NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2660

AN APPROACH TO THE PREDICTION OF THE FREQUENCY DISTRIBUTION OF GUST LOADS ON AIRPLANES IN NORMAL OPERATIONS

By Harry Press

Langley Aeronautical Laboratory
Langley Field, Va.

Washington
April 1952

NATIONAL ADVISORY COMMITTEE FOR AERONA

TECHNICAL NOTE 2660

AN APPROACH TO THE PREDICTION OF THE FREQUENCY DISTRIBUTION OF GUST LOADS ON AIRPLANES IN NORMAL OPERATIONS

By Harry Press

SUMMARY

As a basis for the prediction of the gust-load history of airplanes in service operations, the statistical concepts of random variables and probability distributions are applied to the "sharp edge gust" formula. Expressions are derived for the frequency distribution of gust loads in terms of distributions of the related variables such as effective gust velocity and airspeed. Solutions are obtained under assumptions that appear reasonable on the basis of present practices in gust research. The results are applied in an example and the predicted load experience is compared with the available flight loads data for this case.

INTRODUCTION

The prediction of the gust and gust-load experience of an airplane in operational flight is a problem requiring continuing study for the development of safe and efficient aircraft. After a basic design is selected, one of the problems faced by the designer in setting the ultimate and fatigue strength of the primary structure is the prediction of the airplane gust-load history. Because the load applied to an airplane when encountering a gust is a function of both the gust velocity and a number of associated operating conditions, such as airspeed and altitude, information is needed not only on the characteristics of atmospheric turbulence but also on how and where the airplane will be flown.

In operating practice, the gust intensities encountered vary in an irregular manner over a wide range of intensities; in addition, the airspeeds and altitudes at which gusts are encountered also vary over an appreciable range for a given airplane. The load history of an airplane thus depends upon combinations of conditions in gust encounters. Since the particular combinations of conditions that occur in gust encounters are, within certain physical limitations, largely irregular and beyond control, the load experience exhibits some of the characteristics of chance phenomena and may be described by probability methods.

The statistical nature of the gust-load history has been widely recognized and considerable effort has been expended in the collection and analysis of gust-loads flight data. Until recently, these data were largely collected by use of NACA V-G recorders from which information could be obtained only on the larger loads and gusts. With the development of time-history recorders, such as the VGH recorder of reference 1, suitable for operational use, the entire range of gust and load intensity may be studied. In this connection there exists the problem of developing methods appropriate to the use of these data for purposes of load prediction. This problem is considered in the present paper in which an approach to the prediction problem based on probability methods is presented.

In the present analysis, the statistical theory of random variables and frequency distributions is applied to the form of the sharp-edge-gust equation. The frequency distribution of airplane gust loads is derived in terms of the frequency distributions of related variables such as effective gust velocity and airspeed under assumptions that appear reasonable. The analysis is extended to include the frequency distribution of gust loads as a function of airspeed in order to permit the determination of the frequency distribution of lift and moment forces. The nature and form of the data required for application are indicated. Finally, the methods of application are illustrated in an example in which the expected frequency distributions of loads for particular operations of a modern transport airplane are derived. The results obtained are compared with available flight loads data.

ANALYSIS

Statistical Background

The next few paragraphs will present some of the fundamental statistical concepts and relations used in this paper. These concepts and relations are covered in detail in standard textbooks such as reference 2.

Statistical theory rests upon the concept of a "random variable." This concept assumes that an observation of a variable x may have a multitude of values over a finite or infinite range. For an observation selected at random, all values may not occur with equal probability and the probability of a given value occurring may be defined by a function $f(x)$ such that $f(x)\,dx$ is a measure of the probability that a random value will fall in the interval

$$x - \frac{dx}{2} < x < x + \frac{dx}{2}$$

The function $f(x)$ is herein referred to as the frequency function and is related to the probability-distribution function $F(x)$ by the relation

$$F(x) = \int_{-\infty}^{x} f(x)\, dx \tag{1}$$

where $F(x)$ defines the probability that a random value will fall below a given value x. The probability $F(x)$ may be considered to represent the proportion of all values of x below a given value and is a bounded function $0 \leqq F(x) \leqq 1$. Similarly, the complementary function $1 - F(x)$ defines the probability that a random value will be greater than the given value x.

In a manner analogous to the single-variable case, multivariate functions may be defined to represent several properties of a given random event or observation and the probability distributions of these properties. The relation between the multivariate frequency function and the probability-distribution function is given by

$$F\left(x_1,x_2, \ldots x_n\right) = \int_{-\infty}^{x_n} \cdots \int_{-\infty}^{x_2} \int_{-\infty}^{x_1} f\left(x_1,x_2, \ldots x_n\right) dx_1\, dx_2 \ldots dx_n \tag{2}$$

where $x_1, x_2, \ldots x_n$ are n random variables, $f\left(x_1,x_2, \ldots x_n\right)$ is the multivariate frequency function of the variables $x_1, x_2, \ldots x_n$, and $F\left(x_1,x_2, \ldots x_n\right)$ defines the probability that a random observation will have values less than given values of $x_1, x_2, \ldots x_n$. If

$$F\left(x_1,x_2, \ldots x_n\right) = \int_{-\infty}^{x_n} \cdots \int_{-\infty}^{x_2} \int_{-\infty}^{x_1} f\left(x_1\right) f\left(x_2\right) \cdots f\left(x_n\right) dx_1\, dx_2 \ldots dx_n \tag{3}$$

then the distributions of the variables $x_1, x_2, \ldots x_n$ are said to be statistically independent and the distribution, of x_1 for example, is the same for any fixed values of the other variables. The property of statistical independence provides a useful basis for simplification in many statistical applications and will be referred to subsequently.

If a variable y is a function of several random variables $y = G\left(x_1,x_2, \ldots x_n\right)$, then from equation (2) the probability distribution function of y, $F(y)$, is given by

$$F(y) = \int \cdots \int \int f(x_1, x_2, \ldots x_n)\, dx_1\, dx_2 \ldots dx_n \tag{4}$$

where the limits of integration are restricted to the domain of values

$$G(x_1, x_2, \ldots x_n) < y$$

If, further, the variables x_1, x_2, ... x_n are considered statistically independent, equation (4) becomes

$$F(y) = \int \cdots \int \int f(x_1) f(x_2) \ldots f(x_n)\, dx_1\, dx_2 \ldots dx_n \tag{5}$$

Equation (5) permits the combination of the individual distributions of the several independent variables for determining the distribution of the related function y, the distribution of y depending only on the individual distributions of the variables x_1, x_2, ... x_n and $G(x_1, x_2, \ldots x_n)$.

Sharp-Edge-Gust Equation

In many investigations of gust-loads problems, the peak acceleration increment Δn experienced by an airplane under the action of a vertical gust is given in terms of the so-called sharp-edge-gust equation. Inasmuch as this paper is concerned with statistical consideration of the form of the sharp-edge-gust equation, the equation will be defined and briefly discussed. The sharp-edge-gust equation (reference 3) may be expressed as

$$\Delta n = \frac{1.467\, K a \rho_o U_e V_e}{2W/S} \tag{6}$$

where

a slope of the lift curve, per radian

ρ_o air density at sea level, 0.002378 slug per cubic foot

U_e "effective" gust velocity, feet per second

V_e equivalent airspeed $\sigma^{1/2} V$, where V is the true airspeed measured in miles per hour and σ is the ratio of actual air density to standard air density at sea level ρ_o

W airplane weight, pounds

S wing area, square feet

K alleviation factor used to take into account the effects due to lag in lift, the vertical velocity of the airplane, and the gradient distance of the gust

The corresponding maximum air-load increment (as distinct from load-factor increment) experienced by an airplane under the action of a gust can, from equation (6), be expressed as

$$\Delta L = W\,\Delta n = \frac{1.467\,K\rho_0 a U_e V_e S}{2} \tag{7}$$

If the possible compressibility effects on the slope of the lift curve are neglected, the load increment ΔL is a function only of K, U_e, and V_e for a given airplane type.

Frequency Distribution of Gust Loads

Probability relation for gust loads.- Consideration of the operational gust-load history of an airplane indicates that the variations in the values of K, U_e, and V_e in equation (7) may be treated by probability methods. The variables K, V, and U_e may be considered to be random variables defined by probability or frequency distributions. Statistical theory may then be applied to predict the relative frequency of occurrence of various combinations of values of the variables K, U_e, and V_e and, hence, the associated load increments. The irregular and erratic nature of gust sequences and the many factors influencing the airspeed and K value in a gust encounter would appear to make the approach through probability methods not only the most reasonable but also perhaps the only feasible approach.

The probability-distribution function of the air-load increment $F(\Delta L)$ may, from equation (4), be expressed as

$$F(\Delta L) = \iiint f_1(K,U_e,V_e)\;dK\;dU_e\;dV_e \tag{8}$$

where the limits of integration are restricted to the domain of values of

$$\frac{1.467\,K\rho_0 a U_e V_e S}{2} < \Delta L$$

The function $F(\Delta L)$ defines the proportion of load increments with values less than a given value. Thus, $F(\Delta L)$ is the proportion of all loads having values less than ΔL. The expression $f_1(K,U_e,V_e)$ represents the multivariate frequency function of the random variables K, U_e, and V_e and defines the relative frequency with which all possible combinations of values of the variables occur. (Unfortunately, in practice the function $f_1(K,U_e,V_e)$ is neither known nor feasible to obtain for particular conditions and, furthermore, is subject to change with changes in operating practice or airplane type.)

If the variables K, U_e, and V_e are statistically independent, equation (8) may be expressed from equation (5) as

$$F(\Delta L) = \int\int\int f_2(K) f_3(U_e) f_4(V_e)\ dK\ dU_e\ dV_e \tag{9}$$

where $f_2(K)$, $f_3(U_e)$, and $f_4(V_e)$ are the frequency functions of the random variables K, U_e, and V_e, respectively. For a given set of operations, a knowledge of the applicable individual distributions of K, U_e, and V_e would therefore allow the determination of the frequency distribution of load increments.

Distribution of effective gust velocity.- The effective gust velocity, being a measure of the atmospheric disturbance, is the fundamental variable in the sharp-edge-gust equation that gives rise to the airplane load, whereas the values of the variables K and V_e affect the magnitude of the resultant load. Past work has indicated that the characteristics of atmospheric gustiness can be described in terms of the frequency of occurrence of gusts and the distribution of gust intensities (probability distributions). As indicated in reference 4, the frequency of atmospheric gusts and the distributions of gust intensities appear to vary widely. The information available suggests that the gust characteristics of the atmosphere vary with the type of weather, geographical regions, altitude, and season of the year. Typical probability-distribution functions of effective gust velocity above a threshold of 4 feet per second for various weather conditions are shown in figure 1. The available data for these test conditions appear to approximate simple exponential distributions. The frequency of occurrence of the gusts is also indicated by λ, the average number of gusts per mile of flight. The over-all gust history in normal operations consists of combinations of the distributions of figure 1 and others; the over-all distribution of gust intensities may be expected to depart from linearity and vary appreciably among different types of operations. In view of this variability, it is apparent that the appropriate gust distribution for a given set of operating conditions is required in order to obtain reliable predictions of load experience.

Distribution of airspeeds.- The variable V_e as used in equations (8) and (9) refers to the airspeeds at which gusts are encountered. Experience has indicated that many pilots, in recognition of the relation of load and acceleration to the airspeed at which gusts are encountered, attempt to reduce airspeed in rough air. In these cases, the over-all airspeed time distribution is not independent of the gust velocity U_e and consequently cannot be used for present purposes. Available data indicates that airspeeds in rough air are generally lower than the airspeeds in smooth air and may be independent of the intensity of the turbulence. The over-all distribution of airspeed in rough air would consequently appear satisfactory for determining the function $f_4(V_e)$ in equation (9). The distribution of airspeed in rough air may most easily be obtained from sample records of flight data. In the absence of sample flight data, usable estimates of this distribution may be derived from a consideration of the airplane characteristics and the operating conditions.

Alleviation factor K.- The variable K, the alleviation factor, introduces a serious complication to the analysis inasmuch as the airplane response is a complex function of airplane and gust characteristics. The airplane response has frequently been assumed to be a function of the mass parameter M of the airplane and the gust-gradient distance. Since the mass parameter is a function of airplane weight and air density, its value will vary for a given airplane and operator. For particular airplanes, the factor K may be expected to vary over a range of about ±10 percent. This variation is small and may be expected to have only a minor effect on the load distribution. In addition, prevailing practice in the evaulation of gust statistics are, for simplicity or lack of information, generally restricted to the use of a single K value for a particular set of operations. Thus the effects of the variations in K are already included in the gust statistics. In view of the foregoing discussion which indicates that the variations in values of K are generally small and are already included in published gust statistics, generally no serious error would be introduced by neglecting the variable character of K.

Distribution of gust loads.- Under the assumption of a constant value of K, equation (9) is reduced to

$$F(\Delta L) = \int\int f_3(U_e) f_4(V_e)\ dU_e\ dV_e \tag{10}$$

where the limits of integration are restricted to the domain of values of

$$kU_eV_e < \Delta L$$

where

$$k = 1.467\ K\rho_o a\ \frac{S}{2}$$

If the distribution of effective gust velocity $f_3(U_e)$ is assumed symmetrical about the zero axis which is the usual practice (up and down gusts are assumed to have the same probability distribution), positive gusts alone may be considered since the distribution of associated load increments will likewise be symmetrical. Thus, if only positive gusts are considered and small gusts which have values below some threshold value U_{e_1} are neglected, the integration of equation (10) with respect to U_e yields

$$F(\Delta L) = \int \left[F_3\left(\frac{\Delta L}{kV_e}\right) - F_3\left(U_{e_1}\right)\right] f_4(V_e)\ dV_e \tag{11}$$

Since no gust velocities below the value U_{e_1} are considered, $F_3\left(U_{e_1}\right) = 0$ by definition and equation (11) becomes

$$F(\Delta L) = \int F_3\left(\frac{\Delta L}{kV_e}\right) f_4(V_e)\ dV_e \tag{12}$$

Equation (12) may also be written in terms of the complement of $F(\Delta L)$, $1 - F(\Delta L)$, as

$$1 - F(\Delta L) = \int \left[1 - F_3\left(\frac{\Delta L}{kV_e}\right)\right] f_4(V_e)\ dV_e \tag{12a}$$

This form is frequently more convenient for computational purposes.

The nature of the volume integral equation (12) is illustrated in figure 2. The illustrated surface, $f(U_e,V_e) = f_3(U_e)f_4(V_e)$, represents the joint frequency function of the variables U_e and V_e. A given value of load increment ΔL is represented by the hyperbola $\Delta L = kU_eV_e$ in the U_e,V_e plane. The probability of occurrence of values of ΔL less than a given value in an airspeed interval $V_e \pm \frac{dV_e}{2}$ is given by $F_3\left(\frac{\Delta L}{kV_e}\right)f_4(V_e)\ dV_e$ indicated on the left side of the figure. The term $F_3\left(\frac{\Delta L}{kV_e}\right)f_4(V_e)\ dV_e$ represents the volume of a slice normal to the V_e axis and bounded by the U_e,V_e plane, the $F(U_e,V_e)$ surface, the hyperbolic surface $\Delta L = kU_eV_e$, and the plane $U_e = U_{e_1}$. The volume integral for $F(\Delta L)$ of equation (12) is obtained by the summation of these slices along the V_e axis.

Under the assumption of a constant airspeed $\overline{V}_e$, the $F(U_e,V_e)$ surface of figure 2 reduces to a curve in the plane of constant airspeed and equation (12) reduces to

$$F(\Delta L) = F_3\left(\frac{\Delta L}{k\overline{V}_e}\right) \tag{12b}$$

This simplified solution has sometimes been used in estimating the frequency distribution of loads for a given frequency distribution of effective gust velocity.

Equation (12) yields the probability distribution of gust loads $F(\Delta L)$ which gives the proportion of gust loads having values less than given values of ΔL. For given operations, however, interest centers in the number of loads of a given intensity or in the frequency distribution of gust loads. From the definition of the probability-distribution function

$$F(\Delta L) = \frac{N(\Delta L)}{N} \tag{13}$$

where $N(\Delta L)$ is the number of load increments less than a given value of ΔL and N is the total number of loads (which will, of course, be the same as the number of gusts). The total number of load increments less than a given value $N(\Delta L)$ is consequently given by

$$N(\Delta L) = NF(\Delta L) \tag{13a}$$

and for evaluation requires information on the total number of gusts expected or the average frequency of occurrence of gusts. The total number of load increments in a given interval of load intensity, for example ΔL_1 to ΔL_2 $\left(\Delta L_2 > \Delta L_1\right)$, is from equation (13) given by

$$N\left(\Delta L_1 < \Delta L < \Delta L_2\right) = N\left[F\left(\Delta L_2\right) - F\left(\Delta L_1\right)\right] \tag{13b}$$

Distribution of gust loads by airspeed.- The foregoing analysis provides a means of deriving the total frequency distribution of gust loads from the distributions of effective gust velocity and airspeed. In addition, these results may be extended to derive the frequency distribution of loads separately for various airspeeds. This extension is sometimes desired for use in connection with estimates of the chordwise distribution of loads. If steady flow conditions are assumed to hold, the frequency distribution of applied lift and moment forces may be derived from the frequency distributions of load by airspeed.

From equation (12), consider the function $F_3\left(\frac{\Delta L}{kV_e}\right)f_4(V_e)$. Integrating this function between given values of V_e, between V_{e_1} and V_{e_2} for instance, for various values of ΔL gives

$$F(\Delta L)_{V_{e_{1-2}}} = \int_{V_{e_1}}^{V_{e_2}} F_3\left(\frac{\Delta L}{kV_e}\right)f_4(V_e)\ dV_e \tag{14}$$

where $F(\Delta L)_{V_{e_{1-2}}}$ is the proportion of all loads occurring in the airspeed interval V_{e_1} to V_{e_2}, with values of load less than ΔL. An approximate method of determining the distribution of load increments by airspeed brackets (frequently more convenient for computational purposes than equation (14)) may be derived by integrating equation (10) with respect to V_e for positive gusts above the threshold value U_{e_1} and over the airspeed bracket V_{e_1} to V_{e_2} where the airspeed bracket is reasonably small (5 to perhaps 20 mph). The integration yields

$$F(\Delta L)_{\overline{V}_e} = \int_{U_{e_1}}^{\frac{\Delta L}{k\overline{V}_e}} f_3(U_e)\left[F_4\left(V_{e_2}\right) - F_4\left(V_{e_1}\right)\right]dU_e \tag{15}$$

where $\overline{V}_e$ is an average value of V_e in the airspeed bracket V_{e_1} to V_{e_2}. For present purposes, sufficiently accurate estimates of $\overline{V}_e$ are obtained by using the mean value defined by

$$\overline{V}_e = \frac{\int_{V_{e_1}}^{V_{e_2}} V_e f_4(V_e)\ dV_e}{\int_{V_{e_1}}^{V_{e_2}} f_4(V_e)\ dV_e} \tag{15a}$$

From equation (15), the probability distribution of loads $F(\Delta L)_{\overline{V}_e}$ due to positive gusts above a threshold value of U_{e_1} and in a given airspeed bracket is given by

$$F(\Delta L)_{\overline{V}_e} = F_3\left(\frac{\Delta L}{k\overline{V}_e}\right)\left[F_4\left(V_{e_2}\right) - F_4\left(V_{e_1}\right)\right] \tag{16}$$

In terms of the complement of $F_3\left(\frac{\Delta L}{k\overline{V}_e}\right)$, equation (16) may be given as

$$\left[F_4\left(V_{e_2}\right) - F_4\left(V_{e_1}\right)\right] - F(\Delta L)_{\overline{V}_e} = \left[1 - F_3\left(\frac{\Delta L}{k\overline{V}_e}\right)\right]\left[F_4\left(V_{e_2}\right) - F_4\left(V_{e_1}\right)\right] \tag{16a}$$

which yields the proportion of all loads that occur in a given airspeed bracket and have values greater than given values of ΔL. Equation (16) or (16a) may be evaluated readily to obtain the probability-distribution function of loads for each airspeed bracket. The term $\left[F_4\left(V_{e_2}\right) - F_4\left(V_{e_1}\right)\right]$ in equation (16) represents the proportion of flight time (or distance) spent in the airspeed range V_{e_1} to V_{e_2}. The term $F_3\left(\frac{\Delta L}{k\overline{V}_e}\right)$ transfers the probability function of effective gust velocity to an associated probability function of loads under the assumption that the airspeeds in the airspeed bracket are confined to the mean airspeed for the interval.

In a manner analogous to the case of the total frequency distribution of loads, the number of loads that occur in a given airspeed bracket and have a value less than a given value of ΔL, $N(\Delta L)$, is obtained from equation (16), and the total number of loads (or gusts) N is obtained from the expression

$$N(\Delta L) = NF(\Delta L)_{\overline{V}_e} \tag{17}$$

Numerical Solutions

Over-all probability distribution of gust loads.- For known distributions of U_e and V_e, the distribution of load increments may be obtained from equation (12). The distributions of U_e and V_e which are commonly encountered cannot, however, be readily approximated by simple functions and, further, equation (12) is not readily integrable in closed form for even the simple distribution functions. An approximate and systematic evaluation can generally be made by numerical methods. Using Simpson's rule with an interval of airspeed ΔV permits equation (12) to be expressed as

$$\left.\begin{aligned}
F(\Delta L_1) &= \frac{\Delta V}{3}\left[F_3\left(\frac{\Delta L_1}{kV_{e_1}}\right)f_4\left(V_{e_1}\right) + 4F_3\left(\frac{\Delta L_1}{kV_{e_2}}\right)f_4\left(V_{e_2}\right) + \cdots + F_3\left(\frac{\Delta L_1}{kV_{e_n}}\right)f_4\left(V_{e_n}\right)\right] \\
F(\Delta L_2) &= \frac{\Delta V}{3}\left[F_3\left(\frac{\Delta L_2}{kV_{e_1}}\right)f_4\left(V_{e_1}\right) + 4F_3\left(\frac{\Delta L_2}{kV_{e_2}}\right)f_4\left(V_{e_2}\right) + \cdots + F_3\left(\frac{\Delta L_2}{kV_{e_n}}\right)f_4\left(V_{e_n}\right)\right] \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
F(\Delta L_r) &= \frac{\Delta V}{3}\left[F_3\left(\frac{\Delta L_r}{kV_{e_1}}\right)f_4\left(V_{e_1}\right) + 4F_3\left(\frac{\Delta L_r}{kV_{e_2}}\right)f_4\left(V_{e_2}\right) + \cdots + F_3\left(\frac{\Delta L_r}{kV_{e_n}}\right)f_4\left(V_{e_n}\right)\right]
\end{aligned}\right\} \tag{18}$$

In matrix form this equation may conveniently be written as

$$\begin{bmatrix} F(\Delta L_1) \\ F(\Delta L_2) \\ \cdots \\ \cdots \\ \cdots \\ F(\Delta L_r) \end{bmatrix} = \frac{\Delta V}{3} \begin{bmatrix} F_3\left(\frac{\Delta L_1}{kV_{e_1}}\right) & \cdots & F_3\left(\frac{\Delta L_1}{kV_{e_n}}\right) \\ F_3\left(\frac{\Delta L_2}{kV_{e_1}}\right) & \cdots & F_3\left(\frac{\Delta L_2}{kV_{e_n}}\right) \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ F_3\left(\frac{\Delta L_r}{kV_{e_1}}\right) & \cdots & F_3\left(\frac{\Delta L_r}{kV_{e_n}}\right) \end{bmatrix} \times \begin{bmatrix} f_4\left(V_{e_1}\right) \times 1 \\ f_4\left(V_{e_2}\right) \times 4 \\ f_4\left(V_{e_3}\right) \times 2 \\ \cdots \\ \cdots \\ f_4\left(V_{e_n}\right) \times 1 \end{bmatrix} \tag{18a}$$

Distribution of loads by airspeed.- The determination of the distribution function of load increments by airspeed brackets is straightforward and requires the evaluation of equation (16) or (16a). The mean airspeed for each speed bracket defined by equation (15a) can be estimated by using Simpson's rule and is given by

$$\overline{V}_e = \frac{V_{e_1} f_4\left(V_{e_1}\right) + 4V_{e_{3/2}} f_4\left(V_{e_{3/2}}\right) + V_{e_2} f_4\left(V_{e_2}\right)}{f_4\left(V_{e_1}\right) + 4f_4\left(V_{e_{3/2}}\right) + f_4\left(V_{e_2}\right)} \tag{19}$$

where the subscripts 1, 3/2, and 2 refer to the airspeeds at the beginning, midpoint, and end point, respectively, of each airspeed bracket.

ILLUSTRATIVE PROBLEM

In order to illustrate the application of the foregoing analysis, the results obtained were applied to available data to estimate the expected load history for a twin-engine airplane in transport operations. The example is developed in two parts. In the first part, the over-all probability distribution of load increments is obtained from available data on the distributions of effective gust velocity and airspeed. In the second part of the example, the same basic data are used to derive the distributions of load increment by airspeed brackets. The results obtained are in each part compared with available flight loads data for these operations. The basic data used in the example will be presented and then used in an application to the two parts of the example.

Basic Data

Distribution of effective gust velocity.- Information on the distribution of effective gust velocity for the present operations is available from a limited sample of time-history record of normal acceleration and airspeed covering 281 hours of operations (reference 5). Because of the limited range of the effective gust velocities available from these records, the probability distribution of effective gust velocity $F_3(U_e)$ could not be determined from these data with suitable accuracy, particularly at the higher intensities of effective gust velocity. The available data, however, approximated the shape of the probability-distribution curve labeled curve B and given in reference 4 which was based on more extensive gust statistics. This distribution was consequently used as a basis for the probability-distribution function $F_3(U_e)$ required for the present example. The B curve of the reference was normalized to a threshold value of 4 feet per second, the threshold selected for the present case, and is shown in figure 3. The figure actually gives the complement of $F_3(U_e)$, or $1 - F_3(U_e)$, in order to permit greater accuracy of reading at the higher gust velocities.

Frequency of occurrence of gusts.- Equations (13) and (14) show that, in order to determine the frequency distribution of loads or the number of loads of given intensity in a given period, a total number or an average frequency of occurrence of gusts or loads is required in addition to their probability distributions. From the available time-history data of reference 5, an average frequency of occurrence of effective gust velocities greater than 4 feet per second of 0.7 gusts per mile of flight (0.35 positive and 0.35 negative gusts) was estimated. This value of average gust frequency is used in converting the probability distributions to frequency distributions in the present example.

Distribution of airspeed.- The frequency function of airspeed $f_4(V_e)$ used in the present example is shown in figure 4 and was based on about 130 of the 281 hours of airspeed record in which rough-air conditions existed. The integral of $f_4(V_e)$ over an airspeed interval, V_{e_1} to V_{e_2} for example, indicates the proportion of the total flight distance spent at speeds between V_{e_1} and V_{e_2}. Flight distance was used herein rather than flight time because the available information suggests that flight distance is the more appropriate parameter. (See, for example, reference 4.)

Numerical values.- The numerical values assumed for the calculations of the present example are:

ρ_o, slugs per cubic foot	0.002378
a, per radian	5
S, square feet	864
W (estimated average), pounds	33,900
K	1.16
$k = \frac{1.467\ K\rho_o Sa}{2}$	8.741

Over-all Frequency Distribution of Gust Loads

Probability distribution of gust loads.- The determination of the over-all probability distribution of gust loads requires the evaluation of equation (18) or (18a). By use of an airspeed interval of 10 miles per hour, the numerical values of the arguments $\frac{\Delta L}{kV_e}$ in equation (18a) were obtained and are given in table I(a). The values of ΔL shown in the table are, with the exception of the first value which is the lowest load increment value obtainable from the present case (the load resulting from a 4-fps gust at an airspeed of 120 mph), given in increments of load equivalent to 0.1g load-factor increments starting at 0.2g (6780 lb).

The appropriate values of $F_3\left(\frac{\Delta L}{kV_e}\right)$ and $f_4(V_e)$ are obtained from figures 3 and 4, respectively. In order to reduce the amount of computation, the evaluation was actually carried out for the complementary functions $1 - F(\Delta L)$ and $1 - F_3(U_e)$ as given by equation (12a). The matrix evaluation is shown in table I(b) and the solution $1 - F(\Delta L)$ is shown in figure 5. This figure indicates the probability of a load having a value greater than the indicated value.

Comparison of expected distribution of loads with flight loads data.- The frequency distribution of normal acceleration for values greater than 0.3g was available from the present operations for 834 hours of flight. On the basis of an estimated average airplane weight of 33,900 pounds, these accelerations were converted to loads and the frequency distribution of loads obtained is shown in figure 6 in terms of the number of loads encountered greater than indicated given values. For comparison, the expected frequency distribution of loads for this flight time (834 hr) was derived by using the probability distribution $1 - F(\Delta L)$ given in figure 5 and the average gust frequency of 0.7 gust per mile previously taken as representative of the present operations. If an average airspeed of 200 miles per hour is assumed, a total of $(834)(200)(0.7) = 116,760$ gusts are obtained. The expected number of loads greater than given values of load is then given by the expression $116,760\left[1 - F(\Delta L)\right]$. The results obtained are also shown in figure 6 for comparison with the observed flight loads data.

Distribution of Load Increments by Airspeed Brackets

Probability distribution by airspeed bracket.- In order to illustrate the evaluation of the distribution of load increments by airspeed bracket, the evaluation of equation (16a) was completed for the present example by using increments of 20 miles per hour for the brackets, that is, 120 to 140, 140 to 160, and so forth. The values of the argument $\left(\frac{\Delta L}{kV_e}\right)$ in $F_3\left(\frac{\Delta L}{kV_e}\right)$ for each airspeed bracket for selected values of load increment are given in table II(a), where the mean airspeed $\overline{V}_e$ for each airspeed bracket was obtained by using equation (19) and is also indicated in the table. The values of $\left[1 - F_3\left(\frac{\Delta L}{k\overline{V}_e}\right)\right]$ were obtained by using figure 3 for each of the values of $\Delta L/k\overline{V}_e$ of table II(a) and are given in table II(b). The values of $\left[F_4\left(V_{e_2}\right) - F_4\left(V_{e_1}\right)\right]$ in equation (16) were estimated from figure 4 by using Simpson's rule and are given by the relation

$$F_4\left(V_{e_2}\right) - F_4\left(V_{e_1}\right) = \frac{10}{3}\left[f_4\left(V_{e_1}\right) + 4f_4\left(V_{e_{3/2}}\right) + f_4\left(V_{e_2}\right)\right]$$

where the subscripts 1, 3/2, and 2 refer to the airspeed at the beginning, midpoint, and end point, respectively, of each airspeed bracket. The results obtained for each airspeed bracket are given at the bottom of table II(b). The probability-distribution function of load increment obtained for each airspeed bracket by substituting the numerical values of table II(b) into equation (16a) are given in table II(c). The over-all distribution of load increment for all airspeed brackets $1 - F(\Delta L)$ obtained by adding across each run is also given in the last column of the table. The distributions of load increment by airspeed bracket are shown in figure 7 and indicate the probability that a load will occur in a given airspeed bracket and have a value greater than the values indicated.

Growth of airspeed-load envelope.- Inasmuch as the total load experience for a given number of gusts or loads is defined for each airspeed bracket by the distributions of figure 7, the airspeed-load envelopes (maximum load increments as a function of airspeed) may be estimated directly. For given flight distances, for example 10^5, 10^6, and 10^7 flight miles, a total of 0.35×10^5, 0.35×10^6, and 0.35×10^7 positive (and also negative) gusts, respectively, may be expected for the present operations. For a given number of gusts or loads, the average number of loads that may be expected to exceed a given value is, from statistical theory, given by NP where N is the number of loads and P is the probability of a random value of load exceeding a given value (or the proportion of loads exceeding a given value). Thus, the maximum load that may be expected, on the average, to be exceeded once (the average maximum load of N loads) is the load intensity corresponding to a probability value of $P = \frac{1}{N}$. On this basis, the expected maximum load increments for given numbers of gusts may be obtained from figure 7 by setting the probability P equal to $1/\left(0.35 \times 10^5\right)$, $1/\left(0.35 \times 10^6\right)$, and $1/\left(0.35 \times 10^7\right)$, respectively, and reading the load values at the intersections of these horizontal lines with the probability distributions for each airspeed bracket. The probable maximum load increments obtained in this manner are shown plotted at the mean values of airspeed $\overline{V}_e$ in figure 8. Faired lines are shown to indicate the over-all envelopes that may be expected for the given flight distances. The determination of the negative load increments is, of course, directly parallel and the symmetrical negative envelope is also shown in figure 8.

Comparison of predicted and observed airspeed-load envelopes.- V-G records covering 28,116 hours of flight or 5.62×10^6 miles of flight (obtained by assuming an average airspeed of 200 mph) were available from operations of the present type. An average operating weight of 33,900 pounds was assumed and the over-all load-velocity envelope shown in figure 9 was obtained from a composite of the V-G records. Although V-G records do not permit the separation of the loads due to gusts and maneuvers, an airspeed-load envelope based on the present analysis was derived for comparison. If the average frequency of 0.35 positive gusts per mile of flight is used, a total of about 1.97×10^6 positive (and also negative) gusts may be expected for the period covered by the V-G records. For a value of $P = \frac{1}{1.97 \times 10^6}$, the expected load envelope may be obtained from figure 7 as previously outlined. The results obtained are shown in figure 9 for comparison with the load envelope obtained for the V-G data. The predicted envelope indicates average expected limits of the envelope for the period covered by the present V-G records.

DISCUSSION

The results obtained in the application of the present analysis to a particular set of operations (figs. 5 and 7) illustrates the nature of the probability distributions of load increments due to gusts. Comparison of the predicted frequency distributions of load increment and the measured distribution based on 834 hours of operations (fig. 6) indicates that the results in this case are in good agreement. The predicted numbers of loads appear consistently somewhat larger than actually observed, but differences of the magnitudes observed may be expected from sampling variations since the gust spacing assumed for the present example was based on a limited sample of only 286 hours of time-history record. It is of interest to note that the use of a simplifying assumption that the airplane flies at an average airspeed (equation (12b)) yields an equally good estimate of the observed flight loads in this case. It is easy to show, however, that the good agreement will not be obtained for other gust distributions or for distributions of airspeed that depart appreciably from a symmetrical distribution or have greater scatter.

Although the verification of the complete predicted load history is not possible at this time, the over-all velocity-load envelope has been checked against the more extensive coverage available from V-G records. Comparison of the predicted load envelope with that obtained from the V-G records (fig. 9) indicates that the predicted envelope for 28,116 hours is in reasonable agreement with that measured over the airspeed range of 160 to perhaps 240 miles per hour. In this range, only two large positive loads (at about 175 and 190 mph) exceed the predicted envelope by a significant amount. Inasmuch as the prediction represents an average condition, the differences between the predicted envelope and the V-G data are not considered significant.

Although the agreement between predicted and measured load envelopes appears good in the middle airspeed range, figure 9 indicates that the present analysis does not predict adequately the load envelope for airspeeds below 160 miles per hour or above 240 miles per hour. The inability of the present methods to determine adequately the high- and low-speed part of the load envelopes is not, however, surprising. The present analysis is based on considerations of the loads due to gusts only, whereas the high- and low-speed loads are probably associated with maneuvers. Examination of the 265 V-G records on which the load envelope was based, indicated that six records contributed to the composite above about 265 miles per hour. The adequate estimation of the high- and low-airspeed parts of the load envelope appears consequently to require both a consideration of the maneuver contributions and larger preliminary samples to obtain a measure of the frequency with which the loads associated with the high-speed part of the envelope may be anticipated.

In the application of the present methods to the problem of predicting the history of loads for the service life of an airplane, the major problems involve the selection of the appropriate distributions of effective gust velocity and of airspeed to be used in particular cases. In addition, greater precision in the predictions may be obtained by dividing the service life into significant parts in accordance with information on the parameters which are likely to affect the distributions of K, U_e, V_e or their interrelations. As an example, for a high-altitude transport, the division may be made into three flight stages, climb, level flight, and descent, since available information suggests that the gust experience, the airspeed distribution, and the value of the alleviation factor K may be expected to differ among these flight stages.

The appropriate distribution of effective gust velocities for a particular case is, of course, a fundamental problem which appears, from the variations indicated in figure 1, to require ultimately the collection of gust statistics in order to describe more precisely the distribution of gust velocity for various atmospheric conditions. The immediate practical solution appears to be the study of the gustiness experienced in various types of operations by the collection of samples of gust statistics. Such studies are now underway.

The appropriate airspeed distribution to be used in equation (12), as already pointed out, is the airspeed distribution in rough air. The use of samples of airspeed record even of small size, as indicated by the illustration, appears adequate. When sample data are not available, as in the design stage of a new airplane, other recourses are necessary. The study of airspeed distributions in normal operating practice and the relation of these distributions to airplane characteristics, design parameters, flight stage, and level of gustiness may provide a basis for the estimation of the airspeed distribution appropriate to new airplanes. The assumption, used herein, of independence between gust velocity and airspeed may warrant more precise investigation in particular cases.

CONCLUDING REMARKS

An analysis of the problem of predicting the frequency distribution of gust loads on airplanes has indicated that the frequency distribution can be predicted from a knowledge of the frequency distributions of effective gust velocity and airspeed. The analysis also yields the distribution of load increments for different airspeeds. The application of the methods in an example indicates reasonable agreement between the predicted frequency distribution of gust loads and measured flight loads data. For extended operations, the present methods appeared satisfactory for predicting the airspeed-load envelope in the airspeed cruising range but did not adequately predict the load envelope at the lower and higher airspeeds.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., October 10, 1951

REFERENCES

1. Richardson, Norman R.: NACA VGH Recorder. NACA TN 2265, 1951.

2. Kendall, Maurice G.: The Advanced Theory of Statistics. Vol. I, third ed., Charles Griffin and Co., Ltd. (London), 1947.

3. Donely, Philip: Summary of Information Relating to Gust Loads on Airplanes. NACA Rep. 997, 1950. (Supersedes NACA TN 1976.)

4. Rhode, Richard V., and Donely, Philip: Frequency of Occurrence of Atmospheric Gusts and of Related Loads on Airplane Structures. NACA ARR L4I21, 1944.

5. Steiner, Roy, and McDougal, Robert L.: Summary of Normal Accelerations, Gust Velocities, and Operating Practices from April to August 1949 of a Twin-Engine Airplane in Commercial Transport Operations. NACA RM L50B02, 1950.

TABLE I

EVALUATION OF PROBABILITY DISTRIBUTION OF GUST LOADS

[k = 8.741]

(a) Values of $U_e = \frac{\Delta L}{kV_e}$ for Each Airspeed Bracket

Load increment, ΔL (lb)	$\Delta L/kV_e$ for -														
	V_e = 120 mph	V_e = 130 mph	V_e = 140 mph	V_e = 150 mph	V_e = 160 mph	V_e = 170 mph	V_e = 180 mph	V_e = 190 mph	V_e = 200 mph	V_e = 210 mph	V_e = 220 mph	V_e = 230 mph	V_e = 240 mph	V_e = 250 mph	V_e = 260 mph
4,196	4.00	3.69	3.43	3.20	3.00	2.82	2.67	2.53	2.40	2.29	2.18	2.09	2.00	1.92	1.85
6,780	6.46	5.97	5.54	5.17	4.85	4.56	4.31	4.08	3.88	3.69	3.53	3.37	3.23	3.10	2.98
10,170	9.70	8.95	8.31	7.76	7.27	6.84	6.46	6.12	5.82	5.54	5.29	5.06	4.85	4.65	4.47
13,560	12.93	11.93	11.08	10.34	9.70	9.13	8.62	8.16	7.76	7.39	7.05	6.74	6.46	6.21	5.97
16,950	16.16	14.92	13.85	12.93	12.12	11.41	10.77	10.21	9.70	9.23	8.81	8.43	8.08	7.76	7.46
20,340	19.39	17.90	16.62	15.51	14.54	13.69	12.93	12.25	11.63	11.08	10.58	10.12	9.70	9.31	8.95
23,730	22.62	20.88	19.39	18.10	16.97	15.97	15.08	14.29	13.57	12.93	12.34	11.80	11.31	10.86	10.44
27,120	25.86	23.87	22.16	20.68	19.39	18.25	17.24	16.33	15.51	14.77	14.10	13.49	12.93	12.41	11.93
30,510	29.09	26.85	24.93	23.27	21.82	20.53	19.39	18.37	17.45	16.62	15.87	15.18	14.54	13.96	13.42
33,900	32.32	29.83	27.70	25.86	24.24	22.81	21.55	20.41	19.39	18.47	17.63	16.86	16.16	15.51	14.92
37,290	35.55	32.82	30.47	28.44	26.66	25.09	23.70	22.45	21.33	20.31	19.39	18.55	17.78	17.06	16.41
40,680	38.78	35.80	33.24	31.03	29.09	27.38	25.86	24.49	23.27	22.16	21.15	20.23	19.39	18.62	17.90
44,070	42.01	38.78	36.01	33.61	31.51	29.66	28.01	26.54	25.21	24.01	22.92	21.92	21.01	20.17	19.39
47,460	45.25	41.77	38.78	36.20	33.93	31.94	30.16	28.58	27.15	25.86	24.68	23.61	22.62	21.72	20.88
50,850	48.48	44.75	41.55	38.78	36.36	34.22	32.32	30.62	29.09	27.70	26.44	25.29	24.24	23.27	22.37

NACA

TABLE I - Concluded

EVALUATION OF PROBABILITY DISTRIBUTION OF GUST LOADS - Concluded

(b) Numerical Evaluation of Equation (18a) for the Over-All Probability Distribution of Load Increments

[Exponents in this table refer to factor 10 which was omitted for brevity. Thus, 1.19^{-1} signifies 1.19×10^{-1}.]

$1 - F(\Delta L)$		$1 - F_3\left(\frac{\Delta L}{kV_e}\right)$ for -															$f_4(V_e)$		$1 - F(\Delta L)$
		V_e = 120 mph	V_e = 130 mph	V_e = 140 mph	V_e = 150 mph	V_e = 160 mph	V_e = 170 mph	V_e = 180 mph	V_e = 190 mph	V_e = 200 mph	V_e = 210 mph	V_e = 220 mph	V_e = 230 mph	V_e = 240 mph	V_e = 250 mph	V_e = 260 mph			
$1 - F(4,196)$		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0×1		1.000
$1 - F(6,780)$		1.19^{-1}	2.20^{-1}	2.92^{-1}	4.00^{-1}	5.70^{-1}	7.18^{-1}	7.85^{-1}	9.97^{-1}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00080×4		.892
$1 - F(10,170)$		1.37^{-2}	2.08^{-2}	2.85^{-2}	4.62^{-2}	6.50^{-2}	9.42^{-2}	1.20^{-1}	1.56^{-1}	2.33^{-1}	2.85^{-1}	3.75^{-1}	4.52^{-1}	5.70^{-1}	6.25^{-1}	6.98^{-1}	0.00164×2		.226
$1 - F(13,560)$		1.87^{-3}	3.34^{-3}	5.20^{-3}	7.60^{-3}	1.37^{-2}	1.77^{-2}	2.50^{-2}	3.20^{-2}	4.80^{-2}	6.02^{-2}	7.40^{-2}	1.03^{-1}	1.19^{-1}	1.48^{-1}	2.20^{-1}	0.00325×4		.0474
$1 - F(16,950)$		3.48^{-4}	6.20^{-4}	1.12^{-3}	1.87^{-3}	2.73^{-3}	4.09^{-3}	6.30^{-3}	8.62^{-3}	1.37^{-2}	1.67^{-2}	2.22^{-2}	2.70^{-2}	3.56^{-2}	4.80^{-2}	5.45^{-2}	0.00576×2		.0130
$1 - F(20,340)$		8.85^{-5}	1.70^{-4}	2.80^{-4}	4.66^{-4}	7.44^{-4}	1.27^{-3}	1.87^{-3}	2.43^{-3}	3.81^{-3}	5.20^{-3}	7.15^{-3}	8.58^{-3}	1.37^{-2}	1.58^{-2}	2.19^{-2}	0.00836×4		.00403
$1 - F(23,730)$		2.61^{-5}	5.18^{-5}	8.85^{-5}	1.42^{-4}	2.58^{-4}	3.87^{-4}	5.75^{-4}	8.50^{-4}	1.22^{-3}	1.87^{-3}	2.26^{-3}	3.29^{-3}	4.33^{-3}	6.30^{-3}	7.56^{-3}	0.01185×2		.00140
$1 - F(27,120)$	$= \frac{10}{3}$	8.60^{-6}	1.69^{-5}	3.00^{-5}	5.56^{-5}	8.85^{-5}	1.40^{-4}	2.08^{-4}	3.18^{-4}	4.66^{-4}	6.77^{-4}	9.38^{-4}	1.24^{-3}	1.87^{-3}	2.35^{-3}	3.34^{-3}	0.01525×4	=	.000532
$1 - F(30,510)$		3.19^{-6}	6.30^{-6}	1.17^{-5}	2.00^{-5}	3.47^{-5}	5.60^{-5}	8.85^{-5}	1.31^{-4}	1.98^{-4}	2.80^{-4}	4.15^{-4}	5.40^{-4}	7.44^{-4}	1.10^{-3}	1.32^{-3}	0.01680×2		.000226
$1 - F(33,900)$		1.32^{-6}	2.60^{-6}	4.82^{-6}	8.60^{-6}	1.50^{-5}	2.47^{-5}	3.80^{-5}	5.80^{-5}	8.85^{-5}	1.29^{-4}	1.92^{-4}	2.67^{-4}	3.48^{-4}	4.66^{-4}	6.20^{-4}	0.01469×4		.000104
$1 - F(37,290)$		6.22^{-7}	1.21^{-6}	2.20^{-6}	3.83^{-6}	6.62^{-6}	1.07^{-5}	1.80^{-5}	2.68^{-5}	4.00^{-5}	5.82^{-5}	8.85^{-5}	1.26^{-4}	1.81^{-4}	2.30^{-4}	2.92^{-4}	0.01155×2		.0000485
$1 - F(40,680)$		3.00^{-7}	5.95^{-7}	1.04^{-6}	1.92^{-6}	3.19^{-6}	5.10^{-6}	8.60^{-6}	1.37^{-5}	2.00^{-5}	3.00^{-5}	4.18^{-5}	6.00^{-5}	8.85^{-5}	1.22^{-4}	1.70^{-4}	0.00695×4		.0000240
$1 - F(44,070)$		1.42^{-7}	3.00^{-7}	5.44^{-7}	9.83^{-7}	1.62^{-6}	2.82^{-6}	4.43^{-6}	6.75^{-6}	1.00^{-5}	1.56^{-5}	2.43^{-5}	3.32^{-5}	4.56^{-5}	6.58^{-5}	8.85^{-5}	0.00300×2		.0000128
$1 - F(47,460)$		6.90^{-8}	1.52^{-7}	3.00^{-7}	5.28^{-7}	9.38^{-7}	1.54^{-6}	2.33^{-6}	3.72^{-6}	5.58^{-6}	8.60^{-6}	1.28^{-5}	1.82^{-5}	2.61^{-5}	3.59^{-5}	5.18^{-5}	0.00070×4		.00000700
$1 - F(50,850)$		3.50^{-8}	7.80^{-8}	1.57^{-7}	3.00^{-7}	5.20^{-7}	8.42^{-7}	1.32^{-6}	2.13^{-6}	3.19^{-6}	4.82^{-6}	7.18^{-6}	1.00^{-5}	1.50^{-5}	2.00^{-5}	2.70^{-5}	0×1		.00000390

NACA

TABLE II

EVALUATION OF PROBABILITY DISTRIBUTION OF LOADS

BY AIRSPEED BRACKET

(a) Values of $U_e = \frac{\Delta L}{k\bar{V}_e}$ for Each Airspeed Bracket

Load increment, ΔL (lb)	$\Delta L/k\bar{V}_e$ for airspeed bracket -						
	120 to 140 mph ($\bar{V}_e$ = 133.389 mph)	140 to 160 mph ($\bar{V}_e$ = 152.019 mph)	160 to 180 mph ($\bar{V}_e$ = 171.193 mph)	180 to 200 mph ($\bar{V}_e$ = 190.488 mph)	200 to 220 mph ($\bar{V}_e$ = 209.462 mph)	220 to 240 mph ($\bar{V}_e$ = 227.981 mph)	240 to 260 mph ($\bar{V}_e$ = 244.826 mph)
4,196	3.60	3.16	2.80	2.52	2.29	2.11	1.96
6,780	5.81	5.10	4.53	4.07	3.70	3.40	3.17
10,170	8.72	7.65	6.80	6.11	5.55	5.10	4.75
13,560	11.63	10.20	9.06	8.14	7.41	6.80	6.34
16,950	14.54	12.76	11.33	10.18	9.26	8.51	7.92
20,340	17.44	15.31	13.59	12.22	11.11	10.21	9.50
23,730	20.35	17.86	15.86	14.25	12.96	11.91	11.09
27,120	23.26	20.41	18.12	16.29	14.81	13.61	12.67
30,510	26.17	22.96	20.39	18.32	16.66	15.31	14.26
33,900	29.07	25.51	22.65	20.36	18.52	17.01	15.84
37,290	31.98	28.06	24.92	22.40	20.37	18.71	17.42
40,680	34.89	30.61	27.19	24.43	22.22	20.41	19.01
44,070	37.80	33.17	29.45	26.47	24.07	22.11	20.59
47,460	40.70	35.72	31.72	28.50	25.92	23.82	22.18
50,850	43.61	38.27	33.98	30.54	27.77	25.52	23.76
54,240	46.52	40.82	36.25	32.58	29.62	27.22	25.35
57,630	49.43	43.37	38.51	34.61	31.48	28.92	26.93
61,020	52.33	45.92	40.78	36.65	33.33	30.62	28.51
64,410	55.24	48.47	43.04	38.68	35.18	32.32	30.10
67,800	58.15	51.02	45.31	40.72	37.03	34.02	31.68

NACA

TABLE II - Continued

EVALUATION OF PROBABILITY DISTRIBUTION OF LOADS

BY AIRSPEED BRACKET - Continued

(b) Values of $1 - F_3\left(\frac{\Delta L}{k\overline{V}_e}\right)$ and $F_4\left(V_{e_2}\right) - F_4\left(V_{e_1}\right)$ for each airspeed bracket

[Exponents in this table refer to factor 10 which was omitted for brevity. Thus, 1.19^{-1} signifies 1.19×10^{-1}.]

Load increment, ΔL (lb)	$1 - F_3\left(\frac{\Delta L}{k\overline{V}_e}\right)$ for airspeed bracket -						
	120 to 140 mph	140 to 160 mph	160 to 180 mph	180 to 200 mph	200 to 220 mph	220 to 240 mph	240 to 260 mph
4,196	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6,780	2.27^{-1}	3.88^{-1}	6.78^{-1}	9.43^{-1}	1.00	1.00	1.00
10,170	2.29^{-2}	5.24^{-2}	1.08^{-1}	1.58^{-1}	2.88^{-1}	3.88^{-1}	6.27^{-1}
13,560	3.82^{-3}	8.65^{-3}	1.80^{-2}	3.22^{-2}	5.60^{-2}	1.08^{-1}	1.30^{-1}
16,950	7.70^{-4}	2.05^{-3}	4.20^{-3}	8.65^{-3}	1.72^{-2}	2.58^{-2}	4.40^{-2}
20,340	2.00^{-4}	5.22^{-4}	1.29^{-3}	2.58^{-3}	4.80^{-3}	8.65^{-3}	1.48^{-2}
23,730	6.00^{-5}	1.82^{-4}	4.18^{-4}	8.75^{-4}	2.00^{-3}	3.12^{-3}	4.80^{-3}
27,120	2.08^{-5}	5.80^{-5}	1.48^{-4}	3.32^{-4}	6.78^{-4}	1.29^{-3}	2.08^{-3}
30,510	7.48^{-6}	2.38^{-5}	5.80^{-5}	1.33^{-4}	2.88^{-4}	5.22^{-4}	8.75^{-4}
33,900	3.22^{-6}	9.18^{-6}	2.62^{-5}	6.00^{-5}	1.27^{-4}	2.30^{-4}	4.30^{-4}
37,290	1.52^{-6}	4.22^{-6}	1.18^{-5}	2.70^{-5}	5.80^{-5}	1.20^{-4}	1.90^{-4}
40,680	7.40^{-7}	2.17^{-6}	5.50^{-6}	1.38^{-5}	2.89^{-5}	5.80^{-5}	1.00^{-4}
44,070	3.68^{-7}	1.08^{-6}	2.83^{-6}	7.08^{-6}	1.52^{-5}	2.95^{-5}	5.65^{-5}
47,460	1.90^{-7}	6.00^{-7}	1.62^{-6}	3.80^{-6}	8.33^{-6}	1.68^{-5}	3.08^{-5}
50,850	1.00^{-7}	3.22^{-7}	9.17^{-7}	2.20^{-6}	4.75^{-6}	9.00^{-6}	1.86^{-5}
54,240	5.40^{-8}	1.82^{-7}	5.28^{-7}	1.30^{-6}	2.83^{-6}	5.43^{-6}	9.65^{-6}
57,630	2.82^{-8}	1.02^{-7}	3.13^{-7}	7.85^{-7}	2.19^{-6}	4.45^{-6}	6.05^{-6}
61,020	1.57^{-8}	6.10^{-8}	1.88^{-7}	4.78^{-7}	1.03^{-6}	2.12^{-6}	3.80^{-6}
64,410	8.60^{-9}	3.55^{-8}	1.13^{-7}	3.00^{-7}	6.85^{-7}	1.34^{-6}	2.48^{-6}
67,800	4.89^{-9}	2.04^{-8}	6.70^{-8}	1.88^{-7}	4.38^{-7}	9.00^{-7}	1.60^{-6}
$F_4\left(V_{e_2}\right) - F_4\left(V_{e_1}\right)$	0.0161	0.0680	0.1702	0.2968	0.2884	0.1412	0.0193

NACA

TABLE II - Concluded

EVALUATION OF PROBABILITY DISTRIBUTION OF LOADS

BY AIRSPEED BRACKET - Concluded

(c) Probability Distribution of Load Increments by Airspeed Bracket

$$\left[1 - F_3\left(\frac{\Delta L}{k\overline{V}_e}\right)\right]\left[F_4\left(V_{e_2}\right) - F_4\left(V_{e_1}\right)\right]$$

[Exponents in this table refer to factor 10 which was omitted for brevity. Thus, 1.19^{-1} signifies 1.19×10^{-1}.]

Load increment, ΔL (lb)	$\left[1 - F_3(\Delta L/k\overline{V}_e)\right]\left[F_4(V_{e_2}) - F_4(V_{e_1})\right]$ for airspeed bracket -							$1 - F(\Delta L)$
	120 to 140 mph	140 to 160 mph	160 to 180 mph	180 to 200 mph	200 to 220 mph	220 to 240 mph	240 to 260 mph	
4,196	1.61^{-2}	6.80^{-2}	1.70^{-1}	2.97^{-1}	2.88^{-1}	1.41^{-1}	1.93^{-2}	9.99^{-1}
6,780	3.65^{-3}	2.64^{-2}	1.15^{-1}	2.80^{-1}	2.88^{-1}	1.41^{-1}	1.93^{-2}	8.73^{-1}
10,170	3.69^{-4}	3.56^{-3}	1.84^{-2}	4.69^{-2}	8.31^{-2}	5.48^{-2}	1.21^{-2}	2.19^{-1}
13,560	6.15^{-5}	5.88^{-4}	3.06^{-3}	9.56^{-3}	1.62^{-2}	1.52^{-2}	2.51^{-3}	4.72^{-2}
16,950	1.24^{-5}	1.39^{-4}	7.15^{-4}	2.57^{-3}	4.96^{-3}	3.64^{-3}	8.49^{-4}	1.29^{-2}
20,340	3.22^{-6}	3.55^{-5}	2.20^{-4}	7.66^{-4}	1.38^{-3}	1.22^{-3}	2.86^{-4}	3.91^{-3}
23,730	9.66^{-7}	1.24^{-5}	7.11^{-5}	2.60^{-4}	5.77^{-4}	4.41^{-4}	9.26^{-5}	1.46^{-3}
27,120	3.35^{-7}	3.94^{-6}	2.52^{-5}	9.85^{-5}	1.96^{-4}	1.82^{-4}	4.01^{-5}	5.46^{-4}
30,510	1.20^{-7}	1.62^{-6}	9.87^{-6}	3.95^{-5}	8.31^{-5}	7.37^{-5}	1.69^{-5}	2.25^{-4}
33,900	5.18^{-8}	6.24^{-7}	4.46^{-6}	1.78^{-5}	3.66^{-5}	3.25^{-5}	8.30^{-6}	1.00^{-4}
37,290	2.45^{-8}	2.87^{-7}	2.01^{-6}	8.01^{-6}	1.67^{-5}	1.69^{-5}	3.67^{-6}	4.76^{-5}
40,680	1.19^{-8}	1.48^{-7}	9.36^{-7}	4.10^{-6}	8.33^{-6}	8.19^{-6}	1.93^{-6}	2.36^{-5}
44,070	5.92^{-9}	7.34^{-8}	4.82^{-7}	2.10^{-6}	4.38^{-6}	4.17^{-6}	1.09^{-6}	1.23^{-5}
47,460	3.06^{-9}	4.08^{-8}	2.76^{-7}	1.13^{-6}	2.40^{-6}	2.37^{-6}	5.94^{-7}	6.81^{-6}
50,850	1.61^{-9}	2.19^{-8}	1.56^{-7}	6.53^{-7}	1.37^{-6}	1.27^{-6}	3.59^{-7}	3.83^{-6}
54,240	8.69^{-10}	1.24^{-8}	8.99^{-8}	3.86^{-7}	8.16^{-7}	7.67^{-7}	1.86^{-7}	2.26^{-6}
57,630	4.54^{-10}	6.94^{-9}	5.33^{-8}	2.33^{-7}	6.32^{-7}	6.28^{-7}	1.17^{-7}	1.67^{-6}
61,020	2.53^{-10}	4.15^{-9}	3.20^{-8}	1.42^{-7}	2.97^{-7}	2.99^{-7}	7.33^{-8}	8.48^{-7}
64,410	1.38^{-10}	2.41^{-9}	1.92^{-8}	8.90^{-8}	1.98^{-7}	1.89^{-7}	4.79^{-8}	5.46^{-7}
67,800	7.87^{-11}	1.39^{-9}	1.14^{-8}	5.58^{-8}	1.26^{-7}	1.27^{-7}	3.09^{-8}	3.53^{-7}

NACA

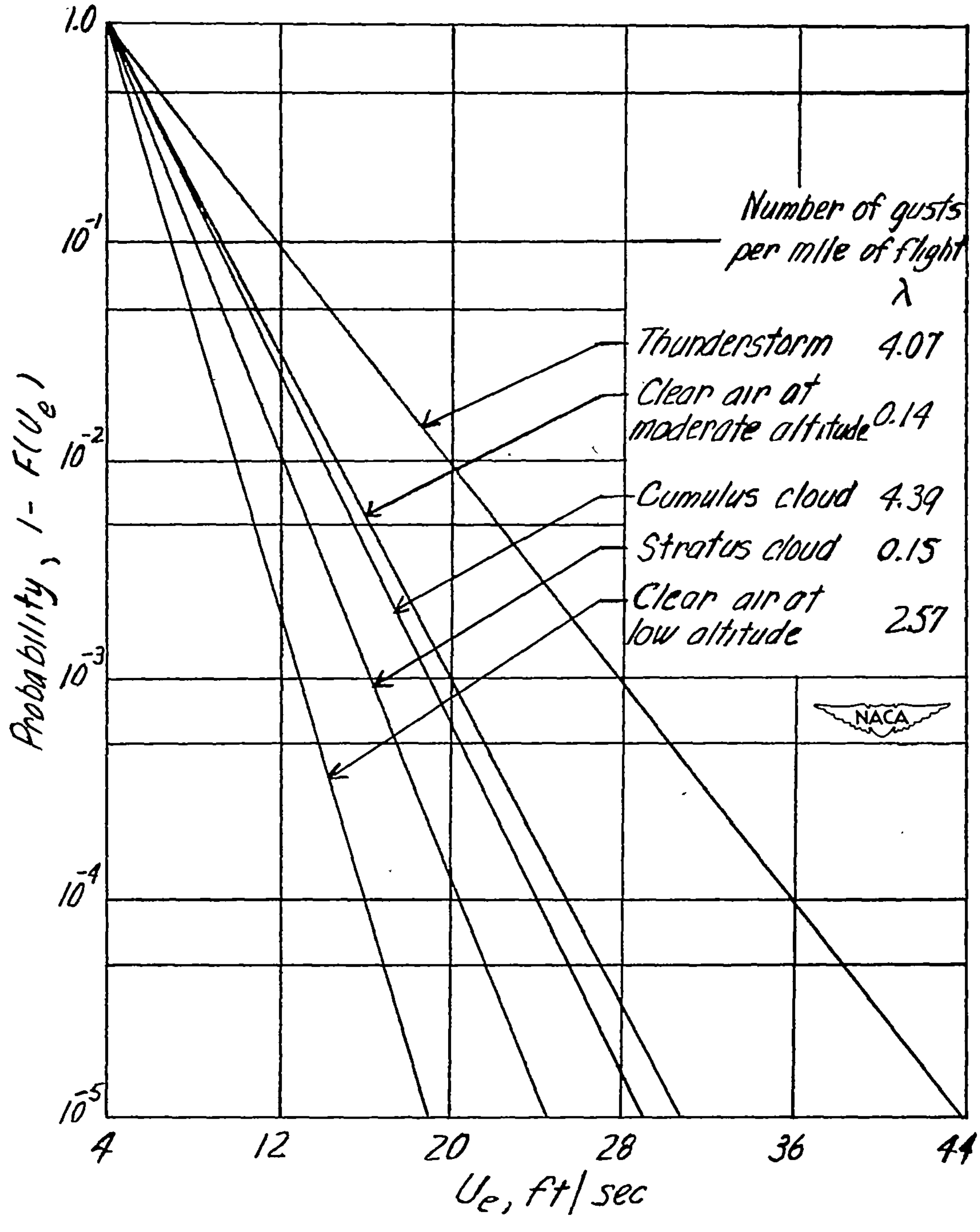

Figure 1.- Probability that an effective gust velocity will exceed a given value for several available flight conditions.

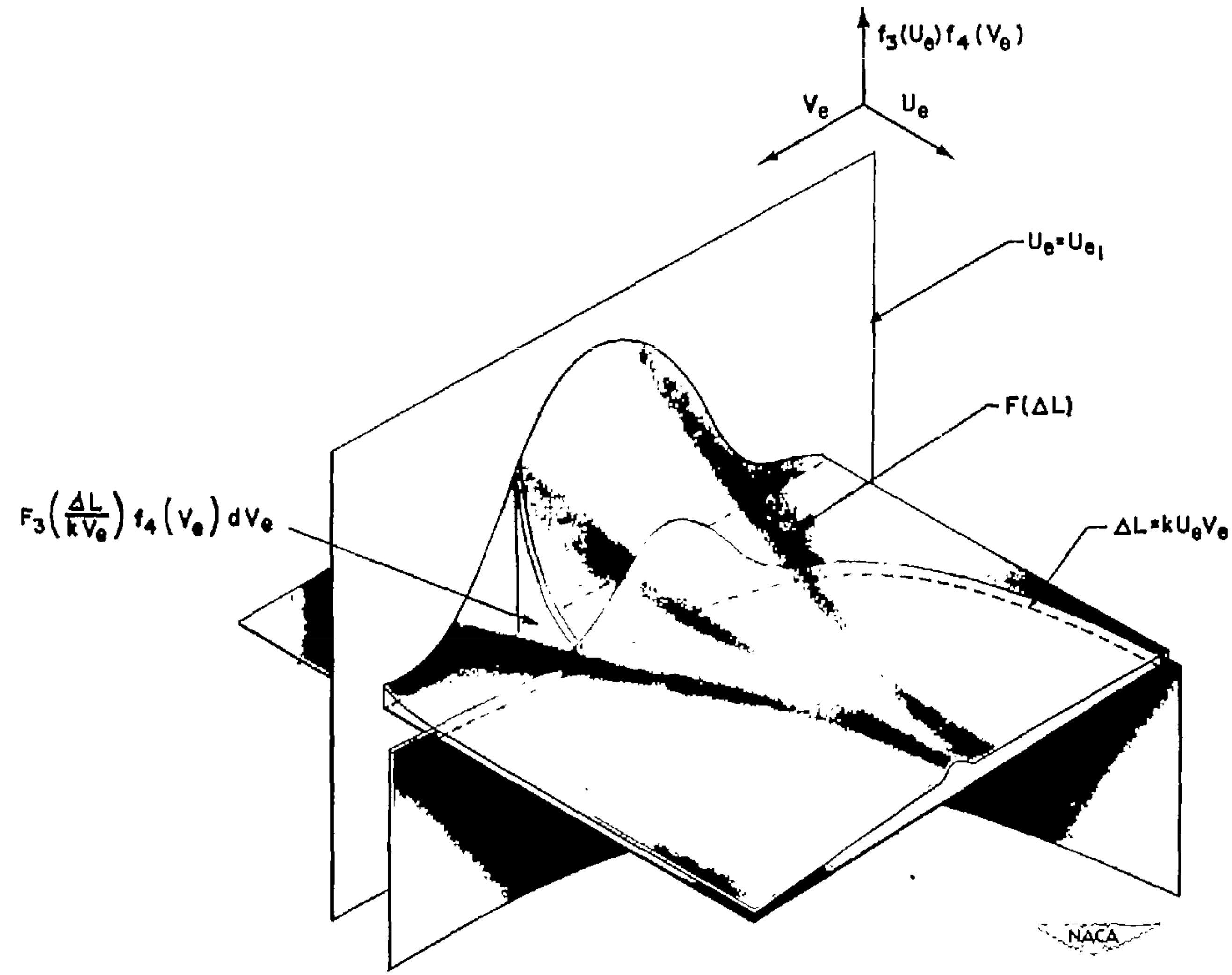

Figure 2.- Pictorial representation of volume integral equation (12),

$$F(\Delta L) = \int F_3\left(\frac{\Delta L}{kV_e}\right) f_4(V_e)\ dV_e.$$

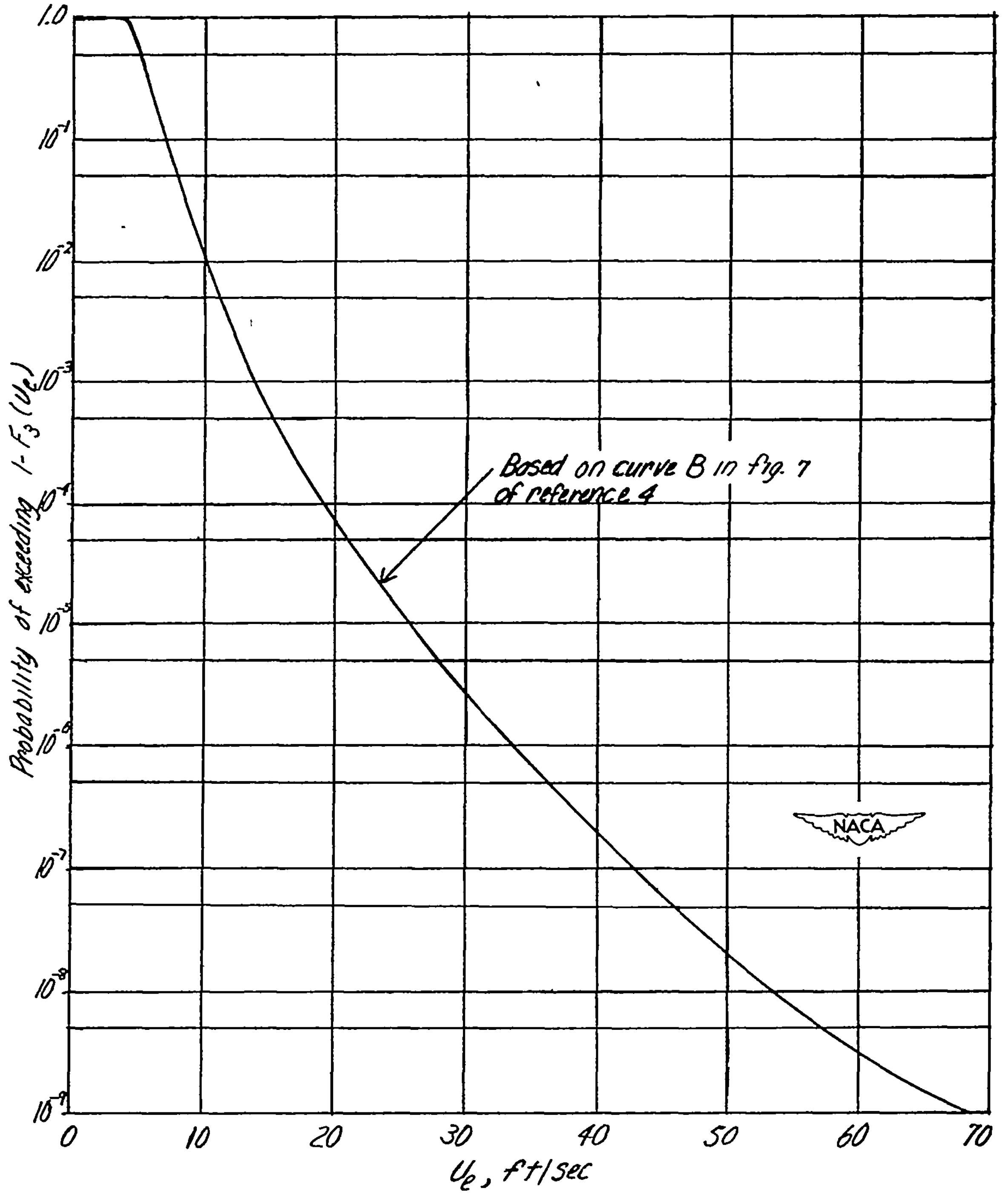

Figure 3.- Probability that a value of effective gust velocity will be greater than indicated value.

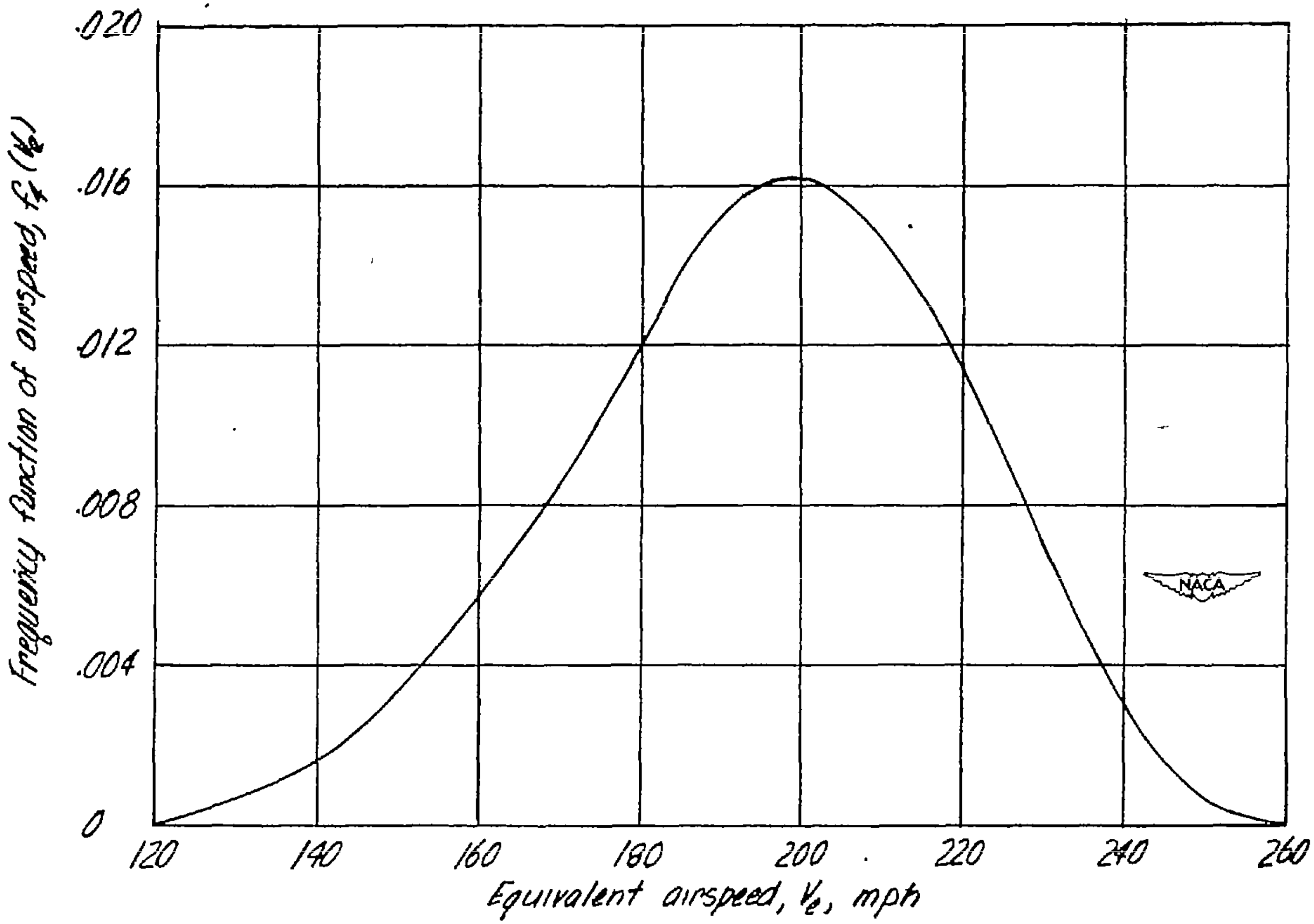

Figure 4.- Frequency function of equivalent airspeed $f_4(V_e)$ indicating the proportion of flight distance covered at given values of airspeed.

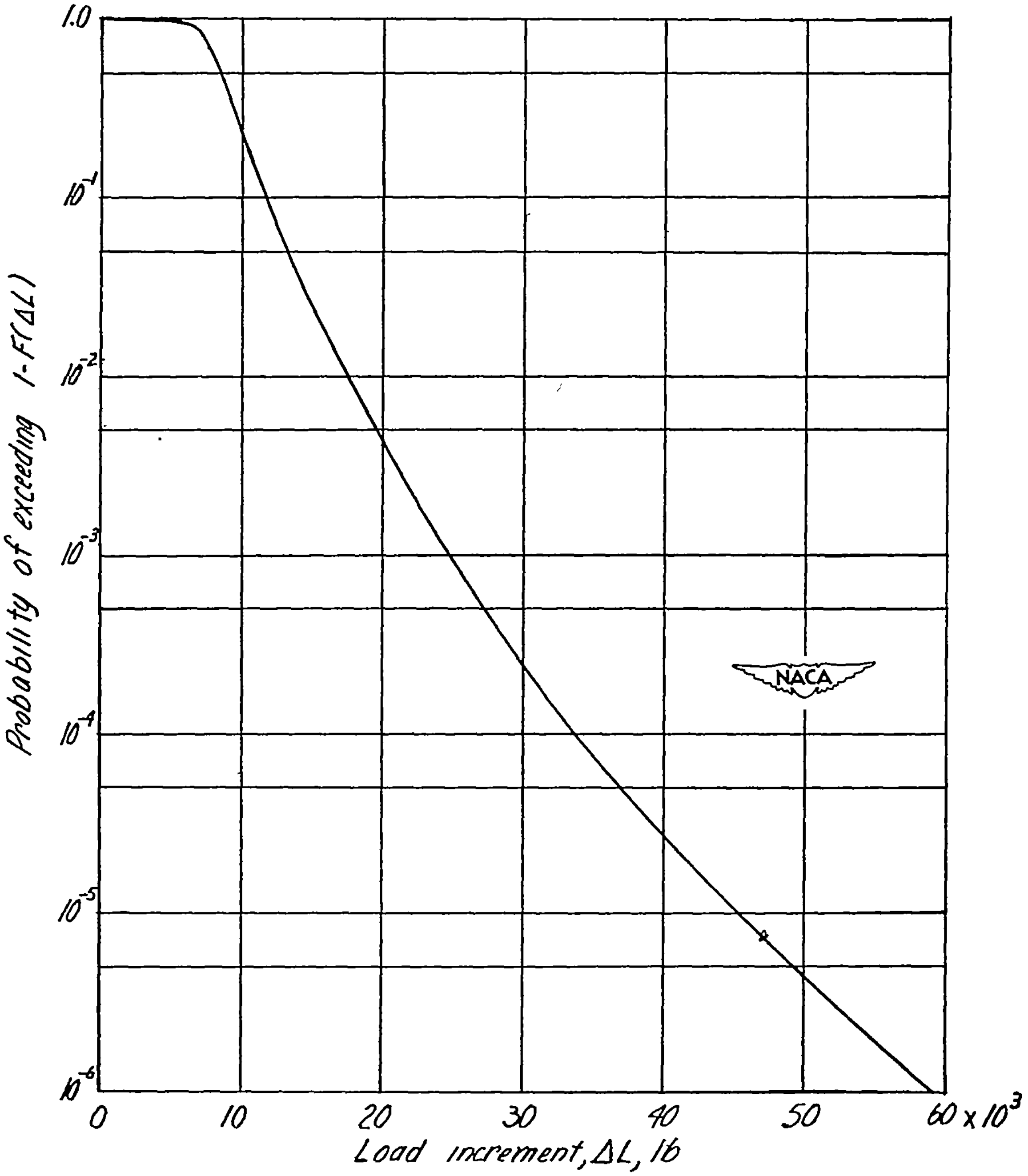

Figure 5.- Probability that a value of load increment will be greater than indicated value.

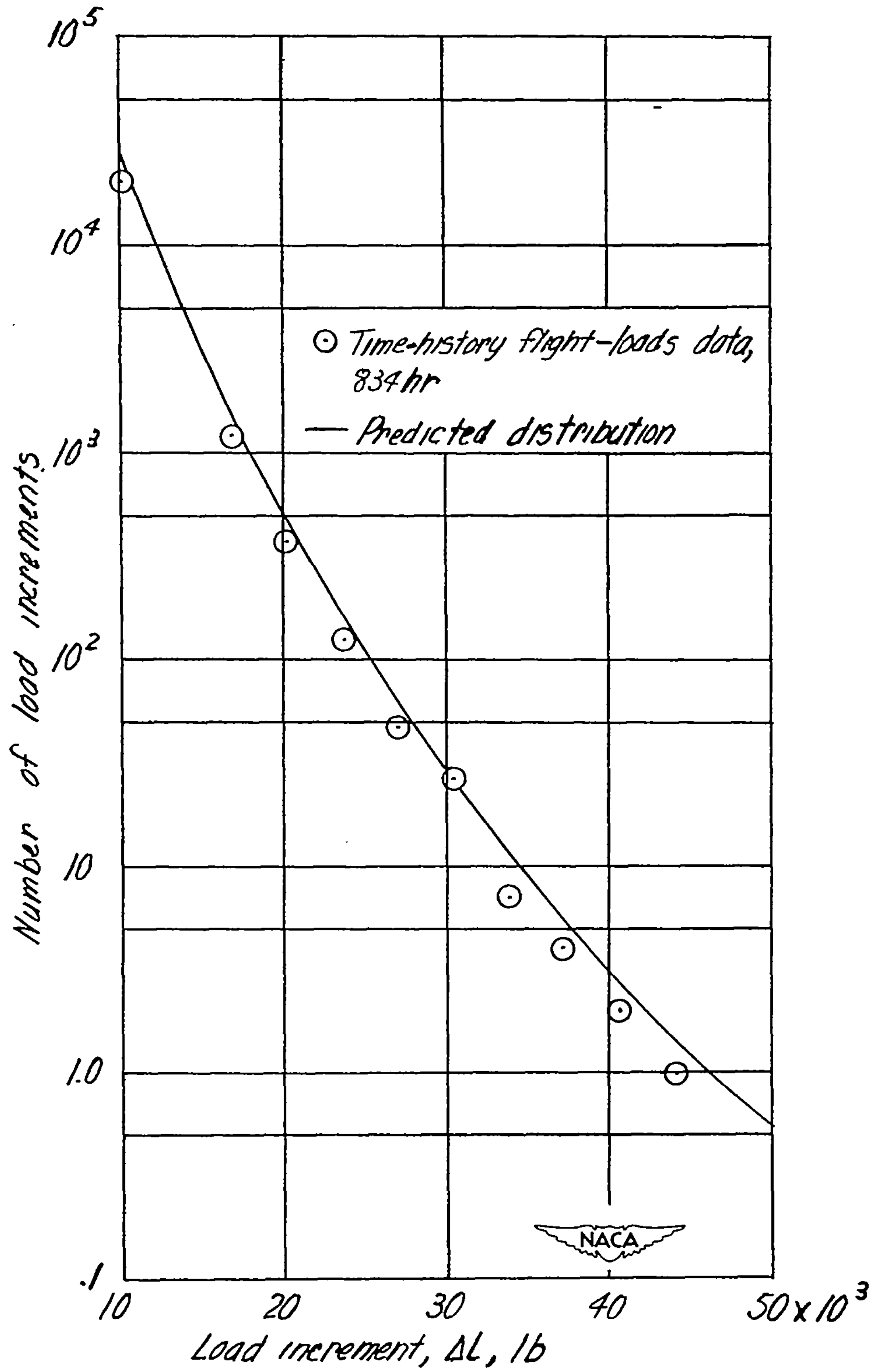

Figure 6.- Comparison of predicted number of load increments greater than indicated value with flight loads data for 834 hours of operation.

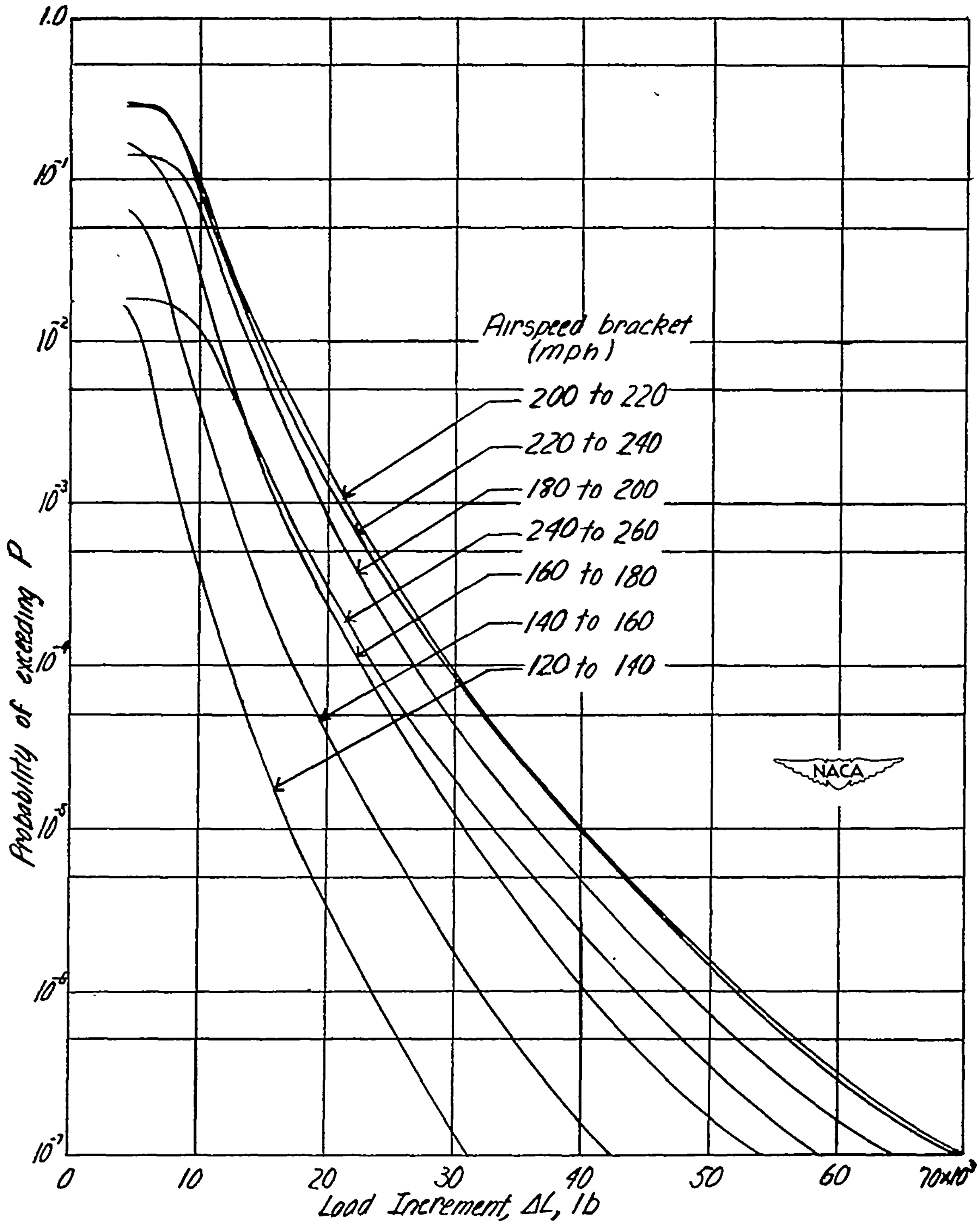

Figure 7.- Probability that a value of load increment will occur in a given airspeed bracket and exceed indicated value.

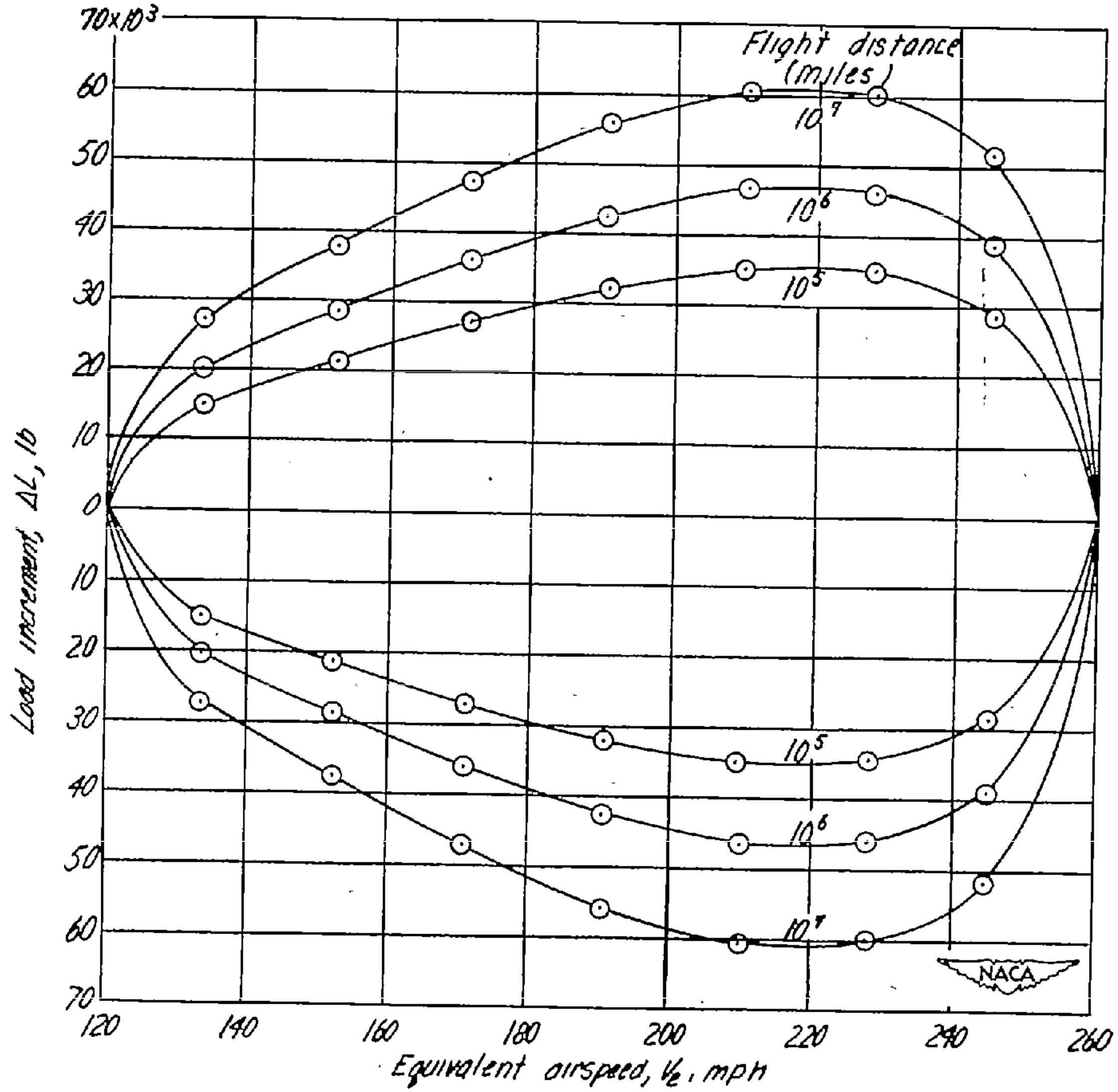

Figure 8.- Airspeed-load-increment envelopes for given flight distances.

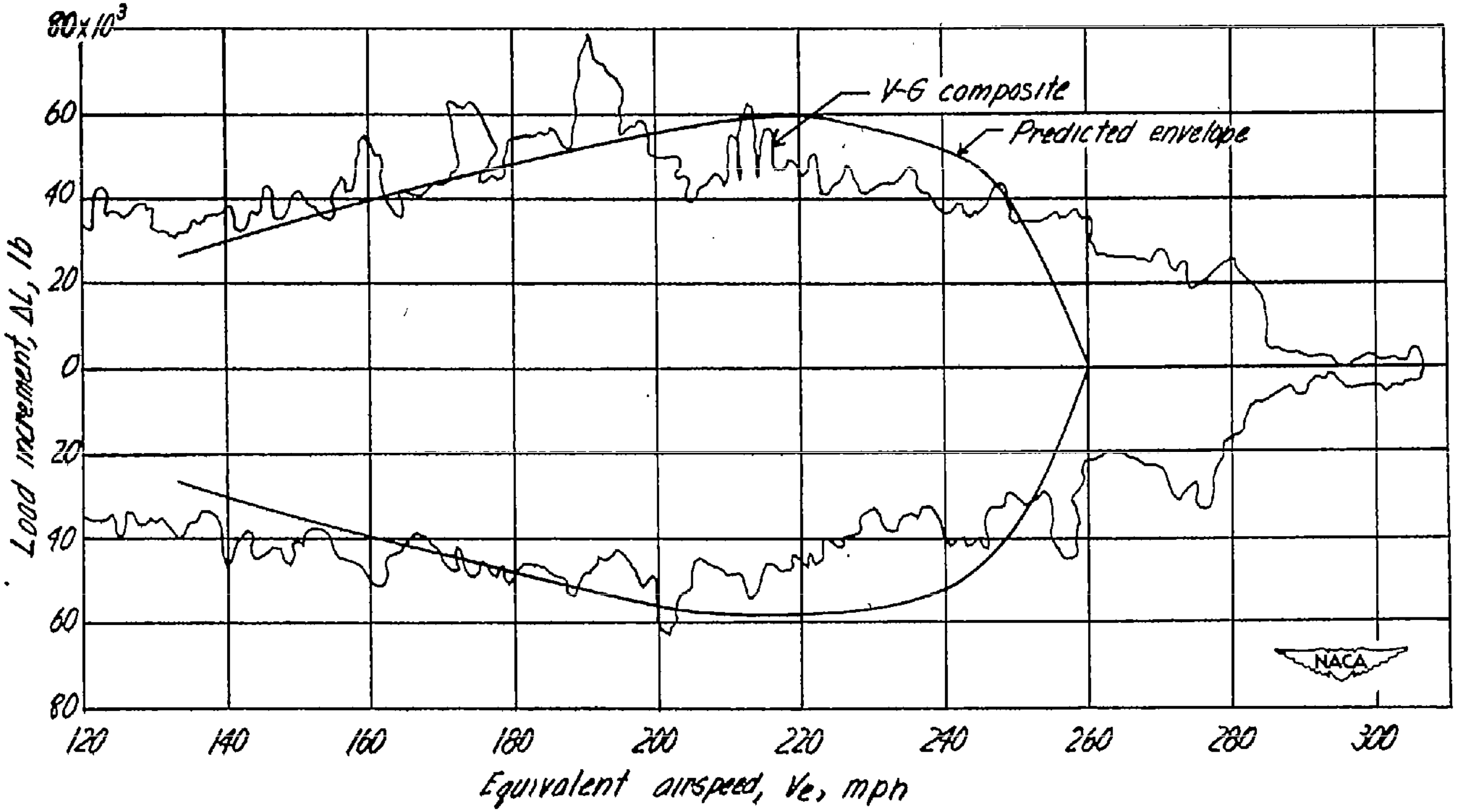

Figure 9.- Comparison of measured and predicted airspeed-load-increment envelopes for 28,116 hours of operation.

NACA-Langley - 4-10-52 - 1000