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THE GAS KINETICS OF VERY HIGH FLIGHT SPEEDS

By Eugen Sanger

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THE GAS KINETICS OF VERY HIGH FLIGHT SPEEDS*

By Eugen Sanger

ABSTRACT

In ordinary gas dynamics we use assumptions which also agree with kinetic theory of gases for small mean free paths of the air molecules. The air forces thus calculated have to be re-examined, if the mean free path is comparable with the dimensions of the moving body or even with its boundary layer. This case is very difficult to calculate. The conditions, however, are more simple, if the mean free path is large compared to the body length, so that the collisions of molecules with each other can be neglected compared to the collisions with the body surface.

In order to study the influence of the large mean free path, calculations are first carried out for the case of extreme rarefaction. Furthermore, the calculations on this "completely ideal" gas will be carried out under consideration of Maxwell's velocity distribution and under the assumption of certain experimentally established reflection laws for the translational and nontranslational molecular degrees of freedom.

The results thus obtained allow us to find, besides the air pressure forces perpendicular to the surface, also the friction stresses parallel to the surface.

Their general results are calculated out for two practically important cases: for the thin smooth plate and for a projectile-shaped body moving axially.

The mathematical part of the investigation was carried out primarily by Dr. Irene Bredt.

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*"Gaskinetik sehr hoher Fluggeschwindigkeiten." Forschungsbericht Nr. 972, May 31, 1938.

OUTLINE

I. INTRODUCTION

With the motion of bodies at very great atmospheric heights, the air can no longer be considered a continuous medium, in the sense of flow theory.

At over 50 kilometers altitude, the mean free path of the air molecules will be of the magnitude of boundary layer thickness and, at over 100 kilometers altitude, of the magnitude of the moving body itself.

The mean free path at greater heights will be definitely greater than the body dimensions of the moving body, and the especially simple conditions of very rarefied "completely ideal" gases are valid where the effect of the collisions of the molecules with each other disappears compared to the effect of the collisions with the moving body.

The air molecules collide then against the moving body as individual particles, independent of each other, and are reflected with a mechanism which deviates more or less from the known Newtonian principle of air drag, as shown by the results of the kinetics of very rarefied gases.

The velocity of the body will be denoted by v (meters per second) in the following discussion.

If we imagine as usual that for the consideration of the flow process the body stands still and the medium moves, then v equals the uniform undisturbed flow velocity of all the air molecules.

The air molecules have also their ideal random thermal motion.

The individual molecular thermal velocities are distributed completely at random in all directions and are of completely arbitrary magnitudes, where the various absolute velocity magnitudes group around a most probable value, c (meters per second), according to Maxwell's distribution, which has the ratio $\sqrt{2}:\sqrt{3}$ to the most applied gas kinetic value of "average molecular speed $\overline{c}$" (square root of the average of the squares of all the velocities present).

According to the known Maxwell speed distribution law, letting $\rho(\mathrm{kg\ s^2/m^4})$ equal the total molecular mass per unit volume, the mass $d\rho$ of those molecules having velocities between c_x and $c_x + dc_x$ is:

$$\frac{d\rho}{\rho} = \frac{4}{\sqrt{\pi}} \frac{c_x^2}{c^3} e^{-\frac{c_x^2}{c^2}} dc_x \tag{1}$$

If we consider only this mass $d\rho$ of an otherwise motionless ($v = 0$) gas, the molecules of which are moving with the particular speed c_x in random directions, then the quantity $d\overline{\rho}$ of molecules striking per second on a unit surface of a motionless plane wall can be calculated, imaging that all velocities c_x are plotted from the center of a sphere with radius c_x. The conditions of figure 1 are the result of an inclination angle ϕ between the wall normal and the velocity direction under consideration, and of the molecular quantity $d\rho \frac{dF}{4c_x^2\pi}$ which passes through the striped area dF of the sphere surface. Then:

$$d\overline{\rho} = \frac{d\rho}{4c_x^2\pi} \int_{\phi=0}^{\pi/2} c_x \cos\phi \, dF = \frac{d\rho}{4} c_x$$

If, with the aid of Maxwell's equation, we include in our calculation all molecules with the various possible speeds c_x between 0 and ∞, the total mass $\overline{\rho}$ of the molecules colliding per second will be:

$$\overline{\rho} = \int_{c_x=0}^{\infty} \frac{c_x}{4}\, d\rho = \frac{\rho}{\sqrt{\pi}} \int_{c_x=0}^{\infty} \frac{c_x^{\,3}}{c^3}\, e^{-\frac{c_x^{\,2}}{c^2}}\, dc_x = \frac{\rho c}{2\sqrt{\pi}} \tag{2}$$

as one can find in any textbook of gas kinetics. The pressure of the motionless gas against the stationary wall can be calculated similarly.

The impulse di_p perpendicular to the wall with which molecules of particular speed c_x are striking the wall at an angle ϕ will be:

$$di_p = \frac{d\rho}{4c_x^{\,2}\pi} \int_{\phi=0}^{\pi/2} c_x^{\,2} \cos^2\phi\, dF = \frac{d\rho}{6}\, c_x^{\,2}$$

and the impulse of all molecules striking the wall:

$$i_p = \int_{c_x=0}^{\infty} \frac{c_x^{\,2}}{6}\, d\rho = \frac{2}{3}\, \frac{\rho}{\sqrt{\pi}} \int_{c_x=0}^{\infty} \frac{c_x^{\,4}}{c^2}\, e^{-\frac{c_x^{\,2}}{c^2}}\, dc_x = \frac{\rho}{4}\, c^2 \tag{3}$$

If we double this impulse value because of the elastic rebound always assumed for motionless gases, then we obtain the gas rest pressure

$$p = \frac{\rho}{2}\, c^2 \quad \text{or} \quad \frac{\rho}{3}\, \overline{c}^2$$

The value of the total impulse is also interesting, i.e., the sum of all single molecular impulses which strike per second on the unit area.

The total impulse of the molecular mass $d\bar{\rho}$ corresponding to a particular c_x will be:

$$d\bar{i} = d\bar{\rho}c_x = \frac{d\rho}{4} c_x^2$$

It is thus 1.5 times greater than the effective impulse di_p against the plate. The total impulse i is also greater in the same ratio.

II. AIR FORCES ON THE FRONT SIDE OF A FLAT PLATE OBLIQUE TO THE AIR STREAM

If we consider again the mass $d\rho$ of molecules with speeds between c_x and $c_x + dc_x$ (almost equal) and if we examine its action on a flat plate in an air stream with an angle of attack α, then this process can be illustrated by figure 2, if we further assume that $v \sin \alpha < c_x$.

The uniform velocity v of the individual molecule combines with the ideal random velocity (which can have any space direction) to give a resultant, the components of which are:

perpendicular to the plate: $v \sin \alpha + c_x \cos \phi$
parallel to the plate and to $v \cos \alpha$: $v \cos \alpha + c_x \sin \phi \cos \psi$
parallel to the plate and perpendicular to $v \cos \alpha$: $c_x \sin \phi \sin \psi$
and for which the absolute value is therefore:

$$w = \sqrt{v^2 + 2vc_x (\sin \alpha \cos \phi + \cos \alpha \sin \phi \cos \psi + c_x^2)}$$

From the sphere of all possible directions of c_x, a spherical sector with the half opening angle $\cos \varkappa = v \sin \alpha / c_x$ is excluded, in which the speed component $v \sin \alpha + c_x \cos \phi$ is directed away from the plate, i.e., the molecules of this ϕ range do not strike the plate. The integration over the velocity directions of all colliding molecules is not from $\phi = 0$ to $\frac{\pi}{2}$ as with the motionless gas, but from $\phi = 0$ to $\pi = \varkappa$.

The molecular mass colliding per second against the unit plate area with the selected speed c_x is therefore:

$$d\overline{\rho} = \frac{d\rho}{4c_x{}^2\pi} \int_{\phi=0}^{\pi-\varkappa} (v \sin \alpha + c_x \cos \psi) 2c_x{}^2\pi \sin \phi \, d\phi$$

$$= \frac{d\rho}{4}\left(c_x + 2v \sin \alpha + \frac{v^2 \sin^2\alpha}{c_x}\right)$$

For $v \sin \alpha > c_x$ the integration extends over the whole sphere from $\phi = 0$ to π and the molecular mass colliding against the plate with selected speed c_x is

$$d\overline{\rho} = \frac{d\rho}{4c_x{}^2\pi} \int_{\phi=0}^{\pi} (v \sin \alpha + c_x \cos \phi) 2c_x{}^2\pi \sin \phi \, d\phi = d\rho v \sin \alpha$$

Both equations naturally give the same value for $v \sin \alpha = c_x$. With the aid of Maxwell's distribution equation the total molecular mass colliding on the unit area per unit time within the speed range $c_x = 0$ to ∞ will be:

$$
\begin{aligned}
\overline{\rho} &= \int_{c_x = v \sin\alpha}^{\infty} \frac{1}{4}\left(c_x + 2v \sin \alpha + \frac{v^2 \sin^2 \alpha}{c_x}\right) d\rho + \int_{c_x=0}^{v \sin \alpha} v \sin \alpha \, d\rho \\
&= \frac{\rho}{\sqrt{\pi}} \int_{c_x = v \sin \alpha}^{\infty} \left(\frac{c_x^3}{c^3} + \frac{2vc_x^2}{c^3} \sin \alpha + \frac{v^2 c_x}{c^3} \sin^2 \alpha\right) e^{-\frac{c_x^2}{c^2}} dc_x \\
&\quad + \frac{4\rho}{\sqrt{\pi}} \int_{c_x=0}^{v \sin \alpha} \frac{vc_x^2}{c^3} \sin \alpha \, e^{-\frac{c_x^2}{c^2}} dc_x \\
&= \frac{\rho}{\sqrt{\pi}} \frac{c}{2} \left(e^{-\frac{v^2 \sin^2 \alpha}{c^2}} + \sqrt{\pi} \frac{v \sin \alpha}{c} + 2 \frac{v \sin \alpha}{c^2} \int_{c_x=0}^{v \sin \alpha} e^{-\frac{c_x^2}{c^2}} dc_x\right)
\end{aligned}
\tag{4}
$$

Thus the number of the colliding molecules is known, and for calculation of the forces acting on the plate, we must now determine what impulse the molecular mass under consideration produces in the directions in question.

The impulse perpendicular to the plate, i_p and the impulse parallel to it i_T will be examined separately.

We find for the impulse per second perpendicular to the plate

1. If $v \sin \alpha < c_x$: $$di_p = \frac{d\rho}{4c_x^2\pi} \int_{\phi=0}^{\pi-\varkappa} (v \sin \alpha + c_x \cos \phi)^2 dF$$

$$= \frac{d\rho}{6} c_x^2 \left(1 + \frac{v}{c_x} \sin \alpha\right)^3$$

2. If $v \sin \alpha > c_x$: $$di_p = \frac{d\rho}{4c_x^2\pi} \int_{\phi=0}^{\pi} (v \sin \alpha + c_x \cos \phi)^2 dF$$

$$= \frac{d\rho}{6} c_x^2 \left(2 + 6 \frac{v^2}{c_x^2} \sin^2\alpha\right)$$

This summation of the impulse components over all possible directions yields the total impulse, perpendicular to the plate, of the air molecules striking the unit area per second with a speed c_x determined by $d\rho$.

The further summation of the impulses over all speeds c_x with the aid of Maxwell's distribution equation results in the total impulse i_p acting per second perpendicular to the plate:

$$i_p = \int_{c_x=0}^{v \sin\alpha} \frac{d\rho}{6} c_x^2\left(2+6\frac{v^2}{c_x^2}\sin^2\alpha\right) + \int_{c_x=v\sin\alpha}^{\infty} \frac{d\rho}{6} c_x^2\left(1+\frac{v}{c_x}\sin\alpha\right)^3$$

$$= \frac{2}{3}\rho\left[\int_{c_x=0}^{v\sin\alpha} \frac{1}{\sqrt{\pi}}\frac{c_x^4}{c^3} e^{-\frac{c_x^2}{c^2}}\left(2+6\frac{v^2}{c_x^2}\sin^2\alpha\right)dc_x\right.$$

$$\left. + \int_{c_x=v\sin\alpha}^{\infty} \frac{1}{\sqrt{\pi}}\frac{c_x^4}{c^3} e^{-\frac{c_x^2}{c^2}}\left(1+\frac{v}{c_x}\sin\alpha\right)^3 dc_x\right]$$

$$= \frac{\rho}{\sqrt{\pi}}\left\{\frac{v\sin\alpha}{c}\left(\frac{1}{3}v^2\sin^2\alpha - \frac{1}{3}cv\sin\alpha + \frac{1}{2}c^2\right)e^{-\frac{v^2\sin^2\alpha}{c^2}}\right.$$

$$\left. + \frac{\sqrt{\pi}}{2}\left(v^2\sin^2\alpha + \frac{1}{2}c^2\right) + \left(v^2\sin^2\alpha + \frac{1}{2}c^2\right)\int_{c_x=0}^{v\sin\alpha} e^{-\frac{c_x^2}{c^2}}\frac{dc_x}{c}\right\} \qquad (5)$$

If we set $\alpha = 0$ then equation (5) gives the impulse of the motionless gas against the motionless wall, equation (3).

Also $\alpha = 0$ gives the impulse of the motionless gas, which will not be influenced through uniform motion of the gas mass parallel to the flat plate. We find the impulse parallel to the plate in the direction of $v \cos\alpha$ in the following manner:

The molecular beam with a particular speed c_x and with a particular direction (the latter determined by the inclination angle ϕ between the velocity c_x and the perpendicular to the plate, and by the angle ψ

between the projection $c_x \cos \phi$ and the direction $v \cos \alpha$) gives the molecular mass colliding per second according to figure 2.

$$d\bar{\rho} = d\rho \frac{c_x^2 \sin \phi \, d\phi \, d\psi}{4 c_x^2 \pi} \left(v \sin \alpha + c_x \cos \phi\right)$$

The velocity component of this beam parallel to the plate is:

$$v \cos \alpha + c_x \sin \phi \cos \psi$$

and the impulse of the beam with a particular c_x, ϕ, and ψ will be

$$d\rho \frac{\sin \phi \, d\phi \, d\psi}{4\pi} \left(v \sin \alpha + c_x \cos \phi\right)\left(v \cos \alpha + c_x \sin \phi \cos \psi\right)$$

If one integrates over all ψ and ϕ, one obtains the impulse of the beam with a particular c_x.

For $v \sin \alpha < c_x$:

$$di_T = \frac{d\rho}{4\pi} \int_{\psi=0}^{2\pi} \int_{\phi=0}^{\pi-\varkappa} \left(v \sin \alpha + c_x \cos \phi\right)\left(v \cos \alpha + c_x \sin \phi \cos \psi\right) \sin \phi \, d\phi \, d\psi$$

$$= \frac{d\rho}{6} c_x^2 \left(\frac{3}{2} \frac{v^3}{c_x^3} \sin^2\alpha \cos \alpha + 3 \frac{v^2}{c_x^2} \sin \alpha \cos \alpha + \frac{3}{2} \frac{v}{c_x} \cos \alpha\right)$$

and for $v \sin \alpha > c_x$:

$$di_T = \frac{d\rho}{4\pi} \int_{\psi=0}^{2\pi} \int_{\phi=0}^{\pi} \left(v \sin \alpha + c_x \cos \phi\right)\left(v \cos \alpha + c_x \sin \phi \cos \psi\right) \sin \phi \, d\phi \, d\psi$$

$$= d\rho \, v^2 \sin \alpha \cos \alpha$$

The integration over all c_x with the aid of Maxwell's distribution equation gives finally the total impulse in the required direction parallel to the plate:

$$i_\tau = \int_{c_x=0}^{v \sin\alpha} d\rho v^2 \sin\alpha\cos\alpha + \int_{c_x=v\sin\alpha}^{\infty} \frac{d\rho}{6} c_x^2\left(\frac{3}{2}\frac{v^3}{c_x^3}\sin^2\alpha\cos\alpha\right.$$

$$\left. + 3\frac{v^2}{c_x^2}\sin\alpha\cos\alpha + \frac{3}{2}\frac{v}{c_x}\cos\alpha\right)$$

$$= \rho\left[\frac{1}{2\sqrt{\pi}} vc\cos\alpha\, e^{-\frac{v^2\sin^2\alpha}{c^2}}\right.$$

$$\left. + \left(\frac{1}{2}v^2\sin\alpha\cos\alpha\right)\left(1 + \frac{2}{c\sqrt{\pi}}\int_{c_x=0}^{v\sin\alpha} e^{-\frac{v^2\sin^2\alpha}{c^2}} dc_x\right)\right] \tag{6}$$

We can start out from the impulse of the gas stream, given by equations (5) and (6), (perpendicular and parallel to the plate) in order to calculate the forces produced by the air on the front side of the plate oblique to the air stream, including the force perpendicular to the plate (normal pressure p) and the force parallel (friction stress τ).

For this calculation, we have to make some assumptions on the transfer of this impulse to the plate and on the change of the kinetic translational energy of the molecules into other energy forms for which present gas kinetics furnish only a partial basis.

If we first assume monatomic gases, so that inner degrees of freedom for energy absorbance do not exist, and further assume that the struck molecules of the wall are in such a temperature condition that they also cannot take over any energy from the colliding molecules, then the molecules have to leave the wall again with the same speed with which they arrived. The collision is thus completely elastic and we have only to derive the direction of reflection.

Gas kinetics distinguishes two different possibilities for this:

<u>Mirror Reflection</u> where the assumption is made (following Newton) that the angle of incidence and the angle of reflection are equal and both beams are in the same perpendicular plane.

<u>Diffuse Reflection</u> where it is assumed (following Knudsen[1]) that the reflection direction is not at all dependent on the direction of the impinging beam and is completely diffuse, i.e., that the colliding molecules first submerge in the wall surfaces, then after a finite time of "adherence" leave again, in a completely arbitrary direction.

This last hypothesis is generally accepted in flow theory, where the adherence of the frictional boundary layer on the surface is explained by diffuse recoil of the molecules.

In the case of a motionless gas ($v = 0$) both assumptions lead to the same distribution of rebound molecules and thus to the same forces on the wall, as the striking molecules are in completely random directions and this then is also true for the rebound molecules under both assumptions.

In the case of a gas in motion, the two assumptions lead to very different air forces.

With elastic "mirror" reflections, the impulse i_T of the gas flow parallel to the wall stays unchanged. Shear stresses on the front side of the plate τ_v are not transferred to the wall. The friction forces are zero.

$$\tau_v = 0 \tag{7}$$

The impulse i_p of the impinging molecules normal to the wall is destroyed completely and an equal but opposite impulse is produced by

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[1]Knudsen, M.: Annalen der Physik, Vol. 28, p. 75, (1909); Vol. 28, p. 114, (1909); Vol. 28, p. 999, (1909); Vol. 31. pp. 205, 633, (1910); Vol. 35, p. 389, (1911); Vol. 34, p. 593, (1911); Vol. 48, p. 1113, (1915); Vol. 50, p. 472, (1916); and Vol. 83, p. 797, (1927).

the completely elastic recoil. The pressure on the wall caused by this process thus equals twice the impulse i_p:

$$p_v = 2i_p \tag{8}$$

With diffuse-elastic reflection, the impulse i_τ of the gas stream parallel to the wall will be given up entirely to the wall and the friction stress equals i_τ:

$$\tau_v = i_\tau \tag{9}$$

The wall normal impulse i_p of the arriving molecules is destroyed again, whereby is created a partial pressure $p_1 = i_p$.

The second part of the pressure, due to the diffuse-elastic recoil, has to be investigated more closely. To imitate the real process, an impulse value of the magnitude of the complete impulse i of the beam striking per unit plate area is distributed evenly on a hemisphere as if all gas particles started from the center of this hemisphere, and finally the resultant of this impulse distribution perpendicular to the plate is ascertained.

The total impulse i of the arriving beam is derived, according to the preceeding impulse calculations, in the following manner:

The molecular mass striking per unit time on the known area section $df = c_x^2 \sin\phi \, d\phi \, d\psi$ selected for a certain c_x, ϕ, and ψ is:

$$d\bar{\rho} = d\rho\left(c_x^2 \sin\phi \, d\phi \, d\psi \middle/ 4c_x^2 \pi\right)\left(v \sin\alpha + c_x \cos\phi\right)$$

The effective speed of this ray is

$$w = \sqrt{v^2 + 2vc_x\left(\sin\alpha \cos\phi + \cos\alpha \sin\phi \cos\psi\right) + c_x^2}$$

and the impulse per second thus:

$$d\overline{\rho w} = d\rho \frac{\sin \phi \, d\phi \, d\psi}{4\pi}\Big(v \sin \alpha + c_x \cos \phi\Big)\sqrt{v^2 + 2vc_x\left(\sin \alpha \cos \phi + \cos \alpha \sin \phi \cos \psi\right) + c_x^{\,2}}$$

This impulse integrated over all c_x, ϕ, and ψ will give finally the total impulse of the beam. The actual carrying out of this integration is so difficult that the impulse will evaluate by successive approximation. It is started with the vector sum of the impulse resultants i_p and i_τ, perpendicular and parallel to the plate.

$$\bar{i}^2 = i_p^{\,2} + i_\tau^{\,2}$$

This impulse resultant is smaller than the total impulse. It is found in connection with equation (3) that the total impulse of a gas at rest ($v = 0$) is 1.5 times the impulse resultant. Total impulse and resultant are equal for uniformity flowing gas without heat motion ($c_x = 0$). For conditions lying in between, we assume a constant relationship for the factor with which the impulse resultant has to be multiplied to obtain the total impulse i. For instance:[2]

$$i = \frac{1.5 + \frac{v}{c} \sin \alpha + 1.5\left(\frac{v}{c}\right)^2}{1 + \frac{v}{c} \sin \alpha + 1.5\left(\frac{v}{c}\right)^2}\,\bar{i}$$

This total impulse, according to our assumption, is now considered as the completely uniform impulse radiation per area unit in all directions outward from the surface.

[2]Interpolation by Prof. Busemann, Braunschweig.

The beam pressure p_2 perpendicular to the surface is then equal to $i/2$, as shown by a simple integration over all normal components.

Thus, the pressure vertical to the wall in the case of diffuse-elastic reflection is:

$$p_v = i + i/2 \tag{10}$$

In order to get a first view on the numerous conditions of the air forces just found, figure 3 shows the relationship between the pressure p or the shear force τ and the dynamic pressure $q = \frac{\rho}{2} v^2$ for a hypothetical atmosphere of monatomic hydrogen with $t = 0^\circ$ C temperature ($c = 2124$ m/s) and for flight speeds up to $v = 8000$ m/s, for either mirrorlike or diffuse recoil.

It is seen how different the air forces can be according to the assumptions made: mirrorlike or diffuse.

In gas kinetics an attempt is made to approach the real conditions by assuming that the reflection for a fraction f of all striking molecules is diffuse, while the remainder $(1 - f)$ will be repelled mirrorlike. The fraction of diffuse reflections depends on the kind of striking molecules and particularly on the material, surface conditions, and temperature of the struck wall.

According to numerous measurements[3], the plate can be considered as completely rough under conditions usually prevailing in flight technique, i.e., the mirror reflected part $(1 - f)$ is negligibly small, so the reflection will be almost completely diffuse.

An experimentally obtained dependence of f on the angle of attack, such that the reflection will be more mirrorlike with smaller angle of attack, is according to previous measurements of flight relations, too insignificant to be considered.

Knauer and Stern[4] assume from optical analogies that the angle of attack at which mirror reflection begins is such that the surface roughness

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[3] For example, Karl Jellinek, Lehrbuch der physikalischen Chemie, Vol. 1, p. 270, 1928.

[4] Knauer, F., and Stern, O.: Zs. f. Phys. Vol. 50, pp. 766, 799 (1929).

height, projected on the beam, must be smaller than De Broglie's wavelength λ of the molecular beam.

With a wave length of 10^{-8} cm and a roughness height of 10^{-5} to 10^{-6}, one obtains for the angle of attack which is of interest $\sin \alpha = \lambda/h = 10^{-3}$, i.e., an angle range of a few minutes, which is insignificant in flight technique.

With regard to the reflection direction, we assume in the following analysis that $f = 1$, i.e., completely diffuse recoil.

So far, for the reflection speed, perfect elasticity of the recoil was assumed, which means individual recoil speed is equal to the colliding speed.

The struck wall will, in fact, be much colder than the gas molecule temperature after its submergence in the wall surface (and so after its complete braking on the plate to the velocity v), so that we have to assume heat transmission to the wall molecules from the colliding molecule which remains a finite time in the plate surface.

Figures 4 and 5 show the internal energies U for molecular nitrogen or hydrogen and their combination from the individual degrees of freedom of the molecular motions as a function of the temperature, starting from an internal energy U_o of the gas at rest corresponding to a temperature of 0°C, with the other internal energy values equal to the kinetic energy corresponding to v.

For this, the relation between U and v is stated as $U = U_o + Av^2/2g$. The graph goes up to a $U = 8000$ kcal/kg, corresponding to a flight speed range up to $v = 8000$ m/s. The specific heat at constant volume c_v was calculated under the usual assumptions on energy absorption by translation, rotation, and oscillation of the molecules (the latter according to Planck's formula) after the collision.

We see from both figures that, especially for the N_2, very high temperatures correspond to the high flight speeds. A complete temperature equalization of the colliding molecule to the wall temperature would be equal to a total annihilation of the recoil speed, or an almost completely inelastic collision. It should be observed that the wall accommodates itself in a short time to the temperature of the colliding molecules, because of the very small heat capacity of the thin metal walls of the moving body.

The molecule mass, colliding on the oblique unit surface per second at very high flight speeds is, for example, about equal to $\rho v \sin \alpha$, and thus the arriving energy $E = U\rho\rho v \sin \alpha$. With $\rho\rho = \gamma = 10^{-6}$ kg/m^3, $\alpha = 7^\circ$, and $v = 8000$ m/s the value of the energy brought in is $E = 7.8$ kcal/m^2 sec, almost independent of the composition of the atmosphere. If the accommodation coefficient of the arriving gas molecules is one, then the wall would obtain this energy in the form of heat and this heat quantity should be given away by radiation, where a temperature increase ΔT of approximately 580° is necessary for black body radiation, i.e., the plate stays in fact pretty cold compared to the colliding molecules, and a very intensive, lasting energy delivery by the colliding molecules is out of the question. According to existing measurements[5], this temperature equalization is not 100 percent, however, an accommodation coefficient of 30 percent was found under certain conditions, i.e., the reflected gas mass contains still 70 percent of its internal energy U which it possessed at the moment of collision.

The reflection velocity for a monatomic atmosphere is established this way.

The remaining internal energy of a molecular atmosphere will distribute itself quite differently over the existing degrees of freedom of the reflected molecule than assumed for the colliding energy, which consisted primarily of kinetic energy $1/2\ Av^2/g$, and only in small measure of the internal energy U_o of the gas at rest, which latter was distributed evenly over all degrees of freedom.

For a diatomic molecule with three translational and two rotational degrees of freedom, the individual shares of U_o, for an "average" velocity $\bar{c}$, are for each kilogram of gas $3/6\ A\bar{c}^2/g$ for the three translational degrees and $2/6\ A\bar{c}^2/g$ for the two rotational degrees of freedom of the molecules.

On collision, all degrees of freedom will take part in the energy distribution change, and it can be assumed for further estimation of the diatomic molecule between the perfectly elastic and the perfectly inelastic collision conditions, for instance, that the total energy $A/g\left(\frac{1}{2}v^2 + \frac{3}{6}\bar{c}^2 + \frac{2}{6}\bar{c}^2\right) = A/g\left(\frac{1}{2}v^2 + \frac{5}{6}\bar{c}^2\right)$ distributes itself on the average evenly over all these five degrees of freedom.

It can be taken from figures 4 and 5 that very high temperatures are associated with the high colliding molecular speeds at which another motional degree of freedom is excited, that of mutual molecular oscillation.

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[5]Wien-Harms, Handbuch der Experimentalphysik, Vol. VIII/2, p. 638, 1929.

The known Boltzmann's equilization rule on the energies is not of value for these oscillational degrees of freedom.

While the three translational and the two rotational degrees of freedom of a diatomic molecule have each the same energy admission

$$U_x = \frac{1}{2} ART = \frac{1}{6} A\bar{c}^2/g \text{ (kcal/kg)}$$

that is together

$$U_{(\text{trans.} + \text{rot.})} = \frac{5}{2} ART = \frac{5}{6} A\bar{c}^2/g$$

the energy admission of the oscillational degree of freedom U_s at low temperatures is practically zero and increases at higher temperatures according to Planck's equation:

$$U_s = \frac{AR\Phi}{e^{\Phi/T} - 1} = \frac{2}{6} \frac{A}{g} \bar{c}^2 \frac{\Phi}{T\left(e^{\Phi/T} - 1\right)}$$

approaching the limiting value, valid for high temperatures, of full exitation of the oscillational degree of freedom,

$$U_s = \frac{2}{2} ART = \frac{2}{6} \frac{A}{g} \bar{c}^2$$

In the last equation:

A = the mechanical equivalent of heat, 1/427 kcal/kg

R = the individual gas constant m/°

T = the absolute temperature °K

Φ = a temperature characteristic for each material, which is, for example, for nitrogen $N_2 = 3350^\circ K$, for hydrogen $H_2 = 6100^\circ K$.

The temperatures of the colliding molecules at high flight speeds are so high (according to figs. 4 and 5) that the molecular gas here already dissociates strongly into its atoms under normal equilibrium conditions.

The transformation of the gases hydrogen and nitrogen, which are of importance in the higher atmospheric layers, into their monatomic, active modification belongs to the most energetic endothermic chemical processes which are known ($H_2 = 2H - 51300$ kcal/kg; $N_2 = 2N - 6050$ kcal/kg), and the dissociation (if it actually occurs) would absorb extraordinary amounts of energy and would make the collision almost completely inelastic.

So far it has not yet been proven by experiments that these molecules really dissociate on collision against a fixed wall at the speeds here considered.

However the results of known tests with electrons colliding against the molecules of very rarefied H_2 or N_2 gases let us guess that the energy of the collision with a molecular speed up to $v = 8000$ m/s is not sufficient to disturb the molecular bond.

With the electrons colliding against N_2 or H_2 molecules dissociations are observed[6] only when the energy of the colliding electron was several times the dissociation energy of the struck molecule. This transferred on our case would yield colliding speeds of over $v = 10,000$ m/s for a nitrogen atmosphere or over $v = 35,000$ m/s for a hydrogen atmosphere, which lies outside of the range of our investigations.

For the calculation of the forces on a plate oblique to the airstream, we shall therefore not assume the dissociation of the colliding molecules.

The degree of elasticity of the recoil will only be derived from the energy distribution of the wall molecules and of the proper rotational and oscillational degrees of freedom of the colliding molecule.

This degree of elasticity, i.e., the ratio of the molecular reflection velocity, when energy division occurs, to the reflection velocity when no energy division occurs, is estimated as follows:

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[6] Wien-Harms; Handbuch der Experimentalphysik, Vol. VIII/1, pp. 704, 706 (1929).

First, corresponding to the measured accomodation coefficient, let 30 percent of the collision energy corresponding to v be communicated to the wall molecules. The remaining collision energy,

$$A/g\left(\frac{0.7}{2}v^2 + \frac{3}{6}\bar{c}^2 + \frac{2}{6}\bar{c}^2\right) = A/g\left(\frac{0.7}{2}v^2 + \frac{5}{6}\bar{c}^2\right)$$

should be distributed over the three translational degrees, the two rotational degrees and the oscillational degree of freedom evenly and according to the exitation degree so that each translational degree of freedom has the following energy:

$$E_r = \frac{A/g\left(\frac{0.7}{2}v^2 + \frac{5}{6}\bar{c}^2\right)}{5 + \frac{2\Phi}{T\left(e^{\Phi/T} - 1\right)}}$$

The condition of motion of the diffusely reflected molecules is the same as assumed with equation (3), only there the energy content of a translational degree of freedom was $A/g\,\frac{1}{6}\,\bar{c}^2$. Here the internal energy belonging to the three translational degrees of freedom is given as $U = 3E_r$ and the translational speed is now:

$$\bar{c}_r = \sqrt{\frac{2gU}{A}} = \sqrt{\frac{6\left(\frac{0.7}{2}v^2 + \frac{5}{6}\bar{c}^2\right)}{5 + \frac{2\Phi}{T\left(e^{\Phi/T} - 1\right)}}}$$

instead of: $\bar{c}_r = \sqrt{v^2 + \bar{c}^2}$ when no energy distribution takes place between the wall and the translational degrees of freedom. (The latter value comes from the energy balance:

$$\frac{1}{2}\bar{c}_r^{\,2} = \frac{1}{2}v^2 + \frac{3}{6}\bar{c}^2)$$

The recoil impulses should fall off approximately like speeds obtained from energetic considerations, so that:

$$\epsilon = \sqrt{\frac{6\left(\frac{0.7}{2} v^2 + \frac{5}{6} \bar{c}^2\right)}{\left[5 + \frac{2\Phi}{T\left(e^{\Phi/T} - 1\right)}\right]\left(v^2 + \bar{c}^2\right)}}$$

$$= \sqrt{\frac{6\left(\frac{0.7}{2} v^2 + \frac{5}{4} c^2\right)}{\left[5 + \frac{2\Phi}{T\left(e^{\Phi/T} - 1\right)}\right]\left(v^2 + \frac{3}{2} c^2\right)}} \tag{11}$$

This degree of elasticity is drawn in figure 6 for nitrogen and hydrogen as a function of the flight speed v, using figure 4 or 5 for the relation between v and T.

The pressure vertical to the wall on the plate oblique to the air stream, in the case of diffuse-inelastic reflection, can be obtained from equation 11:

$$p_v = i_p + \epsilon \frac{i}{2}$$

Corresponding to figure 3, figure 7 shows the relation between the pressure p or the shear τ and the dynamic pressure q for hydrogen ($c = 1508$ m/s) and nitrogen ($c = 406$ m/s) for air angle of attack $\alpha = 4^\circ$ and for flight speeds to $v = 8000$ m/s.

In figure 7 the two impulse contributions due to collision (i_p) and recoil ($\epsilon i/2$) are separated.

The collision impulse could be given without objection from purely mechanical relationships.

The reflection impulse is estimated on the basis of a series of rather arbitrary assumptions on direction and speed of the reflection.

Further information, proceeding out of the conjectures presented here, on the retention of single air molecules after collision with the wall with high speed should be found by experiment. Such tests may be joined with the well-known molecular beam investigations, where the usual beam speeds are to be increased greatly by correspondingly greater energy delivery to the molecules under investigation. One can thus obtain a type of wind tunnel investigation in which single molecules fly with extremely high velocities, and the effects of their collision with a solid body can be observed.

With the help of very rapid molecular beams, a number of questions should be cleared up experimentally: how often a reflection of the molecules from the struck wall actually occurs, what factors change the completely elastic collision into a more or less inelastic collision through the transformation of translational energy into other forms of energy (i.e., rotational, oscillational or dissociational energy of the gas molecules or the wall molecules), and what direction law the final reflection follows, whether mirrorlike reflection, or predominantly diffuse reflection, or following a different law.

These investigations can use to advantage De Broglie's analogy between molecular beams and x-rays.

III. AIR FORCES ON THE FRONT SIDE OF A FLAT PLATE VERTICAL TO THE AIR STREAM

Vertical flow against the flat plate $\left(\alpha = \frac{\pi}{2}\right)$ represents the limiting of oblique flow.

The relations obtained in section 2 are preferably discussed here for this special case.

The total molecule mass, colliding per unit area per unit time is given by equation 4:

$$\bar{\rho} = \int_{c_x=v}^{\infty} \frac{1}{4}\left(c_x + 2v + \frac{v^2}{c_x}\right)d\rho + \int_{c_x=0}^{v} v\,d\rho$$

$$= \frac{\rho}{\sqrt{\pi}}\left[\int_{c_x=v}^{\infty}\left(\frac{c_x^3}{c^3} + \frac{2vc_x^2}{c^3} + \frac{v^2c_x}{c^3}\right)e^{-\frac{c_x^2}{c^2}}dc_x + 4\int_{c_x=0}^{v}\frac{vc_x^2}{c^3}e^{-\frac{c_x^2}{c^2}}dc_x\right]$$

$$= \frac{\rho}{\sqrt{\pi}}\frac{c}{2}\left(e^{-\frac{v^2}{c^2}} + \sqrt{\pi}\frac{v}{c} + 2\frac{v}{c^2}\int_{c_x=0}^{v}e^{-\frac{c_x^2}{c^2}}dc_x\right)$$

$$= \frac{\rho}{\sqrt{\pi}}\frac{c}{2}\left\{\left[1 - \left(1 - \frac{1}{2}\frac{c^2}{v^2} + \frac{1\times 3}{2^2}\frac{c^4}{v^4} - + \ldots\right)\right]e^{-\frac{v^2}{c^2}} + 2\frac{v}{c}\sqrt{\pi}\right\} \tag{4a}$$

The complete integration of equation 4(a) (in contrast to equation 4) is possible by series development, because one can put $v/c \gg 1$ for the high flight speeds under consideration, which was not possible for $v \sin \alpha/c$ in equation 4.

The total impulse of this gas mass striking vertically against the plate per unit time and unit area is given by equation 5:

$$i_p = \int_{c_x=0}^{v} \frac{d\rho}{6} c_x^2\left(2 + 6\frac{v^2}{c_x^2}\right) + \int_{c_x=0}^{\infty} \frac{d\rho}{6} c_x^2\left(1 + \frac{v}{c_x}\right)^3$$

$$= \rho v^2 + \frac{1}{2}\rho c^2 + \frac{\rho}{\sqrt{\pi}}\left[\frac{v^3}{3c} + \frac{vc}{2} - \frac{v^2}{3}\right.$$

$$\left. - \left(v^2 + \frac{c^2}{2}\right)\frac{c}{2v}\left(1 - \frac{1}{2}\frac{c^2}{v^2} + \frac{1\times 3}{2^2}\frac{c^4}{v^4} - + \dots\right)\right] e^{-\frac{v^2}{c^2}} \tag{5a}$$

If, as assumed at the start of the integration $v/c \gg 1$, the terms of the equation multiplied by $e^{-\frac{v^2}{c^2}}$ can be neglected compared with the first two terms of the equation.

For instance, the influence of the variety of absolute molecular velocities (following Maxwell's distribution), which is presented by these higher terms, is less than 0.3 percent of the value of the first two terms, when v/c is 2 or more.

Therefore, for $v/c > 2$, the impulse can be calculated as if all the molecules had the same velocity c; thus, considering only the first two terms of equation 5(a),

$$i_p = \rho v^2 + \frac{1}{2}\rho c^2 \tag{5b}$$

Under the assumption of a completely inelastic collision, this impulse is equal to the required pressure on the plate:

$$p = i_p = \rho v^2 + \frac{1}{2}\rho c^2 \tag{10a}$$

If it is assumed that the air molecules have not lost their velocity after the collision with the plate, but that they rebound perfectly elastically and "mirror-like," then the impulse given to the plate is simply doubled, and equation 10(a) for elastic mirrorlike collision is written:

$$p = 2i_p = 2\left(\rho v^2 + \frac{\rho}{2} c^2\right) \tag{10b}$$

The influence of the random molecular motion, represented by the second term, is at $\frac{v}{c} = 2$ approximately 12.5 percent of the pure Newtonian air force, which is represented by the first term. However, the influence decreases, from $\frac{v}{c} = 5$ on, to under 2 percent of the Newtonian value, contrary to the oblique conditions with small angles of attack, where the influence of the molecular velocity is still great even with high flight speeds.

If it is assumed that the molecule reflection is completely elastic but diffuse, then the pressure on the plate decreases to a value:

$$p = i_p + \frac{i}{2} = i_p\left(1 + 0.75\,\frac{1 + \frac{2}{3}\frac{v}{c}\sin\alpha + \left(\frac{v}{c}\right)^2}{1 + \frac{v}{c}\sin\alpha + \frac{3}{2}\left(\frac{v}{c}\right)^2}\right)$$

$$= \rho\left(v^2 + \frac{1}{2}c^2\right)\left(1 + 0.75\,\frac{1 + \frac{2}{3}\frac{v}{c}\sin\alpha + \left(\frac{v}{c}\right)^2}{1 + \frac{v}{c}\sin\alpha + \frac{3}{2}\left(\frac{v}{c}\right)^2}\right). \tag{10c}$$

If finally the molecule reflection becomes not only diffuse but also partly inelastic in the sense of equation 12, then the pressure on the plate reduces again to:

$$p = i_p + \epsilon \frac{i}{2} = \rho\left(v^2 + \frac{1}{2} c^2\right) 1 + 0.75 \sqrt{\frac{6\left(\frac{0.7}{2} v^2 + \frac{5}{4} c^2\right)}{\left[5 + \frac{2\Phi}{T\left(e^{\Phi/T} - 1\right)}\right]\left(v^2 + \frac{3}{2} c^2\right)}}$$

$$\times \frac{1 + \frac{2}{3}\frac{v}{c} \sin \alpha + \left(\frac{v}{c}\right)^2}{1 + \frac{v}{c} \sin \alpha + \frac{3}{2}\left(\frac{v}{c}\right)^2} \tag{10d}$$

The graphs in figures 8 and 9 show the pressure conditions according to equations 10(a) to 10(d) versus the dynamic pressure of nitrogen or hydrogen.

For the calculation of the pressure of a motionless gas on the walls of the gas container which has the same temperature as the gas kinetics assumes completely elastic collisions for the normal range of molecular speeds, i.e., the molecule keeps the same translational energy, on an average, as it had before the shock. It is not important to know if the reflection is mirrorlike or diffuse, because both assumptions lead to the same molecular picture for the gas at rest.

If the molecules are polyatomic then the distribution of the total energy to each possible degree of freedom is already uniform before the shock; and this uniform distribution need not change after the shock.

In equation 10(a) (for completely inelastic collisions) for the air pressure against a plate vertical to the air stream $p = \rho v^2 + \frac{\rho}{2} c^2$ the first term ρv^2 corresponds to the dynamic pressure of molecules having no random motion against the plate (Newton), while the second term $\frac{1}{2} \rho c^2$ corresponds exactly to the pressure of the static atmospheric air.

However, this explanation of the individual terms is only formally correct, since the resting air pressure is calculated assuming elastic molecular collisions. With the inelastic collision, the decrease in the "stopping" pressure due to loss of the recoil impulse will be offset by the mixed term in the square of the sum of the two speeds (v and c), in the velocity range under consideration.

This condition can be recognized more clearly from equation 10(b) for the completely elastic collision $p = 2\rho v^2 + \rho c^2$ where after subtraction of the static air pressure $\frac{1}{2}\rho c^2$ there remains a pressure of $2\rho v^2 + \frac{1}{2}\rho c^2$, which contains besides the Newtonian term, $2\rho v^2$, also an additional term of $\frac{1}{2}\rho c^2$, which reflects the effect of the mixed term.

No resulting impulse is given parallel to the vertical plate because of the symmetry of the total system, i.e., friction forces are transferred in the plate plane, but the sum of these forces outward is zero.

IV. AIR FORCES ON THE BACK SIDE OF A FLAT PLATE

If the mean free path of a molecule is small compared to the dimensions of an empty space into which the gas is flowing, then the flow-in speed can be greater than the most probable molecule speed. In the flow of diatomic gases into a complete vacuum, the directed velocity, c_{max}, of the total flowing mass can surpass the probable molecule speed c by a factor of about 1.87, according to the laws of gas dynamics.

If, however, the molecular mean free path is comparable to the empty space dimensions or even greater than these as assumed here, then the number of molecule collisions behind the rapidly moving plate during the flow-in is not sufficient to produce the mentioned acceleration, and the molecules move with their usual speed c into the empty space behind the plate.

According to figure 10, collisions between the air molecules and the back side of the plate cannot take place, i.e., the pressure against the "suction side" of the plate must have become zero, as soon as

$$v \geqq \frac{c}{\sin \alpha}$$

This border line is, however, strongly blurred because of Maxwell's distribution.

The effective forces against the back side of the plate can be derived by the same process which led to equations 4 to 12, where now the uniform velocity v is directed away from the plate at the angle α, while before it was directed toward the plate (fig. 11).

It is assumed first that $v \sin \alpha < c_x$.

The uniform velocity v of the individual molecule combines with the ideal random velocity c_x of the molecule (which can have any space direction) to a resultant whose components are:

perpendicular to the plate: $c_x \cos \phi - v \sin \alpha$

parallel to the plate and to $v \cos \alpha$: $v \cos \alpha + c_x \sin \phi \sin \psi$

parallel to the plate and perpendicular to $v \cos \alpha$: $c_x \sin \phi \sin \psi$

From the sphere of all possible directions of c_x, a spherical sector with the half angle $\cos \varkappa = v \sin \frac{\alpha}{c_x}$ is taken, inside of which the velocity component $v \sin \alpha - c_x \cos \phi$ is directed towards the plate, so that the molecules of this ϕ-range do in fact collide against the plate. For all $\phi > \varkappa$ the resulting molecular velocity is directed away from the plate. Therefore no collision with the plate takes place.

The integration over all colliding directions is extended from $\phi = 0$ to $\phi = \varkappa$.

The molecular mass with the chosen speed c_x colliding per second against the unit area of the plate is therefore:

$$d\overline{\rho} = \frac{d\rho}{4{c_x}^2\pi} \int_{\phi=0}^{\varkappa} \left(c_x \cos \phi - v \sin \alpha\right) dF = \frac{d\rho}{4}\left(c_x - 2v \sin \alpha + v^2 \sin^2\alpha / c_x\right)$$

For $v \sin \alpha > c_x$ there are no collisions with the plate, so that this case need not be treated.

The total molecule mass colliding on the unit area per unit time is obtained with the aid of Maxwell's distribution equation for the velocity range $c_x = v \sin \alpha$ to ∞:

$$\bar{\rho} = \int_{c_x=v \sin \alpha}^{\infty} \frac{1}{4}\left(c_x - 2v \sin \alpha + v^2 \sin^2\alpha/c_x\right) d\rho$$

$$= \frac{\rho}{\sqrt{\pi}} \int_{c_x=v \sin \alpha}^{\infty} \left(\frac{c_x^3}{c^3} - 2\frac{vc_x^2}{c^3} \sin \alpha + \frac{v^2 c_x}{c^3} \sin^2\alpha\right) e^{-\frac{c_x^2}{c^2}} dc_x$$

$$= \frac{\rho}{\sqrt{\pi}} \frac{c}{2} e^{-\frac{v^2 \sin^2\alpha}{c^2}} - \frac{\rho}{\sqrt{\pi}} v \sin \alpha/c \int_{c_x=v \sin \alpha}^{\infty} e^{-\frac{c_x^2}{c^2}} dc_x$$

$$= \frac{\rho}{\sqrt{\pi}} \frac{c}{2} \left[e^{-\frac{v^2 \sin^2\alpha}{c^2}} + 2v \sin \alpha/c_x^2 \left(\int_{c_x=0}^{v \sin \alpha} e^{-\frac{c_x^2}{c^2}} dc_x - \frac{\sqrt{\pi}}{2}\right)\right] \tag{13}$$

Similarly for the impulse vertical to the plate:

$$di_p = \frac{d\rho}{4c_x{}^2\pi} \int_{\phi=0}^{\varkappa} \left(c_x \cos\phi - v \sin\alpha\right)^2 dF$$

$$= \frac{d\rho}{6} c_x{}^2\left(1 - 3\frac{v}{c_x}\sin\alpha + 3\frac{v^2}{c_x{}^2}\sin^2\alpha - \frac{v^3}{c_x{}^3}\sin^3\alpha\right)$$

$$i_p = \int_{c_x = v\sin\alpha}^{\infty} \frac{1}{6} c_x{}^2\left(1 - 3\frac{v}{c_x}\sin\alpha + 3\frac{v^2}{c_x{}^2}\sin^2\alpha - \frac{v^3}{c_x{}^3}\sin^3\alpha\right) d\rho$$

$$= \frac{\rho}{\sqrt{\pi}}\left\{\frac{v\sin\alpha}{c}\left(-\frac{1}{3}v^2\sin\alpha + \frac{1}{3}cv\sin\alpha - \frac{1}{2}c^2\right)e^{-\frac{v^2\sin^2\alpha}{c^2}}\right.$$

$$\left. + \left(\frac{1}{2}c + v^2\sin^2\alpha\right)\frac{1}{c}\left(\frac{\sqrt{\pi}}{2}\int_{c=0}^{v\sin\alpha} e^{-\frac{c_x{}^2}{c^2}} dc_x\right)\right\} \tag{14}$$

Finally, for the impulse parallel to the plate:

$$di_{\tau} = \frac{d\rho}{4c_x^2\pi}\int_{\phi=0}^{\varkappa}\left(c_x \cos\phi - v \sin\alpha\right)v \cos\alpha \, dF$$

$$= \frac{d\rho}{4}\left(c_x v \cos\alpha - 2v^2\sin\alpha \cos\alpha + \frac{v^3}{c_x}\sin^2\alpha \cos\alpha\right)$$

$$i_{\tau} = \int_{c_x=v \sin\alpha}^{\infty}\frac{1}{4}\left(c_x v \cos\alpha - 2v^2\sin\alpha \cos\alpha + \frac{v^3}{c_x}\sin^2\alpha \cos\alpha\right)d\rho$$

$$= \frac{\rho}{\sqrt{\pi}} v \cos\alpha\left(\frac{c}{2} e^{-\frac{v^2\sin^2\alpha}{c^2}} - \frac{v \sin\alpha}{c}\int_{c_x=v \sin\alpha}^{\infty} e^{-\frac{c_x^2}{c^2}} dc_x\right)$$

$$= \frac{\rho}{\sqrt{\pi}} v \cos\alpha\left[\frac{c}{2} e^{-\frac{v^2\sin^2\alpha}{c^2}} + \frac{v \sin\alpha}{c}\left(\int_{c_x=0}^{v \sin\alpha} e^{-\frac{c_x^2}{c^2}} dc_x - \frac{\sqrt{\pi}}{2}\right)\right] \quad (15)$$

The impulse resultant vertical to the wall as a consequence of the elastic-diffuse molecular recoil is from the total impulse i of the colliding molecules:

$$i = 1.5\,\frac{1 + \frac{2}{3}\frac{v}{c}\sin\alpha + \left(\frac{v}{c}\right)^2}{1 + \frac{v}{c}\sin\alpha + \frac{3}{2}\left(\frac{v}{c}\right)^2}\sqrt{i_p^2 + i_{\tau}^2}$$

Equation (10) again holds for the total pressure against the suction side of the plate with elastic-diffuse recoil. (Translator's note: Formula missing in original German report.) While for the total shear stress on the suction side of the plate, equation 9 is used:

$$\tau_r = i_\tau$$

The influence of a certain inelasticity of recoil can be estimated, particularly for small angles of attack by the same procedure which led to equation (11), according to which the degree of inelasticity can also be specified.

For the total pressure against the suction side, equation (12) is valid:

$$p_r = i_p + \epsilon \frac{i}{2}$$

Corresponding to figure 7, the graphs in figure 12 show the relation between the pressure p or the shear τ and the dynamic pressure q for hydrogen and nitrogen at an angle of attack $\alpha = 4^\circ$ and flight speeds up to $v = 8000$ m/s.

V. APPLICATION EXAMPLES

With the help of the previously mentioned relations, it is possible to estimate all the air forces acting on the surfaces of a flying body of any shape, which is moving at flight altitudes of over 100 km with speeds between about 2000 m/s and 8000 m/s, if definite assumptions are made on the composition of the air at this height.

The air forces were differentiated into those which act perpendicular to the surface under observation (pressures), and those which act parallel to the surface (friction).

The pressure stresses as well as the shear stresses were found to be a function only of the angle of attack and the flight speed, for a particular gas.

In figures 13 and 14 is shown this dependence of the air forces on all possible angles of attack and on flight speeds between $v = 2000$ m/s and $v = 8000$ m/s for an atmosphere of molecular hydrogen.

It is to be noted in figure 13 that the air pressure vertical to the plate strongly increases with increasing velocity, even with an angle of attack $\alpha = 0$, if the molecular recoil is diffuse.

This representation can be used as a basis for the calculation of air force coefficients for certain flight bodies in hydrogen, treating each flat surface section separately, with its own angle of attack, or, if the body surface is curved, analyzing it into a great number of small areas with individual angles of attack (flat areas or symmetrical cone areas), and then investigating these.

As the simplest example, the flat, infinitely thin plate will be treated first. The usual symbols are

A lift

W drag

F wing surface

and the air force coefficients are:

$$c_a = \frac{A}{qF} = \left(\frac{p_v}{q} - \frac{p_r}{q}\right)\cos\alpha - \left(\frac{\tau_v}{q} + \frac{\tau_r}{q}\right)\sin\alpha$$

$$c_w = \frac{W}{qF} = \left(\frac{p_v}{q} - \frac{p_r}{q}\right)\sin\alpha + \left(\frac{\tau_v}{q} + \frac{\tau_r}{q}\right)\cos\alpha$$

and the glide ratio:

$$\epsilon = \frac{c_w}{c_a} = \frac{\left(\frac{p_v}{q} - \frac{p_r}{q}\right)\sin\alpha + \left(\frac{\tau_v}{q} - \frac{\tau_r}{q}\right)\cos\alpha}{\left(\frac{p_v}{q} - \frac{p_r}{q}\right)\cos\alpha - \left(\frac{\tau_v}{q} + \frac{\tau_r}{q}\right)\sin\alpha}$$

In figure 15 are drawn the lift coefficients and in figure 16 the glide ratios of the flat, thin plate according to the above relations.

On account of the extraordinarily great shear forces very bad glide ratios result, which are approximately $\epsilon = \frac{c_w}{c_a} = 1.9$ at 2000 m/s with the most favorable angle of attack and which got worse at higher speeds, for example, at 8000 m/s, ϵ = about 2.7.

The most favorable angles of attack are comparatively great at small speeds, i.e., at $v = 2000$ m/s, $\alpha = 25^\circ$ approximately, and decrease with increasing speed to about 7° at $v = 8000$ m/s.

Similarly to the infinitely thin plate, high speed profiles of finite thickness can also be calculated, i.e., wedge-shaped and lenticular airfoils. Their air force coefficients hardly deviate from those of the smooth plate, if they are of moderate thickness.

In general, the wings investigated here in the gas kinetics flow range behave worse than in the gas dynamics range, where already the glide ratios are worse than in the usual aerodynamical flow region.

The full effect of this unfortunate behavior will be corrected to some extent by a flight technique such that at the high flight velocities under consideration, inertial forces are developed by the concave downward flight path, which support the wing.

Figure 17 treats the question of how great the air drag is in the gas kinetics flow range for a body of rotation (projectile form) moving axially, with an ogival nose of three calibers radius and cylindrical body, and how far the air drag can be improved by a truncated cone bevel at the end of the missile.

These questions can be easily answered with the aid of figures 13 and 14 if the ogive is divided into a large number of truncated cones, each of which represents a surface with a definite angle of attack.

The extraordinary value of the drag coefficient is again striking; it can be traced to the very great friction forces in the extremely rarefied air.

A noticeable improvement of the drag coefficient could be obtained by beveling the end of the projectile; the improvement is about 7 percent of the original value.

Somewhat more tediously but in basically the same manner, the air forces on a projectile, airship, etc., at an oblique angle of attack can be determined, using figures 13 and 14.

VI. SUMMARY

The air forces on bodies of arbitrary shape are investigated when the bodies move with speeds of 2000 to 3000 m/s in such thin air that the mean free path of the air molecules is greater than the dimensions of the moving body.

The air pressure acting perpendicular to the body surface, as well as the friction forces acting parallel to the surface, are derived with the aid of the calculation procedure of gas kinetics for surfaces facing both toward and away from the air stream at any angle.

The air forces for an atmosphere of definite composition (molecular hydrogen) are calculated as a function of the flight velocity at all possible angles of attack of a surface and shown in graphs.

Thereby the friction stresses between air and body surface prove to be of the same magnitude as the dynamic pressure and as the air pressures vertical to the body surface, i.e., 300 times greater than in the aerodynamic flow range.

The application of the general calculation results to particular technically important cases, like thin airfoils and projectile shapes, results in extraordinarily high air drag coefficients and poor glide ratios even for the theoretically best wing sections

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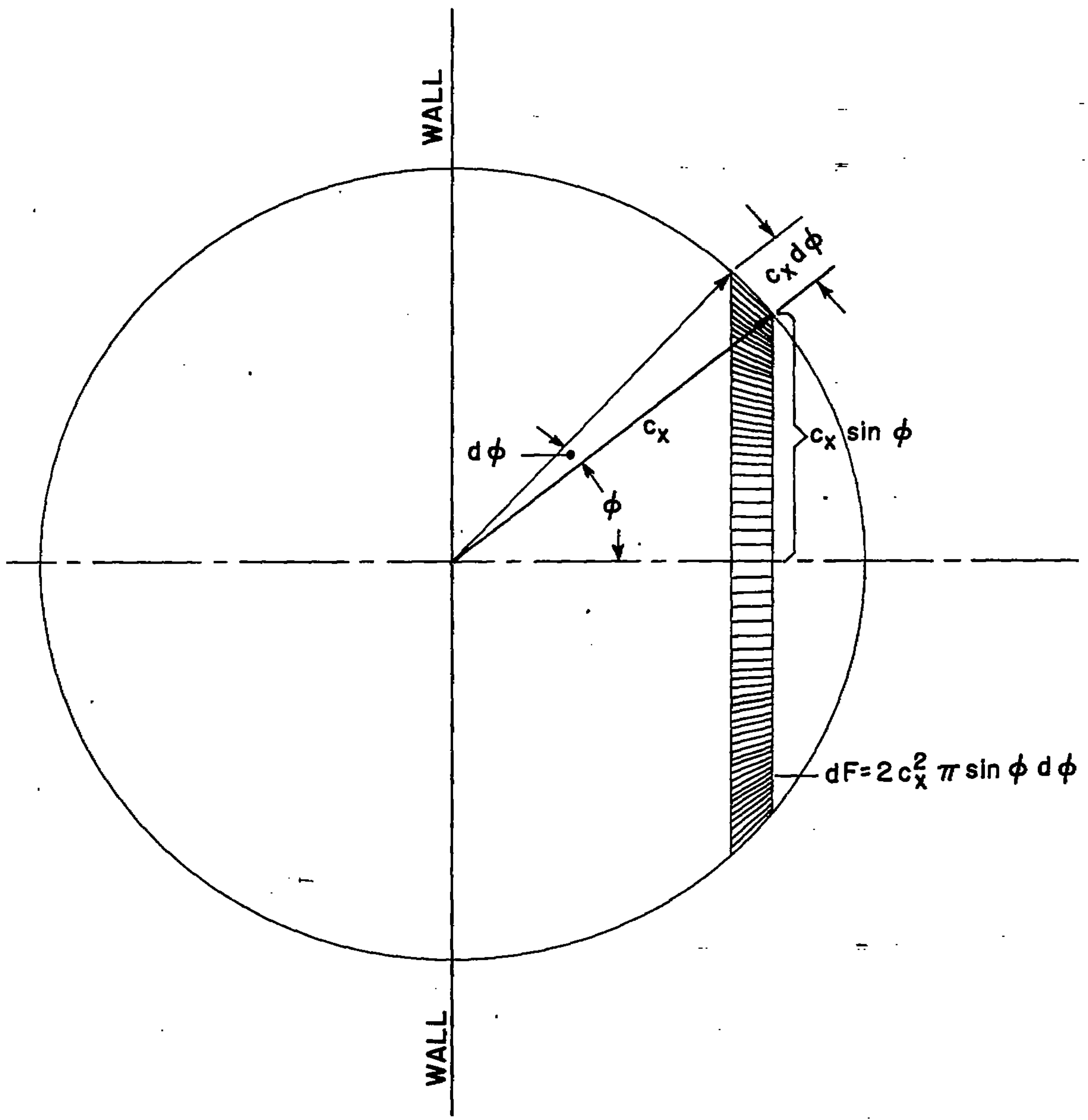

Figure 1.- Velocity vectors of the thermal motion of molecules of a motionless gas and their position relative to a fixed boundary wall.

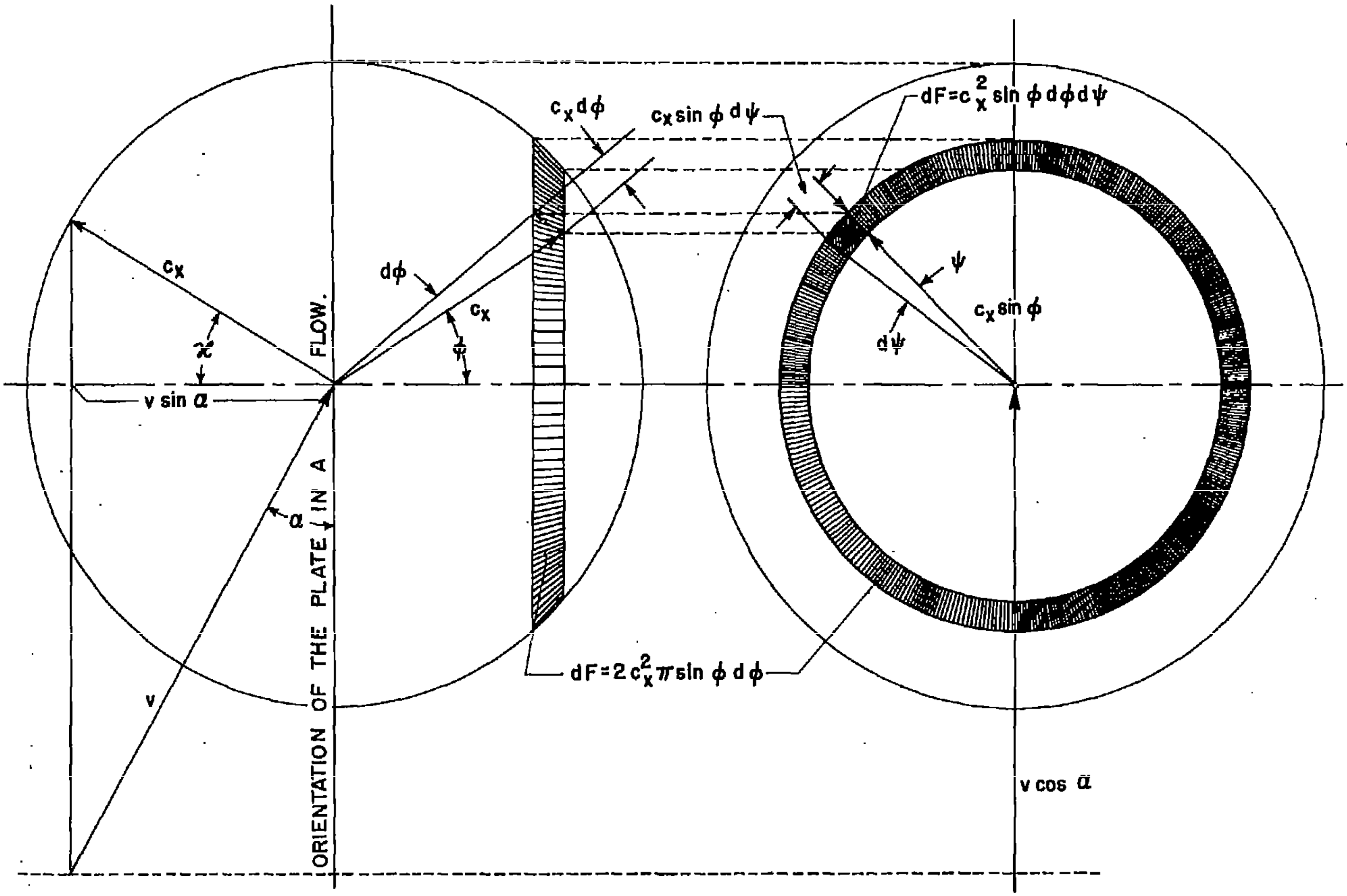

Figure 2.- Velocity vectors of the total molecular motion of a gas flowing obliquely against a flat plate.

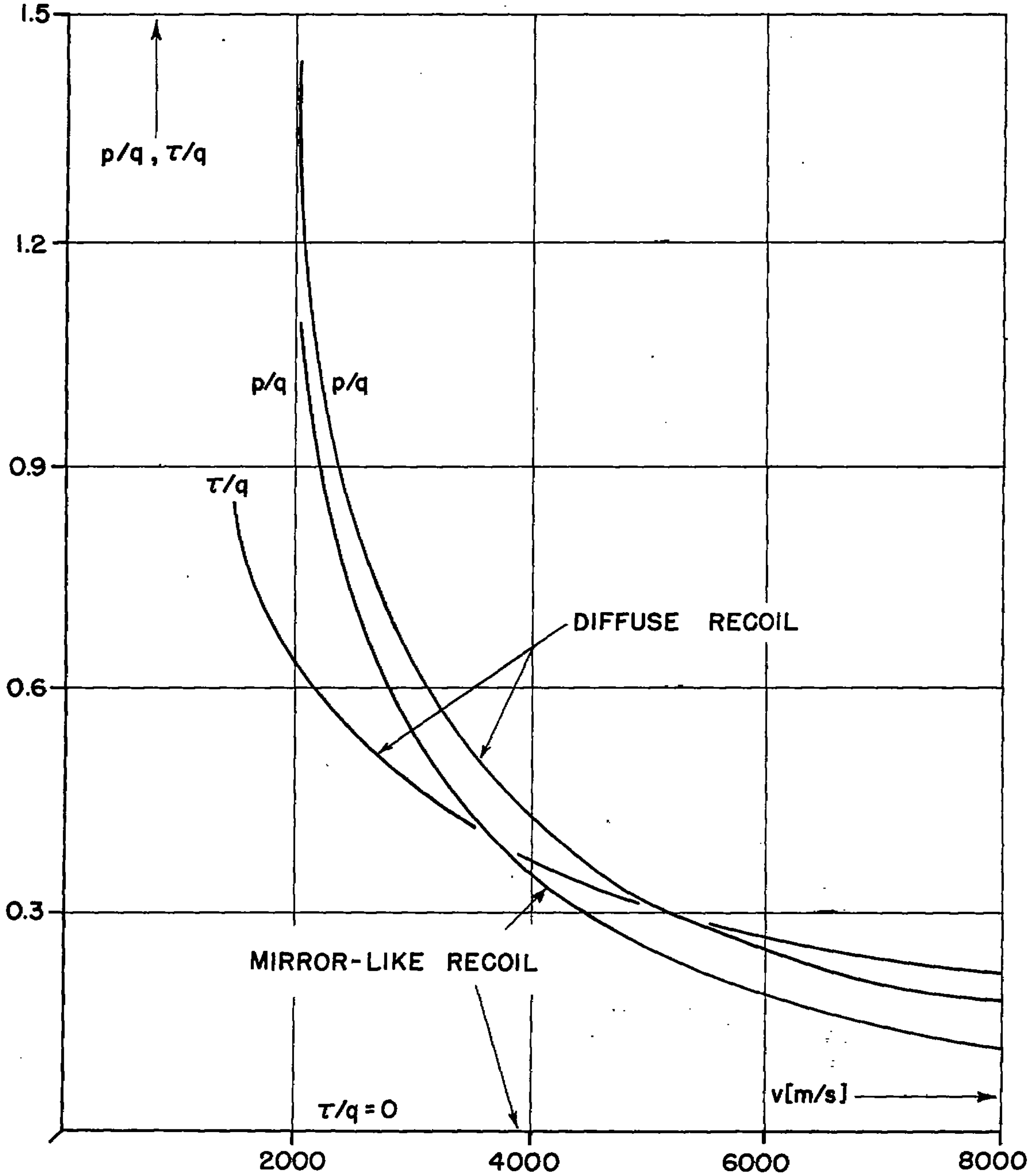

Figure 3.- Air pressures p and shear stresses τ on the front side of a flat plate at 4° angle of attack in an atmosphere of atomic hydrogen under the assumption of elastic diffuse or mirror-like recoil of the atoms from the wall.

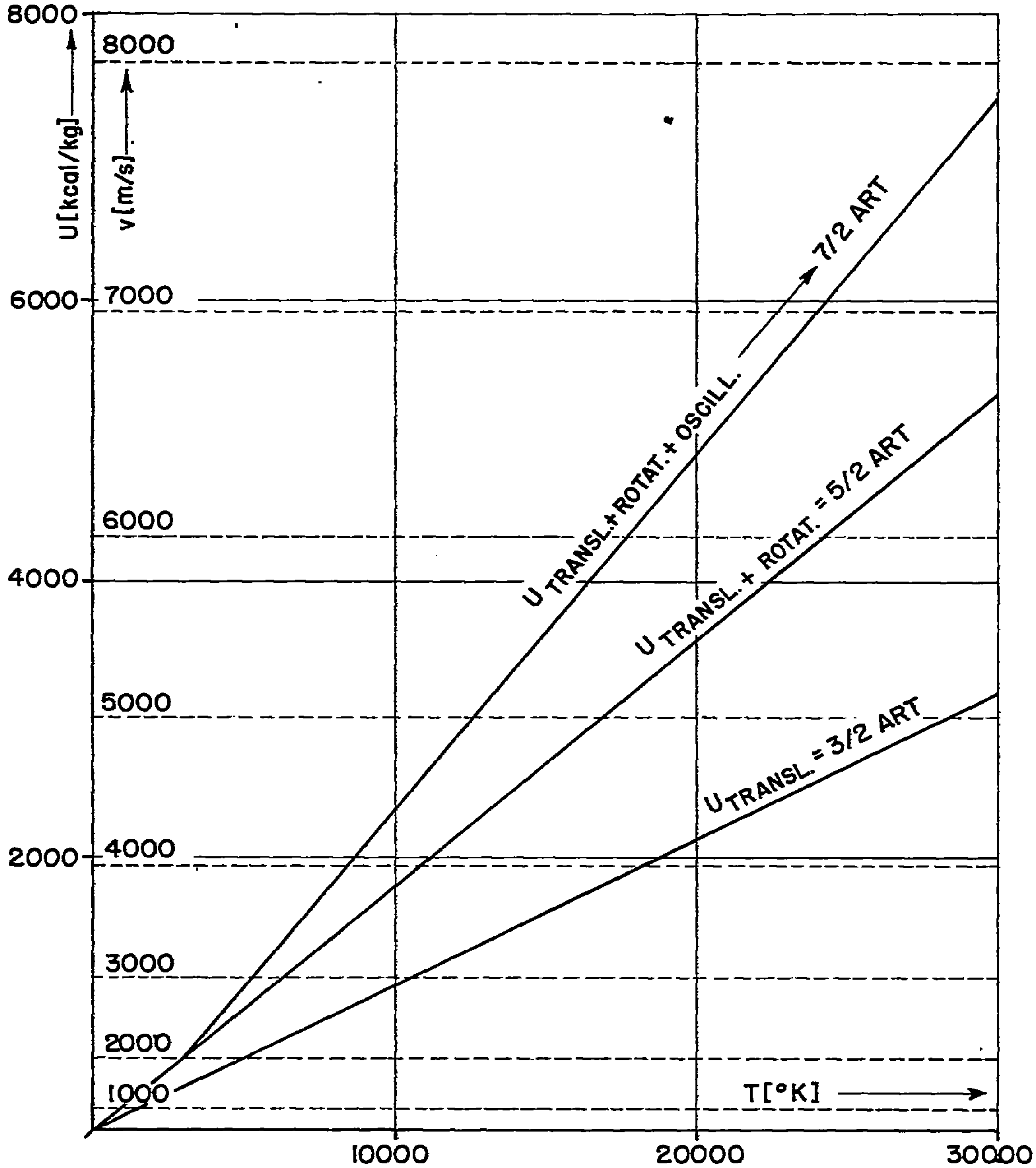

Figure 4.- Colliding speed and associated internal energy of molecular nitrogen in relation to the colliding temperature of the gas.

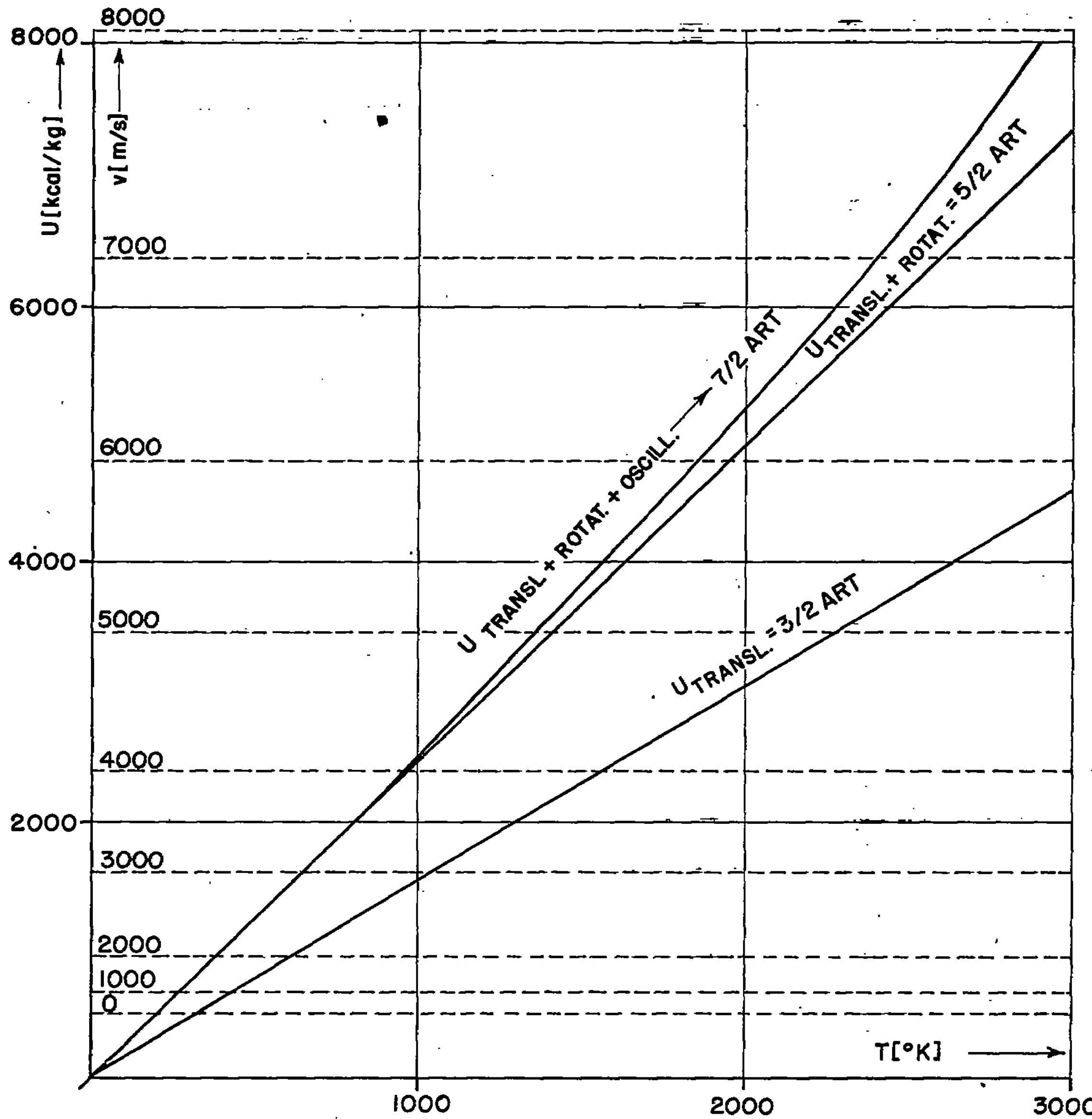

Figure 5.- Colliding speed and associated internal energy of molecular hydrogen in relation to the colliding temperature of the gas.

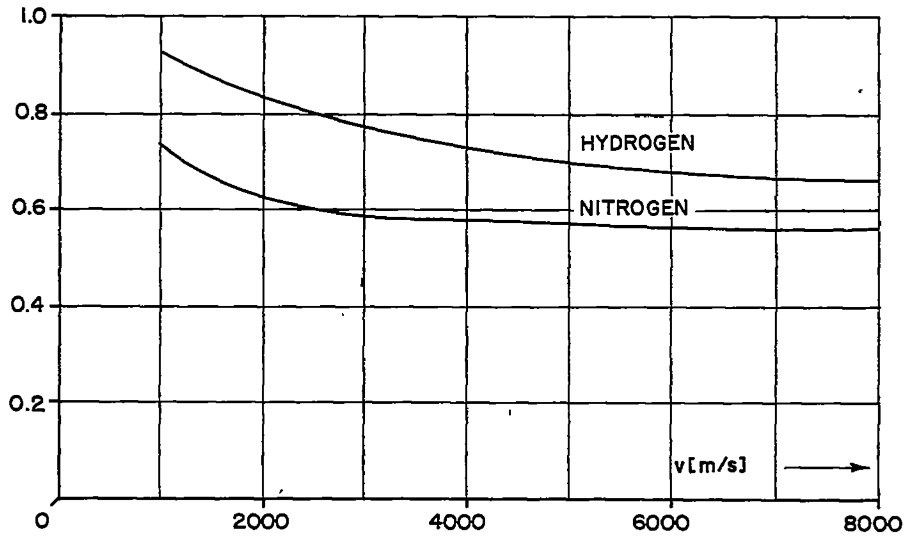

Figure 6.- Degree of elasticity of recoil for nitrogen or hydrogen molecules from the struck wall.

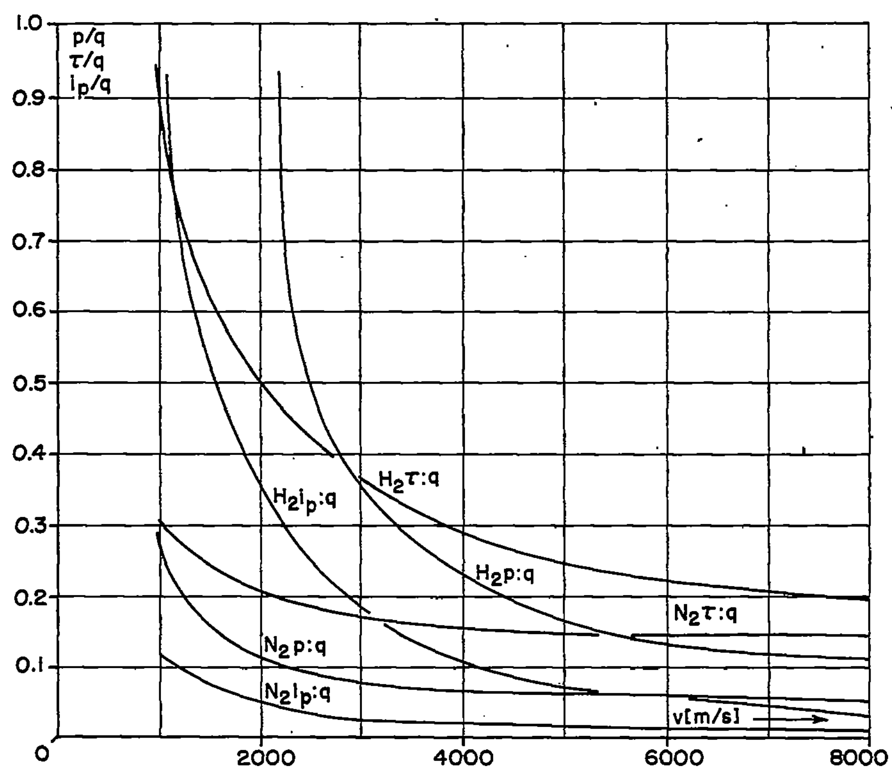

Figure 7.- Air pressure p and shear stress τ on the front side of a flat plate at 4° angle of attack in an atmosphere of molecular hydrogen or nitrogen under the assumption of diffuse and semielastic molecular recoil from the wall.

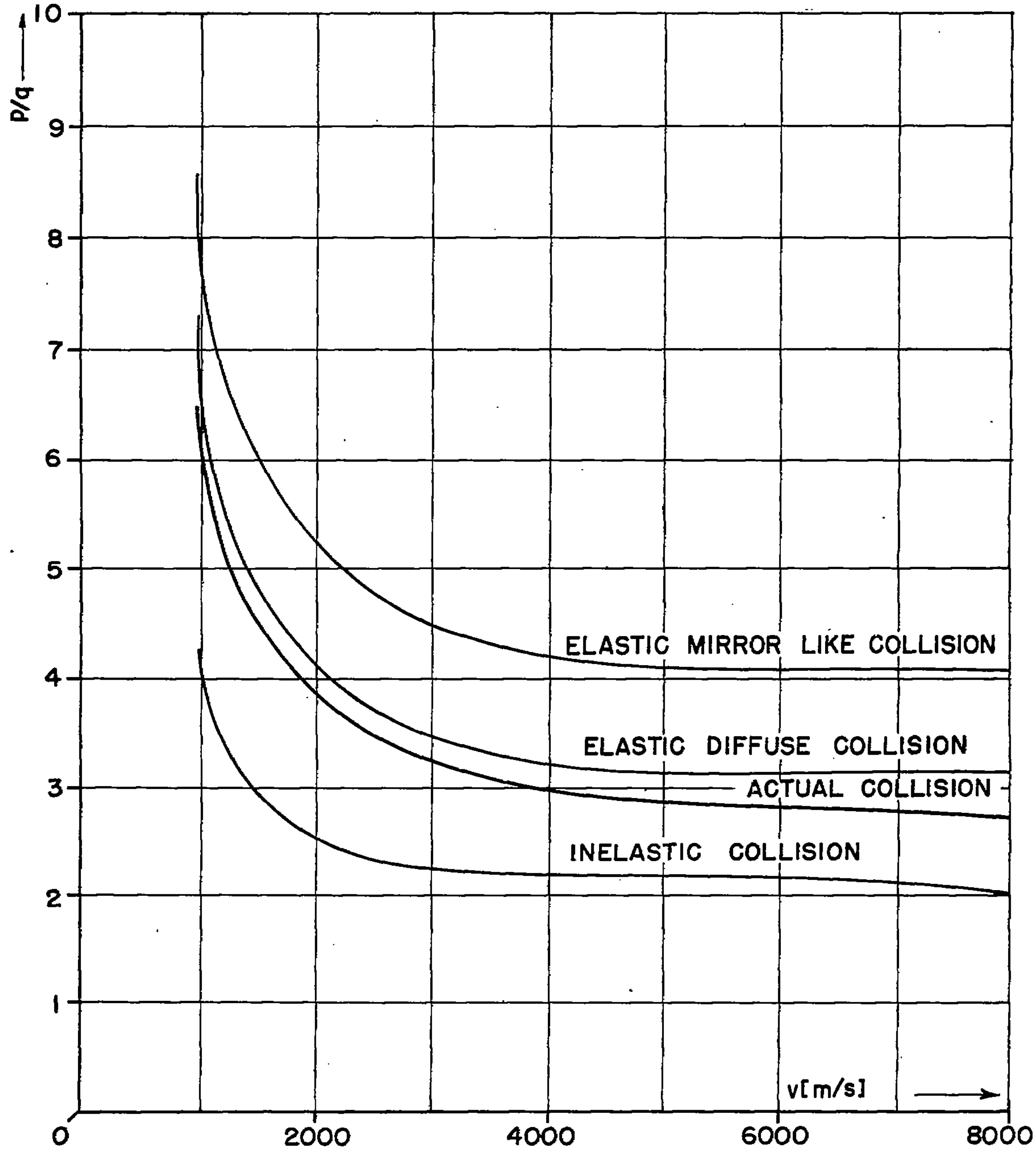

Figure 8.- Air pressure p on the wall vertical to a stream of molecular hydrogen, under various assumptions on the collision process.

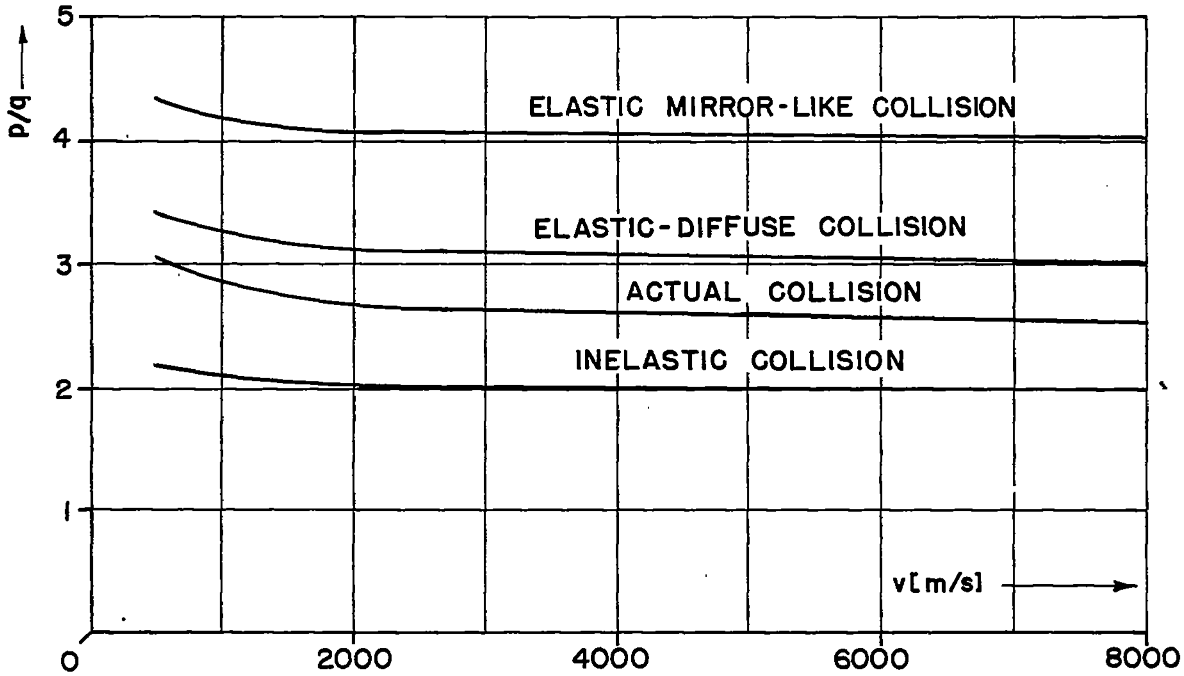

Figure 9.- Air pressure p on the wall vertical to a stream of molecular nitrogen, under various assumptions on the collision process.

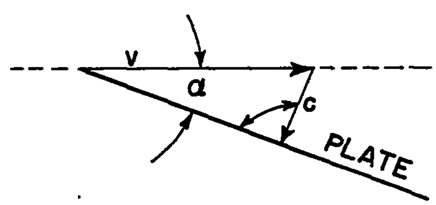

Figure 10.- Velocity vectors of the molecular motion for collision on the back side of the flat plate.

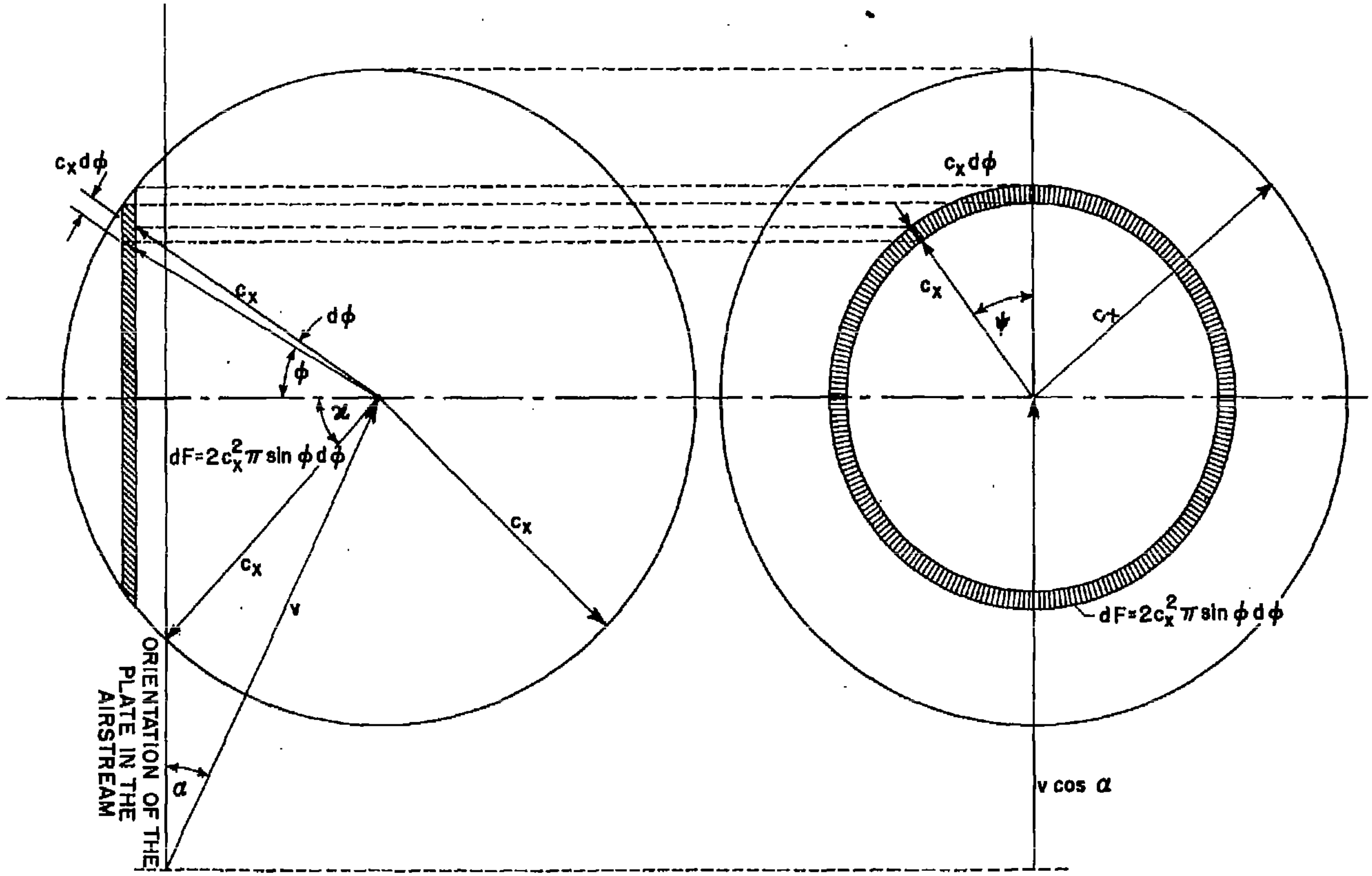

Figure 11.- Velocity vectors of the molecular motion for collision on the back side of the flat plate, and their location with respect to the plate.

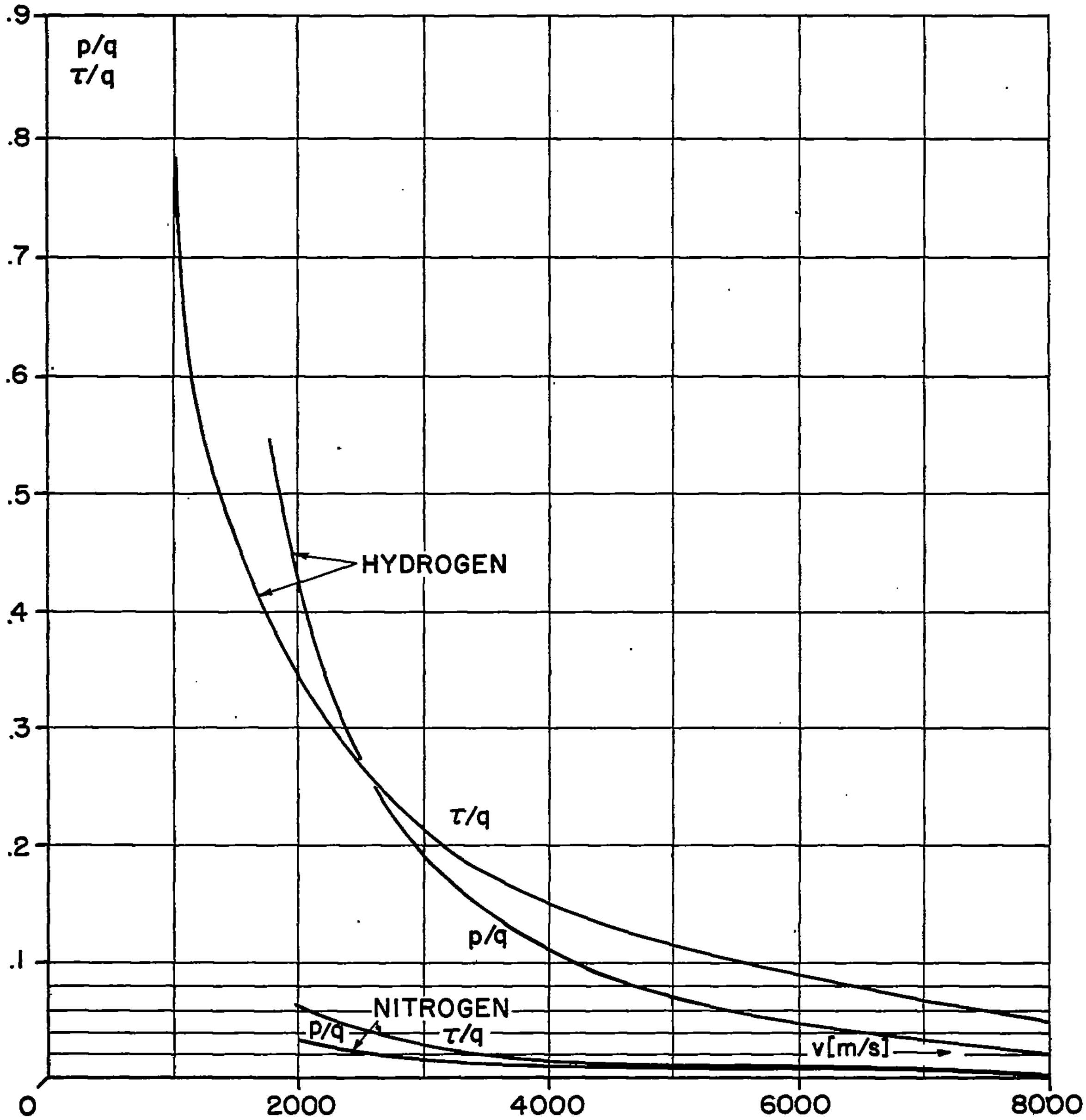

Figure 12.- Air pressures p and shear stress τ on the back side of a flat plate at 4° angle of attack in an atmosphere of molecular hydrogen or nitrogen, under the assumption of diffuse and semielastic molecular recoil from the wall.

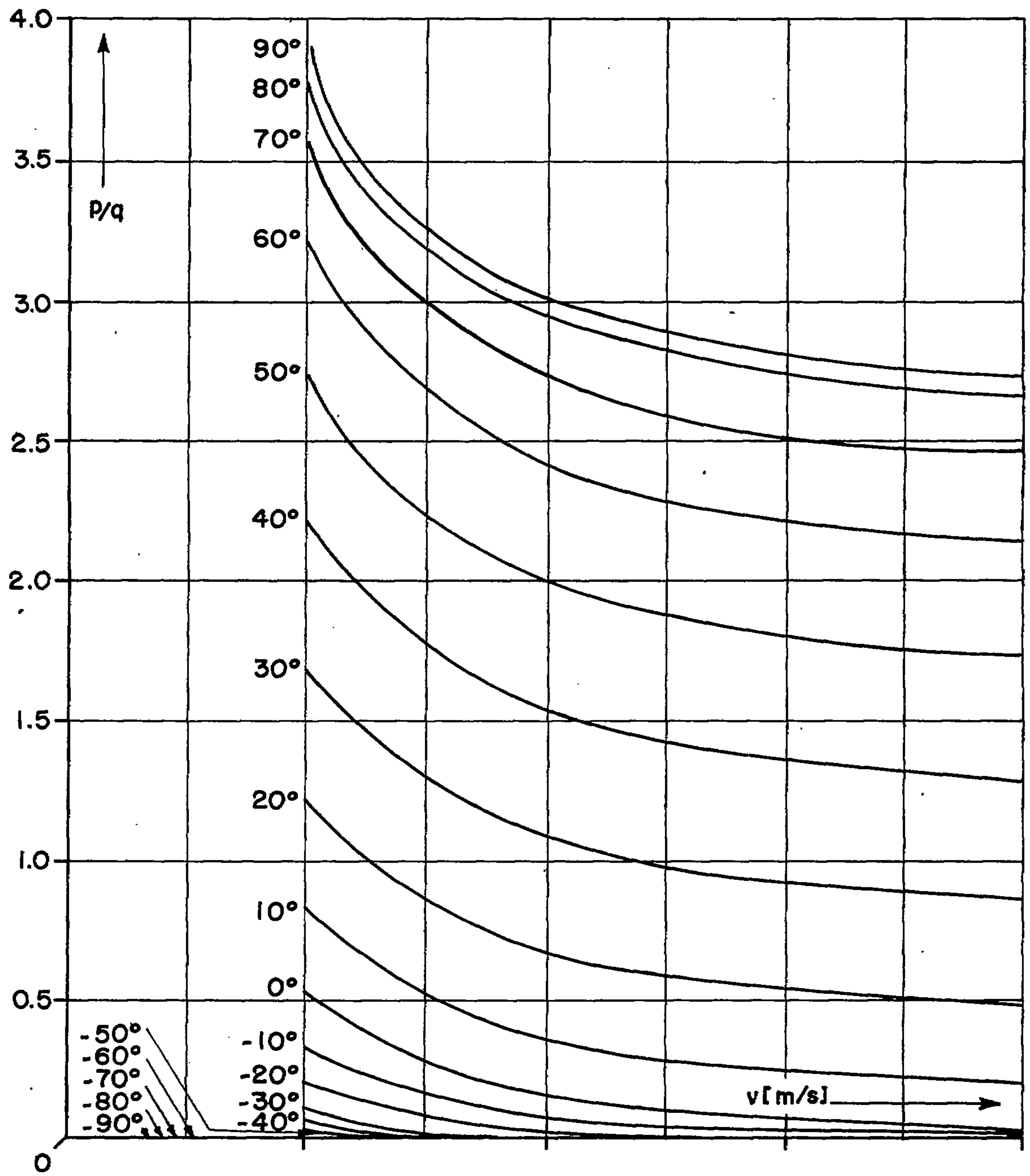

Figure 13.- Coefficient p/q of the air pressure vertical to the plate for all angles of attack and for flight speeds between 2000 m/s and 8000 m/s in atmosphere of molecular hydrogen.

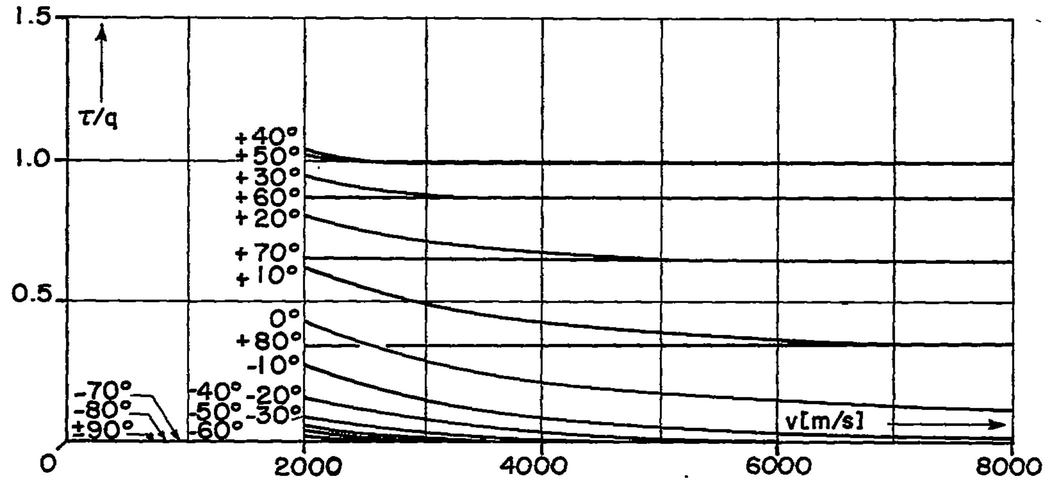

Figure 14.- Coefficients τ/q of the shear stress between air and plate for all angles of attack and for flight speeds between 2000 and 8000 m/s in an atmosphere of molecular hydrogen.

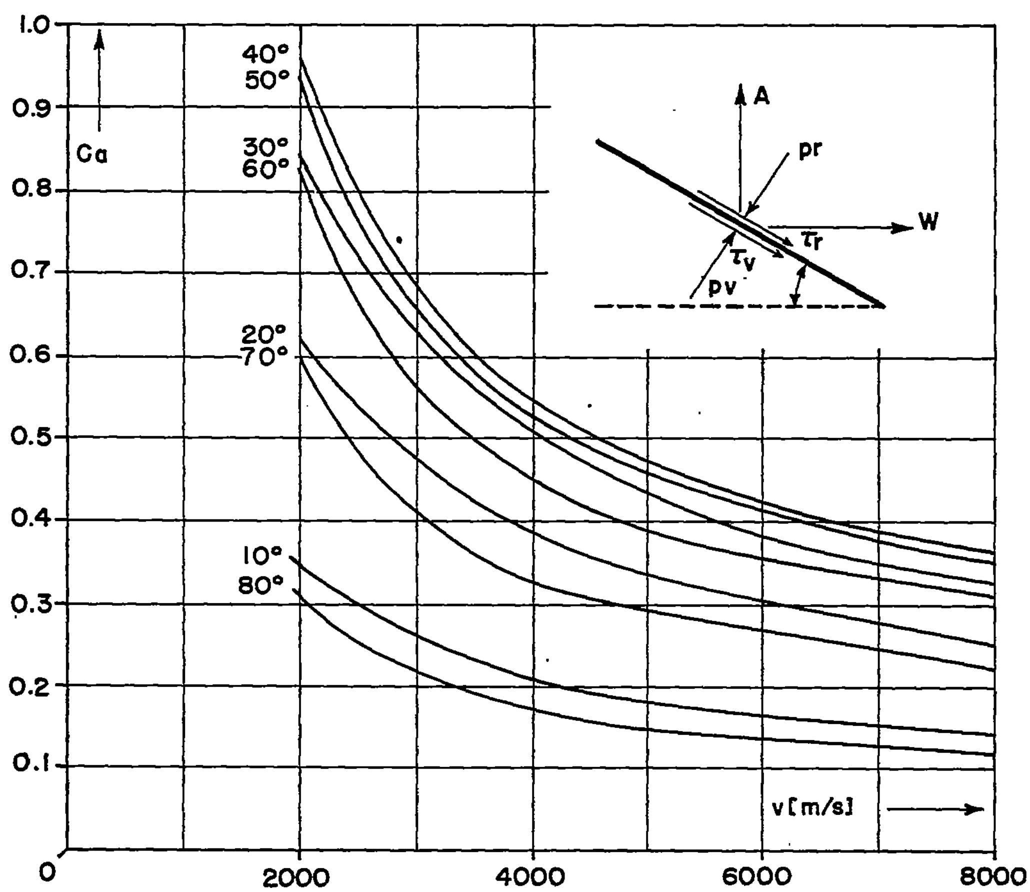

Figure 15.- Lift coefficients for the flat infinitely thin plate.

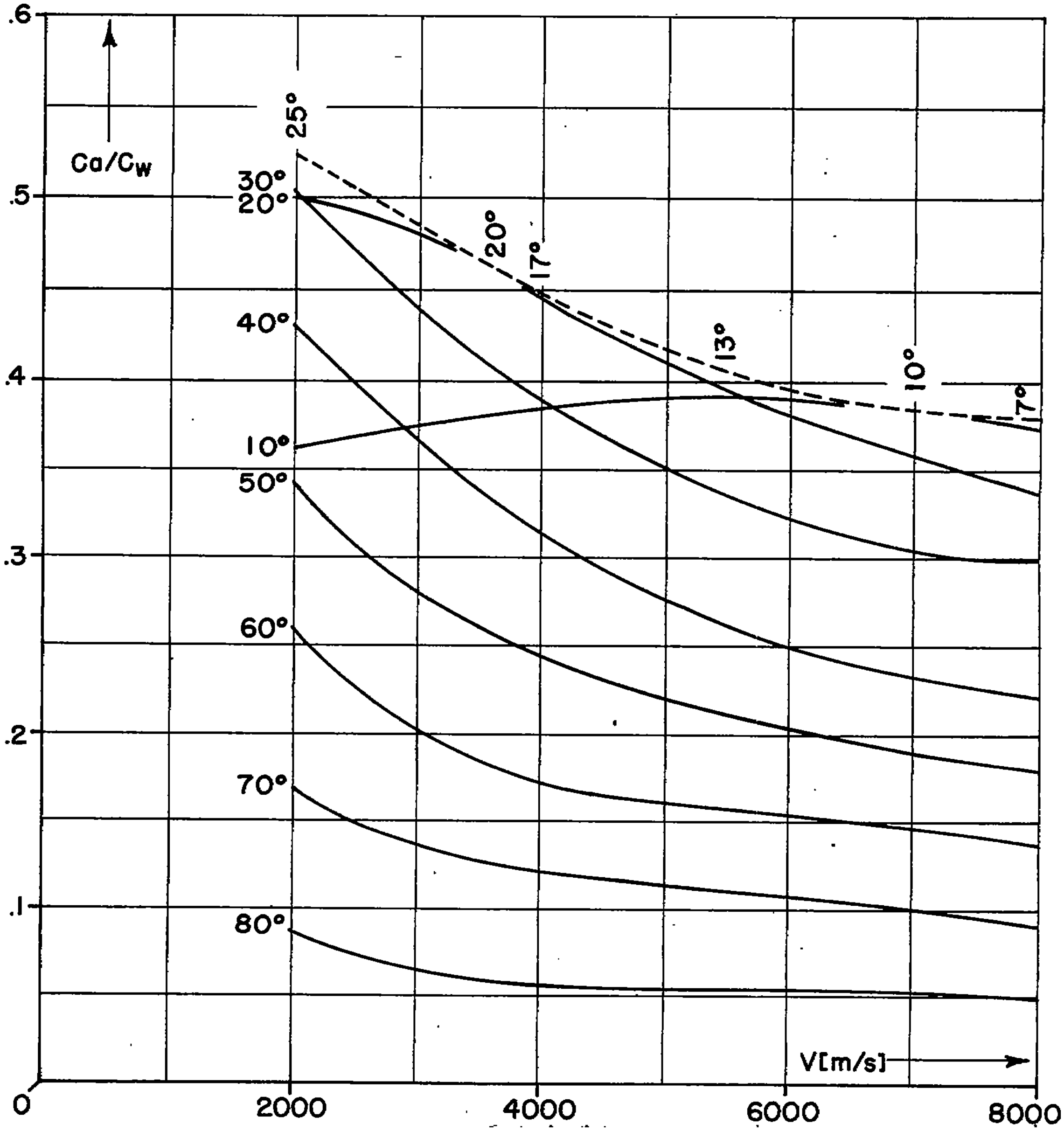

Figure 16.- Reciprocals of the glide ratios for the flat infinitely thin plate, and best values of the glide ratio (dotted line) with corresponding angles of attack.

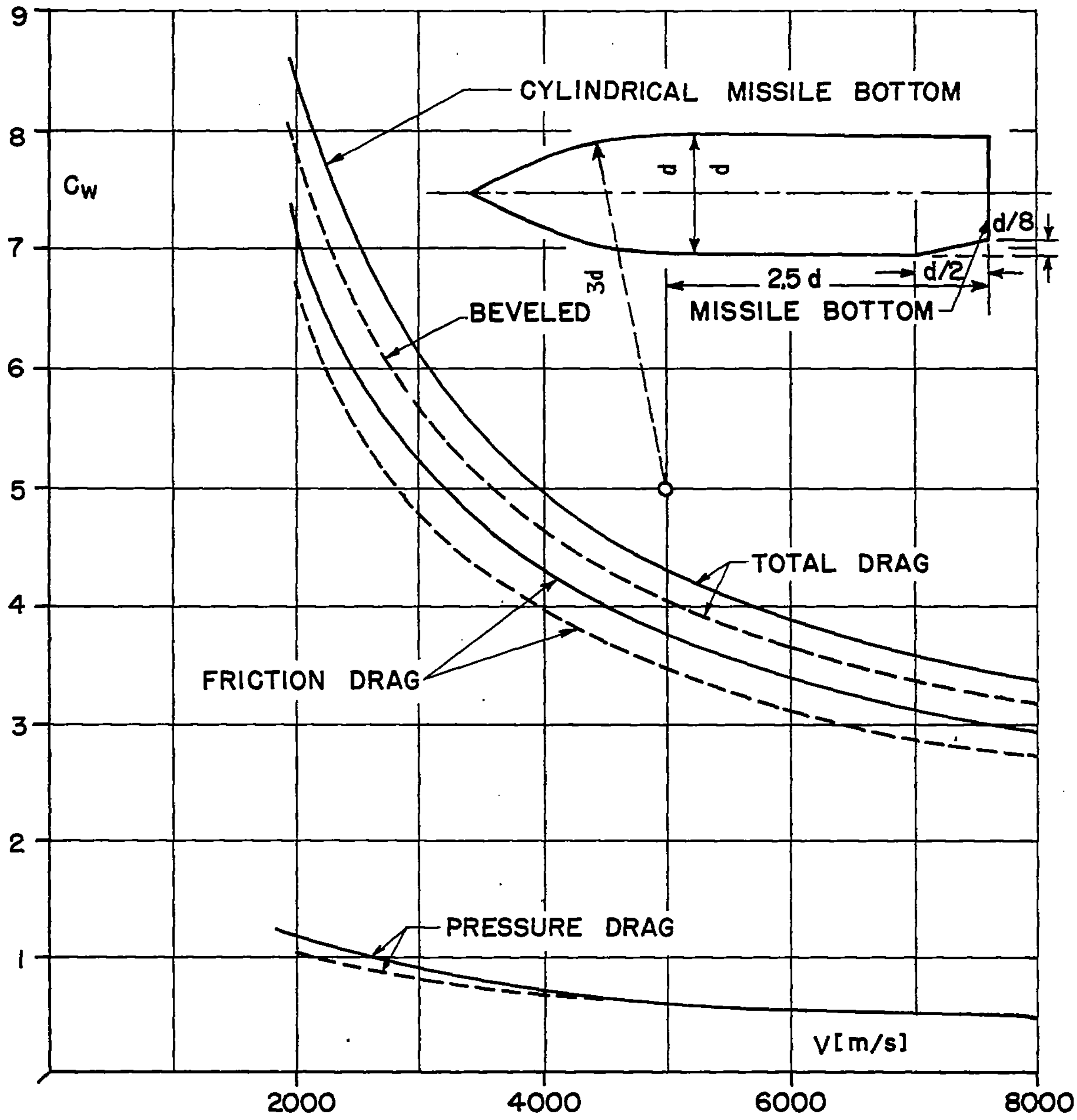

Figure 17.- Coefficients of the pressure drag, friction drag, and total drag for a projectile-shaped body of rotation, with different missile bottoms.