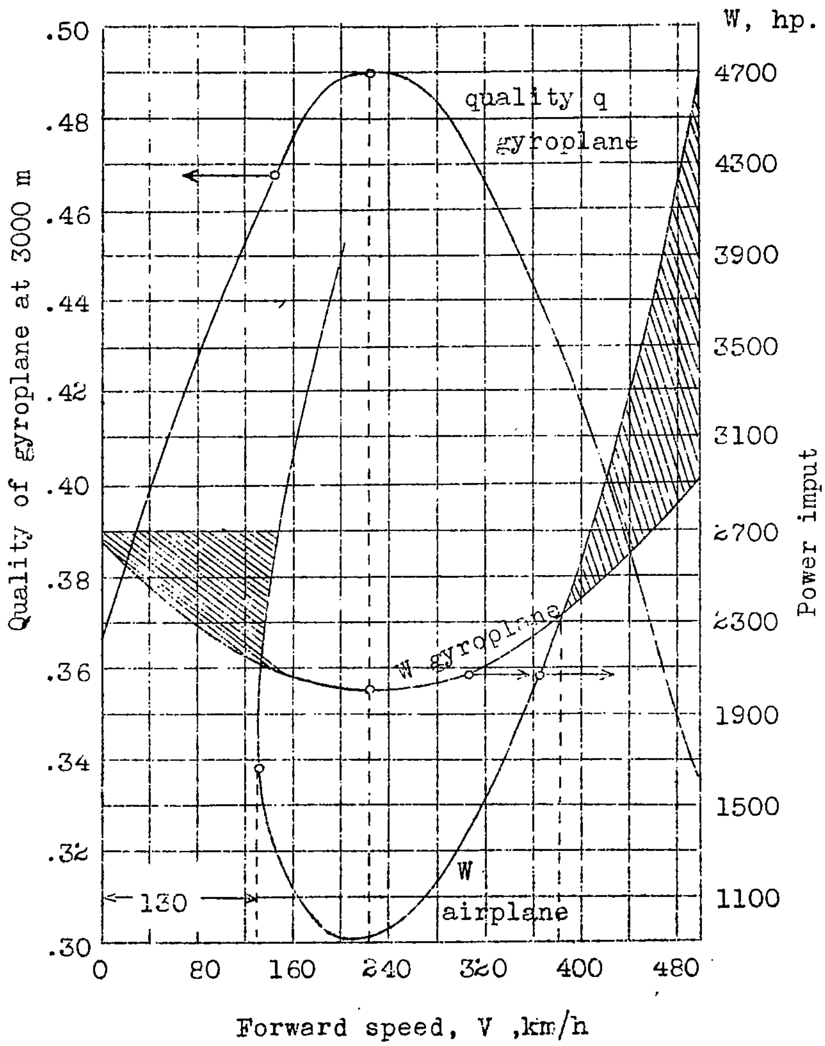
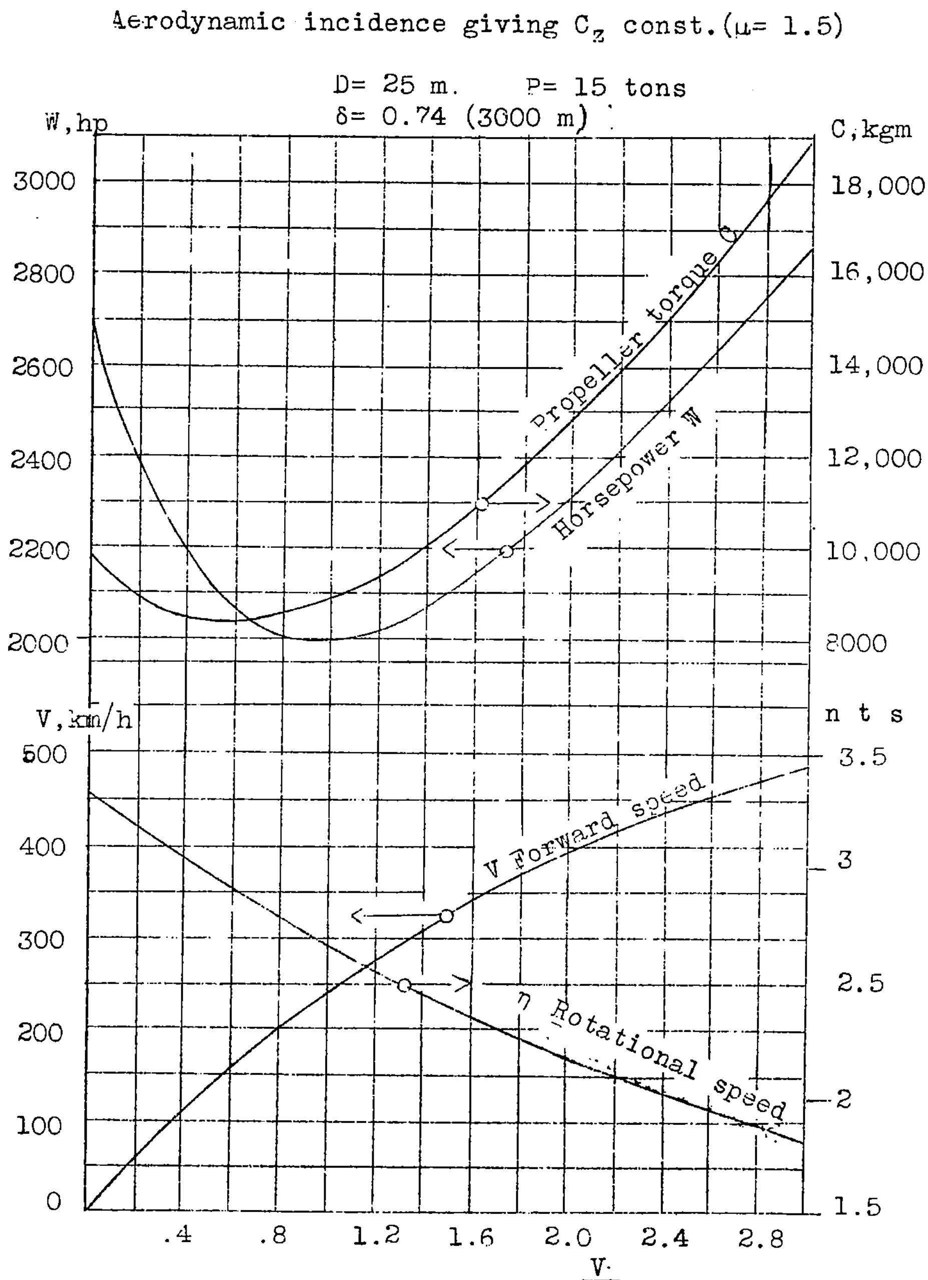


Figure 16.- Gyroplane of the future and airplane of the same weight. Power absorbed and quality q of gyroplane at 3000 m.



Parameter of translation, $\frac{V}{nD}$

Figure 17.

$$\frac{\beta}{\alpha_z} = 2\pi \frac{C}{DP}$$

D, Diameter
P, Total weight

$h_o = 0.068$, $h_r = 0.015$, $c_{x_o} = 0.011$,
$\mu = 1.5$, $N = 4$, $\sigma/D^2 = 1/2000$

$h_o = 0.07$, $h_r = 0.015$, $c_{x_o} = 0.009$,
$\mu = 1.5$, $N = 6$, $\sigma/D^2 = 1/15000$

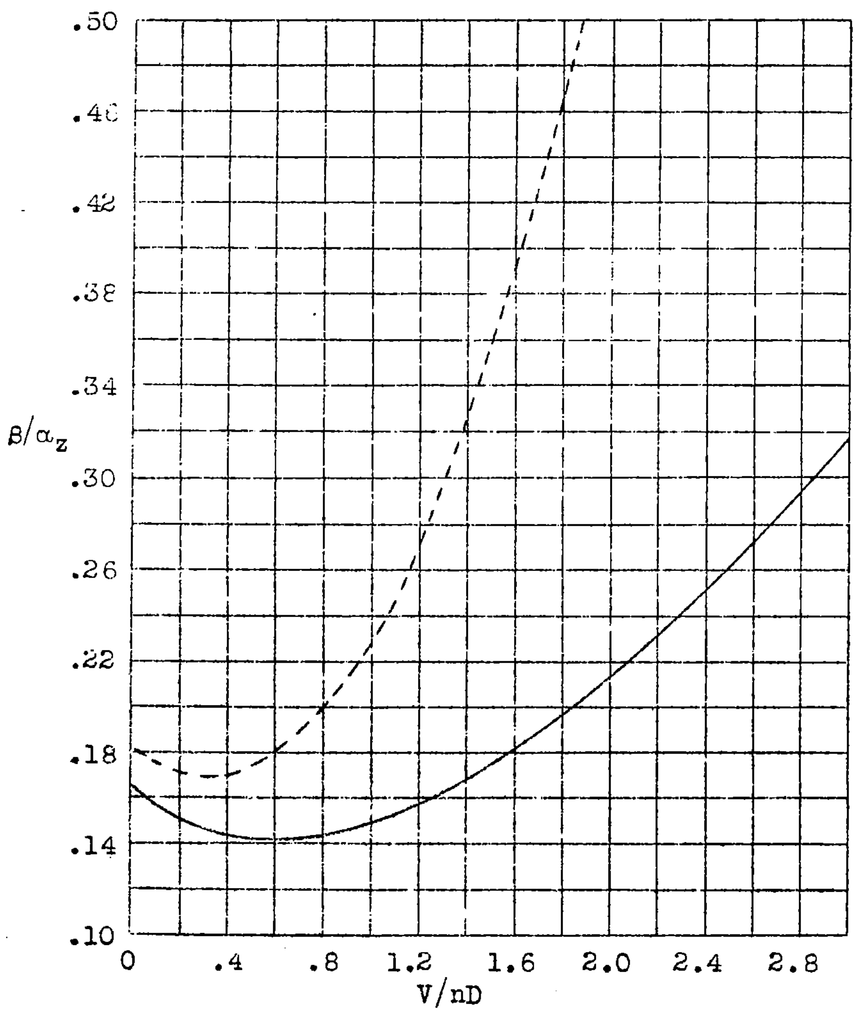


Figure 18.- Coefficient of propeller torque.

$\alpha_z/\gamma^2 = P/\delta D^2 V^2$

------ $h_o = 0.068$, $h_r = 0.015$, $c_{x_o} = 0.011$, $\mu = 1.5$

$N = 4$ blades, $\sigma/D^2 = 1/2000$

———— $h_o = 0.07$, $h_r = 0.015$, $c_{x_o} = 0.009$, $\mu = 1.5$

$N = 6$ blades, $\sigma/D^2 = 1/15000$

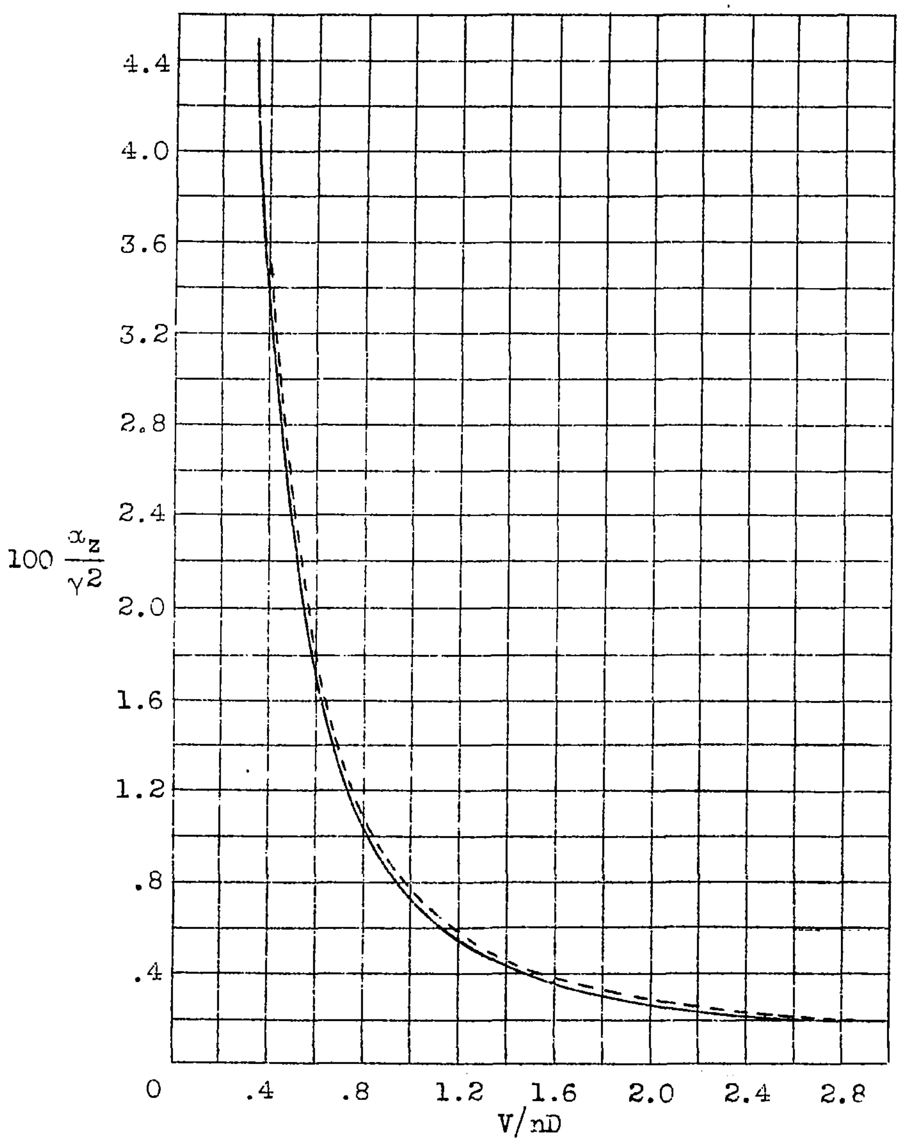
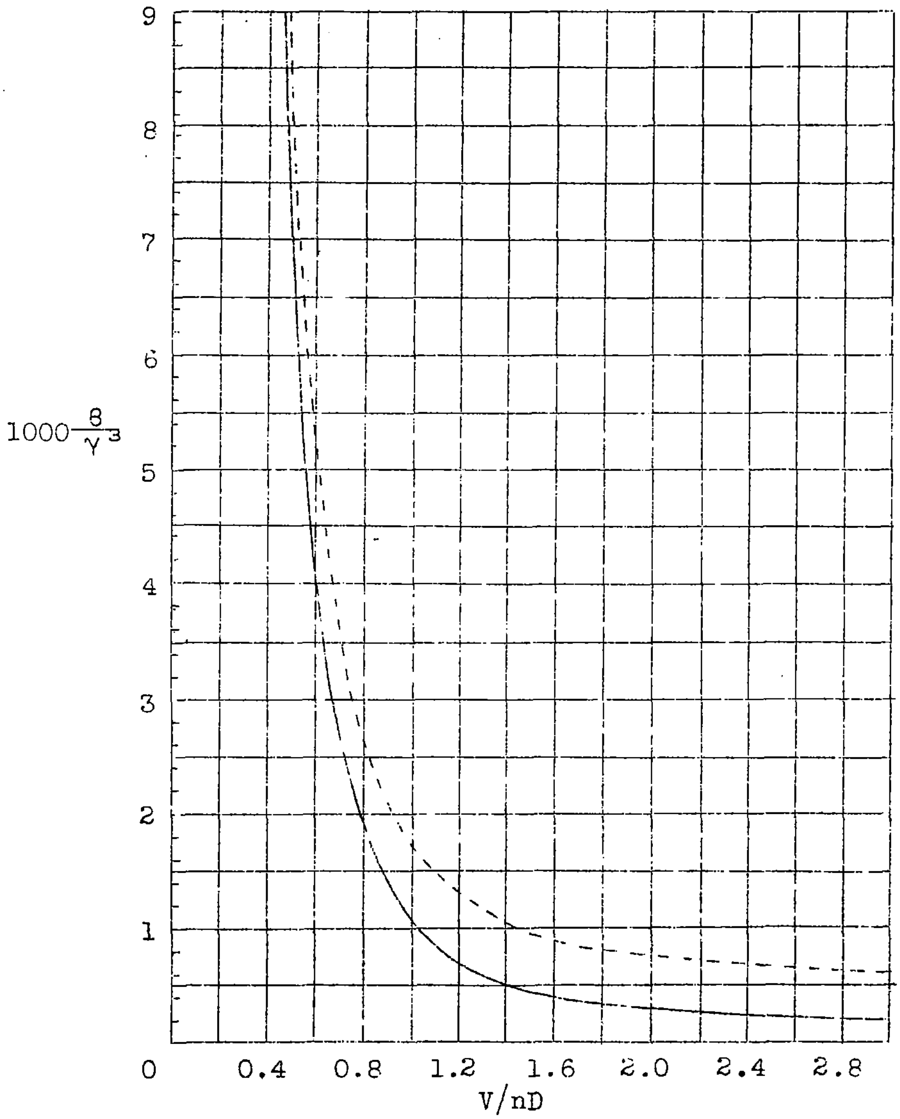


Figure 19.- Lift coefficient.

	h_o	c_{x_o}	N	h_r	μ	$\frac{\sigma}{D^2}$
- - - - - -	0.068	0.011	4 blades	0.015	1.5	1/2,000
————	0.07	0.009	6 blades	0.015	1.5	1/15,000



$$\frac{\beta}{\gamma^3} = \frac{W}{\delta D^2 V^3}$$

Figure 20.- Power coefficient.

$$P=\frac{\delta}{16}\,C_z S V^2 \qquad C_z=\frac{64}{\pi}\,\frac{\alpha_z}{\gamma^2}$$

$$W=\frac{\delta}{16}\,C_x S V^3 \qquad C_x=\frac{64}{\pi}\,\frac{\beta}{\gamma^3} \qquad \operatorname{Tan}\phi=\frac{C_x}{C_z}$$

I $h_o = 0.068$, $h_r = 0.015$, $c_{x_o} = 0.011$, $\mu = 1.5$, $N=4$, $\frac{\sigma}{D^2} = \frac{1}{2000}$

II $h_o = 0.07$, $h_r = 0.015$, $c_{x_o} = 0.009$, $\mu = 1.5$, $N=6$, $\frac{\sigma}{D^2} = \frac{1}{15000}$

III Froude induced parabola $C_x = \frac{C_z^{3/2}}{2}$

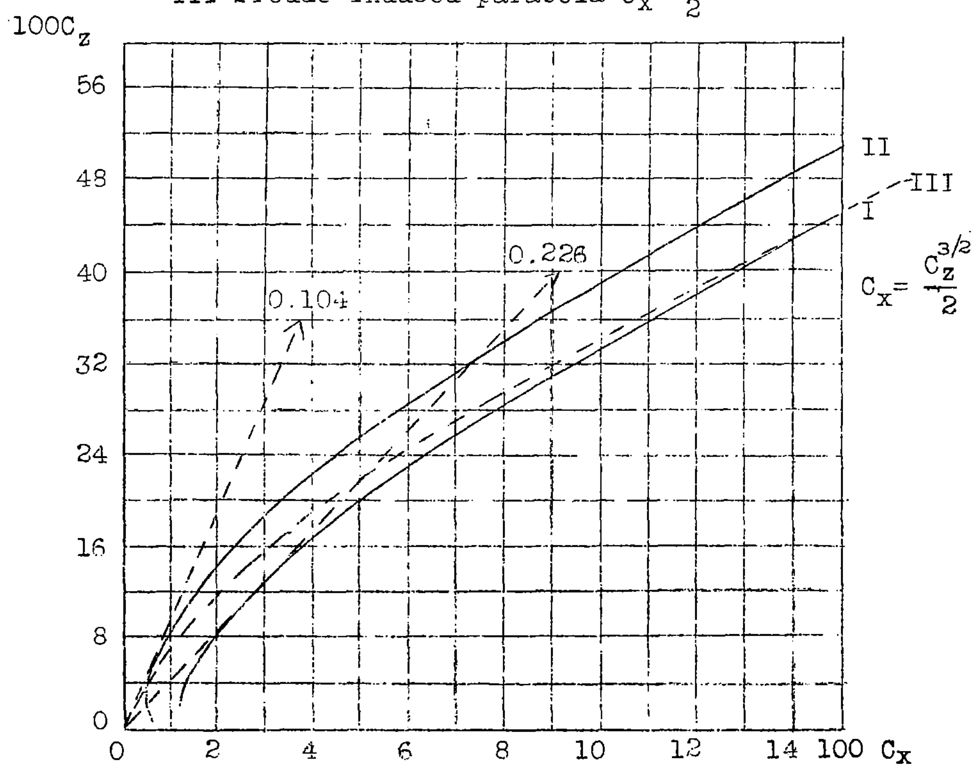
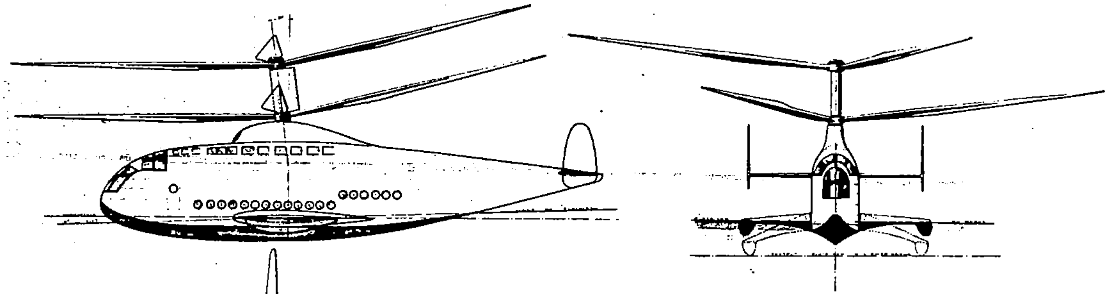
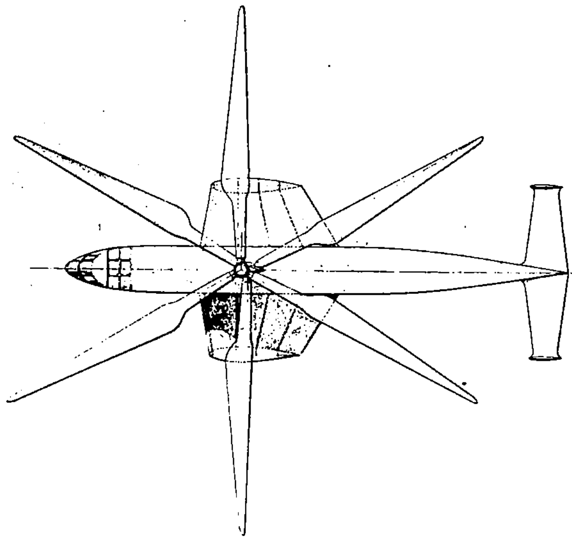


Figure 21.- Polar of gyroplane referred to area $S=\frac{\pi D^2}{4}$ of swept disk.





GYROPLANE LOUIS BREGUET

Design of a 3-engine amphibian gyroplane with retractable skids forming ballonets and with a hull.
Total weight:- 16,000 kg(35,273 lb.)
Propeller diameter(3 blades each):- 25 m (82.02 ft.)
Total blade area:- 34 m² (365.97 sq.ft.)
Total maximum:-
Power output at 3,000 m (9843 ft.): 3600 hp.
Speed at 3,000 m :-
using 2900 hp.:500 km/h(311 m.p.h.)
" 2400 hp.:400 " (249 ")
" 2000 hp.:250 " (155 ")
Hovering at 3,000 m ,power imput.2650hp

Figure 22. Gyroplane of the future.

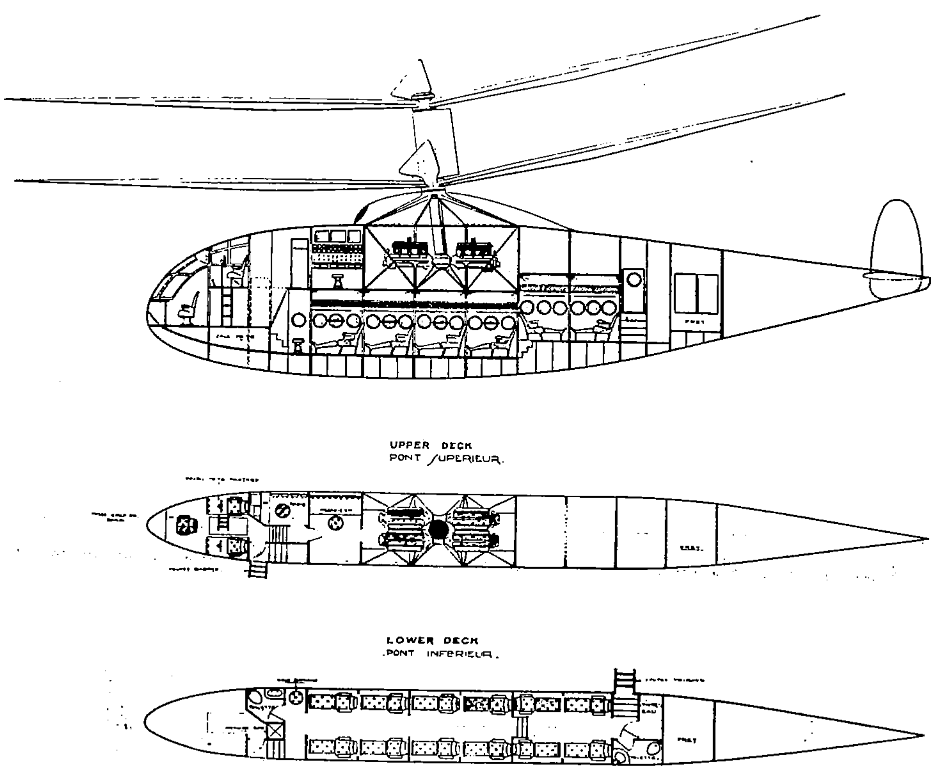


Figure 23. Gyroplane of the future.

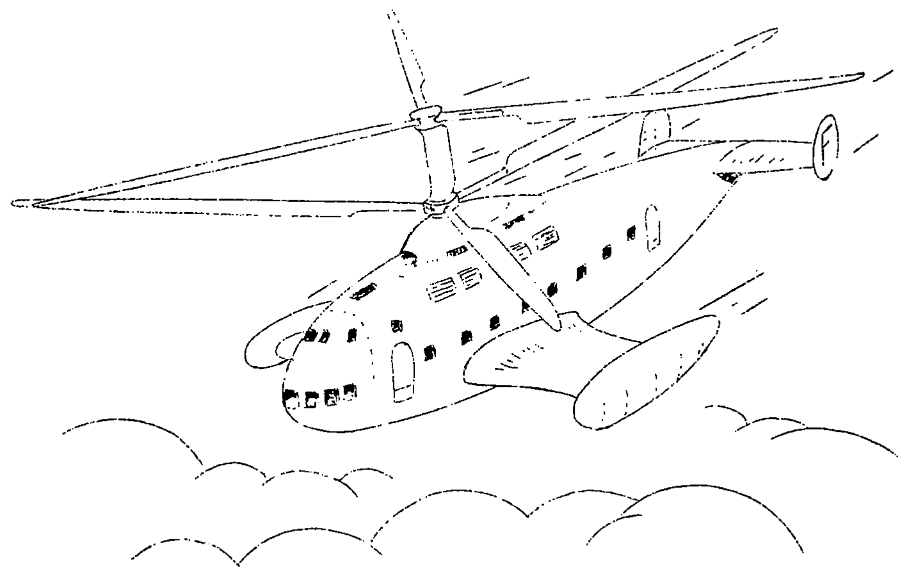


Figure 24.- A transatlantic gyroplane.