NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3860

METHOD FOR CALCULATING EFFECTS OF DISSOCIATION ON FLOW VARIABLES IN THE RELAXATION ZONE BEHIND NORMAL SHOCK WAVES

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SUMMARY

Generalized expressions and charts which depend on the shock Mach number, the initial state of the gas, and an enthalpy parameter (the enthalpy divided by the ratio of the pressure to the density) are presented for the temperature, pressure, density, and flow velocity behind a shock wave. The charts and an enthalpy plot for dissociated air have been used to find the relation in graphical form between the degree of dissociation in air and the enthalpy parameter. Plots are presented of the resulting dependence of the flow variables on the degree of dissociation.

Because the chemical reaction rates needed to predict the dependence of degree of dissociation on distance behind the shock are not known, order-of-magnitude estimates of their values have been used in a numerical example, the purpose of which is to illustrate the use of reaction-rate equations to predict relaxation time and distance behind the shock front.

INTRODUCTION

One of the problems associated with flight at hypersonic speeds is the determination of the effects on the air of the high temperatures produced by strong shock waves. Among these effects, dissociation of the diatomic molecules O_2 and N_2 is of considerable concern because the large amount of energy required for dissociation constitutes a heat sink which reduces the air temperature, sometimes by thousands of degrees, from its undissociated value. On the other hand, the tendency of atoms to recombine on a surface and yield the heat of dissociation may constitute an additional heat-transfer mechanism which could cause an increase in aerodynamic heating.

Because dissociation behind a shock wave proceeds at a finite rate, a transition zone exists in which the gas properties gradually approach their equilibrium values at some distance behind the shock front. Similar relaxation zones exist for other degrees of freedom (such as vibration, electronic excitation, and ionization) but this report is concerned only with the effects of dissociation.

Several tabulations of flow properties behind shock waves have been published for air (refs. 1, 2, and 3). Although these calculations take into account the increase in specific heat due to excitation of additional degrees of freedom (including dissociation), the gas is assumed to be in thermal equilibrium; thus, the calculations do not apply in the relaxation zone.

The approach to equilibrium is discussed in reference 1 and approximate expressions valid for small deviations from equilibrium are developed. These expressions are exponential in character. The only treatment found which was essentially different from that of reference 1 was that of reference 4 where expressions derived from kinetic-theory rate equations were integrated to trace the course of the flow variables in the relaxation zone.

This paper presents a method for calculating the variation in the properties of a real gas in the relaxation zone behind a strong shock wave as a function of the degree of dissociation. When numerical results are desired, rate equations can be introduced as a final step to find the variation of the properties with distance behind the shock front.

SYMBOLS

a_1 speed of sound in air at 300° K, 3.475×10^4 cm/sec

D dissociation energy, 117,960 cal/mole for O_2 and 225,080 cal/mole for N_2

d distance behind shock, cm

$G = \frac{N_o}{\bar{R}T_1}$ or $2.45 \times 10^{19}\ \frac{\text{molecules}}{\text{cm}^3}\ \text{atm}^{-1}$ for $T_1 = 300^\circ$ K

g mole fraction

K equilibrium constant based on partial pressures, atm

k_d specific reaction rate for dissociation, $\left(\frac{\text{molecules}}{\text{cm}^3}\right)^{-1} \text{sec}^{-1}$

$k_d' = \dfrac{k_d T^{-\frac{1}{2}}}{7.59 \times 10^{-12}}$, dimensionless	
k_r	specific reaction rate for recombination, $\left(\dfrac{\text{molecules}}{\text{cm}^3}\right)^{-2} \text{sec}^{-1}$
$\overline{M}$	dimensionless velocity based on sound speed in undisturbed gas, v/a_1
M_1	shock Mach number, v_1/a_1
N_o	Avogadro's number, 6.025×10^{23} $\dfrac{\text{molecules}}{\text{mole}}$
p	pressure, atm
R	gas constant per unit mass, $\overline{R}/\mu_1$
$\overline{R}$	universal gas constant, $82 \dfrac{\text{cm}^3\text{-atm}}{\text{mole}} \text{deg}^{-1}$ or $1.987 \dfrac{\text{cal}}{\text{mole}} \text{deg}^{-1}$ where appropriate
r	gross reaction rate, $\dfrac{\text{molecules}}{\text{cm}^3} \text{sec}^{-1}$
T	temperature, $^{\circ}$K
t	reaction time, sec
v	velocity relative to shock front, cm/sec
[X]	total concentration of atoms and molecules, molecules/cm^3
$y = D_{O_2}/\overline{R}T$	
α	degree of dissociation
β	dimensionless enthalpy parameter, enthalpy divided by p/ρ
γ_1	ratio of specific heats in undisturbed gas
μ_1	molecular weight of undissociated gas mixture, $\dfrac{\text{gm}}{\text{mole}}$
ρ	density, gm/cm^3

[ ]	concentration, molecules/cm^3

Subscripts:

1	conditions in undisturbed gas
2	conditions at beginning of dissociation process
d	dissociation of molecule
j	component of gas mixture
r	recombination of atoms
A	atomic form
M	molecular form
N, N_2	atomic and molecular nitrogen, respectively
O, O_2	atomic and molecular oxygen, respectively
R	rare gas

Superscripts:

e	"equilibrium" value
I	dissociation of O_2, N_2 undissociated
II	dissociation of N_2, O_2 completely dissociated

DEVELOPMENT OF THE METHOD

The method for calculating the variation in the properties of a real gas in the relaxation zone behind a strong shock wave is developed in three main steps as follows:

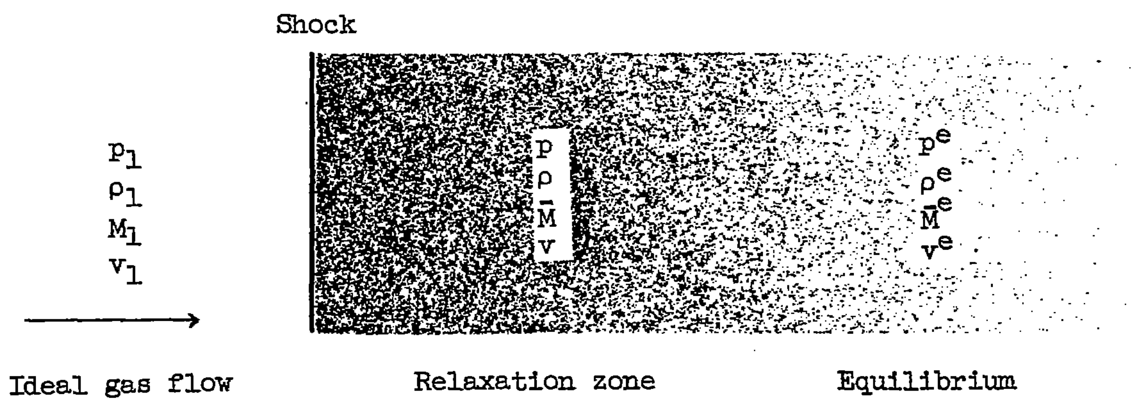
(1) As in the example of reference 1, the normal-shock equations for a gas with variable specific heat are developed in a form which presents the flow properties as a function of the enthalpy parameter. By using an initial γ of 1.4, the relations obtained have been numerically evaluated over suitable ranges of shock Mach number and enthalpy-parameter values; the results are presented in both tabular and graphical form.

(2) For the case of air, enthalpy data from the literature have been used to prepare two plots of the enthalpy parameter as a function of temperature for arbitrarily chosen values of the degree of dissociation. One plot applies to the dissociation of oxygen; the other, to the dissociation of nitrogen. Curves representing a unique relation between temperature, enthalpy parameter, and degree of dissociation behind a shock wave have been found with the aid of the results of the first step and are presented on the two enthalpy parameter plots. Graphical representations of the resulting dependence of the flow variables on degree of dissociation are presented.

(3) From the assumed reaction-rate equations, expressions are developed which give the dependence of the degree of dissociation (and therefore the flow variables) on time (or distance) behind a shock wave in air.

Normal-Shock Equations For a Gas With Variable Specific Heat

In order to establish the normal-shock equations for a gas with finite relaxation time due to dissociation, consider the flow as indicated in the following sketch:



When a coordinate system fixed with respect to the shock wave is used, the oncoming gas undergoes an abrupt change in flow variables upon passing through the shock front. This change is followed by a gradual relaxation zone in which the flow variables at any point behind the shock wave are related to the initial state parameters by:

Energy equation:

$$\beta_1 \frac{p_1}{\rho_1} + \frac{1}{2} v_1^2 = \beta \frac{p}{\rho} + \frac{1}{2} v^2 \tag{1}$$

Momentum equation:

$$p_1 + \rho_1 v_1{}^2 = p + \rho v^2 \tag{2}$$

Continuity equation:

$$\rho_1 v_1 = \rho v \tag{3}$$

In equation (1) the parameter β (ref. 1) is the enthalpy divided by the ratio p/ρ. For a perfect gas with constant specific heat $\beta = \frac{\gamma}{\gamma - 1}$. The value $\frac{\gamma}{\gamma - 1}$ for β is assumed to apply ahead of the shock wave since in most applications this gas is in the range of perfect gas flow. Combining equations (1) to (3) with the equation of state for a partially dissociated gas, which is

$$\frac{p}{\rho} = RT(1 + \alpha) \tag{4}$$

yields the following expression for the flow velocity in the relaxation zone:

$$\bar{M} = \frac{\frac{1 + \gamma_1 M_1{}^2}{\gamma_1 M_1}\beta - \sqrt{\left(\frac{1 + \gamma_1 M_1{}^2}{\gamma_1 M_1}\beta\right)^2 - (2\beta - 1)\left(M_1{}^2 + \frac{2}{\gamma_1 - 1}\right)}}{2\beta - 1} \tag{5}$$

where $\bar{M} = \frac{v}{a_1}$ is the local velocity divided by the free-stream speed of sound ahead of the shock wave. The ratios of the corresponding flow variables are

$$\frac{p}{p_1} = \gamma_1 M_1\left(\frac{1 + \gamma_1 M_1{}^2}{\gamma_1 M_1} - \bar{M}\right) \tag{6}$$

$$\frac{\rho}{\rho_1} = \frac{M_1}{\bar{M}} \tag{7}$$

$$(1 + \alpha)\,\frac{T}{T_1} = \gamma_1\bar{M}\left(\frac{1 + \gamma_1 M_1{}^2}{\gamma_1 M_1} - \bar{M}\right) \tag{8}$$

In equations (5) to (8) the variation of $\bar{M}$, p/p_1, ρ/ρ_1, and $(1 + \alpha)\,\frac{T}{T_1}$ throughout the relaxation zone are, for a given shock Mach number M_1, uniquely related to the variation of β. All values of β must exist from the initial value $\beta = \beta_1 = \frac{\gamma_1}{\gamma_1 - 1}$ to the final value β^e determined by the equilibrium conditions. The relations expressed by equations (5) to (8) are applicable to any gas obeying the assumed equation of state and are given in table I for a range of M_1 from 1 to 20 and β from 3.5 to 10.1. Typical curves are shown in figure 1. For these calculations and curves the value $\gamma_1 = 1.4$ has been used. The point corresponding to $\beta = 3.5$ in each case yields the usual perfect-gas normal-shock results for air.

Determination of the Dependence of β and the Flow Variables on Degree of Dissociation in the Relaxation zone

The variation of β in the relaxation zone is established by the determination of reaction paths in the β,T plane which connect the initial and equilibrium states and along which β, α, and T are uniquely related. In order to do this, it is necessary to relate the α and β values of a mixture to the α_j and β_j of its components by the following relations (ref. 1):

$$\alpha = \sum g_j \alpha_j \tag{9}$$

$$\beta(1 + \alpha) = \sum g_j \beta_j (1 + \alpha_j) \tag{10}$$

$$\beta_j = \frac{(1 - \alpha_j)(\beta_M)_j}{1 + \alpha_j} + \frac{\alpha_j}{1 + \alpha_j}\left[\frac{D_j}{\bar{R}T} + 2(\beta_A)_j\right] \tag{11}$$

Figure 2 shows β_j as a function of T for O_2, O, N_2, and N, which are the principal constituents of partially dissociated air.

Reference 5 was the data source for enthalpy below 8,000° K and reference 6, above 8,000° K. Both sources assume that rotation, vibration, and electronic excitation are in equilibrium. (Some new β_j values (ref. 7) which disagree with ref. 6 were published while this report was in preparation. These data (ref. 7) are also shown in fig. 2.) For the rare gases, a value of β_j of 2.5 was used. The dissociation energies were taken from references 8 and 9. The composition of air was assumed to be as follows: $g_{N_2} = 0.7805$; $g_{O_2} = 0.2103$; and $g_R = 0.0092$.

Equations (9) and (10) show that α and β are not uniquely related if more than one α_j is a variable. Fortunately, the rates of dissociation of nitrogen and oxygen are of different orders of magnitude (ref. 4) so that it is usually possible to treat the dissociation of oxygen by ignoring the dissociation of nitrogen and to treat the dissociation of nitrogen by considering oxygen to be completely dissociated. Ionization and the formation of nitric oxide have been neglected.

Dissociation of O_2, N_2 undissociated.- The first case considered is O_2 dissociated, N_2 undissociated. This condition is denoted by the superscript I. From equation (9),

$$\alpha = g_{O_2}\alpha_{O_2} \tag{12}$$

From equations (10) and (11),

$$\beta^I(1 + \alpha) = g_{O_2}\left[\left(1 - \alpha_{O_2}\right)\beta_{O_2} + \alpha_{O_2}\left(\frac{D_{O_2}}{RT} + 2\beta_O\right)\right] + g_{N_2}\beta_{N_2} + g_R\beta_R$$

or, by introducing equation (12),

$$\beta^I = \frac{\beta_2^I + \alpha f_{O_2}}{1 + \alpha} \tag{13}$$

where

$$\beta_2^I = g_{O_2}\beta_{O_2} + g_{N_2}\beta_{N_2} + g_R\beta_R$$

and

$$f_{O_2} = \frac{D_{O_2}}{RT} + 2\beta_O - \beta_{O_2}$$

Dissociation of N_2, O_2 completely dissociated.- The second case considered is the dissociation of N_2, O_2 completely dissociated. This condition is denoted by the superscript II. From equation (9),

$$\alpha = g_{O_2} + g_{N_2}\alpha_{N_2} \tag{14}$$

From equations (10), (11), and (14)

$$\beta^{II} = \frac{\beta_2^{II} + \alpha f_{N_2}}{1 + \alpha} \tag{15}$$

where

$$\beta_2^{II} = g_{O_2}\left(\frac{D_{O_2}}{\overline{\overline{R}}T} + 2\beta_O\right) + \left(g_{N_2} + g_{O_2}\right)\beta_{N_2} - g_{O_2}\left(\frac{D_{N_2}}{\overline{\overline{R}}T} + 2\beta_N\right) + g_R\beta_R$$

and

$$f_{N_2} = \frac{D_{N_2}}{\overline{\overline{R}}T} + 2\beta_N - \beta_{N_2}$$

Figure 3 shows β^I and β^{II} plotted against T at constant α. For $\alpha = 0.21$, $\beta^I = \beta^{II}$ for all values of T. If the same scale were used in both parts of the figure, they could be joined along the $\alpha = 0.21$ line to form a single plot.

For given values of M_1 and T_1, there is only one point on each of the curves of figure 3 which is consistent with the β,α,T relation shown in figure 1(d). A curve drawn through these points thus traces uniquely in the β,T plane the reaction path appropriate to the shock Mach number and the initial temperature. Its course is independent of the initial pressure except that it terminates on the curve for which $\alpha = \alpha^e$. Since α^e is a function of temperature and pressure, the length of a reaction path depends on the pressure. Some typical reaction paths are shown in figure 3. Also shown in figure 3 are lines representing the relation between α^e, T^e, and p^e. The intersection of one of these "equilibrium" isobars with a reaction path gives the terminal point on the reaction path for the value of p^e concerned. Quotation marks are placed around the word "equilibrium" to indicate that the equilibrium referred to is that which would be attained under the assumptions of cases I and II. The "equilibrium" isobars were plotted from the relations (ref. 1, eq. (1.14)):

$$K_{O_2} = 4p^e \frac{(\alpha^e)^2}{\left(1 + \alpha^e\right)\left(g_{O_2} - \alpha^e\right)} \tag{16}$$

$$K_{N_2} = 4p^e \frac{\left(\alpha^e - g_{O_2}\right)^2}{\left(1 + \alpha^e\right)\left(g_{N_2} + g_{O_2} - \alpha^e\right)} \tag{17}$$

The values of K_{O_2} and K_{N_2} are given in table II. Interpolation to find values not given in the table was done by using the relation

$$\log K \propto 1/T$$

The information contained in the reaction paths is more clearly illustrated by figures 4, 5, and 6, which are cross plots showing β, T, p/p_1, ρ/ρ_1, and $\bar{M}$ as functions of M_1. On these figures any vertical line between two curves of constant α is a reaction path. Some general trends of the effect of dissociation on the flow variables are evident from figure 5. The temperature drop through the dissociation region is almost constant (fig. 5(d)) for a given degree of dissociation. Pressure increases slightly with dissociation (fig. 5(b)) at low shock Mach numbers, the increase becoming smaller as M_1 increases. The changes in density and velocity (figs. 5(a) and 5(c)) exhibit pressure-dependent maxima as the shock Mach number increases. For example, a maximum change in the density ratio occurs at about $M_1 = 10.5$ when the equilibrium pressure is 10^{-2} atmospheres and at about $M_1 = 13$ when $p^e = 100$ atmospheres.

The curves of figure 6 are similar to those of figure 5 but, because of the larger dissociation energy of N_2, a greater range of M_1 values would be required to obtain curves as complete as those of figure 5.

Reference 10 asserts that for sufficiently strong shocks it is possible for pressure to decrease in the relaxation region. Figures 1(b) and 3(a) show that, for air obeying the assumptions of case I, this effect should occur for shock waves that produce a temperature of the order of

12,300° K behind the shock front after adjustment of translation, rotation, vibration, and electronic excitation to equilibrium. The corresponding shock Mach number is 16.9. The shock Mach number at which this effect would occur for case II is greater than the largest considered herein $(M_1 = 20)$. A more general argument for the possibility that a reversal in sign of the pressure change (and also the density change) occurs for sufficiently strong shocks is given in the appendix.

Determination of the Dependence of the Flow Variables on Distance Behind a Normal-Shock Wave

In order to determine the variation of the flow variables with distance behind a normal shock wave, the time variation of the degree of dissociation must be determined from reaction-rate equations. Unfortunately, their correct forms are not known with certainty. The equations used here are the same as those of reference 4 except for the assumption that all reaction partners are equally effective. Dissociation is treated as the result of a two-body collision in which only one of the bodies need be an oxygen molecule and in which the specific rate for the reaction is assumed to be the same for all partners. Recombination is treated similarly, the difference being that three-body collisions are involved and two of the collision partners must be oxygen. According to these assumptions, the expression for the rate of change of oxygen concentration with time during dissociation is

$$\frac{d}{dt}\left[O_2\right] = -\frac{1}{2}\frac{d}{dt}\left[O\right] = k_r^I\left[O\right]^2\left[X\right] - k_d^I\left[O_2\right]\left[X\right] = r_r^I - r_d^I = -r^I \tag{18}$$

The k terms are specific reaction rates and are assumed to be functions of temperature only. The r terms with subscripts are gross reaction rates for recombination and dissociation. The r term without a subscript is the net reaction rate.

Combining equations (6) and (8) with the gas law (eq. (4)) written in the following form:

$$p = \left[X\right]\frac{\overline{R}T}{N_o} \tag{19}$$

yields the concentrations appearing in equation (18) in terms of α and $\underline{M}$ as follows:

$$[X] = GM_1p_1 \frac{1 + \alpha}{\bar{M}} \tag{20}$$

$$[O] = GM_1p_1 \frac{2\alpha}{\bar{M}} \tag{21}$$

$$[O_2] = GM_1p_1 \frac{g_{O_2} - \alpha}{\bar{M}} \tag{22}$$

At equilibrium the net reaction rate is zero and the following relations hold:

$$\frac{k_d^I}{k_r^I} = \frac{[O]^2}{[O_2]} = GT_1 \frac{K_{O_2}}{T} \tag{23}$$

Values of the equilibrium constant are given in table II. Values for $T \leq 8{,}000^\circ$ K were taken from reference 5. Correction to latest value of dissociation energy was accomplished by subtracting $\frac{0.4343\Delta D}{1.987T}$ from log K by using the following values:

	D, cal/mole	ΔD, cal/mole
O_2	117,960	788
N_2	225,080	54,860

The value of D for O_2 was taken from reference 8; for N_2, from reference 9. Values of the equilibrium constant for $T > 8{,}000^\circ$ K were calculated from the equilibrium compositions of pure N_2 and O_2 given in reference 6. Correction to latest value of dissociation energy was made for N_2 only, by using $\Delta D = 53{,}960$.

Similar relations can be determined for the dissociation of nitrogen and are as follows:

$$\frac{d}{dt}[N_2] = -\frac{1}{2}\frac{d}{dt}[N] = k_r^{II}[N]^2[X] - k_d^{II}[N_2][X] = r_r^{II} - r_d^{II} = -r^{II} \tag{24}$$

$$[N] = GM_1 p_1 \frac{2(\alpha - g_{O_2})}{\bar{M}} \tag{25}$$

$$[N_2] = GM_1 p_1 \frac{g_{N_2} + g_{O_2} - \alpha}{\bar{M}} \tag{26}$$

$$\frac{k_d^{II}}{k_r^{II}} = \frac{[N]^2}{[N_2]} = GT_1 \frac{K_{N_2}}{T} \tag{27}$$

where $[X]$ is given in equation (20).

Another problem in the application of these rate equations is the lack of well-established values for the specific reaction rates (k terms) There are no experimental values and the theoretical estimates available are not in good agreement, as is illustrated by the curves of figure 7 where $\log_{10} k_d' + 10$ is plotted against $\frac{1}{T}$. The expressions for k_d' shown below are for O_2 and were derived from the rate equations given in the literature:

Source	k_d'
Reference 1 (eq. (2.28))	$\frac{1}{100} ye^{-y}$
Reference 4 (eq. (16))	e^{-y} (approx.)
Reference 11 (p. 710)	$\frac{1}{6} y^3 e^{-y}$
This report	$\frac{9.7K_{O_2}}{T^{3/2}}$

The expression from reference 11 differs from those in references 1 and 4 in two respects:

(1) It was not derived specifically for dissociation but rather for a typical homogeneous bimolecular reaction between simple molecules.

(2) The internal energy of the molecules is considered to be available as activation energy; usually, only the translational energy is taken in account.

The k_d^I values used in this report were obtained from equation (23) by assuming that k_r^I is independent of temperature and has the order of magnitude value $k_r^I = 10^{-32}(\text{molecules/cm}^3)^{-2}\text{sec}^{-1}$. The weak temperature dependence of the atomic recombination rate as compared with the molecular dissociation rate is pointed out in reference 4. Recombination rates for atoms in general are usually of the order of magnitude chosen (refs. 12, 13, and 14). The value of k_d^{II} (for N_2) was obtained from equation (27) by using $k_r^{II} = 10^{-32}(\text{molecules/cm}^3)^{-2}\text{sec}^{-1}$.

With these assumptions regarding the specific reaction rates, the gross reaction rates are

$$r_r^I = \left(GM_1p_1\right)^3 \frac{4\alpha^2(1 + \alpha)}{\bar{M}^3} 10^{-32} \tag{28}$$

$$r_d^I = \frac{GT_1K_{O_2}}{T}\left(GM_1p_1\right)^2 \frac{\left(g_{O_2} - \alpha\right)(1 + \alpha)}{\bar{M}^2} 10^{-32} \tag{29}$$

$$r_r^{II} = \left(GM_1p_1\right)^3 \frac{4\left(\alpha - g_{O_2}\right)^2(1 + \alpha)}{\bar{M}^3} 10^{-32} \tag{30}$$

$$r_d^{II} = \frac{GT_1K_{N_2}}{T}\left(GM_1p_1\right)^2 \frac{\left(g_{N_2} + g_{O_2} - \alpha\right)(1 + \alpha)}{\bar{M}^2} 10^{-32} \tag{31}$$

The time required to traverse any small portion of the reaction path is from equation (18)

$$\Delta t^I = \int_T^{T+\Delta T} \frac{1}{2r^I}\left(\frac{d}{dT}[O]\right) dT \tag{32}$$

In the cases examined, $\frac{d}{dT}[O]$ along the reaction path was so nearly constant that the following expression was used:

$$\Delta t^{I} = \frac{1}{2}\left(\frac{d}{dT}[O]\right)_{av} \int_{T}^{T+\Delta T} \frac{dT}{r^{I}} \tag{33}$$

where the average was taken over the temperature interval ΔT. Since $\overline{M}$ varies slowly along the reaction path, the distance covered by a particle in the time Δt^{I} is

$$\Delta d^{I} = a_1 \overline{M}_{av} \, \Delta t^{I} \tag{34}$$

Similar equations were found to hold for dissociation of N_2. In the evaluation of these equations for the specific cases to be discussed subsequently, ΔT was chosen to be some convenient temperature interval, say 200° K. The integral in equation (33) was evaluated graphically. Time and distance measured from the shock front were obtained by summing the appropriate values of Δt and Δd.

As has been previously mentioned, the course of a reaction path is independent of the initial pressure except in length. The net reaction rate r^{I}, which is given by the difference between equations (28) and (29), is proportional to $p_1{}^2$ when $r_r^{I} \ll r_d^{I}$. This inequality is always satisfied at the initial point of the reaction path and is satisfied over practically all the reaction path when the pressure and temperature are such that "equilibrium" corresponds to practically complete dissociation of the component concerned. Equations (33) and (34) show that t and d are proportional to $1/p_1$ when the above inequality holds and lead to the useful observation that t and d calculated at one initial pressure can, with certain limitations, be used at other pressures if they are multiplied by the ratio of the first value of p_1 to the second value.

APPLICATION TO A MACH NUMBER 14 SHOCK WAVE MOVING INTO AIR AT $T_1 = 300°$ K AND $p_1 = 10^{-4}$ ATMOSPHERES

The results of the previous sections are now applied to a Mach number 14 shock wave in air at 300° K and 10^{-4} atmospheres. As has been previously discussed, the dissociation of O_2 is essentially completed prior to the start of the dissociation of N_2. Hence, it is convenient to divide the calculation into two parts, the dissociation of O_2 and the dissociation of N_2.

Dissociation of O_2, N_2 Undissociated

For the first part of the calculation, O_2 is considered to be dissociated and N_2, undissociated. Tabulation of the values of T, β, p/p_1, $\overline{M}$, and α along the $M_1 = 14$ reaction path were made from figures 1 and 3(a). The strength of a Mach 14 shock is about 240 (fig. 5(b)) so that the "equilibrium" pressure should be about 2.4×10^{-2} atmospheres. Figure 3(a) shows that the reaction path intersects the "equilibrium" isobar at $\alpha = 0.21$. Thus, the dissociation of O_2 is practically complete. At each chosen point the gross reaction rates were calculated from equations (28) and (29) and their difference was taken to find the net reaction rate. The time and distance variation of T was then evaluated from equations (33) and (34).

Dissociation of N_2, O_2 Completely Dissociated

The treatment of the condition where N_2 is dissociated and O_2 completely dissociated is similar to the first case except that the "equilibrium" value of α had to be calculated since N_2 does not dissociate completely. This value was determined by plotting r_d^{II} and r_r^{II} against α and reading off the value of α^e from the intersection of the two curves.

The numerical results obtained are given in table III and figure 8. The time required for 90-percent completion of the dissociation of O_2 at 2.4×10^{-2} atmospheres is shown in table III to be 3.3×10^{-6} seconds; the corresponding distance behind the shock is 1.8×10^{-1} centimeters. The time required for 90-percent completion of the dissociation of N_2 at 2.4×10^{-2} atmospheres is 2.9×10^{-2} seconds; the corresponding distance is 1.2×10^3 centimeters.

The example described was chosen in order to illustrate well the types of calculations usually encountered and was not intended to produce typical relaxation distances for O_2 and N_2. In fact, there are no typical relaxation distances, since the reaction rate is dominated by the Boltzmann factor $e^{-D_j/RT}$. This statement means that any relaxation distance obtained for dissociation of O_2 can also be obtained for N_2 by raising the temperature.

A comparison with the result of reference 4 is made in figure 9. All the curves are for $M_1 = 14$, $T_1 = 300^\circ$ K, and N_2 undissociated and are plotted against $t[O_2]_2$, where $[O_2]_2$ is the concentration of O_2 at the beginning of the dissociation process.

This quantity is defined as:

$$[O_2]_2 = GM_1p_1\frac{g_{O_2}}{\overline{M}_2} \tag{35}$$

Dividing equation (29) by $\left\{[O_2]_2\right\}^2$ and inserting the result in equation (18) yields

$$\frac{d\left\{[O_2]/[O_2]_2\right\}}{d\left\{t[O_2]_2\right\}} = -\frac{(1+\alpha)\left(g_{O_2}-\alpha\right)}{\left(g_{O_2}\right)^2}\left(\frac{\overline{M}_2}{\overline{M}}\right)^2\frac{GT_1K_{O_2}}{T}10^{-32} \tag{36}$$

The r_r^I term in equation (18) is negligibly small in this case and has been omitted. Equation (16) from reference 4 written in terms of the same same symbols is

$$\frac{d\left\{[O_2]/[O_2]_2\right\}}{d\left\{t[O_2]_2\right\}} = -\Big[3.25\left(g_{O_2}-\alpha\right) + 4.79(2\alpha) +$$

$$7.1\left(g_{N_2}\right)\Big]\frac{\left(g_{O_2}-\alpha\right)}{\left(g_{O_2}\right)^2}\left(\frac{\overline{M}_2}{\overline{M}}\right)^2\left(1.15\times10^{-12}\right)\sqrt{T}\,e^{-\frac{59,400}{T}}$$

$$\approx -\frac{(1+\alpha)\left(g_{O_2}-\alpha\right)}{\left(g_{O_2}\right)^2}\left(\frac{\overline{M}_2}{\overline{M}}\right)^2(6.6)\left(1.15\times10^{-12}\right)\sqrt{T}\,e^{-\frac{59,400}{T}} \tag{37}$$

One of the curves of figure 9 was taken directly from reference 4. The other two were calculated by the method of this report, one by using equation (36) and the other by using equation (37). The use of equation (37) corresponds to the use of the specific rate from reference 4. The difference remaining after the effect of the different choice of specific rates is removed is attributed to the use in reference 4 of fixed average values of β_j for the components of air.

The large difference between the numerical results obtained here and those in reference 4 illustrates the fact that current computations of relaxation times must be regarded as highly tentative.

CONCLUDING REMARKS

Generalized expressions and charts have been presented for the temperature, pressure, density, and flow velocity behind a shock wave that depend on the shock Mach number, the initial state of the gas, and an enthalpy parameter (the enthalpy divided by the ratio of pressure to density). The variation of this enthalpy parameter behind the shock was related directly to variation of the degree of dissociation.

Order-of-magnitude chemical reaction rates were used to illustrate the application of these results for the prediction of the variation of flow properties with distance behind a Mach number 14 shock in air at 300° K and 10^{-4} atmospheres. This calculation yielded a relaxation time (based on 90-percent completion of the composition change) of 3.3×10^{-6} seconds for oxygen. This relaxation time is much shorter than that obtained in NACA TN 3634; however, if the same reaction rate is used, the relaxation time agrees with that of NACA TN 3634.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., August 29, 1956.

APPENDIX

AN ARGUMENT WHICH SHOWS THAT DECREASING PRESSURE AND DENSITY BEHIND A SHOCK WAVE CAN BE CONSISTENT WITH DECREASING TEMPERATURE

The momentum equation and the continuity equation can be combined to give

$$dp = v^2 d\rho \tag{A1}$$

This relation shows that the sign of the pressure change is always the same as that of the density change. The following equation is derived from the equation of state:

$$\frac{dp}{p} - \frac{d\rho}{\rho} = \frac{dT}{T} + \frac{d\alpha}{1 + \alpha} \tag{A2}$$

Under the assumptions of case I the effect of dissociation dominates ordinary heat-capacity effects so that

$$dT = c\,d\alpha \tag{A3}$$

where c is a slowly varying negative parameter.

Consider the effect of a fixed change in α, $d\alpha > 0$, at a fixed value of α but over a wide range of T-values. The following assertions can be made:

(a) $\dfrac{d\alpha}{1 + \alpha} = \text{constant} > 0$

(b) $\left|\dfrac{dp}{p}\right| \leq \left|\dfrac{d\rho}{\rho}\right|$, since $\left|\dfrac{dp}{p}\right| = \left|\dfrac{\rho v\ dv}{p_1 + (\rho v)(v_1 - v)}\right| \leq \left|\dfrac{dv}{v_1 - v}\right| \leq \left|\dfrac{dv}{v}\right| = \left|\dfrac{d\rho}{\rho}\right|$

where $v_1 \gg v$

(c) At low Mach number it is known that the left-hand side of equation (A2) is negative. This relation implies that

$$\left|\frac{dT}{T}\right| > \left|\frac{d\alpha}{1 + \alpha}\right|$$

(d) If a reversal in trend of p and ρ relaxation occurs, $dp = 0$ and $d\rho = 0$ occur simultaneously. At this point,

$$-\frac{dT}{T} = \frac{d\alpha}{1 + \alpha}$$

(e) The reversal in trend would require that at high Mach number the left-hand side of equation (A2) be positive. This relation implies that

$$\left|\frac{dT}{T}\right| < \left|\frac{d\alpha}{1 + \alpha}\right|$$

These considerations show that a reversal of sign in the density and pressure changes can occur without a reversal in sign of the change in temperature. Numerical values taken from figure 5(d) agree with these statements. For example, when $1 + \alpha = 1.02$ and $d\alpha = 0.00102$:

$$\frac{d\alpha}{1 + \alpha} = 10^{-3}$$

At $5,000^\circ$ K, $c = -13,550$ and

$$\left|\frac{dT}{T}\right| = \frac{13.83}{5,000} = 2.77 \times 10^{-3} > \frac{d\alpha}{1 + \alpha}$$

At $16,000^\circ$ K, $c = -7850$ and

$$\left|\frac{dT}{T}\right| = \frac{8.00}{16,000} = 0.5 \times 10^{-3} < \frac{d\alpha}{1 + \alpha}$$

The reversal temperature is different for each choice of α. For the value chosen ($\alpha = 0.02$), it is 11,300° K. For $\alpha = 0$, it is 11,200° K; for $\alpha = 0.21$, it is 12,500° K.

At 11,300° K, $c = -11,080$ and

$$\frac{dT}{T} = \frac{11.3}{11,300} = 10^{-3} = \frac{d\alpha}{1 + \alpha}$$

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TABLE I.- FLOW VARIABLES BEHIND A NORMAL SHOCK WAVE AS A FUNCTION OF THE ENTHALPY PARAMETER

Enthalpy parameter, β	Values of $\bar{M}$ for values of M_1 of -																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3.5	1.000	0.750	0.778	0.875	1.000	1.139	1.286	1.438	1.593	1.750	1.909	2.069	2.231	2.393	2.556	2.719	2.882	3.046	3.211	3.375
3.7	.780	.691	.724	.817	.935	1.066	1.204	1.347	1.492	1.640	1.789	1.939	2.091	2.243	2.395	2.548	2.702	2.855	3.009	3.164
3.9	.693	.642	.678	.767	.879	1.002	1.132	1.266	1.404	1.543	1.683	1.825	1.967	2.110	2.254	2.398	2.542	2.687	2.832	2.977
4.1	.630	.600	.637	.723	.829	.945	1.068	1.195	1.325	1.456	1.589	1.723	1.857	1.993	2.128	2.264	2.401	2.537	2.674	2.812
4.3	.581	.563	.601	.683	.784	.895	1.011	1.132	1.255	1.379	1.505	1.632	1.759	1.887	2.016	2.145	2.274	2.404	2.533	2.663
4.5	.540	.530	.569	.648	.744	.849	.960	1.075	1.191	1.310	1.429	1.550	1.671	1.793	1.915	2.037	2.160	2.283	2.407	2.530
4.7	.506	.502	.541	.616	.708	.808	.914	1.023	1.134	1.247	1.361	1.476	1.591	1.707	1.824	1.940	2.057	2.174	2.292	2.409
4.9	.476	.476	.515	.587	.675	.771	.872	.976	1.082	1.190	1.299	1.408	1.519	1.629	1.740	1.852	1.963	2.075	2.187	2.300
5.1	.449	.453	.491	.561	.645	.737	.834	.933	1.035	1.138	1.242	1.347	1.452	1.558	1.665	1.771	1.878	1.985	2.092	2.200
5.3	.426	.432	.470	.537	.617	.706	.799	.894	.992	1.090	1.190	1.291	1.392	1.493	1.595	1.697	1.800	1.902	2.005	2.108
5.5	.405	.413	.450	.514	.592	.677	.766	.858	.952	1.047	1.142	1.239	1.336	1.433	1.531	1.629	1.728	1.826	1.925	2.023
5.7	.387	.395	.432	.494	.569	.651	.737	.825	.915	1.006	1.098	1.191	1.284	1.378	1.472	1.566	1.661	1.756	1.851	1.945
5.9	.370	.379	.415	.475	.548	.627	.709	.794	.881	.969	1.058	1.147	1.237	1.327	1.418	1.508	1.599	1.691	1.782	1.873
6.1	.354	.365	.400	.458	.528	.604	.684	.766	.849	.934	1.020	1.106	1.192	1.280	1.367	1.454	1.542	1.630	1.718	1.806
6.3	.340	.351	.385	.442	.509	.583	.660	.739	.820	.902	.984	1.068	1.151	1.235	1.320	1.404	1.489	1.573	1.659	1.744
6.5	.327	.339	.372	.427	.492	.563	.638	.714	.792	.872	.951	1.032	1.113	1.194	1.276	1.357	1.439	1.521	1.604	1.686
6.7	.315	.327	.360	.413	.476	.545	.617	.691	.767	.843	.921	.999	1.077	1.155	1.234	1.313	1.393	1.472	1.552	1.631
6.9	.303	.316	.348	.400	.461	.528	.598	.669	.743	.817	.892	.967	1.043	1.119	1.196	1.272	1.349	1.426	1.503	1.580
7.1	.293	.306	.337	.387	.447	.512	.579	.649	.720	.792	.865	.938	1.011	1.085	1.159	1.234	1.308	1.383	1.458	1.532
7.3	.283	.296	.327	.376	.434	.496	.562	.630	.699	.769	.839	.910	.982	1.053	1.125	1.197	1.270	1.342	1.415	1.487
7.5	.274	.287	.317	.365	.421	.482	.546	.612	.679	.747	.815	.884	.954	1.023	1.093	1.163	1.233	1.304	1.374	1.445
7.7	.266	.279	.308	.354	.409	.469	.531	.595	.660	.726	.793	.860	.927	.995	1.063	1.131	1.199	1.268	1.336	1.405
7.9	.258	.271	.300	.345	.398	.456	.516	.579	.642	.706	.771	.836	.902	.968	1.034	1.100	1.167	1.233	1.300	1.367
8.1	.250	.263	.292	.335	.387	.444	.503	.563	.625	.688	.751	.814	.878	.942	1.007	1.071	1.136	1.201	1.266	1.331
8.3	.243	.256	.284	.327	.377	.432	.490	.549	.609	.670	.731	.793	.856	.918	.981	1.044	1.107	1.170	1.233	1.297
8.5	.237	.249	.277	.318	.368	.421	.477	.535	.594	.653	.713	.773	.834	.895	.956	1.018	1.079	1.141	1.202	1.264
8.7	.230	.243	.270	.311	.359	.411	.466	.522	.579	.637	.696	.755	.814	.873	.933	.993	1.053	1.113	1.173	1.233
8.9	.224	.237	.263	.303	.350	.401	.455	.509	.565	.622	.679	.737	.794	.852	.911	.969	1.028	1.086	1.145	1.204
9.1	.219	.231	.257	.296	.342	.392	.444	.498	.552	.607	.663	.719	.776	.833	.890	.947	1.004	1.061	1.118	1.176
9.3	.213	.226	.251	.289	.334	.383	.434	.486	.540	.594	.648	.703	.758	.814	.869	.925	.981	1.037	1.093	1.149
9.5	.208	.220	.245	.283	.327	.374	.424	.475	.528	.580	.634	.687	.741	.796	.850	.904	.959	1.014	1.069	1.124
9.7	.203	.215	.240	.276	.320	.366	.415	.465	.516	.568	.620	.672	.725	.778	.831	.885	.938	.992	1.045	1.099
9.9	.199	.211	.234	.270	.313	.358	.406	.455	.505	.556	.607	.658	.710	.762	.814	.866	.918	.971	1.023	1.076
10.1	.194	.206	.229	.265	.306	.351	.398	.446	.495	.544	.594	.644	.695	.746	.797	.848	.899	.950	1.002	1.053

TABLE I.- FLOW VARIABLES BEHIND A NORMAL SHOCK WAVE AS A FUNCTION OF THE ENTHALPY PARAMETER - Continued

Enthalpy parameter, β	Values of p/p_1 for values of M_1 of -																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3.5	1.00	4.50	10.33	18.50	29.00	41.83	57.00	74.50	94.33	116.50	141.00	167.83	197.00	228.50	262.33	298.50	337.00	377.83	421.00	466.50
3.7	1.31	4.66	10.56	18.82	29.45	42.44	57.80	75.52	95.60	118.04	142.85	170.02	199.55	231.44	265.70	302.32	341.30	382.64	426.35	472.42
3.9	1.43	4.80	10.75	19.10	29.85	42.98	58.51	76.42	96.72	119.40	144.48	171.94	201.80	234.04	268.67	305.69	345.09	386.89	431.07	477.64
4.1	1.52	4.92	10.92	19.35	30.20	43.46	59.13	77.21	97.71	120.61	145.93	173.66	203.79	236.34	271.31	308.68	348.46	390.66	435.26	482.28
4.3	1.59	5.02	11.07	19.57	30.51	43.88	59.69	77.92	98.59	121.69	147.22	175.19	205.58	238.41	273.66	311.35	351.47	394.03	439.01	486.43
4.5	1.64	5.12	11.21	19.77	30.79	44.27	60.19	78.56	99.39	122.66	148.39	176.56	207.19	240.26	275.79	313.76	354.19	397.06	442.39	490.16
4.7	1.69	5.20	11.33	19.95	31.05	44.61	60.64	79.14	100.11	123.54	149.44	177.81	208.64	241.94	277.71	315.94	356.64	399.81	445.44	493.54
4.9	1.73	5.27	11.44	20.11	31.28	44.92	61.05	79.67	100.76	124.34	150.40	178.94	209.96	243.46	279.45	317.92	358.87	402.30	448.21	496.61
5.1	1.77	5.33	11.54	20.26	31.49	45.21	61.43	80.15	101.36	125.07	151.27	179.97	211.17	244.86	281.04	319.73	360.90	404.58	450.75	499.41
5.3	1.80	5.39	11.63	20.40	31.68	45.47	61.77	80.58	101.90	125.73	152.07	180.92	212.27	246.13	282.50	321.38	362.77	406.67	453.07	501.98
5.5	1.83	5.44	11.71	20.52	31.85	45.71	62.09	80.99	102.41	126.35	152.81	181.79	213.29	247.31	283.85	322.91	364.49	408.59	455.21	504.35
5.7	1.86	5.49	11.79	20.63	32.02	45.93	62.38	81.36	102.87	126.91	153.49	182.59	214.22	248.39	285.08	324.31	366.07	410.36	457.18	506.53
5.9	1.88	5.54	11.86	20.74	32.17	46.14	62.65	81.70	103.30	127.44	154.11	183.33	215.09	249.39	286.23	325.61	367.54	412.00	459.00	508.55
6.1	1.90	5.58	11.92	20.84	32.30	46.33	62.90	82.02	103.70	127.92	154.70	184.02	215.90	250.32	287.30	326.82	368.90	413.52	460.70	510.42
6.3	1.92	5.62	11.98	20.93	32.43	46.50	63.13	82.32	104.07	128.38	155.24	184.66	216.65	251.19	288.29	327.95	370.16	414.94	462.27	512.17
6.5	1.94	5.65	12.04	21.01	32.55	46.67	63.35	82.60	104.41	128.80	155.75	185.26	217.35	252.00	289.21	329.00	371.35	416.26	463.75	513.80
6.7	1.96	5.68	12.09	21.09	32.67	46.82	63.55	82.86	104.74	129.19	156.22	185.82	218.00	252.75	290.08	329.97	372.45	417.50	465.12	515.32
6.9	1.98	5.72	12.14	21.16	32.77	46.97	63.74	83.10	105.04	129.56	156.67	186.35	218.62	253.46	290.89	330.90	373.49	418.66	466.41	516.75
7.1	1.99	5.74	12.18	21.23	32.87	47.10	63.92	83.33	105.33	129.91	157.08	186.84	219.19	254.13	291.65	331.76	374.46	419.75	467.63	518.09
7.3	2.00	5.77	12.23	21.30	32.96	47.23	64.09	83.54	105.59	130.24	157.48	187.31	219.73	254.75	292.37	332.58	375.38	420.78	468.77	519.35
7.5	2.02	5.80	12.27	21.36	33.05	47.35	64.25	83.75	105.85	130.55	157.85	187.75	220.25	255.35	293.04	333.34	376.24	421.74	469.84	520.54
7.7	2.03	5.82	12.31	21.41	33.14	47.46	64.40	83.94	106.09	130.84	158.20	188.16	220.73	255.90	293.68	334.07	377.06	422.66	470.86	521.67
7.9	2.04	5.84	12.34	21.47	33.21	47.57	64.54	84.12	106.31	131.11	158.53	188.55	221.18	256.43	294.29	334.75	377.83	423.52	471.82	522.73
8.1	2.05	5.86	12.38	21.52	33.29	47.67	64.67	84.29	106.52	131.37	158.84	188.92	221.62	256.93	294.86	335.40	378.56	424.34	472.73	523.74
8.3	2.06	5.88	12.41	21.57	33.36	47.77	64.80	84.45	106.73	131.62	159.14	189.27	222.03	257.40	295.40	336.02	379.26	425.12	473.60	524.70
8.5	2.07	5.90	12.44	21.62	33.42	47.86	64.92	84.61	106.92	131.86	159.42	189.61	222.42	257.86	295.92	336.60	379.92	425.85	474.42	525.60
8.7	2.08	5.92	12.47	21.66	33.49	47.95	65.04	84.75	107.10	132.08	159.69	189.92	222.79	258.28	296.41	337.16	380.54	426.56	475.20	526.47
8.9	2.09	5.94	12.50	21.70	33.55	48.03	65.14	84.89	107.28	132.29	159.94	190.23	223.14	258.69	296.87	337.69	381.14	427.22	475.94	527.29
9.1	2.09	5.95	12.52	21.74	33.61	48.11	65.25	85.03	107.44	132.50	160.19	190.51	223.48	259.08	297.32	338.20	381.71	427.86	476.65	528.08
9.3	2.10	5.97	12.55	21.78	33.66	48.18	65.35	85.15	107.60	132.69	160.42	190.79	223.80	259.45	297.75	338.68	382.25	428.47	477.33	528.82
9.5	2.11	5.98	12.57	21.82	33.71	48.26	65.44	85.28	107.75	132.87	160.64	191.05	224.11	259.81	298.15	339.14	382.77	429.05	477.97	529.54
9.7	2.12	6.00	12.59	21.85	33.76	48.32	65.53	85.39	107.90	133.05	160.85	191.30	224.40	260.15	298.54	339.58	383.27	429.61	478.59	530.22
9.9	2.12	6.01	12.62	21.89	33.81	48.39	65.62	85.50	108.04	133.22	161.06	191.54	224.68	260.47	298.91	340.00	383.75	430.14	479.18	530.88
10.1	2.13	6.02	12.64	21.92	33.86	48.45	65.70	85.61	108.17	133.38	161.25	191.77	224.95	260.78	299.27	340.41	384.20	430.65	479.73	531.51

TABLE I.- FLOW VARIABLES BEHIND A NORMAL SHOCK WAVE AS A FUNCTION OF THE ENTHALPY PARAMETER - Continued

Enthalpy parameter, β	Values of ρ/ρ_1 for values of M_1 of -																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3.5	1.000	2.667	3.857	4.571	5.000	5.268	5.444	5.565	5.651	5.714	5.762	5.799	5.828	5.851	5.870	5.885	5.898	5.909	5.918	5.926
3.7	1.283	2.895	4.141	4.895	5.345	5.628	5.814	5.941	6.032	6.098	6.149	6.188	6.218	6.242	6.262	6.279	6.292	6.304	6.313	6.322
3.9	1.443	3.116	4.424	5.215	5.690	5.987	6.185	6.317	6.413	6.485	6.536	6.576	6.608	6.634	6.655	6.672	6.687	6.699	6.709	6.718
4.1	1.586	3.336	4.706	5.536	6.034	6.347	6.552	6.693	6.795	6.867	6.922	6.965	6.999	7.026	7.048	7.066	7.081	7.094	7.104	7.114
4.3	1.721	3.554	4.988	5.856	6.379	6.706	6.921	7.069	7.174	7.251	7.309	7.354	7.389	7.418	7.441	7.459	7.475	7.489	7.500	7.509
4.5	1.851	3.772	5.269	6.176	6.725	7.065	7.290	7.444	7.554	7.635	7.696	7.743	7.780	7.809	7.833	7.855	7.870	7.883	7.895	7.905
4.7	1.978	3.988	5.549	6.496	7.067	7.424	7.659	7.820	7.934	8.019	8.082	8.131	8.170	8.201	8.226	8.247	8.264	8.278	8.291	8.301
4.9	2.102	4.203	5.830	6.816	7.410	7.783	8.027	8.195	8.315	8.403	8.469	8.520	8.560	8.592	8.619	8.640	8.658	8.673	8.686	8.697
5.1	2.225	4.417	6.110	7.136	7.754	8.142	8.396	8.571	8.695	8.786	8.855	8.909	8.950	8.984	9.011	9.034	9.052	9.068	9.081	9.093
5.3	2.347	4.631	6.389	7.456	8.098	8.500	8.765	8.946	9.075	9.170	9.242	9.297	9.341	9.376	9.404	9.427	9.447	9.463	9.477	9.489
5.5	2.467	4.844	6.669	7.775	8.441	8.859	9.133	9.322	9.456	9.554	9.628	9.686	9.731	9.767	9.796	9.821	9.841	9.858	9.872	9.884
5.7	2.587	5.057	6.948	8.095	8.785	9.218	9.502	9.697	9.836	9.938	10.015	10.074	10.121	10.159	10.189	10.214	10.235	10.253	10.267	10.280
5.9	2.706	5.270	7.227	8.414	9.128	9.576	9.871	10.073	10.216	10.322	10.401	10.463	10.511	10.550	10.582	10.608	10.629	10.647	10.663	10.676
6.1	2.825	5.483	7.506	8.735	9.472	9.935	10.259	10.448	10.597	10.706	10.788	10.852	10.902	10.942	10.974	11.001	11.023	11.042	11.058	11.072
6.3	2.943	5.695	7.785	9.052	9.815	10.293	10.608	10.823	10.977	11.089	11.175	11.240	11.292	11.333	11.367	11.395	11.418	11.437	11.454	11.468
6.5	3.061	5.907	8.064	9.371	10.158	10.652	10.976	11.199	11.357	11.473	11.561	11.629	11.682	11.725	11.759	11.788	11.812	11.832	11.849	11.863
6.7	3.178	6.118	8.342	9.690	10.502	11.010	11.345	11.574	11.737	11.857	11.948	12.017	12.072	12.116	12.152	12.182	12.206	12.227	12.244	12.259
6.9	3.296	6.330	8.621	10.009	10.845	11.369	11.713	11.949	12.117	12.241	12.334	12.406	12.463	12.508	12.545	12.575	12.600	12.621	12.640	12.655
7.1	3.412	6.541	8.900	10.328	11.188	11.727	12.082	12.325	12.498	12.625	12.720	12.794	12.853	12.899	12.937	12.968	12.994	13.016	13.035	13.051
7.3	3.529	6.752	9.178	10.647	11.531	12.086	12.450	12.700	12.878	13.008	13.107	13.183	13.243	13.291	13.330	13.362	13.389	13.411	13.430	13.447
7.5	3.646	6.963	9.456	10.966	11.875	12.444	12.818	13.075	13.258	13.392	13.493	13.572	13.633	13.682	13.722	13.755	13.783	13.806	13.826	13.842
7.7	3.762	7.175	9.735	11.285	12.218	12.803	13.187	13.450	13.638	13.776	13.880	13.960	14.023	14.074	14.115	14.149	14.177	14.201	14.221	14.238
7.9	3.878	7.386	10.031	11.604	12.561	13.161	13.555	13.826	14.018	14.160	14.266	14.349	14.414	14.465	14.508	14.542	14.571	14.596	14.616	14.634
8.1	3.994	7.596	10.291	11.923	12.904	13.519	13.924	14.201	14.398	14.543	14.653	14.737	14.804	14.857	14.900	14.936	14.965	14.990	15.012	15.030
8.3	4.110	7.807	10.570	12.241	13.247	13.878	14.292	14.576	14.779	14.927	15.039	15.126	15.194	15.248	15.293	15.329	15.360	15.385	15.407	15.426
8.5	4.226	8.018	10.848	12.560	13.590	14.256	14.660	14.952	15.159	15.311	15.426	15.514	15.584	15.640	15.685	15.723	15.754	15.780	15.802	15.821
8.7	4.342	8.228	11.126	12.879	13.933	14.595	15.029	15.327	15.539	15.695	15.812	15.903	15.974	16.031	16.078	16.116	16.148	16.175	16.198	16.217
8.9	4.458	8.439	11.404	13.197	14.277	14.953	15.397	15.702	15.919	16.078	16.199	16.291	16.364	16.423	16.470	16.510	16.542	16.570	16.593	16.613
9.1	4.573	8.650	11.682	13.516	14.620	15.311	15.766	16.077	16.299	16.462	16.585	16.680	16.755	16.814	16.863	16.903	16.936	16.964	16.988	17.009
9.3	4.689	8.860	11.960	13.835	14.963	15.670	16.134	16.452	16.679	16.846	16.971	17.068	17.145	17.206	17.256	17.297	17.331	17.359	17.384	17.404
9.5	4.804	9.070	12.238	14.153	15.306	16.028	16.502	16.828	17.059	17.230	17.358	17.457	17.535	17.597	17.648	17.690	17.725	17.754	17.779	17.800
9.7	4.919	9.281	12.516	14.472	15.649	16.386	16.871	17.203	17.439	17.613	17.744	17.846	17.925	17.989	18.041	18.083	18.119	18.149	18.174	18.196
9.9	5.035	9.491	12.794	14.791	15.992	16.744	17.239	17.578	17.820	17.997	18.131	18.234	18.315	18.380	18.433	18.477	18.513	18.544	18.570	18.592
10.1	5.150	9.701	13.072	15.109	16.335	17.103	17.607	17.953	18.200	18.381	18.517	18.623	18.706	18.772	18.826	18.870	18.907	18.938	18.965	18.988

TABLE I.- FLOW VARIABLES BEHIND A NORMAL SHOCK WAVE AS A FUNCTION OF THE ENTHALPY PARAMETER - Concluded

Enthalpy parameter, β	Values of $(1 + \alpha)T/T_1$ for values of M_1 of -																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3.5	1.000	1.688	2.679	4.047	5.800	7.941	10.469	13.387	16.693	20.388	24.471	28.943	33.805	39.055	44.694	50.722	57.138	63.944	71.139	78.722
3.7	1.020	1.612	2.549	3.847	5.510	7.542	9.942	12.711	15.849	19.356	23.232	27.478	32.092	37.075	42.428	48.150	54.241	60.701	67.530	74.728
3.9	.991	1.541	2.430	3.664	5.246	7.179	9.462	12.097	15.082	18.419	22.107	26.146	30.536	35.278	40.370	45.814	51.609	57.755	64.255	71.101
4.1	.957	1.475	2.321	3.496	5.005	6.847	9.025	11.537	14.383	17.565	21.081	24.932	29.118	33.639	38.495	43.686	49.211	55.071	61.267	67.797
4.3	.922	1.414	2.220	3.343	4.784	6.544	8.624	11.024	13.744	16.783	20.143	23.822	27.822	32.141	36.780	41.739	47.019	52.618	58.537	64.776
4.5	.888	1.356	2.127	3.201	4.581	6.266	8.257	10.554	13.157	16.066	19.282	22.804	26.632	30.767	35.207	39.954	45.007	50.367	56.032	62.004
4.7	.856	1.303	2.042	3.071	4.394	6.009	7.918	10.121	12.617	15.407	18.490	21.867	25.538	29.502	33.760	38.312	43.157	48.296	53.728	59.455
4.9	.825	1.253	1.962	2.951	4.221	5.772	7.606	9.721	12.118	14.798	17.759	21.002	24.528	28.335	32.424	36.796	41.449	46.385	51.602	57.102
5.1	.796	1.207	1.888	2.839	4.061	5.553	7.316	9.351	11.657	14.234	17.082	20.202	23.593	27.255	31.188	35.393	39.869	44.616	49.634	54.924
5.3	.768	1.164	1.820	2.736	3.912	5.349	7.048	9.008	11.229	13.711	16.454	19.459	22.725	26.253	30.041	34.091	38.402	42.975	47.809	52.904
5.5	.743	1.124	1.756	2.639	3.774	5.160	6.798	8.688	10.830	13.224	15.870	18.768	21.918	25.320	28.974	32.880	37.038	41.448	46.110	51.024
5.7	.718	1.086	1.696	2.549	3.644	4.983	6.565	8.390	10.459	12.770	15.326	18.124	21.166	24.451	27.979	31.751	35.766	40.025	44.527	49.272
5.9	.696	1.051	1.641	2.465	3.524	4.818	6.347	8.112	10.111	12.346	14.816	17.522	20.463	23.639	27.050	30.696	34.578	38.695	43.047	47.634
6.1	.674	1.018	1.588	2.386	3.411	4.663	6.143	7.851	9.786	11.949	14.340	16.958	19.804	22.878	26.179	29.708	33.465	37.449	41.661	46.101
6.3	.654	.986	1.539	2.312	3.304	4.518	5.952	7.606	9.481	11.576	13.892	16.429	19.186	22.164	25.362	28.781	32.420	36.280	40.361	44.662
6.5	.635	.957	1.493	2.242	3.205	4.381	5.772	7.376	9.194	11.226	13.472	15.931	18.605	21.493	24.594	27.909	31.438	35.182	39.138	43.309
6.7	.617	.929	1.449	2.176	3.111	4.253	5.602	7.159	8.924	10.896	13.076	15.463	18.058	20.861	23.871	27.088	30.514	34.147	37.987	42.035
6.9	.599	.903	1.408	2.114	3.022	4.131	5.442	6.955	8.669	10.584	12.702	15.021	17.542	20.264	23.188	26.314	29.641	33.170	36.901	40.834
7.1	.583	.878	1.369	2.056	2.938	4.016	5.291	6.761	8.428	10.290	12.349	14.603	17.054	19.701	22.544	25.582	28.817	32.248	35.875	39.698
7.3	.568	.855	1.332	2.000	2.859	3.908	5.148	6.578	8.200	10.012	12.015	14.208	16.593	19.168	21.933	24.890	28.037	31.375	34.904	38.623
7.5	.553	.832	1.297	1.948	2.783	3.805	5.012	6.405	7.984	9.748	11.698	13.834	16.155	18.662	21.555	24.234	27.298	30.548	33.984	37.605
7.7	.539	.811	1.264	1.898	2.712	3.707	4.883	6.241	7.779	9.498	11.397	13.478	15.740	18.183	20.806	23.611	26.597	29.763	33.110	36.639
7.9	.526	.791	1.233	1.850	2.644	3.614	4.761	6.084	7.584	9.260	11.112	13.141	15.346	17.727	20.285	23.019	25.930	29.017	32.281	35.721
8.1	.513	.772	1.203	1.805	2.580	3.526	4.645	5.936	7.398	9.033	10.840	12.819	14.970	17.294	19.789	22.456	25.296	28.308	31.491	34.847
8.3	.501	.754	1.174	1.762	2.518	3.442	4.534	5.794	7.222	8.818	10.581	12.513	14.613	16.881	19.316	21.920	24.692	27.632	30.739	34.015
8.5	.490	.736	1.147	1.721	2.459	3.362	4.428	5.659	7.053	8.612	10.335	12.221	14.272	16.487	18.866	21.409	24.116	26.987	30.022	33.221
8.7	.478	.719	1.121	1.682	2.403	3.285	4.327	5.530	6.893	8.416	10.099	11.943	13.947	16.111	18.436	20.921	23.566	26.372	29.338	32.464
8.9	.468	.703	1.096	1.644	2.350	3.212	4.231	5.407	6.739	8.228	9.874	11.676	13.636	15.752	18.025	20.454	23.041	25.784	28.683	31.740
9.1	.458	.688	1.072	1.609	2.299	3.142	4.139	5.289	6.592	8.049	9.658	11.422	13.338	15.408	17.631	20.008	22.538	25.221	28.058	31.047
9.3	.448	.674	1.049	1.574	2.250	3.075	4.050	5.176	6.451	7.877	9.452	11.178	13.054	15.079	17.255	19.581	22.057	24.683	27.458	30.384
9.5	.439	.660	1.027	1.541	2.203	3.011	3.966	5.068	6.316	7.712	9.255	10.944	12.781	14.764	16.894	19.171	21.595	24.166	26.884	29.749
9.7	.430	.646	1.006	1.510	2.158	2.949	3.884	4.964	6.187	7.554	9.065	10.720	12.519	14.461	16.548	18.779	21.153	23.671	26.334	29.140
9.9	.421	.635	.986	1.480	2.114	2.890	3.807	4.864	6.063	7.402	8.883	10.505	12.267	14.171	16.216	18.402	20.728	23.196	25.805	28.555
10.1	.413	.621	.967	1.451	2.073	2.833	3.732	4.768	5.943	7.257	8.708	10.298	12.026	13.892	15.897	18.039	20.320	22.739	25.297	27.992

TABLE II.- EQUILIBRIUM CONSTANTS BASED ON PARTIAL PRESSURES

T, °K	K_{O_2} for $O_2 \rightleftarrows 2O$	K_{N_2} for $N_2 \leftrightarrows 2N$
1,500	1.623×10^{-11}	
2,000	4.413×10^{-7}	
2,500	2.070×10^{-4}	
3,000	1.246×10^{-2}	1.899×10^{-10}
3,500	2.398×10^{-1}	4.817×10^{-8}
4,000	2.182×10^{0}	3.087×10^{-6}
4,500	1.221×10^{1}	7.936×10^{-5}
5,000	4.842×10^{1}	1.077×10^{-3}
5,500	1.498×10^{2}	9.198×10^{-3}
6,000	3.848×10^{2}	5.551×10^{-2}
7,000	1.706×10^{3}	9.629×10^{-1}
8,000	5.258×10^{3}	8.503×10^{0}
10,000	2.525×10^{4}	4.950×10^{2}
12,000	7.086×10^{4}	4.014×10^{3}
15,000	1.976×10^{5}	3.149×10^{4}

TABLE III.- VARIATION OF FLOW PROPERTIES

BEHIND A NORMAL-SHOCK WAVE

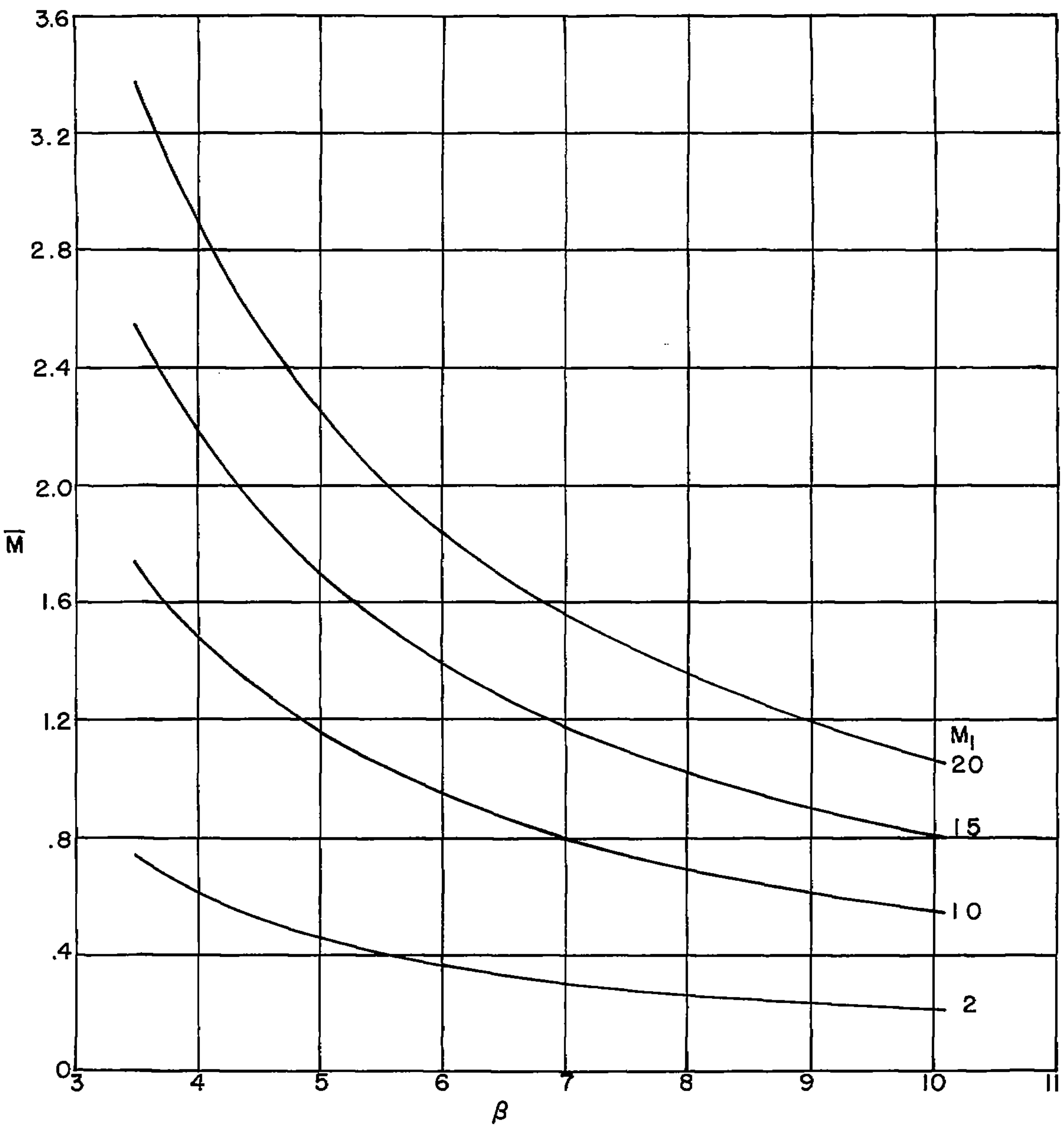
$\left[M_1 = 14;\ T_1 = 300^\circ\ K;\ p_1 = 10^{-4}\ \text{atmospheres}\right]$

(a) O_2 dissociation

T, °K	α	β	p, atm	ρ/ρ_1	$\bar{M}$	t, sec	d, cm
9,140	0.0000	4.55	0.0240	7.90	1.770	0.000 × 10^{-6}	0.0000
9,000	.0120	4.56	.0241	7.92	1.765	.026	.0016
8,800	.0280	4.59	.0241	7.98	1.755	.070	.0043
8,600	.0450	4.62	.0241	8.04	1.740	.123	.0075
8,400	.0610	4.66	.0242	8.12	1.725	.190	.0115
8,200	.0780	4.70	.0242	8.20	1.710	.275	.0166
8,000	.0945	4.75	.0242	8.30	1.688	.383	.0229
7,800	.1110	4.80	.0243	8.40	1.665	.524	.0310
7,600	.1275	4.86	.0243	8.52	1.640	.716	.0420
7,400	.1440	4.93	.0244	8.65	1.618	.990	.0574
7,200	.1600	5.00	.0244	8.79	1.590	1.398	.0799
7,000	.1750	5.08	.0245	8.95	1.562	2.075	.1166
6,800	.1895	5.16	.0245	9.10	1.535	3.300	.1820
6,540	.2103	5.28	.0246	9.34	1.498	∞	

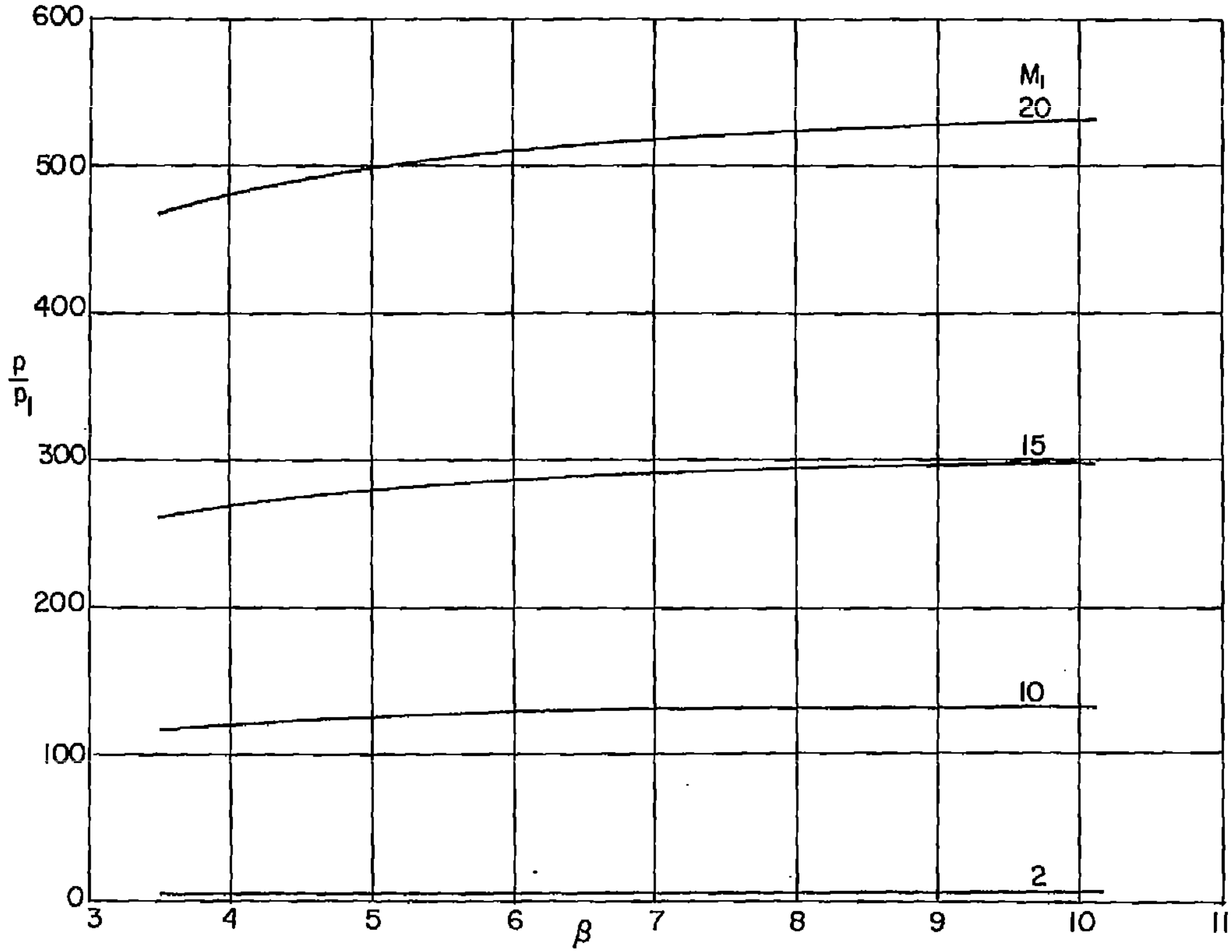
(b) N_2 dissociation

T, °K	α	β	p, atm	ρ/ρ_1	$\bar{M}$	t, sec	d, cm
6,540	0.2103	5.28	0.0246	9.34	1.498	0.00 × 10^{-3}	0.0
6,400	.2155	5.37	.0247	9.52	1.470	.08	4.0
6,200	.2235	5.52	.0247	9.80	1.430	.27	13.7
6,000	.2320	5.66	.0248	10.08	1.390	.68	33.2
5,800	.2400	5.82	.0249	10.39	1.350	1.44	69.0
5,600	.2485	6.00	.0250	10.75	1.303	2.99	139.1
5,400	.2570	6.18	.0251	11.10	1.262	6.35	286.6
5,200	.2650	6.37	.0251	11.47	1.221	13.34	583.1
5,000	.2730	6.58	.0252	11.88	1.178	28.69	1212
4,870	.2792	6.73	.0253	12.18	1.148	∞	



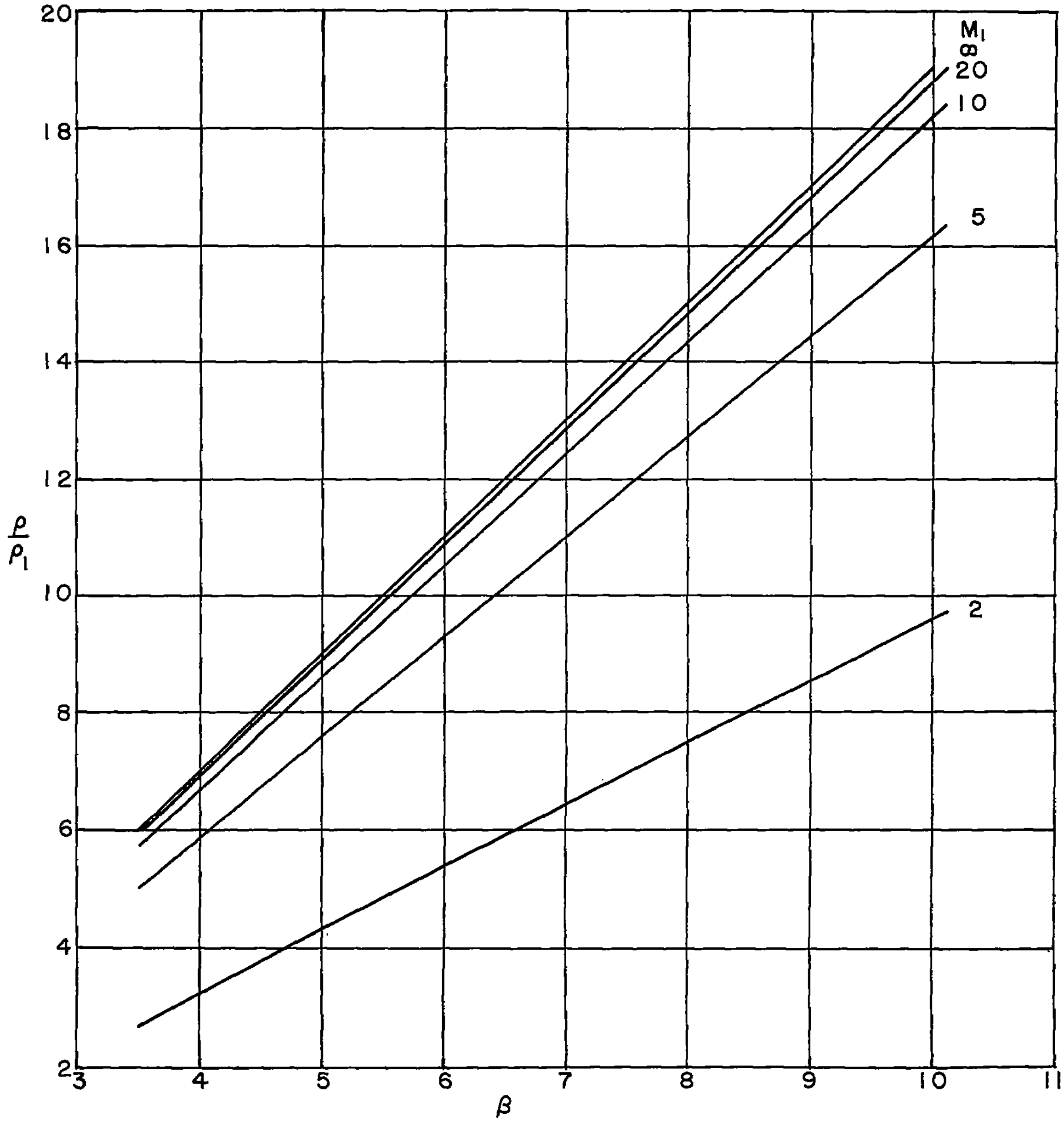
(a) $\overline{M} = v/a_1$.

Figure 1.- Flow variables behind a normal shock wave as a function of the enthalpy parameter β for various shock Mach numbers. $\gamma = 1.4$.



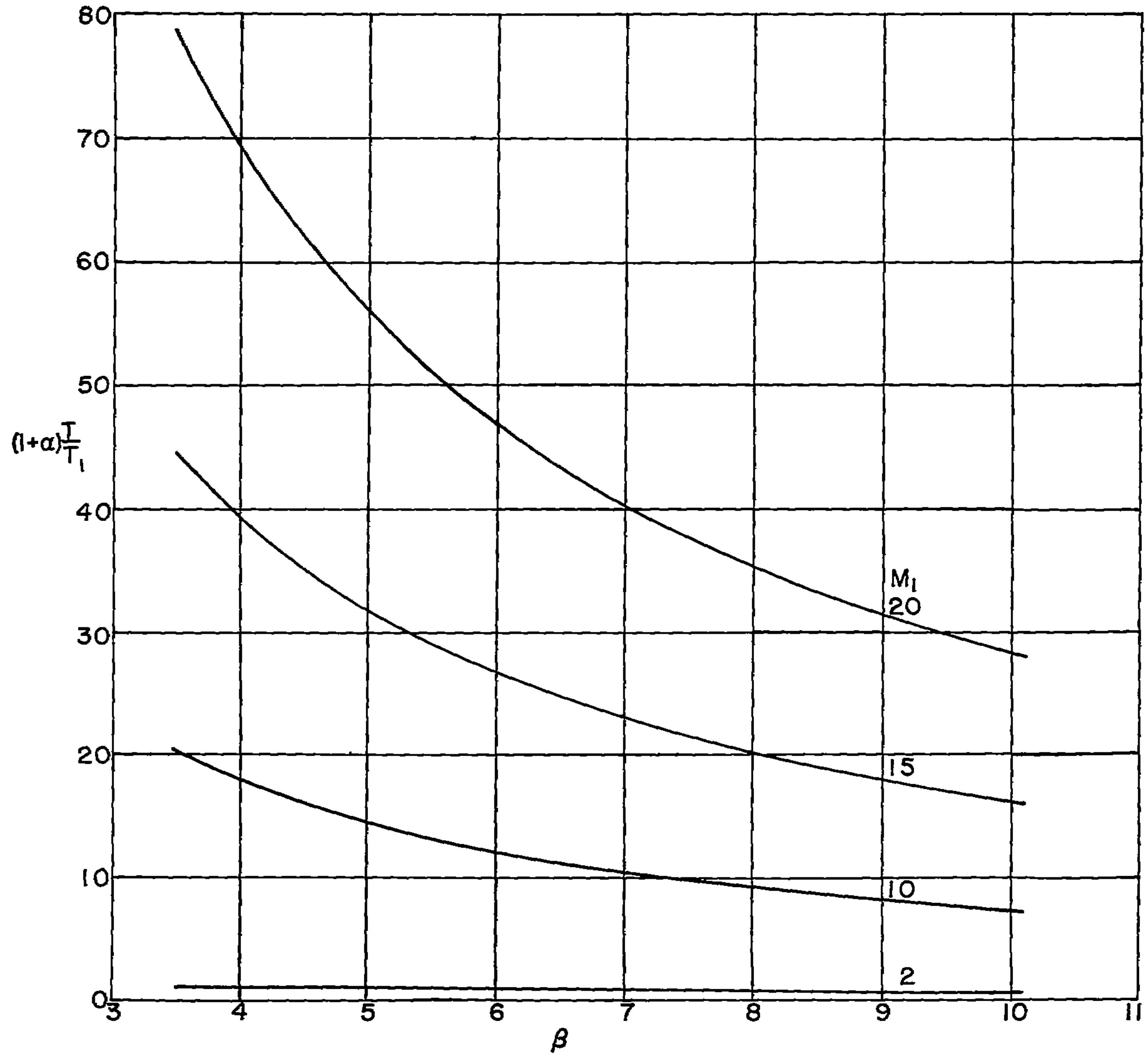
(b) p/p_1.

Figure 1.- Continued.



(c) ρ/ρ_1.

Figure 1.- Continued.



(d) $(1 + \alpha)\frac{T}{T_1}$.

Figure 1.- Concluded.

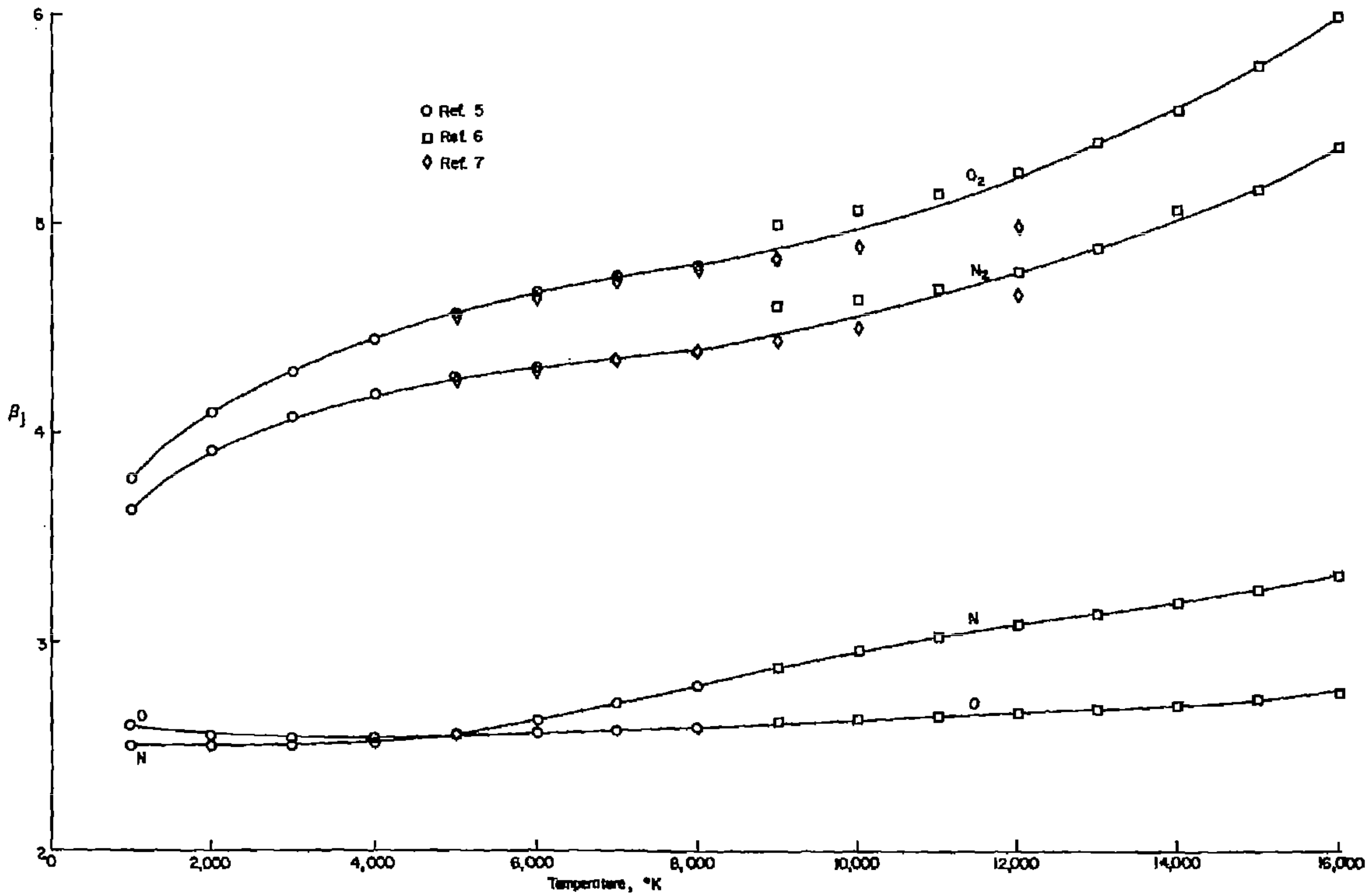
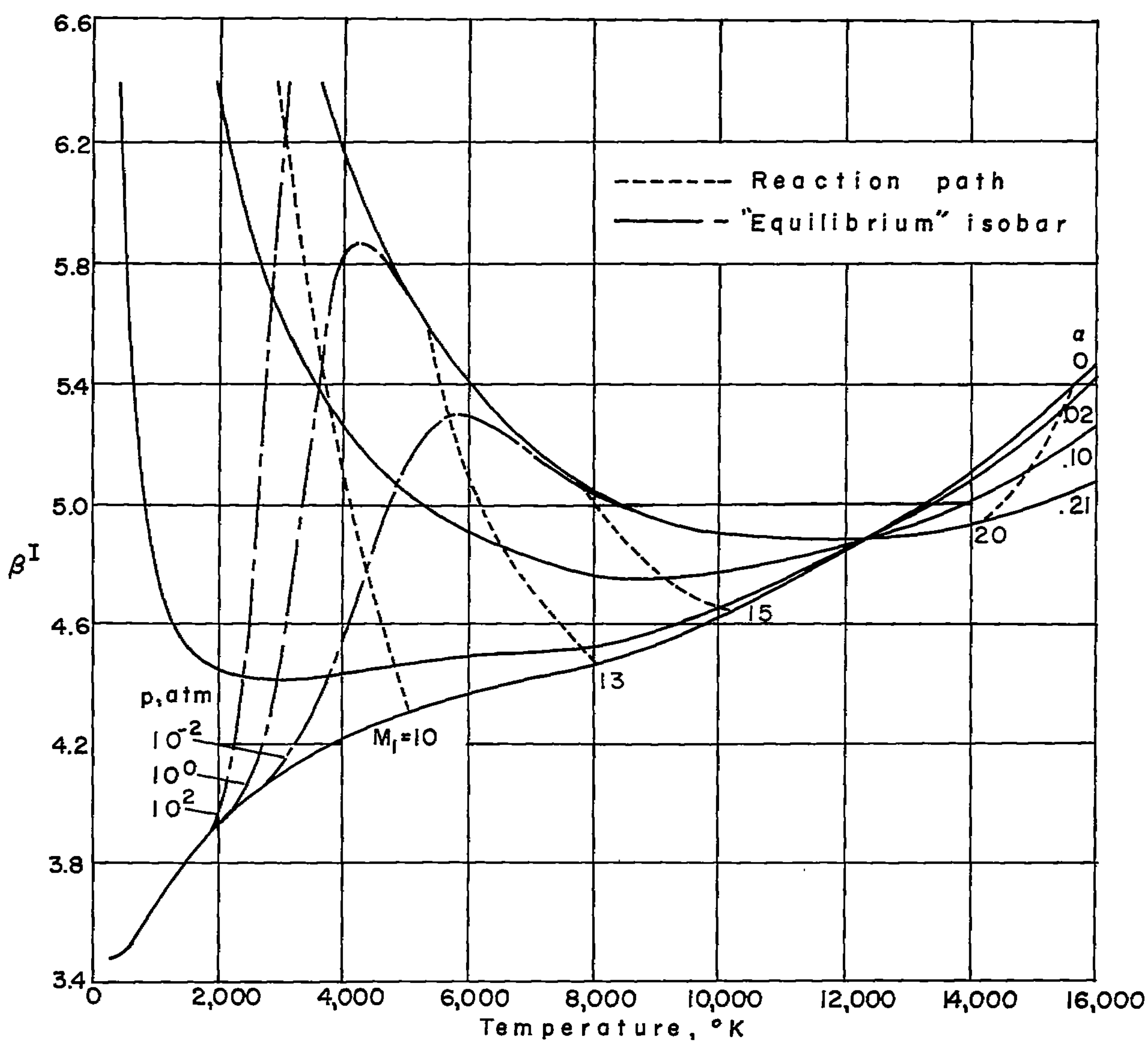
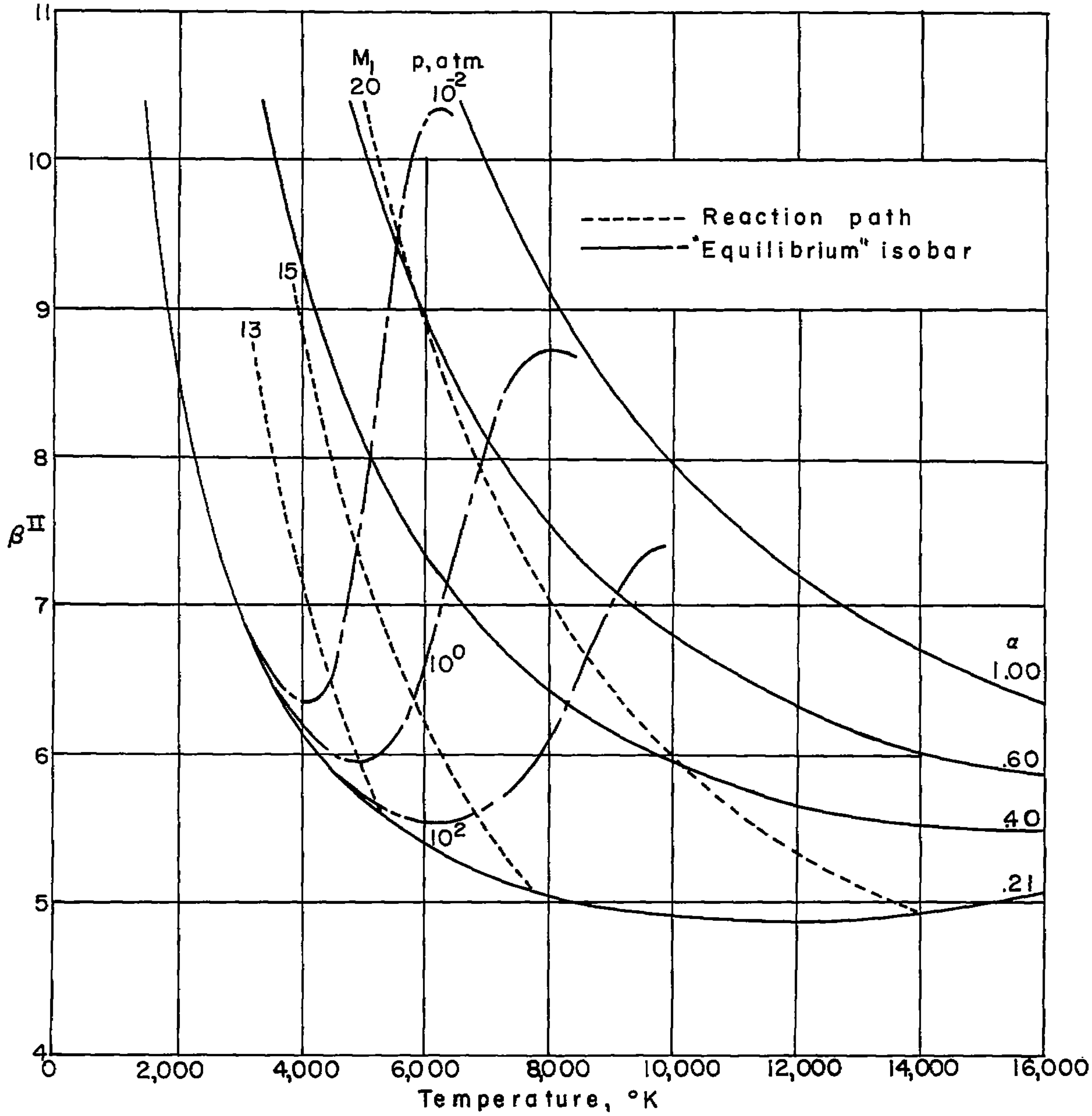


Figure 2.- Enthalpy parameter β_j as a function of temperature.



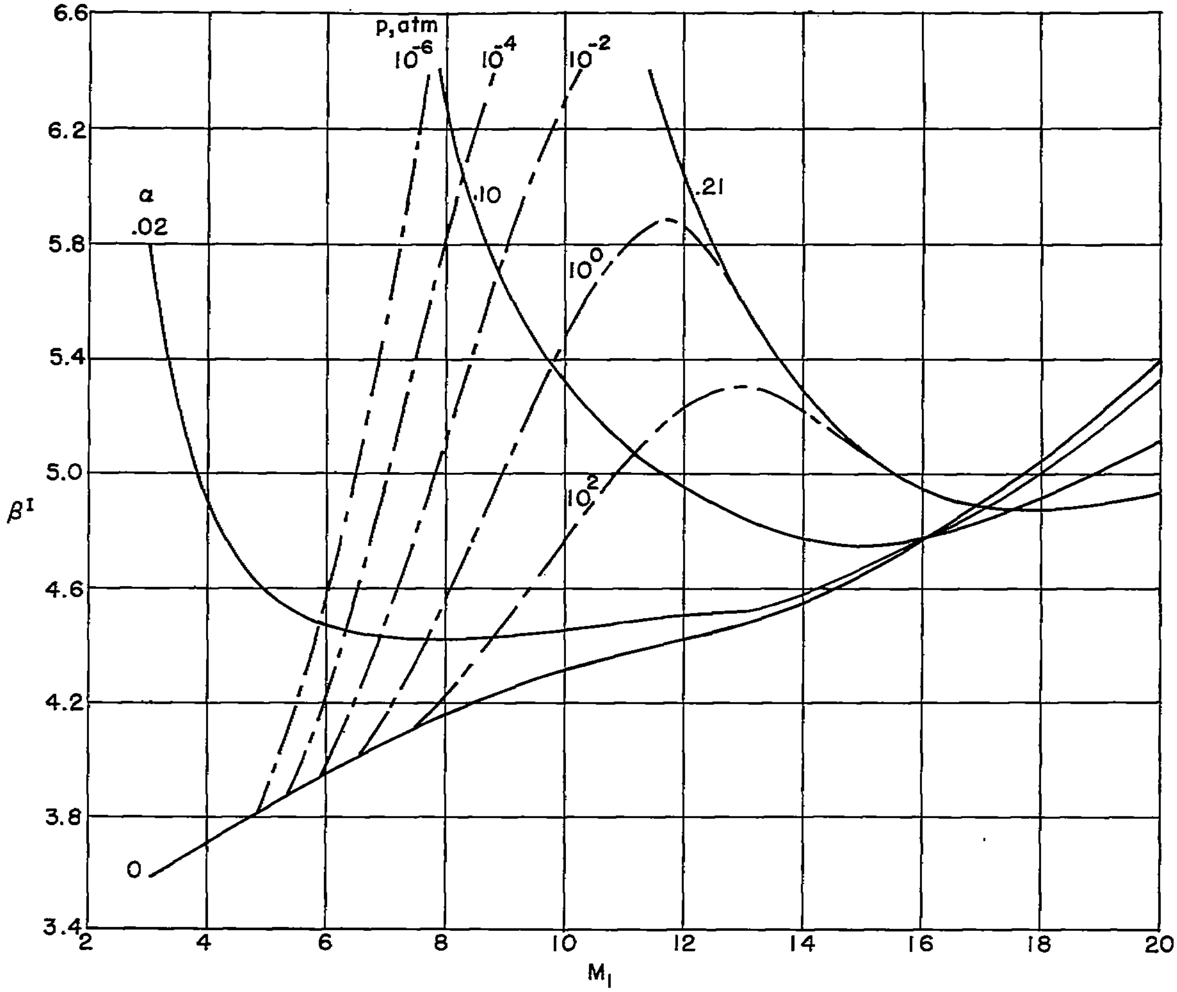
(a) N_2 undissociated.

Figure 3.- Energy content of air as a function of temperature and degree of dissociation. Reaction paths and "equilibrium" isobars. $T_1 = 300^\circ$ K.



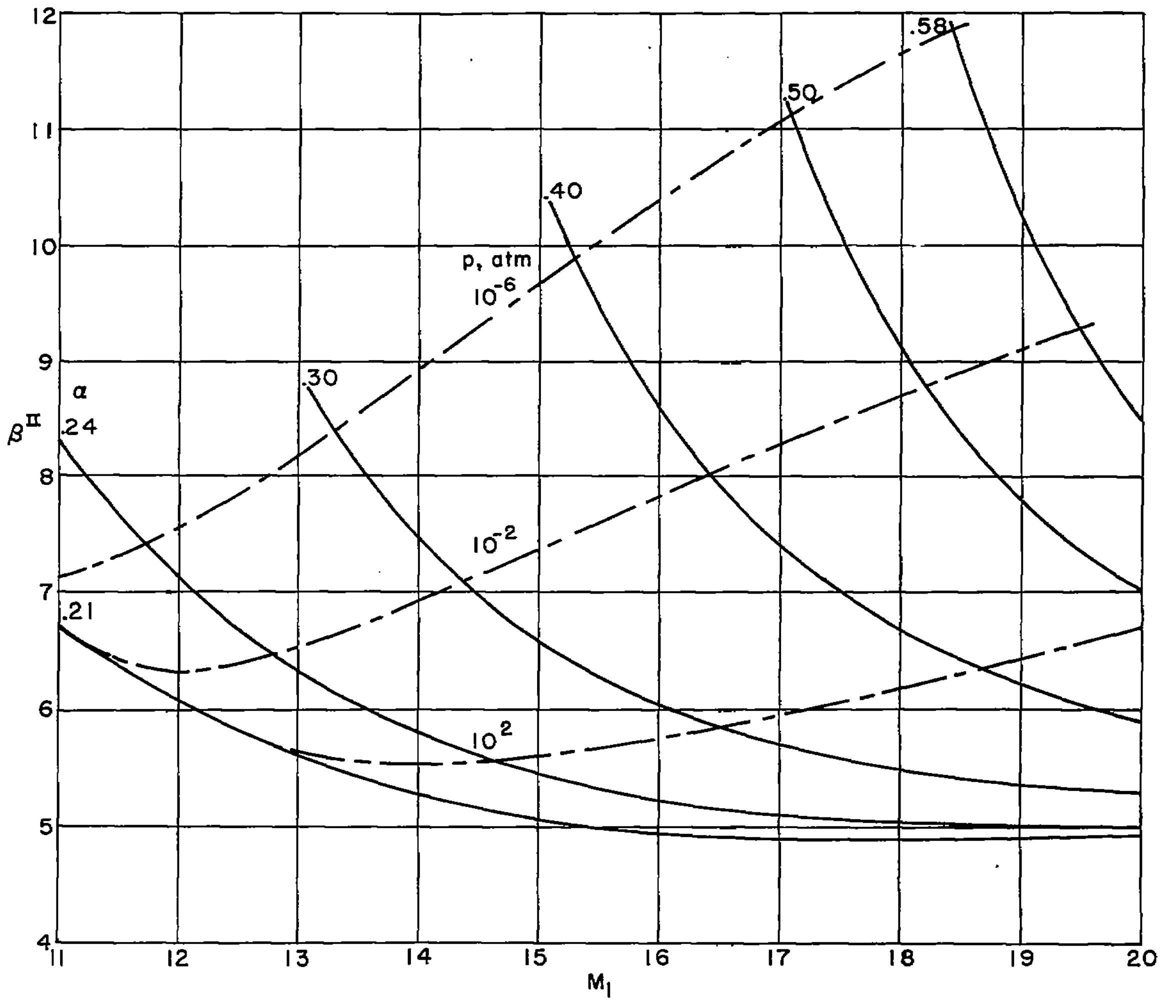
(b) O_2 completely dissociated.

Figure 3.- Concluded.



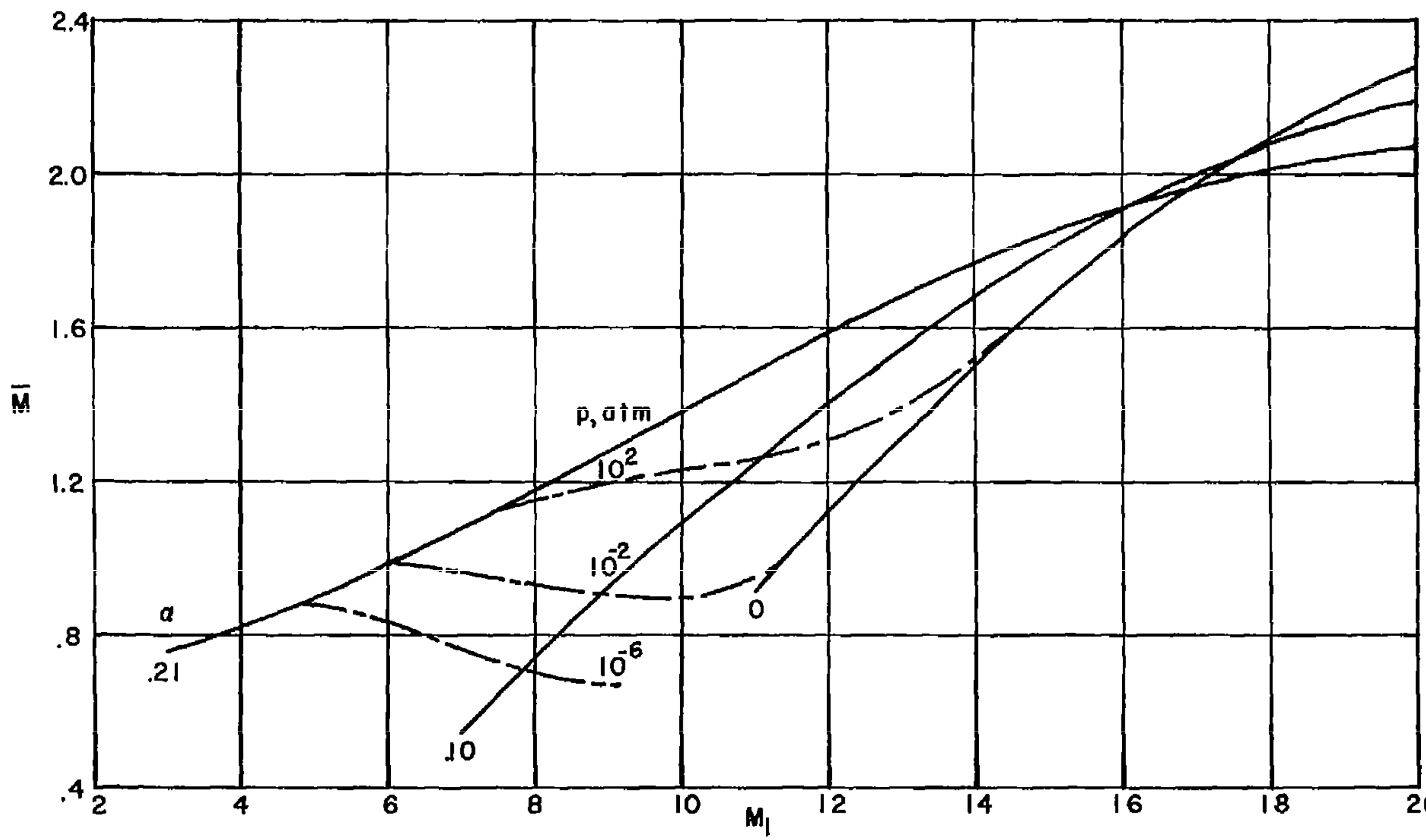
(a) N_2 undissociated.

Figure 4.- Energy content of air as a function of shock Mach number and degree of dissociation. Reaction paths (vertical lines) and "equilibrium" isobars. $T_1 = 300^\circ$ K.



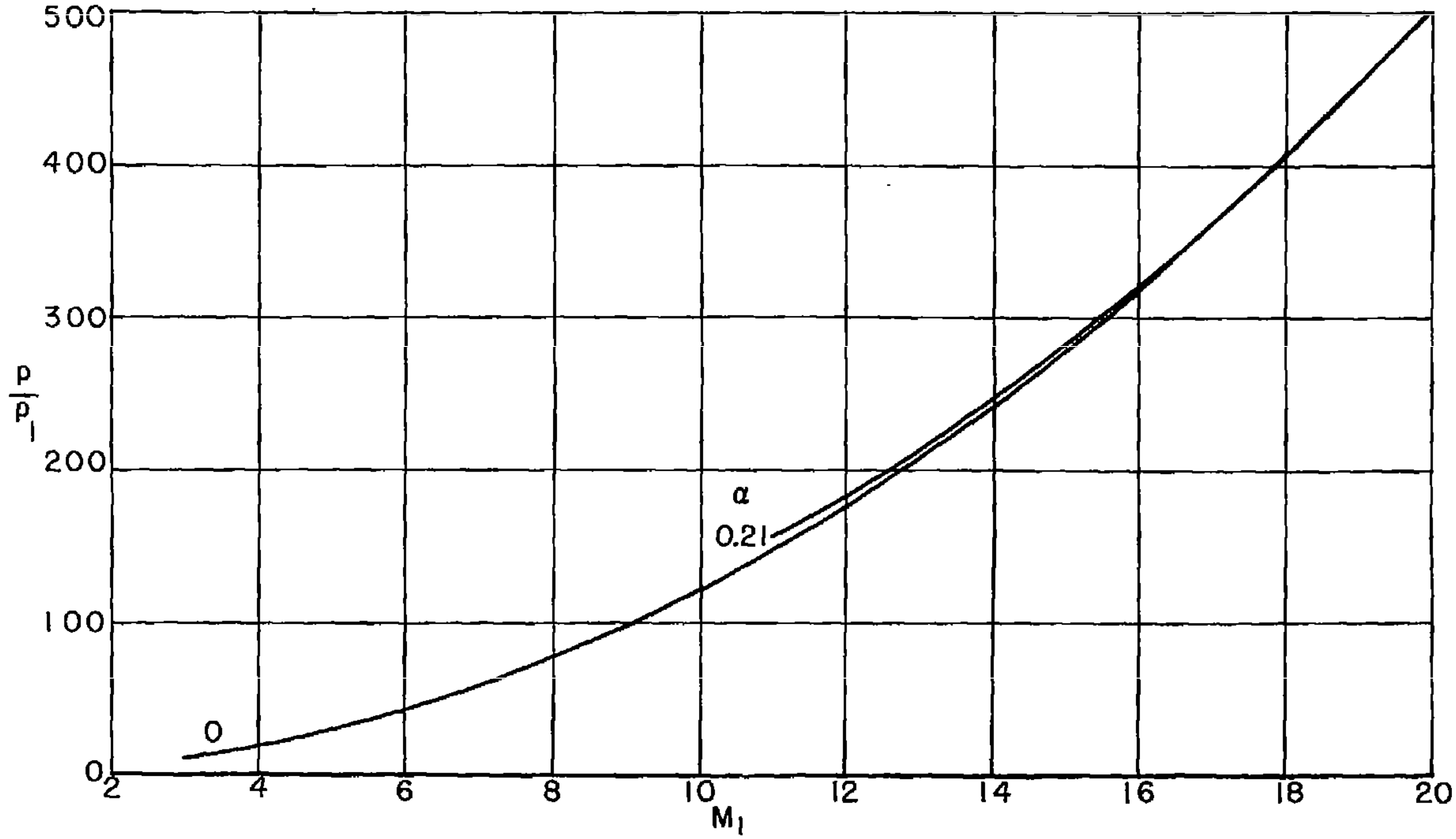
(b) O_2 completely dissociated.

Figure 4.- Concluded.



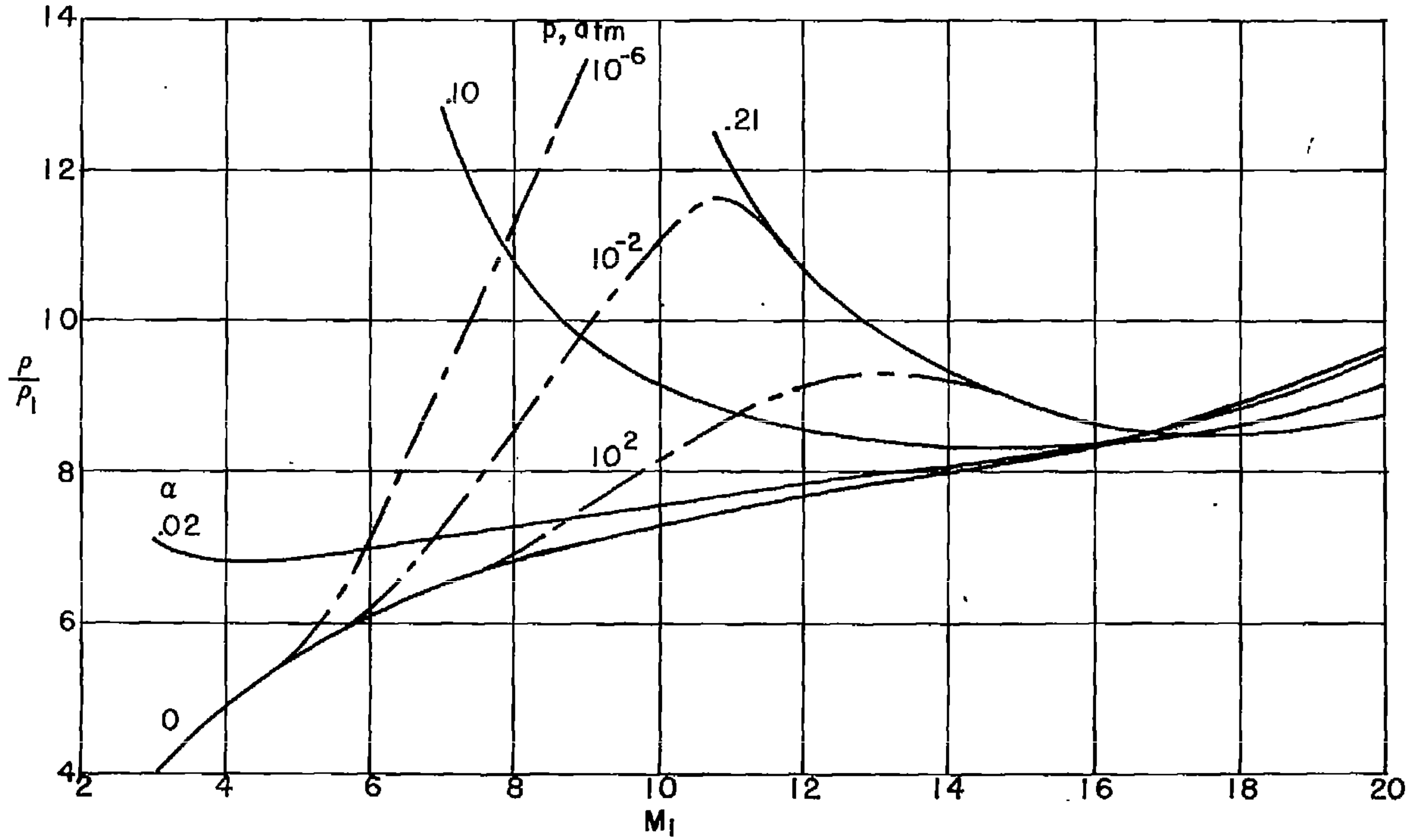
(a) $\bar{M} = v/a_1$.

Figure 5.- Flow variables as a function of shock Mach number and degree of dissociation. Reaction paths (vertical lines) and "equilibrium" isobars. N_2 undissociated; $T_1 = 300^\circ$ K.



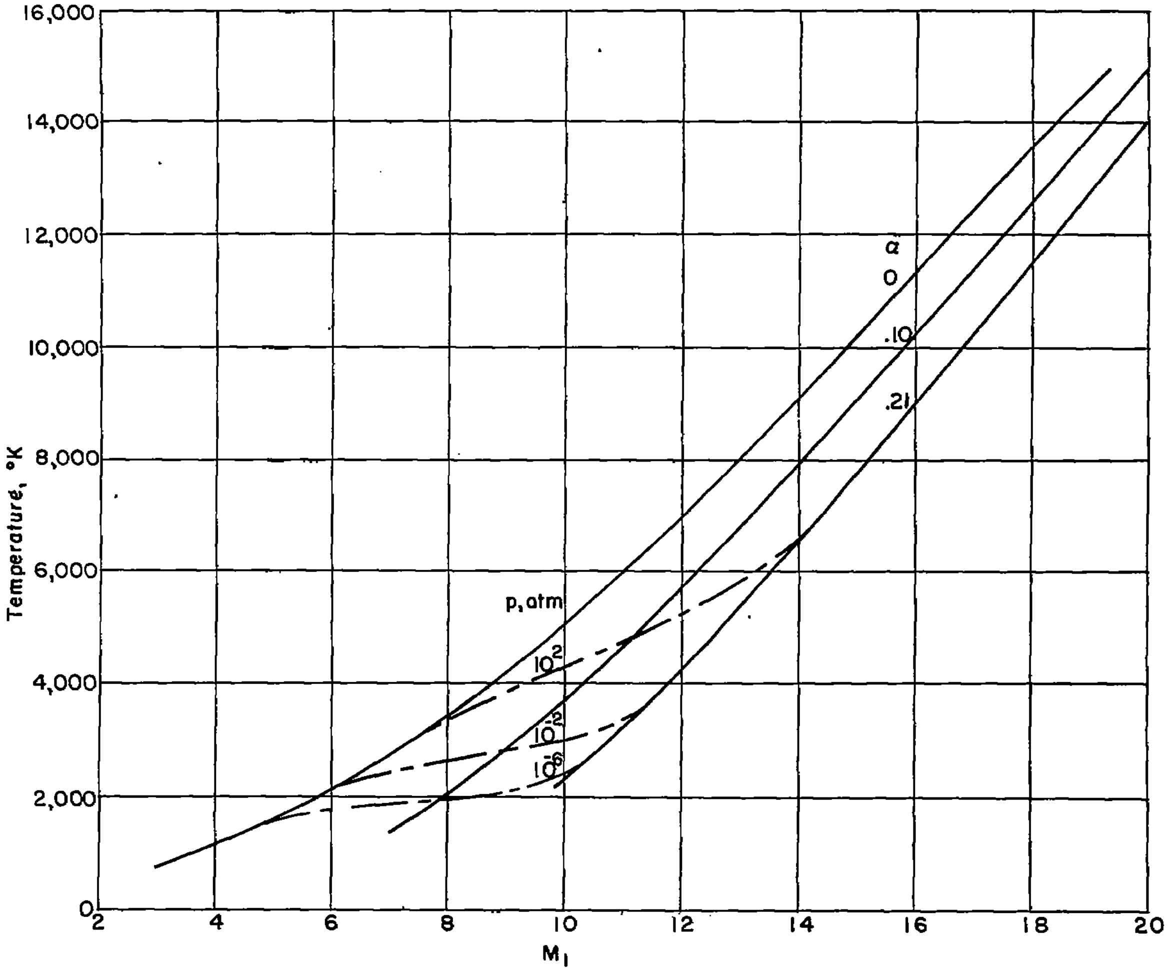
(b) p/p_1.

Figure 5.- Continued.



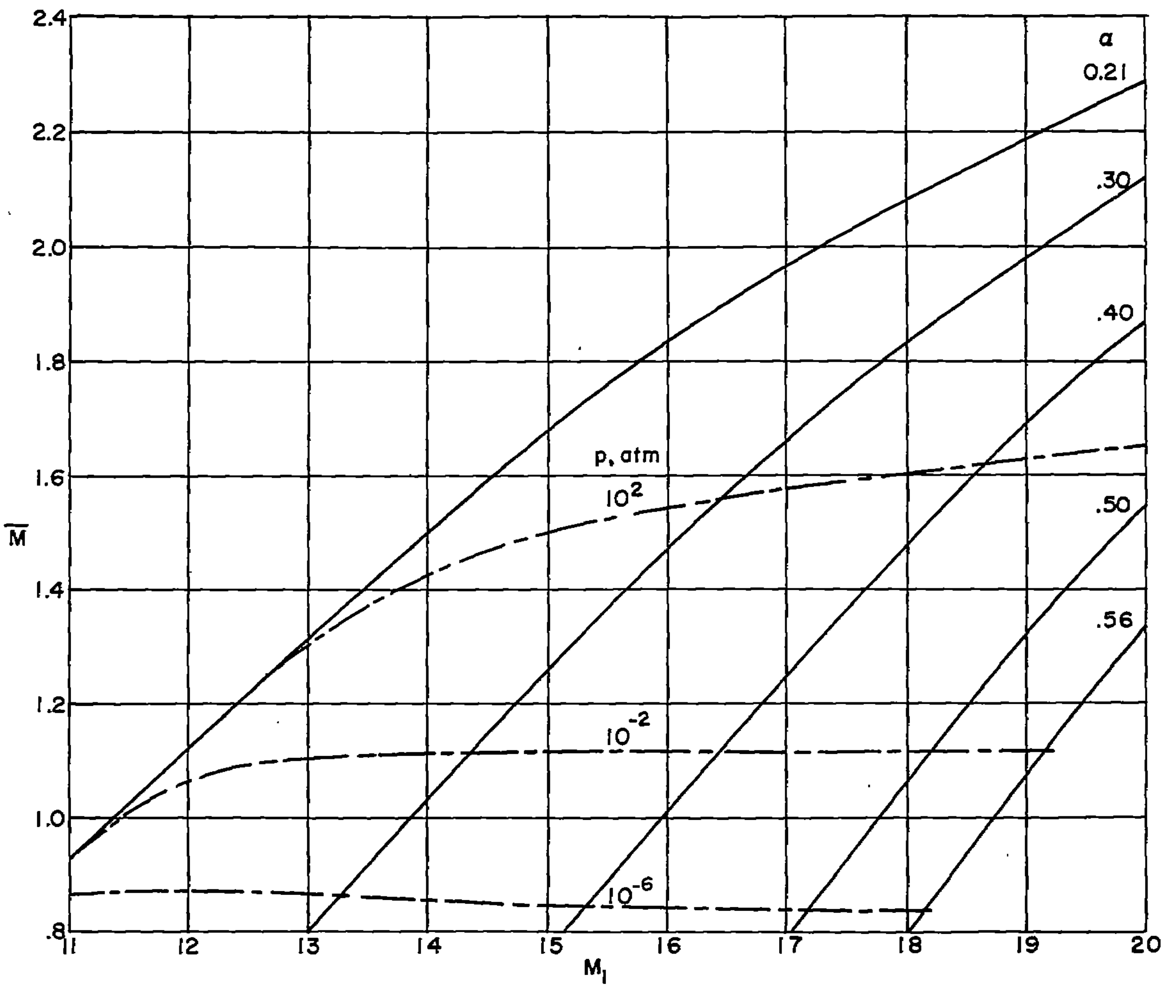
(c) ρ/ρ_1.

Figure 5.- Continued.



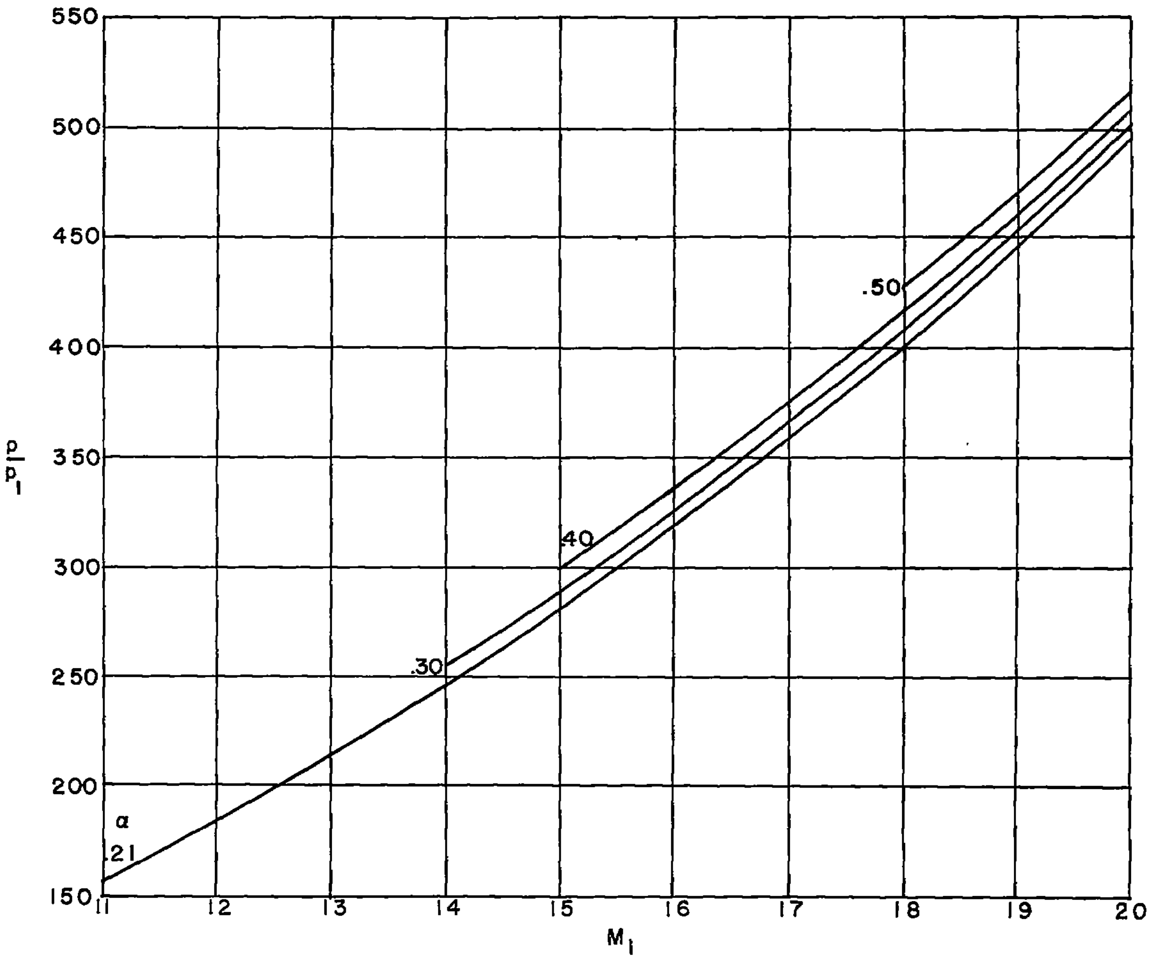
(d) Temperature.

Figure 5.- Concluded.



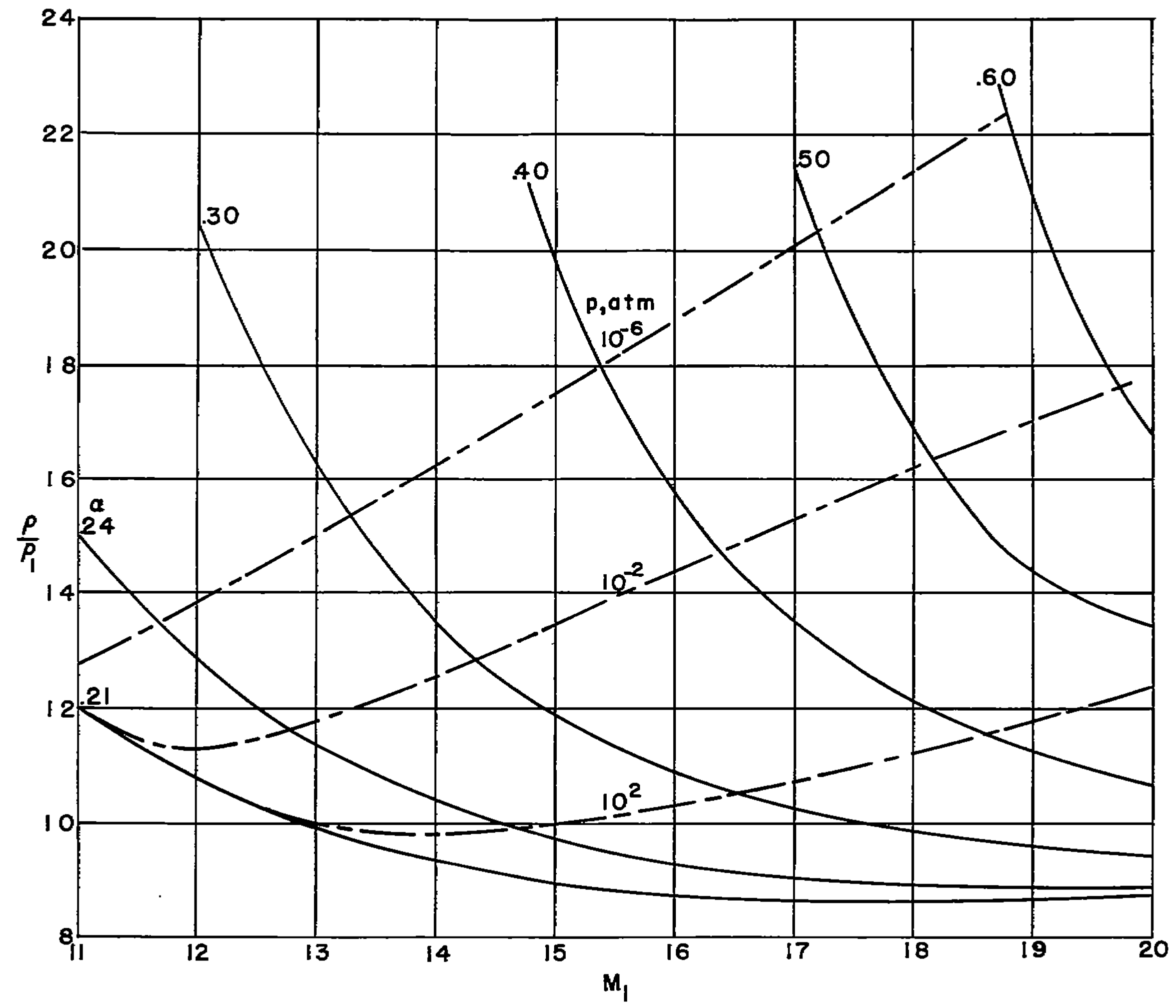
(a) $\overline{M} = v/a_1$.

Figure 6.- Flow variables as a function of shock Mach number and degree of dissociation. Reaction paths (vertical lines) and "equilibrium" isobars. O_2 completely dissociated; $T_1 = 300^\circ$ K.



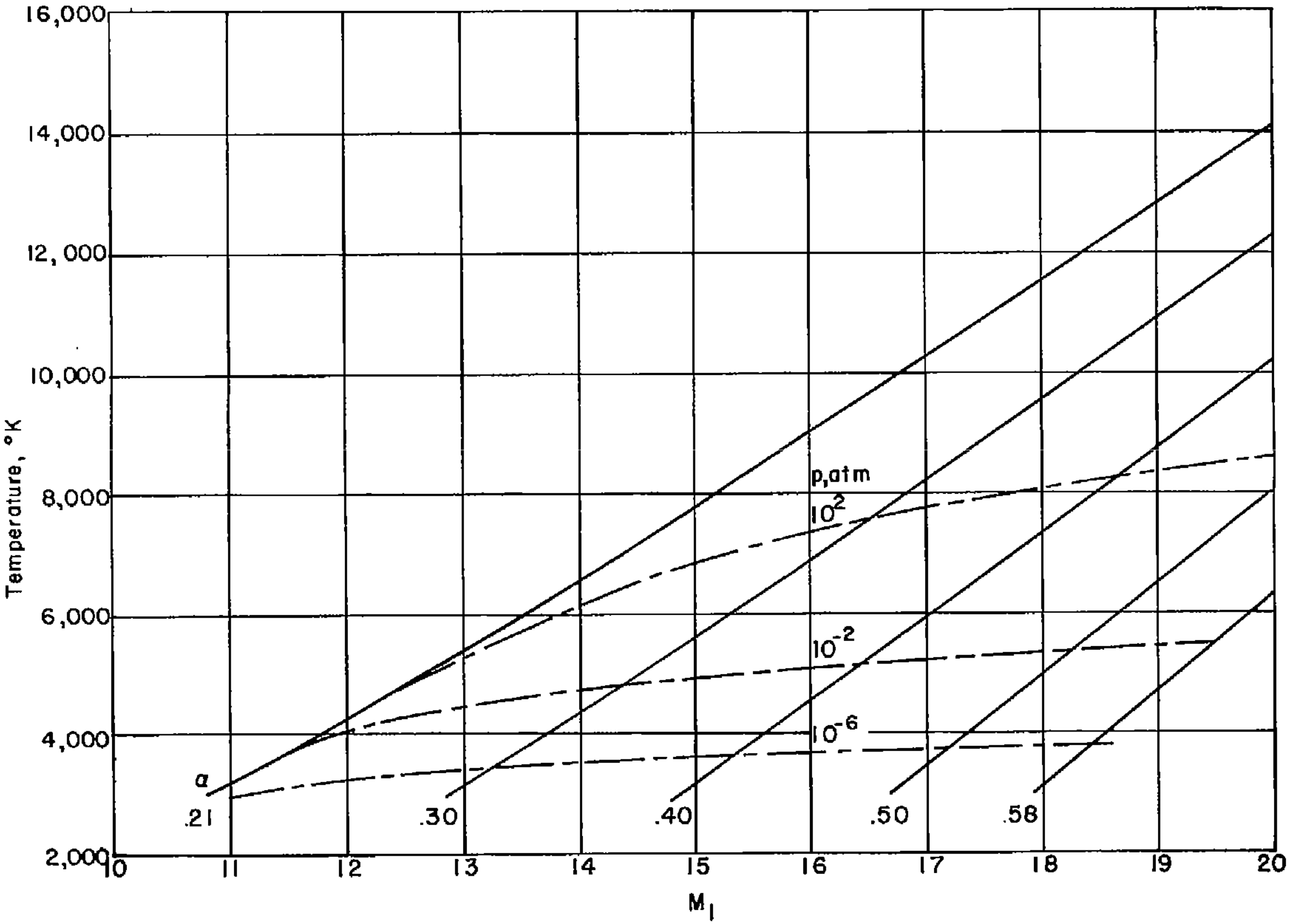
(b) p/p_1.

Figure 6.- Continued.



(c) ρ/ρ_1.

Figure 6.- Continued.



(d) Temperature.

Figure 6.- Concluded.

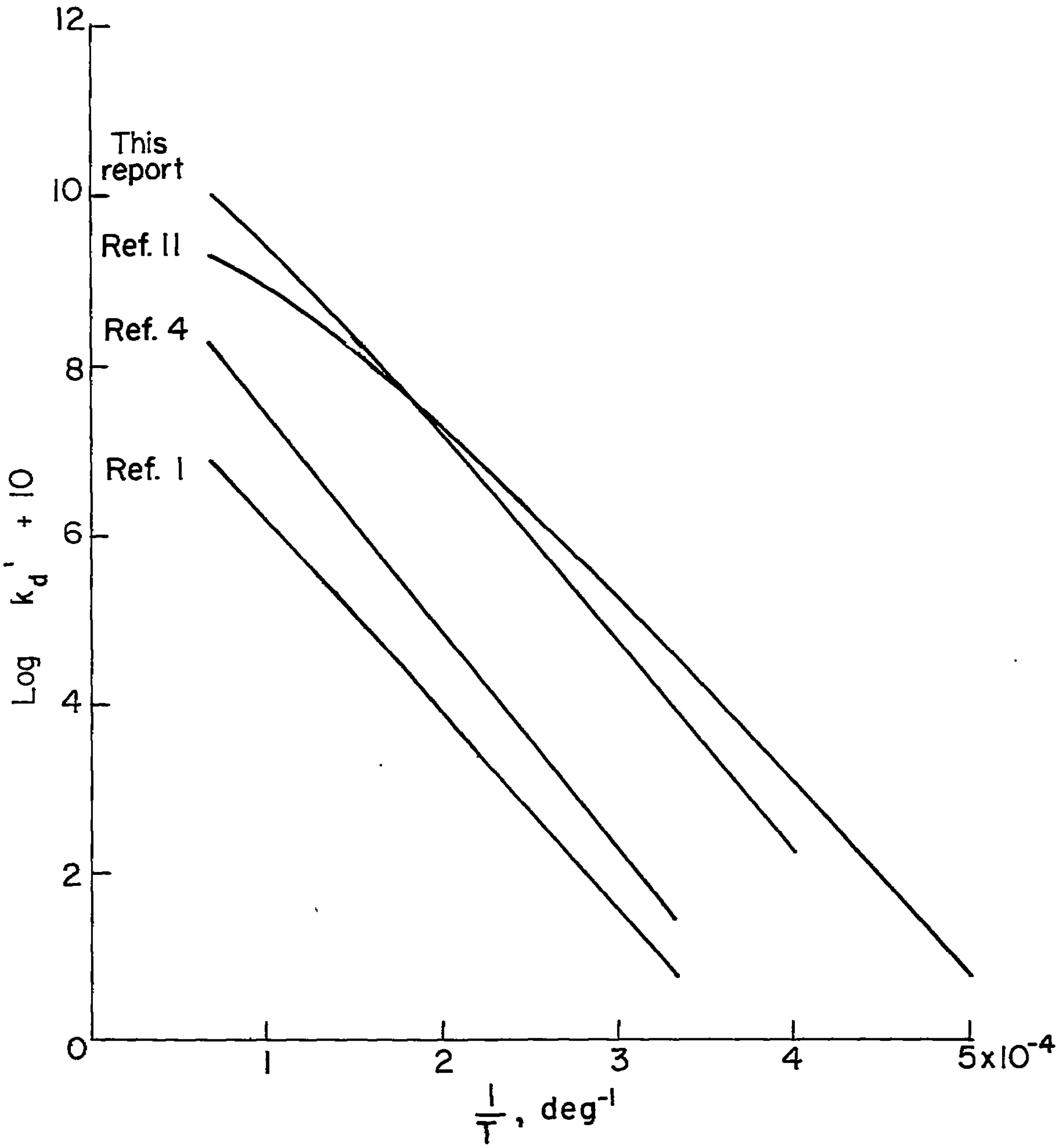
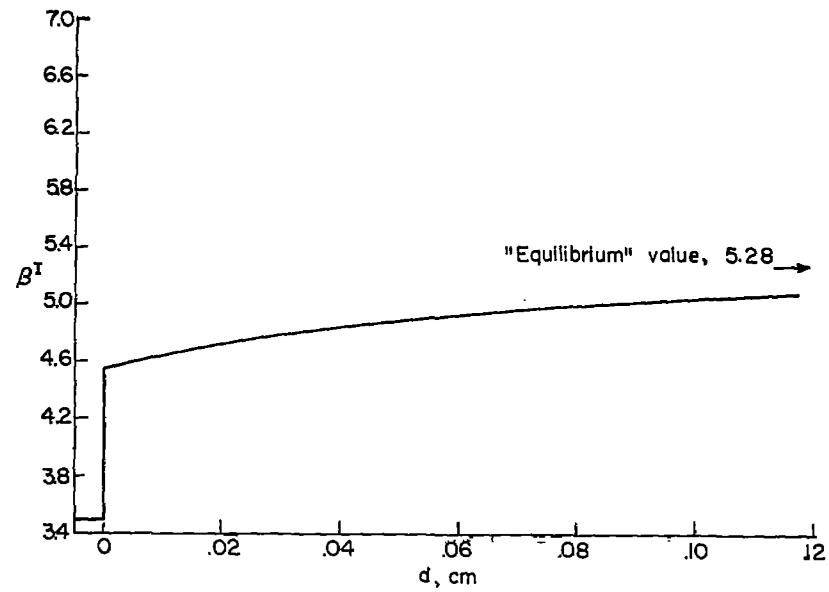
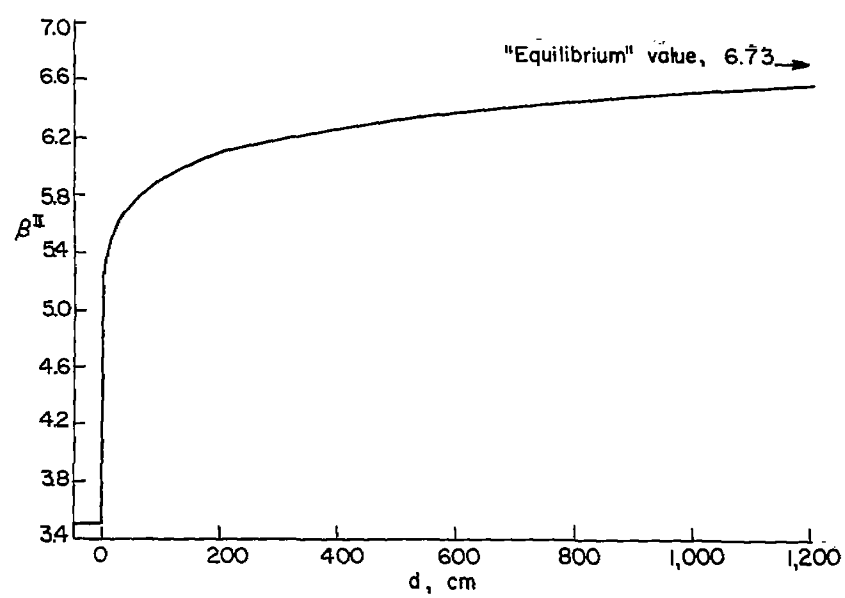


Figure 7.- Comparison of theoretical estimates of specific reaction rates for dissociation of O_2.

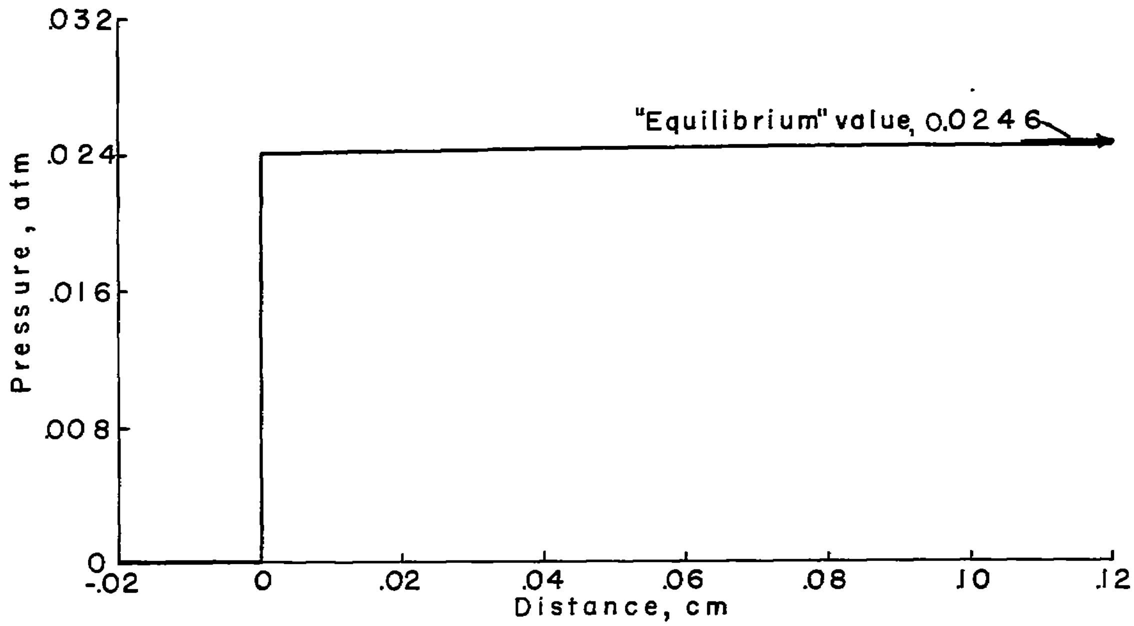


(a) Enthalpy parameter β^{I}. N_2 undissociated.

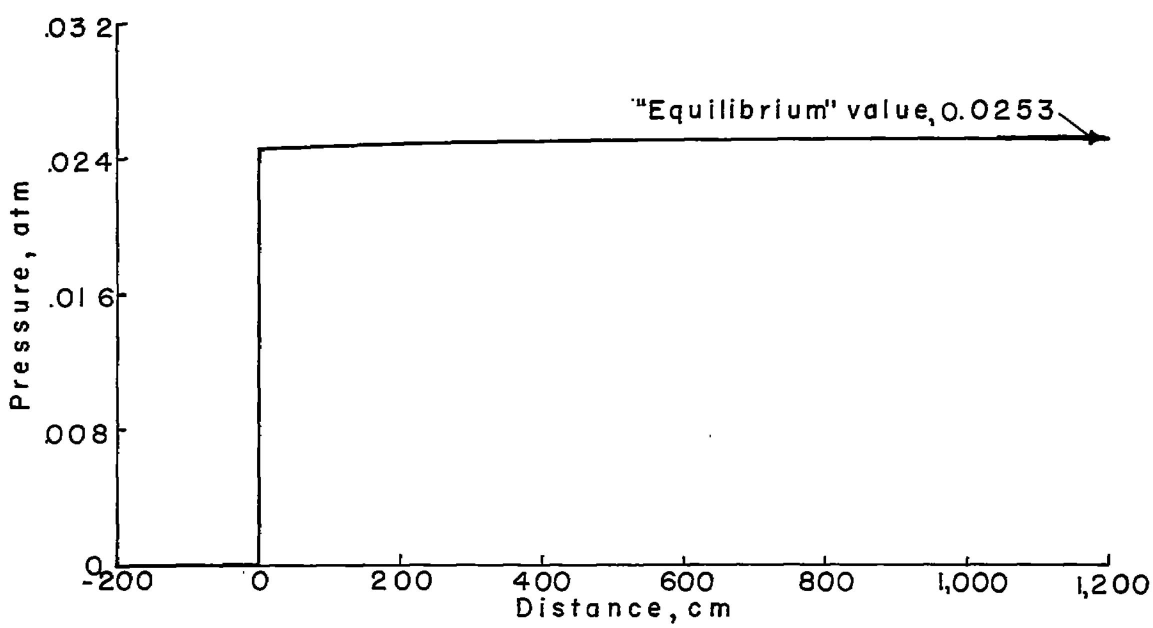


(b) Enthalpy parameter β^{II}. O_2 completely dissociated.

Figure 8.- Variation of flow properties with distance behind a shock wave in air. $M_1 = 14$; $T_1 = 300^\circ$ K; $p_1 = 10^{-4}$ atmospheres.

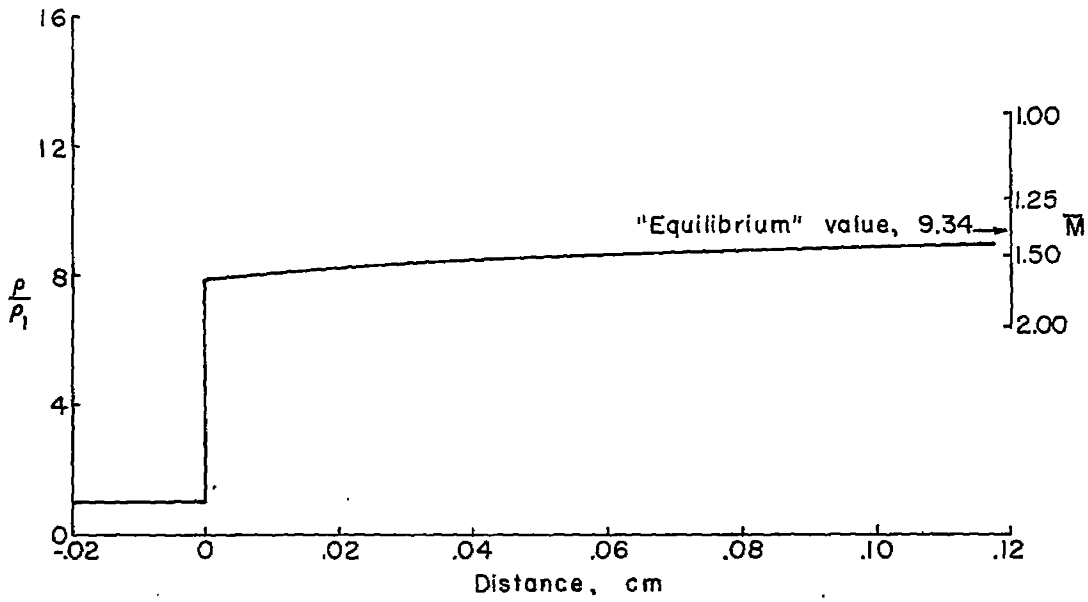


(c) Pressure. N_2 undissociated.

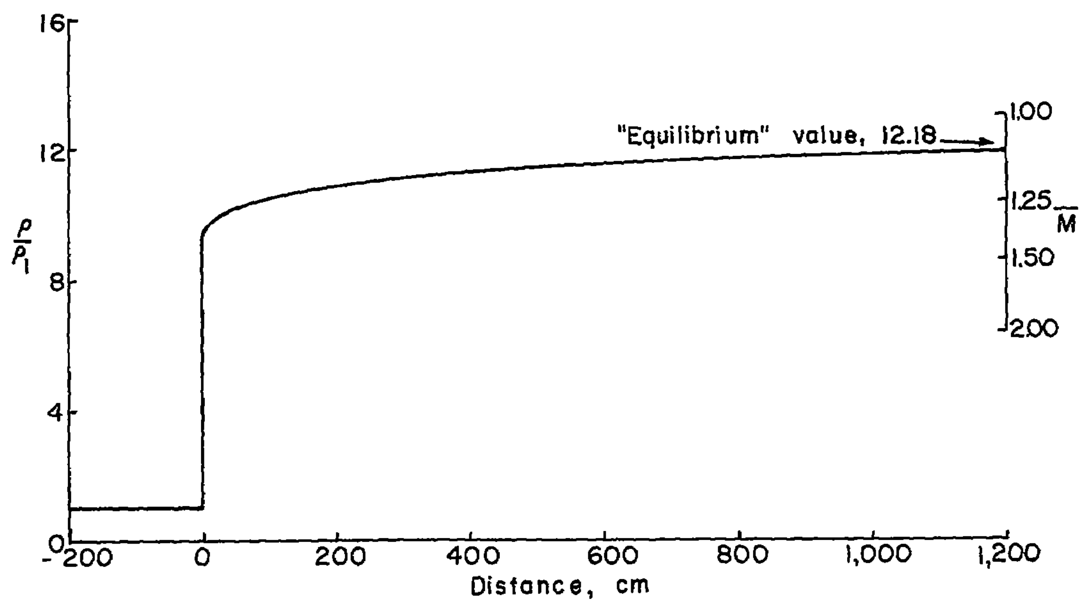


(d) Pressure. O_2 completely dissociated.

Figure 8.- Continued.

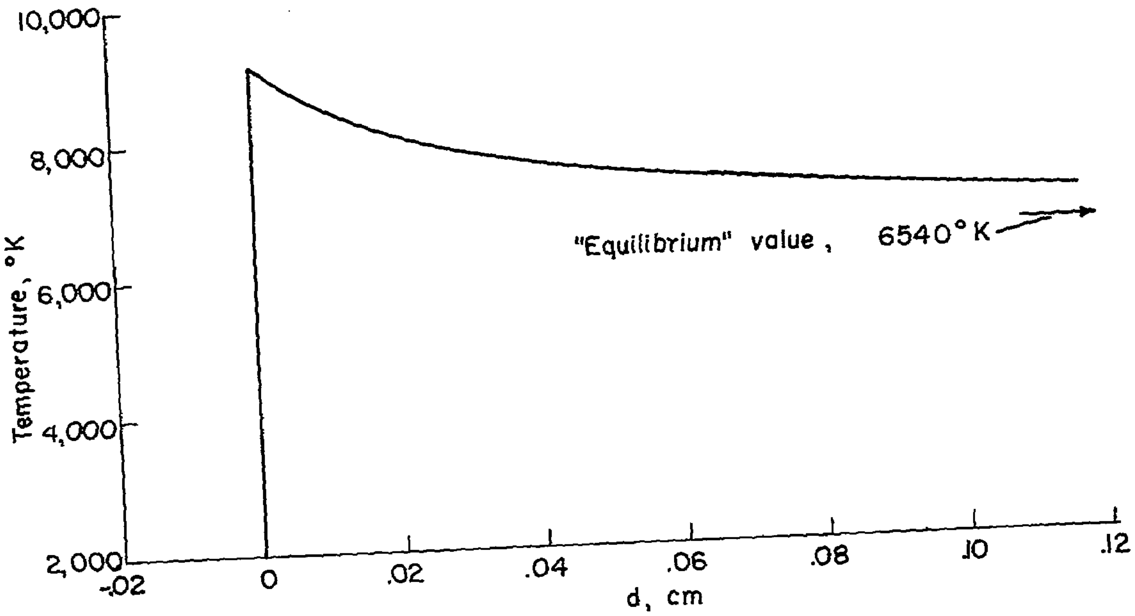


(e) Density and velocity. $\overline{M} = v/a_1$; N_2 undissociated.

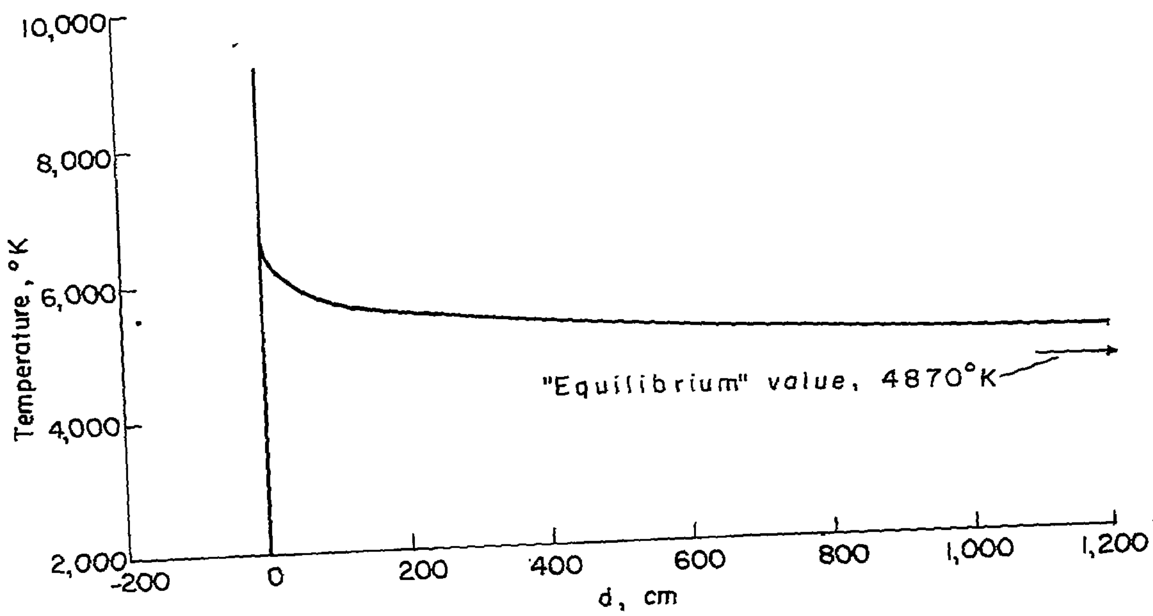


(f) Density and velocity. $\overline{M} = v/a_1$; O_2 completely dissociated.

Figure 8.- Continued.



(g) Temperature. N_2 undissociated.



(h) Temperature. O_2 completely dissociated.

Figure 8.- Concluded.

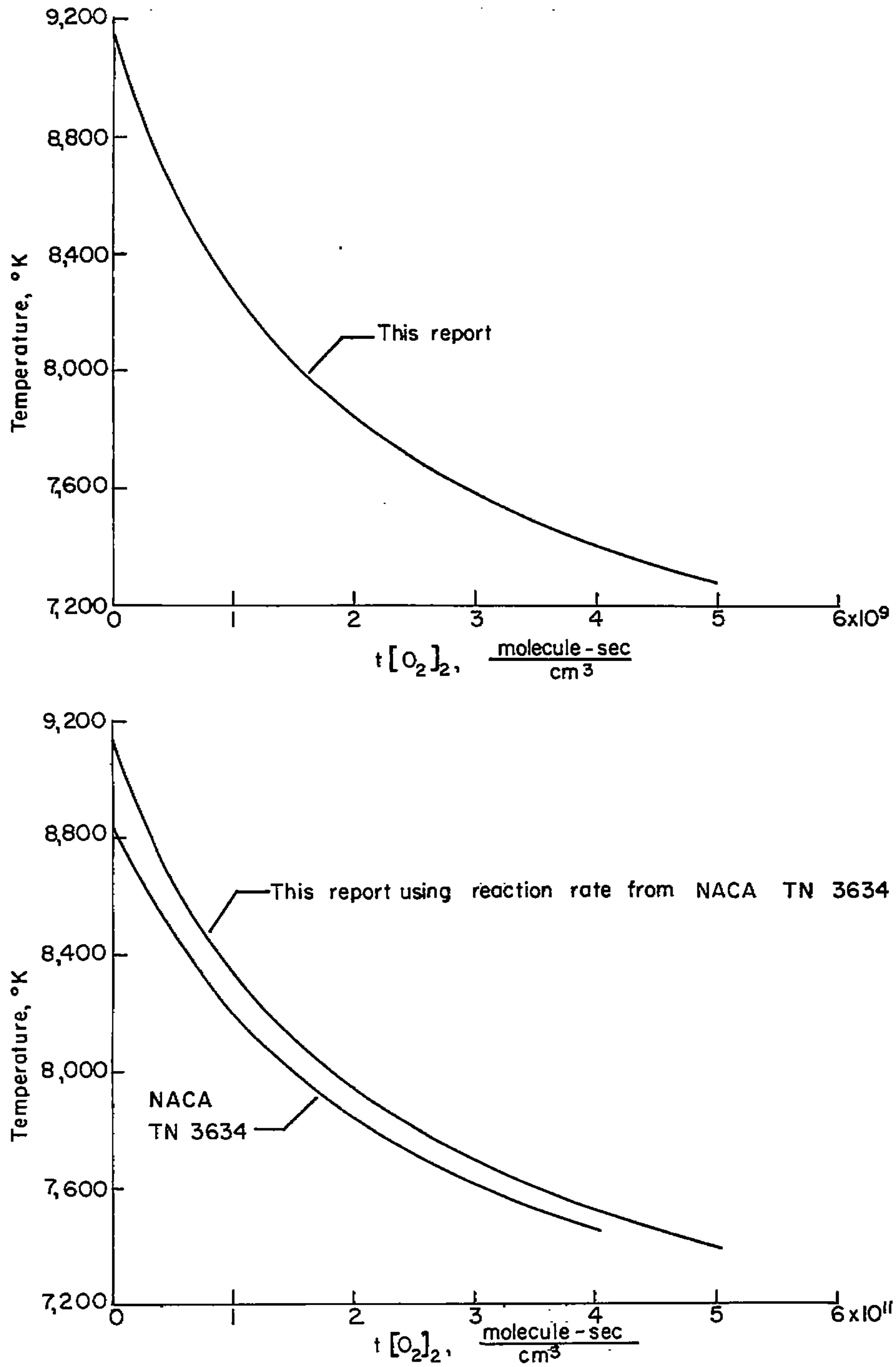


Figure 9.- Comparison of the results of this paper with those of NACA TN 3634. $M_1 = 14$; $T_1 = 300^\circ$ K; N_2 undissociated.

NACA - Langley Field, Va.