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AUTOMATIC STABILITY OF AIRPLANES

By Fr. Haus

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PART I

INTRODUCTION TO THE PROBLEM OF AUTOMATIC STABILITY

Since the early days of aviation, the problem of airplane stability was a subject of investigation which absorbed the activity of many scientists. It is endeavored below to give a full outline of the problem and to classify the proposed solutions systematically. Longitudinal stability, which can be studied separately, is considered first. The combination of lateral and directional stabilities, which cannot be separated, will be dealt with later.

I. THE PITCHING MOMENT

Let M be the moment exerted by the air forces about an axis originating in the center of gravity and perpendicular to the plane of symmetry. This moment is often called the pitching moment. It is positive when it tends to nose the airplane over. It is expressed by the following formula

$$M = C_m S l \frac{\rho V^2}{2g}$$

where C_m is a nondimensional factor,

S , the wing area of the airplane,

l , the mean wing chord.

C_m is a well-known factor determined by wind-tunnel model tests. It is, in general, called coefficient, which should not, however, lead to the belief that its value is

*"Stabilité automatique des avions." L'Aéronautique, Nos. 156-9, May, June, July and August, 1932.

constant. It is, on the contrary, an actual function depending on several variables, namely:

The airplane incidence i ,

" elevator setting β ,

" propeller revolutions, denoted by $\gamma = \frac{V}{nD}$.

If, moreover, the airplane motion is not merely translational, but comprises an angular pitching velocity q about the transverse axis, the factor C_m is a function of the auxiliary variable η :

$$\eta = \frac{q l'}{V}$$

where l' is a characteristic length of the airplane, namely, the distance between the center of the tail unit and the airplane c.g.

In that case, η is the relation between the linear velocity of the tail unit, due to the pitching motion, and the translational velocity of the airplane. The variation of C_m , as a function of these variables, can be determined for any airplane type. The partial derivatives C_m' of the function, with respect to the different variables, have then the following significations:

$\frac{\partial C_m}{\partial i}$ denotes static or inherent stability, i.e., the tendency of an airplane oscillating about its c.g. to settle at a constant angle of incidence with respect to the relative wind. We put $\partial C_m / \partial i = \mu$ and call this expression the coefficient of static stability.

$\frac{\partial C_m}{\partial \beta}$ denotes the action of the elevator on the magnitude of the moment M . We put $\partial C_m / \partial \beta = \nu$.

$\frac{\partial C_m}{\partial \gamma}$ defines the influence of the propeller slipstream on this moment and $\partial C_m / \partial \eta$ denotes the damping of the pitching vibrations of the airplane oscillation about its presumably fixed c.g.

II. STATIC STABILITY AND ELEVATOR EFFICIENCY

The exact value of C_m as a function of the variables i and β can be easily determined. This result may be achieved for any airplane type by systematic flight tests. After building an airplane with good stability characteristics, it is interesting to express these characteristics by numerical relations which permit anticipating the maneuverability of airplanes under design. In general, these data are obtained by scale-model wind-tunnel tests, the results of which, although less accurate, still form interesting approximations.

Under these conditions, the i values are plotted as the abscissas and the C_m values as the ordinates of a system of axes. (Fig. 1.) This permits plotting the curves corresponding to the various values, whence the classical diagram of the C_m as a function of the variables i and β is obtained. This diagram can also be found by calculation when the aerodynamic characteristics of the different airplane components and the position of its c.g. are known. This diagram differs, however, from the actual conditions more than that obtained by wind-tunnel tests.

An airplane is statically stable when $\mu > 0$, since a deflection from its position of equilibrium then produces a restoring moment in the desired direction. The value of μ equals the angular coefficient of the tangents to the C_m curves of Figure 1.

Experience shows that the static stability grows with the incidence within the range of current flight angles. The maximum value of the coefficient of static stability is often reached at incidences near the maximum lift of the airplane.

If the curves corresponding to different β values were parallel, the coefficient of static stability would be related to the incidence only, but independent of the elevator setting. This, however, is not quite correct, since the curves are not quite parallel. Roughly speaking and considering the main parameters of static stability only, namely, the position of the c.g. and the relative size of the tail surfaces, one may write, below maximum lift:

$$\frac{\partial C_m}{\partial i} = \frac{dC_z}{di} \left(0,25 - \frac{p}{l} \right) + \frac{S' l'}{S l} \frac{dC_z'}{di'} \frac{di'}{di} k, \quad (a)$$

where p is the distance between the c.g. and the leading edge,

S' , the area of the tail surface,

i' , " incidence of the tail surface,

C_z , " lift of the wings,

C_z' , " " " " tail surface,

k , " relation between the square of the true air speed about the tail surface (making allowance for the airplane wake and the propeller slipstream) and the square of the airplane speed.

The derivatives $\frac{dC_z}{di}$ and $\frac{dC_z'}{di'}$ do not vary materially. The most efficient means of increasing static stability is:

a reduction of p by a forward shifting of the c.g.

an increase of $S'l'$ or the geometric characteristics of the tail surface. The partial derivative denoting the efficiency of the elevator may be written

$$\frac{\partial C_m}{\partial \beta} = \frac{S'l'}{S\ l} \frac{dC_z'}{d\beta} k. \quad (b)$$

III. PROPELLER ACTION

The moment M , produced by the air force on the wing, depends on the parameter γ of the propeller r.p.m., owing to the action of the slipstream on the tail surfaces. The moment of the external forces about the c.g. comprises, in addition to the air-force moment M , a moment $T s$ exerted by the thrust of the propeller if its axis lies outside of the c.g., s being its lever arm, which is positive when the propeller axis lies above the c.g. When s is different from zero, the total moment about the c.g. is

$$M + T s$$

and equilibrium is possible only when

$$M + T s = 0.$$

Each of these two terms is affected by variations of the propeller speed. The effect of a variation of γ on M is a characteristic of practical importance. All flight tests show its outstanding influence. This fact has been confirmed by laboratory tests which permit estimating the numerical value of the derivative $\partial C_m / \partial \gamma$. The measured values vary between 0.05 and 0.22. The fact that they are positive shows that the moment is nose-heavy when $\gamma = V/nD$ increases, i.e., when the revolution number decreases at constant speed or when the airspeed increases at constant r.p.m.

It is now impossible to calculate $\partial C_m / \partial \gamma$ in advance, owing to insufficient knowledge of the mechanism of mutual interference. The moment T_s is a function of the lever arm of the propeller axis about the c.g. The effect of this moment can be easily foreseen. Besides, many airplanes have a very small s .

IV. DAMPING OF THE PITCHING MOTION

The damping of the pitching motion can be easily defined. If it is assumed to be due to displacements of the horizontal tail surface only, we have

$$\frac{\partial M}{\partial q} = - \frac{dC_z'}{di'} S' l'^2 k \frac{aV}{2g}, \quad (c)$$

which permits writing

$$\frac{\partial C_m}{\partial \eta} = \frac{S' l'}{S l} \frac{dC_z'}{di'} k \quad (d)$$

where the derivative dC_z'/di' must necessarily be written with the radian as the angular unit. The tail surface produces part of the total damping only. The wings, the fuselage, and all the surfaces tend to produce a similar effect.

The effect of the tail surface is, however, predominant. All the other causes of damping can be expressed by multiplying the term corresponding to the tail surface by a factor k' , the mean numerical value of which is of the order of 1.25.

V. DYNAMIC STABILITY

We have thus briefly summarized our knowledge of the pitching moment. No final conclusions regarding longitudinal stability can be reached on the ground of these data only. If our investigations are confined to the conditions of equilibrium, a series of problems may be taken up by studying moment M only. If, on the contrary, our study extends to the stability of motion or dynamic stability, which, in the final analysis, should be our aim, the study of the moment M is insufficient. The motion is not determined by the equilibrium of the moments only, but also by the equations which govern the equilibrium of the forces in the plane of symmetry. Stability cannot be studied as though the airplane were suspended from its presumably stationary c.g., since a disturbed equilibrium of the forces modifies the trajectory and affects the airplane motion in the same way as the moments.

A survey of longitudinal stability clearly shows that a statically stable airplane tends to maintain a constant speed and modify its trajectory to the extent required for the maintenance of the speed. The opinion on this question of Mr. A. Sée, a well-known prewar technician, expressed by him in 1912, is given below.* According to him, stability of form roughly insures the stability of the relative speed and the stability of the angle of incidence, which are more or less quickly recovered by the airplane after losing either of them. The mechanism for producing this effect is well known. If the relative speed of an airplane in equilibrium is suddenly reduced either by a temporary reduction of the engine speed or by a back squall, the airplane makes a downward motion and assumes the following successive attitudes;

1. The relative speed being reduced, the lift grows deficient and the airplane begins to fall;
2. This fall increases the angle of incidence;
3. With increasing incidence, the inherent stability produces a moment which noses the airplane over;

*Sée, A.: En quoi consiste la Stabilité? L'Aerophile, October 15, 1912.

4. In diving, the airplane recovers its incidence, but the fall continues and its speed increases;

5. The lift increases with the speed and the airplane stops falling;

6. As soon as the downward motion is stopped, the incidence is reduced and the static stability creates a tail-heavy moment which levels off the airplane.

There is little to be changed about such an explanation of the phenomenon which Mr. Séé sums up as follows:

"An airplane has longitudinal stability, not when it opposes pitching motions, but when it conforms to the pitching motions necessitated for stability of speed."

In this explanation, which we quote as it was given in 1912, no attention was paid to the variations of the pitching moment due to modifications of γ . If this action is taken into consideration, it is found to oppose the pitching motions necessary for the constancy of the relative speed when the disturbances of the latter are due to external causes. In fact, when $\partial C_m / \partial \gamma$ is positive, a reduction of V tends to stall the airplane. On the other hand, when the decrease of the speed is due to the stopping of the engine, a decrease in the speed V is accompanied by a much greater reduction of n . The stopping of the engine or a reduction of its speed causes an increase of γ . In this case the effect of the derivative $\partial C_m / \partial \gamma$ contributes toward creating the required nose-heavy moment.

The conclusion reached by Mr. Séé may be considered as a program. A statically stable airplane conforms to this program under any condition provided the effect of the static stability overcomes the action of the variation γ which is sometimes detrimental. The possible action of the moment T_s must also be taken into consideration when the thrust axis lies outside the c.g.

VI. ANALYTICAL REPRESENTATION

New data on these motions are obtained by an analytical study of the phenomenon. In view of their outstanding interest the mathematical theory is outlined below.

The equation is established on the ground of a system of axes OX , OY , OZ integral with the airplane and originating in the c.g. (Fig. 2.) The axes OX and OZ are in the plane of symmetry. The former, directed forward, is parallel to the wing chord, and the latter, perpendicular to OX , is directed upward. The OY axis, perpendicular to the plane of symmetry, is directed toward the left. The position of the airplane in space is determined by the following elements:

1. The coordinates of its c.g. with reference to a system of fixed axes;
2. The orientation of the movable axes OX , OY , OZ , integral with the airplane, with reference to a system of fixed axes.

The study of stability does not require the coordinates of the c.g. to be known. The knowledge of the three angles which determine the direction of the trihedral integral with the airplane, relative to a fixed trihedral, is necessary. Let φ , θ , ψ be these angles which are defined later in connection with the study of lateral stability. The airplane motion, at any moment, is determined by:

1. The air speed V of its c.g., the projections of V of the three axes being u , v , and w ;
2. Its angular velocity Ω , whose projections on the axes are p , q , and r .

The position of the airplane on its trajectory is defined by two angles which are functions of the above values, namely, the angle of incidence i and the angle of sideslip j . By definition, we have

$$\tan i = -\frac{w}{u}, \quad \tan j = +\frac{v}{u}.$$

For small angles this may be written:

$$i = -\frac{w}{u}, \quad j = +\frac{v}{u}.$$

These conventions are of a sufficiently general character to permit a subsequent study of lateral and directional stability. The number of variables is smaller for

the study of longitudinal motion. (Fig. 3.) The following parameters only are required:

Two linear velocities u and w ;
 One angular velocity q ;
 One angle of orientation θ ,

which defines the inclination of the OX axis to the horizontal. This angle is positive when the OX axis is directed downward.

VII. STUDY OF THE LONGITUDINAL MOTION

The question may now be studied analytically. Let F_x and F_z (fig. 3) be the projections of the resultant of the air forces on the axes integral with the airplane. These projections are positive when they define forces acting in the positive direction of the axes and inversely. M is the aerodynamic moment about the transverse axis. It is easily seen that F_x , F_z , and M are only functions of u , w , q (or of the equivalent values V , i , η), when the propeller interference is disregarded.

The variables u , w , q , θ satisfy the equations of motion with reference to the axes integral with the airplane

$$\left. \begin{aligned} T + F_x + P \sin \theta &= \frac{P}{g} \left(\frac{du}{dt} + q w \right), \\ F_z - P \cos \theta &= \frac{P}{g} \left(\frac{dw}{dt} - q u \right), \\ M + T s &= \frac{P r^2}{g} \frac{dq}{dt} \end{aligned} \right\} \quad (1)$$

where r is the radius of gyration. The relation $q = \frac{d\theta}{dt}$, which results from the definitions, must be added to these equations. These four equations form the system (1). The inclination of the airplane and its motion in its plane of symmetry can be defined at any moment by these four variables.

If this motion is disturbed by a sudden increase of a variable, δu , for example, the other variables are like-

wise automatically subjected to modifications. During the period immediately following the initial disturbance, positive or negative increments δw , δq , $\delta \theta$ are incepted. These increments δu , δw , δq , $\delta \theta$ are all functions of the time. In the paragraph quoted above, Mr. Sée showed that disturbances δw , δq , $\delta \theta$ are automatically incepted when the initial disturbance is $\delta u < 0$. Their action, on a statically stable airplane, opposes the initial disturbance δu , whereupon they vanish, after thus fulfilling their purpose. Each of the possible initial disturbances can, of course, be studied individually, as was done by Mr. Sée, who studied the successive attitudes of the airplane for the purpose of determining whether, in all the possible cases, the increments δu , δw , δq , $\delta \theta$ tend toward zero with increasing time. In the affirmative, the motion is stable, since any disturbance finally vanishes. The airplane may be considered dynamically stable. This result is easily achieved by mathematical analysis. The method of small motions forms a criterion which permits determining whether the increments δu are eventually eradicated.

The airplane is assumed, in what follows, to be a rigid glider with locked controls. It is furthermore assumed that the engine controls are not touched by the pilot and that the revolution speed of the engine conforms instantaneously to the airplane speed V . This leads to the conclusions and calculations summarized below. The assumptions on which they are based have been discussed elsewhere.*

The second version of the system of equations reads as follows:

$$\left. \begin{aligned} \frac{du}{dt} &= -qw + \frac{g}{P} (T + P \sin \theta + F_x), \\ \frac{dw}{dt} &= +qu + \frac{g}{P} (F_z - P \cos \theta), \\ \frac{dq}{dt} &= \frac{g}{Pr^2} (M + T s), \\ \frac{d\theta}{dt} &= q. \end{aligned} \right\} \quad (2)$$

*Haus: Stabilité et Maniabilité des avions. Gauthier-Villars et Cie.

The second members of the equations are functions f_1, f_2, f_3, f_4 of the variables u, w, q, θ .

$$\left. \begin{aligned} \frac{du}{dt} &= f_1 (u, w, q, \theta), \\ \frac{dw}{dt} &= f_2 (u, w, q, \theta), \\ \frac{dq}{dt} &= f_3 (u, w, q, \theta), \\ \frac{d\theta}{dt} &= f_4 (u, w, q, \theta). \end{aligned} \right\} (3)$$

The motion is modified when infinitely small increments $\delta u, \delta w, \delta q, \delta \theta$ are added to the variables which, at the moment t_0 , have the values u_0, w_0, q_0, θ_0 satisfying the system (3). The resulting motion necessarily satisfies the general equations, so that we may write:

$$\left. \begin{aligned} \frac{d(u_0 + \delta u)}{dt} &= f_1 (u_0 + \delta u, w_0 + \delta w, q_0 + \delta q, \theta_0 + \delta \theta), \\ \frac{d(w_0 + \delta w)}{dt} &= f_2 (u_0 + \delta u, w_0 + \delta w, q_0 + \delta q, \theta_0 + \delta \theta), \\ \frac{d(q_0 + \delta q)}{dt} &= f_3 (u_0 + \delta u, w_0 + \delta w, q_0 + \delta q, \theta_0 + \delta \theta), \\ \frac{d(\theta_0 + \delta \theta)}{dt} &= f_4 (u_0 + \delta u, w_0 + \delta w, q_0 + \delta q, \theta_0 + \delta \theta). \end{aligned} \right\} (4)$$

Developing (4) by Taylor's equation and subtracting system (3) into which the values u_0, w_0, q_0, θ_0 have been introduced, we obtain by neglecting the terms of the second order, a differential system, the only variables of which are the disturbances

$$\begin{aligned}
 \frac{d(\delta u)}{dt} &= \frac{\partial f_1}{\partial u} \delta u + \frac{\partial f_1}{\partial w} \delta w + \frac{\partial f_1}{\partial q} \delta q + \frac{\partial f_1}{\partial \theta} \delta \theta, \\
 \frac{d(\delta w)}{dt} &= \frac{\partial f_2}{\partial u} \delta u + \frac{\partial f_2}{\partial w} \delta w + \frac{\partial f_2}{\partial q} \delta q + \frac{\partial f_2}{\partial \theta} \delta \theta, \\
 \frac{d(\delta q)}{dt} &= \frac{\partial f_3}{\partial u} \delta u + \frac{\partial f_3}{\partial w} \delta w + \frac{\partial f_3}{\partial q} \delta q + \frac{\partial f_3}{\partial \theta} \delta \theta, \\
 \frac{d(\delta \theta)}{dt} &= \frac{\partial f_4}{\partial u} \delta u + \frac{\partial f_4}{\partial w} \delta w + \frac{\partial f_4}{\partial q} \delta q + \frac{\partial f_4}{\partial \theta} \delta \theta.
 \end{aligned}
 \tag{5}$$

This system establishes a relation between the disturbances, their derivatives with respect to time, and the partial derivatives $\partial f_1/\partial u, \dots, \partial f_4/\partial \theta$, the value of which is determined by the aerodynamic characteristics of the airplane and by the conditions of flight. This linear system can be easily integrated. The general integral is of the form

$$\begin{aligned}
 \delta u &= C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + C_3 e^{\lambda_3 t} + C_4 e^{\lambda_4 t}, \\
 \delta w &= l_1 C_1 e^{\lambda_1 t} + l_2 C_2 e^{\lambda_2 t} + l_3 C_3 e^{\lambda_3 t} + l_4 C_4 e^{\lambda_4 t}, \\
 \delta q &= m_1 C_1 e^{\lambda_1 t} + m_2 C_2 e^{\lambda_2 t} + m_3 C_3 e^{\lambda_3 t} + m_4 C_4 e^{\lambda_4 t}, \\
 \delta \theta &= n_1 C_1 e^{\lambda_1 t} + n_2 C_2 e^{\lambda_2 t} + n_3 C_3 e^{\lambda_3 t} + n_4 C_4 e^{\lambda_4 t}.
 \end{aligned}
 \tag{6}$$

According to the theory of differential equations, the exponents λ are found after solving the characteristic equations of the system

$$\begin{vmatrix}
 \frac{\partial f_1}{\partial u} - \lambda & \frac{\partial f_1}{\partial w} & \frac{\partial f_1}{\partial q} & \frac{\partial f_1}{\partial \theta} \\
 \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial w} - \lambda & \frac{\partial f_2}{\partial q} & \frac{\partial f_2}{\partial \theta} \\
 \frac{\partial f_3}{\partial u} & \frac{\partial f_3}{\partial w} & \frac{\partial f_3}{\partial q} - \lambda & \frac{\partial f_3}{\partial \theta} \\
 \frac{\partial f_4}{\partial u} & \frac{\partial f_4}{\partial w} & \frac{\partial f_4}{\partial q} & \frac{\partial f_4}{\partial \theta} - \lambda
 \end{vmatrix} = 0
 \tag{7}$$

This equation may be written as follows:

$$\lambda^4 + A_1 \lambda^3 + A_2 \lambda^2 + A_3 \lambda + A_4 = 0
 \tag{8}$$

It can always be determined as a function of the variables u, w, q, θ , of the applied forces and moments, provided the derivatives are known.

VIII. RESULTS OF THIS INVESTIGATION

The general integral (6), which can be studied for various purposes, shows the variation of the disturbances $\delta u, \dots, \delta \theta$ as functions of the time.

1. We merely seek to determine whether the motion is stable or not. It is then unnecessary to solve equation (8).

The airplane motion is stable when the $\delta u, \dots, \delta \theta$ values, given by the general integral, tend toward zero with increasing t , irrespective of the value of the coefficients. To this end it is necessary and sufficient that the λ 's be negative when they are real, or that their real part be negative when they are imaginary. Routh showed that these conditions are fulfilled when the following five inequalities are satisfied, the equation being written in the form (8).

$$A_1 > 0, \quad A_2 > 0, \quad A_3 > 0, \quad A_4 > 0,$$

$$A_2 - \frac{A_3}{A_1} - \frac{A_1 A_4}{A_3} > 0.$$

These conditions are the criterion of dynamic stability.

2. We may seek to determine the general characteristics of the motion. The equation of the fourth degree must then be solved. Its four roots are imaginary in most cases,

$$\lambda_{1.2} = a \pm b i, \quad \lambda_{3.4} = a' \pm b' i$$

and define two oscillatory motions. The periods are given by $T = 2\pi/b$, and the damping is characterized by the factor a . The duration required, in order that the value of the amplitude of oscillation may reach half the initial amplitude, is

$$\tau_{\frac{1}{2}} = \frac{\ln 2}{a}, \quad (\ln \text{ being the Napierian logarithm}).$$

The four disturbances $\delta u, \delta w, \delta q, \delta \theta$ are represented by damped sinusoids.

3. Lastly, the integration constants C_1, \dots, C_4 , corresponding to any given initial disturbance, may be sought and the factors l_1, \dots, n_4 calculated. The Lagrange method for the integration of systems of linear equations is then used, and the amplitude of each of the motions and the phase difference of the sinusoids are determined in that case. We shall not go further into the details of these calculations, several results of which are given below.

IX. NATURE OF THE OSCILLATIONS

The two oscillations determined by the preceding analysis differ materially. One of these oscillations is found to correspond chiefly to the rotations of the airplane about its c.g. This oscillation has a relatively short period (2 to 8 seconds) and is very rapidly damped ($\tau_{\frac{1}{2}}$ of the order of 0.2 second). The second oscillation is much slower. It has a long period ($T > 20$ seconds) and much less damping ($\tau_{\frac{1}{2}} > 12$ seconds). It is chiefly caused by irregularities resulting from a disturbed equilibrium of the forces about the airplane. These two phenomena can be studied separately by methods of approximation. Munk* showed that the rapid oscillation corresponds to

$$\lambda^2 + B_1 \lambda + B_2 = 0,$$

where

$$B_1 = \frac{g}{V} \frac{l}{C_z} \left(\frac{dC_z}{di} + \frac{dC_z'}{di'} \frac{S' l'^2}{S r^2} \right)$$

and

$$B_2 = \frac{g^2}{V^2} \frac{l}{C_z^2} \frac{dC_z}{di} \frac{dC_z'}{di'} \frac{S' l'^2}{S r^2} + \frac{l}{C_z} \frac{gl}{r^2} \frac{dC_m}{di}.$$

Inasmuch as

$$\lambda = a \pm b i$$

*Munk, Max M.: The Tail Plane. T.R. No. 133, N.A.C.A., 1922.

we have $a = -\frac{B_1}{2}$ and $b = \sqrt{B_2 - \frac{B_1^2}{4}}$,

whence the main characteristics of these oscillations can be derived. The explanation is easy. When μ varies without causing a modification of

$$\frac{S' l'}{S l} \frac{dC_z'}{di'}$$

the damping is independent of μ , while the period is inversely proportional to the static stability. It is easily found that a statically stable airplane follows, so to speak, its angle of attack under the action of the rapid oscillation. The slow oscillation differs in this respect and can be studied by determining the characteristics of the trajectory of the c.g. It is then found that, as soon as the equilibrium of the applied forces is disturbed, the trajectory becomes a sinusoid. This trajectory rises and falls alternately, the speed along it being necessarily variable.

The airplane reaches its maximum speed when it passes through the low points of this trajectory and its minimum speed when passing through an apex. These oscillations cause rather strong variations of the orientation θ of the airplane in space, but do not affect the incidence to the same extent. An experimental study of slow oscillations was made long ago. The best determination was made by the National Advisory Committee for Aeronautics for a VE-7 airplane and resulted in the diagram shown in Figure 4. The analytical study shows that the roots a' and b' , characterizing the damping and the period, are complex functions of μ and increase the difficulty of the explanation.

Gates nevertheless demonstrated in a striking manner that these roots depend on the characteristics $\frac{\partial C_m}{\partial i}$ and $\frac{\partial C_m}{\partial \eta}$ defined above. Referring back to expressions (a) and (d) (pages 3 and 5), it is found that the values most easily subject to modifications are

Let $\left(0.25 - \frac{p}{t}\right)$ and $\frac{S' l'}{S l}$.

$$x = \left(0.25 - \frac{p}{t}\right) \quad \text{and} \quad y = \frac{S' l'}{S l}$$

we may then write

$$\frac{\partial C_m}{\partial i} = A x + B y, \quad \frac{\partial C_m}{\partial \eta} = C y$$

where A , B , and C are roughly constant. Gates* succeeded in plotting a diagram giving the values of a' and b' as functions of x and y . A diagram of this type is shown in Figure 5. No generalization is possible, since these curves are not independent of all the other variables of the problem. They were plotted for a given airplane with a specific radius of gyration. The curves of a' have a common tangent, the equation of which is $Ax + By = 0$ and which is the locus of the points of zero static stability. Parallels corresponding to a series of values of the coefficient μ may be added in the diagram. The points of the range corresponding to the usual x and y values are considered below. Let A be a point representing an airplane with a small degree of stability. If its stability is increased by a forward shifting of the c.g., the displacement must be parallel to the x axis. During this motion the curves with increasing b' are successively encountered. Hence, the period decreases with increasing stability. The a' curves first have decreasing values, but they increase after a certain shifting. Consequently, the damping is first reduced and then slightly increased by an increase of static stability. It would likewise be possible to anticipate the effect of an increase in stability resulting from an action on the tail surface, by traveling on a parallel to the y axis.

Gates' diagram permits foreseeing the modifications of the characteristics of the slow oscillation, without requiring much calculation. This method of representation may be used for the study of the equation of the rapid oscillation, but the explanation is too simple to necessitate the application of this method.

We cannot now go further into the study of the motion corresponding to a zero or negative coefficient of static stability. It may be mentioned, however, that when μ becomes negative, the long-period oscillation grows unstable before the short-period oscillation.

*Gates, S. B.: A Survey of Longitudinal Stability below the Stall, with an Abstract for Designer's Use. R. & M. No. 1118, British A.R.C., 1927.

X. APPLICATIONS OF THE METHOD

The above method is not confined to the study of the return to the initial position of equilibrium. It permits studying the motions by which an airplane tends toward a new position of equilibrium, either under the action of a control or under that of an atmospheric disturbance.

Four airplanes, with a wing loading of 40 kg/m² (8.2 lb./sq.ft.) flying at 40 m/s (131.2 ft./sec.) with a lift of $C_z = 0.40$ are considered below. They have, however, different coefficients of stability which are, respectively,

$$\mu = 0.002 \quad \mu = 0.004 \quad \mu = 0.006 \quad \mu = 0.008,$$

the angles being expressed in degrees. The periods and the $\tau_{\frac{1}{2}}$ of each oscillation were calculated for these four airplanes, and the following table was thus obtained.

	Fast oscillation		Slow oscillation	
	T (sec.)	$\tau_{\frac{1}{2}}$ (sec.)	T (sec.)	$\tau_{\frac{1}{2}}$ (sec.)
$\mu = 0.002$	6.70	0.186	44.5	13.6
$\mu = 0.004$	3.38	0.185	33.0	15.9
$\mu = 0.006$	2.51	0.185	29.5	16.8
$\mu = 0.008$	2.08	0.185	26.4	17.0

This calculation was made without taking the derivative $\partial C_m / \partial \gamma$ into account, thus assuming the latter to be nil. This series of airplanes was used for the determination of the amplitude of the oscillations produced by three different initial disturbances.

A. Airplane Flying in Still Air

The incidence of the airplane is assumed to be disturbed by a local vortex at one of its extremities or by a wrong maneuver of the pilot. The curves of Figure 6 are obtained by calculation. They define the motion which develops after an initial disturbance characterized by

$$\delta w = - 8 \text{ m/s}$$

$$\delta \theta = - 0.2 \text{ rad.},$$

i.e., after a disturbance of 11.4° of the incidence. These curves include the ordinates of both oscillations. δu and $\delta \theta$ are represented during the 45 seconds and δw during the 10 seconds following the beginning of the motion. Besides, the derivatives $d(\delta w)/dt$ and $d(\delta \theta)/dt$ are represented on a large scale during the first seconds. The derivative $d(\delta w)/dt$ represents the acceleration of the airplane in m/s^2 along its OZ axis; the derivative $d(\delta \theta)/dt$ represents the angular velocity, i.e., the disturbance δq .

The initial disturbance imparts a vertical acceleration to the airplane and exerts a nose-heavy moment. Both effects tend to reduce the incidence. The latter is rapidly restored by these phenomena to about its normal value, but slow long-period motions are set up which affect chiefly the speed u and the inclination θ .

B. Disturbances of the Surrounding Medium

Instead of being motionless, the surrounding air is now assumed to have a translational velocity U' , the components of which are u' and w' , U being the absolute speed of the airplane with the components u and w . The relative speed is then

$$V = \sqrt{(u - u')^2 + (w - w')^2}$$

and the incidence is

$$i = - \frac{w - w'}{u - u'}.$$

It is always possible to determine, for a known airplane type, the motion corresponding to the equilibrium of the forces in air with any absolute speed, since the relative speed and the incidence of flight are independent of the speed of displacement. Hence, the airplane motion resulting from a disturbance of the surrounding medium can be easily determined. Let R_1 be the flight condition of the airplane in still air when

$$u_1' = 0, \quad w_1' = 0.$$

This condition corresponds to the values u_1 and w_1 of the absolute airplane speed. In disturbed air A_2 , in which

$$u_2' \neq 0 \quad \text{and} \quad w_2' \neq 0,$$

the airplane, flying at the same incidence and relative speed, has a flight condition R_2 characterized by different absolute speeds u_2 and w_2 , but determined by

$$u_2 = u_1 + u_2' \quad \text{and} \quad w_2 = w_1 + w_2'.$$

The airplane loses its equilibrium when, flying under condition R_1 , it passes suddenly from atmosphere A_1 to atmosphere A_2 . It cannot maintain the condition R_1 and tends toward the condition R_2 . The motions of the airplane during this change of attitude can be calculated by taking R_2 as the condition of equilibrium from which the airplane is deflected by

$$\delta u = -u_2',$$

$$\delta w = -w_2'.$$

Hence, the airplane motions can be studied by the above method and result from a cumulation of the oscillatory motions of predetermined period and damping.

C. Rising Gust

We first propose to study the case of a rising gust such that

$$u_2' = 0,$$

$$w_2' > 0,$$

about an airplane in level flight.

The condition R_1 in atmosphere A_1 , assumed to be motionless, is characterized by a certain number of given values u_1 , w_1 , θ_1 . (Fig. 7.) The condition R_1 in atmosphere A_2 is transitory, not constituting a state of equilibrium. The angle of incidence is much too large (fig. 8) and the airplane trajectory will change.

The airplane tends toward a new condition R_2 where it is in equilibrium in atmosphere A_2 . It is clear that

this condition is characterized by the same values of the relative speed, incidence and inclination of the airplane to the horizontal as condition R_1 , but by a different absolute speed. The latter increases, since the speed of displacement of the surrounding medium is imparted to the airplane. (Fig. 9.)

We have calculated, for the four airplanes studied above, the course and amplitude of the disturbances δu , δw , δq , $\delta \theta$ which bring the airplane into the position R_2 after it enters a region of rising currents having a velocity of 8 m/s (26.2 ft./sec.):

$$\delta w = -w_2' = -8$$

The results of the calculation are shown in Figure 10. In this case also the airplane will be violently raised and will begin a nose dive. These two motions tend to close the angle between the directions OX and V of Figure 8, the two axes approaching each other.

Airplanes with a high degree of static stability oscillate faster, the vector OX covers a greater part of the distance and the incidence of flight is reached by a displacement $\delta \theta$, greater than if the airplane had only a low degree of stability. Static stability increases the disturbance of inclination $\delta \theta$ following a vertical gust. In its final state, the airplane must, however, recover its initial inclination θ . (Fig. 9.) A great static stability thus contributes toward temporarily deflecting the airplane from its final inclination. This explains why speed disturbances δu , following rising gusts, are more pronounced in airplanes with great static stability than in airplanes with a low degree of stability.

D. Horizontal Gust

The effect on an airplane in flight of a horizontal gust of 10 m/s (32.8 ft./sec.), opposed to the airplane motion, was calculated by the same method. The initial disturbance is an increment of δu . When the effect of this disturbance is neutralized, the airplane returns to level flight at its original incidence but at a different altitude. The phenomena take place in the following order. A vertical acceleration of approximately 5 m/s² (16.4 ft./sec.²), which decreases rapidly, is first imparted to the airplane. Then the airplane is stalled, thus starting the characteristic motions of the slow oscillation. (Fig. 11.)

These diagrams illustrate the complete effect of the initial disturbances on four airplanes differing in their coefficient of static stability. The diagrams can be combined when the initial disturbance does not correspond to one of the three cases considered. They are very useful for the study of stabilizing units or methods of stabilization.

XI. AUTOMATIC STABILITY

We now reach the point where the problem of automatic longitudinal stability may be clearly defined. All the flying qualities which a normal airplane is expected to have, are often incorporated in present-day airplane types by rule-of-thumb methods. This result is achieved by shifting the c.g. and using wings of carefully selected form and size. Fairly satisfactory airplane characteristics are developed by these means. This cannot be denied, since throughout the world thousands of airplanes fulfill the purpose for which they are designed. Even the danger of stalling is greatly reduced by good aerodynamic designs.

In good weather the longitudinal stability of most present-day airplane types permits flying with locked elevator controls. The study of dynamic stability shows that the motions initiated by an airplane under these conditions are finally damped. The pilot's efforts to expedite the damping of these motions may be useful, but not essential. If only stability in still air were considered, the problem would consist in damping the long-period oscillations as quickly as possible.

Conditions are different in moving air. Left to itself, with locked controls in bad weather, even the most carefully designed airplane behaves much worse than when it is flown by a pilot. Practically no airplane can fly satisfactorily in moving air with locked controls. It needs a pilot.

The reason is obvious. When the whole airplane is subjected to a disturbance of the surrounding air, there exists a position of equilibrium toward which it tends, while accomplishing a series of motions, the succession of which can be determined by the preceding analysis. If only a part of the airplane, e.g., a tip, is subjected to the action of an atmospheric disturbance, the momentums corresponding to angular velocities will have to be studied.

Besides, the airplane will start motions which, however, do not tend toward a final state of equilibrium until the whole airplane enters a region where every point of the air has the same absolute velocity. This problem involves difficult calculations.

It is certain, however, that, in moving air, the disturbances may recur at short intervals, long before the airplane has recovered its position of equilibrium. The airplane tends toward a continually varying equilibrium. It is badly tossed about and flies along an irregular trajectory. In that case the above diagrams may be of interest only during the first few seconds following a disturbance. It is obvious that the rapidity of the changes in the surrounding medium makes any regularity of the airplane trajectory impossible 15 or 30 seconds after the initial disturbance. The problem is no longer concerned with secondary stresses but with immediate effects.

Inasmuch as the strongest action of the air forces is exerted on airplanes with a high degree of stability, they are the most violently tossed about. The regularity of the trajectory of an airplane cannot be improved by increasing its static stability. It seems rather that this result may be achieved by reducing the stability. However, in consideration of safety, such practice cannot be followed. An airplane with released controls is generally less stable than one with locked controls. Below a certain degree of stability, airplanes may become unstable with released controls. Other difficulties may arise, such as the inversion of the controls. The conclusion is thus reached that it may be advisable to drop the usual methods of research and to dissociate phenomena which, under normal conditions, are related with one another. These results can be reached only by abandoning the practice of rigid and indeformable airplanes.

1. The assumed distortion of the airplane may be assumed to result in a deflection of the control surfaces. This actually occurs in practice. In moving air, the pilot neutralizes by this means certain, if not all, effects of external forces. The effect of changes in the elevator setting β on longitudinal stability is studied below, particularly as regards the possibility of building an automatic stabilizer designed to carry out these maneuvers and to produce the necessary deflections of the control surfaces.

2. It is easily seen that the rigidity of a wing cell may be more or less reduced by basic changes in its design. We are thus in the presence of two different solutions which have each been the object of numerous researches. It is proposed to study first the solutions of the first group in which the only considered deformation of the airplane is a change in the elevator setting β . We shall seek to determine the effect on the airplane motion of the various laws which may affect this setting. This problem justifies the study of the dynamic stability as outlined above.

The motions imparted to an airplane by an automatic stabilizer differ from those which it performs without the stabilizer. Logically, this study should begin with the natural reactions of the wing cell.

We have hitherto studied only statically stable wing cells. Some designers think that airplanes with satisfactory flying ability may be built by combining statically unstable wing cells with automatic stabilizers. This idea can be put into practice and several solutions are considered below.

Our program, however, is more modest. The aerodynamic study of stability shows that satisfactory positive coefficients of static stability can be developed by constructional methods. In this case the action of the stabilizer is confined to correcting irregularities of the trajectory. The airplane is allowed to display its good qualities which are improved by the stabilizer which combats, if necessary, certain detrimental effects of inherent stability.

Blind flying indicates that a solution of the problem is possible. In instrument flying, all the maneuvers are made by the pilot on the basis of the combined readings of a few instruments. Inasmuch as each combination of the readings of these instruments corresponds to a specific maneuver, it does not seem impossible to have part or all of the work done by a mechanical device.

However, a solution of this problem cannot be attempted, unless it be clearly expressed. A list of all maneuvers must be drawn up, giving the respective amplitudes of the control deflections. It must then be demonstrated that, under all possible conditions of flight, none of the proposed maneuvers can be wrong.

In a paper on "Automatic Stability," read at Ghent in 1913, by Mr. Sée, clearly indicated the necessity of making a program of the maneuvers before attempting a study or construction of the actual stabilizer. We shall work along the same lines. Our knowledge of the natural reactions of a rigid airplane forms the basis for the establishment of such a program. We shall then see how closely existing stabilizers agree with this program.

Notes: 1.- All the statements made in the first part of this paper are based on the assumption of instantaneous atmospheric disturbances. The accuracy of this assumption may be questioned, but it affords the advantage of conclusions which are unaffected by the law establishing the disturbances. It is obvious that if a disturbance develops according to a sinusoidal law of the same period as one of the airplane oscillations, the resulting resonance masks the essential facts.

2.- In order that our explanations may be more explicit, we have assumed very pronounced initial disturbances exceeding the narrow limits of application of the method of small motions. Since, however, all the equations are linear, it is only necessary to divide the initial disturbance and its effects by the same number, in order to render the method applicable.

PART II

AUTOMATIC LONGITUDINAL STABILITY

XII. PROGRAM

The natural forces about a statically stable airplane are:

a) When the incidence is increased by an accidental displacement of the airplane in still air, or when the airplane enters a region of up-currents, it always receives a strong acceleration along the OZ axis. It is violently lifted, but oscillates at the same time about its c.g. and dives. This dive is prolonged when the initial disturbance

is a displacement of incidence, but it is, on the contrary, very short and followed by a slight stall when the initial disturbance is a modification of the inherent speed of the surrounding medium.

b) Any reduction of the incidence produces reactions opposite in sign to the above.

c) When the airplane meets a frontal gust, corresponding to an increase of the relative speed, it is lifted and stalled so far as the effect of static stability is not neutralized by the derivative $\partial C_m / \partial \gamma$.

d) Forces opposed to those defined above are produced by a reduction in the relative speed. Besides, in each of these cases, the speed along the OX axis is subject to slow disturbances, the effect of which is felt for a very long time. These various air forces are shown in the above diagrams.

Which of these irregularities of the trajectory may be suppressed? It is easily found that the accelerations along OZ are due to the rigidity of the airplane. They are the direct consequence of any change of incidence and cannot be avoided by a deflection of the elevator alone. This result might be reached, if the whole wing were deformable. The pitching motions, on the contrary, depend on the elevator setting and may be controlled by the proposed stabilizing unit. The variations of the velocity along OX, practically coincident with the speed along the trajectory, are rather slow. They may likewise be influenced by elevator settings which are secondary effects of the pitching motions.

In spite of the detrimental effect of the accelerations called forth by the vertical inflections of the trajectory, the latter cannot be prevented in moving air, our action being thus confined to the pitching motions. What is the program for the use of the automatic stabilizer? Is it necessary to maintain the direction of the natural airplane rotations? The answers to these questions are given in three groups, each defining a program.

First Program

Many technicians, including Colonel Etévé, consider that a stabilizing unit must stimulate the natural rotations of an airplane. If we accept this proposition, there

immediately arise various possibilities of application.

a) Certain reactions may be accentuated and others reduced. The analytical study shows that an airplane with a small degree of static stability may not be entirely stable if $\partial C_m / \partial \gamma$ is large enough, the stability of speed being deficient under certain conditions. It may be advantageous to increase this stability without increasing the coefficient of static stability μ , by using a stabilizing unit which is free from the troubles inherent in an increase of μ .

b) It is always useful to increase the damping of slow oscillations which, normally, are only little damped. An attempt may even be made at producing aperiodically recurrent motions.

c) We may seek to reduce the sensitivity of the airplane to transitional disturbances.

Second Program

Other technicians do not agree with the above program. They suggest, on the contrary, a modification of the natural airplane rotations in one or the other direction. A. See and Boykow agree on the following point. The airplane should not rise vertically under the action of a frontal gust, the latter being often of short duration, so that the stall may leave the airplane in a difficult position when the gust ceases. According to them, the airplane should face a frontal gust without nosing up, but rather diving slightly.

Third Program

According to a third version, the airplane motions should be nearly completely neutralized and the stability of speed sacrificed in order to keep the direction of the airplane axes fixed in space. This result may be achieved by means of gyroscopic stabilizing units which suppress pitching, without, however, straightening the trajectory in the vertical plane. Hence, these devices correspond to a quite different program, in which a deliberate attempt is made to modify the basic characteristics of the airplane motion. This program may be justified on the assumption that the origin of the disturbances is more frequently an angular displacement of the airplane than a change in the components of the absolute motion of the air. Com-

promises between these various principles have been sought by the combined use of gyroscopes and anemometers for guarding airplanes equipped with a stabilizing unit of the third type, against excessive disturbances of speed.

XIII. ELEMENTS OF A STABILIZING UNIT

All longitudinal stabilizing units have one or more disturbance detectors which control the elevator deflections. The detectors are sensitive to one of the variables characterizing the motion, or to a combination of them.

The main detectors available for this purpose are listed in Table I. The direct-control or servo-motor stabilizing units now available are listed in Table II.

TABLE I. Disturbance Detectors

Instrument	Parameter to which the instrument is sensitive	Recorded variable
1. Anemometer	Relative speed	V
2. Wind vane	Incidence	$i = w/u$
3. Free gyroscope suspended at its c.g.	Inclination in space	θ
4. Motor-driven gyroscope with a precessional moment	Angular velocity	q
5. Pendulum or accelerometer along OX	Direction of apparent gravity	$\frac{du}{dt}$ and $\sin \theta$
6. Accelerometer along OZ	Magnitude of apparent gravity	$\frac{dw}{dt}$ and $\cos \theta$
7. Lift indicator	Magnitude of lift	$i V^2$ or uw
8. Variometer	Speed along the vertical	$\frac{da}{dt}$

TABLE II. Principal Stabilizer Types

Parameter to which the stabilizer is sensitive	Control			
	Direct	Servo-motor		
Simple	Speed	Etévé (1914)	S.T.Aé. Regnard (1910), Sperry	
	Incidence	{ Etévé (1910), Constantin }		
	Inclination	-		
	Angular velocity	{ Lucas- Girardville }		
	Direction of apparent gravity	Moreau (1912)		S.E.C.A.T.
	Magnitude of apparent gravity	-		-
	Speed + incidence	Etévé		-
Compound	Speed + direction of apparent gravity	-	{ Mazade, Askania et Dautre (1911)	
	Speed + magnitude of apparent gravity	-	{ Dautre (1913), Boykow	
	Speed + inclination	-	Marmonier	
	Speed + angular velocity	-	Boykow	

In view of the danger of a possible breakdown of the stabilizing unit, or its ill-timed action, a quick-release clutch, restoring the control of the airplane to the pilot, is necessary and has been incorporated in nearly all the devices built or proposed to this day.

No description is given of the electric or compressed-air servo-motors, whereas a short account is given of the devices combining, eventually, the readings of two or more detectors and insuring, under all conditions, settings of

the controls closely meeting the requirements.

The disturbance detector may be connected with an index whose deviation from a median position is proportional to the disturbance. (Fig. 12.) A deflection of the elevator by servo-motor, proportional to or simply a function of the travel of the index, is a problem solved long ago. The index, moving along two segments, closes an electric circuit, thus starting the servo-motor in one or the other direction, as soon as it leaves its zero position. Such a device operates the control in the proper direction, as soon as the disturbance is felt and the index comes in contact with the segment. The result, however, is purely qualitative, since the servo-motor runs at the same speed for all the positions of the index within the limits x_0 and x_m . The elevator deflection does not depend on the instantaneous position x of the index, but on the time during which the circuit is closed, namely, on the time elapsed since the first contact of the index with the segment at x_0 .

Such a device is useless without a complementary unit establishing a given relation between the deflection β and the position x of the index, or introducing a relation between these two parameters according to a given law. This unit forms the actual control device.

Let, for example, an upward shifting of the index through an angle α increase the setting of the control surface, on the understanding that a deflection β of the latter, corresponding to the angle α , be $\beta = k \alpha$. When the servo-motor which deflects the control surface downward, likewise shifts the segment upward through an angle α , it is obvious that when the control reaches the desired position β , the index will be opposite the dead point of the segment and the servo-motor will stop.

With decreasing disturbance, the index comes in contact with the lower segment and starts the motor in the opposite direction. The segment then moves downward. The instant the disturbance ends, the index having resumed its initial position, the segment must likewise return to its original position and the control surface to the neutral position. This result cannot be reached without a shifting of the segment which, however, need not necessarily be proportional to the deflection of the elevator. The law of displacement of the segment can be modified by suitable cams. This makes it possible to control the relation be-

tween the displacements of the index and the deflections of the control surface. The principle of this device is necessarily incorporated in all the stabilizing devices with servo-motors. A similar arrangement can be easily imagined with the servo-motor operated by compressed air.

There are simple stabilizing units by which the elevator is deflected as a function of a single variable, and compound stabilizing units depending on the readings of several detectors according to which they operate the servo-motors. These devices require a "combiner." Several examples of this type are given later.

Lastly, the stabilizing unit may also be used for putting the airplane through a complete maneuver resulting in a change of the flying attitude. The maneuvers are started and guided by the pilot, who, by means of the gas throttle, may use the stabilizing unit to obtain an entirely automatic control of all the phases of the airplane flight.

As regards lateral stability, which is not studied in this paper, the airplane should assume automatically the bank of a correct turn when the rudder is deflected by the pilot. The function of the stabilizing unit is thus extended and it becomes a sort of automatic pilot.

The principles outlined above permit a general study of stabilizing units to be made. The simplest devices and their anticipated effect on the trajectory are investigated first. Compound stabilizing units are studied next. The difficulty of the task is not so much due to the shortage of data on this subject as to the impossibility of procuring reliable information on the test results obtained with these devices.

XIV. LONGITUDINAL STABILIZATION AS A FUNCTION OF THE SPEED

Static stability always insures stability of speed, by the effect of the slow oscillation, when no disturbing action of the propeller is to be feared. The latter can, however, produce two important effects. Its slipstream exerts an aerodynamic action on the tail surfaces and its thrust produces a moment when its axis does not pass through the c.g. The first of these effects produces the following result. In most cases, the horizontal tail sur-

faces produce negative lift. For an equal incidence and translational speed, this action increases with the intensity of the propeller slipstream, which increases with respect to the translational speed when the latter decreases or when the revolution number increases. Hence, a reduction of the factor $\gamma = V/nD$ results in a stall and justifies the sign of $dC_m/d\gamma$. (Fig. 13.) A reduction of speed stalls the airplane and produces a detrimental action, whereas a reduction of the engine speed noses the airplane down and exerts a favorable action. The effect of decentering the thrust axis is favorable or detrimental according to the cause of the disturbance. A reduction of the airplane speed increases the thrust, whereas it is reduced by a dropping of the engine speed. The airplane must be nosed down in both cases. When the thrust axis lies below the c.g., the speed reduction produces an effect which is detrimental in the first case and favorable in the second. A contrary conclusion is reached when the thrust axis lies above the c.g. These remarks show that the stability of speed and, lastly, the dynamic stability of airplanes are materially affected by the propeller. The coefficient of static stability required by an airplane in order that its effect be prevalent and that dynamic stability be always insured, may be found by calculation.

Considering the complex character of the natural reactions on the airplane, which are sometimes modified by secondary effects, it is not surprising that many designers sought to insure constancy of speed by direct means, a mechanical device tending to depress the elevator and nose the airplane down as the speed decreases. The sensitive component of such a device is an anemometer. Its reading is proportional to the square of the speed. Colonel Etévé built and actually tested in 1914 an anemometric stabilizer (fig. 14), a brief description of which was given by the inventor in *L'Aéronautique*, No. 143, page 120.

The effect of such a device on the trajectory can be easily understood. According to the calculation, a stabilizing unit, sensitive to speed alone, has no effect on short-period oscillations, but exerts a considerable influence on slow oscillations. It reduces their period and damping. This fact is easily understood by referring to the diagram of the δu and $\delta \theta$ values.

The $\delta \theta$ curve is known to be offset 90° relative to the δu curve, (Fig. 15.) When the speed grows excess-

ive (from A to B), the airplane tends to stall. The stabilizing unit increases the stall. From B to C the speed is too small and the airplane noses down. The stabilizing unit increases the dive. Hence, when the speed stabilizer has no secondary effect of the propeller to neutralize, it amplifies the pitching motions and affects the oscillation as indicated above.

When the anemometric stabilizing device fulfills its purpose, it also amplifies the long-period oscillations. Theoretically, it is well suited for automatic piloting, but, although very attractive in principle, it is imperfect.* Many attempts have been made to use it in combination with a unit for correcting its defects. The proposed solutions will be examined farther on.

XV. LONGITUDINAL STABILIZATION AS A FUNCTION OF THE INCIDENCE

The vane is the element sensitive to incidence variations. The stabilization of an airplane, as a function of the incidence, may be achieved by the Etévé or Constantin methods, using vanes for the direct control of the elevator; or by a vane controlling a servo-motor similar to that built by the S.T.Aé. In both cases the deflections produced by the vane, when it is balanced about its axis, are functions of the airplane incidence alone. In the first case, the deflecting moment of the vane and the resisting moment of the control are both proportional to V^2 . The final position of the control is therefore independent of V and depends on the incidence only. In the second case, when the vane controls a servo-motor, the work it has to do is practically nil, and the vane always takes a position corresponding to the zero moment about its axis. This position depends on the incidence only. The Etévé and Constantin power vanes have already been described in

*The review of the French patent 696,338 (May 31, 1930), published in L'Aéronautique No. 155, page 113, is recalled in this connection. The patent was issued to the Société des Etablissements Liore et Olivier, for "improvements in flying machines." This patent describes a device for automatic longitudinal stabilization, in which a vane, carrying an anemometric plate near its hub, controls the variations in the stabilizer setting, so as to maintain a constant speed.

L'Aéronautique. Their principles are recalled in the legends of Figures 16 and 17.

The S.T.Aé. vane operates the elevator through the intermediary of a servo-motor. It consists of an ordinary airfoil instead of two hollow superposed blades. It applies the principle of the quadrilateral with two unequal sides, thus forming a stable wind vane, while the airfoil is still located forward of its hinge axes. One of these axes carries a brush sliding along a distributor and controlling the action of the servo-motor. The distributor is controlled by the deflections of the control surface. We cannot give a detailed description of the extremely ingenious electrical devices designed by Mr. Granat and built by the Saint-Chamond company.

A wind vane maintains the airplane at a given angle of incidence in flight. An additional unit, not described in Figures 16 and 17, is put at the pilot's disposal in case the airplane would have to be flown at other speeds and angles. It is not believed that stabilization by means of wind vanes gives essentially different results from those obtained by a simple increase of the static stability. The C_m of the airplane is a function of the setting β , which, again, is a function of the incidence, so that, as a first approximation, it looks as though the total coefficient of static stability were

$$\mu_t = \frac{\partial C_m}{\partial i} + \frac{\partial C_m}{\partial \beta} \frac{d\beta}{di}$$

where $d\beta/di$ depends on the law of operation of the stabilizing unit. If the latter had no inertia, the question might be studied theoretically by introducing the value μ_t , given above, into the calculation. It would then be found that the stabilizing unit acts chiefly on rapid oscillations. Its action on slow oscillations is limited and agree with Gates' diagram. We disagree with the statement that stabilization by means of the vane may suppress the slow oscillation and transform it into an aperiodic oscillation.

Theoretically speaking, the same results should be obtained by a unit stabilizing the incidence and by an increase of the static stability achieved by ordinary aerodynamic means. Good results have been obtained with incidence stabilizing devices, which were tested on airplanes with very little or no stability and which enabled the pi-

lot to fly with released controls. The following two questions are examined for the purpose of determining the relative merits of inherent stability and of the incidence stabilizing device.

1. What does a high coefficient of static stability call for? A forward shifting of the c.g. is the best means of increasing the stability of an airplane. When, however, the c.g. of an airplane with standard wings is located at 30 per cent of the chord, the c.p. lies aft of the c.g. at high speeds and requires the use of a tail unit which produces negative lift and detrimentally affects the performances. This point was particularly stressed by Constantin. The advantage of the vane is that it permits reaching a suitable μ_t factor without having to shift the c.g. forward or increase the area of the tail surface. The vane permits locating the c.g. at the rear, but insures a degree of stability obtainable only by a forward location of the c.g. This, however, does not apply to wings with a zero C_{m_0} which always permit a forward location of the c.g.

2. Is the action of the vane quicker or slower than that of the inherent stability? Considering that the vane may be located ahead of the airplane, one may be led to believe that its action is started as soon as it is reached by a disturbance, a fraction of a second earlier than the tail is reached by the gust. On the other hand, the stabilizing unit and the elevator are affected by inertia; hence their action is retarded. It is thought that, in general, the action of the stabilizer is slower than that of the inherent stability. It was seen, however, that rather violent forces are set up when a statically stable airplane is restored to a given incidence. Under these conditions the progressive action of a control deflection is to be preferred to the immediate effect of the inherent stability.

The principle of wind-vane stabilization has been confirmed by experience. It is therefore desirable to determine the stability characteristics by systematic tests, as functions of the $\partial C_m / \partial i$, $\partial C_m / \partial \beta$, and $d\beta / di$ values.

XVI. LONGITUDINAL STABILIZATION AS A FUNCTION OF THE INCLINATION .

Stabilization, as a function of the inclination of the airplane to the horizontal, forms the subject of the third program covering the cases in which the natural airplane motions are opposed instead of being developed. A gyroscope hung on gimbals in a fixed position relative to the horizontal and capable of changing its position with respect to the airplane without encountering appreciable resistance, produces a deflection of the control surface proportional to the inclination θ .

It is easily seen that, following angular displacements of the airplane, due to ill-timed maneuvers of the pilot or to the action of a gust on only part of the airplane (e.g., the tail unit), the gyroscopic stabilizer can start a maneuver contributing toward restoring the airplane to its original position. This solution was anticipated by Regnard, who, in 1910, sought to apply it to airplanes.

The action of the gyroscopic stabilizer is less satisfactory when the disturbance, instead of changing the inclination of the airplane, alters the translational speed of the entire surrounding medium. In this case the airplane has the same inclination θ both in its initial and final positions, but its absolute speeds u and w may differ. The successive phases of the airplane in reaching normally the speeds u and w , corresponding to the new conditions of flight, are accompanied by modifications of inclination $\delta\theta$ and changes of altitude. The gyroscopic stabilizer opposes the $\delta\theta$ variations and modifies the course of the return to equilibrium without, however, preventing it completely.

On the other hand, in case of engine trouble, the stabilizer prevents the dive required for the maintenance of the flying speed. This is a serious defect of the gyroscopic stabilizer which, however, can be easily remedied.

Sperry, the great expert on gyroscopes, built a stabilizer in 1914, which was sent in for the competition of the Union for Aerial Safety and gave encouraging results. This device incorporated an anemometric blade which offset the control and stabilized the airplane on a predetermined

downward trajectory when the speed dropped below a given minimum value. More recently, Sperry placed on the American market the device described in Figure 18. He seems to have abandoned the arrangement designed to bring the airplane down in case of engine trouble, probably relying on the progress achieved in engine construction.

The gyroscopic stabilizer functions normally as long as the axis of the gyroscope remains vertical. When the gyroscope is suspended at its c.g., it is subject to the action of no moment which is a function of its position. It is in neutral equilibrium. Hence, the gyroscope should be started only in the horizontal position. Besides, any cause of disorientation should be avoided. The moments of disorientation can never be completely avoided. The friction of the suspension pivots and of the index on the segment are sufficient causes for the disorientation of the gyroscope.

The fundamental features of gyroscopes are briefly recalled below. Let OZ be the axis of rotation of a gyroscope, I its inertia moment, and ω its angular velocity of rotation about this axis. (Fig. 19.) When a moment L is exerted on the gyroscope about the OX axis, the system revolves at an angular velocity q about the OY axis. The axis of the gyrostat tends to settle along OX . This phenomenon is the well-known precessional motion which tends to cause the gyroscope axis and the axis of the deflecting moment to coincide. The angular precessional velocity is given by

$$q = \frac{L}{I\omega}.$$

We now seek to determine the effect of the pitching motions on the gyroscope. Under the action of these motions, the gyroscope turns about the pivots parallel with the OX axis. This rotation cannot take place without friction, so that a moment L is automatically set up and the gyroscope axis travels unavoidably in the XOZ plane. The moments about the OY axis produce a precession which tends to displace the gyroscope axis in the YOZ plane. Hence, the gyroscope cannot maintain its initial position when it is subjected to the action of external moments. Its axis must be vertical for a good functioning of the stabilizing unit. A cumulation of the effects of the external moments must therefore be prevented and the gyroscope axis restored, if necessary, to its initial position.

To obtain this result, the gyroscope must always be subjected to the action of gravity.

Instead of suspending the gyroscope at its c.g., it may be hung from a point above its c.g. This forms a gyroscopic pendulum or weighted gyroscope, which is sensitive to the apparent gravity. Such an arrangement, however, increases the causes of disorientation, since any linear acceleration of the airplane changes the direction of the apparent gravity and introduces new disturbing moments. The gyroscopic pendulum tends to settle in the direction of the apparent gravity, but this tendency is opposed by the properties of the gyroscope. Hence, the device precesses about this direction and describes a cone growing gradually narrower.

For the purpose under consideration it is obviously improper to increase the sensitivity of the gyroscope to gravity by weighting it. This result must be attained in some other way. Sperry uses a sort of independent pendulum whose deflection from the gyroscope axis opens or closes nozzles yielding passage to an air flow. When the gyroscope axis coincides with the direction of the pendulum, the gyroscope is subjected to the action of four equal air jets. When the pendulum axis makes an angle with the gyroscope axis, i.e., when the apparent gravity is inclined forward (fig. 20), the output of one of the nozzles is reduced, thus subjecting the gyroscope to the action of a moment. This contrivance introduces very small moments which become effective only after long application. Hence, short-period oscillations of the pendulum, due to modifications in the direction of the apparent gravity, are without effect. The correction is effective only in the long run, namely, when the gyroscope is permanently deflected from the vertical, which is the mean direction of the positions of the pendulum. This is, in principle, the restoring device which tends to insure the constancy of orientation of the gyroscope axis.

Another gyroscopic stabilizer is the device known in England as the "robot," incorporated in the equipment of the "Fairey" long-distance airplane. The commercial production of this instrument is said to be in the hands of the Smith company, but no description has thus far been published.

XVII. LONGITUDINAL STABILIZATION AS A FUNCTION OF THE ANGULAR VELOCITY

An apparatus, sensitive to the angular velocity, can be easily conceived. The second fundamental property of gyroscopes is recalled in this connection. A gyroscope, with an inertia moment I , revolving about its OZ axis at the angular velocity ω and driven by forced rotation at a speed q about its OY axis, exerts a moment about OX which is proportional to $I\omega q$. This moment may serve to deflect the elevator in a predetermined direction.*

The idea of using the moment produced by the forced motions of the gyroscope for the operation of control surfaces seems to be due to Colonel Lucas Girardville, who, in 1910, built a direct-drive device, in which he used a gyroscope weighing 5.8 kg (12.8 lb.) and rotating at 6,000 revolutions per minute. Although mounted on an airplane, this device does not seem to have been actually tested in flight.

It is easily realized that the operation of the elevator, as a function of the angular pitching velocity, is not an actual method of stabilization. In practice, such an arrangement becomes effective only when the pitching motions due to atmospheric disturbances begin. Hence, it cannot be used for controlling the pitching motions proper, a task assumed by earlier stabilizer types. It can be used, however, for the damping of motions produced by other causes, provided the mechanism is arranged in such a manner that the air-force moment set up by a stalling motion depresses the elevator and vice versa. Hence, the stabilizing unit depending on the angular velocity has the same action as a device which increases the damping artificially. This feature is interesting, but not essential.

The increase of damping does not eliminate the fundamental question of the direction of rotation, but at most, completes it. Under these conditions the device sensitive to angular velocity must be considered as an accessory capable of being combined with another stabilizing unit controlling the rotations. The latter might possibly limit

*The gyroscope axis may also be oriented along the OX axis of the airplane. The forced rotation about the lateral OY axis then exerts precessional moments about OZ .

its effect by contributing toward rendering certain motions aperiodically recurrent, which, otherwise, would be oscillatory.

XVIII. LONGITUDINAL STABILIZATION AS A FUNCTION OF THE DIRECTION OF THE APPARENT GRAVITY

The apparent gravity is the resultant of gravity and the forces of inertia which act on the masses. Assuming a pendulum mounted on an airplane, let θ' (fig. 21) be the angle made by the pendulum axis with the extension of the OZ axis. The angle θ' is positive when the pendulum is forward of the axis. For a uniform airplane motion

$$\theta' = \theta.$$

By imparting an acceleration du/dt to the airplane, the pendulum is deflected backward through an angle equal to $\text{arc tan } \frac{1}{g} \frac{du}{dt}$. It is easily seen that, at any moment, the direction corresponding to the position of equilibrium of the pendulum is determined by

$$\theta' = \theta - \text{arc tan } \frac{1}{g} \frac{du}{dt}.$$

This is the direction of the apparent gravity. When the angles are expressed in degrees and the accelerations in m/s^2 , it becomes

$$\theta' = \theta - 5.84 \frac{du}{dt}.$$

A pendulum, installed on an airplane, does not always occupy its position of equilibrium, but may oscillate about it. If these oscillations are very pronounced, the pendulum cannot be used as the directing unit of an automatic stabilizer. If, on the contrary, the pendulum motion is rendered periodic by suitable damping, it can be used as a disturbance detector. The damping should not, however, exceed the strictly necessary limit, so as to avoid too much delay* in the pendulum reaching its position of equilibrium.

*The same problem arises in the construction of accelerometers. See Bouny, "Bulletin du Service Technique belge de l'Aéronautique," No. 7.

This consideration applies to all disturbance detectors. They can be satisfactorily used only when they do not oscillate about their position of equilibrium. No such difficulties are encountered with vanes and anemometers, owing to their high degree of natural damping. This restriction applies to the pendulum only. When the pendulum (or any other indicator of the direction of the apparent gravity) forms the sensitive unit of a stabilizer, the control deflection is a function of

$$\theta' = \theta - 5.84 \frac{du}{dt}.$$

The previously published diagrams permit an easy determination of the magnitude of this auxiliary variable for an indeformable airplane with given aerodynamic characteristics.

Considering the three typical disturbances specified above, we shall now seek to determine, for an airplane with a static stability of $\mu = 0.002$ flying with locked controls, the magnitude of the speed along OX and of its derivative; of the inclination θ of the airplane to the horizontal; of the readings θ' of an aperiodic pendulum carried in flight. (Fig. 22.) The diagram is confined to a study of the motion during the first 10 seconds.

First case.— The airplane is deflected through $\delta\theta$ in still air. The effect of the acceleration on the pendulum opposes the action of the airplane's inclination, but the latter nevertheless, prevails. The angle θ' has the same direction as the inclination θ but a smaller value.

Second case.— The airplane is subjected to a vertical gust. The effect of acceleration opposes the action of θ . During the first second or two of the motion the effect of the acceleration prevails and the angle θ' is opposite to θ . The effect of θ prevails after the rather rapid decrease in the acceleration. The angle θ' then has the same sign as θ , but is smaller.

Third case.— The airplane is subjected to a horizontal gust. The effect of the acceleration prevails and neutralizes the effect of the inclination. The pendulum is assumed to take up its position of equilibrium θ' immediately and to control a servo-motor which stalls the airplane when θ' increases, and conversely. The direc-

tion in which a stabilizer mounted on the airplane acts, may be determined and the modification which it introduces in the forces about the airplane may be anticipated by means of the above diagrams.

First case.- The stabilizer noses the airplane down for an initial disturbance of $\delta\theta < 0$. Its action is correct, but not so efficient as that of the stabilizer whose action is a function of the true inclination θ to the horizontal.

Second case.- At the beginning of the motion, the stabilizer tends to nose the airplane over. It then produces a slight stall which corresponds to the natural motions of a statically stable airplane.

Third case.- The stabilizer stalls the airplane and contributes toward the development of the natural reaction. The stabilizer, function of the direction of the apparent gravity, works correctly in the case of external disturbances. It contributes toward producing the undulations of the trajectory which lead to stability of speed. On the other hand, its action is detrimental in case of engine trouble, since it tends to stall the airplane. Curves θ' of the figure correspond to a given airplane with a given coefficient $\mu = 0.002$. According to these curves, the conclusions are not materially affected by greatly differing values of μ .

A parasol monoplane with a direct-drive pendulum stabilizer, known as the "aerostable," was built by Moreau in 1912. The pilot's seat, suspended below the wing, oscillated back and forth. The pendulum was thus formed by the pilot whose displacements controlled the elevator. This airplane is not believed to have given good results. It obviously lacked one of the fundamental characteristics of any disturbance indicator, namely, aperiodic recurrence. The mass of the pendulum was much too large for the damping forces involved. The defective functioning of the Moreau device and the excessive oscillations of its pendulum are chiefly attributable to deficient damping.

It is believed that, more recently, a stabilizer with servo mechanism, using an aerodynamically balanced pendulum device, was built and successfully tested by the S.E.C.A.T. (Société d'Etude et de Construction d'Appareils de Télémechanique). Unfortunately, no information is available on this device.

The hesitation to use the pendulum as a disturbance indicator, owing to its sensitivity to accelerations, is certainly exaggerated and it is believed that a carefully designed pendulum would give interesting results.

XIX. STABILIZATION AS A FUNCTION OF THE LIFT

Two methods may be adopted for the construction of a stabilizer as a function of the lift:

a) By measuring the magnitude of the lift by means of an auxiliary surface. This magnitude is a function of i and V^2 . So long as the critical incidence is not exceeded, the lift is a function of the product $i V^2$ or $-uw$.

b) By measuring the magnitude of the apparent gravity along the OZ axis. According to the equation of equilibrium of the forces along the OZ axis, the component F_z balances the apparent gravity. The latter is obtained by measuring $\frac{dw}{dt} + g \cos\theta$ by means of an accelerometer.

No stabilizing unit based on the exclusive use of the sustentometer or accelerometer, was ever proposed.* On the other hand, several designers thought of using these devices in combination with other contrivances. A stabilizer of this type is described later. It may be finally stated that no designer ever used a statescope alone.

XX. COMPOUND STABILIZERS - VANE ANEMOMETERS

Many designers, considering that none of the simple stabilizers can be perfect, built compound devices, combining the features of two or more disturbance recorders.

The six indicators used, namely, the anemometer, wind vane, free gyroscope, power-driven gyroscope, pendulum,

*The British Aeronautical Research Committee have just published the description of a control-stick acceleration alarm tested at the Royal Aircraft Establishment. When the acceleration exceeds a given value (2.5 or 3 g) a hinge is released near the handle and prevents the control stick from being pulled farther back. This device is not an actual stabilizer.

and vertical accelerometer or sustentometer, permit a great number of combinations to be made. Fortunately, designers of stabilizing units have made a selection of the possible combinations, thus reducing the scope of our study.

The speed indicator is used in all cases, since one of the main tasks of designers is the prevention of the stall and of ensuing accidents. This simplifies our work, since it permits confining our investigation to the combinations of the anemometer with one or more of the other indicators. The first solution, given by Colonel Etévé, is the vane anemometer, a combination of an anemometer with a wind vane.

This stabilizing unit incorporates the characteristics of the first two types studied. It acts on fast oscillations by the stabilization of the incidence, which it causes, and on slow oscillations through the intermediary of a component sensitive to speed. Its action may be anticipated from Figures 6, 10, and 11.

The principle and the calculus of vane anemometers formed the subject of investigations published by Colonel Etévé in L'Aéronautique. The action of the anemometric vane is intensified by a neat combination. This device uses the surface a (fig. 23), balanced about the c axis, as a servo-motor intensifying the action of the wind on the surface d . Theory, checked by wind-tunnel tests, shows that the useful moment about b , due to a variation du of the speed, is proportional to $K_1 u^3 du$, or to the cube of the speed. On the other hand, the useful moment produced by a variation of the incidence, is proportional to the square of the speed and is expressed by $K_2 u^2 di$.

XXI. AIRPLANE FLYING WITH RELEASED CONTROLS

Airplane flight with released controls may be reasonably introduced in the study of the automatic stability. The elevator setting varies during such flights. We shall seek to determine the law of these variations. J is the hinge moment, assumed to be positive when it tends to raise the elevator. This moment may be expressed by

$$J = C_j S_m' l_m' \frac{\rho v^2}{2g}$$

where C_j is a characteristic coefficient, S_m' the area of the elevator, l_m' the elevator chord, k a factor expressing the difference between the air speed about the tail unit and the translational speed of the airplane.

Experiments show that C_j is expressed by

$$C_j = k' i' + k'' \beta$$

C is the moment exerted by the weight of the elevator, which it tends to depress. For a tail surface but little inclined to the horizontal and for small deflections, we have $C = P_m d$. (Fig. 24.)

If no action is exerted by the controls on the control surfaces, the elevator settles in a position of equilibrium about its hinge, for which $J = C$.

First case.— When the control surface is balanced about its hinge, d is nil, we always have $J = 0$ and

$$C_j = k' i' + k'' \beta = 0$$

whence

$$\beta = - i' \frac{k'}{k''}$$

The total coefficient of static stability μ_t , making allowance for variations in the setting, is

$$\mu_t = \frac{\partial C_m}{\partial i} + \frac{\partial C_m}{\partial \beta} \frac{d\beta}{di}$$

But

$$\frac{d\beta}{di} = \frac{d\beta}{di'} \frac{di'}{di} = - \frac{k'}{k''} \frac{di'}{di}$$

whence

$$\mu_t = \mu - v \frac{di'}{di} \frac{k'}{k''}$$

For example, for

$$v = 0.008, \quad \frac{di'}{di} = 0.5, \quad k' = 0.007 \quad \text{and} \quad k'' = 0.01,$$

the coefficient of static stability (expressed in degrees) is reduced by 0.0028.

Second case.— The elevator is not balanced about its hinge, $C \neq 0$. We always have

$$(k' i' + k'' \beta) S_m' l_m' \frac{a v^2}{2g} k = C,$$

$$\beta = - \frac{k'}{k''} i' + \frac{C}{k'' S_m' l_m' \frac{a v^2}{2g} k}.$$

β is no longer a function of i only, but also of V . Static stability is reduced by the same amount as before, but a stabilization which is a function of the speed takes place, since β increases with decreasing V .

The elevator is deflected by its own weight with changing airplane speed. This effect is similar to the action of a stabilizer sensitive to speed. Hence, the act of releasing an elevator weighted aft of its hinge axis may be identified with that of increasing inherent airplane stability by:

1. A stabilizing unit, function of the incidence, having a detrimental action;
2. A stabilizing unit, function of the speed, having a favorable action.

The first unit exerts its action on the fast, and the second on the slow oscillation. When, with locked controls, the airplane has enough static stability to support a reduction of the stability of incidence without great inconvenience, the effect of the speed stabilizer on the slow oscillation may convey the impression of an increase of the total airplane stability.

This was shown theoretically by Schenk and checked experimentally by Hübner, who measured the slow oscillation of a Junkers airplane with locked and released controls. (Fig. 25.)

The characteristics of the oscillations are

<u>Control</u>	<u>Locked</u>	<u>Released</u>
T	42 sec.	32 sec.
$\tau \frac{1}{2}$	28.5	26.6

The reduction of the period, with released control shows that, in this case, the airplane is more vigorously restored to its normal position.

XXII. PENDULUM ANEMOMETERS

The combination of anemometer and pendulum was made by at least three designers. All three used devices with indirect drive and servo-motor, but their methods of construction differed materially. When a pendulum is used as a disturbance detector, it usually controls the servo-motor in such a manner that an increase of θ' results in a stall. Our study of the simple pendulum stabilizer is based on this assumption. The action of this device is favorable when the effect of external disturbances is concerned, but it starts a contradictory maneuver when the airplane speed is reduced by engine trouble. A speed stabilizer is used to obviate this defect. The correction is made by establishing a suitable relation between the position of the pendulum and the speed, thus forming a pendulum anemometer.

A particularly good solution was proposed and successfully tested by Mazade and Aveline in 1922. Its principle is recalled in the legend of Figure 26. A device based on similar principles has been patented by the Askania company (fig. 27).

The Doutré stabilizer, built and tested in 1911, likewise combined the anemometer and the pendulum, but for a different purpose. It was designed to use the horizontal inertia forces to bring the airplane down in case of engine trouble, the relation between θ' and the setting being thus reversed. An increase of θ' resulted in a dive, contrary to the result obtained with the stabilizing units described above. The device (fig. 28) consisted of an anemometric plate combined with two equal masses m and m' sliding along shafts parallel with the longitudinal airplane axis. These masses, subject to the action of the inertia forces $\left(m \frac{du}{dt}\right)$ and of the component $mg \sin \theta$ of gravity, were maintained by springs. An increase of θ' , shifting the masses forward, and reductions in the relative speed, pushing the plate forward, deflected the elevator through the intermediary of a compressed air servo-motor, thus nosing the airplane over.

A reduction of θ' , due, for example, to a positive acceleration of the airplane, stalled the latter. An individual increase of the relative speed, however, produced no effect, since the rearward motions of the plate were checked by a stop. Under these conditions, a frontal gust increasing the relative speed and reducing the absolute speed, noses the airplane down, thus driving it through the disturbance at a small incidence.

In this respect the device met the requirements of the Second Program. In the case of engine breakdown, the action of the two indicators cumulates in nosing the airplane over.

The descriptions published at that time show that the designer sought to use the effect of the inertia forces on the masses, but endeavored to eliminate the action of the longitudinal component of gravity. This action would have been detrimental under given circumstances, inasmuch as it introduced a stabilization of negative inclination. It is easily seen that the effects of inertia and gravity cannot be separated. Hence, the Doutré stabilizer sometimes caused ill-timed maneuvers. In spite of this defect, these devices have a historical interest, since they prove that the true function of stabilizing units had already been clearly visualized at that time.

XXIII. LONGITUDINAL STABILIZATION AS A FUNCTION OF THE

SPEED AND LIFT

In order to protect stabilizing units against the influence of the inclination θ , Mr. Doutré built in 1913, a modified device comprising an anemometric vane and an accelerometer, sensitive to accelerations along OZ. Such a device is practically insensitive to the inclination θ , the action of which is exerted only by its cosine which approximates unity and varies but little. This device raised little comment, and no description of it is available.

Another device, the Boykow anemo-accelerometer (fig. 29) combines the disturbances δu and $d(\delta w/dt)$. A segment S revolves about an axis under the action of the dynamic pressure on a plate P and of the inertia forces exerted on a mass M. Springs r and r' tend continu-

ally to restore P to its initial position. The segment has a series of contact blocks. An index i , likewise hinged about the axis, makes contact with the blocks. It is under the action of a mass M' , a spring R , and a damper A .

The segment is shifted in the direction of the arrow by an acceleration increasing the apparent weight of the mass M . If this acceleration is temporary, the interval is too short on account of the damper, for the force of inertia to exert its action on the mass M' , and the index i remains stationary. This results in a displacement of the segment relative to the index. If the acceleration is of long duration (e.g., during a turn), M' and i have the same travel and there is no longer a relative displacement between the index and the segment.

An increase of the absolute speed causes, on the contrary, the segment to revolve in an opposite direction to the arrow. This system permits combining the action of the comparatively slow disturbances δu with the rapid disturbances $d(\delta w)/dt$.

An increase of incidence corresponds always to an increase in lift and to a positive acceleration $d(\delta w)/dt$. The displacement of the segment in the direction of the arrow depresses the elevator. The action of the blade is then exerted in the desired direction, since any increase of dynamic pressure tends to raise the elevator.

A frontal gust corresponding to a sudden increase of the relative speed, produces two antagonistic phenomena, namely, a withdrawal of the blade and, on the other hand, a dropping of the mass under the action of the airplane acceleration.

By stressing the second effect, the Second Program, according to which an airplane is nosed down by a line squall, is again approximated. However, attention is called to the difference between the durations of the two disturbances. The variation of the acceleration forms, to a great extent, part of the fast oscillation; it has a much shorter duration than the disturbance δu . The balancing or the predominance of the acceleration can be achieved only at the beginning of the phenomenon. After one or two seconds the anemometer exerts its action all by itself.

XXIV. GYROSCOPE AND ANEMOMETER

It is recalled that the first Sperry stabilizing unit had an anemometric blade which nosed the airplane over as soon as the relative speed dropped below a certain limit.

It is possible to obtain a continuous cumulation of the gyroscope and anemometer readings. This solution is proposed by Mr. Marmonier. (Fig. 30.) The gyroscopic unit of Mr. Marmonier is very complex. (Fig. 31.) It is weighted at the bottom and has four gyroscopes. Longitudinal stability is controlled only by the two gyroscopes shown in Figure 30. They can both move through a certain angle about vertical axes. However, these displacements are limited by springs not shown in the drawing. It is rather difficult to work out the theory of this unit, since the gyroscopes are affected by displacements of the airplane in azimuth. It is believed, however, that as long as these displacements are small, the plane of the external ring of the gyroscopic system, integral with the index, remains horizontal. A description of the lateral stability component of the Marmonier device is beyond the scope of this paper.

XXV. LONGITUDINAL STABILITY AS A FUNCTION OF THE RELATIVE AND ANGULAR VELOCITIES

The advantage of increasing the damping of airplane oscillations was realized by Boykow who designed a stabilizer for this purpose, which is illustrated in Figure 32.

By means of a component sensitive to speed, the Boykow stabilizer produces a pitching moment of suitable direction, but brakes this motion as soon as it is incepted, in order to prevent the position of equilibrium from being passed. Yet a difficulty is brought into evidence by the figures illustrating the course of the disturbances of the airplane motion. The angular velocities corresponding to the slow oscillations are small as compared with the angular velocities of the fast oscillation at the beginning of a disturbance. The former are of the order of 1 degree per second, whereas the rapid oscillations involve ten times faster pitching motions. The con-

trol of slow oscillations requires very sensitive devices. This sensitivity may be excessive for fast oscillations, which have a sufficient degree of inherent damping and do not require much additional damping.

The above description applies to the device as it was seen in 1928 at the Berlin Salon. It may be further improved by connecting with the dynamometer circuits any desired type of disturbance detector cumulating its action with that of the anemometer and gyroscopes. A weighted gyroscope, not shown in the figure, was proposed for this purpose. The anemo-accelerometer described above might likewise be substituted to the simple anemometer. The principle is subject to many variations, since the fundamental idea of this device is to increase the damping by using the gyroscopes as tachometers recording very small angular velocities.

XXVI. POSSIBILITY OF MATHEMATICAL STUDY

The great variety of systems proposed for the longitudinal stabilization of airplanes appears from the preceding study, which illustrates the effect of these devices on the airplane trajectory. Useful indications are afforded by calculation, which permits making a theoretical study of the effect exerted on the trajectory by stabilizing units which are functions of the variables u , w , q , and θ defining the motion.

As previously stated, the method of small motions requires the derivatives of forces and moments to be expressed as functions of these four variables. The derivatives of the moment for a standard airplane are

$$\begin{aligned} \frac{dM}{du} = S l \frac{aV^2}{2g} \frac{dC_m}{du} \\ + S l C_m \frac{a}{2g} \frac{dV^2}{du} . \end{aligned}$$

V can be identified with u , and we can write

$$\frac{dC_m}{du} = \frac{\partial C_m}{\partial i} \frac{di}{du} + \frac{\partial C_m}{\partial \gamma} \frac{d\gamma}{du} .$$

Now
$$\frac{di}{du} = \frac{w}{u^2} \quad \text{and} \quad \frac{d\gamma}{du} = \frac{1}{nD},$$

on the assumption that the number of revolutions n of the engine is independent of u , or roughly, $1:2 nD$, making allowance for the variation of n corresponding to current values of the propeller characteristics. Hence we have, for the second assumption,

$$\frac{dC_m}{du} = \frac{\partial C_m}{\partial i} \frac{w}{u^2} + \frac{\partial C_m}{\partial \gamma} \frac{1}{2nD},$$

and, finally,

$$\frac{dM}{du} = s l \frac{aV}{2g} \left(-i \frac{\partial C_m}{\partial i} + \frac{\gamma}{2} \frac{\partial C_m}{\partial \gamma} + 2 C_m \right).$$

Likewise,

$$\frac{dM}{dw} = s l \frac{aV^2}{2g} \frac{\partial C_m}{\partial w} = -s l \frac{aV}{2g} \frac{\partial C_m}{\partial i}.$$

We know already that

$$\frac{dM}{dq} = s l l' \frac{aV}{2g} k \frac{\partial C_m}{\partial \eta}.$$

Lastly $dM/d\theta$ is nil, since the forces and aerodynamic moments about an airplane are functions of its motion relative to the air, but independent of its absolute position in space.

The four derivatives of the moment M may thus be determined as functions of the characteristics $\partial C_m/\partial i$, $\partial C_m/\partial \gamma$, $\partial C_m/\partial \eta$, which were defined at the beginning of this report. It is recalled that the term with $dC_m/\partial \gamma$ had been neglected in the calculated example. When an airplane is equipped with a stabilizing unit whose action is instantaneous and which deflects the elevator in accordance with the function of one of the variables u , w , q , θ , the expression of the derivatives need only to be completed by the terms,

$$\frac{dM}{d\beta} \frac{d\beta}{du} = s l \frac{av^2}{2g} \frac{\partial C_m}{\partial \beta} \frac{d\beta}{du}$$

$$\frac{dM}{d\beta} \frac{d\beta}{dw} = s l \frac{av^2}{2g} \frac{\partial C_m}{\partial \beta} \frac{d\beta}{dw} = - s l \frac{av}{2g} \frac{\partial C_m}{\partial \beta} \frac{d\beta}{di}$$

$$\frac{dM}{d\beta} \frac{d\beta}{dq} = s l \frac{av^2}{2g} \frac{\partial C_m}{\partial \beta} \frac{d\beta}{dq}$$

$$\frac{dM}{d\beta} \frac{d\beta}{d\theta} = - s l \frac{av^2}{2g} \frac{\partial C_m}{\partial \beta} \frac{d\beta}{d\theta}$$

The derivatives $\frac{d\beta}{du}$, $\frac{d\beta}{di}$, $\frac{d\beta}{dq}$, $\frac{d\beta}{d\theta}$ incorporated in the latter conform to the law of operation of the stabilizing unit. This method of investigation was used by Garner and Gates in working out the problem of automatic stabilization.

The calculation was not extended beyond the determination of the periods and of the damping. It stressed the advantages afforded by stabilizing units working as functions of the angular velocity. It is to be regretted that this calculation was not extended to the determination of the amplitudes, which requires much more time. Interesting results may be obtained by a further development of this calculus, on the understanding, however, that it is valid only for an instantaneous action of the stabilizing unit. The method of small motions is based on the assumption that the air forces are functions of the variables u , w , q , θ and independent of their derivatives. It is unsuitable for anticipating the effect of stabilizing units working as functions of an acceleration.

None of the stabilizing units known at present fully solves the problem proposed at the beginning of this study. The problem is too complex, inasmuch as the action of the stabilizing unit often depends on the characteristics of the airplane on which it is mounted. We have outlined the principles of operation of longitudinal stabilizing units showing that, regardless of the adopted law of elevator deflection, only a part of the problem can be solved, when the airplane is controlled by the elevator alone. It is impossible to stabilize simultaneously all the variables characterizing the airplane motion. Each stabilizing unit controls one or two of these variables leaving the others free.

The regularity of the airplane trajectory may be increased if, in addition to the elevator, other portions of the plane, e.g., the wings, are deflected. This leads to the problem of the flexible or hinged wing, to which much attention was devoted by Breguet, de Monge, and Leyat, who built airplanes to suit this program. A study of these airplanes is beyond the scope of this paper, but attention is invited to the new solutions made possible by these instruments, the theoretical study of which grows constantly more complex on account of the new variables involved. (Figs. A and B.)

THE D.V.L. ZEISS PHOTOCHRONOGRAPH

This device, which is chiefly used for the study of landing and take-off trajectories, was designed by the D.V.L. (Deutsche Versuchsanstalt für Luftfahrt) and built by Zeiss.

Photochronographic device.— The cylindrical light-metal case has a "Tele-Tessar" lens of 40 cm (15.75 in.) focal length with an aperture of 1:6.3. This lens is vertically offset, a space of only 1 cm (0.4 in.) height being left for the foreground below the horizon, while the field in which the recorded maneuvers take place has a height of 8 cm (3.15 in.). The shutter is of the four-plate type for 1/100 second exposures. The chronograph, for readings to 1/50 second, is shown in the upper opening of the case. A second lens reproduces its image, as well as that of a pad placed beside it, at the bottom of each record.

Film magazine.— A film strip of 12.5 x 0.08 m (41 ft. x 3.15 in.) for 80 negatives of 7 x 12 cm (2.75 x 4.72 in.) suffices for four take-offs and landings.

Method of application.— For a take-off the photochronograph is placed 50 m (164 ft.) behind the airplane ready to take off against the wind. In landing, the airplane flies over the instrument. Hence the plane of symmetry of the airplane must be perpendicular to the plane of the film. For a known span, the trajectories can be easily interpreted by successive measurements of the prints with

a comparator. The accuracy, of the order of ± 4.5 cm (1.77 in.) for pictures taken at horizontal distances of over 100 m (328 ft.) and of 30 cm (11.8 in.) at 450 m (1476 ft.), is quite remarkable.

Left: Complete device.- The black bag at the rear contains the supposed film, any length of which can be cut off for development.

Right: Elements of the photochronograph.- From left to right.- Rear view of case, film magazine, and rear cover, winding device, and film bag. The locking arrangement of the bag is impervious to light. Total weight with tripod: 14 kg (31 lb.).

LEGENDS

FIGURE 14.- Diagram of the Etévé anemometric stabilizing unit (1914). This device consists of an anemometric surface n , mounted on an upright pivoting about the axis o , and of a combination of two equal masses q and r , mounted on another upright pivoted about the axis p . These two masses, equidistant from the axis p , form an inertia balance opposing rotational motions. A spring s , connects the anemometer with the inertia balance, and an adjustable spring t , balances the action of the wind on n . Shaft p , of the balance, can be thrown into gear with the shaft of the airplane control stick.

FIGURE 16.- Etévé power vane.- The Etévé stabilizer, placed behind the wing, consists of a vane a , hinged about an axis b . The displacements of the vane control the setting of the elevator e , hinged about f , through the intermediary of the rod cd . The elevator is automatically depressed with increasing angle of incidence (case of the figure). A conveniently chosen lever arm deflects the elevator 4° when the vane is deflected 1° . The vane can be operated by means of a suitable control, which permits the position of equilibrium to be changed. The elevator can thus be controlled by the vane. A Wright biplane was stabilized by this device in 1910.

FIGURE 17 - Constantin power vane.- The Constantin vane has a comparatively small area and consists of a combination of two superposed hollow vanes. It is hinged to a deformable quadrilateral ABCD, and balanced by counterweights. A maximum vane moment is reached by: a) using

vanes with sections having the largest possible dC_z/di value; b) producing very favorable angular displacements, or much work for small displacements. The first condition is fulfilled by Mr. Constantin by using two thin vanes in biplane arrangement. These vanes are bent to arcs of a circle, the insides of the cambers facing each other. It is proved that the dC_z/di of such a combination is very large. The second condition is fulfilled by the inventor by using a quadrilateral ABCD with slightly different sides AD and BC, AB, and CD being equal. When BC, which is a little larger than AD, is located at the rear, the vanes are pivoted by any upward displacement and their incidence is thus slightly reduced. Hence the vane is stable as if its pivot axis were located in front. The relation is such that, for a variation of 1° in the angle of incidence of the vane, the lever arms AB and CD are deflected through approximately 20° . This amplification corresponds to a fictitious lengthening of the lever arm from 1 to 20 and permits a considerable stress to be exerted by the vane. This arrangement permits the direct control of the elevator by means of a vane with a very small area.

FIGURE 18 - Sperry device for automatic longitudinal stabilization by means of a gyroscope.- The latest Sperry device operates the three controls and consists of two electrically driven gyroscopes revolving at 15,000 r.p.m. The gyroscope with vertical axis controls the longitudinal stability. It is hung on gimbals and oscillates about axes parallel with OX and OY. Its displacements about OY are accompanied by an index i moving along the distributor segment S and making electrical contacts in one or the other direction, according to the inclination θ of the airplane with respect to the gyroscope, which is motionless in space. Through these contacts the elevator is operated by the following method, common to all three controls. An impeller M, of constant speed, drives a main shaft A and three sets of bevel gears, such as the couple p_1-p_2 mounted idle on this shaft. Contact between the upper bevel wheel p_1 or the lower wheel p_2 and the main shaft A is established, when necessary, by sliding stops operated by the fork F. This is achieved by pivoting the fork lever under the action of an electromagnet E, controlled by the contacts of the index with the segment. According to the inclination of the gyroscope, the upper or lower bevel wheel follows the motion of the shaft. The third bevel gear p_3 , always meshed with the

other two wheels, revolves in one or the other direction and operates the control surface. Simultaneously, through the intermediary of the worm gear V, the segment is restored to its neutral position, thus insuring full control over the device. Automatic piloting is achieved by an auxiliary device not shown in the figure, which changes the position of the segment with respect to the suspension rings of the gyroscope.

FIGURE 23 - Etevé vane anemometer.- a, guiding surface, supported by an arm bc, hinged about the b axis, a being rotatable about c. When a is locked to its axis, it acts like a simple wind vane, conforming to the direction of the wind, provided, however, that the c.g. of the entire unit is on the b axis. In order to facilitate this balancing action, Colonel Etevé placed the anemometer in front. It consists of a drag plate d, mounted on a rod de, hinged about the f axis, this axis being itself supported by the rod bc. The bar gh, is integral with a. The rod eg, connects the anemometer with the wind vane. kl is the elevator control and i the adjustment spring operated by the pilot through a cable j. The guide pulley can be placed at a fixed point such as m, the figure bchm then forming a deformable trapezium which permits increasing the efficiency of the wind vane.

FIGURE 26 - Mazade and Aveline device.- The pendulum consists of a mass of mercury contained in a U-shaped tube located in the plane of symmetry. The top of the rear branch of the U tube is open. The forward branch is connected with a Venturi tube. The mercury in this branch is subjected to the action of a negative pressure increasing as the square of the speed and its level is simultaneously a function of the apparent vertical and of the relative speed. An electrode enters each tube. Its length is adjusted so as to be slightly above the surface of the mercury in normal flight. When the mercury shifts place, one or the other electrode comes in contact with it and closes a circuit which, by a relay, controls a servo-motor. The Mazade stabilizing unit has a recall device A. It may be assumed that an increase of the relative speed or a displacement of the apparent gravity in the direction of the increasing θ' values makes the mercury rise in the forward tube. The servo-motor raises the elevator as long as the circuit remains closed. A rotation of the tube about a transverse axis exerts the necessary action to

avoid excessive elevator settings. This rotation raises the front tube and its electrode breaks the circuit. This system may be identified with the segment used in the case when a sliding index is controlled by the disturbance indicator. The oscillations of the mercury are damped by stops placed in the tube and permitting the oscillations to be transformed into aperiodically recurrent motions. The relation for the cumulation of disturbances $\delta\theta'$ and δV^2 is given by the pitch of the branches of the U and by the cross section of the Venturi. A device conforming to a given law can thus be constructed.

FIGURE 27 - Askania device.-- This figure was published in our Patent Review, No. 143, page 138. A Venturi produces a negative pressure in a chamber C, closed by a flexible diaphragm 3. This diaphragm is connected with a pendulum 5, and the displacements of the whole system are transmitted to an oscillating nozzle 6, traversed by a jet of compressed air. The direction of this jet controls the elevator deflections. The recall motion is produced by displacements of receiving pipes opposite the oscillating nozzle. In the Askania stabilizer the speed and θ' increments act in the same direction. The system is completed by pressure boxes 1, operating throttle valve 2, and thus eventually stopping the air supply. The Venturi produces a suction in the cavity according to the density of the air. The air intake is controlled by the pressure boxes according to the pressure of the surrounding air. This device has a great variety of adjustments. Thus, the effect, on the diaphragm, of variations in the density of the air, may be balanced. On the other hand, these effects can also be amplified, thus eventually rendering the stabilizer sensitive to variations in altitude (application of the eighth disturbance indicator).

FIGURE 30 - Diagram of the Marmonier device for automatic longitudinal stability by gyroscopes and anemometer.-- A system of two gyroscopes G_1 and G_2 , mounted on gimbals, is assumed to be always in a horizontal plane. The external ring a, of this unit, controls the displacements of an index i on a segment s through the intermediary of two differentials P_1 and P_2 . When the airplane stalls, the gyroscope moves the index in the direction of the arrow. The servo-motor S is thus started and the elevator depressed. Simultaneously, the recall device shifts s.

The variations of the relative speed exert an action

on the horizontal pinion of the differential P_1 . A reduction of the relative speed results in a forward shifting of the index. The amplitude of this displacement depends on the characteristics of the opposing springs of the anemometric blade. Beyond a certain limit, this blade is insensitive to speed increments, its backward motions being checked by a stop. The control stick operates the horizontal pinion of the second differential P_2 . A forward motion of the stick shifts the index in the same direction. The displacements of the gyroscopic unit are transmitted to the segment only when the pinions of the differential revolve about their axis without the latter traveling about its hinge point on the main shaft. The blade and stick controls must therefore be irreversible or at least have a much greater resistance than that due to displacements of the index.

FIGURE 32 - Boykow automatic stabilizing unit, detecting relative and angular velocities.- Two gyroscopes G_1 and G_2 , revolving in opposite directions, precess about vertical axes A_1 and A_2 , interconnected by master and articulated connecting rods B , but restored to their initial position by springs r . An angular pitching velocity corresponding to the stalling motion, turns the axes of precession in the direction of the arrows 1 when the gyroscopes revolve in the direction 2. One of the axes of precession controls the displacements of an index i . The other is, on the contrary, subjected to the action of a moment which is function of the air speed. This moment is interpreted by an electro-dynamometer E , controlled by a speed indicator I . With decreasing speed, the dynamometer produces a moment about its axis in the direction of the arrow 1. Hence the effect of the electro-dynamometer and the precessional motion of the gyroscopes cumulate on the index. A displacement of the latter in the direction of the arrow produces a positive elevator deflection β . The index I slides over an electrical resistance ab . The wiring system, which forms a Wheatstone bridge, shows how a moment depending on the position of the index is exerted on the dynamometer axis. The index modifies the relation between the resistances a and b , sending a variable current through the diagonal consisting of coil c . The resistances d and e are variable, and f represents the winding of the exciting electromagnets E .

Translation by W. L. Koporindé,
National Advisory Committee
for Aeronautics.

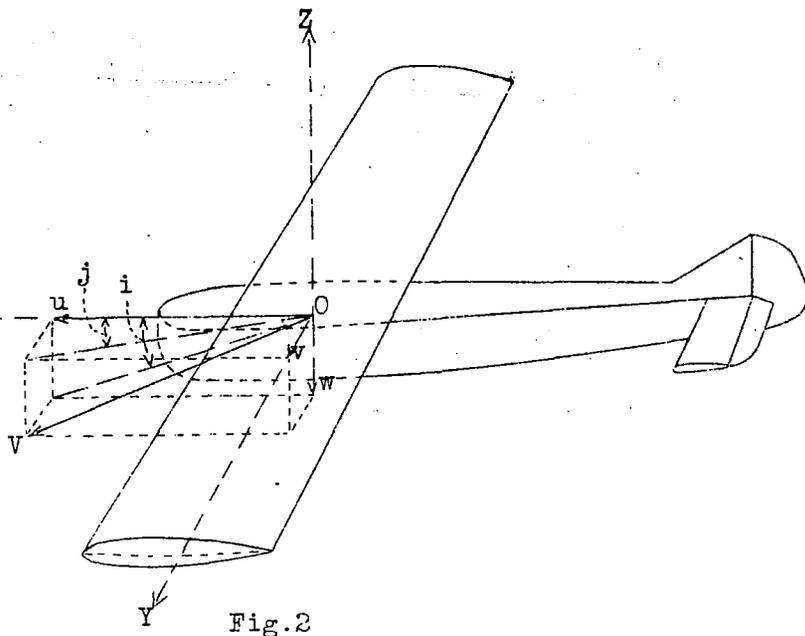
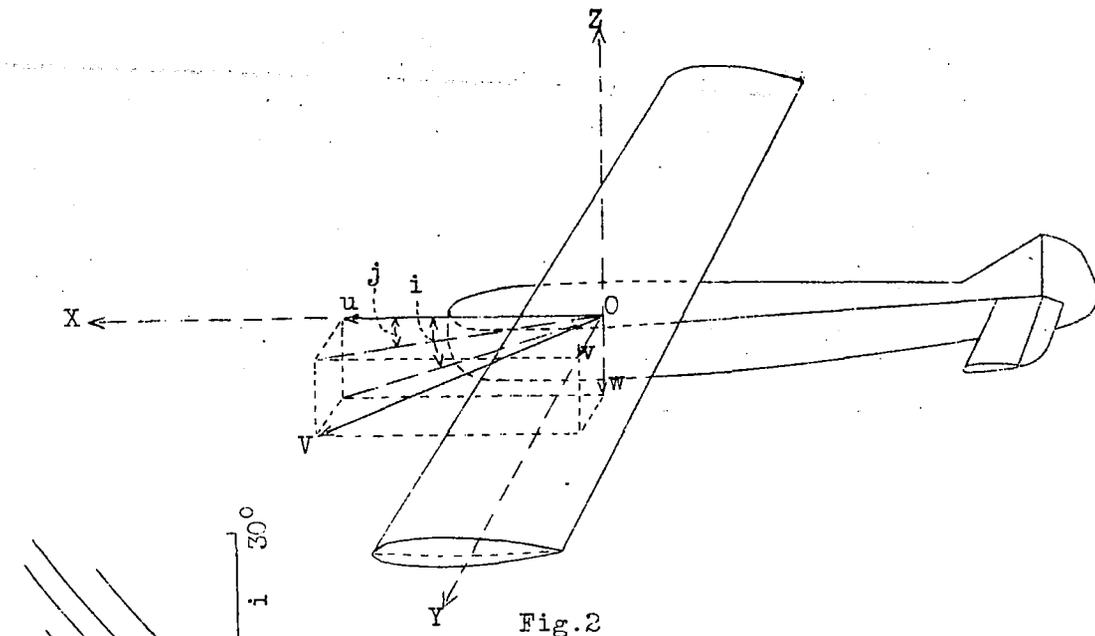


Fig. 2

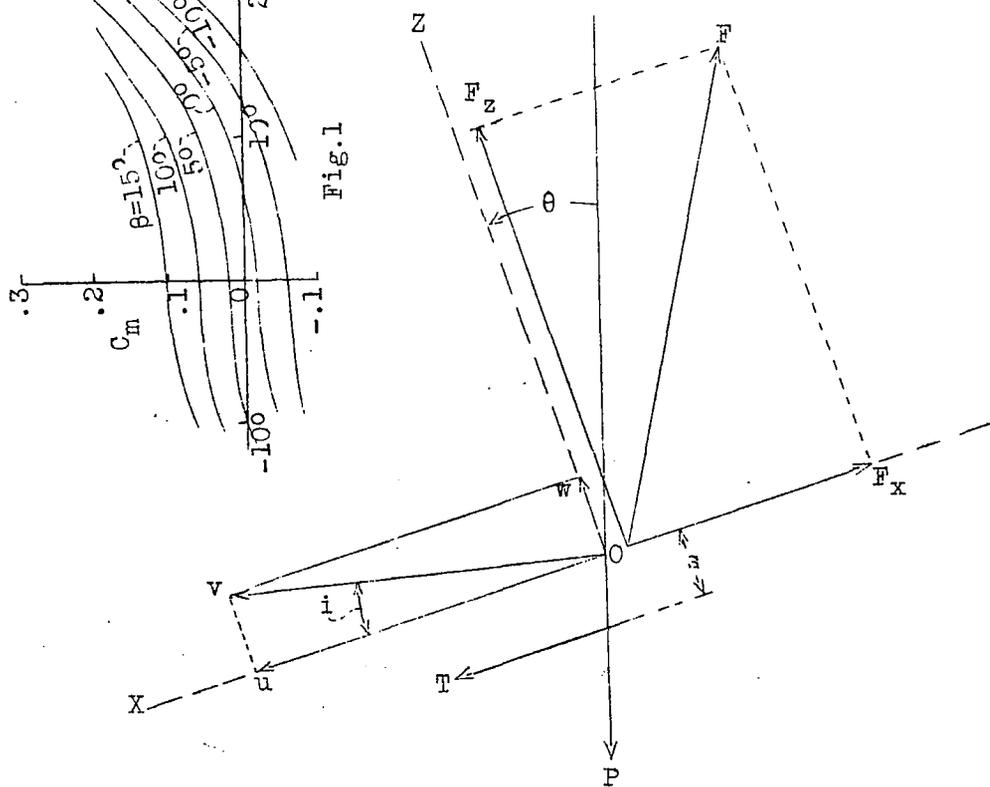


Fig. 3

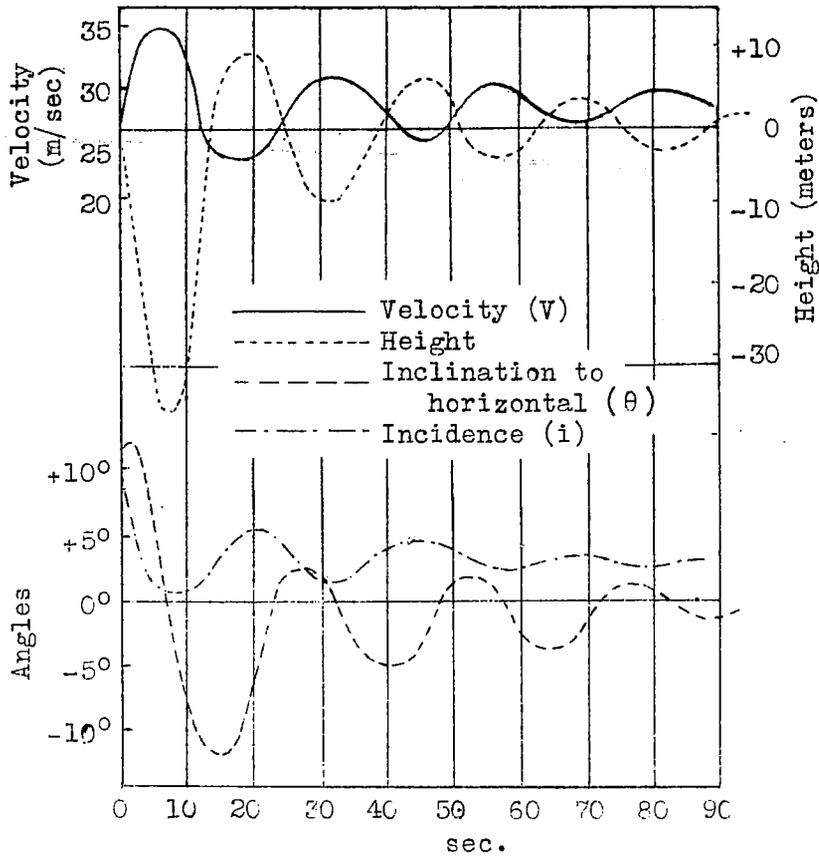


Fig. 4 Experimentally recorded oscillations

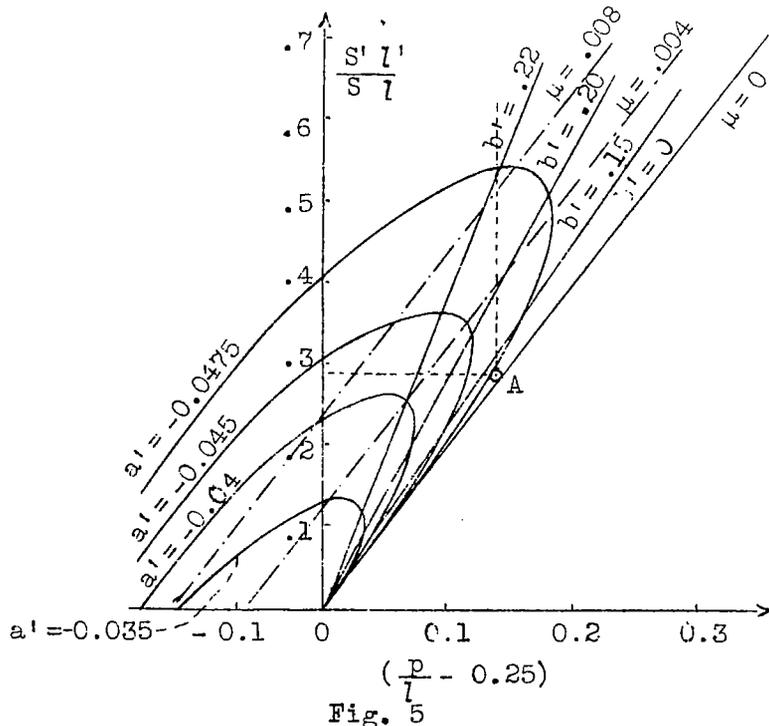


Fig. 5

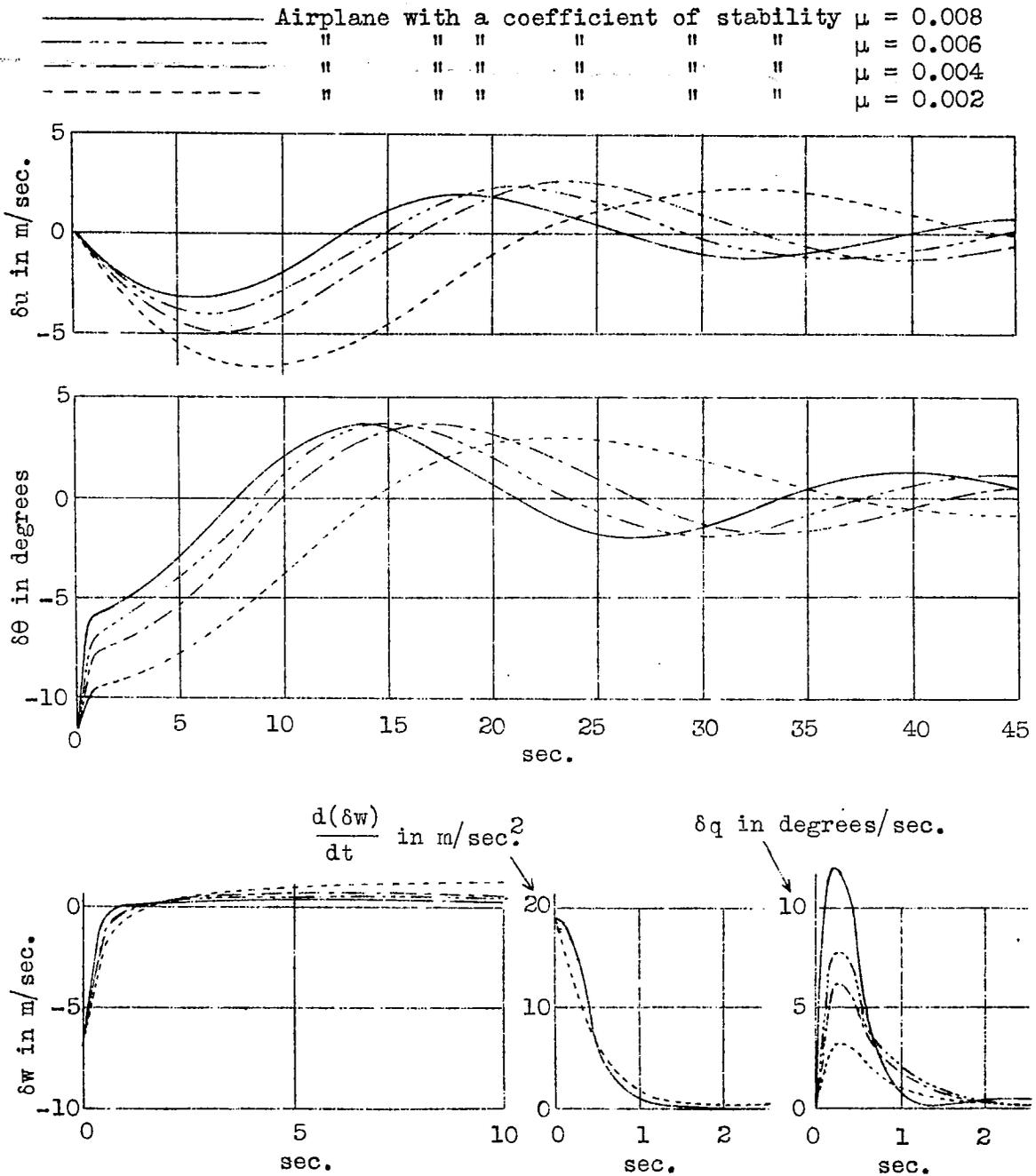


Fig.6 Oscillations established after an initial disturbance characterized by $\delta_w = -8$ m/s and $\delta\theta = -0.2$ radian.

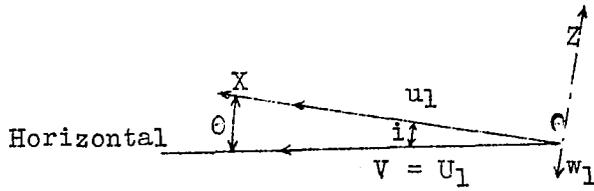


Fig.7

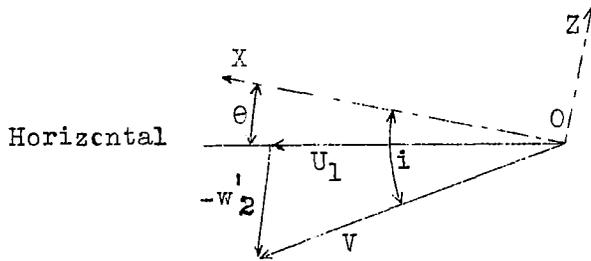


Fig.8

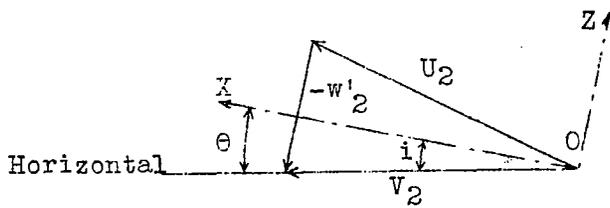


Fig.9

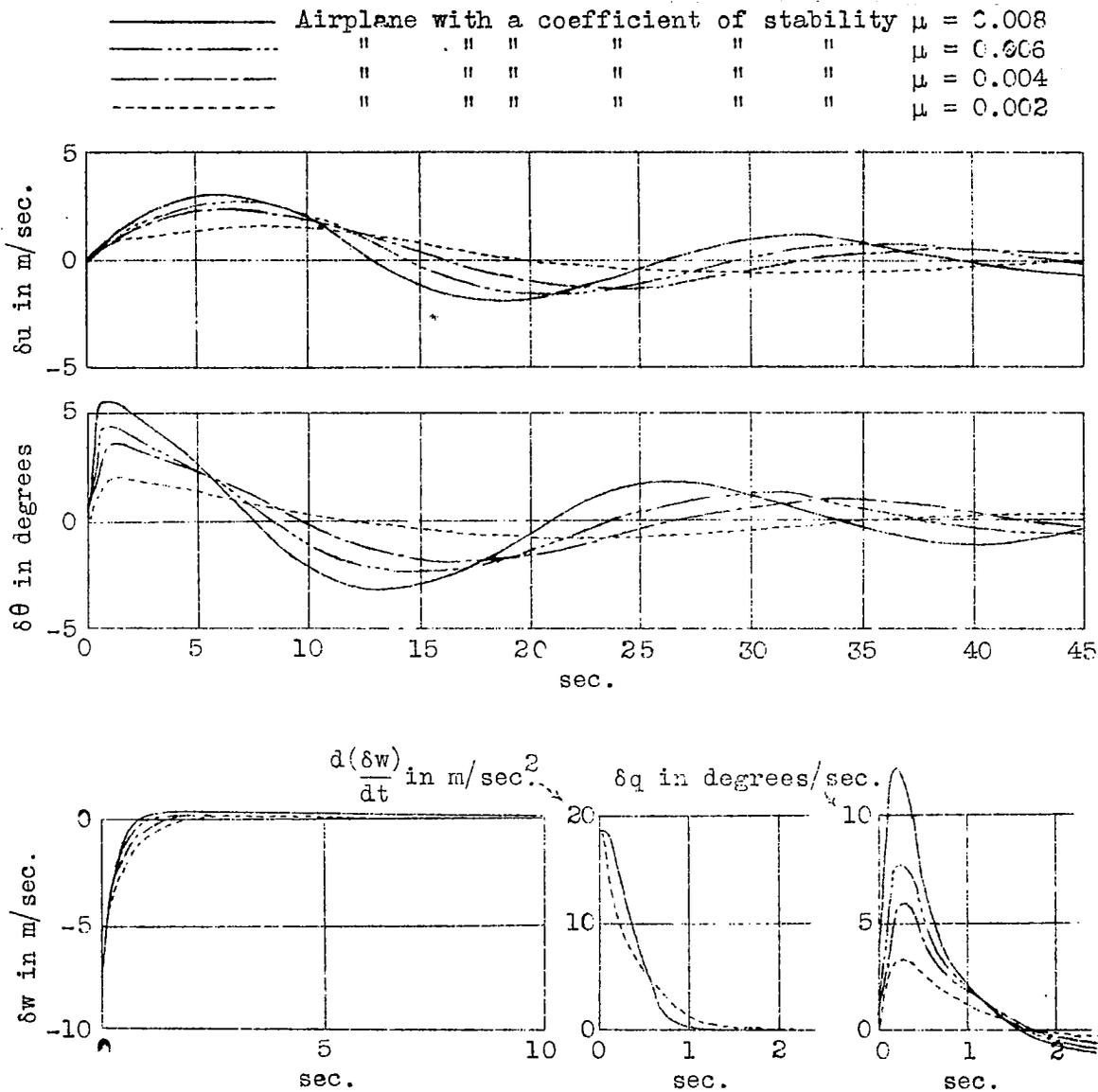


Fig. 10 Oscillations established after an initial disturbance characterized by $\delta w = -8$ m/sec. (ascending current).

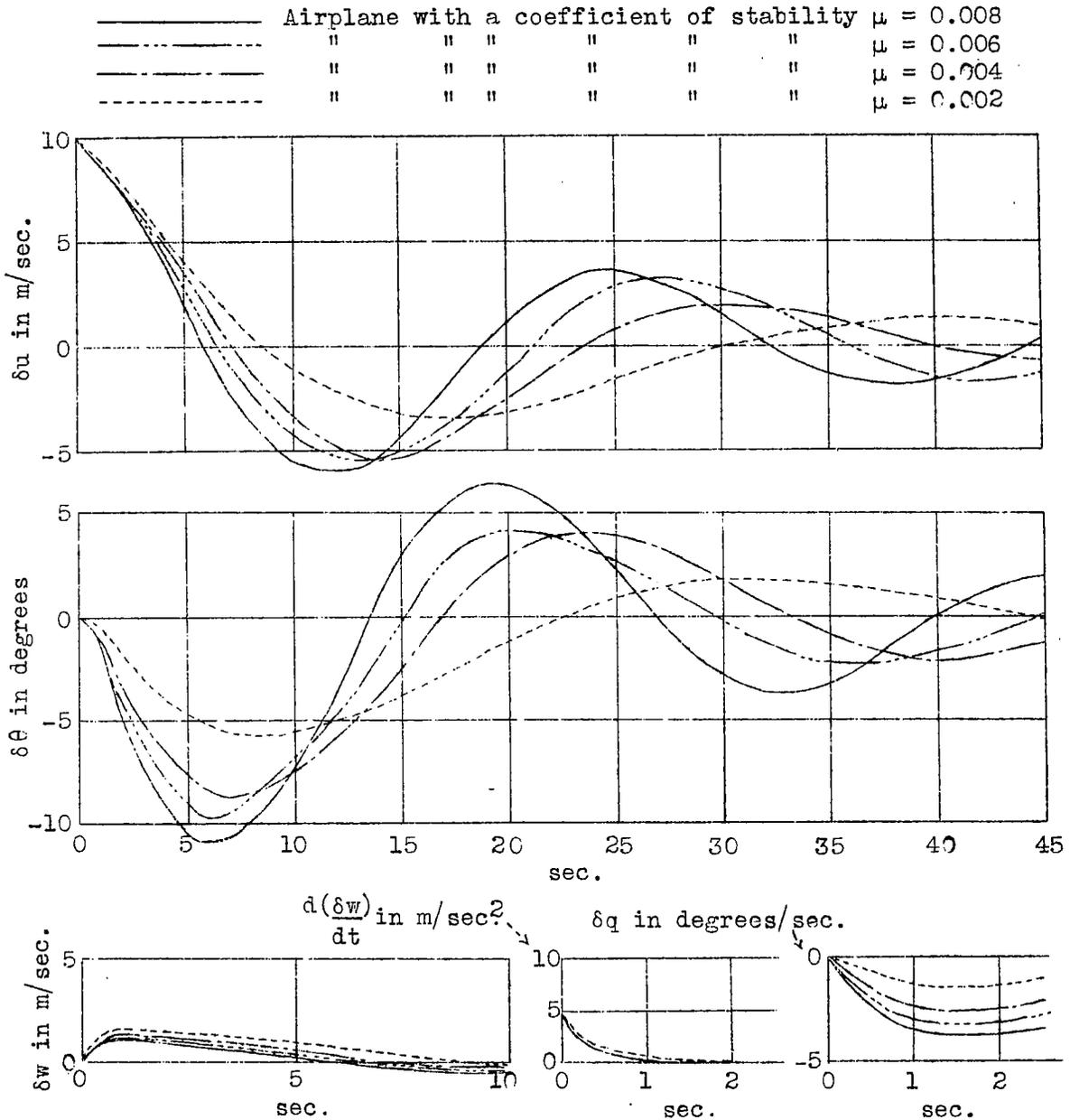


Fig. 11 Oscillations established after an initial disturbance characterized by $\delta u = 10$ m/sec. (horizontal gust opposed to motion of airplane.)

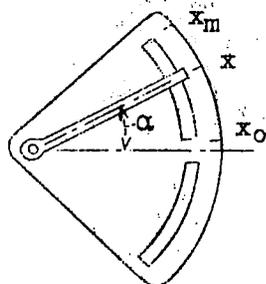


Fig. 12

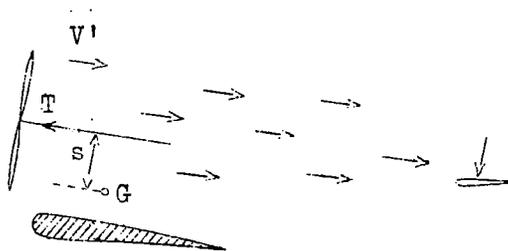


Fig. 13

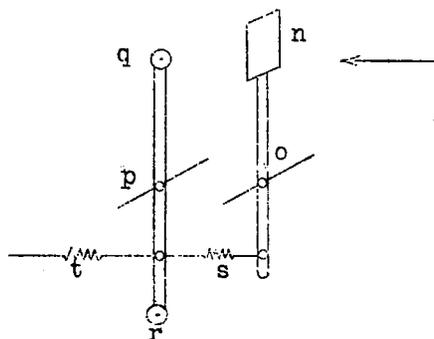


Fig. 14 Diagram of the Étévé anemometric stabilizing unit (1914).

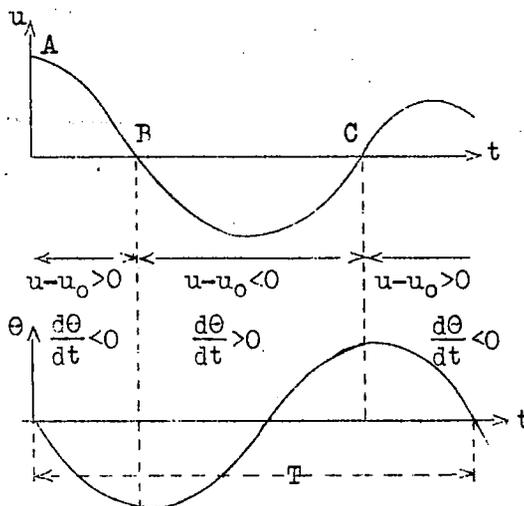


Fig. 15

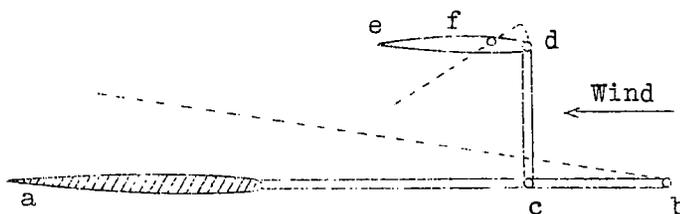


Fig. 16 Étévé power vane.

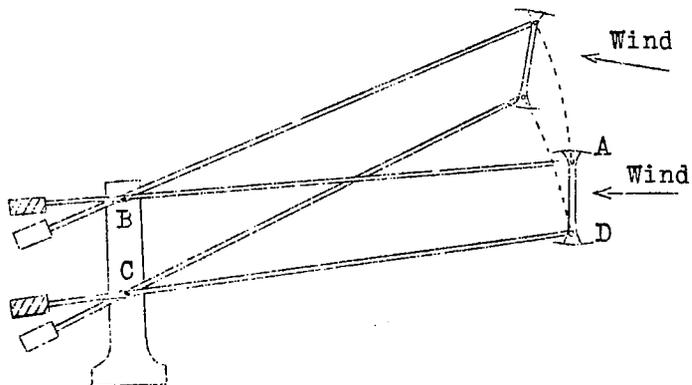


Fig. 17 Constantin power vane.

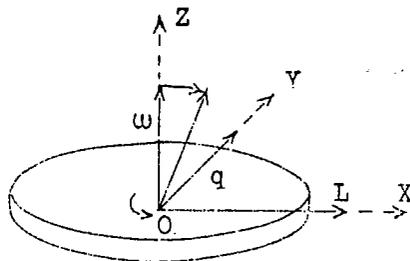


Fig. 19

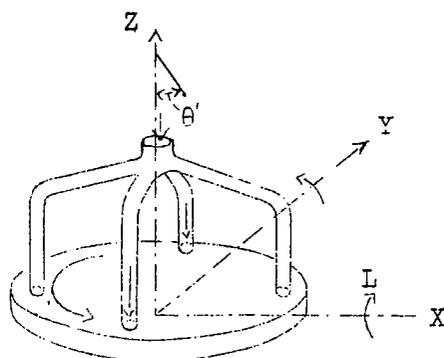


Fig. 20

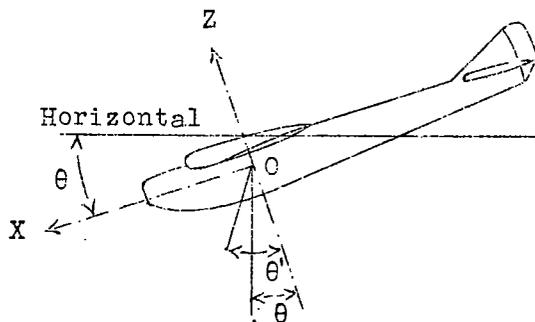


Fig. 21

Left, $\delta\theta = -0.2$ radian and $\delta w = -8$ m/s,

Center, $\delta w = -8$ m/s;

Right, $\delta u = 10$ m/s.

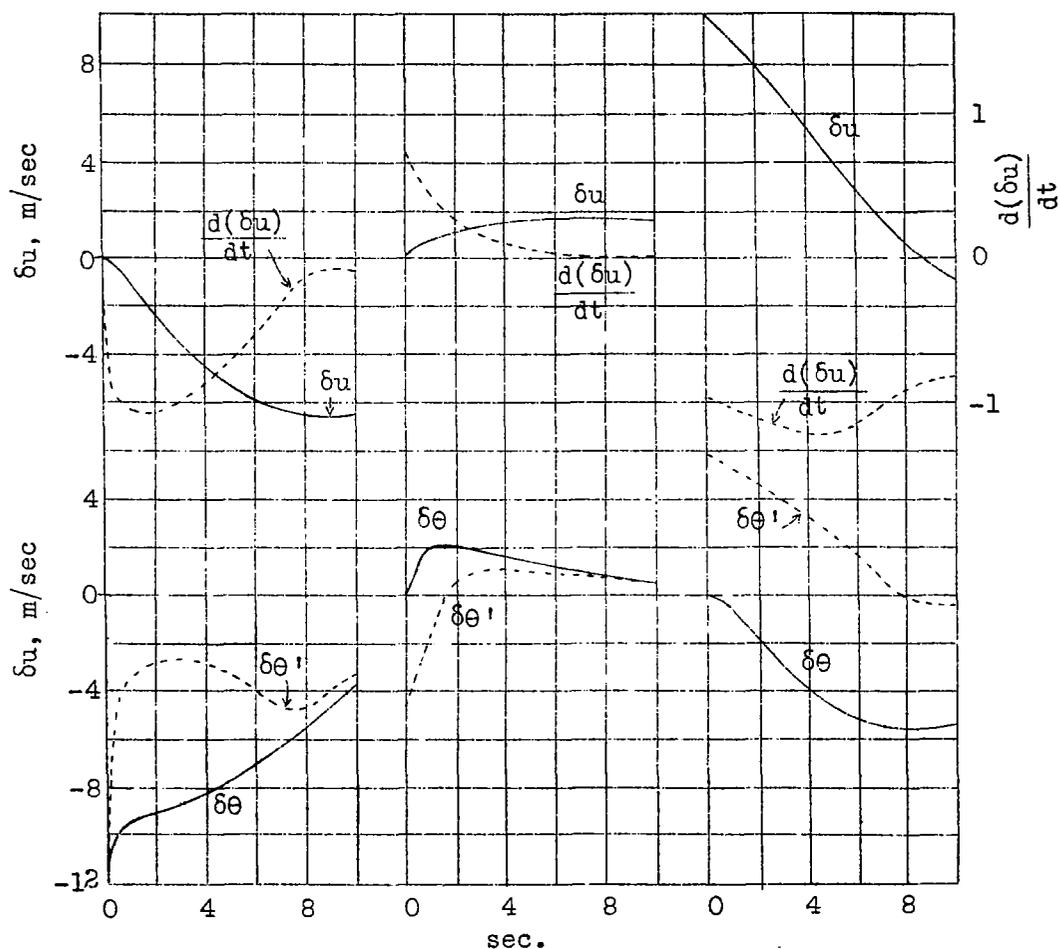


Fig.22 Variations of the speed along OX (δu), of the derivative $\frac{d(\delta u)}{dt}$ of this speed, of the inclination θ of the airplane to the horizontal and of the inclination θ' of an aperiodic pendulum carried in flight, for three types of initial disturbances.

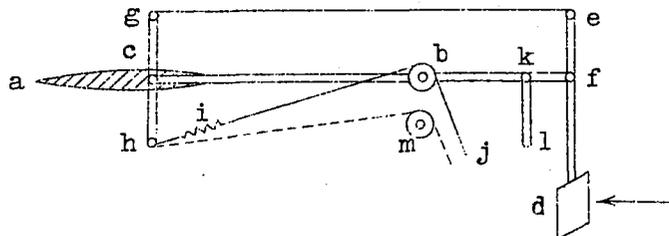


Fig. 23 Étévé vane anemometer

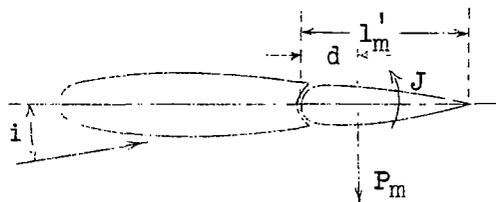


Fig. 24

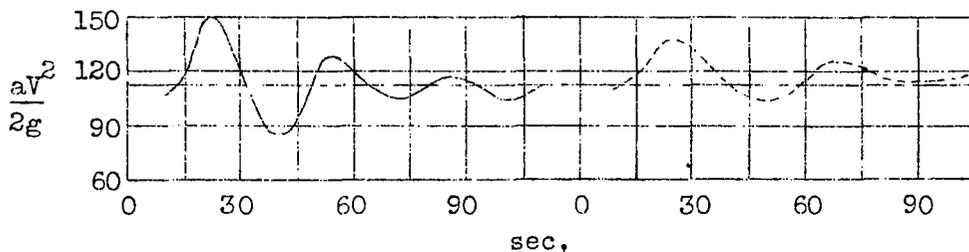


Fig. 25 Slow oscillations measured on Junkers airplane. Left:- Control locked. Right:- Control released.

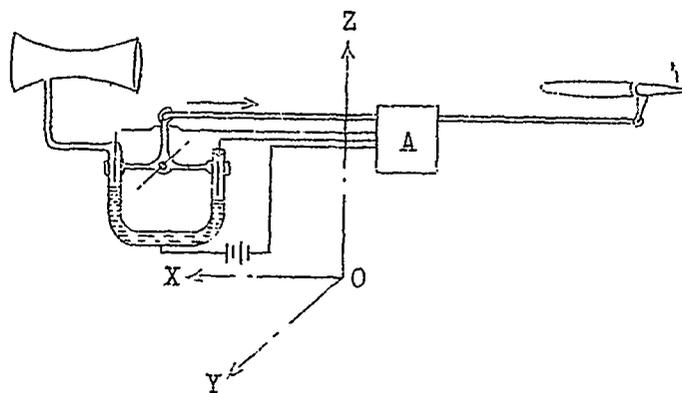


Fig. 26 Mazade and Aveline device.

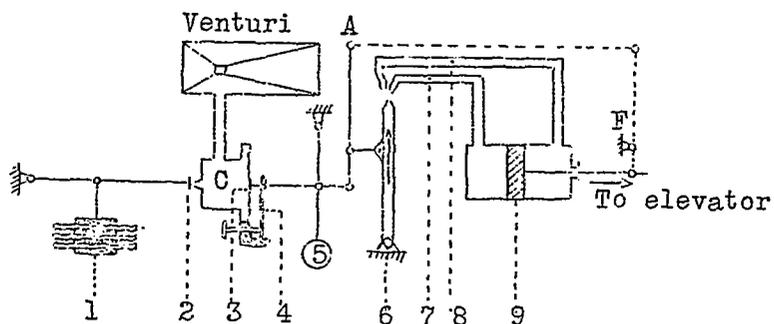
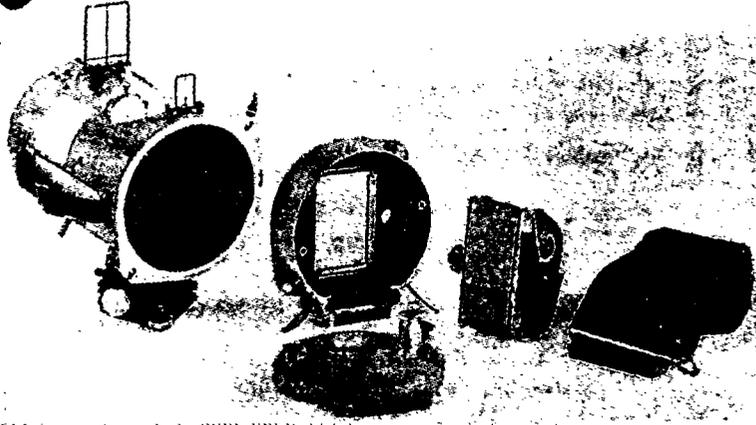


Fig. 27 Askania device.

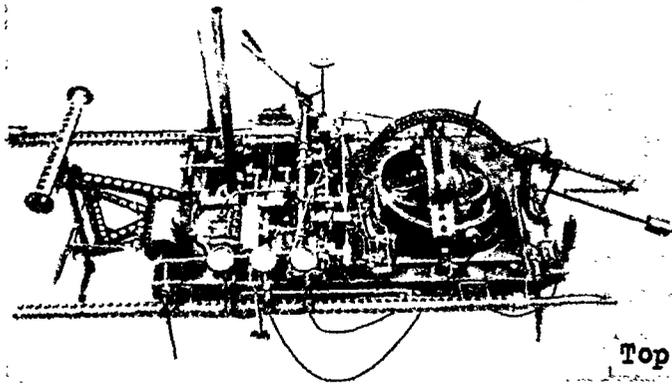


Left, Complete device. The black bag at the rear contains the exposed film, any length of which can be cut off for development.

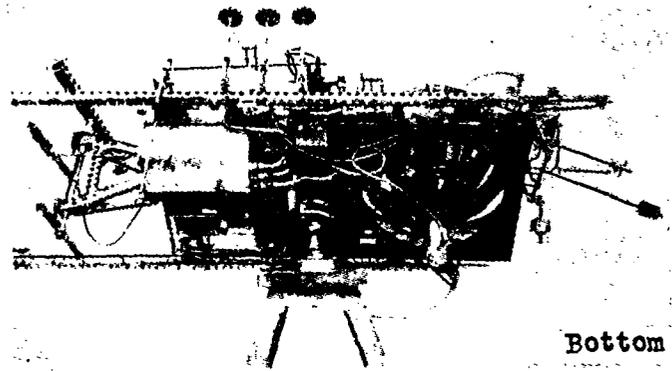
Right, Elements of the photochronograph. From left to right. Rear view of case, film magazine and rear cover, winding device and film bag. The locking arrangement of the bag is impervious to light. Weight with tripod, 14 kg (31 lb.)



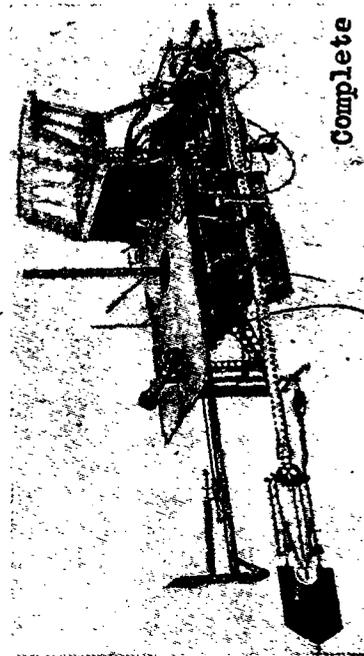
Figs. A,B The D.V.L. Zeiss photochronograph. This device, which is chiefly used for the study of landing and take-off trajectories, was designed by the D.V.L.(Deutsche Versuchsanstalt für Luftfahrt)and built by Zeiss.



Top



Bottom



Complete

Figs.31a,31b,31c Top, bottom and complete views of Marmonier stabilizer unit.