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# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1284

INSTRUMENT FOR MEASURING THE WALL SHEARING STRESS  
OF TURBULENT BOUNDARY LAYERS

By H. Ludwig

Translation of "Ein Gerät zur Messung der Wandschubspannung  
turbulenter Reibungsschichten"



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INSTRUMENT FOR MEASURING THE WALL SHEARING STRESS  
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## SUMMARY

It is shown that at a smooth wall in a turbulent boundary layer the velocity profile next to the wall is dependent, aside from the material constants of the flowing medium, only on the shearing stress transmitted to the wall, even with pressure rise or with pressure drop. Consequently, the heat transfer of a small element that is built into the wall and has a higher temperature than that of the flowing medium is a measure of the wall shearing stress. Theoretical considerations indicate that the wall shearing stress of the boundary layer can be defined by means of a heat-transfer measurement with an instrument mounted in the wall. Such an instrument is described. The calibration curve and its directional sensitivity curve are indicated. It permits the determination of the wall shearing stress in magnitude and direction.

## I. INTRODUCTION

The technique in aerodynamic measurements frequently involves the problem of defining the wall shearing stress of a turbulent boundary layer, since it is of decisive importance for the entire flow process. But its measurement presents great difficulties. Direct measurement by means of a balance, as carried out by Schultz-Grunow (reference 1), is feasible only in special cases, because of the large amount of instrumental equipment required. In general, it is restricted to flows with approximately constant pressure in the zone of the experimental plate, since, otherwise, uncontrollable slot flows occur, which introduce considerable measuring errors. Another method, employed up to now, consists in exploring the entire boundary layer with a fine pitot tube, and then computing the wall shearing stress by the momentum method. But this method calls for considerable expenditure of labor, since the flow velocity must be determined in magnitude and direction over a wide range.

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Moreover, the boundary layers are so thin in many cases that the experimental determination of the velocity distribution in the boundary layer cannot be effected at all. The accuracy of measurement of this method is very poor for complicated flow processes, since the test value (the wall shearing stress) must be determined by differentiation of slightly variable quantities (loss of momentum of boundary layer), which, as is known, leads to inaccurate results, even when the quantities to be differentiated are themselves measured comparatively correct.

Another method has been cited by Fage and Falkner (reference 2). The special feature of this method is the pressure orifice at the point of the wall where the shearing stress is to be measured. Approximately 1/20 millimeter above this orifice is a sharp knife edge. The portion of the velocity near the wall (the laminar sublayer) is then dammed up between knife edge and wall. The pressure rise below the knife edge with respect to the undisturbed static pressure gives then a measure for the wall shearing stress, since the velocity distribution in wall proximity is definitely correlated to the shearing stress. However, in view of the difficulty in handling and due to the extremely sensitive test probe, this method has not made much headway.

According to the method described in the present report the shearing-stress measurement is reduced to a heat-transfer measurement.

## II. PHYSICAL PRINCIPLES OF THE SHEARING-STRESS MEASUREMENT

The part of the velocity profile adjacent to the wall, whether for the turbulent boundary layers on a smooth, flat plate without pressure gradients in flow direction, or for turbulent boundary layers in smooth pipes or channels with constant section, can be represented in the following form (reference 3).

$$\frac{u}{u^*} = f\left(\frac{yu^*}{\nu}\right) = f(y^*) \quad (1)$$

$f$  being the same function in all cases;  $u$ , the flow velocity;  $y$ , the wall distance;  $\nu$ , the kinematic viscosity;  $u^*$ , the so-called shearing-stress velocity defined by the equation  $u^* = \sqrt{\tau_w/\rho}$ ;  $\tau_w$ , the shearing stress transferred to the wall; and  $\rho$ , the density;  $\frac{yu^*}{\nu}$  is abbreviated to  $y^*$ . This relation, derived on the basis of a dimensional analysis, is very satisfactorily confirmed by measurements (reference 3). For  $y^*$  values exceeding 50, the shearing stress is practically completely transferred by the turbulent exchange, while the contribution of the internal

friction to the shearing-stress transfer is no longer worth mentioning. Equation (1) assumes here the form

$$\frac{u}{u^*} = a \log y^* + b \quad (2)$$

known as the logarithmic velocity law, with  $a$  and  $b$  as universal constants.

In direct proximity of the wall, that is, for very small  $y^*$  values, the turbulent exchange is voided by the presence of the wall, and the shearing stress is then transmitted solely by the internal friction of the flowing medium. From the equation  $\tau = \mu \frac{\partial u}{\partial y}$ <sup>1</sup> defining the internal friction and the boundary condition  $u = 0$  for  $y = 0$ , it then follows, that for these small  $y^*$  values, equation (1) assumes the following form

$$\frac{u}{u^*} = \frac{\gamma u^*}{\nu} = y^* \quad (3)$$

This purely laminar layer next to the wall is called the laminar sublayer of the turbulent boundary layer.

Between these two parts of the boundary layer, there is also a corresponding transition zone, where the shearing stress is transferred in part by turbulent exchange, and in part by internal friction.

With a view to ascertaining the thickness of this laminar sublayer and the variation of the function in equation (1), in the transition zone, Reichardt (reference 4) has made a number of velocity-profile measurements extending into the laminar sublayer. However, since this sublayer is, as a rule, very thin, he was forced to make the measurements at very small  $u^*$  values, which means at small flow velocities where the sublayer was thick enough for exploration with fine hot wires and pitot tubes. The measurements indicated that the laminar law, equation (3), is rigorously valid only up to  $y^*$  values of from about 1.5 to 2. At  $y^* = 5$  the velocity differs by about 10 percent and at  $y^* = 10$  by about 25 percent from the law given by equation (3).

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<sup>1</sup> $\tau$  = shearing stress,  $\mu$  = viscosity.

All the existing measurements and theoretical investigations, which show that the velocity distribution in wall proximity can be represented in the form of equation (3), refer to the two specific cases: developed turbulent flow in a pipe or channel, and flow past a wall at constant speed outside of the boundary layer (constant pressure in direction of flow). But, for the shearing-stress measurements under consideration, the velocity distribution close to the wall in general cases, that is, in flows with considerable pressure rise or drop in flow direction, is exactly the point of greatest interest. Still, it can be assumed that equation (1) is approximately valid here also for points nearest to the wall. This is readily proved for the laminar sublayer. It is true that the shearing stress  $\tau$  at a short distance from the wall differs a little from the wall shearing stress  $\tau_w$ , since for points near the wall Prandtl's general boundary-layer equations give  $\frac{\partial \tau}{\partial y} = \frac{dp}{dx}$ . But, for the normally appearing pressure increases and decreases and the very small thickness of this sublayer, this increase and decrease of the shearing stress within the laminar sublayer is so small that  $\tau = \tau_w = \text{constant}$  still is closely approximate and equations (1) and (3) remain applicable. But it is also anticipated that the transition zone from the purely laminar to the turbulent part is closely approximated by equation (1) because this layer, too, is still so thin that the variation in shearing stress due to the pressure gradient is trifling. Even the state of flow departing substantially from the law, equation (1), at greater wall distances, is not indicative of an effect in wall proximity; for the velocity profile in plate flow without pressure rise and that for flow in pipes or channels are markedly different at great wall distances and still are reproduced very satisfactorily in wall proximity by equation (1). The same holds true in rough approximation for the adjoining purely turbulent zone in wall proximity.

So these considerations show that the same general speed law as for constant pressure (equation (1)) is applicable also to boundary layers with pressure gradients in flow direction in wall proximity, although it is to be expected that the departures from this law start at so much smaller wall distance as the pressure gradient is greater.

A certain experimental proof of the validity of equation (1) can be found in Wieghardt's measurements on boundary layers with different pressure gradients (reference 5). For it is shown that the velocity  $u$  near the wall is approximately proportional to  $y^{1/7.7}$  for all velocity profiles. Now the general law, equation (1), which applies at constant pressure, can be approximated, as is known by a power formula

$$\frac{u}{u^*} = c \left( \frac{u^* y}{\nu} \right)^{1/n} \quad (4)$$

(reference 3), where  $n$  and  $C$  are constants still somewhat dependent on the  $y^*$  range, in which this general law, equation (1), is to be approximated as closely as possible. In the range involved in Wieghardt's measurements,  $n$  is a number of around 7 to 8; hence it may be assumed that the law, equation (1), in this small adjacent zone is applicable also with pressure rise or pressure drop. But this finding is not conclusive, since in Wieghardt's measurement the factor  $u^*$  is not known, so that in equation (4) the power of  $y$  can be proved but not the numerical factor  $C$ . At this point reference is made to a report by H. Ludwig and W. Tillmann, shortly to be published,\* in which it will be shown that, for the pressure gradients involved in practice, the general speed law, equation (4) and equation (1), respectively, reproduces the velocity distribution rather closely and up to comparatively great wall distances.

With validity of the general velocity law, a shearing-stress measurement will be a simple matter, in theory. It simply calls for a measurement of velocity  $u$  at any distance  $y$  followed by insertion of the two values in the equation (1) resolved with respect to  $u^*$ . The result is  $u^*$  and with it the wall shearing stress  $\tau_w$ . The only drawback is that the velocity must be measured at very short wall distance (at best, within the laminar sublayer) because it is the only place where the general velocity law is still applicable with the necessary exactness. Considering the fact that the thickness of the laminar sublayer in air currents with the usual velocities is, as a rule, only a few hundredths to tenths of a millimeter, it is readily apparent that the customary mechanical aids (pitot tube, hot wire) are useless for such measurements. An attempt was therefore made to assess the velocity distribution in direct wall proximity by means of a heat-transfer measurement. The method is explained by way of the diagrammatic drawing, figure 1. A fluid or a gas with turbulent boundary layer flows past a solid wall C; its velocity profile is shown at the left-hand side. The sublayer (straight streamlines in fig. 1) is laminar in wall proximity, the outer part of the flow is turbulent (wavy stream lines). A small, heat conducting metal block A is inserted in the solid wall C (considered heat resistant, for the present). A small electric heater raises the temperature of the block A above that of the fluid which is to have the same temperature as the wall C. Starting from the forward edge of block A, a warm boundary layer (layer with higher temperature) is built up within the boundary-layer flow, indicated by crosshatching in figure 1. By making the length of block A short enough, the thickness of the warm boundary layer can be kept small. The amount of heat transferred to the fluid is then defined, by the temperature of the block A, by the known material constants of the flowing medium, and by the velocity distribution in the immediate proximity of the wall. But, by equation (1), this velocity distribution is, aside from the material constants, only affected by the shearing stress velocity  $u^*$ , that is by the wall shearing stress  $\tau_w$ , so that, with given material constants and temperature of block A, a unique correlation of shearing stress and heat transfer of the block is obtained.

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\*This paper is available as NACA TM 1285.

This relationship can then be determined by a calibration measurement with known shearing stresses. In the following, this relationship is investigated in the light of the differential equation of the heat transfer.

### III. THEORETICAL CONSIDERATIONS ON THE RELATIONSHIP BETWEEN SHEARING STRESS AND HEAT TRANSFER

The first problem is to establish, in the light of the differential equation and the boundary conditions, how the relationship between shearing stress and heat transfer can be expressed nondimensionally with the most general validity.

The solid wall in figure 1, regarded as absolutely heat resistant, is to coincide with the  $x$  axis. The heated block A of constant temperature  $T_w$  is to reach from  $x = 0$  to  $x = \lambda$ . The fluid, so far as it is unaffected by the heating element, has a temperature  $T_\infty$ . The coordinate at right angle to the wall is denoted with  $y$ .

To simplify matters, it is assumed that the flow field is not affected at all by the temperature field. Theoretically, this can always be obtained with any degree of accuracy by choosing  $(T_w - T_\infty)$  small enough.

The differential equation for the heat transfer reads then

$$\rho c_p (\underline{w} \text{ grad } T) - \text{div} (\lambda_{\text{eff}} \text{ grad } T) = 0 \quad (5)$$

where  $\underline{w}$  is the vector of the flow velocity with the components  $u$  and  $v$ , and  $c_p$  the specific heat at constant pressure. The thermal conductivity is expressed here by an effective value  $\lambda_{\text{eff}}$  in view of the apparent increase in thermal conductivity as a result of the turbulent exchange outside of the laminar sublayer. In consequence,  $\lambda_{\text{eff}}$  is affected by  $y$ . In the immediate vicinity of the wall, where the entire heat-transfer problem takes place, the general law (equation (1)) can be applied to  $\underline{w}$ . Thus

$$\begin{aligned} u &= u^* f(y^*) \\ v &= 0 \end{aligned} \quad (6)$$

The velocity component  $v$  at right angle to the wall is equated to zero in the immediate vicinity of the wall, because  $\tau_w$  and  $u^*$ , and therewith the velocity profile itself, vary very slowly. From dimensional considerations, it follows that the effective coefficient of heat conduction  $\lambda_{\text{eff}}$  must be representable in the following form

$$\lambda_{\text{eff}} = \lambda g(y^*, \text{Pr}) \quad (7)$$

where  $\lambda$  is the normal heat conductivity factor,  $\text{Pr} = \frac{\mu c_p}{\lambda} = \frac{v}{a}$ , the Prandtl number, and  $g$ , an unknown function. Introducing equation (6) and equation (7) in equation (5) and replacing  $x$  and  $y$  by the variables

$$\xi = \frac{u^* x}{\sqrt{v a}}$$

$$\eta = \frac{u^* y}{\sqrt{v a}} = y^* \sqrt{\text{Pr}} \quad (8)$$

gives

$$\sqrt{\text{Pr}} f \left( \frac{\eta}{\sqrt{\text{Pr}}} \right) \frac{\partial T}{\partial \xi} - g \left( \frac{\eta}{\sqrt{\text{Pr}}}, \text{Pr} \right) \left( \frac{\partial^2 T}{\partial \xi^2} + \left( \frac{\partial^2 T}{\partial \eta^2} \right) - \frac{\partial g \left( \frac{\eta}{\sqrt{\text{Pr}}}, \text{Pr} \right)}{\partial \eta} \frac{\partial T}{\partial \eta} \right) = 0 \quad (9)$$

with the boundary conditions

$$\begin{aligned} T &= T_w & 0 \leq \xi \leq \bar{l} & & \text{for } \eta = 0 \\ \frac{\partial T}{\partial \eta} &= 0 & -\infty \leq \xi \leq 0 & \quad \bar{l} \leq \xi \leq \infty & \text{for } \eta = 0 \\ T &= T_\infty & & & \text{for } \eta = \infty \end{aligned} \quad (10)$$

with, for abbreviation,  $\frac{l u^*}{\sqrt{v a}} = \bar{l}$ .

From the homogeneity of this differential equation in  $T$ , the form of the coefficients, and the form of the boundary conditions, it follows

that the temperature field can be represented in the following form:

$$T = T_{\infty} + (T_w - T_{\infty}) h(\xi, \eta, Pr, \bar{l}) \quad (11)$$

Since directly at the wall the heat transfer is solely by conduction, the heat volume  $Q$  transferred in unit time is

$$Q = b\lambda \int_0^{\bar{l}} \left( \frac{\partial T}{\partial y} \right)_{y=0} dx = b\lambda \int_0^{\bar{l}} \left( \frac{\partial T}{\partial \eta} \right)_{\eta=0} d\xi = b\lambda (T_w - T_{\infty}) k(\bar{l}, Pr) \quad (12)$$

$b$  is the width of the element,  $k$ , a function not further identified.

Now, when the mean transfer factor  $\bar{\alpha}$

$$\bar{\alpha} = \frac{Q}{\bar{l}b(T_w - T_{\infty})} \quad (13)$$

and the corresponding dimensionless heat-transfer factor, the so-called Nusselt number  $\bar{Nu} = \frac{\bar{\alpha}\bar{l}}{\lambda}$ , are introduced

$$\bar{Nu} = k(\bar{l}, Pr) \quad (14)$$

Thus it is seen that, on the assumption of a constant Prandtl number, a unique relationship exists between quantity  $\bar{l} = \frac{u^*\bar{l}}{\sqrt{v_a}} = \left( \frac{\bar{l}^2 \tau_w}{\mu a} \right)^{1/2}$  and the Nusselt number  $\bar{Nu} = \frac{\bar{\alpha}\bar{l}}{\lambda}$ . The Nusselt number  $\bar{Nu}$  is defined by a measurement of  $Q$  and  $(T_w - T_{\infty})$ , and then  $\bar{l}$ ,  $u^*$  and  $\tau_w$  can be computed, when the function  $k$  is known. Theoretically, this function  $k$ , that is, the relationship between  $\bar{Nu}$ ,  $\bar{l}$ , and  $Pr$ , could be determined by integration of equation (9); it would merely involve some assumptions identifying the variation of the function  $g(y^*, Pr)$ . In view of the uncertainty of this assumption and the fact that in the construction of a measuring element the ideal forms serving as a basis of the calculations cannot be maintained, this complicated calculation process is not worth while. The connection between  $\bar{Nu}$  and  $\bar{l}$  is much better obtained by the calibration measurement, which does not have to be made

with the same flowing medium for which the measuring element is to be used later, although both mediums must have the same Prandtl number.

If the lengths  $l$  and  $\bar{l}$  are chosen small enough so that the warm boundary layer remains completely within the laminar sublayer, the theoretical connections are simplified substantially. According to equation (3), the expression  $f\left(\frac{\eta}{\sqrt{\text{Pr}}}\right) = \frac{\eta}{\sqrt{\text{Pr}}}$  can be then put in equation (9) which gives, since the turbulent exchange is also absent,

$$\begin{aligned} g\left(\frac{\eta}{\sqrt{\text{Pr}}}, \text{Pr}\right) &= 1 \\ \frac{\partial g\left(\frac{\eta}{\sqrt{\text{Pr}}}, \text{Pr}\right)}{\partial \eta} &= 0 \end{aligned} \quad (15)$$

Hence by equation (9)

$$\eta \frac{\partial T}{\partial \xi} - \left( \frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial \eta^2} \right) = 0 \quad (16)$$

with the corresponding boundary conditions

$$\begin{aligned} T &= T_w & 0 \leq \xi \leq \bar{l} & & \text{for } \eta = 0 \\ \frac{\partial T}{\partial \eta} &= 0 & -\infty \leq \xi \leq 0 & \quad \bar{l} \leq \xi \leq \infty & \text{for } \eta = 0 \\ T &= T_\infty & & & \text{for } \eta = \infty \end{aligned} \quad (17)$$

Thus  $\bar{l}$  (dimensionless depth of element) remains the sole parameter of the solution in the  $\xi, \eta$  system. At  $\bar{l}$  values not too small, the thickness of the warm boundary layer is small compared to its length. The entire forward portion of this layer up to  $\xi$  values approaching those for  $\bar{l}$  is then entirely unaffected by  $\bar{l}$ . Therefore the solution of (16) for the boundary conditions

$$\begin{aligned} T &= T_w & \text{for } 0 \leq \xi \leq \infty & & \text{and } \eta = 0 \\ \frac{\partial T}{\partial \eta} &= 0 & \text{for } -\infty \leq \xi \leq 0 & & \text{and } \eta = 0 \end{aligned}$$

gives, at the same time, the correct solution for the correct boundary conditions.

For sufficiently great  $\xi$  values, the usual omission of the boundary-layer theory in the differential equation leaves  $\frac{\partial^2 T}{\partial \xi^2} \ll \frac{\partial^2 T}{\partial \eta^2}$ ; hence

$$\eta \frac{\partial T}{\partial \xi} - \frac{\partial^2 T}{\partial \eta^2} = 0 \quad (18)$$

Leveque (reference 6) already transformed and solved this partial differential equation by substitution of

$$\bar{\eta} = \frac{\eta}{(9\xi)^{1/3}} \quad (19)$$

into the ordinary differential equation

$$3\bar{\eta}^2 \frac{dT}{d\bar{\eta}} + \frac{d^2 T}{d\bar{\eta}^2} = 0 \quad (20)$$

with the boundary conditions

$$T = T_w \quad \text{for } \bar{\eta} = 0$$

$$T = T_\infty \quad \text{for } \bar{\eta} = \infty$$

Transferred and resolved, the temperature field is

$$T = T_w - (T_w - T_\infty)F(\bar{\eta}) \quad (21)$$

with

$$F(\bar{\eta}) = \frac{\int_0^{\bar{\eta}} e^{-\bar{\eta}^3} d\bar{\eta}}{\int_0^\infty e^{-\bar{\eta}^3} d\bar{\eta}} \quad (22)$$

Therefore, the temperature depends solely on the parameter  $\bar{\eta}$ , or, in other words, the temperature profiles, the thickness of which increases with  $\xi^{1/3}$ , are similar in sections  $\xi = \text{const.}$  Computing the heat

volume  $Q$  transferred from it, then the Nusselt number  $\bar{Nu}$  as function of  $\bar{l}$  gives

$$\bar{Nu} = \frac{\bar{\alpha}l}{\lambda} = \frac{(9\bar{l})^{2/3}}{6 \int_0^\infty e^{-\bar{\eta}^3} d\bar{\eta}} = 0.807 \bar{l}^{2/3} \quad (23)$$

or, when  $\bar{l}$  is replaced again by the original quantities,

$$\bar{Nu} = \frac{\bar{\alpha}l}{\lambda} = 0.807 \left( \frac{l^2}{\mu a} \right)^{1/3} \tau_w^{1/3} \quad (24)$$

The Nusselt number  $\bar{Nu}$  and the heat-transfer factor  $\bar{\alpha}$  are in this case, proportional<sup>2</sup> to the third root of the wall shearing stress. On assuming that the warm boundary layer remains within the laminar sub-layer, the dependence of  $\bar{Nu}$  on the Prandtl number cancels out altogether.

In figure 2, the temperature field  $\frac{T_w - T}{T_w - T_\infty} = F(\bar{\eta})$  is plotted against  $\bar{\eta}$  according to equation (22). Defining the wall distance at which the tangent to the temperature profile in point  $\bar{\eta} = 0$  and the asymptote to the temperature profile meet (fig. 2) as thickness of the thermal boundary layer, the latter follows as

$$\delta_w = 1.86 \left( \frac{\mu x}{\tau_w} \right)^{1/3} \quad (25)$$

The thickness of the laminar sublayer is given by the following relations

$$\frac{u^* \delta_L}{\nu} = C \quad (26)$$

where  $C$  is a constant, which, depending upon the demands made on the laminarity, ranges between 1.5 and 10. By equations (25) and (26) the ratio  $\frac{\delta_w}{\delta_L}$  follows as

$$\frac{\delta_w}{\delta_L} = \frac{1.86}{C} \left( \frac{c_f}{2} \right)^{1/6} \left( \frac{Re_L}{Pr} \right)^{1/3} \quad (27)$$

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<sup>2</sup>(Derived independently by Reichardt.)

where  $c_f' = \frac{\tau_w}{\rho/2U^2}$  is the local coefficient of friction, that is, the wall shearing stress  $\tau_w$  made dimensionless with the dynamic pressure outside of the boundary layer. The factor  $Re_l$  is the Reynolds number formed by  $l$ , the velocity  $U$  outside of the boundary layer and the kinematic viscosity  $\nu$ , and  $Pr$  the Prandtl number. Since  $c_f'$  varies rather little as a rule, (ordinarily ranging from 0.002 to 0.003,  $(\frac{c_f'}{2})^{1/6}$  is practically a constant.

On entering the material constants for air, the velocities usually occurring in air and the practical element length  $l$  of about 1 mm, into equation (27), it is seen that the thermal boundary layer generally extends a little beyond the laminar sublayer. Nevertheless, it is anticipated that with the use of small  $l$ , the law, equations (23) and (24), still reproduces the relationship between heat transfer and shearing stress approximately because the change in heat transfer due to turbulence occurs only in the outer zones of the thermal boundary layer, where the temperature gradient is small in any case. Furthermore, two effects, compensating in part, occur in this case. The turbulent exchange is accompanied by a greater heat transfer and the exchange of momentum by a decrease in the mean flow velocity, which is equivalent to a reduction in heat transfer.

Incidentally, it should be borne in mind that this dipping of the thermal boundary layer into the turbulent part detracts in no way from the validity of the relation between heat transfer and shearing stress, save for the change in the form of equations (23) and (24) which has no effect on the present measurements, since the relationship between heat transfer and shearing stress is to be determined by a calibration measurement anyhow.

In the derivation of equation (24), the assumption that the thermal boundary layer remains entirely within the laminar sublayer was supplemented further by the assumption  $\frac{\partial^2 T}{\partial \xi^2} \ll \frac{\partial^2 T}{\partial \eta^2}$ , which is certainly justifiable for greater  $\xi$ , while, for very small values of  $\xi$  quite near the forward edge of the element,  $\frac{\partial^2 T}{\partial \xi^2}$  is no longer negligible with respect to  $\frac{\partial^2 T}{\partial \eta^2}$ . But an iteration, in which  $\frac{\partial^2 T}{\partial \xi^2}$  is replaced by the value from Leveque's solution as first approximation, indicates readily that substantial variations in heat transfer occur only for  $\xi$  values less than 5. So, when  $\bar{l}$  is considerably greater than 5, as is the case for air flows with the usual velocities, the omission of term  $\frac{\partial^2 T}{\partial \xi^2}$  plays no

essential part. Therefore, it is expected, according to equation (24), that in the tests which are to be made in air, the third root of the shearing stress is approximately proportional to the coefficient of heat transfer. Obviously, for very small  $\bar{l}$  values, where the omission of term  $\frac{\partial^2 T}{\partial x^2}$  is no longer permissible, the relationship between heat transfer and shearing stress remains unique.

#### IV. DESCRIPTION OF SHEARING STRESS INSTRUMENT

Figure 3 represents the instrument for measuring the shearing stress, which has proved very practical in the shearing measurements in air, described in section V. The construction and mode of operation is explained by way of this drawing. A steel ring D is fitted and screwed tight into the smooth wall C on which the shearing stress of the air streaming past is to be measured. It is essential that D insure the best possible heat conduction with the wall C (large contact area), in order that the heat passing from the measuring instrument to the wall as a result of imperfect heat insulation, does not heat up ring D. The measuring element is fitted into the hole of ring D as closely as possible and held by a hard rubber lock nut F. To obviate the use of an instrument for each test station, dummy plugs may be used. Naturally, all pieces must be fitted flush so as to leave no edges at the joints which might disturb the boundary-layer profile of the flow.

The instrument itself consists of a brass casing B in whose hole the 2- by 9- by 6-mm copper block A is mounted. The block is held by a celluloid diaphragm E of about 1/10-mm thickness cemented on the 2- by 9-mm surface, which is cemented over the opening of the casing as smoothly as possible. A pressure-equalizing hole H in the wall of the housing prevents the diaphragm from bulging during a pressure difference between inner and outer space. A thread 1/10 mm deep, cut in the casing B at the seat of the diaphragm, insures a very smooth surface. This method of mounting the block A provides adequate heat insulation relative to housing B, because with the small dimensions of the hole in the casing B, for which the convection produces no essential contribution to the heat transfer, the air forms an excellent heat insulator, and the diaphragm itself, being of little thickness and low conductivity, transfers no great volume of heat to the casing A. The heat transfer from block A to the air, on the other hand, is little affected by the celluloid diaphragm because it is thin. Block A also carries a little electric heater of about 0.13 watt. In addition, the temperature of the block can be measured by a thermocouple whose junction is located near the heat-transferring surface. The four wires of about 1/10-mm gage pass insulated through the bottom of the casing A. Back of the bottom, the

wires have a greater cross section. The wires pass through the hard rubber cap A to keep the casing B from being heated by the heat of the operator's hand when changing the instrument. An indicator K and dial L marked off in degrees above ring A complete the setup. This way the direction by which the block A is fitted can be read from the outside.

#### V. MODE OF SHEARING-STRESS MEASUREMENT AND DETERMINATION OF THE CALIBRATION CURVE

According to Section III, a definite relationship exists between  $\bar{Nu} = \frac{\bar{\alpha}l}{\lambda}$  and  $\bar{l} = \left( \frac{l^2 \tau_w}{\mu a} \right)^{1/3}$  when the Prandtl number  $Pr$  is given. It was also indicated that the exact form of the relationship for the present instrument was to be determined by a calibration measurement. The first problem consists in finding how the quantities  $\bar{\alpha}$  and  $\bar{Nu}$  can be measured with the instrument. It calls for the measurement of the heat volume  $Q$  transferred from the instrument A in unit time and the temperature difference  $(T_w - T_\infty)$ . The heat volume  $Q$  is readily measured by applying a certain electric voltage, and with it also heat input at block A, and waiting until the steady state is reached; for the amount of heat transfer must be equal to the input, which is readily measured. The temperature difference  $(T_w - T_\infty)$  is best determined by using a second instrument, which is installed in the same wall, as cold junction of the thermocouple when the heating is turned off. The appearing thermocouple voltage, which is proportional to  $(T_w - T_\infty)$ , is measured with a potentiometer or a sensitive ammeter. In the second case, the voltage drop due to the finite resistance of the lead-in wires must be taken into consideration as a rule. From  $Q$  and  $(T_w - T_\infty)$  the value of  $\bar{\alpha}$  and  $\bar{Nu}$  can then be computed.

However, it is to be noted that the amount of heat given off by the block A consists of two portions, the heat volume transferred direct from the block to the flowing medium and that transferred to the wall C as a result of the imperfect heat insulation of the block A. The determination of  $\bar{\alpha}$  and  $\bar{Nu}$  just indicated, comprises both portions, while the theoretical considerations of Section III refer only to the first portion. However, since the second depends only on the instrument itself and is unaffected by the transmitted shearing stress, it merely results in a parallel displacement of the calibration curve.

To provide a known shearing stress for the calibration measurement, the instrument to be calibrated was installed in the rectangular test length described by Schultz-Grunow (reference 1) in a flat sheet steel wall, 6 meters in length and 1.4 meters wide. The opposite, movable

wall was set for constant pressure over the entire test length. A boundary layer, like on an infinitely thin flat plate in parallel flow, forms then at the wall. The friction coefficients, and hence the shearing stresses, have already been computed by various writers (references 1, 7, 8, and 9) by various methods and are therefore fairly accurately known. The present calibration tests were based on the Schultz-Grunow test data, since they had been secured in the same test length by direct force measurements, and so any defects in the experimental setup do not involve the calibration measurements. Now, according to the arguments in Section III, the assumption of a fixed Prandtl number makes  $\bar{Nu}$  a single-valued function of  $\bar{l}$ , but the derivation was made on the assumption that the temperature rise  $(T_w - T_\infty)$  is so small that the material constants within the thermal boundary layer still can be regarded as constant. For instrumental reasons,  $(T_w - T_\infty)$  cannot be made so small that this assumption is rigorously correct. For this reason, the relationship between  $\bar{Nu}$  and  $\bar{l}$  is somewhat different, depending upon what temperature difference  $(T_w - T_\infty)$  is chosen. Aside from that, it also depends somewhat on whether or not the dimensionless quantities  $\bar{Nu}$  and  $\bar{l}$  are formed with the material constant corresponding to  $T_\infty$  or  $T_w$ . This difficulty is overcome by stipulating that the material constants corresponding to  $T_\infty$  be made dimensionless, and also that the same temperature difference  $(T_w - T_\infty)$  always be used. The second requirement is replaced, for reasons of measuring technique, by the stipulation that the operation always be carried out with the same heat input. This also ensures a definite relationship between  $\bar{Nu}$  and  $\bar{l}$ . The adjustment of the fixed heat input is much more convenient than the adjustment of the fixed temperature  $T_w$ , where it is necessary to await the slowly approaching steady state first before an adjustment can be made. As calibration curve,  $\bar{l}^{2/3}$  is then plotted as abscissa against  $\bar{Nu}$  as ordinate. On the assumption that the thermal boundary layer does not extend appreciably beyond the laminar sublayer, the calibration curve is, according to equation (23), approximately a straight line, which, however, does not pass through the origin of the coordinate because of the amount of heat passing through the imperfect heat insulation onto the wall. Figure 4 represents the calibration curve for this instrument. It shows the approximately rectilinear variation of the calibration curve over a wide  $\bar{l}$  range ( $\bar{l}$  range equivalent to a shearing-stress range of about 1:223). The curvature is largely attributable to the presence of the celluloid diaphragm between the surface of the copper block and the flowing air. Measurements with other instruments fitted with glass diaphragms (greater heat conduction) exhibited much straighter calibration curves, but poorer heat insulation relative to the casing. The straight line anticipated by equation (23) is shown as a dashed line. The variation of the shearing stress and of quantity  $\bar{l}$  was effected once by varying the flow velocity and then by shifting the position of the instrument. The instrument was first mounted 1.78 meters

from the front edge of the wall, then 5.28 meters from the front edge. In both cases the total speed range was covered. The points of both test series are seen to be in good agreement. The slight systematic difference is not necessarily attributable to the instrument, since it is not greater than the measuring accuracy of the Schultz-Grunow measurements used as basis of the calculation. According to the theoretical considerations in Section III, it was possible to carry out the calibration measurements with a different flowing medium also as long as the Prandtl number was the same in both cases. When the imperfect heat insulation of the block A is taken into account, this is no longer possible as is readily apparent from the following reasoning:

When quantity  $\bar{t}$  is given, the Nusselt number corresponding to the direct heat transfer onto the flowing medium is fixed, but the Nusselt number corresponding to the direct heat transfer onto the wall is somewhat different for various flowing mediums, since not all of the heat flows through the casing into the chamber, but a part passes directly through the celluloid diaphragm and through the heating and thermocouple wires. For this reason, the calibration and the principal measurements are carried out as much as possible on the same medium and at the same temperature, since the different temperatures correspond to different material constants of the medium, and hence the effect is the same. However, this temperature effect is quite small so that temperature fluctuations of  $\pm 5^\circ \text{C}$  have no measurable effect. In measurements at greater temperature fluctuations, the relation of calibration curve and temperature must be determined separately.

## VI. DIRECTIONAL SENSITIVITY OF THE INSTRUMENT AND MEASUREMENT OF THE DIRECTION OF THE SHEARING STRESS

The shearing stress transmitted by the flowing medium on the wall is a vectorial quantity; hence its exact identification is predicated upon knowing its absolute magnitude and direction. In many cases, the direction is automatically given by the direction of the flow outside the boundary layer, that is, when no pressure gradient perpendicular to the direction of flow exists, because then a two-dimensional flow is formed in the boundary layer. In such cases, only the magnitude of the shearing stress is of interest. The instrument is then mounted in such a way that the narrow side of the surface of the block is parallel to the direction of the shearing stress. In this case, it is desirable that the instrument have a low directional sensitivity in order that minor angular errors during mounting of the instrument do not result in erroneous measurements. In figure 5, the measured  $\bar{Nu}$  divided by the Nusselt number at angle  $\varphi = 0$ ,  $\frac{\bar{Nu}}{Nu_{\varphi=0}}$ , is plotted against the angle  $\varphi$

(angle between shearing stress and direction of the narrow side of the surface of block A). The more than satisfactory directional sensitivity of the probe is readily apparent. Up to angles of  $\pm 15^\circ$ , there is no error at all, and even at greater angles it is very small.

But frequently there are also flows with pressure gradient at right angles to the flow. In that case, the flow within the boundary layer has a different direction at different wall distances. The direction of the shearing stress is then determined by the direction of the flow in the immediate proximity of the wall. This is a case where the direction of the shearing stress is not given to begin with and must be ascertained by measurement. The same instrument can be used, but it is mounted in such a way that the direction of the shearing stress is approximately parallel to the long side of the surface of block A. A heat-transfer measurement gives the directional dependence of the Nusselt number represented in figure 6; it shows a distinct minimum when the shearing stress is parallel to the long side of block A. A few measurements at three or four points on either side of the minimum give this minimum, and with it the direction of the shearing stress, fairly accurately.

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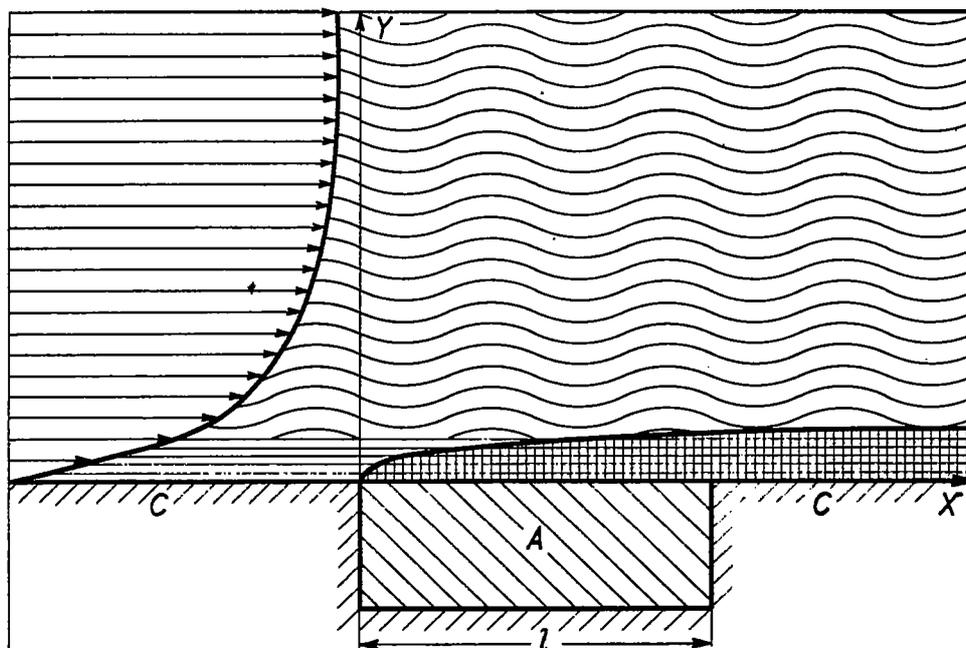


Figure 1.- Diagrammatic representation of the test method. Laminar sublayer (straight streamlines), thermal boundary layer (cross hatching), turbulent part of boundary layer (wavy streamlines).

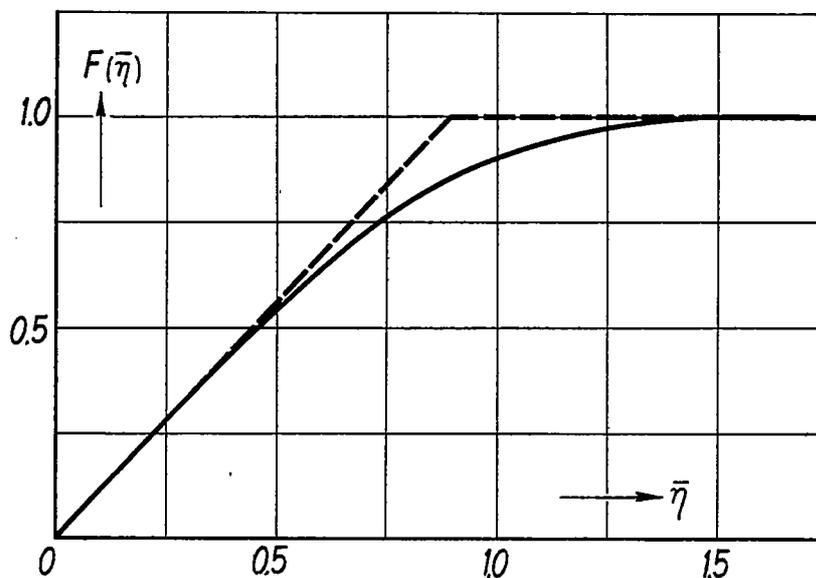


Figure 2.- Temperature profile of thermal boundary layer for the case of thermal boundary layer contained entirely within the laminar sublayer (according to equation (22));  $\delta_w$  defined as thickness of thermal boundary layer.

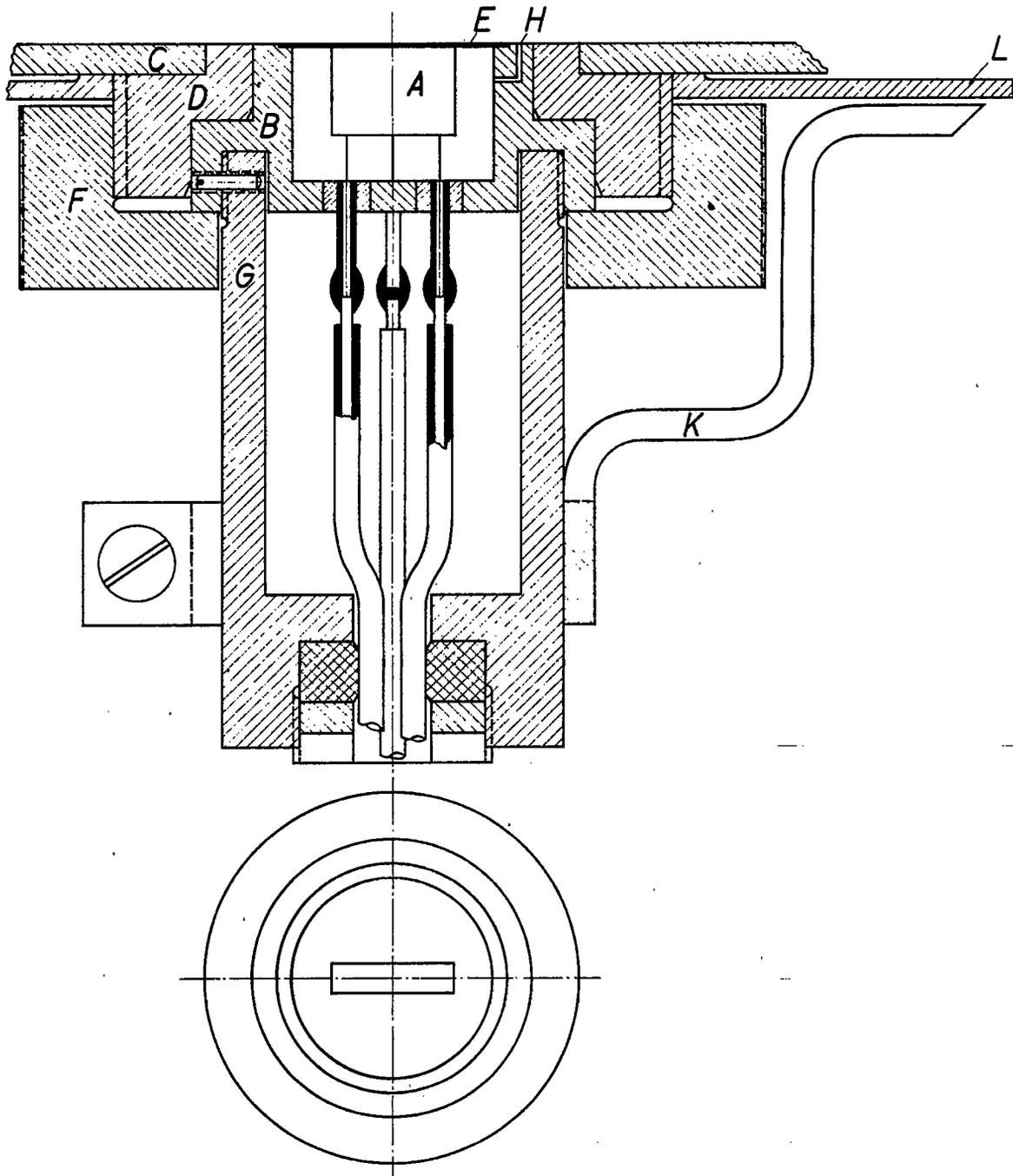


Figure 3.- Instrument designed for measuring wall shearing stress.

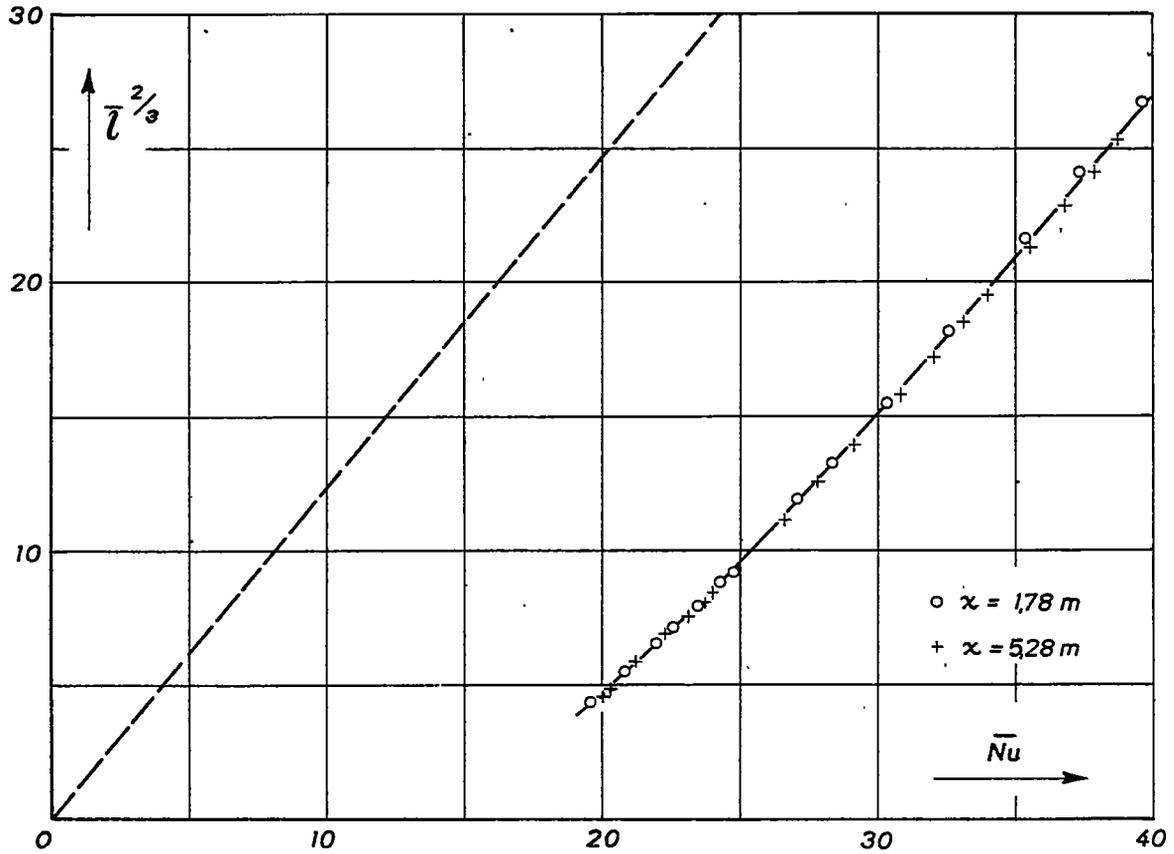


Figure 4.- Calibration curve of instrument shown in figure 3. The parallel displacement between the two curves is due to the fact that the direct transfer of heat to the wall is not taken into account for the theoretical curve.

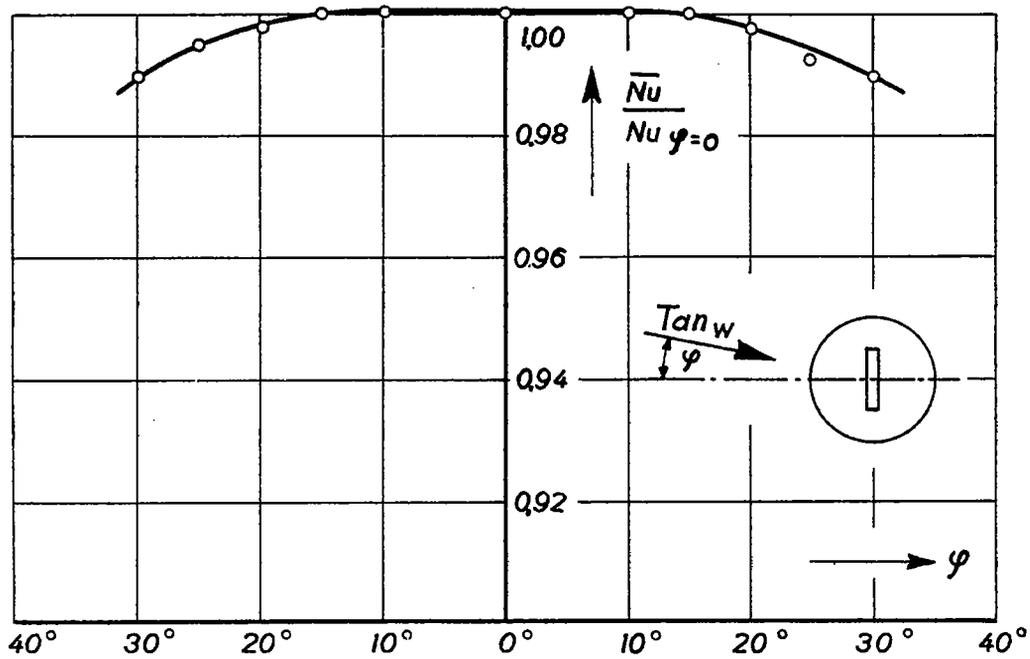


Figure 5.- Directional sensitivity in flow parallel to the small side of the heat-transferring rectangle.

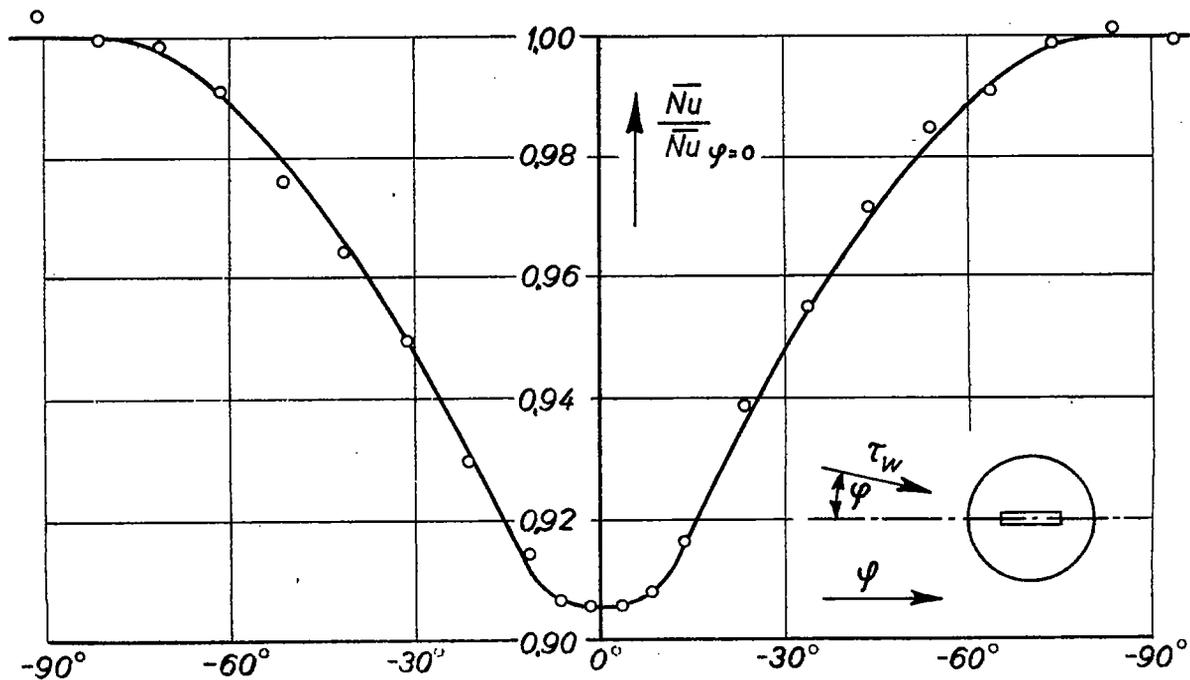


Figure 6.- Directional sensitivity in flow parallel to the long side of the heat-transferring rectangle.