REPORT No. 211

WATER MODEL TESTS FOR SEMIRIGID AIRSHIPS

By L. B. TUCKERMAN
Bureau of Standards

REPORT No. 211

WATER MODEL TESTS FOR SEMIRIGID AIRSHIPS

By L. B. Tuckerman

INTRODUCTION

This report is based on a study made by the writer as a member of the Special Committee on Design of Army Semirigid Airship *RS-1* appointed by the National Advisory Committee for Aeronautics.

The semirigid airship such as the *Roma*, the *Forlanini*, the Italian military type, or the *RS-1* now building for the United States Army, depends, for its strength to resist static and aerodynamic forces, partly on the envelope under pressure and partly on the articulated (Italian military type) or "rigid" (*Roma, Forlanini, RS-1*) keel.

Theoretical considerations show that the interaction of keel and envelope may be partly favorable and partly unfavorable. As a combined beam they unite to resist bending moments, distributing the bending moments between them, but the "breathing" of the envelope, or poor fit of keel to envelope, cause them to act against each other, setting up additional "internal" stresses balanced between keel and envelope.

Obviously an accurate knowledge of the character of the interaction of envelope and keel, and the relative magnitude of these two effects is of importance in the refinement of the design of the semirigid airship.

Although the theory indicates clearly the existence of both these effects, attempts to calculate their magnitude from theoretical considerations have failed on account of mathematical difficulties involved. Mr. Pagon and Professor Hovgaard (of the National Advisory Committee for Aeronautics *RS-1* committee) have both made simplifying assumptions and secured interesting results, but the assumptions they found necessary are such as to place in doubt even the order of magnitude of their numerical results.

A careful study of their work has not shown any feasible way of removing this difficulty. All assumptions tried which seem reasonably definite, lead to a maze of simultaneous equations involving elliptic integrals. As a double differentiation of the solution of these equations, with respect to pressure and distance, is involved in the determination of the shear stresses, it seems doubtful whether existing tables of elliptic integrals are adequate for their computation, and even if the tables were adequate, the computations would be unreasonably time consuming. Crocco's mechanical computer, although satisfactory for laying out the envelope, is similarly inadequate for the computation of these stresses due to interaction of keel and envelope.

It is therefore worth while to inquire what information regarding this interaction of keel and envelope can be gained from a water model.

Water models have frequently been used for determining the shapes and strengths of balloons and airships and their deformations under static load. The model built to scale, of the same fabric as is used in the ship, is hung upside down and filled with water under pressure and its behavior under different water pressures and different applied loads is studied.

The effect of kinetic loads—such as the wind forces acting on airships in flight, can not be directly determined by water-model tests. It is necessary to determine these wind forces independently by theoretical computations, or by observation on airships in flight or on models in a wind tunnel. The effect of these kinetic forces is then determined by subjecting the model to equivalent static forces. The theory of such model tests can be found in numerous publications,[1] but, so far, I have seen no discussion of models with flexible keels designed to simulate the flexibility of the keel structure in the semirigid airship.

THE FLEXIBLE KEEL WATER MODEL

A flexible-keel water model will of course be subject to all the conditions of size, pressure, loads, etc., which are necessary in balloon and nonrigid airship models and in addition, to conditions specifying the relations which the elastic constants of the keel in the model should bear to those of the ship. The derivation of these additional relations is the primary object of this paper.

BUCKINGHAM'S Π THEOREM

The law of physical similitude, or of dynamic similarity (as it is known when the problems are purely mechanical in their nature) first stated by Newton and developed in recent years by Reynolds, Rayleigh, and others, underlies all theories of model tests. Buckingham[2] has formulated this law in a theorem, the "Π theorem," which is especially convenient for the routine handling of these problems. The complete application of the theorem requires the listing of all the physical quantities ($Q_1, \cdot\cdot Q_j \cdot\cdot Q_n$) involved in the dynamic behavior to be studied, together with the dimensions of each in terms of some convenient system of (m) fundamental units. Buckingham's Π theorem then states that any equation connecting these (n) quantities, may be written in the form

$$f\,(\Pi_1, \Pi_2, \Pi_k, \cdot\cdot \Pi_{n-m}) = O$$

where

$$\Pi_1, \Pi_2, \cdot\cdot \Pi_k \cdot\cdot \Pi_{n-m}$$

are any ($n-m$) independent products of the form $Q_1^{a_1}, Q_2^{a_2}, \cdot\cdot\cdot Q_J^{a_J} \cdot\cdot\cdot Q_n^{a_n}$ dimensionless in terms of the fundamental units chosen,[3] $a_1, a_2 \cdot\cdot\cdot a_n$ being pure numbers. Some of these Π's are well known in aerodynamic theory, such as the Reynolds Number $\frac{LV\rho}{\mu}$, the lift and drag coefficients of airplanes $\frac{R}{\frac{1}{2}\delta V^2 A}$, the fineness ratios of airfoils and airships $\frac{L}{L'}$, etc.

The advantage of the formulation of the law of dynamic similarity in the form of Buckingham's Π theorem lies in the fact that the attention can be concentrated on the purely physical aspects of the problem, that is, on listing, with their dimensions, all of the quantities upon which the particular dynamic behavior under investigation materially depends.

The formation of ($n-m$) independent Π_k's is then a matter of routine. After any set of ($n-m$) independent Π_k's has been found, the arrangement of them into physically more significant groupings is a matter of simple inspection.

SCOPE OF DISCUSSION

Although the conditions for the nonrigid water model could be assumed and the additional relations for the flexible keel separately determined it seemed easier to carry through a systematic discussion on the basis of the Π theorem.

The following discussion, then, is intended to include all of the essential factors of water-model design and will, therefore, in large part, reproduce the well-known results of previous water-model theories in developing the conditions necessary for a flexible-keel water model.

[1] References: Crocco, La Technique Aerienne, June 1, 1911; Haas and Dietzius, N. A. C. A. Report No. 16, 1917. F. D. Swain, Air Service (War Dept.) Engineering Division, McCook Field Report No. 2067, Apr. 22, 1922; J. C Hunsaker, Navy Dept., Bureau of Aeronautics, Technical Note No. 1; Upson, Unpublished memorandum of Goodyear Tire & Rubber Co.

[2] E. Buckingham, Phys. Rev. Vol. IV, p. 345, 1914; Journal A. S. M. E., 1915; Phil. Mag. Vol. 43, p. 696, 1921.

NOTE.—This theorem in a somewhat modified form has recently been used in an extended discussion of model tests by A. H. Gilsen—"The Principle of Dynamical Similarity with Special Reference to Model Tests in Engineering (Land)," Vol. 117, pp. 325, 357, 391, and 422, 1924.

[3] For a discussion as to the limitations on the choice of these units the reader is referred to Buckingham's papers.

GROUPING OF PHYSICAL QUANTITIES

In listing these physical quantities, those of the same physical dimensions, which can be conveniently discussed together, will be listed together in a group. There will, in general, be several groups having the same physical dimensions. Thus, for example, the flexural modulus of the fabric, the flexural and torsional strengths of the keel and the bending moments of the load all have the same physical dimensions (FL). Their relations to the behavior of the model are, however, so different that they can not be conveniently discussed together and they are therefore listed in three separate groups in spite of the fact that they have the same dimensions.

COMPLETE GEOMETRICAL SIMILARITY UNNECESSARY

In so far as shape affects the behavior of the airship, dynamical similarity requires that the model be exactly geometrically similar to the full-sized original; but if the action of a certain member such as a wire or girder depends only on its elasticity or strength, its visible external form is a matter of no importance. The fluid forces on the envelope are of vital importance and, since they depend on the form of the envelope, the model must, in this respect, be geometrically similar to the full-sized ship. But if the fluid forces which act *directly* on the keel are of negligible importance in comparison with the forces between the keel and the envelope, the only strictly geometrical condition imposed on the keel is that its points of attachment to the envelope be similarly situated to those in the full-sized keel. All that is required of the model keel is that its elastic and strength constants shall be suitably adjusted, and its actual shape aside from the positions of the envelope attachments is immaterial because it has no effect on what happens.

ASSUMPTIONS CONCERNING FABRIC OF ENVELOPE

Thus, in a water model one-thirtieth the length of an airship, geometrical similarity would demand an envelope one-thirtieth the thickness of the airship envelope. As this is clearly not feasible, it is usual to assume that the thickness of the envelope is geometrically a negligible factor and that the actual envelope could be replaced by an envelope of any other thickness (small in comparison with the other dimensions of the ship) without affecting its dynamical behavior. Experiments show that this assumption is ordinarily reasonable. This other envelope, however, must be dynamically equivalent to the actual envelope, i. e., it must, considered as an elastic surface, offer the same resistance to deformations as the actual surface. This implies that all the elastic constants of the envelope, tension moduli, shear modulus, tensile strength, and flexural resistance, may be sufficiently specified in terms of forces and moments per unit length instead of per unit area. This is, of course, common in textile measurements, where the strength of a fabric is expressed as a force per linear (not square) inch.

FUNDAMENTAL UNITS

Since the water model is subjected only to static loads, only two fundamental units are needed. For convenience we shall adopt length (L) and force (F) as our fundamental units.

FABRIC CONSTANTS

The *fabric* of the envelope will then be characterized dynamically by the following fabric constants:

1. Its weight per unit area, μ dimensions (FL^{-2})

$$\left.\begin{array}{ll}\text{2. 2 tension moduli} & F_1, F_2 \\ \text{1 shear modulus} & F_3 \\ \text{2 tensile strengths} & F_4, F_5\end{array}\right\} (F_h) \text{ dimensions } (FL^{-1})$$

3. A flexural modulus σ dimensions (FL)

The modulus of normal shear is negligible in all practical cases.

FABRIC STRESSES

There will be induced in the fabric certain tensile and shear stresses measured as

4. Force per unit length T_1, T_2, T_3 (T_h) dimensions (FL^{-1})

ASSUMPTIONS CONCERNING KEEL

Similarly it is obviously impossible to reproduce the keel structure in detail. Only the outer surface of the model keel will reproduce the geometrical shape of the airship keel.

It seems reasonable to assume that the keel will be adequately represented dynamically by a thin elastic rod in which shear deformation and shear stresses are not negligible.

This, perhaps, needs a more detailed explanation. In the theory of the deformations of thin elastic rods, it is assumed that any portion of the rod is equivalent to any other, differing in material or distribution of material through the cross section, provided that the curvatures and twists produced in the two by the same bending moments and the same axial torque are identical, and provided that rupture, permanent deformation, or other failure will occur under identical axial loads, bending moments, and torque. Due to its low stiffness in shear the curvature of the keel of the airship at any place will depend appreciably not only upon the bending moments, but also upon the local distribution of shears. These shears are assumed to be of negligible importance in the ordinary theory of thin rods. Consequently, an adequate representation of the characteristics of the keel must include in addition two shear moduli.

KEEL CONSTANTS

The *keel* will then be characterized dynamically by the following keel constants given as functions of the distance along the keel measured as a fraction of its total length:

5. Weight per unit length m dimensions (FL^{-1})
6. 2 flexural moduli K_1, K_2; 1 torsional modulus K_3 $\}$ (K_h) dimensions (FL^{-2})
7. 2 flexural strengths H_1, H_2; 1 torsional strength H_3 $\}$ (H_h) dimensions (FL)
8. 2 shear moduli S_1, S_2 (S_h) dimensions (F)
9. 2 shear strengths S_1', S_2' (S_h') dimensions (F)
10. 1 stretch modulus s dimensions (F)
11. 1 stretch strength s' dimensions (F)

It is unnecessary to consider a compressive strength since in airship construction lightness requires large compression members to be so flexible that compressive failure will only occur in flexure.

WIRE CONSTANTS

Since the weight of the *suspender wires* is a very minor element in the design and their strength is always easily made adequate, it seems reasonable to assume that each can be adequately represented dynamically by a single

12. Wire or cordage stretch constant $W_1, W_2, \ldots (W_h)$, dimensions (F).

SUFFICIENCY OF CONSTANTS

These structural constants are thought to include all the dynamical characteristics of the material and the structure which are of significance in the problem. As a matter of fact, some of these given will be shown to be unnecessary for the purpose. Others are almost certainly of negligible importance. Still others impose conditions on the model which are impracticable so that their disturbing influence must be carefully considered. The list was made unnecessarily full merely to insure that no really significant characteristics were omitted. If, however, any significant structural constants have been omitted the conclusions will be uncertain to the extent that such omitted constants are of importance.

LOADS

Aside from these constants of the material and structure, the forces are essential elements in the problem. These can be applied as concentrated loads, including shears (load differences); they represent weights of cars, fuel tanks, and useful load, propeller thrust, etc.

13. Loads P_1, P_2, . . . (P_k) dimensions (F). Bending moments can of course be calculated back to the forces from which they arise. It is, however, frequently desirable to treat them as independent load elements especially when studying the effect of forces remote from their point of application, or of aerodynamic moments whose force distributions are not accurately known. It is therefore convenient to introduce

14. Bending moments M_1, M_2, . . . (M_k) dimensions (FL). That these in part duplicate the forces listed under (13) is no objection since a redundant list does not interfere with the validity of the Π theorem. The gas and air pressure might also be included under the forces (13) but because of their manner of application they are more conveniently listed separately.

15. Pressures, gas and aerodynamic p_1, p_2, . . . (p_k) dimensions (FL^{-2}).

DEFORMATIONS

The behavior of the ship under these loads may be studied: First, by

16. Deflections δ_1, δ_2, . . . (δ_k) dimensions (L) which measure its deformation under load, and determine the mode of interaction of its various parts. These are what would be determined in deformation or shape tests.

Volume changes could also be separately listed, but as these always change as the cube of a linear dimension (dimensions (L^3)) a separate term seemed unnecessary.

STRENGTH

Second, by its failure in whole or in part due (a) to tensile stresses, T_k, in the fabric exceeding the corresponding tensile strengths, F_4, F_5; (b) to bending moments or torques in the keel exceeding the corresponding strengths, H_1, H_2, H_3, or to shears exceeding its shear strength, S', or axial forces exceeding its stretch strength, s'. Tests which determine these conditions of failure are strength tests.

ADVANTAGE OF LIMITATION TO DEFORMATION TESTS

Even in structures of this type there is, over a considerable range, approximate proportionality between load and deflections, so that deflection experiments at low loads, where there is no danger of failure of any part, may be expected to give a satisfactory picture of the dynamic interaction of these parts. This is important, because if low-load tests are adequate for the purpose, it is not necessary to specify the strength constants of the model which will greatly simplify its design.

Moreover, if low-load tests give an adequate understanding of the dynamic interaction of the various parts, strength tests may be unnecessary since the strength of individual parts can be sufficiently well estimated by elementary theory if the laws of interaction of the various parts are known.

In this estimation judgment must be used. It would not be safe to calculate strengths directly on an assumption that loads and deflections are proportional up to failure. Allowance must be made for the deviations from proportionality at high loads. It seems probable, however, that the general nature of these deviations can be determined from low-load tests. The problem is similar to that involved in beam design where the stresses are calculated from elastic theory—allowance being made for the known deviation from Hooke's law at high stresses.

SIZE

The size of the ship (and model) may be characterized by its overall

17. Length L dimensions (L). All other significant dimensions are of course proportional to this.

FLUID DENSITIES

The buoyant (or loading) effect of the fluids used is determined by the density differences between internal (gas, water) and external (air) fluids. It is equal to this density difference multiplied by the acceleration of gravity.

18. Buoyancy B dimensions (FL^{-3}).

FORMATION OF THE Π'S

From any complete list of the (n) physical quantities (Q_r) involved in a physical phenomenon an indefinite number of products (Π) dimensionless in the (m) fundamental units can be found. Only $(n-m)$ of them, however, are independent. From any group of $(n-m)$ independent Π's any other Π can be formed by multiplication, division, and extraction of roots.

When several of the physical quantities (Q_r) have the same dimensions, and any Π has been found containing one of them as a factor, other Π's independent of each other can be found directly from this one by replacing this (Q_r) in turn by each of the others of the same dimensions. Thus since $\frac{BL^2}{F_1}$ is dimensionless, so also are $\frac{BL^2}{F_2}$, $\frac{BL^2}{F_3}$, $\frac{BL^2}{F_4}$ and $\frac{BL^2}{F_5}$ and these form a group of five mutually independent Π's all of the same type. The quantities F_1, F_2, F_3, F_4, and F_5 have been listed in a group (2) under the general symbol F_h. For convenience in discussing them together we shall represent this group of Π's which are all of the same type by the single symbol $\Pi_1 = \frac{BL^2}{F_h}$. In what follows then Π_k will represent not a single dimensionless product but the group of all the mutually independent Π's of the same type formed from the corresponding groups of Q_r's. Obviously if there are n' groups of Q_r's there will be $(n'-m)$ independent types of Π's.

APPLICATION OF THE Π THEOREM

With this notation, if the quantities which are arranged in these 18 groups are an adequate specification of the dynamic characteristics of the ship, the law of dynamic similarity as expressed in Buckingham's Π theorem states that any equation representing a dynamic behavior of the structure can be expressed in the form,

$$f(\Pi_1, \Pi_2, \ldots \Pi_k \ldots \Pi_{16}) = O$$

where f is a function characterizing the particular dynamic behavior in question and $\Pi_1, \ldots \ldots \Pi_{16}$ represent $(n-2)$ Π's of any 16 independent types, dimensionless in the chosen units $(F, L,)$ formed from all the (n) quantities of the 18 types by multiplication and division. For complete dynamic similarity to exist, all except one of these Π's must be given the same value in the model as in the ship. The other will then necessarily have the same value. Each of the Π's then represents a condition to be imposed on the model and any Π_k represents a group of such conditions,[4] including the obvious one that all quantities of the same group should have the same ratio in model as in ship.

The following seem to be the simplest expressions of these conditions. They have been chosen so that the first determines the model length in terms of the buoyancy (B) and the others determine the remaining quantities in terms of the length:

	Determines
$\Pi_1 = BL^3 \frac{1}{F_h}$	Model length.
$\Pi_2 = \frac{P_h}{L} \frac{1}{F_h}$	Model loads.
$\Pi_3 = \frac{M_h}{L^2} \frac{1}{F_h}$	Model moments.
$\Pi_4 = \frac{P_h L}{F_h}$	Model pressures.
$\Pi_5 = \frac{\delta_h}{L}$ or its equivalent $\Pi'_5 = \frac{\text{Volume change}}{L^3}$	Scale of model deformations.
$\Pi_6 = \frac{T_h}{F_h}$	Fabric tensions in model.

These six relations are usually given in the elementary theory of water models.

[4] For another method of treatment of groups of quantities of the same dimensions, see Buckingham l. c.

	Determines
$\Pi_7 = \frac{W_h}{L}\frac{1}{F_h}$	Size of model wire or cordage used for suspensions.
$\Pi_8 = \mu L \frac{1}{F_h}$	Fabric counterweight.

Π_8 is considered by Hunsaker (l. c.) but Π_7 has been found only in the unpublished memorandum of Upson.

The next eight, relating to the flexible keel and to a caution with reference to the fabric have not been found in previous discussions.

	Determines
$\Pi_9 = m \frac{1}{F_h}$	Keel counterweight.
$\left.\begin{array}{l}\Pi_{10} = \frac{K_h}{L^3}\frac{1}{F_h} \\ \Pi_{11} = \frac{S_h}{L}\frac{1}{F_h} \\ \Pi_{12} = \frac{s}{L}\frac{1}{F_h}\end{array}\right\}$	Elastic constants of model keel.
$\left.\begin{array}{l}\Pi_{13} = \frac{H_h}{L^2}\frac{1}{F_h} \\ \Pi_{14} = \frac{S_h'}{L}\frac{1}{F_h} \\ \Pi_{15} = \frac{s'}{L}\frac{1}{F_h}\end{array}\right\}$	Strength constants of model keel.
$\Pi_{16} = \frac{\sigma}{L^2}\frac{1}{F_h}$	Flexural rigidity of model envelope.

In the following discussion we shall use the subscript s for the ship and m for the model. Dynamic similarity then requires that $\Pi_{km} = \Pi_{ks}$. This, as will be seen, can not be completely realized.

Π_1 DENSITY DIFFERENCE, SIZE, AND FABRIC CONSTANTS

The buoyance, B_s, for the ship varies somewhat with flying conditions. For hydrogen, at present, it is usual to assume 68 pounds per 1,000 cubic feet, and for helium, 64 pounds per 1,000 cubic feet. The (negative) buoyancy B_m for the water model is, for all practical purposes, the buoyancy of water at O° C, 62.4 pounds per cubic foot. Henco the ratio is

$$\frac{B_s}{B_m} = \begin{cases} 0.00109 \text{ for hydrogen} \\ 0.001025 \text{ for helium} \end{cases}$$

The fabric constants F_1, F_2, F_3, F_4, and F_5, should have the same ratio in model as in ship. It is technically impossible to produce two markedly different fabrics for which this is true. Consequently, it is customary to use the *same* fabric in model as in ship, assuming $F_{hs} = F_{hm}$. In practice this is not perfectly realized. Equal strength demands equal overlap at seams in model and in ship. As the seams are a much greater portion of the area of the model, this results in an effectively stiffer model envelope, i. e., $F_{1s} < F_{1m}$, $F_{2s} < F_{2m}$, $F_{3s} < F_{3m}$, while the strength constants $F_{4s} = F_{4m}$, and $F_{5s} = F_{5m}$. This discrepancy, although not great, is still not negligible, amounting to about 15 to 20 per cent in the case of the *RS-1*. So far as deformations are concerned, this could probably be adequately allowed for by correcting F_1, F_2, and

F_8, by the ratio of seam area to total envelope area in model and ship (this is approximately 1 per cent in the *RS-1* and 17 per cent in the water model tested at Akron) and correspondingly increasing the scale of the model. Such a procedure would lead to an underestimate of strength if it were used for a strength test.

If, however, we relinquish strength tests on the model, F_{4m} and F_{5m} may safely be much smaller than F_{4s} and F_{5s}. In a conversation, Mr. Zimmerman of the Goodyear Co. estimated that if we were content with an equivalent of 2 to 2½ inches ship pressure, the width of overlap in the model could probably be reduced to one-fourth that in the ship, making the correction involved less than 4 per cent, which is probably negligible.

As the bursting strength can be fairly well determined from laboratory tests on the fabric, it would seem preferable to do this.

Where suspension patches are used a similar difficulty is involved. Since the strength of their attachment to the envelope depends almost entirely on the shear resistance of the cement film, equal strength requires that this area be approximately one-thirtieth as great in model as in ship instead of one nine-hundredth as required by geometrical similarity. This discrepancy also can be reduced if strength tests are not required, but in any case the shape and stress of the envelope near patches must be expected to differ considerably in model and in ship.

Assuming, with these qualifications

$$F_{hs} = F_{hm}$$

the requirement that $\Pi_{1m} = \Pi_{1s}$ gives

$$\frac{L_m}{L_s} = \sqrt{\frac{B_s}{B_m}} = \begin{cases} 0.033 \text{ for hydrogen} \\ 0.032 \text{ for helium} \end{cases}$$

the well-known ratio of approximately 1:30. The small correction for seam overlap indicated above could readily be made if it seemed desirable. As F_h appears in practically all the Π's, this would mean a slight correction to nearly all the model constants. For simplicity of discussion, it will be omitted, and $F_{hm} = F_{hs}$ assumed. The factor $\frac{1}{F_h}$ appearing in the succeeding Π's is then constant and need not be discussed further.

Π_2, Π_3, Π_4, Π_5, Π_6—PRESSURES, FABRIC TENSIONS, LOADS, MOMENTS, AND DEFLECTIONS

These show pressures varying as $\frac{1}{L}$, fabric tensions independent of L, loads and deflections proportional to L, moments proportional to L^2 and volume changes to L^3. As these conditions have been fully discussed in previous publications, they do not need further discussion.

Π_7 SUSPENDER WIRES OR CORDS

Since the W_h's are directly proportional to the cross-sectional area of the wires (nearly so for cords of similar construction) the model wires and cords, if of the same material and construction, will have diameters varying approximately as $\sqrt{L}$. As the stretch of suspenders usually constitutes only a small portion of the total deflections of the ship, this condition ordinarily need not be accurately fulfilled. It is merely necessary to choose from available standard wires and cords those which fit the conditions most nearly.

Π_8 AND Π_9 COUNTERWEIGHTS OF KEEL AND ENVELOPE

Here the model differs radically from the ship. In the ship the gas inside is less dense than the air outside, so that the weight of keel and fabric (downward) is opposite in direction of the gas lift (upward). To simulate this condition in the water model it would be necessary to immerse the model in a tank of water and fill it with air under pressure. As this would be difficult experimentally the model is turned upside down and filled with water. The weight of the keel and fabric (downward) is now in the same direction as the water load (downward) which is directly opposite to the condition in the ship. To compensate for this counterweighting may be employed. Theoretically, each portion of the envelope should be counterweighted by $\left(\frac{L_s}{L_m}+1\right)=\frac{\mu_s+\mu_m}{\mu_m}$ times its weight and each portion of the keel by $\frac{(m_s+m_m)}{m_m}$ times its own weight.

Where the actual shape of the envelope is sought from the model test, the accurate individual distributed counterweighting of fabric and keel is important. This, however, requires complicated devices. For fabric counterweighting air bags and netting suspensions have been used which give rough approximations. It seems possible that distributed buoyant material sewed to the inside of the bag might be used. Mr. C. P. Burgess has suggested that this compensation might also be effected by making the model proportionally smaller. Any such change, however, should be used with caution and only relied upon after an investigation of all the other relations involved.

If only *changes* of shape under *changing* loads are desired, it would seem that the complications of separate fabric counterweighting might safely be omitted. This would give a model shape differing from the shape of the ship by less than the changes in shape experienced in normal conditions under changing superpressure. This difference would presumably cause only negligible second order differences in the measured changes under changing load.

If accurate distribution of counterweighting be not necessary, the total counterweighting indicated by Π_8 and Π_9 is automatically insured by the static equilibrium of the model.

Π_{13}, Π_{14}, Π_{15} KEEL STRENGTH

These conditions in connection with Π_{10}, Π_{11}, and Π_{12} seem practically impossible of realization. If, however, we confine our attention to deflection tests at low (safe) loads they can be ignored.

Π_{16} FLEXURAL STIFFNESS OF ENVELOPE

Observations of some model tests lead me to believe that this condition may sometimes be of importance in interpreting them. It requires that the flexural stiffness of the model fabric should be only approximately one nine-hundredth part of that of the ship. The flexural stiffness of the fabric in the ship is safely negligible but that does not mean that a fabric 900 times as stiff in flexure (other properties the same) would not take an appreciably different shape. In fact, it seems certain that it would. The general character of the difference is clear. The stiffer fabric would smooth out changes in curvature of the envelope, rounding off more flatly the portions of sharper curvature. In particular the stiffer fabric would tend to iron out wrinkles so that it is not safe to conclude from the absence of wrinkles in the model that they would not appear in the ship under corresponding conditions. These differences have been noted by others but no discussion of their cause has been found.

Π_{12} LONGITUDINAL STRETCH OF KEEL

The longitudinal stretch of the individual portions of the keel is negligible.

Π_{10} AND Π_{11}, KEEL FLEXIBILITY

The three relations contained in the form Π_{10} require that the two flexural moduli K_1 and K_2 (these are sometimes called "flexural rigidities") and the torsion modulus K_3 (sometimes called "torsional rigidity") of the keel all vary as L^5. For an isotropic solid section

$$K_1=EI_1 \text{ and } K_2=EI_2$$

while the torsion modulus K_3 is for a fairly compact isotropic solid section

$$K_3=M\frac{A^4}{4\Pi^2 I} \text{ approximately}$$

where M is the shear modulus of the natural A the area and I the polar moment of inertia of the cross section.

If the keel were an absolutely similar structure on a smaller scale these constants would vary as L^4. The requirements evidently call for a relatively somewhat stiffer keel construction in model than in ship. This is, of course, due to the fact that the envelope is proportionately stiffer in model than in ship. The two relations contained in the form Π_{11} require that the two shear moduli of the keel S_1 and S_2 vary as L. For an isotropic solid cross section

$$S_1 = S_2 = MA$$

For absolutely similar structures these would vary as L^2, so that the requirements demand that the keel of the model be also relatively stiffer in shear than in the ship.

In adequately meeting the conditions imposed by these Π's on the five keel constants K_1, K_2, K_3, and S_1 and S_2 lies the possibility of satisfactorily studying the dynamic interaction of keel and envelope.

In the articulated keel of the Italian military type it seems easily possible. Here the vertical flexural modulus $K_1 = 0$ and the vertical shear modulus $S_1 = 0$. The torsional modulus K_3 is small and can probably be safely assumed to be zero. This leaves only the horizontal flexural (K_2) and shear (S_2) moduli to be fitted to the model conditions. The vertical stiffness of the keel is furnished by the car suspensions which can easily be adjusted to meet the wire stretch conditions of Π_7.

Whether an adequate approximation to these five constants can be fitted to a model of the "rigid" keel of the *Roma* or *RS-1* is a question. The values for the ship can be adequately computed from the design data. Theoretically it is possible by properly slotting and boring out a solid keel of the requisite external dimensions to fit it to any value and any ratio of these constants. Practically, it can only be done by a series of cut and try operations continually controlled by measurement. How accurately this needs to be done, in order to secure an adequately representative model, can only be determined by experience.

Even if an accurate fitting is impracticable, it may be possible by experimenting with a number of model keels differing sufficiently in their elastic constants, to work out empirical laws in which these constants appear separately and thus compute back to the actual ship. Even in this case reliable results can only be expected if the flexibility of the model keel does not differ too much from the values indicated by the theory.

CONCLUSIONS

SCOPE OF TESTS

A test on a flexible-keel water model seems to promise valuable information concerning the interaction of keel and envelope in the case of a semirigid airship.

The model should, for best results, be constructed solely for the purpose of studying the change of shape under load.

Any attempt to combine strength tests with deformation tests in the same model would lead to many compromises between conflicting requirements, resulting in less certain results.

ENVELOPE CONSTRUCTION

For these deformation tests all seams and patches should be made as small as possible, consistent with sufficient strength to resist the stresses under relatively low pressures (perhaps 2 to 2½ inches ship pressure) and loads not exceeding the actual loads carried by the ship. By this means the envelope of the model can be made to represent more closely the elastic behavior of the ship's envelope.

CAUTION IN INTERPRETATION

Even when this is done it should be remembered that the model envelope is relatively stiffer than the ship envelope and especially so in flexure. Consequently the shape of the model in regions of sharp curvature, or wrinkling, or in the neighborhood of seams or patches, should not be expected to reproduce accurately the corresponding portions in the ship.

Allowance should also be made for the departure from proportionality between loads and deformations, when strengths are estimated from deformation tests.

COUNTERWEIGHTING

It may be desirable to attempt fairly accurate counterweighting of envelope and keel but for the first trials it would seem desirable to avoid this experimental complexity by confining the attention to *changes* of shape under *changing* loads, which would obviate the necessity of accurate envelope counterweighting.

As a supplementary experiment it is suggested that it might be worth while to attempt envelope counterweighting by means of distributed cork floats or similar devices sewed inside the envelope. The suggestion of a smaller model should only be attempted after a more detailed analysis of the problem.

The distributed counterweighting of the keel is not particularly complicated so that it should certainly be included in supplementary tests.

KEEL CONSTRUCTION

From a construction standpoint this will be the most difficult. The following is a suggested procedure:

Construct a solid keel of the requisite shape and dimensions of an easily worked material (probably wood). Subject this to measured bending moments, torques, and shears, measuring at uniformly spaced stations along it, the curvatures, twist, and shear deformations (these last will probably be negligible in the solid model). Calculate the moduli K_{1m}, K_{2m}, K_{3m}, S_{1m} and S_{2m} and plot them as ordinates with distance along the axis (as fractions of total length) as abscissæ. Plot the corresponding moduli calculated for the keel of the ship, K_{1s}, K_{2s}, and K_{3s} on a scale $\left(\frac{L_m}{L_s}\right)^3$ as large and S_{1s} and S_{2s} on a scale $\frac{L_m}{L_s}$ as large. To satisfy the conditions the plotted curves of K_{1m} should be identical with that of K_{1s}, of K_{2m} with that of K_{2s}, etc. Where the bending moduli K_{1m} and K_{2m} are too high, transverse saw cuts should be made. Where the torsion modulus is too high longitudinal saw cuts should be made.

For the solid model keel, S_{1m} and S_{2m} will probably be practically infinite. Transverse holes bored or cut through the model keel will reduce these values. Rectangular holes with sides parallel to the axis will be more effective than round ones in proportion to the amount of material removed. If sufficiently low shear moduli can not be obtained in this way a built-up keel model will be necessary.

These adjustments must be carefully carried out since the types of cut mentioned, although lowering in greatest measure the constants indicated, at the same time lower all of the elastic constants of the keel. Consequently the process of adjustment will be by a series of successive approximations until the desired constants are obtained.

How accurately this can be done practically can hardly be surmised in advance. The adjustment should be carried to the point at which it seems that further labor would be wasted. If only the same order of magnitude is obtained the test should still give useful information.

SUMMARY OF CONSTRUCTION DATA

1. Fabric same as in ship. Seams and patches as small as possible.
2. All lengths $\frac{L_m}{L_s}=\sqrt{\frac{B_s}{B_m}}=\begin{cases}0.033 \text{ for hydrogen}\\ 0.032 \text{ for helium}\end{cases}$
3. Supender wires or cords $\frac{W_{hm}}{W_{hs}}=\frac{L_m}{L_s}$
4. Keel constants, flexure, and torsion $\frac{K_{1m}}{K_{1s}}=\frac{K_{2m}}{K_{2s}}=\frac{K_{3m}}{K_{3s}}=\left(\frac{L_m}{L_s}\right)^3$

   Shear $\frac{S_{1m}}{S_{1s}}=\frac{S_{2m}}{S_{2s}}=\frac{L_m}{L_s}$

SUMMARY OF TEST DATA

LOADING

5. Fabric counterweighting proportional to $\mu_s\left(\frac{L_s}{L_m}+1\right)$
6. Keel counterweighting proportional to $\frac{m_s+m_m}{m_m}$
7. Loads $\frac{P_{hm}}{P_{hs}}=\frac{L_m}{L_s}$
8. Moments $\frac{M_{hm}}{M_{hs}}=\left(\frac{L_m}{L_s}\right)^2$
9. Pressures $\frac{p_{hm}}{p_{hs}}=\frac{L_s}{L_m}$

STRESSES AND DEFLECTIONS

10. Fabric stresses $\frac{T_{hm}}{T_{hs}}=1$
11. Deflections $\frac{\delta_{hm}}{\delta_{hs}}=\frac{L_m}{L_s}$

The other requirements of the theory are either unnecessary for deflection tests or impracticable. The important effects of the failure to meet these requirements are summarized in the conclusions under the heading "Caution in interpretation."