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No. 758

THE PROCESSES IN SPRING-LOADED INJECTION VALVES OF
SOLID INJECTION OIL ENGINES

By O. Lutz

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THE PROCESSES IN SPRING-LOADED INJECTION VALVES OF
SOLID INJECTION OIL ENGINES*

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SUMMARY

On the premise of a rectangular velocity wave arriving at the valve, the equation of motion of a spring-loaded valve stem is developed and analyzed. It is found that the stem oscillates, the oscillation frequency being consistently above the natural frequency of the nozzle stem alone, and whose amplitudes would increase in the absence of damping. The results are evaluated and verified on an example. The pressure in the valve and the spray volume are analyzed and several pertinent questions are discussed on the basis of the results.

1. INTRODUCTION

According to recent researches on the phenomena accompanying the injection and combustion in solid injection oil engines (reference 1), the process of the injection is not uniform (corresponding to the delivery of the fuel pump), but is rather in mountains and ridges as indicated by the pressure and valve-stem travel of a spring-loaded injection valve in figure 1. At first glance one is apt to attribute this to the fuel pipes (reference 1), especially when - as in Lieb's report - it refers to open valves; that is, valves without stem.

But with closed valves (as the Bosch type, shown in figure 2), it must be verified whether or not the valve system itself is perhaps subject to oscillations (as this so-called "rattling" of the stem seems to indicate), which

*"Die Vorgänge in federbelasteten Einspritzdüsen von kompressorlosen Olmaschinen." Ingenieur-Archiv, vol. IV, no. 2, pp. 153-168.

would place the cause in the valve itself. This investigation forms the subject of this report, along with the injection lag, the shape of the injection curve and the pressure during the spray period.

It will be seen that the valve assembly - spring-loaded valve stem and compressible content - actually can oscillate. The necessary equations for the mathematical treatment are given in section 2, subject to several simplifying approximations. The obtained general differential equation of the 3d order is resolved in section 3; it yields the natural frequency of the valve assembly. Section 4 treats the injection lag, section 5, the boundary equations which afford the special solution of the motion equation.

The general evaluation of the formulas - effect of dimensions and operating conditions of the valve on the motion process - is illustrated on an example in sections 6 and 7. The investigations thus far justify some pertinent statements (section 8). Unpublished data of the Robert Bosch laboratory, graciously placed at our disposal, were also analyzed. Lastly, several particular questions related to the work, such as pressure course in the valve, course of sprayed quantity, etc., are discussed.

2. THE MOTION EQUATION OF THE VALVE STEM (fig. 3)

The motion of the stem from rest position ($x = 0$) is figured positive when upward. The loads on the stem (positive when downward) in vertical direction are:

1) The spring tension $F = c_0 + c_1 x$ with initial tension c_0 and spring constant c_1 (kg m^{-1});

2) The pressure - pf_n due to fuel pressure (figured with whole stem section f_n , which thus includes the approximated quota, even if slight, of the momentum $\rho f v^2$ of the emerging spray);

3) A damping force $\vartheta \frac{dx}{dt}$ (ϑ in $\text{kg m}^{-1} \text{ sec.}$) proportional to the speed.

4) Lastly, a constant frictional force R which,

for rest position prior to stem lift will be greater (R_0) than when in motion, where it also changes direction ($\pm R_1$).

Then

$$F - p f_n + \delta \frac{dx}{dt} + R = - m \frac{d^2x}{dt^2} \quad (1)$$

with m = mass of oscillating part of valve stem and spring. Moreover, the continuity condition demands that the excess of the incoming quantity $f_l v_l$ over the out-flowing f_v be equal to the space relinquished through the movement of the fuel stem in unit time, together with the decrease in fuel volume effected by the pressure in unit time or, in other words, that

$$f_l v_l - f_v = f_n \frac{dx}{dt} + k \frac{dp}{dt} \quad (2)$$

The elastic elongation of the wall material of valve and fuel pipe is disregarded; it may be figured in with factor k . With E = elasticity modulus of the fuel and V = volume participating in the pressure fluctuations, k becomes

$$k = \frac{V}{E} \quad (3)$$

because the relative reduction in volume is $\epsilon = \frac{\Delta V}{V} = \frac{p}{E}$.

For the present analysis the discharge quantity f_v is assumed proportional to the travel x of the valve stem, that is, with a constant r (in $m^2 \text{ sec.}^{-1}$)

$$f_v = rx.$$

This approaches reality very closely according to the experiments of the R. Bosch Co. Besides, it is the only assumption which renders the produced differential equation (5) linear, that is, tractable in closed form. This is important for the reason that only through it the general conclusions, of prime significance here, can be readily and comprehensively drawn. The more exact course may be determined from the stem travel and the valve pressure, with the approximate calculation as basis. When combining the spray section f of the discharged spray through $\zeta u x$ with travel x and valve opening u , the Eulerian

equation $\frac{\rho}{2} v^2 = p - p_1$ (ρ = specific gravity of fuel, p_1 = pressure in cylinder) gives for r :

$$r = \zeta u \sqrt{\frac{2}{\rho} (p - p_1)} \quad (4a)$$

From (1) to (4) follows the differential equation for the valve-stem motion:

$$m \frac{d^3 x}{dt^3} + \vartheta \frac{d^2 x}{dt^2} + \left(c_1 + \frac{f_n^2 E}{V} \right) \frac{dx}{dt} + \frac{f_n r E}{V} x = \frac{f_n f_l E}{V} v_l \quad (5)$$

The friction R_1 is disregarded since it does not appear during the stem movement; neither does the initial spring tension. But it does become effective with each change of motion direction as a displacement of the center of oscillation in the sense of lowering the amplitudes of natural oscillations. It has no effect on the natural frequency. In forced oscillations with periodical interference force, the conditions may become more complicated (reference 2); which is, however, outside the scope of the present report.

Since equation (5) does not contain the initial spring tension c_0 , it follows that it has no influence on the motion process of the valve stem as soon as the movement, i.e., the injection, has begun. On the other hand, it naturally influences the valve pressure and the injection lag.

3. THE GENERAL SOLUTION OF THE MOTION EQUATION

Equation (5) is written as

$$\alpha x''' + \vartheta x'' + \beta x' + \gamma x = \delta \quad (6)$$

the right-hand side giving the time rate of the speed in the tube produced by the pump or, if fluctuations in the pipes change the speed, the course of the tube velocity v_l existent on the valve. The delivery of the conventional types of pumps requires only a fraction of the whole travel of the pump plunger, so that for this reason the speed during delivery does not change very much. For the following, it is assumed that it remains constant for the duration of delivery (fig. 4). For the rest, the solution can be found for any course by a variation of the

constants. But the calculation would become very complicated even for elementary assumptions without effecting any basic changes.

The natural oscillations are expressed with:

$$\alpha x_1''' + \wp x_1'' + \beta x_1' + \gamma x_1 = 0 \quad (7)$$

One appropriate form of the general solution of this equation is

$$x_1 = A e^{\lambda_1 t} + e^{\lambda_2 t} (B \sin \lambda_3 t + C \cos \lambda_3 t) \quad (8)$$

with

$$\lambda_1 = -\frac{\wp}{3\alpha} + y + z, \quad \lambda_2 = -\frac{\wp}{3\alpha} - \frac{y+z}{2}, \quad \lambda_3 = \frac{\sqrt{3}}{2} (y-z) \quad (9)$$

wherein y and z represent the terms

$$\left. \begin{aligned} y &= \sqrt[3]{-\frac{\gamma_1}{\alpha} + \sqrt{\frac{\gamma_1^2}{4\alpha^2} + \frac{\beta_1^3}{27\alpha^3}}} \\ z &= \sqrt[3]{-\frac{\gamma_1}{\alpha} - \sqrt{\frac{\gamma_1^2}{4\alpha^2} + \frac{\beta_1^3}{27\alpha^3}}} \end{aligned} \right\} \quad (10)$$

and β_1 and γ_1 are bound up with the original quantities α , β , γ , and \wp through

$$\beta_1 = \beta - \frac{\wp^2}{3\alpha}, \quad \gamma_1 = \gamma + \frac{2\wp^3}{27\alpha^2} - \frac{\wp\beta}{3\alpha} \quad (11)$$

From (10) follows that y is positive, z negative, but greater than y in absolute value; $y + z$ is therefore negative, $y - z$ positive. In conjunction with this, equation (8) manifests: the valve assembly can oscillate; its natural frequency is

$$\lambda_3 = \frac{\sqrt{3}}{2} (y-z) = \frac{\sqrt{3}}{2} \left[\sqrt[3]{-\frac{\gamma_1}{\alpha} + \sqrt{\frac{\gamma_1^2}{4\alpha^2} + \frac{\beta_1^3}{27\alpha^3}}} + \sqrt[3]{-\frac{\gamma_1}{\alpha} - \sqrt{\frac{\gamma_1^2}{4\alpha^2} + \frac{\beta_1^3}{27\alpha^3}}} \right] \quad (12)$$

The amplitudes increase ($\lambda_2 > 0$) with little damping, decrease when the damping is great, and remain unchanged when $\lambda_2 = 0$, δ to be defined from

$$\frac{\delta}{3\alpha} = -\frac{y+z}{2} \quad (13)$$

as explained elsewhere in the report. The center of oscillation approaches on an exponential curve, the position prescribed by the pump feed (term

$$A e^{\lambda_1 t}, \lambda_1 < 0).$$

The result is graphed in figure 5.

Premised on δ constant in equation (6), the general solution of the complete equation (6) is then as follows:

$$x = x_1 + x_0 = A e^{\lambda_1 t} + e^{\lambda_2 t} (B \sin \lambda_3 t + C \cos \lambda_3 t) + \frac{\delta}{\gamma} \quad (14)$$

with (9) to (11) valid for λ_1 , λ_2 , and λ_3 ; in other words, simply the term $x_0 = \delta/\gamma$ is added.

4. INJECTION LAG

The valve stem does not immediately open upon arrival of the velocity wave, but rather only after the pressure in the valve chamber and fuel tube has risen so that the pressure on the stem section f_r equals the combined initial spring tension c_0 and position R_0 . (The minor pressure on the stem tip due to cylinder pressure is disregarded.) The conditions then, are:

$$p f_r \geq c_0 + R_0.$$

Another result of (2) for the closed valve is:

$$f_l v_l = k \frac{dp}{dt}$$

which, integrated, gives:

$$k (p - p_0) = f_l \int_{-t_0}^0 v_l dt;$$

here p_0 is the residual pressure in the injection tube between injections. The time is counted from the beginning of stem travel and between the arrival of the velocity wave and the travel of the stem, i.e.; the injection lag is designated by t_0 . According to (15), we then have:

$$f_r \left(\frac{f_l}{k} \int_{-t_0}^0 v_l dt + p_0 \right) = c_0 + R_0 \quad (16)$$

from which the injection lag t_0 may be determined. In the general case the integral is solved graphically; but since the velocity wave was assumed rectangular its value equals $v_l t_0$ and with regard to (3), we have:

$$t_0 = \frac{V (c_0 + R_0 - p_0 f_r)}{f_r f_l v_l E} \quad (17)$$

Accordingly, the injection lag is greater as the valve content V , initial spring tension c_0 , and the stem friction R_0 are greater, and as the residual pressure p_0 , the annular section f_r , the strength v_l of the arriving velocity wave, and the elasticity modulus E of the fuel are smaller. Being at rest, the mass of the valve stem has no effect.

In practice the time t_0 itself is less important than the lag in degrees of crank angle. It is defined by

$$\varphi_0 = 6n t_0 = \frac{6n V (c_0 + R_0 - p_0 f_r)}{f_r f_l v_l E} \quad (17a)$$

Since for one and the same pump the velocity v_l increases linearly with the revolution speed, the lag in degrees of crank angles by itself remains the same at different speeds, excepting, of course, the minor variations due to R_0 and p_0 .

5. BOUNDARY EQUATIONS AND SPECIAL SOLUTION OF THE MOTION EQUATION

At the instant of opening R_0 changes to the usually much lower kinematic friction R_1 ; aside from that, the opening pressure p ($= \frac{c_0 + R_0}{f_r}$, compare (15)) now acts on

f_r as well as on the much greater section f_n' existing on the orifice (which for single-stem atomizers is less by the amount of the single-stem section than of the whole stem section f_n), so that the stem is accelerated by a force of $(R_0 - R_1) + p (f_n' - f_r)$. Then the boundary equations at the instant $t = 0$ of injection are:

$$x = 0, \quad x' = 0, \quad x'' = \frac{(R_0 - R_1) + p (f_n' - f_r)}{m} \equiv b \quad (18)$$

At the pump cut-off ($t = t_1$) the motion process is expressed with x_1 , x_1' , and x_1'' ; from then on the system oscillates according to its natural oscillation:

$$x_1 = A_1 e^{\lambda_1 t} + e^{\lambda_2 t} (B_1 \sin \lambda_3 t + C_1 \cos \lambda_3 t) \quad (14a)$$

This new phase is expressed in new time coordinates with x_1 , x_1' , and x_1'' as boundary conditions for $t = 0$.

With (18) and the abbreviation $x_0 = \frac{\delta}{\gamma}$, equation (14) then gives for A, B, and C:

$$0 = A + C + x_0,$$

$$0 = A \lambda_1 + B \lambda_3 + C \lambda_2,$$

$$b = A \lambda_1^2 + 2 B \lambda_2 \lambda_3 + C (\lambda_2^2 - \lambda_3^2),$$

whence

$$\left. \begin{aligned} A &= - \frac{x_0 (\lambda_2^2 + \lambda_3^2) - b}{(\lambda_1 - \lambda_2)^2 + \lambda_3^2}, \\ B &= \frac{x_0 \lambda_1 (\lambda_3^2 - \lambda_2^2 + \lambda_1 \lambda_2) - b (\lambda_1 - \lambda_2)}{\lambda_3 [(\lambda_1 - \lambda_2)^2 + \lambda_3^2]}, \\ C &= \frac{x_0 \lambda_1 (2 \lambda_2 - \lambda_1) - b}{(\lambda_1 - \lambda_2)^2 + \lambda_3^2} \end{aligned} \right\} \quad (19)$$

For the closing phase, we obtain with x_1 , x_1' , and x_1'' the analogous equations for the new quantities A_1 , B_1 , and C_1 :

$$x_1 = A_1 + C_1,$$

$$x_1' = A_1 \lambda_1 + B_1 \lambda_3 + C_1 \lambda_2,$$

$$x_1'' = A_1 \lambda_1^2 + 2B_1 \lambda_2 \lambda_3 + C_1 (\lambda_2^2 - \lambda_3^2),$$

whence

$$\left. \begin{aligned} A_1 &= \frac{x_1 (\lambda_2^2 + \lambda_3^2) - 2x_1' \lambda_2 + x_1''}{(\lambda_1 - \lambda_2)^2 + \lambda_3^2} \\ B_1 &= \frac{x_1 \lambda_1 (\lambda_2^2 - \lambda_3^2 - \lambda_1 \lambda_2) + x_1' (\lambda_1^2 - \lambda_2^2 + \lambda_3^2) + x_1'' (\lambda_2 - \lambda_1)}{\lambda_3 [(\lambda_1 - \lambda_2)^2 + \lambda_3^2]} \\ C_1 &= \frac{x_1 \lambda_1 (\lambda_1 - 2\lambda_2) + 2x_1' \lambda_2 - x_1''}{(\lambda_1 - \lambda_2)^2 + \lambda_3^2} \end{aligned} \right\} (20)$$

6. EVALUATION OF FORMULAS

a) No damping. - The damping factor $\delta \frac{d^2 x}{dt^2}$ is set = 0. Then β_1 and γ_1 become:

$$\beta_1 = \beta = c_1 + \frac{f_n^2 E}{V}, \quad \gamma_1 = \gamma = \frac{f_n r E}{V} \quad (21)$$

conforming to (11), (6), and (5).

Equation (12) is valid for the natural frequency λ_3 , shown in figure 6 plotted against β_1/α and γ_1/α , whereby

$$\frac{\beta_1}{\alpha} = \frac{\beta}{\alpha} = \frac{c_1}{m} + \frac{f_n^2 E}{m V}, \quad \frac{\gamma_1}{\alpha} = \frac{\gamma}{\alpha} = \frac{f_n r E}{m V} \quad (22)$$

since $\delta = 0$.

It is seen that the natural frequency increases with β_1/α and γ_1/α , from which we infer, according to (22): The natural frequency of the oscillatory system - spring-loaded valve together with compressible valve content is higher as the stem section f_n , elasticity modulus E of

the fuel, spring constant c_1 , valve opening u , and valve pressure p are greater (the latter are contained in r according to (4a)) and as the oscillating mass m and the fuel volume V are smaller.

For low γ_1/α values the natural frequency is as that of the valve stem alone, that is, frequency ω , with which the stem would swing under the effect of the spring alone, since according to figure 6 and equations (6) and (12) the natural frequency approaches $\lambda_3' = \sqrt{\beta_1/\alpha}$ when the γ_1/α values are small. In this case, however, the second summand $\frac{f_n^2 E}{mV}$ in (22) also disappears, so that

$$\lambda_3' = \sqrt{\frac{c_1}{m}} = \omega \quad (23)$$

That is, the natural frequency of the valve stem only, follows. Another deduction is: The natural frequency of the valve system is always higher than that of the valve stem alone. It approaches this frequency so much closer as f_n , E , valve opening u , and spray velocity v ; that is, as the valve pressure is smaller and the fuel volume V is greater.

The term $A e^{\lambda_1 t}$ in (14) describes the shift of the median position of the oscillating valve stem when the balance of the system has been disturbed by extraneous interferences. The new position is reached so much faster and at a steeper angle as the absolute amount (always negative) of λ_1 is higher.

Figure 7 shows $\lambda_1' = y + z = \lambda_1 + \frac{\beta}{3\alpha}$ versus β_1/α and γ_1/α . This graph reveals and equation (10) confirms that the λ_1' curves become the straight lines $\lambda_1' = -\frac{\gamma_1}{\beta_1}$ with a 45° slope for $\sqrt[3]{\frac{\gamma_1^2}{\beta_1^3}}$, and become the vertical lines $\lambda_1' = -\sqrt{\frac{\gamma_1}{\alpha}}$ when $\sqrt[3]{\frac{\gamma_1^2}{\beta_1^3}} > 1000$. The slope is never less than 45° , so that with proportional increase of β_1/α and γ_1/α the amount of λ_1' becomes greater. Aside from that, it increases when β_1/α becomes smaller and γ_1/α greater.

Now we conclude from (22): The valve stem travels the arriving velocity wave so much faster as c_1 (the stiffness of the spring) and m are smaller and the valve opening and valve pressure are greater. This statement should not be confused with the injection lag in section 4; figure 8 shows the pertinent part in shaded lines.

The effect of f_n , E , and V is not uniform. So long as the first sum c_1/m in (22) is substantially preponderant, λ_1 increases with increasing f_n and E and decreasing V ; the stem travels so much faster, even if γ_1 is substantially greater than β_1 . However, cases of opposite effect are not impossible.

The factor $e^{\lambda_2 t}$ in (14) expresses the time rate of amplitude of the natural oscillations. If λ_2 is positive, the amplitude increases; if λ_2 is negative, they decrease. Equation (9) gives:

$$\lambda_2 = -\frac{\lambda_1}{2} \quad (24)$$

for $\delta = 0$. Accordingly, λ_2 may also be taken from figure 7; without damping, it is always positive, the amplitudes increase. For the rest, the statements retain their validity as for λ_1 ; the oscillations increase so much faster as c_1 , m , etc., are smaller.

The coefficients A , B , and C in (14) express the magnitude of the amplitudes. The general relationship between A , B , and C on the one hand, and α , β_1 , and γ_1 on the other, expressed in (10), (19), and (20) is extremely complex besides being nonuniform in places. For this reason, the explanation is confined to the scope of practical application. As a rule, the amplitudes are lower as the denominator $(\lambda_1 - \lambda_2)^2 + \lambda_3^2$ is higher, that is, as the natural frequency λ_3 is higher and the faster the stem travels the arriving velocity wave. Contrariwise, the higher the initial acceleration b , that is, the greater the gap between static and sliding friction of the valve stem, and the smaller the annular stem section relative to the stem section, the higher the amplitudes. We distinguish between three cases (fig. 9). For

$$b = x_0 (\lambda_2^2 + \lambda_3^2) \quad (25)$$

according to (19) there is no shifting of the median position of oscillation (II) - if b is smaller it approaches x_0 (I) from below; if b is greater, the stem shoots at the very first instant far beyond this position (III).

The acceleration of b being unaffected by the force x_0 of the arriving wave, the most adverse case III occurs so much more rarely as x_0 is greater, or in other words, the valve operates more evenly with high, than with low load.

The same arguments hold for the closing process, with the difference, however, that case III may become beneficial; the valve closes so much faster as the unequation (compare (20)):

$$2x_1' \lambda_2 - x_1'' > x_1 (\lambda_2^2 + \lambda_3^2) \quad (26)$$

is more pronounced. but by virtue of the impossibility of arbitrarily influencing the instantaneous speed x_1' and acceleration x_1'' at cut-off of delivery, this case is of only secondary importance as far as the general argument is concerned.

b) With damping.- A valve whose stem executes growing oscillations with constant fuel delivery is unserviceable. Without damping in proportion to the stem velocity, the natural oscillations rise consistently even with constant feed, as previously explained. Damping contingent upon the path modifies only the natural frequency but not the growth of the oscillations, and even the mechanical friction itself falls short in explanation, as shown elsewhere on an example.

From (11) follows that damping causes β_1 to decrease consistently; γ_1 also decreases but only so long as $\frac{2}{9} \left(\frac{\delta}{\alpha}\right)^2 < \frac{\beta}{\alpha}$. Thus, figure 6 reveals that damping generally diminishes the natural frequency λ_3 ; but for very high damping factors δ the natural frequency may rise again.

To determine the damping effect on λ_1 and λ_2 , we proceed as follows:

$$\lambda_1 = -\frac{\delta}{3\alpha} + \lambda_1' = -\left(\frac{\delta}{3\alpha} - \lambda_1'\right) \quad (9a)$$

and

$$\lambda_2 = \lambda_2' - \frac{\delta}{3\alpha} = -\frac{\lambda_1'}{2} - \frac{\delta}{3\alpha} \quad (9b)$$

Figure 7 gives $\lambda_1' = y + z$, which defines λ_1 and λ_2 for numerical interpretation.

It will be remembered that according to (7) and (8) λ_1 is the real root of the equation of the 3d order,

$$\alpha \lambda_1^3 + \delta \lambda_1^2 + \beta \lambda_1 + \gamma = 0,$$

whose coefficients and, in particular, the terms $\delta \lambda_1^2$, and γ are all positive. By rewriting the equation as

$$\alpha \lambda_1^3 + \beta \lambda_1 = -\gamma - \delta \lambda_1^2,$$

it is seen that the amount of λ_1 increases with δ . One surprising result then is that the valve stem travels the arriving velocity wave so much faster as the damping is higher.

For λ_2 , $\lambda_1 = -2\lambda_2 - \frac{\delta}{\alpha}$, according to (9a) and (9b).

Writing this term in the above-mentioned equation of the 3d order gives a corresponding equation with λ_2 as a real root:

$$8\alpha \lambda_2^3 + 2\beta \lambda_2 = \gamma - \delta \left(8\lambda_2^2 + 2\frac{\delta}{\alpha} \lambda_2 + \frac{\beta}{\alpha}\right)$$

It is seen that λ_2 is at first positive for small damping δ , equals zero for the boundary value

$$\frac{\delta}{\alpha} = \frac{\gamma}{\beta} \quad (27)$$

or

$$\frac{\delta}{m} = \frac{f_n r E}{c_1 V + f_n^2 E} \quad (27a)$$

and negative for higher δ values.

Thus the growth of the oscillation amplitudes, expressed with $e^{\lambda_2 t}$ in (14), is reduced by the damping; for ϑ (from (27)) the amplitudes remain the same; for higher values they decrease. Conformably, a damping proportional to the speed of the valve stem is beneficial. The valve stem travels the velocity wave more quickly; the natural oscillations (which without damping would increase) are damped.

7. EXAMPLE

For illustration, we use the data of the well-known Bosch valve, shown in figure 2. The type DN 4 S 1 has the following characteristics:

Weight of moving parts of the stem (inclusive of 50 percent of the spring weight):	$23.1 \text{ g} = 0.0231 \text{ kg}$
Spring constant:	$c_1 = 115 \text{ kg/cm} = 1.15 \times 10^4 \text{ kg m}^{-1}$
Valve-stem section:	$f_n = 19.7 \text{ mm}^2 = 19.7 \times 10^{-6} \text{ m}^2$
Annular section of valve stem:	$f_r = 12.5 \text{ mm}^2 = 12.5 \times 10^{-6} \text{ m}^2$
Volume of valve:	$V_1 = 740 \text{ mm}^3 = 0.74 \times 10^{-6} \text{ m}^3$
Pump volume V_2 between pump plunger and fuel line:	$V_2 = 1830 \text{ mm}^3 = 1.83 \times 10^{-6} \text{ m}^3$
Volume of 1-meter tube having 2 mm diameter:	$V_3 = 3.14 \times 10^{-6} \text{ m}^3$
Elasticity modulus (average):	$E = 2 \times 10^8 \text{ kg m}^{-2}$
r (see equation (4)) (reference 1) for $n=400$ r.p.m.:	$r = 4.15 \times 10^{-2} \text{ m}^2 \text{ sec}^{-1}$
Likewise, $f_l v_l$:	$f_l v_l = 18.54 \times 10^{-6} \text{ m}^3 \text{ sec}^{-1}$

The premises are that all the fuel between pump plunger and valve participates in the pressure fluctuation, so that $V = V_1 + V_2 + V_3 = 5.71 \times 10^{-6} \text{ m}^3$. Then the values of the coefficients in (5) are (compare (5)):

$$\alpha = \frac{0.0231}{9.81} = 2.35 \times 10^{-3} \text{ kgm}^{-1} \text{ sec}^2$$

$$\beta = 1.15 \times 10^4 + \frac{19.7^2 \times 10^{-12} \times 2 \times 10^8}{5.71 \times 10^{-6}} = 2.51 \times 10^4 \text{ kgm}^{-1}$$

$$\gamma = \frac{19.7 \times 10^{-6} \times 4.15 \times 10^{-2} \times 2 \times 10^8}{5.71 \times 10^{-6}} = 0.287 \times 10^8 \text{ kgm}^{-1} \text{ sec}^{-1}$$

$$\delta = \frac{19.7 \times 10^{-6} \times 18.54 \times 10^{-6} \times 2 \times 10^8}{5.71 \times 10^{-6}} = 1.28 \times 10^4 \text{ kg sec}^{-1}$$

a) Without damping.— We formulate:

$$\frac{\beta_1}{\alpha} \equiv \frac{\beta}{\alpha} = \frac{2.51 \times 10^4}{2.35 \times 10^{-3}} = 10.68 \times 10^6 \text{ sec}^{-2}, \quad \frac{\gamma_1}{\alpha} \equiv \frac{\gamma}{\alpha} = \frac{0.287 \times 10^8}{2.35 \times 10^{-3}} = 12.22 \times 10^9 \text{ sec}^{-3}$$

and read the natural frequency

$$\lambda_3 = 3410 \text{ sec}^{-1}$$

from figure 6.

On page 40 of Heinrich's report (reference 1) is given a natural frequency period of between 1.25×10^{-3} and 1.66×10^{-3} sec. as against our slightly higher 1.84×10^{-3} sec. Now, it is physically impossible that with such high oscillations, the entire fuel content in the pump line participates on the pressure fluctuation, because a pressure change can only propagate at sound velocity (about 1,500 m/s). For this reason it is assumed that aside from the valve content, only half of the tube volume participates (a more accurate figure would have to be obtained by experiment). Then $V = V_1 + 0.5 V_3 = 0.74 \times 10^{-6} + 1.57 \times 10^{-6} = 2.31 \times 10^{-6} \text{ m}^3$, from which follows

$$\beta = 4.51 \times 10^4 \text{ kg m}^{-1}, \quad \gamma = 0.708 \times 10^8 \text{ kg m}^{-1} \text{ sec}^{-1},$$

$$\delta = 3.165 \times 10^4 \text{ kg sec}^{-1};$$

likewise,

$$\frac{\beta_1}{\alpha} \equiv \frac{\beta}{\alpha} = 19.18 \times 10^6 \text{ sec}^{-2}, \quad \frac{\gamma_1}{\alpha} \equiv \frac{\gamma}{\alpha} = 30.1 \times 10^9 \text{ sec}^{-3},$$

for which figure 6 gives $\lambda_3 = 4510 \text{ sec}^{-1}$,

corresponding to a period of $1.394 \times 10^{-3} \text{ sec}$ or 3.34° crank angle, which is in close accord with the test. The further calculations are made with these new figures.

Alone, the natural frequency of the valve stem (see (23)) would be:

$$\omega = \sqrt{\frac{c_1}{m}} = \sqrt{\frac{1.15 \times 10^4}{2.35 \times 10^{-3}}} = 2210 \text{ sec}^{-1},$$

that is, only about half of the natural frequency of the valve system. The partial exponent λ_1 of the term for the shifting of the oscillation center $A e^{\lambda_1 t}$ is according to figure 7:

$$\lambda_1 = -1430 \text{ sec}^{-1}$$

and for λ_2 (24),

$$\lambda_2 = -\frac{\lambda_1}{2} = 715 \text{ sec}^{-1}.$$

The injection lag may be computed from (17). Since the valve, at rest, is adjusted through the change in initial spring tension to the prescribed valve pressure, the initial tension c_0 computed therefrom includes at the same time the friction R_0 . The residual pressure p_0 is $p_0 = 55 \text{ atm.}$ for 400 r.p.m., according to Heinrich. The fuel volume is, of course, that of the entire volume $V = V_1 + V_2 + V_3$ between pump plunger and valve stem. Then, according to (17) the valve pressure for 100 atm. is:

$$t_0 = \frac{5.71 \times 10^{-6} (12.5 - 55 \times 0.125)}{12.5 \times 10^{-6} \times 18.54 \times 10^{-6} \times 2 \times 10^8} = 0.687 \times 10^{-3} \text{ sec}$$

or the lag in degrees of crank angles

$$\varphi_0 = 6 \times 400 \times 0.687 \times 10^{-3} = 1.65^\circ \quad (c_0 + R_0 = 100 \times 10^4 \times f_r = 12.5 \text{ kg})$$

The coefficients A, B, and C are defined according to (17), while the initial acceleration b is disregarded, since the amplitudes are apparent even without initial acceleration. Then we have:

$$x_0 = \frac{\delta}{\gamma} = \frac{3.165 \times 10^4}{0.708 \times 10^8} = 4.47 \times 10^{-4} \text{ m}$$

and

$$A = - \frac{4.47 \times 10^{-4} (715^2 + 4510^2)}{2145^2 + 4510^2} = -3.74 \times 10^{-4} \text{ m},$$

$$B = \frac{-4.47 \times 10^{-4} \times 1430 (4510^2 - 715^2 - 1430 \times 715)}{4510 (2145^2 + 4510^2)} = -1.068 \times 10^{-4} \text{ m},$$

$$C = -x_0 - A = -4.47 \times 10^{-4} + 3.74 \times 10^{-4} = -0.73 \times 10^{-4} \text{ m}.$$

In this manner the start of the stem motion is shown in figure 10 (--- curve). It is seen that, without damping, the oscillations would grow rapidly; at the second lift (indicated by arrow in figure 10) the stem would already hit the stem stop.

b) With damping.-- Owing to the lack of experimental data, the damping factors are estimations which are far from reliable. We stipulate a damping δ of $4 \text{ kg m}^{-1} \text{ sec}$. (Reason: the mean stem velocity is about 0.25 m/s , for which a damping force of 1 kg is to appear.) Then we have, according to (11):

$$\beta_1 = 4.51 \times 10^4 - \frac{4^2}{3 \times 2.35 \times 10^{-3}} = 4.283 \times 10^4 \text{ kg m}^{-1}$$

$$\gamma_1 = 0.708 \times 10^8 + \frac{2 \times 4^3}{27 \times 2.35^2 \times 10^{-6}} - \frac{4 \times 4.51 \times 10^4}{3 \times 2.35 \times 10^{-3}} = 0.461 \times 10^8 \text{ kg m}^{-1} \text{ sec}^{-1};$$

that is,

$$\frac{\beta_1}{\alpha} = 18.25 \times 10^6 \text{ sec}^{-2}, \quad \frac{\gamma_1}{\alpha} = 19.6 \times 10^9 \text{ sec}^{-3},$$

equivalent to a natural frequency of $\lambda_3 = 4390 \text{ sec}^{-1}$ according to figure 6.

Damping lowers the natural frequency. Besides, we find from figure 7 that

$$\lambda_1' = -1020 \text{ sec}^{-1},$$

whence

$$\lambda_2' = 510 \text{ sec}^{-1}$$

and

$$\lambda_1 = \lambda_1' - \frac{\beta}{3\alpha} = -1020 - \frac{4}{3 \times 2.35 \times 10^{-3}} = -1585 \text{ sec}^{-1},$$

$$\lambda_2 = \lambda_2' - \frac{\beta}{3\alpha} = -55 \text{ sec}^{-1}.$$

Without allowance for b , equations (19) then give:

$$A = - \frac{4.47 \times 10^{-4} (55^2 + 4390^2)}{1530^2 + 4390^2} = - 3.98 \times 10^{-4} \text{ m},$$

$$B = \frac{-4.47 \times 10^{-4} \times 1585 (4390^2 - 55^2 + 55 \times 1585)}{4390 (1530^2 + 4390^2)} = - 1.44 \times 10^{-4} \text{ m},$$

$$C = - 4.47 \times 10^{-4} + 3.98 \times 10^{-4} = - 0.49 \times 10^{-4} \text{ m}.$$

shown in figure 10 as full-drawn curve.

For the closing phase, it is simply assumed that the pump feed stops at the instant the stem swings through the median position; that is (see fig. 10):

$$x_1 = x_0, \quad x_1' = \pm a \lambda_2 \approx 1.1 \times 10^{-4} \times 4390 = \pm 0.483 \text{ m sec}^{-1}, \quad x_1'' = 0.$$

With these values follow from (20) the quantities A_1 , B_1 , and C_1 at

$$A_1 = 4.61 \times 10^{-4} \text{ m}, \quad B_1 = 2.54 \times 10^{-4} \text{ m}, \quad C_1 = - 0.14 \times 10^{-4} \text{ m},$$

when the stem swings exactly upward through the zero position ($x_1' > 0$) and

$$A_1' = 3.27 \times 10^{-4} \text{ m}, \quad B_1' = 0.34 \times 10^{-4} \text{ m}, \quad C_1' = 1.20 \times 10^{-4} \text{ m},$$

when the stem swings exactly downward through the zero position ($x_1' < 0$). Both processes are shown in figure 10.

In the first case there is an after-spray of more than double the amount of fuel, so that the pressure can be much more decreased than in the second case. From this follows: The reduction of pressure in the fuel line (the residual pressure p_0) is very much dependent upon the momentary motion attitude of the stem at the end of the pump feed. As the latter differs for each control setting and speed, it is possible that the experiment may not be able to reveal any regular relationship between residual pressure p_0 on one hand, and speed and control setting on the other.

The effect of friction R_1 on the damping of the natural oscillations may now be estimated. The acceleration in a reversal point is, according to figure 10:

$$b_1 = a \lambda_3^2 \approx 1.1 \times 10^{-4} \times 4390^2 = 21.2 \times 10^2 \text{ m sec}^{-2}$$

necessitating a load of

$$P_1 = b_1 m = 21.2 \times 10^2 \times 2.35 \times 10^{-3} = 5 \text{ kg}$$

on the valve stem. The friction R_1 , which appears with the change in direction can, however, be only of the approximate order of the stem weight; that is, less than 1/100 of the acceleration, so that its effect will not be great. Only in the case of lateral pressure on the stem may friction appear which could effect any substantial damping of the oscillation.

A comparison of figure 10 with figure 1 manifests a qualitative agreement between theory and experiment. It should be borne in mind that the speed of the fuel wave is not uniform owing to the expansion processes in the line, so that the median location of the stem itself oscillates. (The peaks in figure 1 are the result of connecting the test points by straight lines.)

3. COMPARISON WITH AVAILABLE EXPERIMENTAL DATA

The general results given in section 6 should be checked by systematic experiments. Some statements, however, may be made on the basis of the data already available (reference 1):

a) Injection lag (section 4).-- Heinrich finds that the lag, expressed in degrees of crank angle, is increased as the valve content, the valve pressure - that is, the initial spring tension - and the speed are increased and as the residual pressure between injections and the velocity wave are decreased. The example worked out in section 7, likewise is quantitatively in agreement.

b) Natural frequency (section 6).-- Heinrich's strobograms distinctly reveal the oscillations of the valve stem. The natural frequency computed in section 7 is in very close agreement with the experiment. Admittedly, the measurement does not reveal whether the oscillations are natural oscillations or whether they are perhaps in part motivated by the expansion processes in the tubes. On the other hand, special experiments made by the Robert Bosch Co. attest to actual oscillations in the valve system. The same valve was fitted to the same pump once without fuel line, and with two tubes of 845 mm and 1,245 mm length. The stem-travel curve is fundamentally the same in all three arrangements - a proof that the oscillations are not due to processes in the tube. The oscillation superposed on the basic form had a frequency of around $4,300 \text{ sec}^{-1}$, which checks very closely with our calculation. The amplitudes were not as high as those given by Heinrich, who used a tube of 1 meter length (fig. 1). It is possible that coupling processes existed, in his case, between stem oscillation and expansion oscillation in the fuel tube which, according to the calculation, would invariably occur with this tube length as explained in section 9.

Berg and Rode's diagrams also show stem oscillations, although considerably damped; likewise they reveal clearly the exponential course at start and cut-off of pump feed.

9. PRESSURE COURSE IN THE VALVE. SPECIAL PROBLEMS

The pressure in the valve is, according to (2) and (4)

$$k \frac{dp}{dt} = f_l v_l - r x - f_n \frac{dx}{dt} \quad (2a)$$

The speed v_l is assumed constant, the course of the stem travel x is ascertained (14), so that the course

of p may be determined. The limiting condition is a valve pressure of 100 atmospheres, 200 atmospheres, etc., depending upon the initial spring tension at start of injection ($t = 0, x = 0$).

We forego the complicated calculations, preferring an approximation which has the advantage of being more comprehensive. The chief point is the phase lag between pressure and stem oscillation. If both are in phase, that is, at high pressure concurrently with large injection, the sprayed volume fluctuates extremely with respect to time; with a 180° phase lag, the discrepancies in the spray volume will be small.

As customary in alternating-current technique, the oscillations are given in vectors (fig. 11), the constant term being disregarded, and a harmonic oscillation of the valve stem is assumed. The diagram is plotted with the figures of the example in section 7.* We see: the pressure oscillation leads the stem oscillation by more than $1/4$ period. The lead is so much greater as the stem section is greater and as the valve opening and the spray velocity, that is, the valve pressure, is lower. According to that, a greater lead assures a more uniform injection. The course of the sprayed volume can be determined by computing the spray velocity v from the pressure (according to (2a)) by means of the Eulerian equation:

$$\frac{\rho}{2} v^2 = p - p_1 \quad (p_1 = \text{pressure in cylinder chamber}).$$

Assuming constant remaining valve factor, the injected quantity then follows from the spray velocity and the travel of the fuel stem, which may be readily computed from the results given herein.

*As known, the first derivation has a lead of $1/4$ period or 90° ; its amplitude is ω -fold ($\omega = \text{frequency}$). In the graph the amplitude x of the stem oscillation is assumed at 0.1 mm; all values are given according to magnitude. The pressure fluctuation leads the stem oscillation by 165° or 1.53° crank angle (period of whole oscillation, 3.34° crank angle (compare section 7)); its amplitude is 18.8 atmospheres per 0.1 millimeter of stem amplitude. The figures are in very close accord with Heinrich's data. (See fig. 1.)

From the design point of view, the high natural frequency is of particular significance. It has been shown that the natural frequency of the valve system is about twice as high as that of the valve stem alone. If the valve stem consists of separate parts not rigidly fastened together, it may happen that the natural oscillation of the part connected to the spring will still fall below the frequency of the valve system. In this case the stem parts would lift each other from the particular place, causing increased wear.

Another important point is the possibility of resonance-like compound effects between the expansion oscillation in the fuel line and the natural oscillation of the valve system. The fuel wave requires a time interval of $t_1 = \frac{2l}{c}$ (l = length of tube in m, c = wave velocity \approx 1,500 m/s), to speed through the fuel tube (forward and back again). Resonance in the conventional term, that is, increase in oscillation amplitudes, occurs when this time interval coincides with the period of the natural oscillation $T = \frac{2\pi}{\lambda_3}$ or its multiples; that is, when

$$l = i \frac{\pi c}{\lambda_3} \quad (i = 1, 2, 3 \dots)$$

and when the phases are identical. In our example in section 7, it would for $i = 1$ entail

$$l = \frac{\pi \times 1500}{4390} = 1.07 \text{ m,}$$

a figure well within normal practice. Consequently, it is entirely possible that resonances may occur which raise the stem oscillations. But as to the rest - with different phase - the motion processes may be extremely irregular.*

*For example, the following may occur: By virtue of the opening shock, the stem oscillates at first at natural frequency. Now assume that the tube oscillation has the same frequency but a phase such that the stem - if it followed it - would oscillate in the phase at 1/2 different period. Then the tube oscillation and the damping itself slows up the motion to be followed by the stem; the oscillations therefore decrease first, then increase again.

Lastly, there is the so-called "after spray" of the valve. It is substantially a matter of processes in the fuel tubes (reference 1). The question still remains as to whether with the assumed rectangular velocity wave (no oscillations in tubes), an after spray is possible. It is possible when the stem is so forcibly flung on its seat during the cut-off period as to make it bounce back, or when the natural oscillation superposed on the exponential curve of the closing motion makes an upward oscillation directly before reaching the seat of the stem. The more detailed analysis may be made with (20).

Translation by J. Vanier,
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Translator's note: The reader is also referred to:

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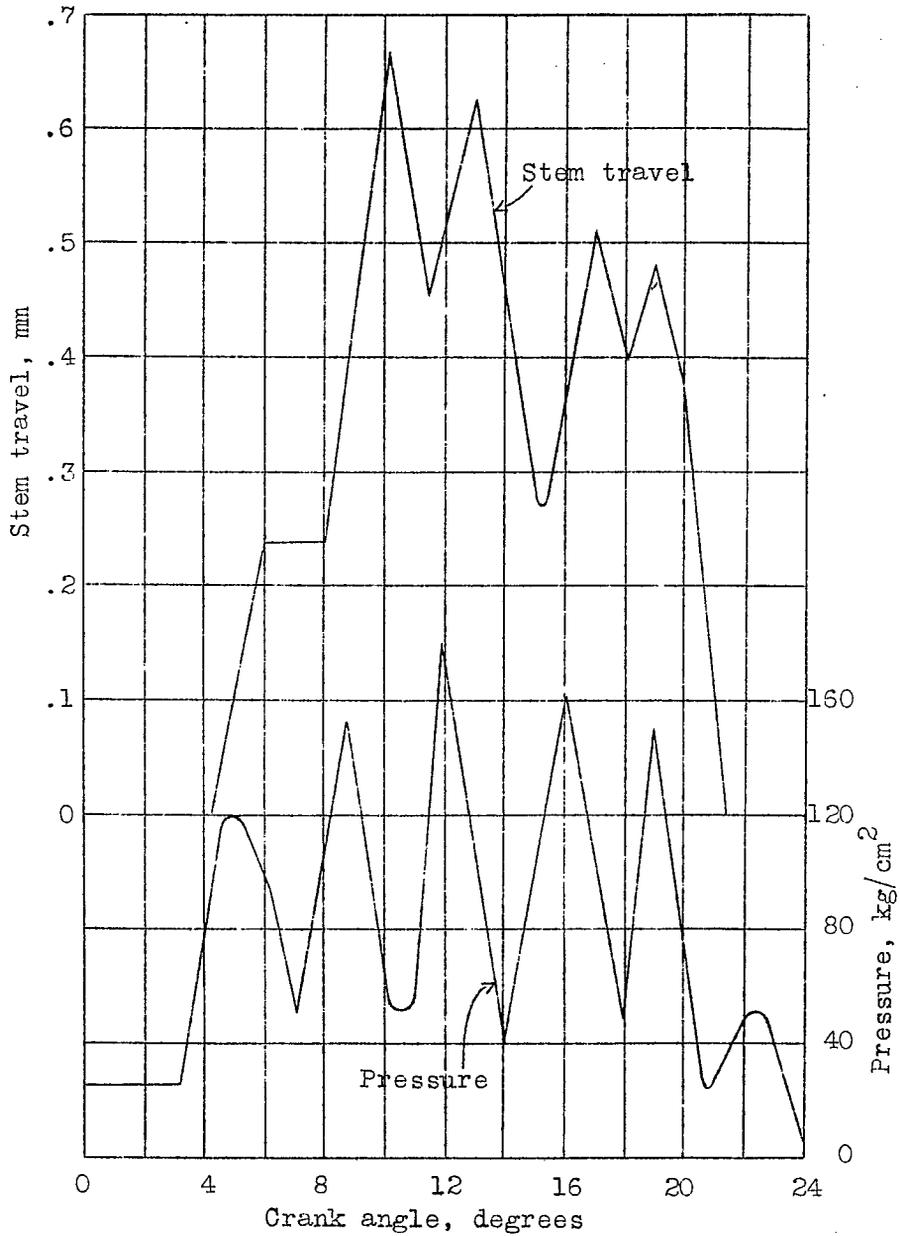


Figure 1.- Pressure curve and stem travel of a spring-loaded fuel injection valve (Heinrich).

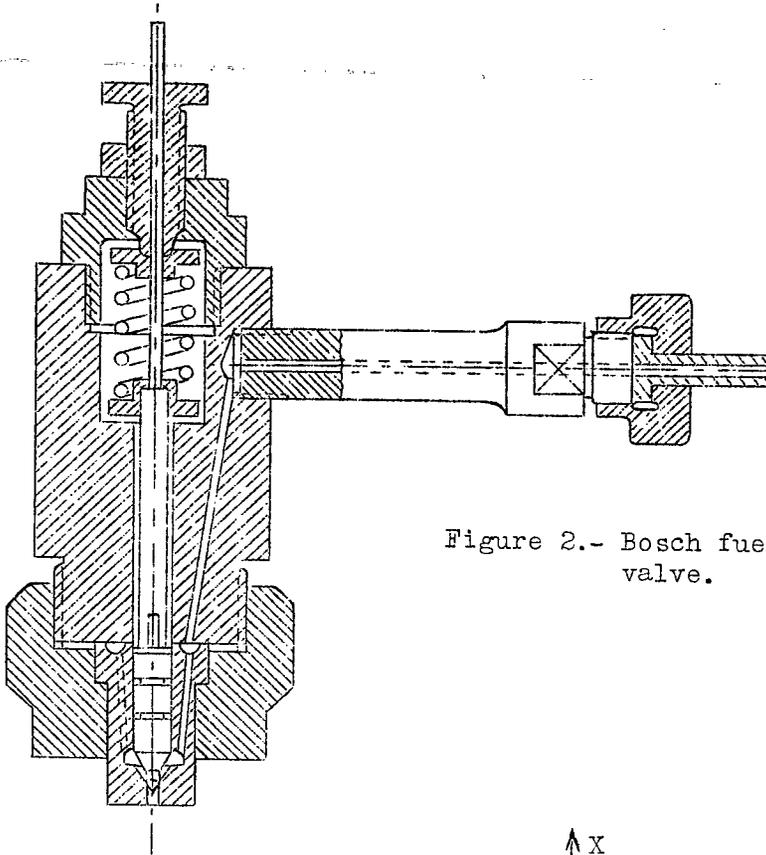


Figure 2.- Bosch fuel injection valve.

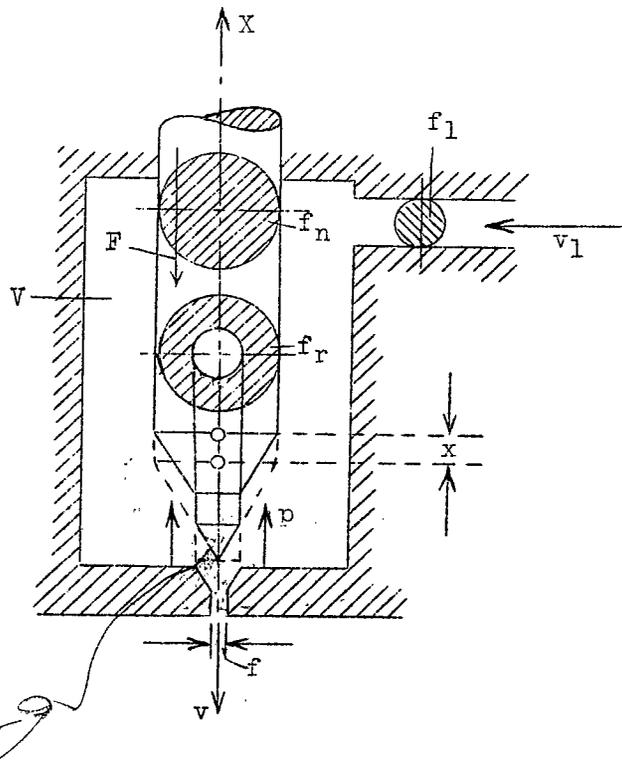


Figure 3.- Schematic view of spring-loaded injection valve.

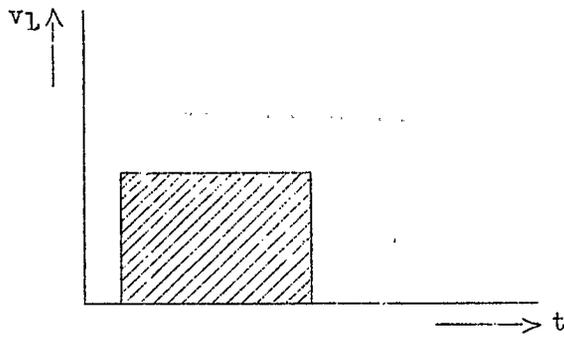


Figure 4.-Assumed velocity course in the tube ahead of the valve.

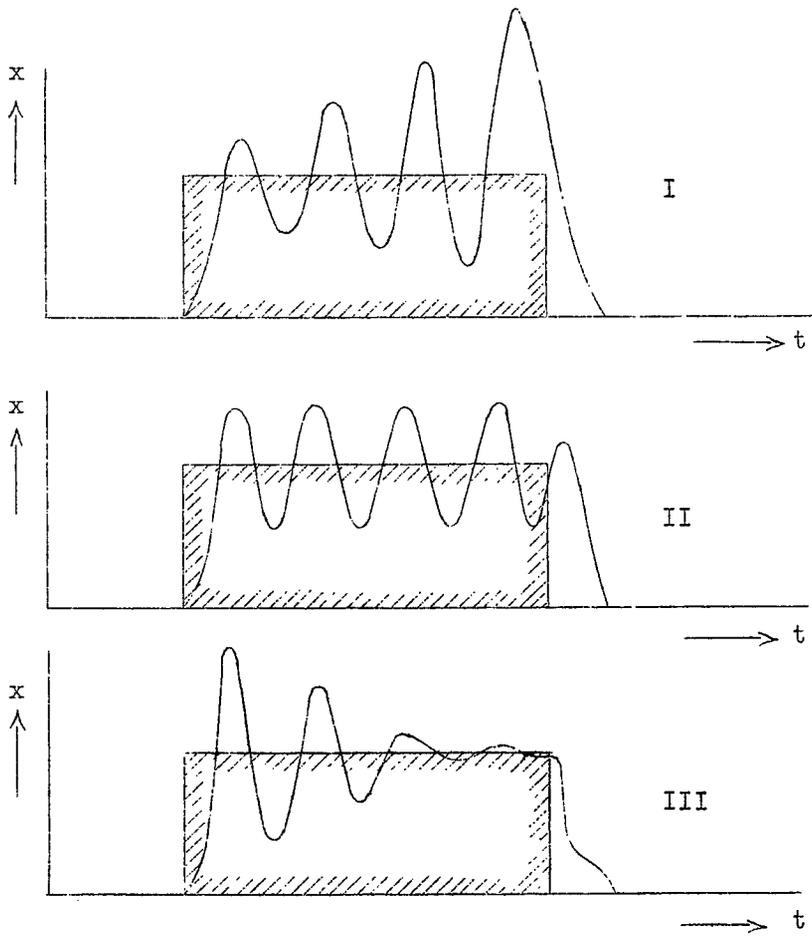


Figure 5.-Diagram of stem travel.

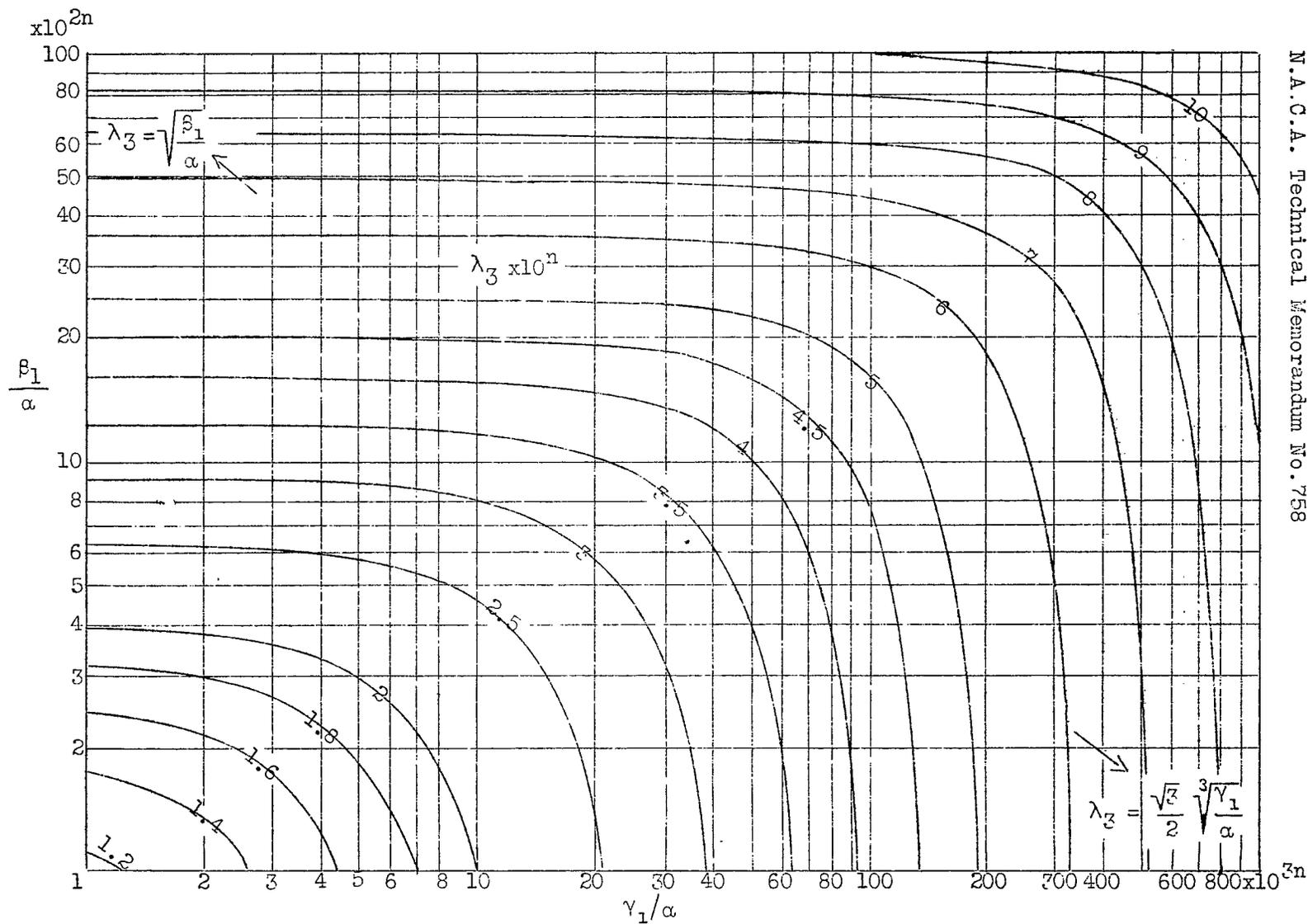


Figure 6.-Natural frequency λ_3 , versus β_1/α and γ_1/α .

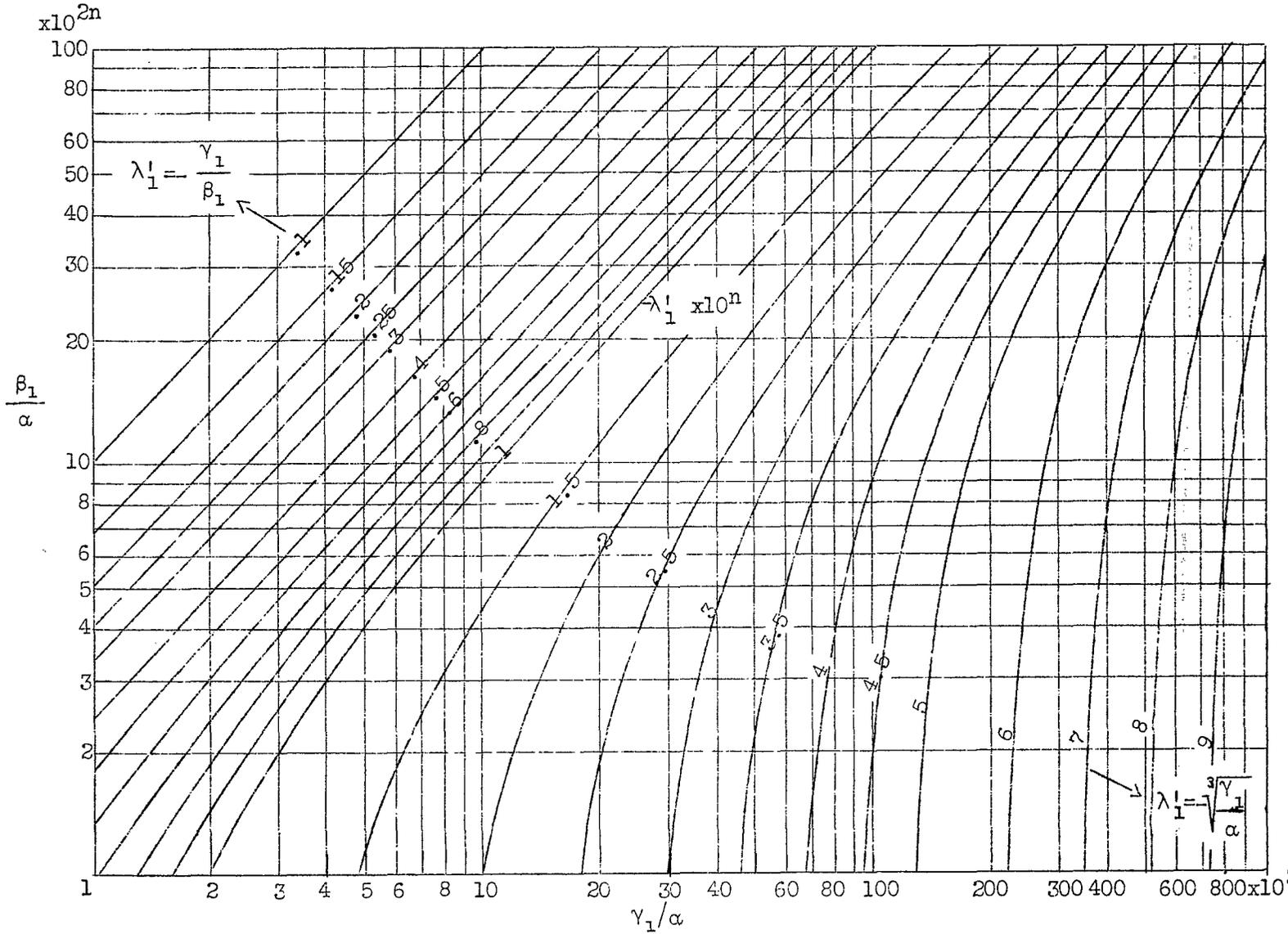


Figure 7. λ_1' , versus β_1/α and γ_1/α .

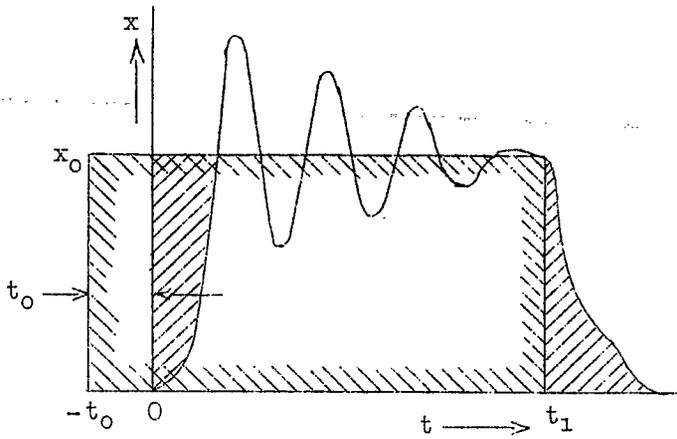


Figure 8.-
Injection lag
 t_0 versus
stem travel.

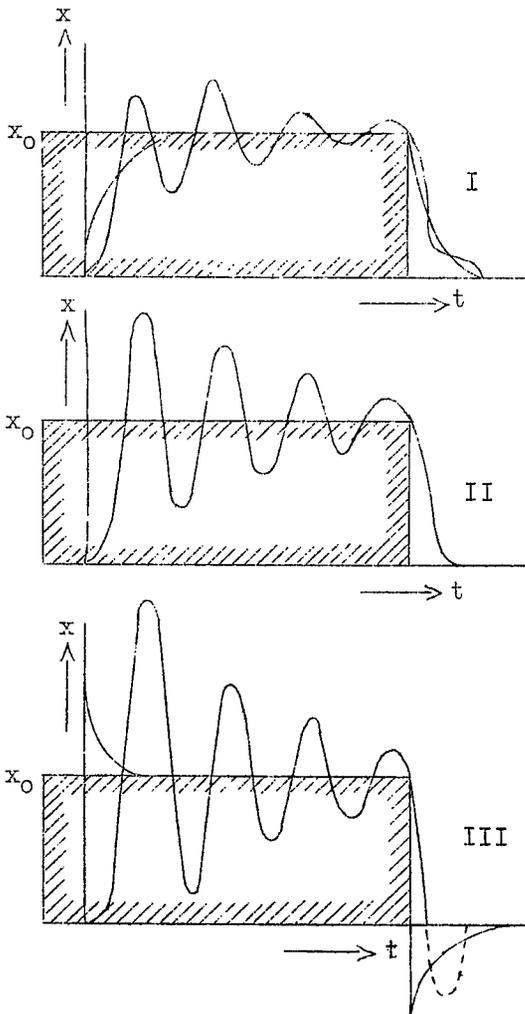


Figure 9.-
Stem travel
versus
different
initial
accelerations.

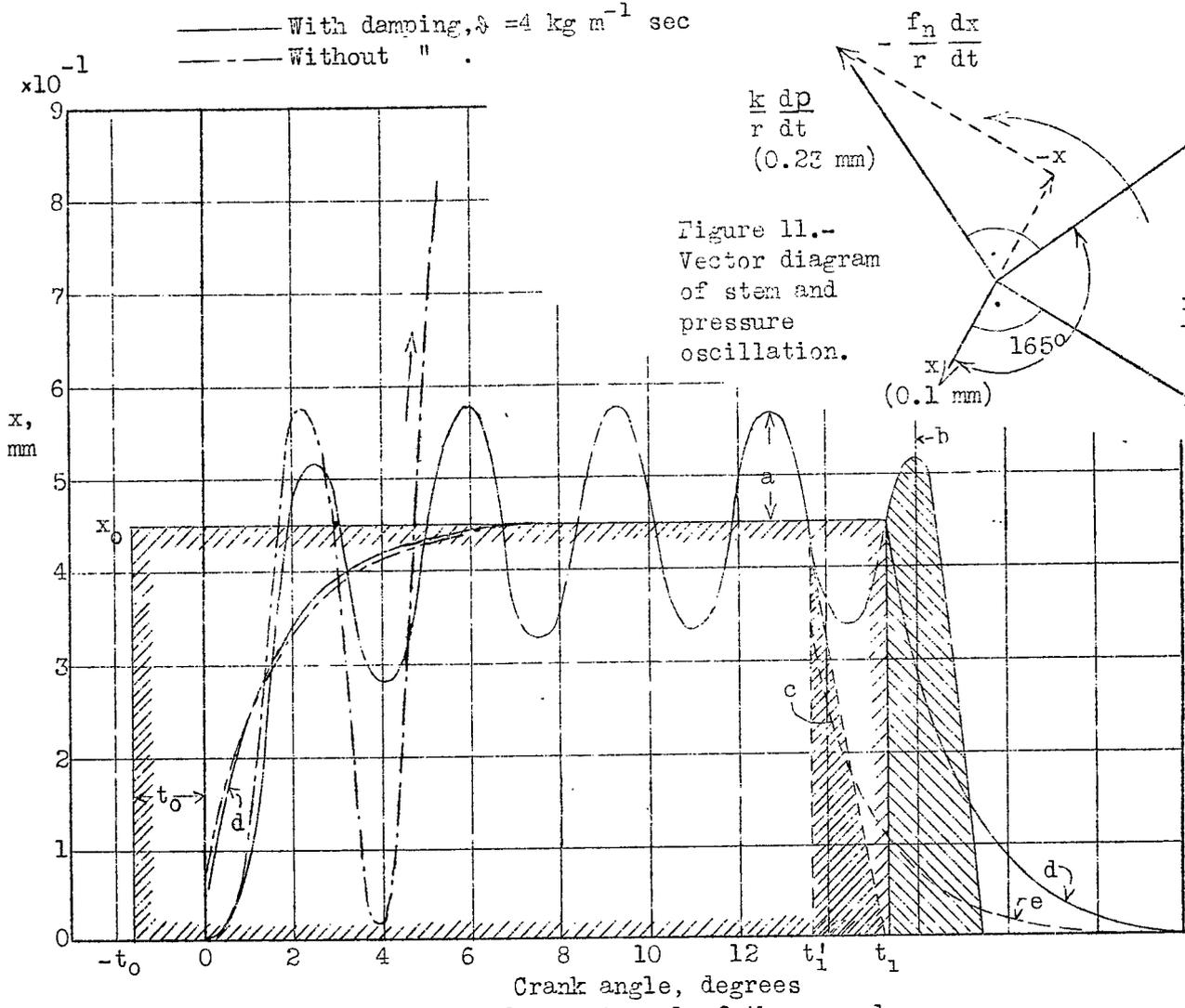
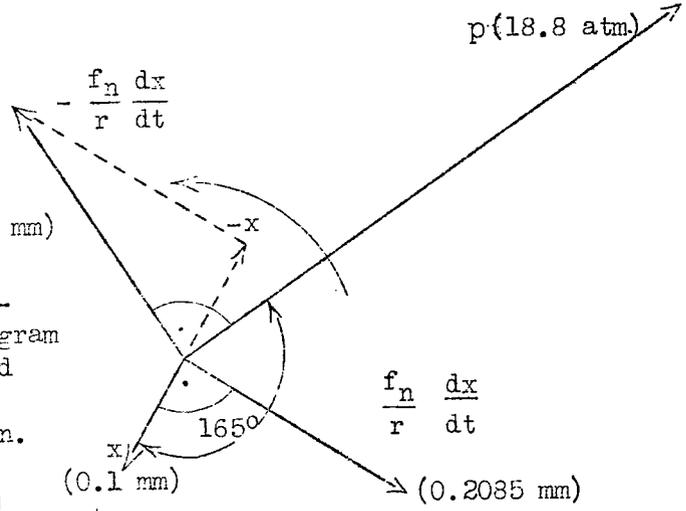


Figure 10.-Curve of stem travel of the example.

Figure 11.- Vector diagram of stem and pressure oscillation.



b, 7.85×10^{-2} mm \times crank angle, degrees
 c, 3.5×10^{-2} mm \times crank angle, degrees

d, $a_1 e^{\lambda_1 t}$
 e, $a_1' e^{\lambda_1 t}$

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