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TECHNICAL MEMORANDUM 165

LOCATION OF CENTER OF PRESSURE OF AIRPLANE WINGS.

By Mises.

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LOCATION OF CENTER OF PRESSURE OF AIRPLANE WINGS.*

By Mises.

In the first volume of the "Ergebnisse aus der aerodynamischen Versuchsanstalt zu Göttingen" (Results obtained in the Göttingen Aerodynamic Laboratory), data are given in considerable detail on the resulting air force moment exerted on various wing sections at different angles of attack. A hydrodynamic theory, which (on the assumption of uniform motion and hence of infinitely wide wings) renders it possible to compute this moment as a function of the angle of attack, was first published by me in 1917 and supplemented in 1920 ("Zeitschrift für Flugtechnik und Motorluftschiffahrt," 1917, pp. 157-163, 1920, pp. 68-73 and pp. 87-89). The Göttingen data can well serve to test the theory and it may be remarked in advance that experiment and computation compare very favorably. It seems especially noteworthy that a portion of the demonstrable agreement is entirely independent of the diminution undergone by the air forces in the transition to wings of finite width.

The theoretical results may be summed up as follows: Let μ represent the air density $\frac{\gamma}{g}$, v the velocity of the air in the wind tunnel (flight speed), and α the "effective" angle of attack, i.e. the angle between the speed vector and a fixed direction in the wing section, so selected that the lift vanishes for $\alpha = 0$ (direction of the first axis of the wing section). Then the lift for the unit width of wing

$$A = 4\pi \mu v^2 a \sin\alpha \qquad (1)$$

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and the moment of the lift for a given pivot has the value

$$M = 2 \pi \mu v^2 c^2 \sin 2 (\alpha - \gamma) \tag{2}$$

in which a, c and γ are certain fixed values of the wing section, which will be discussed later. If the pivot is moved from the particular position assumed in equation (2), a distance l in the direction forming the angle φ with the "first axis" of the wing section (Fig. 1), the moment becomes

$$\left.\begin{array}{l} M_1 = M - A\, l \cos (\alpha - \varphi) = \\ 2 \pi \mu v^2 [c^2 \sin 2 (\alpha - \gamma) - 2 a\, l \sin\alpha \cos (\alpha - \varphi)] \end{array}\right\} \tag{3}$$

If we divide A by the chord b and the so-called dynamic pressure, i.e. $\mu \frac{v^2}{2}$, we obtain the dimensionless "coefficient of lift."

$$c_a = \frac{2 A}{\mu v^2 b} = 8 \pi \frac{a}{b} \sin \alpha \tag{4}$$

and if we reduce M_1 by the square of b and the dynamic pressure and change the sign, we obtain Prandtl's "coefficient of moment."

$$\left.\begin{array}{l} C_m = - \frac{2 M}{\mu v^2 b^2} = \\ 4 \pi [\frac{2 a l}{l^2} \sin \alpha \cos (\alpha - \varphi) - \frac{c^2}{b^2} \sin 2 (\alpha - \gamma)] \end{array}\right\} \tag{5}$$

By a very simple transformation in the right-hand member of equation (5), we can introduce expression 4 and thus obtain the formula

$$\left.\begin{array}{l} C_m = C_a [\frac{l}{b} \cos (\alpha - \varphi) - \frac{c^2}{a\, b} \cos \quad \cos (\alpha - \gamma)] \\ \qquad + 8 \pi \frac{c^2}{b^2} \sin \gamma \cos \alpha \cos (\alpha - \gamma) \end{array}\right\} \tag{6}$$

which is adapted for comparison with the formula resulting from the Göttingen experiments. In all practical cases, α, γ and φ are such small angles that it is permissible to consider all cosines in equation (6) as equal to unity. Hence, equation (6) becomes

$$c_m = c_a \left[\frac{l}{b} - \frac{c^2}{ab}\right] + 8\,\pi\,\frac{c^2}{b^2}\sin\gamma \tag{7}$$

i.e. the coefficient of moment c_m, represented as a function of the coefficient of lift c_a, gives a straight line. If we assume that, as a result of the finite width of the wing at every point of the chord extension, only the λ-fold of the forces calculated for "uniform motion" is effective ($\lambda < 1$, about 2/3 for ordinary cases) and put $c_m' = \lambda c_m$, $c_a' = \lambda c_a$, equation (7) becomes

$$c_m' = c_a' \left[\frac{l}{b} - \frac{c^2}{ab}\right] + \lambda\,8\,\pi\,\frac{c^2}{b^2}\sin\gamma \tag{8}$$

The inclination of the line c_m/c_a is therefore independent of the diminution factor λ. We accordingly recognize in the practical agreement of the computed and experimentally determined inclination a touchstone for the theory.

Fig. 2 represents a wing section, for which all the characteristics of lift and moment have been accurately computed. If we choose, as was the case in the Göttingen experiments, the forward end of the chord for the center of moments for M_1, the dimensions determining the inclination of the line have the values

$$a = 0.44,\quad b = 1.62,\quad l = 0.79,\quad c^2 = 0.1789.$$

From these the coefficient of c_a in equation (7), or c_a' in equation (8), is found to be 0.237. In order to be able to judge of the degree of approximation of equation (7) to the exact equation (6), the precise values of the coefficients of c_a in equation (6) are given for the values $\gamma = 3.5^\circ$ and $\varphi = 12^\circ$ here involved. They are for

$\alpha =$	5°	0°	5°	10°	15°
	0.218	0.227	0.233	0.238	0.242

The variations are therefore in fact small.

A glance at the Göttingen report (pp. 83-101) shows immediately: (1) that all c_a'/c_m' lines are nearly straight, excepting for a systematic deflection at the smallest angles of attack; (2) that all these lines are almost equally inclined, about 4 : 1, thus corresponding to the coefficient 0.25.

The theory also teaches that, within the range of the wing sections in practical use today, the determining ratios for the lines of moment, c : a, b : a, and l : a vary only very slightly. The limiting values, which apply strictly only for an infinitely thin straight wing section but from which in actual practice there is but little deviation, are

$$a = c = \frac{l}{2} = \frac{b}{4}$$

from which follows the values 0.25 of the coefficient in equations (7) and (8), in agreement with the mean experimental value.

The relation of the position of the line of moments with respect to the origin of the coordinates is not so clear. The consecond member of equation (7) contains the quantity γ, the angle between the "first" and "second" axis of the wing section. This quantity depends on the shape of the entire wing section and may be determined in each individual case, as soon as the conformable image on the unit circle has been found. As a starting point, to which reference has already been made, γ may be considered equal to about one-fourth of the angle of camber of the mean section line. The diagrams in the report show in fact that the c_m'/c_a' line moves toward the right, when the camber is increased. In order to test the amount of the displacement with respect to the origin of the ordinates, wing section 397 (Fig. 3) of the report was chosen, in which the experimental points of the straight line, computed according to equation (8) with $\lambda = 2/3$ (or really of the exact line according to equation (6), conform closely (Fig. 4). The camber of wing section 397 (Fig. 3) is doubtless smaller than that of the theoretically tested one in Fig. 2, so that it must be assumed that the former has a considerably smaller γ. From this it would follow that the value of λ obtained from the ratio of the lift values $c_a' : c_a$ is too small. The systematic experiments for discovering the influence exerted by the aspect ratio of a rectangular wing on the air forces, led to the result (see p. 50 of the report) that c_m', considered a function of c_a', is practically independent of the aspect ratio. Since, on the other hand, the ratio $c_a' : c_a$ depends greatly on the aspect ratio of the wing, we are led to conclude from the result of these experiments, that λ must equal unity ($\lambda = 1$) in equation (8). Comparison with the wing section in Fig. 2 and others in the Göttingen report does not gainsay this conclusion in the least.

We may sum up by saying that the hydrodynamic theory, in close agreement with observation for the relation between the coefficient of lift and moment, gives a straight line with the inclination of 1 : 4, whose distance from the origin of the coordinates increases with the camber. The degree of dependence on the camber and the possible influence of the aspect ratio of the wing still need to be cleared up.

Translated by the National Advisory Committee for Aeronautics.

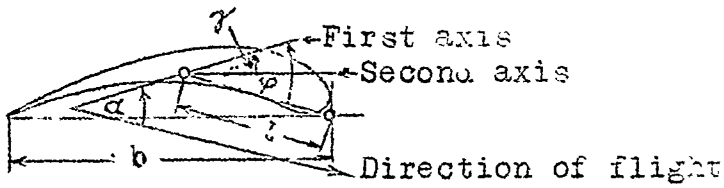

Fig.1


Fig.2


Wing section No.397
Fig.3

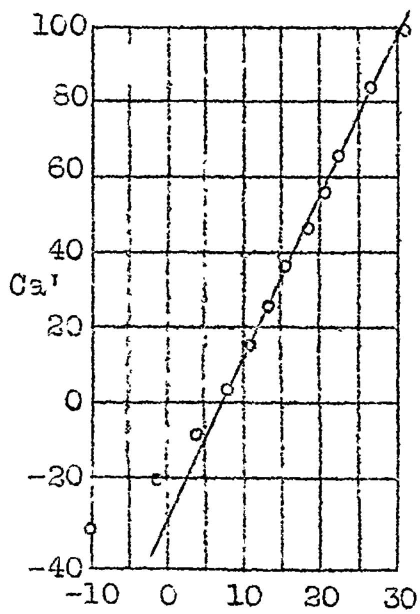

Fig.4