REPORT NO. 29.

THE GENERAL THEORY OF BLADE SCREWS.

INCLUDING PROPELLERS, FANS, HELICOPTER SCREWS, HELICOIDAL PUMPS, TURBO-MOTORS, AND DIFFERENT KINDS OF HELICOIDAL BRAKES.

By George de Bothezat.

INTRODUCTION.

The present theory gives a complete picture and an exact quantitative analysis of the whole phenomenon of the working of the blade screw.[1] This theory not only includes all cases of applications of blade screws, but also unites in a continuous whole the entire scale of states of work conceivable for a blade screw.

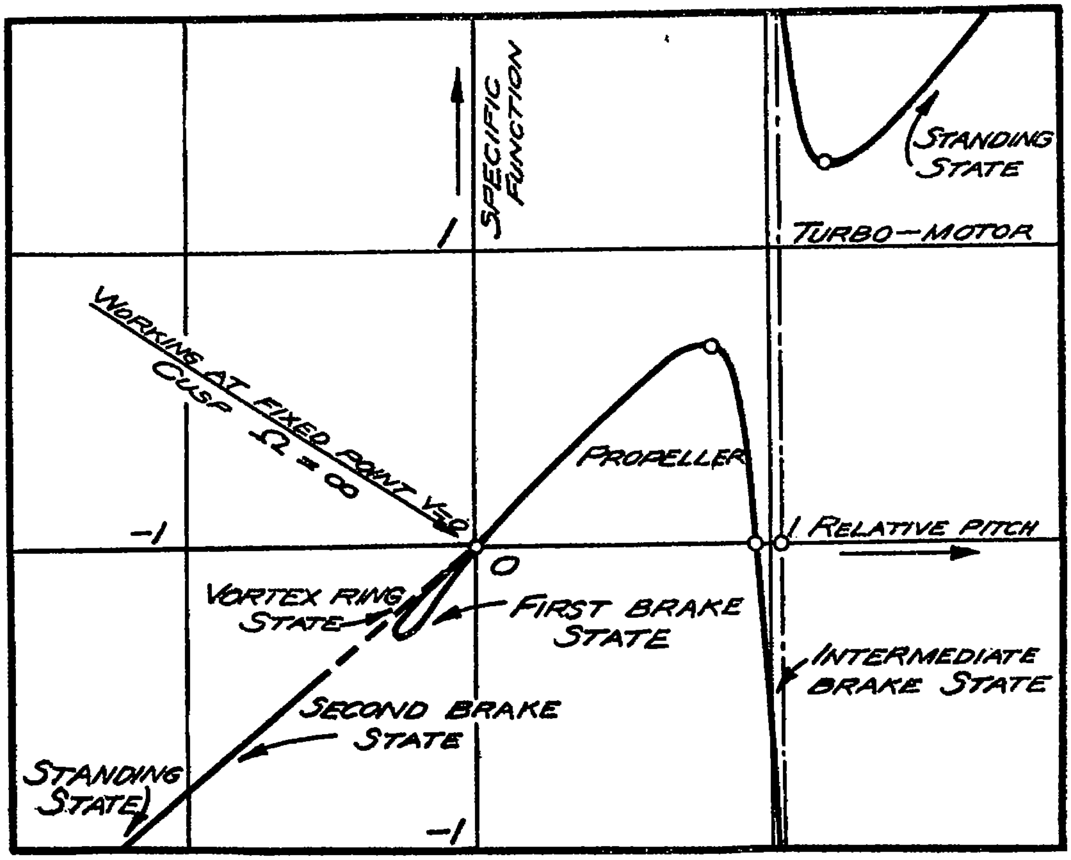
For the study of the phenomenon of the working of blade screws, I adopt as fundamental parameter a quantity which I call *relative pitch*. The relative pitch is the pitch of the trajectory of a section of the screw blade, measured by taking the pitch of the blade section itself as unity. I call *specific function* the ratio of the thrust power to the torque power of the blade screw. The curve of the specific function, as shown in the annexed illustration,[2] unfolds the complete cycle of all the states of the work possible for a screw. For negative values, great in absolute value, of the relative pitch, the specific function is directed toward the origin of the coordinates by a sensibly rectilinear parabolic branch. Here we find ourselves in the region of the screw working as a brake, characterized by the property that the fluid stream crossing the area swept by the blades of the screw has the same sense as the velocity of the fluid current directed on the screw. The segment of this branch of the specific function which is close to the origin and is indicated by dots on the annexed drawing corresponds to a phenomenon discovered in a purely analytical manner, for the first time, by the present theory, which I have named the *vortex ring working state*. This phenomenon takes place in the following order: One imagines the screw working in the above-mentioned brake state and considers the progressive lessening of its translational speed. Under these conditions a moment arrives when a surface of separation is formed in the wake of the screw across which there is no fluid flow. Directly after its formation the surface of separation resolves itself into two surfaces; and a vortex ring, the axis of which coincides with the axis of the screw, appears in the space thus formed. The two surfaces of separation which inclose the vortex ring move progressively apart, and a moment arrives when one of these surfaces crosses the space swept by the blades of the screw. This moment corresponds to the change of sense of the fluid stream crossing the plane of the screw, and at that moment the screw tends to make an infinite number of revolutions. The curve of the specific function reaches the origin by a cusp. This is the *whirling phenomenon*,[3] immediately followed by a new brake state of work represented by a loop on the curve of the specific function. This latter state of brake work is terminated by the work of the screw at a fixed point, when the specific function once more reaches the origin. The blade screw fulfills then the functions of a ventilator, a helicoidal pump or a helicopter. When we enter the region of positive values of the relative pitch, the screw

[1] The author allows himself to introduce the term "blade screw" as general designation of any kind of screw fitted with blades.

[2] See also figure A, p. 40.

[3] The whirling phenomenon is often observed in the braking of ships by means of the screw. There is a moment when a sudden jerk on the engines is observed, and the engines themselves tend to accelerate. In practical navigation this phenomenon has been considered as accidental. It is, however, a quite regular phenomenon, which detects the moment of change in the direction of the stream of water flowing across the screw.

becomes propulsive and the specific function represents the efficiency of the propeller. After having passed a maximum, the specific function decreases rapidly, and, passing through zero value, brings us to a short interval of breakage, which asymptotically goes over to the turbine work of the screw. In this latter interval the specific function represents the inverse of the efficiency of the turbo-motor. After having passed a minimum which corresponds to the maximum of the turbo-motor efficiency, the specific function, by a parabolic branch, quasi-rectilinear, disappears into infinity, which corresponds to the stoppage of the screw in a current directed on the screw. All this sequence of phenomena corresponds to the rotation of the screw in one sense. By the rotation of the screw in an inverse sense, we obtain the series of phenomena of reverse rotation, which forms, as it were, the reflected image of the phenomena



of direct rotation. The general equation of the specific function thus obtained leads directly to the determination of the most favorable conditions of screw working in all the series of its applications. The maxima and minima of the specific function correspond exactly to the maximum of efficiency of the different working states of a blade screw, separated from one another by zero or infinite values of the specific function. We are thus naturally brought to methods of calculation of blade screws in conditions of maximum efficiency. The system of fundamental equations obtained by us thus shows all the properties of the blade screws in all the variety of their working conditions. We thus obtain a complete solution of the whole series of those important problems which have been standing so long owing to the requirements of practice in the applications of blade screws, and which have, up to the present, remained without any satisfactory solution.

I have arrived at all these results, on the one hand, by the conceptional definition of the screw problem of which the normal working conditions are the expression, and, on the other hand, by the employment of a method of solving hydrodynamic problems, which I call the *empirical-theoretical method*. These two sides of the question are of such importance that I must stop to examine them generally.

What is exactly meant by speaking of the exact solution of a problem? When a new problem is raised, before proceeding to its solution two stages should be distinguished. The first, the most difficult to reach, is that in which the thought seeks to formulate the statement of the question. It is only afterwards, when the problem has been formulated, that we can, properly speaking, approach a solution. All the great scientific conquests of human thought have begun by a powerful conception of the problem to be solved. The conceptional definition of a problem is distinguished by the fact that it is only an abstraction from the world of our sensations, only a mental approximation to the reality of the external world. A simple example will suffice to give point to my idea. Let us take the problem of the motion of a rigid body. It is a well-known fact that in nature no solids exist in the absolute meaning of mechanics. So all the mechanics of the solid is only an approximation to reality; but the whole value of this approximation lies in the fact that numerous natural bodies approach in certain conditions so nearly to an absolute rigidity that the established laws of the mechanics of solids give a description of actual solids, which, in general, exceeds all the demands of the applied sciences. The problem once stated, an exact solution can be sought. It is only of the exactitude of the solution that there can be question. All problems in themselves can only be approximations to reality. That is why we should never insist too much on finding exact solutions of problems which present too considerable difficulties. The whole value of numerous methods of approximation lies in the fact that the results obtained are, so to speak, of the same degree of exactitude as the conception of the problem. Important problems remain long without being solved only because their very conception has not been sufficiently thought out. The blade screw is an example of such. A more thorough conception, while making the solution easier, often brings us still nearer to reality.

The empirical-theoretical method to which I have had recourse for the solution of the screw problem, presents a certain analogy to the general method of solving problems of the theory of elasticity. At one time scientists tried to deduce the elastic properties of solid bodies starting from the hypothesis of the molecular structure of bodies. But real progress in the theory of elasticity was only obtained when this risky method was abandoned. In order to establish the elastic properties of solids, the modern theory of elasticity has recourse to direct experiment, and, based on the data of this latter, it connects the complex cases with the simple one by the help of the fundamental propositions of mechanics. This, in my opinion, is what should be done with regard to the solution of the problem of hydrodynamic resistance. Find out the factors which depend on the physical nature of the fluid and the surfaces in contact, and for their numerical values fall back on direct experiment. Then from the knowledge of these factors, once they are determined, the results which mechanics allow to be established must be drawn. I know well all the methods which have been proposed for the solution of the problem of hydrodynamic resistance of fluids. These methods have all the following scheme: First of all, by aid of some hypothesis the fundamental characteristics of the flow around the solid in motion are sought. Afterwards the distribution of velocities in the fluid mass is calculated. From the latter one finds the pressure distribution, the resultant of which at the surface of the body ought to represent the hydrodynamic resistance of the fluid. Thus Euler's method consists in supposing the flow of the fluid to be continuous and allowing a potential function for the fluid velocity. This conception of the phenomenon leads to the conclusion that all bodies immersed in a fluid do not show any hydrodynamical resistance, which is in flagrant contradiction to experiment. In order to explain the phenomenon of fluid resistance, Helmholtz has supposed the formation behind the body of surfaces of discontinuity, to which he had been led in studying the flow of fluids through orifices. This method has been developed by Kirchhoff and Lord Rayleigh. The value of the hydrodynamic resistance obtained by this method is, however, less than that furnished by experiment. The cause of this divergence lies in the fact that this type of flow is unstable, the viscosity of the fluid destroying the surfaces of discontinuity. Of late years W. M. Kutta,[1] having established, in the case of movements parallel to a plane, the relation between the circulation over a contour embracing a cylindrical solid and its hydrodynamical resistance, tried to determine this latter by studying some types of flow around solids, which, although stream-lined, furnished a finite value of the circulation around the cylinder. To Messrs. S. Tchapliguine and N. Joukowski[2] we owe numerous developments and applications of this method. The authors of this theory have been able to calculate the lift furnished by the cylindrical body, but the value obtained does not fully agree with experiment. As for the drag, it escaped their investigations. I have therefore endeavored to give a general demonstration of the theorem of circulation which explains this misunderstanding and which will be found in Note IV at the end of this memoir. This theorem referred to above does not furnish a zero value of the drag, and its authors arrived at this conclusion only by the fact of supposing the fluid to be perfect, a hypothesis quite superfluous and entirely unnecessary for the establishment of this theorem. But this theorem, when understood in its widest sense, does not lead to the solution of the problem of hydrodynamic resistance, since the values obtained for the circulation depend on the type of flow assumed, which still remains to be determined. This latter question of the type of flow is excellently stated by M. V. Karman,[3] who proposes to determine the hydrodynamical resistance starting from the estimation of the momentum of the vortices in quincunx, which are formed behind the cylindrical solid in uniform rectilinear motion in a fluid. This theory, applied up to the present only to the most simple cases, gives results which agree better than all the other theories with experiment. All the attempts enumerated above, although quite erudite, can not give us the value of the hydrodynamical resistance for all the cases demanded by technique, and we are always obliged to resort to experiment for its determination. How ought we to proceed when a problem of hydrodynamical resistance bars the way to our investigations? It is by the empirical-theoretical method that I find the means of circumventing this difficulty. This method really consists in reversing the question. We do not propose to calculate the hydrodynamical resistance starting from the type of flow of the fluid, but, inversely, it is the flow of the fluid that we shall try to determine, starting with the knowledge of the hydrodynamical resistance measured experimentally. In general, the empirical-theoretical method can be characterized as follows: All the space in which a hydrodynamical phenomenon takes place is divided into two kinds of regions. In some of these regions the hydrodynamical resistances are, so to speak, concentrated; in the others they are absent. The hydrodynamical resistances once experimentally measured, the connections between the two kinds of regions are established by means of the general theorems of mechanics and hydrodynamics, the phenomena which take place in the second kind of region being considered as under the laws of perfect fluids.

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[1] See W. M. Kutta, "Illustrirte Aeronautische Mitteilungen," 1902, and "Sitzungsberichte der Koeniglichen Bayerischen Akademie der Wissenschaften," Munich, 1910 and 1911.

[2] See "L'aerodynamique," by N. Joukowski, Paris, 1916, Ch. VI, §§ 18, 19, 20.

[3] See V. Karman, "Nachrichten von der Koeniglichen Gesellschaft der Wissenschaften zu Göttingen," 1911, and "Physikalische Zeitschrift, 1912.

Returning after the preceding general considerations to the examination of our screw problem, I shall begin by its definition. Like all conceptional definitions, this will only approximate reality to a certain degree. But the value of our formulation of the problem lies in the fact that it leads us to a solution of this latter which satisfies all the demands of the technique of the application of blade screws.

The following, in accordance with the empirical-theoretical method, is my conception of the blade-screw problem. In order to fix the ideas, I will assume that the screw is a propeller. I divide the slip stream created by the rotation of the blade screw into three domains. The first is that part of the stream which is disposed forward of the screw and up to the section of the stream which the local phenomena created by the rotation of the blades have not reached. The second domain, which contains the screw, immediately follows the first and incloses that part of the stream immediately disturbed by the rotation of the screw blades. I define this second domain by the condition that the differences of pressures on the two limiting sections are actually equal to the thrust produced by the screw. The third domain is the direct prolongation of the second counted up to the narrowest section of the slip stream created by the screw rotation. I assume that the flow of the fluid in the first and third domains obeys the laws of perfect fluids, while the phenomena taking place in the second region are estimated by direct experiment. As regards the fluid stream running out of the third region, I assume that its velocity is progressively dissipated by the viscosity of the fluid. The above enumerated conditions constitute what I call *the normal conditions of the working of a blade screw.* I call *neighboring conditions* all the circumstances which deviate from normal conditions.

The conception of a problem can only be judged by the conclusions to which it leads. The results stated in this memoir will, I hope, be the most eloquent evidence in favor of our conception of the screw problem. I should like to mention that it has been quite impossible for me to deal with all the questions which my conception of the screw problem raises. I have concentrated my efforts above all on the problems which appear to me to be the most important for practice. Time itself, as it passes, will, no doubt, reveal, better than I may have been able to do here, many sides of the widespread screw problem upon which I have often only touched. In many cases I may have only raised the veil of mystery which up to the present has concealed so jealously from our eyes many sides of the phenomenon of the blade screw working, and have outlined only their general picture. But I allow myself to believe that the results which I have obtained are fully sufficient for the exact calculation, in full certitude, of blade screws of the highest possible efficiency for the states of work submitted by me to a detailed study.

It is also to be mentioned that, strictly speaking, the blade-screw theory can only be an integral theory, because in principle the problem of calculation of the hydrodynamical resistance is defined by integral relations. But the present theory is rather a differential theory, in the sense that it is based on a system of differential relations. The possibility of such a simplification is the result of some assumptions which seem to be so close to reality, by the results to which they lead, that the transition to a necessarily more complicated integral[1] theory is not practically demanded.

To some it may seem that this theory contains many assumptions. But I must say that the present theory contains fewer assumptions than any earlier theory. I have only devoted special attention to indicate all the assumptions made, which was often neglected. And I will also ask that one consider all the assumptions made, not so much in themselves as in the consequences to which they lead.

[1] The general outlines of the blade-screw integral theory will be found in Note VI at the end of this Memoir.

I wish also to point out to those about to verify the present theory by experiment two circumstances which disturb, as it were, the purity of the phenomenon: on the one hand, the deformation of the screw blades, on the other hand, the deviation of the fluid resistance from the square law for the velocity. It often happens that when the angular velocity of the screw increases, the blades undergo a certain distortion or flexion owing to the load to which they are subjected. This causes a modification of the general shape of the blades, which although generally small, has an immediate effect on the results of testing. As to the square law for the velocity, it is well known to be only a first approximation, and can be applied only in certain intervals of the velocity variation for which the coefficients of resistance ought to be directly measured. When the coefficients of resistance are taken as constant in large intervals of the velocity variation the results of the calculations raise differences which have to be attributed to the deviation of the fluid resistance from the square law for the velocity.

In conclusion I shall give a general summary of the chief results obtained at this time in this memoir.

Chapter I is devoted to the establishment of the system of fundamental equations relating to the blade screw. The theorems of momentum and of moments of momentum are submitted to a critical examination in their application to the screw. A complete picture of the flow of the fluid in the slip stream created by the rotation of the screw is given. The examination of the distribution of the pressures in this fluid stream leads to the generalization of Bernouilli's theorem shown in Note II at the end of this memoir. The reasons which make negligible the mutual influence of the different sections of a blade are indicated. It is shown that the *effective pitch* alone, as opposed to the *constructive pitch*, can serve to describe the properties of the screw. The fundamental theorem registering the losses in the work of the screw is established. The explicit expressions of the velocities in the slip stream produced by the rotation of the screw—which I call *slip and race velocities*—are calculated both forward of the screw and in its wake, as functions of the dimensions of the screw and the coefficients of resistance. Rigorous demonstration is given of the fact already known, but generalized by us, that the specific function is a function of the relative pitch alone. All the general data of the empirical laws of fluid resistance of which use is made are stated in Note III at the end of this memoir.

Chapter II contains the general discussion of the 16 states of work which may establish themselves for a blade screw. The existence of the vortex ring state and the whirling phenomenon are established. All the fundamental functions which enter the blade-screw theory are submitted to a general analytical discussion. The general outline of the curve of the specific function is examined. Finally, I have pointed out two limited cases of the work of the screw; the screw with a zero constructive pitch and the screw with an infinite constructive pitch. The consideration of the effective pitches explains the paradoxes apparently realized by these cases.

Chapter III is devoted to the study of the propulsive screw or propeller. I give, first of all, a comparative summary of the general formulæ for the working of the screw when advancing and when standing at a fixed point. I establish the fundamental proposition that *when a screw is working at a fixed point the angles of attack of all the sections are constant, independently of the angular velocity of rotation of the screw.* Then the losses of the screw's working power are estimated. These I divide into three classes: the fan losses, the vortex losses and the resistance losses. The most favorable working conditions of a blade section are established. I establish the approximate proposition that when a blade section works at its maximum of partial efficiency, its slip measures the losses, its efficiency is equal to its relative pitch. *An exact standard is given for choosing the most profitable outlines to adopt for screw blade sections.* I deal with the question of the limiting dimensions of the blades, their limited number and mutual interference. It is shown that, for given working conditions, there exists a limit power which a screw can employ usefully. In the analysis of these questions the new notion of *breadth ratio* is naturally evolved. Among other experiments, those of G. Eiffel with two coupled screws, the bringing of which nearer together in the inverse sense of their rotation has increased the efficiency, find a direct explanation. I then proceed to the valuation of the total work of all the sections of the blades. A geometrical interpretation is given to the question of the total efficiency of a blade screw, which establishes the direct relations between the partial and total efficiencies. I examine the question of the effective pitch of the entire screw. I then pass to the integration of the work of different sections of the blade, and give a general discussion of the different conditions which may occur in this integration. After an examination of certain properties of the integrals obtained, I compare the working of the propeller in forward motion with its working at a fixed point. The question of investigation of the best contour to give to the blades is stated as a problem of calculus of variation. The problem of design and the calculus of the dimensions of propellers is made the subject of detailed study. In order to solve the fundamental relations which give the value of the angle of attack effectively established in each section and which can not be solved by ordinary methods, I have prepared a monogram with four parameters, according to M. d'Ocagne's method of parallel tangent coordinates. A second monogram has been prepared in order to facilitate the calculus of the function az and the load efficiency q, but this evidently has not the importance of the former, since the relations for which it gives numerical values may be calculated directly. The problem of the calculus and design of propulsive screws is thus entirely solved in the widest sense for all the demands of practice. Finally, I handle the important question of the selection and adaptation of screws. I am led to establish the new notion of *uniform families of screws* divided into *varieties*. Up to the present this has generally been limited to screws geometrically similar. I introduce the notion of screws which are, so to speak, hydrodynamically alike. When we compare screws among themselves, it is natural to imagine the different sections of blades in similar working conditions, what directly leads to functional relations connecting all screws of the same variety. Hydrodynamic similarity is realized when homologous sections of the blades of the screws of the family under consideration are geometrically similar and when the relative fluid current is directed upon them under the same incidences. It is thus that the notion of variety of a uniform family is revealed and characterized by the similarity of homologous blade sections, independent of their effective pitches, and by the identity of the system $S(i)$ of effective angles of attack of all the sections of each blade of these screws. But the introduction of the system of angles of attack $S(i)$ as a fundamental characteristic became possible when explicit relations were established between the effective angles of attack, the geometrical and hydrodynamical characteristics of screws, and their working conditions, results attained for the first time in this thesis. That is why we can now fix in this way the mutual orientations of the different sections of blades whose evolutes in the plane are geometrically similar. It is the latter possibility which forms the basis of the theory of uniform families and which leads us to the solution of the delicate problem of the selection and adaptation of screws. I am thus brought to divide screws into three kinds—major screws, optima or maxima screws, and minor screws—all of which essentially differ in their general properties. I establish three fundamental relations connecting all the screws of one variety and allowing of a direct solution, by the reading of a simple diagram, of all the infinite series of screws satisfying the conditions of speed, power, and number of revolutions for a given case. I indicate the process of the *testing screw* for choosing propellers in case the drag or head resistance of the vehicle of locomotion in view is unknown, which is usually the case in practice. The influence of the number of revolutions on the efficiency and size of screws is examined in outline. Note

V, at the end of this article, gives the geometrical basis of the conventions used for screw drawings.

Chapter IV summarizes a new method of determining the coefficients of fluid resistance based on the properties of the screw revealed by the present theory. This method forms, so to speak, the basis of all the experimental data necessary for the calculus of screws in exactly the same conditions of screw working. This is one of the most convenient methods, since it only demands tests at a fixed point of screws with plane-radial blades. I give a brief summary of the general properties of this new type of plane-radial screws, of which the method I stated above establishes an important application. This short incursion into the domain of screws working at a fixed point easily shows us how copiously the working of the screw in all the deviations of its applications, of which the working at a fixed point has seemed until now the most difficult to grasp, has been effectually included in the present theory. We find to the contrary in the light of the actual theory that it is the most simple case.

This first memoir thus contains, besides a general summary of the whole screw problem, a detailed study of the propulsive screw—that is, the propeller—and the different questions in connection with it. A second memoir, directly continuing this one, will contain a special study of screws at a fixed point in their different applications, principally when used as fans and as helicopters, as well as a detailed study of the turbo-motor screws, especially as aeroturbines, that is to say, as windmills.

Finally, I can not refrain from expressing the wish to see special laboratories set apart for the special study, in the light of the present theory, of the domain of the blade screw, still so new, so widespread, and important from the point of view of universal social economy. It is sufficient to bear in mind for one moment the important uses to which blade screws may be applied, if only in shipping and aeronautics, without mentioning other applications, such as fans, turbines, etc.—to imagine the enormous supplies of energy which the screw is the instrument of utilizing—to see the importance arising from its study. Every percentage gained in the efficiency of screws is expressed by an equivalent total of multimillions of fuel economy. All the power of marine and aerial fleets is directly based on the perfection of the screws employed. The screw thus appears as an important State Question, and that is why nothing that can contribute to its perfection should be neglected. The results obtained by the present theory will be valued the more quickly and powerfully the more rapidly are created special organizations furnished with all the necessary material for the pursuit of the possibilities here developed. The program of activity of such laboratories is already drawn up. Tools and instruments for all the indispensable tests should be collected, and every effort concentrated to obtain the whole of the experimental data necessary for the calculation of screws. The principal aim of such an establishment should be the standardizing of all screws necessary for the development of the technical arts of the State. The screw problem is of such importance that groups of competent specialists should be devoted to its special study and charged to watch over its highest and most perfect development. Will those to whom the importance of the creation of such special laboratories—for the study of the blade screw—is more than evident excuse me for these pleas in their favor which I have allowed myself to express here?

The main results contained in this memoir were in the hands of the author already at the end of 1915. Their publication in Russian was begun in 1916, but only the first two chapters and the first half of the third chapter were edited at the beginning of 1917, further publication having been stopped by the outbreak of the revolution in Petrograd.

GEORGE DE BOTHEZAT.

Washington, D. C.
November 1918.

NOTATION.

UNIFORMLY EMPLOYED IN THE PRESENT MEMOIR.

D	screw diameter.
H	effective pitch of a blade section.
r	distance of a blade section from screw axis.
b	breadth of a blade section.
ΔA	area of a blade element; $\Delta A = b\Delta r$.
n	number of blades.
N	number of revolutions of the screw per second.
Ω	angular velocity of the screw; $\Omega = 2\pi N$.
V	translatory speed of the screw along its axis.
ΔC	partial torque due to the elements, of all the blades, disposed at the same distance from the screw axis.
ΔQ	partial thrust due to these same blade elements.
C	resultant torque applied to the screw axis.
Q	resultant thrust.
S, S', S''	sections through the slip stream.
r, r', r''	distances to the screw axis of a point taken in the surfaces S, S', S''.
$\Delta S, \Delta S', \Delta S''$	annular elements of the surfaces S, S', S''.
v, v', v''	slip velocities in the sections S, S', S''.
$r\omega, r'\omega', r''\omega''$	race velocities in the sections S, S', S''.
ΔM	fluid mass flowing in a unit of time through $\Delta S, \Delta S', \Delta S''$.
δ	mass density of the fluid in which the screw is working.
W	resultant velocity of the fluid relative to a blade section.
ΔR	fluid resistance of a blade element.
i	effective angle of attack (measured from zero line).
α	constructive angle of attack (measured from chord).
φ	effective blade angle (inclination of the zero line of one blade section to the plane of rotation of the screw).
ψ	constructive blade angle (inclination of the chord of one blade section to the plane of rotation of the screw).
γ	angle between chord and zero line of a section. $i = \alpha + \gamma$; $\varphi = \psi + \gamma$.
K_y	lift coefficient.
K_x	drag coefficient.
$K_t = k_t\delta$	coefficient of the resultant fluid resistance. $\Delta R = K_t \Delta A W^2 = k_t \delta \Delta A W^2$; $K_t \cong Ki$; $k_t \cong ki$
β'	angle between fluid resistance ΔR and zero line.
β_H, β_T	angles of ΔR with the normal to the zero line.
β	notation used for either β_H or β_T.
i'	value of the angle of attack for which the fluid resistance ΔR is normal to the zero line.
a	breadth ratio; $a = nb/2\pi r$.
ρ	"specific function," equal to the partial efficiency in the case of a propeller.
x	relative pitch; $x = V/NH$.
μ	advance; $\mu = V/N$.
ξ	relative advance; $\xi = V/ND$.
s	slip; $s = 1 - x$.
q	load coefficient; $\Delta Q = q\delta\Delta S V^2$.
η	total efficiency of the screw.
i_0	angle of attack of a blade section of a screw working at a fixed point.
ρ_0	partial efficiency at a fixed point.

All the quantities relating to the work of the screw at a fixed point are marked by a sub zero.

ρ_v	fan efficiency.
p_v	fan losses.
p_t	vortex losses.
p_r	resistance losses.
p	total losses; $p = p_v + p_t + p_r$
L_u	total thrust power developed by a propeller.
L_a	total torque power absorbed by a propeller.
$S(i)$	system of angles of attack under which the blade sections are working.

Chapter I.

THE FUNDAMENTAL EQUATIONS.

Let us consider an unlimited fluid mass, in which is immersed a blade screw rotating with the uniform angular velocity Ω $^1/_{sec.}$ around its axis and having a uniform translation with the velocity V *mt/sec.* along that axis. Let us examine, in their general outlines, the flow phenomena produced by the blade screw rotation in the surrounding fluid medium. We shall assume, to fix the ideas, that we have to do with a propulsive screw or propeller.

The relativity principle of hydrodynamics allows us to consider either the screw moving with the uniform velocity V in an immobile fluid mass or the translationless screw plunged in a fluid stream directed with the velocity V in inverse sense on the screw parallel to its axis. Considering the latter case, viz, the screw immersed in a fluid stream parallel to its axis, the following is observed: The screw rotation creates a fluid stream, generally called *slip stream,* whose section in the neighborhood of the screw is very nearly equal to the area swept by the blades of the screw. The velocity of the flow inside that slip stream differs from the velocity V. A velocity increase is already observed in front of the screw, but it is in the wake, in the narrowest section of the slip stream, that the largest increase of velocity is observed. Beyond its translational motion, the fluid in the slip stream has also a rotational motion, so that the motion of the fluid particles in the slip stream is a helicoidal one.

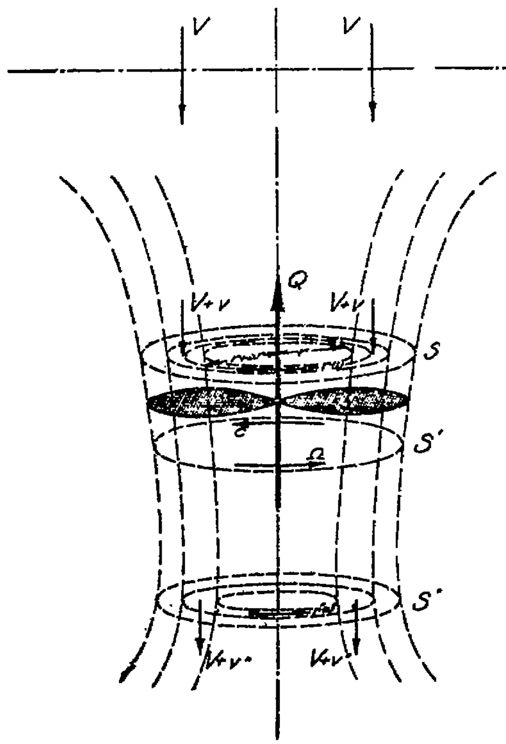

FIG. 1.

Let us divide the slip stream created by the rotation of the screw into three domains. The first domain is constituted by the part of the slip stream disposed in front of the screw. This domain is included between the section S_0 of the slip stream taken at such a distance from the screw that the flow velocity in it is still equal to V, and the section S directly in front of the screw, but, however, at such a distance from the latter that the flow in it is not disturbed by the local phenomena created by the rotation of the blades of the screw. The exact position of this last section S will appear in the following: The second domain contains the screw and is included between the sections S and S' of the slip stream defined by the condition that the sum of the differences of the pressure in these sections S and S' is equal to the resultant thrust of the screw. These sections will be submitted in the following to a supplementary condition which will specify them exactly The third domain is formed by the slip stream running off the screw and is included between the section S' and the narrowest section S'' of the slip stream. The sections S, S', and S'' will be, in the general case, surfaces having the axis of the screw as axis of symmetry. In figure 1 these sections are represented in a purely conventional manner.

Let us decompose the velocities of the fluid particles in the section S into two components; one axial (see fig. 1) which we will designate by $V+v$, the velocity v representing the increase of the flow velocity already existing in the section S; the other tangential component, which we will designate by $r\omega$ where r is the distance to the screw axis of a fluid particle crossing the section S. As for the radial components of the velocities of the particles in the section S, as well as in the sections S' and S'' of the slip stream, we will consider them as negligible, these velocities having very small values for the states of work of the screw which are of practical importance. The states of work of the screw for which the radial velocities have sensible values will appear, besides, from the later developments of this memoir.[1]

We will call *slip velocity* the velocity v, and *race velocity* the velocity $r\omega$. The slip and race velocities have generally different values in different points of the section S. These velocities are, besides, periodical functions of the time, whose period depends from the period of the screw rotation multiplied by the number of blades. But we will agree to consider v and rw as the mean values of the real periodical velocities, and under such conditions the velocities v and $r\omega$ can be considered as constant in time and in space for all the points of the section S at equal distances from the axis of the blade screw—evidently only for a determinate state of work of the screw.[2]

For the distribution of the pressures, just as for the distribution of the velocities, we will only consider the mean values instead of the real periodical values. For all points situated in one plane normal to the screw axis and at equal distances from it, the pressures will thus be considered as equal.

Let us decompose in a similar way the velocities of the fluid particles in the sections S' and S'' into axial components

$$V+v' \text{ and } V+v''$$

and tangential components

$$r'\omega' \text{ and } r''\omega''$$

The velocities v' and v'' will be named slip velocities in the sections S' and S'', and $r'\omega'$ and $r''w''$ race velocities in these same sections. To these last slip and race velocities have to be applied all the remarks we have made in relation to the velocities v and $r\omega$.

As for the velocities of the particles of the slip stream behind the section S'', we will admit, in agreement with experiment, that they are progressively dissipated by viscosity.

Let us divide the whole slip stream into a series of regions of infinitely small thickness, limited by surfaces of revolution, the locus of the stream lines of the mean velocities of the slip stream, and having the screw axis as axis of symmetry. These annular regions will cut off on the surfaces S, S', and S'' annular areas which we will designate respectively by

$$\Delta S,\ \Delta S',\ \Delta S''$$

and which are limited by circumferences having radii

$$r+\frac{\Delta r}{2},\ r-\frac{\Delta r}{2}$$

$$r'+\frac{\Delta r'}{2},\ r'-\frac{\Delta r'}{2}$$

$$r''+\frac{\Delta r''}{2},\ r''-\frac{\Delta r''}{2}$$

[1] The radial velocities have nonnegligible values only for states of work in the neighborhood of the vortex ring state.

[2] The relations which exist between the real periodical velocities and their mean values have been studied by *G. A. Crocco* in his memoir "Sulla teoria analitica della eliche e su alcuni metodi sperimentali"—Rendiconti degli studi ed esperienze eseguite nel laboratorio di costruzioni aeronautiche del battaglione specialist. Anno I, No. 1, 30 Novembre, 1911. This memoir is reproduced almost completely in French in the "Technique Aeronautique," Tome VI, 1912. No. 67, p. 194. No. 71, p. 321. No. 72, p. 363.

Let us designate by ΔM the fluid mass which flows in a unit of time through one of these annular regions. On account of the continuity of the flow we must have

$$\Delta M = \Delta S(V+v)\delta = \Delta S'(V+v')\delta = \Delta S''(V+v'')\delta \tag{1}$$

where δ is the fluid mass density which we consider as constant in the whole fluid mass. The constancy of the density is evident when we have to do with an incompressible fluid, such as water, for example. But the density can also be considered as constant for a compressible fluid, such as air, for example, so long as the flow velocities do not exceed values of the order of about a hundred meters a second, because under such conditions the observed pressure differences will be low, and, accordingly, the density variation negligible.

Throughout this memorandum we will use the metric units:

klg.—weight; meter; second.

In this system of units, for normal conditions of pressure and temperature (760 mm. and 15° centigrade) the density has the values

$$\text{for water } \delta \simeq 100 \frac{klg\ sec^2}{mt^4}$$

$$\text{for air } \delta \simeq 1/8 \frac{klg\ sec^2}{mt^4}$$

Let us designate by ΔI and $\Delta I''$ the moments of inertia relative to the screw axis, of the fluid mass ΔM considered in the annular sections ΔS and $\Delta S''$ respectively. We have:

$$\Delta I = \Delta M . r^2; \ \Delta I'' = \Delta M . r''^2 \tag{2}$$

Taking into account the relations (1) and assuming the similitude of the flow conditions [1] in the sections S and S'' we get

$$\frac{\Delta S}{\Delta S''} = \frac{V+v''}{V+v} = \frac{r^2}{r''^2} = \frac{\Delta I}{\Delta I''} \tag{3}$$

from which follows

$$\Delta I'' = \Delta I \ \frac{V+v}{V+v''} \tag{4}$$

---

[1] Exactly speaking, we want to say that if all the Δr's are taken equal in the section S, we admit all the $\Delta r''$'s in section S'' to be also equal. Under such conditions

$$\frac{\Delta S}{\Delta S''} = \frac{8\pi r \Delta r}{8\pi r'' \Delta r''} = \frac{r \frac{r}{n}}{r'' \frac{r''}{n}} = \frac{r^2}{r''^2} \tag{1}$$

n being the number of Δr's and $\Delta r''$'s respectively contained in r and r''. This hypothesis is justified by the following considerations: Exactly speaking, we have

$$\frac{\Delta S}{\Delta S''} = \frac{r \Delta r}{r'' \Delta r''} = \frac{V+v''}{V+v}$$

or going over from finite differences to differentials we get

$$r dr = \frac{V+v''}{V+v} r'' dr''$$

and integrating

$$r^2 = \int \frac{V+v''}{V+v} d(r''^2)$$

when

$$\frac{V+v''}{V+v} \simeq \text{const} \tag{2}$$

we have

$$\frac{r^2}{r''^2} = \text{const; hence } \frac{r}{r''} = \text{const}$$

or $r/r'' = dr/dr''$ and consequently

$$\frac{r dr}{r'' dr''} = \frac{r^2}{r''^2}$$

All the foregoing is only the characteristic of the flow in the slip stream from a purely kinematical standpoint. We will now proceed to the fundamental equations which connect the work of the blade-screw with the motion of the surrounding fluid. We shall begin by an examination of the pressure distribution in the slip stream and of the conditions which exist on its boundary.

In each cross section of the slip stream the pressure is not constant, being generally lower in the middle of the cross section, than on the periphery; this is on account of the rotation of the fluid in the slip stream. In Note II, at the end of this memorandum, it is indicated in general outlines how this pressure distribution can be calculated, and its general course is represented in Fig. 2. In nearly all the practically important applications of blade-screws the pressure differences in the slip stream cross sections are small, on account of the fact that the fluid rotation is slow, and, besides, the pressure differences produced by the fluid rotation are partially compensated by the curvature of the flow surfaces in the meridional planes of the slip stream.

It is thus easy to see that in the section S the pressure is necessarily inferior to the outside pressure p_0. This follows from the fact that the velocity of the flow in the slip stream is increasing as we approach the section S. We shall see in the following that when one passes from section S to section S' the pressure rises, and in the section S' is greater than p_0. But from S' to S'' the slip stream velocity is still increasing on account of the narrowing of the slip stream, and therefore the pressure decreases, and in the section S'' its departure from the pressure p_0 is generally very small. In the definition, given in the following, of the normal conditions of work of a screw, we shall assume the pressure in the section S'' to have recovered its original value, that is, retaken the value p_0. This means that the action of the considered blade screw is not to produce a difference of pressure, but consists in communicating a certain momentum to the fluid. Under such conditions, beyond the section S'' the slip stream diffusion must go on at a quasi-constant pressure. The case of work of the screw with "pressure step" will form the subject of a separate investigation.[1]

The existence of a pressure and a flow velocity difference between the inside and the outside of the slip stream in the sections S' and S'' leads us necessarily to admit, as follows from the considerations shown in Note II at the end of this memoir, that the boundary of the slip stream must be a vortex sheath maintaining these pressure and velocity differences. The vortex intensity and the curvature at each point of the slip stream vortex sheaths can be estimated when the pressure and velocity differences on both its sides are known. The existence of the slip stream vortex sheaths follows also from the fact that from each blade tip there must run off vortex filaments, which dispose themselves on the slip stream boundary. This directly follows from William

which confirms the relation (1). Let us now show that in the great majority of practically important cases the condition (2) is satisfied:

I. For $V=o$, and remarking as will be shown in the following that $v''=2v$, we have

$$\frac{V+v''}{V+v}=\frac{v''}{v}=\frac{2v}{v}=2=\text{const}$$

This case corresponds to the work of the screw at a fixed point.

II. For V having a large value relative to v

$$\frac{V+v''}{V+v}=\frac{V+2v}{V+v}\cong\frac{V}{V}=1=\text{const}$$

This case corresponds to propellers and to turbines.

III. For V of the same order as v

$$\frac{V+v''}{V+v}\cong\frac{v+2v}{v+v}=\frac{3v}{2v}=3/2=\text{const}$$

IV. For $v=\text{const}$ in the whole section of the slip stream.
In this case we evidently have

$$\frac{V+v''}{V+v}=\frac{V+2v}{V+v}=\text{const}$$

[1] I call "pressure step" the pressure difference which can exist between the outside pressure p_0 and the pressure p'' in section S'', and which in some special applications of blade screws can have very sensible values.

Thomson's (Lord Kelvin) theorem on the invariability of the circulation along a contour accompanying the fluid in its motion. When following such a contour embracing a blade of the screw and moving with the flow relative to the blade, the circulation along this contour must maintain its value—so far as the fluid can be considered as perfect—which is fixed by Kutta's theorem.[1] Vortex tubes must thus run off the tips of the blades and dispose themselves on the slip stream boundaries. The fluid in the slip stream having also a rotational motion, there must also be formed a central vortex tube along the screw axis.[2]

We have thus reached a general picture of the flow in the slip stream created by the blade-screw rotation. Let us now consider the slip stream as represented in figure 2 and apply to it the momentum theorem as well as the theorem of moments of momentum. In the case of steady motion of a fluid these theorems can be expressed by the following unique proposition.[3]

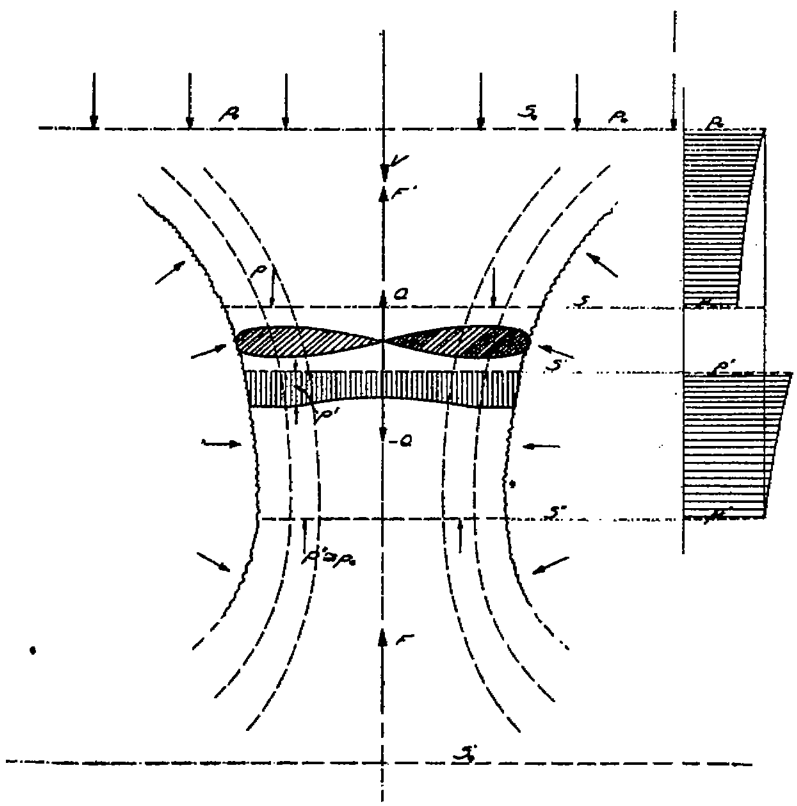

FIG. 2.

When a fluid mass is in a state of steady motion the resultant of the wrench of the system of ALL THE EXTERIOR FORCES *applied to a portion of the fluid mass limited by a closed surface, and of the screw of the inflow fluid momentum (the outflow fluid momentum having to be taken in reversed sense) is equal to zero.*

Let us introduce the following notations: We will call *partial thrust* and designate by ΔQ the axial component of the resultant fluid pressure on all the blade elements contained in an annular volume $(\Delta S, \Delta S')$. The moment ΔC of this resultant pressure referred to the screw axis will be named *partial torque*. The resultant thrust of the blade screw and the resultant torque applied to its axis will be respectively designated by Q and C. The senses of the blade-screw translation and rotation, when the latter is propulsive, will be adopted as positive senses along the screw axis and around it. Let us, besides, designate by p, p', and p'' the pressures respectively in the sections S, S' and S'', the exterior pressure being designated by p_0, as has already been mentioned. As for the stresses on the boundaries of the slip stream, we shall decompose them into components normal to the boundary surface, whose value is p_0', and into components tangential to the boundary surface, the last being produced by the slip stream friction against the surrounding fluid medium. It is easy to see that in the sections S_0 and S'_0

[1] See Note IV at the end of this memoir.

[2] See the stereotypical photographs of *Oswald Flamm.* The air bubbles seen on these photographs dispose themselves along the axis of the vortex tubes.

[3] See Note I at the end of this memoir.

(see fig. 2) the stresses will admit only normal components equal to p_0, so that the tangential stress components will have a sensible value only at the lateral boundaries of the slip stream. We will designate by F' and F the projections, on the screw axis, of the resultant of the tangential components developed on the lateral surface of the slip stream for its portions respectively included between S_0, S'' and S'', S'_0. The resultant moments, relative to the screw axis, of these tangential components will be respectively designated by C'_F for the portion of the slip stream surface between S_0 and S'', and by C_F for the portion of the slip stream surface between S'' and S'_0.

Let us first apply the theorems of momentum and moments of momentum to the slip stream portion between the extreme sections S_0 and S'_0. The only exterior forces acting on this volume are. On one hand the thrust $-Q$ and the torque $-C$ (resultant action of the blade screw on the fluid); on the other hand the friction forces developed on the boundaries of the volume considered, whose resultants are $F+F'$ along the screw axis and $C_F+C'_F$ around the screw axis, this under condition that the exterior pressure exerted on all the volume considered has a resultant equal to zero. As there is no fluid momentum variation for the volume considered, we must have

$$F+F'-Q=0$$
$$C_F+C'_F-C=0$$

or

(5a) $$Q=F+F'$$
(5b) $$C=C_F+C'_F$$

We will consider as negligible the friction forces developed in the slip stream between the sections S_0 and S'', that is, admit

$$F'\cong 0;\ C'_F\cong 0$$

because it is between the sections S'' and S'_0, where the slip stream diffusion takes place, that is developed nearly the whole totality of the friction forces. Under such conditions we will have

(6a) $$Q\cong F$$
(6b) $$C\cong C_F$$

which means that the friction forces developed between these sections S'' and S'_0 equilibrate the thrust and the torque of the blade screw.

Let us now apply the same theorems to the slip stream portion included between the sections S'' and S_0'. The exterior forces applied to this volume are the resultants F and C_F of the friction forces and the resultant of the pressures normal to the surface of this volume. This last resultant is seen to be equal to $-\Sigma\Delta S''\ (p''-p_0)$ when it is remarked that the uniform exterior pressure p_0 considered as applied to the whole volume (S'', S'_0) must equilibrate itself. The inflow fluid momentum, for this volume, has for its resultant along the screw axis

$$-\Sigma\Delta M(V+v'')+\Sigma\Delta MV=-\Sigma\Delta Mv''$$

and for the resultant torque around the screw axis

$$-\Sigma\Delta I''\omega''.$$

We thus must have

$$F-\Sigma\Delta S''(p''-p_0)-\Sigma\Delta Mv''=0$$
$$C_F-\Sigma\Delta I''\omega''=0$$

But as we consider that in the section S'' the pressure has already reached the value of the exterior pressure p_0 we must simply have

7a) $$F=\Sigma\Delta Mv''$$
(7b) $$C_F=\Sigma\Delta I''\omega''.$$

Let us apply the above-mentioned theorems to the portion of the slip stream between the sections S_0 and S''. The exterior forces applied to this volume are: The friction forces whose resultants are F' and C'_F; the exterior normal pressures with the resultant $\Sigma\Delta S''(p''-p_0)$; the thrust $-Q$ and the torque $-C$. The inflow momentum has for resultant

$$\Sigma\Delta M(V+v'')-\Sigma\Delta MV=\Sigma\Delta Mv''$$

$$\Sigma\Delta I''\omega''$$

We thus must have

$$-Q+F'+\Sigma\Delta S''(p''-p_0)+\Sigma\Delta Mv''=0$$

$$-C+C'_F+\Sigma\Delta I''\omega''=0$$

But as we admit F', C'_F, and $\Sigma\Delta S''(p''-p_0)$ to be negligible, we have

(8a) $$Q=\Sigma\Delta Mv''$$

(8b) $$C=\Sigma\Delta I''\omega''$$

It must be remarked that effectively it is for the section S'' that the fluid momentum variation reaches its greatest value. These last relations (8a) and (8b) also follow from the comparison of the relations (6) and (7).

Let us also apply our two theorems to the annular volume $(\Delta S_0, \Delta S'')$. As the friction forces have been considered as negligible for the volume (S_0, S''), they have also to be considered as negligible for the volume $(\Delta S_0, \Delta S'')$. The resultant of the normal pressures being also considered as negligible, we have

(9a) $$\Delta Q=\Delta Mv''$$

(9b) $$\Delta C=\Delta I''w''$$

and, comparing with the relations (8) we see directly that

(10a) $$Q=\Sigma\Delta Q$$

(10b) $$C=\Sigma\Delta C$$

This last consequence is of first importance. It justifies the partition of the slip stream into annular regions and shows that the resultant thrust Q and the resultant torque C of the blade screw can be considered as equal to the sums of the partial thrust ΔQ and partial torques ΔC under the hypothesis made.[1] The relations (10) also establish the possibility of integrating the partial thrust and torque along the blade. In other words the relations (10) show that *the mutual interference of the sections of the same blade* can be admitted as negligible. What is, in reality, the mechanism of the transmission of this blade section interference? It is specially expressed by the pressure differences in the section S''. Thus the working conditions of blade elements included in an annular volume such as $(\Delta S_0, \Delta S'')$ are submitted to the influence of the pressure difference $\Delta S''(p''-p_0)$ which is, exactly speaking, variable along the blade. But this last pressure difference being negligible in comparison with the other forces acting on the blade elements considered, the mutual interference of the blade sections turns out to be also negligible.

Let us finally apply the momentum theorem to the fluid mass contained in the annular volume $(\Delta S, \Delta S')$. The exterior forces applied to this volume are the pressure of the blades on the fluid, whose resultant along the screw axis is equal to $-\Delta Q$, and the resultant of the exterior pressure acting on this volume, equal to

$$p'\Delta S'-p\Delta S,$$

[1] It will be easy to see that the same conclusion would have been reached if neither the friction forces nor the pressure differences $\Sigma\Delta S''(p''-p_0)$ had been neglected.

when one neglects the friction forces acting on the boundaries of this volume. On the other hand, as in the most important practical applications of blade screws the sections S and S' come out to be close to the blade screw, and as we neglect the radial velocities we have

$$\Delta S \cong \Delta S' \text{ and } v \cong v'$$

and on account of the flow continuity we will thus have

$$v \cong v'$$

The fluid momentum variation for the annular volume $(\Delta S, \Delta S')$ is thus equal to zero. The axial resultant of all the exterior forces applied to this volume is therefore equal to zero, so that we will have

$$p'\Delta S' - p\Delta S - \Delta Q = (p'-p)\Delta S - \Delta Q = 0$$

or

(11) $$p' - p = \frac{\Delta Q}{\Delta S}$$

This last relation will be considered as being the definition of the surface S' when the surface S will be known.

We can now see that the pressure distribution along a stream line crossing the space swept by the screw blades will have the general course represented on the right-hand side of figure 2.

Finally the following fact must be noted. If the theorem of moments of momentum were applied to the slip stream portion included between the sections S_0 and S, it would appear that the fluid contained in this portion has no rotation. The rotation of this portion of the slip stream can thus be due only to viscosity and to the periodicity of the pressure distribution in the section S. It is also for these last reasons that there can be a variation of the moment of momentum of the fluid between the sections S' and S''.

We will say BY DEFINITION *that a blade screw is working under* NORMAL CONDITIONS *when the relations (8) and (9) can be considered as sufficient approximations of the thrust and the torque.*

This definition is justified by the fact that in the most important practical applications of the blade screw the *normal conditions* are realized.

We will call *neighborhood conditions* all the circumstances which can remove us from the normal conditions.

In some blade-screw applications, the neighborhood conditions have a primordial influence. These special cases of blade-screw applications will be submitted to a separate investigation.

Substituting in the relations (9a) and (9b) the above values of ΔM and $\Delta I''$ we get:

(12) $$\Delta Q = \Delta S(V+v)v''\delta.$$

(13) $$\Delta C = \Delta S r^2 \omega'' \frac{(V+v)^2}{V+v''}\delta$$

These expressions will give us the values of the partial thrust ΔQ and the partial torque ΔC produced by the considered blade elements only when the slip and race velocities v, v'', and ω'' will be determined.

Let us agree to call *specific function* the quantity

(14) $$\rho = \frac{V\Delta Q}{\Omega \Delta C} = \frac{V}{r\Omega} \cdot \frac{v''(V+v'')}{r\omega''(V+v)}$$

which represents the ratio of the work $V\Delta Q$ of the partial thrust to the work $\Omega\Delta C$ of the partial torque. It is easy to see that this ratio is nothing other than the *partial efficiency* of a blade element of the blade screw considered when the last is propulsive. But I have considered it necessary to adopt for this ratio a more general name, because we will have to consider it far out of the limits, where it has the meaning of the efficiency of a propulsive screw. We shall see in the following that this ratio specifies by its numerical value the type of machine which the blade screw realizes.

Let us now pass to the direct evaluation of the fluid pressure on the elements of the blades of the screw.

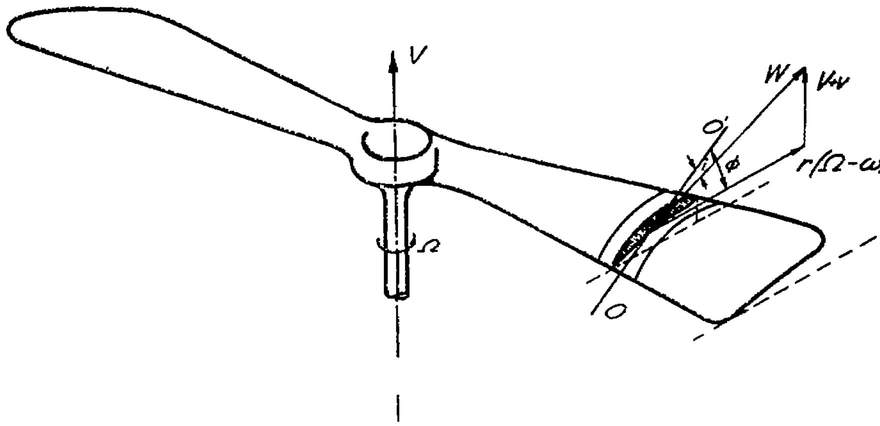

FIG. 3.

Figure 3 gives a general picture of one of the blade elements considered. The relative velocity W of the fluid in regard to the blade element is the resultant of the velocities $V+v$ and r $(\Omega-\omega)$. The line OO' is the *zero line*,[1] to which is referred the angle of attack i; φ is the angle between the zero line and the plane of rotation of the blade screw; it is by this angle, called *effective blade angle*, that we fix the inclination of the blade elements on the screw rotation plane.

The relation between the effective blade angle φ and the pitch H of a blade section is to be directly seen from figure 4. We have

$$H=2\pi r \operatorname{tg} \varphi \tag{15}$$

It is easy to see that the numerical value of the pitch depends upon the reference line adopted to fix the inclination of the blade element considered.

The pitch H counted from the zero line will be called *effective pitch*, in opposition to the *constructive pitch* measured from any other reference line—the chord of the blade section profile, for example—whose consideration can be more convenient in some cases, as for the workshop drawings of blade screws.

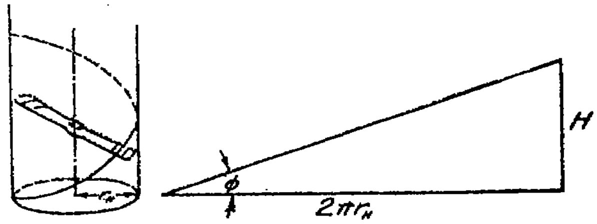

FIG. 4.

As far as I know it, in nearly all the blade-screw investigations it was the constructive pitch, measured from the blade section chord, that was always considered; but, as follows in full evidence from what is said in Note III at the end of this memoir, the constructive pitch is no other than a quantity arbitrarily chosen. Therefore we can not adopt this quantity to describe the blade-screw properties. The blade properties depending upon pitch can only be referred to the effective pitch, which is a perfectly defined hydrodynamical characteristic, fully

[1] For the definition of the zero line and in general all concerning the laws of fluid resistance, see Note III at the end of this memorandum.

independent of the screw blade section profile. To equal constructive pitches can correspond very easily unequal effective pitches, and vice versa. Under such conditions it is easy to conceive all the difficulties which the consideration of the constructive pitch can bring into the analysis of the screw-blade problem. Thus, when using the constructive pitch, we can often find negative values for the slip, while the effective pitch will always give positive values of the last, as must be from the physical meaning of the slip. We therefore see how important is the consideration of the effective pitch.[1]

All the quantities which are necessary in order to specify the value of the fluid resistance ΔR of a blade element are represented on figure 5. It is by the angle β_H that we will fix the inclination of the resistance ΔR to the normal to the zero line. As is well known, we have

$$\Delta R = k_i \delta \Delta A W^2 \tag{16}$$

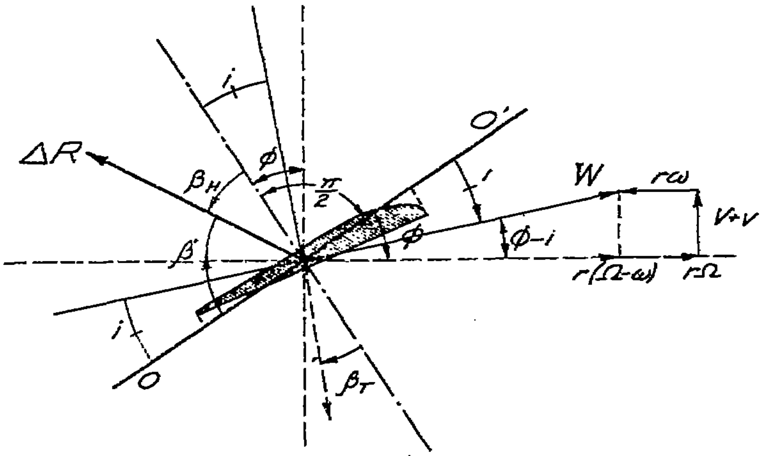

FIG. 5.

where k_i is an empirical function of the angle of attack depending upon the blade section profile considered; δ the fluid mass density; ΔA the area of a blade element, equal in a sufficient approximation to

$$\Delta A = b \Delta r \tag{17}$$

b being the breadth of the blade element considered.

The velocity W is equal to

$$W^2 = (V+v)^2 + r^2(\Omega - \omega)^2 \tag{18}$$

and we also have

$$W \sin(\varphi - i) = V + v; \quad W \cos(\varphi - i) = r(\Omega - \omega) \tag{19}$$

$$\operatorname{tg}(\varphi - i) = \frac{V+v}{v(\Omega - \omega)} \tag{20}$$

For small angles of attack the formula (16) reduces to

$$\Delta R = k \delta \Delta A W^2 i \tag{21}$$

In Note III will be found all the restrictions in the use of the formulæ (16) and (21).

[1] For example, the effective pitch gives at once the right understanding of the working conditions of a boomerang. A regular boomerang, whose two asymmetrical blades are not twisted, but have aerofoil sections, will have an effective pitch of a certain value for rotations in both senses in the plane of its blades, and thus when thrown with an initial rotation will produce a thrust to which all interesting boomerang properties are due. A boomerang is, exactly speaking, nothing more than a propeller left free to move in space.

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If the screw considered has n blades, we will have included in the annular space considered n blade elements, giving each a resistance ΔR. Projecting these forces ΔR on the screw axis and on its rotation plane, we will find the values of the partial thrust ΔQ and partial torque ΔC produced by the elements considered:

$$\Delta Q = n\Delta R \cos(\varphi+\beta_H) \tag{22}$$

$$\Delta C = nr\Delta R \sin(\varphi+\beta_H) \tag{23}$$

These last formulæ assume that the values adopted for the empirical functions k_i and β_H take account of the possible mutual interference of blades of the screw. In the following we will return, more in detail, to this last question.

Comparing the relations (22) and (23) we find

$$\Delta C = \Delta Q r \operatorname{tg}(\varphi+\beta_H) \tag{24}$$

and for the specific function we get the value

$$\rho = \frac{V\Delta Q}{\Omega \Delta C} = \frac{V}{r\Omega}\,\frac{1}{\operatorname{tg}(\varphi+\beta_H)} \tag{25}$$

It must be noted that the last expression of the specific function is fully independent of any hypothesis.

The expressions (12) and (13) of the partial thrust ΔQ and the partial torque ΔC, found in the foregoing by the general consideration of the fluid motion around the blade screw, must evidently be equal to the expressions (22) and (23) of these same quantities found by the direct evaluation of the fluid pressure on the blades of the screw. We thus have:

$$\Delta Q = \Delta S(V+v)v''\delta = n\Delta R \cos(\varphi+\beta_H) = nk_i\,\delta\Delta A\, W^2 \cos(\varphi+\beta_H)$$

$$\Delta C = \Delta S r^2 \omega'' \frac{(V+v)^2}{V+v''}\delta = nr\Delta R \sin(\varphi+\beta_H) = nrk_i\Delta A\, W^2 \sin(\varphi+\beta_H)$$

with

$$\Delta S = 2\pi r\Delta r;\ \Delta A = b\Delta r$$

Introducing these last values in the above formulæ we get:

$$v''(V+v) = \frac{nb}{2\pi r}k_i W^2 \cos(\varphi+\beta_H) \tag{26}$$

$$r\omega''\frac{(V+v)^2}{V+v''} = \frac{nb}{2\pi r}k_i W^2 \sin(\varphi+\beta_H) \tag{27}$$

In these last relations appears the expression

$$\frac{nb}{2\pi r}$$

which is the ratio of the total breadth of the blade sections considered to the circumference by them described. We will designate this ratio by a and give it the name *breadth ratio*. Thus we will state

$$a = \frac{nb}{2\pi r} \tag{28}$$

Introducing the notation (28) in the formulæ (26) and (27), and taking account of the relation (19), we finally get

$$\frac{v''}{V+v}=\frac{ak_1\cos(\varphi+\beta_H)}{\sin^2(\varphi-i)} \tag{29}$$

$$\frac{r\omega''}{V+v}=\frac{ak_1\sin(\varphi+\beta_H)}{\sin^2(\varphi-i)} \tag{30}$$

These last two relations constitute the first two equations of the general blade-screw theory.

Let us now calculate the work of the fluid resistance of the blade elements considered in their motion relative to the flow meeting them. We have

$$n\Delta R\,W\cos(\Delta R,W)=n\Delta R\sin(\varphi+\beta_H)\cdot r(\Omega-\omega)-n\Delta R\cos(\varphi+\beta_H)\cdot(V+v)$$

and on account of the relations (22) and (23) we get

$$n\Delta R\cos(\Delta R,W)=\Delta C(\Omega-\omega)-\Delta Q\,(V+v)=\Omega\Delta C-V\Delta Q-\omega\Delta C-v\Delta Q. \tag{31}$$

In the last member of this relation, $\Omega\Delta C$ is the work of the partial torque; $V\Delta Q$ the work of the partial thrust; $\omega\Delta C$ the work communicated to the fluid in its rotational motion; $v\Delta Q$ the work communicated to the fluid in its translatory motion. Accordingly, the quantity $n\Delta R\cos(\Delta R,W)$ represents the work spent in the displacement in the fluid of the blade elements considered. It is evident that the same relation holds for any other blade elements. We are thus brought to the following fundamental theorem.

THEOREM I.—*The work of the fluid resistance of the blades in their motion relative to the flow meeting them is equal to the work spent for the displacement of the blades in the fluid, that is, equal to the work spent in shocks, friction, etc., of the fluid flow against the blades.*

This last fact established, we are now able to apply the kinetic energy theorem to the annular space containing the blade elements considered. We thus have

$$\Omega\Delta C=V\Delta Q+\tfrac{1}{2}\Delta Mv''^2+\tfrac{1}{2}\Delta I''\omega''^2+[\Omega\Delta C-V\Delta Q-\omega\Delta C-v\Delta Q] \tag{32}$$

from which follows directly

$$v\Delta Q+\omega\Delta C=\tfrac{1}{2}\Delta Mv''^2+\tfrac{1}{2}\Delta I''\omega''^2. \tag{33a}$$

This last relation is in reality evident of itself, because it expresses the fact that the work communicated by the screw to the fluid is equal to the kinetic energy of the fluid in the section S''. But it was my intention to show, so to speak, the whole genesis of the last relation, on account of the distribution of energy absorbed by the screw working. It must also be noted that the relation (33a) is a direct consequence of our definition of the normal working conditions of a blade screw, according to which the losses between the sections S_0, S and S', S'' are considered as negligible. The relation (33a) can also be written in the following form: substituting in (33a) for the partial thrust ΔQ and the partial torque ΔC their values given by the relations (9), we get

$$v''(2v-v'')=r''^2\omega''(\omega''-2\omega). \tag{33b}$$

We shall now show that on account of the normal screw working conditions not only the relations (33a) occur, but also that we have, separately, besides

$$v\Delta Q=\tfrac{1}{2}\Delta Mv''^2 \tag{34a}$$

and

$$\omega\Delta C=\tfrac{1}{2}\Delta I''\omega''^2. \tag{34b}$$

In effect, let us apply the Bernouilli theorem between the sections S_o and S. Neglecting the interior losses between these sections and neglecting also the radial velocities in section S, we have [1]

$$p_o+\frac{\delta V^2}{2}=p+\frac{\delta(V+v)^2}{2}$$

or

$$p_o-p=\frac{\delta}{2}(2Vv+v^2). \tag{35a}$$

Let us apply once more, to the same approximation, the Bernouilli theorem between the sections S' and S''; we have

$$+\frac{\delta}{2}(V+v)^2=p''+\frac{\delta}{2}(V+v'')^2.$$

But as we admit the pressure p'' to have already reached the value p_o in the section S'', we will have

$$p'-p_o=\frac{\delta}{2}(2Vv''+v''^2-2Vv-v^2). \tag{35b}$$

Adding term by term the relations (35a) and (35b), we get

$$p'-p=\frac{\delta}{2}(2V+v'')v''$$

On the other hand, on account of the relations (11) and (12), we have

$$p'-p=\frac{\Delta Q}{\Delta S}=\delta(V+v)v''.$$

From the direct comparison of these last two relations we get

$$\frac{\delta}{2}(2V+v'')v''=\delta(V+v)v''$$

or

$$v''=2v$$

which, on account of the relation (33b), has as a direct consequence

$$\omega''=2\omega.$$

But according to the relation (9a) we have $\Delta Q=\Delta Mv''$.

---

[1] When the radial velocities are neglected, the motion of the fluid in the slip stream, in cylindrical coordinates, is expressed between the sections S_o, S and S', S'' by the following system of equations:

$$\delta(V+v)\frac{\partial(V+v)}{\partial z}=-\frac{\partial p}{\partial z};\quad \delta r\omega^2=\frac{\partial p}{\partial r};\quad \delta(V+v)\frac{\partial(r\omega)}{\partial z}=0$$

v and $r\omega$ being here the slip and race velocities at any point of the slip stream at a distance r from screw axis, p the pressure at the point considered, z the cylindrical coordinate parallel to the screw axis, the last being an axis of symmetry for the whole phenomenon.

The third of these equations means that the radial components of the vortices in the slip stream can be considered as negligible. The second of these equations justifies the calculation of the pressure distribution in a cross section of the slip stream indicated in Note II. The first of these equations, integrated along a stream line, gives

$$\frac{\delta(V+v)^2}{2}+p=\text{const}$$

We thus see that when the radial velocities are neglected the race velocity comes out to be negligible in the calculation of the pressure distribution along a stream line of the slip stream.

We thus will have

$$v\Delta Q = \Delta Mv''v = \tfrac{1}{2}\Delta Mv''^2$$

The relation (34a) is thus justified. On account of the relation (33a) the relation (34b) follows directly.

The two relations

$$v'' = 2v;\ \omega'' = 2\omega \tag{36}$$

constitute the other two equations of our blade-screw theory.[1]

These last relations show us that the slip and race velocities in the section S'', that is, in the outdraught, are exactly the double of the corresponding slip and race velocities in the section S, that is, in the indraught. The exact position of the section S to which has to be referred the velocity W, used for the calculation of the fluid resistance of the blades, is thus fixed exactly.[2]

As far as I know, the relations (36) are in full agreement with all the experiments made up to the present day on the velocity distribution in the slip stream.

Thus N. Joukowski, in his analytical interpretation of Flamm experiments, comes to the same results.[3]

G. Eiffel has observed the slip velocities in the indraught and outdraught of a propeller for values of V included in the interval of 10 m./sec. to 25 m./sec.; and his experiments verify with an accuracy of 1 or 2 per cent the relations (36).[4]

The relations (36) thus appear as a fundamental characteristic of the flow in the slip stream.

Substituting in the equations (29) and (30) the values found for v'' and ω'' we get:

$$\frac{2v}{V+v} = \frac{ak_t \cos.(\varphi+\beta_H)}{\sin^2(\varphi-i)} \tag{37}$$

$$\frac{2r\omega}{V+2v} = \frac{ak_t \sin(\varphi+\beta_H)}{\sin^2(\varphi-i)} \tag{38}$$

The expression of the specific function takes the form

$$\rho = \frac{V}{r\Omega}\cdot\frac{v\,(V+2v)}{r\omega\,(V+v)} = \frac{V}{r\Omega}\cdot\frac{1}{\operatorname{tg}(\varphi+\beta_H)} \tag{39}$$

The equations (37), (38), and (39) with the relations

$$v'' = 2v;\ \omega'' = 2\omega \tag{36}$$

constitute the system of fundamental equations of the general blade-screw theory, which embraces all the blade-screw properties.[5]

All the following chapters of this memoir will be devoted to deducing the blade-screw properties by the analysis of this system of equations. It is in the consequences obtained that there will be found the best confirmation of the system of equations established.

Before passing to this analysis, I will establish the explicit expressions of the principal quantitites which are used to characterize the work of the blade-screw elements considered.

---

[1] The establishment of the relations (36) for the middle part of the slip stream where the r's are small, needs only the hypothesis that the losses between the sections S_0, S, and S', S'' are negligible.

[2] Effectively: The position of the section S'' is exactly known, as it is the narrowest section of the slip stream; the section S is the one disposed in the indraught where the slip and race velocities are exactly the halves of the slip and race velocities in the section S''. The position of the section S' is fixed by the relation (11).

[3] N. Joukowski. "Vortex Theory of the Propulsive Screw," relations (20) and (21) on pp. 11 and 12. Moscow, 1912 (in Russian).

[4] G. Eiffel. "Nouvelles recherches sur la resistance de l'air et l'aviation faites au laboratoire d'Auteuil," Paris, 1914. See the table on p. 379.

[5] A critical discussion of this system of equations will be found in Note VI at the end of this memoir.

We will designate by N the number of turns of the blade screw per second, and call relative pitch the quantity

$$x=\frac{V}{NH} \tag{40}$$

which expresses the ratio of the pitch of the trajectory of the blade element considered to its own pitch.

Let us besides designate by μ and call *advance per turn*, or, shorter, the *advance*, the ratio V/N.

We have

$$\mu=\frac{V}{N}=\frac{2\pi V}{\Omega}=Hx \tag{41}$$

As $H=2\pi r$ tg φ we also have

$$\frac{V}{r\Omega}=\frac{V}{2\pi r N}=\frac{V \operatorname{tg}\varphi}{NH}=\frac{\mu \operatorname{tg}\varphi}{H}=x \operatorname{tg}\varphi \tag{42}$$

The specific function takes the form

$$\rho=x\frac{\operatorname{tg}\varphi}{\operatorname{tg}(\varphi+\beta_H)} \tag{43}$$

and we have

$$\rho \operatorname{tg}(\varphi+\beta_H)=x \operatorname{tg}\varphi=\frac{V}{r\Omega} \tag{44}$$

Let us introduce the notation

$$\frac{a\, k_4 \cos(\varphi+\beta_H)}{2\sin^2(\varphi-i)}=a\, z \tag{45}$$

The equations (37) and (38) reduce then to

$$\frac{v}{V+v}=a\, z \tag{46}$$

$$\frac{r\omega}{V+2v}=az \operatorname{tg}(\varphi+\beta_H) \tag{47}$$

From these last equations we find directly the values of the slip and race velocities v and $r\omega$:

$$v=\frac{V a z}{1-a z}=r\Omega\frac{a z}{1-a z}x \operatorname{tg}\varphi \tag{48}$$

$$r\omega=\frac{a z(1+a z)}{1-a z}V \operatorname{tg}(\varphi+\beta_H)=r\Omega\frac{a z(1+a z)}{1-a z}x \operatorname{tg}\varphi \operatorname{tg}(\varphi+\beta_H) \tag{49}$$

Introducing these last values in the expression (20) of tg $(\varphi-i)$ we find:

$$\operatorname{tg}(\varphi-i)=\frac{V+v}{r(\Omega-\omega)}=\frac{1}{(1-a z)\left[\frac{1}{x \operatorname{tg}\varphi}-\frac{a z(1+a z)}{1-a z}\operatorname{tg}(\varphi+\beta_H)\right]}$$

from which relation we get the value of the relative pitch

$$x=\frac{(1-a z)\operatorname{tg}(\varphi-i)}{\operatorname{tg}\varphi\,[1+a z(1+a z)\operatorname{tg}(\varphi+\beta_H)\operatorname{tg}(\varphi-i)]} \tag{50}$$

Introducing this last value of the relative pitch in the expression (43) of the specific function, we find:

$$\rho=\frac{(1-az)\operatorname{tg}(\varphi-i)}{\operatorname{tg}(\varphi+\beta_H)[1+az(1+az)\operatorname{tg}(\varphi+\beta_H)\operatorname{tg}(\varphi-i)]} \tag{51}$$

We thus see that the relative pitch x and the specific function ρ of a blade element are functions of the angle of attack i only. We can therefore consider the specific function as being a function of the relative pitch only. We are thus brought to the following theorem:

THEOREM II.—*The specific function ρ of a blade element is a function of the relative pitch of the same blade element only.*

Substituting in the expression (12) of the partial thrust, the value (48) of v we get:

$$\Delta Q=2\Delta S\,v(V+v)\,\delta=\frac{2V^2az}{(1-az)^2}\Delta S\,\delta \tag{52}$$

and introducing the notation

$$\frac{2az}{(1-az)^2}=q \tag{53}$$

we get

$$\Delta Q=q\,\delta\,\Delta S\,V^2 \tag{54}$$

which expression of the partial thrust is similar to the expression of the fluid resistance

$$\Delta R=k_i\delta\Delta A\,W^2$$

We will call *load coefficient* the coefficient q defined by the relation (53).

Introducing the value found for the partial thrust ΔQ in the expression (22), we find for ΔR the value

$$\Delta R=\frac{\Delta Q}{n\cos(\varphi+\beta_H)}=\frac{q\,\delta\,\Delta S\,V^2}{n\cos(\varphi+\beta_H)} \tag{55}$$

And for the partial torque ΔC we find the value:

$$\Delta C=\Delta Q\,r\operatorname{tg}(\varphi+\beta_H)=r\,q\,\delta\,\Delta S\,V^2\operatorname{tg}(\varphi+\beta_H) \tag{56}$$

The work developed in a unit of time and the power absorbed by the blade elements considered are equal to

$$V\Delta Q=q\,\delta\,\Delta S\,V^3 \tag{57}$$

$$\Omega\Delta C=\frac{q\,\delta\,\Delta S\,V^3\operatorname{tg}(\varphi+\beta_H)}{x\operatorname{tg}\varphi} \tag{58}$$

Between the slip s and the relative pitch x exists the relation

$$s=\frac{H-\frac{V}{N}}{H}=1-\frac{V}{NH}=1-x \tag{59}$$

from which follows

$$x=1-s \tag{60}$$

Let us finally agree always to consider the indraught and the outdraught relative to the slip stream which the blade-screw rotation tends to produce.

CHAPTER II.

THE STUDY OF THE SPECIFIC FUNCTION.

We will make the present discussion in the following way: On one hand we will direct our attention to the blade elements; on the other, we will follow the general picture of the phenomenon by aid of our system of fundamental equations. For the general view of the different states of work of the blade-screw, which we have in mind here, it will be more convenient to fix the orientation of the fluid resistance ΔR relative to the blade element by aid of the angle

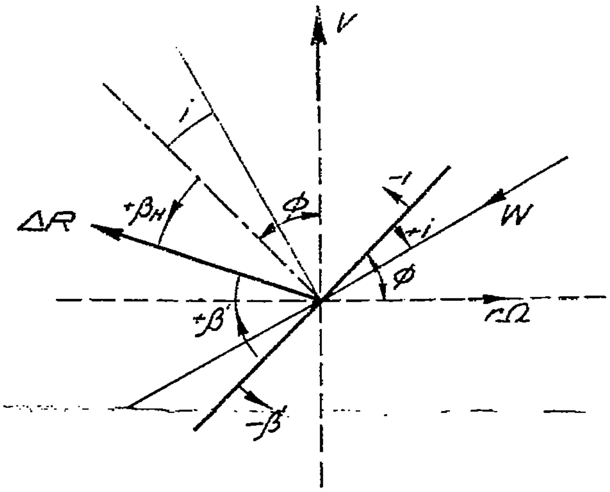


FIG. 6.

β' between ΔR and the zero line. The senses adopted as positive for the angles β' and i are indicated on figure 6. Substituting for β_H in the formulæ (45), (50), (43), (48), (49), (52) and (56) its value

$$\beta_H = \frac{\pi}{2} - \beta'$$

we get

(61) $$az = \frac{ak_i \sin(\beta' - \varphi)}{2 \sin^2(i - \varphi)} = \frac{v}{V + v}$$

(62) $$x = \frac{1 - az}{\operatorname{tg} \varphi [az(1 + az)\operatorname{ctg}(\beta' - \varphi) - \operatorname{ctg}(i - \varphi)]} = \frac{V}{NH}$$

(63) $$\rho = \frac{x \operatorname{tg} \varphi}{\operatorname{ctg}(\beta' - \varphi)} = \frac{V \Delta Q}{\Omega \Delta C}$$

(64) $$v = V \frac{az}{1 - az} = r\Omega \frac{az}{1 - az} x \operatorname{tg} \varphi$$

(65) $$r\omega = V \frac{az}{1 - az}(1 + az) \operatorname{ctg}(\beta' - \varphi) = r\Omega x \operatorname{tg} \varphi \frac{az(1 + az)}{1 - az} \operatorname{ctg}(\beta' - \varphi)$$

(66) $$\Delta Q = 2 \Delta S v (V + v) \delta = \frac{2az}{(1 - az)^2} \delta \Delta S V^2 = q \delta \Delta S V^2$$

(67) $$\Delta C = \Delta Q \, r \operatorname{ctg}(\beta' - \varphi) = rq\delta \Delta S V^2 \operatorname{ctg}(\beta' - \varphi)$$

In these formulæ figure the two empirical functions k_i and β', the general course of which is nearly the same for all the aerofoil profiles. For variations of the angle of attack i starting from zero, the empirical function β' increases very rapidly, and even for small values of the angle i reaches values near to 90°, which value this function maintains till the angle of attack approaches 180°; for values of the angle of attack near 180° the angle β' rapidly reaches also the value of 180°. The empirical function k_i also increases rapidly with the angle of attack, up to a certain value of the latter, after which the increase of k_i becomes moderate; after the angle of attack has reached the value of 90°, the empirical function k_i decreases, first moderately, afterwards rapidly, and approaches the value zero when i approaches 180°. (See Note III at the end of this memoir.)

In the present discussion it is the general course of the whole phenomenon of the working of a blade screw that I wish to establish. The quantitative side of the question will be taken up in full detail in the following chapters. This is why in this chapter, for the simplicity of the analysis and the symmetry of the results, we will assume that the blade elements considered are simply constituted of flat plates with the blade angle φ equal to 45°.

I will begin by two general remarks.

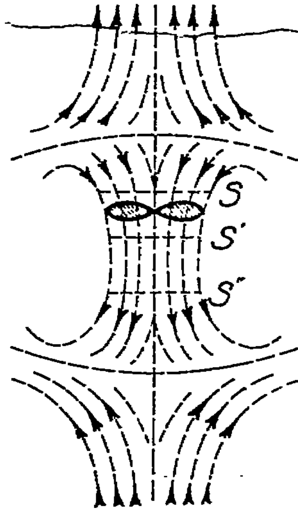


FIG. 7a.

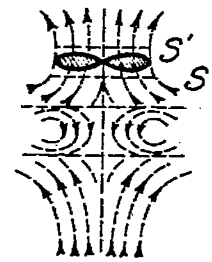


FIG. 7b.

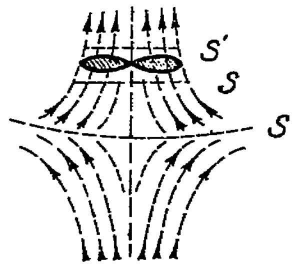


FIG. 7c.

Remark I.—The expressions (9a) and (9b) of page 174 will give us in magnitude and sense the values of the partial thrust ΔQ and the partial torque ΔC when ΔM and $\Delta I''$ are always taken as positive. But the relations (1) will give for ΔM positive values only when $(V+v)>O$ or $(V+v'')>O$, in dependence upon the expression adopted. We must therefore, in the expressions of ΔM and $\Delta I''$, change the signs before $(V+v)$ and $(V+v'')$ when these last expressions will become negative. This corresponds to a change in the sign before az in the equation (46), when $(V+v)$ becomes negative, and in the equation (47) when $(V+v'')=(V+2v)$ becomes negative. Thus, for $(V+v)<O$ and $(V+2v)<O$ the second members of the equations (46) and (47) change their signs. It is only with such changes in signs that the equations (12) and (13) from page 23 become compatible with the equations (22) and (23) of page 26, which always give ΔQ and ΔC in magnitude and sense.

Let us examine in their general outlines the phenomena which accompany the change of signs of $(V+v)$ and $(V+v'')$. Let us consider a blade screw working at a fixed point, and let us communicate to the blade screw a translation along its axis of increasing velocity in the sense inverse to the sense of the thrust produced by the screw; or let us consider a fluid current running on a blade screw working at a fixed point, with an increasing velocity, uniform in the whole current, parallel to the screw axis and directed on the screw in the sense of its t Since in the slip stream created by the blade screw the slip and race velocities decrease as we move away from the blade screw, starting from the sections S and S'', there must be formed, as soon as a fluid stream is directed on the blade screw in the sense above indicated, two *surfaces of separation* through which there must be no flow and between which the screw will be included. This state of things is schematically represented on figure 7a. When the velocity of the fluid stream directed on the blade screw is increased, the two surfaces of separation will approach one another, and there will be a moment when one of these surfaces of separation (it will be the one disposed in the indraught) will cross the space swept by the blades of the screw. This moment will correspond to the change of the sense of the fluid stream crossing the space swept by the blades of the screw. At this moment the sections S and S' of the slip stream will be interchanged. The crossing of the space swept by the screw blades by the surface of flow separation is characterized by the conditions

$$(V+v)=o \text{ and } i=\varphi$$

which bring with them $az=\infty$ and therefore $x=o$ for $V\neq o$. We thus see that we must have $N=\infty$ t is, the blade screw will show a tendency to take an infinite rotation. The thrust and the torque of the blade screw have the tendency to disappear. In the case of a ship propeller, in the state of brake work of the propeller, the *whirling phenomenon* is often observed and corresponds to the conditions just described. When this critical point of work is passed, a new state of work establishes itself, for which the two surfaces of flow separation are disposed on the same side of the blade screw. We have $(V+v)<o$ but $(V+v'')>o$. Between the two surfaces of flow separation there will appear a vortex ring, stationary relative to the blade screw, whose axis coincides with the axis of the screw, as schematically represented in figure 7b. I have named this last state of work of the blade screw the *vortex ring state of work*. This state of work is included in the interval $(V+v)=o$ and $(V+v'')=0$. We will designate by $\varphi+\psi$ the value of the angle of attach which corresponds to this last condition. At the moment $(V+v'')=0$, the fusion of the two surfaces of flow separation takes place (see fig. 7c) which is immediately followed by their disappearing as soon as $(V+v'')$ has changed its sign, which corresponds to the change of sense of the whole stream crossing the screw. From this moment on, the slip stream created by the blade screw is, so to speak, vanquished by the outside stream directed on the screw. The ensemble of the phenomena is just that which accompanies the change of sense of the stream crossing the blade screw.

For the vortex ring state of work, and for the states of work near to the last, the radial velocities, near the space swept by the screw blades, have sensible values. Under such conditions our system of equations (61)–(67) can give only an approximate characteristic of these states of work, for the detailed study of which the radial velocities have to be taken into account. I will limit myself here to the establishment of the existence of the vortex ring state of work and will not go into its detailed study.[1]

Remark II.—The complete cycle of states of work of the blade screw includes the states of *direct rotation*, that is rotation in one sense, and of *reversed rotation*, that is rotation in the inverse sense. The states of work of direct rotation are separated from the states of work of reversed rotation by the *standing states*. When the blade screw is stopped in a fluid current, the angles of attack of the blade elements have for values

$$i=i'_a=-\left(\frac{\pi}{2}-\varphi\right) \text{ or } i=i_a=\frac{\pi}{2}+\varphi$$

We thus see that the states of work with rotation in one sense are included in an interval of variation of the angle of attack i equal to π. On the other hand, it is easy to see that the states of work of rotation in one sense can only be the reproduction of the states of work of rotation in the other sense, when the screw blades are of identical configuration on both sides. Under such conditions all the quantities characterizing the blade screw working must be periodical functions with a period equal to π. This remark will allow us in the present case, i. e., of blade elements constituted by flat plates, to judge of the values of the function considered in an interval of variation of i equal to 2π, when this function shall have been studied in an interval of variation of i equal to π.

[1] The author has been deprived of the possibility of reproducing experimentally this interesting vortex ring state of work. All the foregoing description of the phenomenon has been obtained by its purely analytical discussion.

As for the standing states above mentioned, it must be noted that, the screw having no rotation, the phenomenon of the slip stream disappears, and it is to be expected that our system of equations will give only an approximate characteristic of these standing states. But when the standing states establish themselves we have simply to do with an immobile screw plunged in a fluid current directed along its axis, and, accordingly, this standing state can be very easily submitted to a direct experimental study, since we have only to measure the drag of the blade screw and the torque necessary to prevent its rotation.

Having established the existence of the vortex ring state of work and the periodicity of the function describing the blade-screw work, we shall study first from a purely analytical standpoint the general character of variation of the principal functions occurring in the blade-screw theory.

Let us first examine the general course of the functions az and ctg $(\beta'-\varphi)$ which figure in all the formulæ (62)–(67), and which depend upon the empirical functions k_i and β'.

We have

(68)
$$az=\frac{ak_i \sin(\beta'-\varphi)}{2\sin^2(i-\varphi)}=\frac{v}{V+v}$$

For $i=\varphi$, $az=+\infty$; the function az has an asymptote parallel to the axis of ordinates and we have $(V+v)=0$. The function az is equal to zero for $\beta'=\varphi$ which corresponds to a very small angle of attack $i=\epsilon$. For values of the angle of attack included between $\varphi\geq i\geq\epsilon$ the function az takes positive values. For $(V+v'')=0$, we have $az=-1$, the angle of attack having the value $i=-[\pi-(\varphi+\psi)]$. In the interval $-[\pi-(\varphi+\psi)]\leq i\leq\varphi$, the function az takes negative values. The general course of the function az is represented in figure 8, where the sign of az has been changed in the interval $i<-(\pi-\varphi)$ and $i>(\varphi+\psi)$. The function az appears then as a periodical function with a period equal to π, in complete agreement with the foregoing remarks, and under such conditions the system of equations (62)–(67) can be considered in the whole interval of variation of i between 0° and ±180°, with the exception of the interval corresponding to the vortex ring state of work. For this last state of work the portion of the curve of az is plotted in dots, in agreement with the change of sign in the equations (46) and (47) indicated above. In this same figure 8 is represented the general course of the function $\text{ctg}(\beta'-\varphi)$, which directly follows from the general course of the empirical function β'.

After having established the general character of variation of the functions az and $\text{ctg}(\beta'-\varphi)$, it will be easy to follow the general course of the functions:

(69)
$$\frac{v}{V}=\frac{az}{1-az}$$

(70)
$$\frac{r\omega}{V}=\frac{az(1+az)}{1-az}\text{ctg}(\beta'-\varphi)$$

(71)
$$x=\frac{1-az}{\text{tg}\,\varphi[az(1+az)\text{ctg}(\beta'-\varphi)-\text{ctg}(i-\varphi)]}=\frac{V}{NH}$$

(72)
$$\rho=\frac{1-az}{\text{ctg}(\beta'-\varphi)[az(1+az)\text{ctg}(\beta'-\varphi)-\text{ctg}(i-\varphi)]}=\frac{x\,\text{tg}\,\varphi}{\text{ctg}(\beta'-\varphi)}$$

(73)
$$\frac{v}{r\Omega}=x\,\text{tg}\,\varphi\,\frac{az}{1-az}$$

(74)
$$\frac{\omega}{\Omega}=x\,\text{tg}\,\varphi\,\frac{az(1+az)}{1-az}\text{ctg}\,(\beta'-\varphi)$$

(53)
$$q=\frac{2az}{(1-az)^2}$$

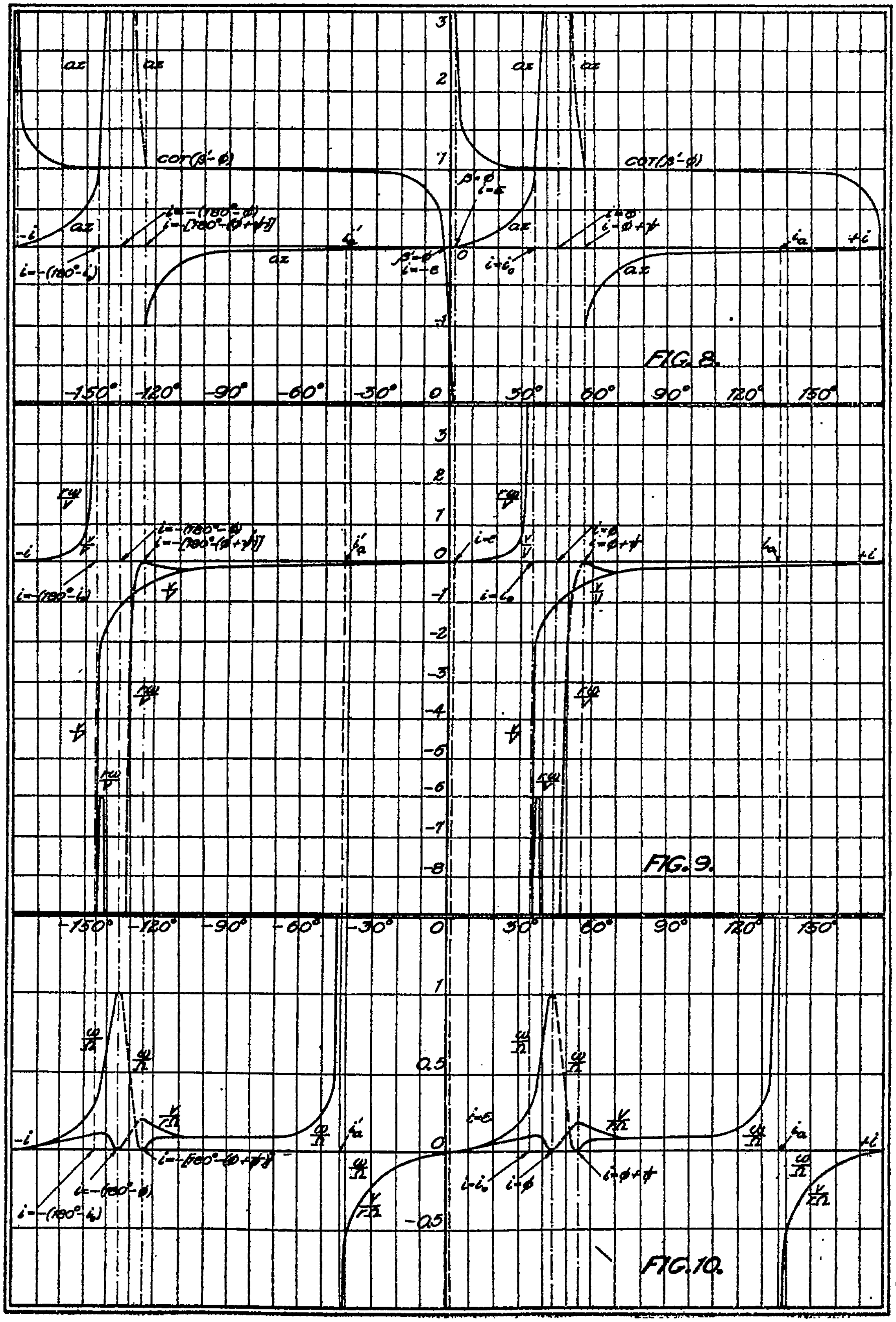


FIG. 8.
FIG. 9.
FIG. 10.

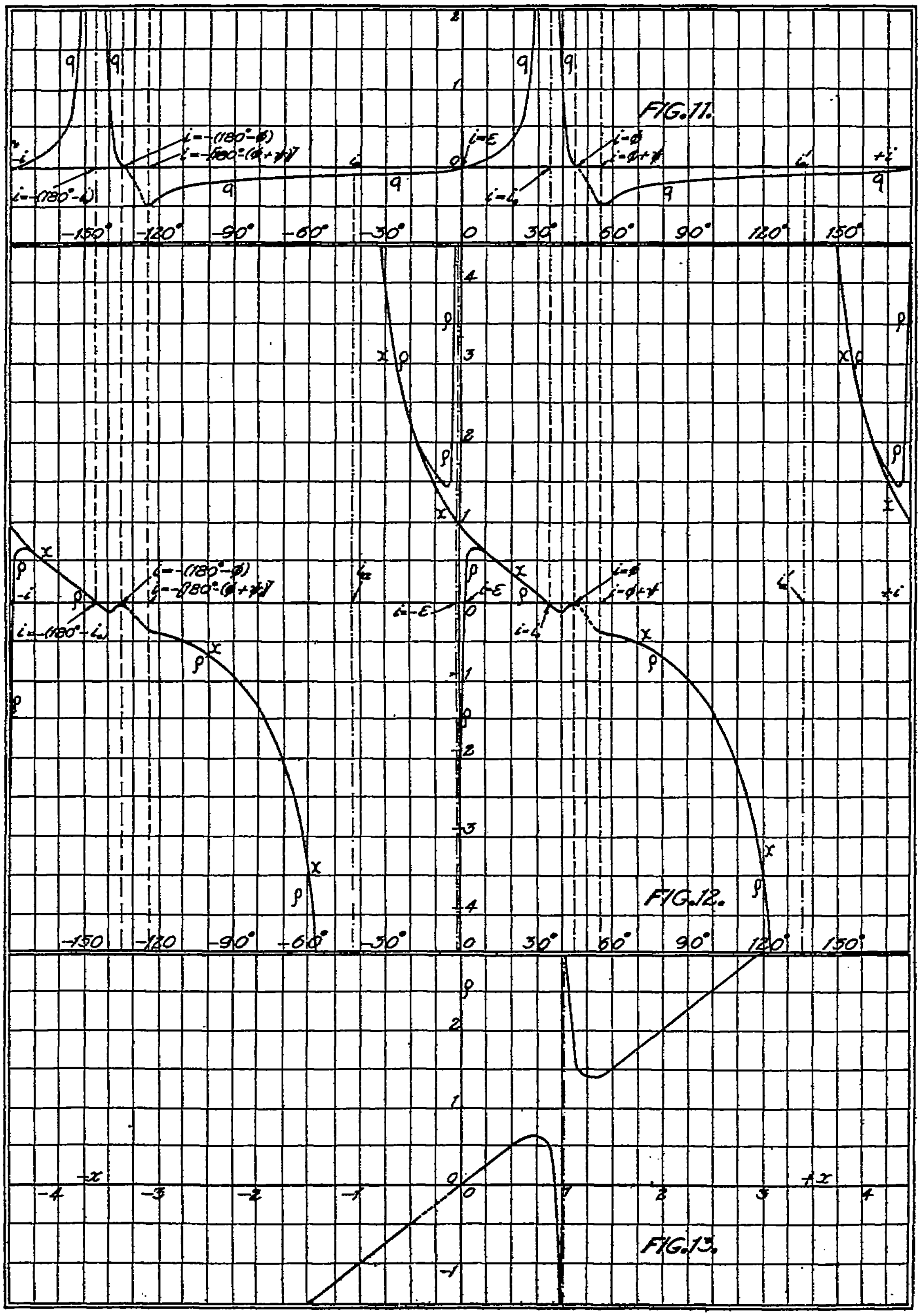


FIG. 11.

FIG. 12.

FIG. 13.

By aid of these functions we can in all the cases appreciate the values of the slip and race velocities and follow the variation of the specific function and the partial thrust. In all the following diagrams the parts of the curve corresponding to the vortex ring state of work are represented by dots.

In figure 9 are represented the functions v/V and $r\omega/V$. These functions have an asymptote in common for $az=1$, that is, $V=0$; the function $r\omega/V$ has also an asymptote parallel to the axis of ordinates for $az=\infty$; that is $(V+v)=0$. Both functions are equal to zero for $az=0$. The function $r\omega/V$ is equal to zero once more for $az=-1$; that is $(V+v')=0$. A maximum and minimum of the function $r\omega/V$ are fixed by the condition

$$\frac{d}{d(az)}\left[\frac{az\ (1+az)}{1-az}\right]=0$$

taking into account that ctg $(\beta'-\varphi)\cong 1$ for angles of attack having values not too close to 0° or $\pm 180°$. From the foregoing equation we find

$$\left(\frac{r\omega}{V}\right)_{\substack{max \\ min}}=\frac{(1\pm\sqrt{2})\ (2\pm\sqrt{2})}{\mp\sqrt{2}} \text{ for } az=1\pm\sqrt{2}$$

In Fig. 12 are represented the functions x and ρ. For $i=o$, the relative pitch x being smaller than unity has values near unity. The relative pitch x is equal to zero for $az=1$ and $az=\infty$ and goes through a maximum between the values of az which correspond to values of the angle of attack included between $i=i_o$ and $i=\varphi$. As by definition $x=V/NH$, the relative pitch can take the value zero only for $V=0$ or $N=$. But to the value $i=\varphi$ corresponds the beginning of the vortex ring state of work with $N=\infty$; as a consequence, to the value $i=i_o$ will correspond $V=0$, that is, the state of work at a fixed point. The relative pitch $x=\infty$, that is, admits an asymptote parallel to the axis of ordinates, for

$$az(1+az)\text{ ctg }(\beta'-\varphi)=\text{ctg }(i-\varphi)$$

This last relation gives two values for the angle of attack i, one positive, the other negative, which are approximate values of the angles of attack corresponding to the standing states, while for $x=\infty$ we have $N=0$ on account of the relation $x=V/NH$. It is easy to see that the angles of attack of the standing states have for exact values

$$i'_a=-\left(\frac{\pi}{2}-\varphi\right);\ i_a=\frac{\pi}{2}+\varphi$$

In the interval $i'_a<i<i_o$ the relative pitch takes positive values. In the interval $i_o<i<i_a$ the relative pitch takes negative values. For angles of attack whose difference from the preceding values are equal to 180° the relative pitch takes the same values. For values of i for which ctg $(\beta'-\varphi)\cong 1$ we have $\rho\cong x$, while we admit $\varphi=45°$. The specific function $\rho=\infty$, that is, admits an asymptote parallel to the axis of ordinates for ctg $(\beta'-\varphi)=0$, which corresponds to $i=-\epsilon$. In the interval $i'_a<i<-\epsilon$ the specific function ρ has a minimum greater than unity, and has a maximum less than unity in the interval $-\epsilon<i<i_o$.

In figure 10 are represented the functions ω/Ω and $v/r\Omega$. It is easy to see that for small values of az we have $\omega/\Omega\cong v/r\Omega$. These functions have the same asymptotes, parallel to the axis of ordinates, as the relative pitch x, and are equal to zero for $az=0$, that is, $i=\epsilon$. When i tends toward its value $i=\varphi$ the function ω/Ω tends toward unity, and the function $v/r\Omega$ tends toward zero. The function ω/Ω is equal to zero for $az=-1$.

In fig. 11 is represented the general course of the function q. This function is equal to zero for $az=0$ that is, $i=\epsilon$ and for $az=\infty$, that is, $i=\varphi$. This function has an asymptote parallel to the axis of ordinates for $az=1$, that is, $i=i_o$. This function takes positive values in the interval $\epsilon<i<\varphi$ and negative values in the interval $\epsilon>i>-(\pi-\varphi)$.

After these preliminary considerations we can pass to our general discussion.

On figure A the specific function ρ is represented as a function of the relative pitch x. On figure B is represented the complete system of states of work of the blade screw, whose continuous sequence we shall establish by the study of the specific function.[1]

We shall start our discussion from the moment when the screw rotates at a fixed point with the angular velocity Ω (see fig. B, 1). We have $V=0$. Under such conditions the blade screw can fulfill the functions of a *fan*, or a *helicoidal pump*, or be a *lifting screw* (helicopter screw). The relative pitch x and the specific function ρ are both equal to zero. The function az, as directly follows from relation (61), is equal to unity.

$$az=\frac{ak_t \sin (\beta'-\varphi)}{2\sin^2(i-\varphi)}=1. \tag{76}$$

This last relation fixes the value of the angles of attack of the blade elements considered, for the work of the screw at a fixed point. We will designate by i_o the angle of attack defined by the relation (76), as has already been mentioned in the foregoing. It is easy to see that this last value of the angle of attack is independent of the angular velocity Ω of the screw rotation.

The slip velocity in the indraught being equal to

$$v=r\Omega\,\frac{az}{1-az}\,x\ \mathrm{tg}\ \varphi=\frac{az\cdot r\Omega}{az(1+az)\ \mathrm{ctg}\ (\beta'-\varphi)-\mathrm{ctg}\ (i-\varphi)} \tag{77}$$

substituting $az=1$ we get

$$v_o=\frac{r\Omega}{2\ \mathrm{ctg}\ (\beta'-\varphi)-\mathrm{ctg}\ (i_o-\varphi)} \tag{78}$$

When the values of i_o and v_o are known, the relations (66) and (67) give the value of the partial thrust ΔQ and partial torque ΔC of a blade screw working at a fixed point.

We will designate by *fan velocity* the slip velocity in the section S'', that is $v''=2v$. It is to be noted that this fan velocity is in direct connection with the thrust. If the blade screw produces a thrust, there must necessarily be a fan velocity; and, inversely, when there is a fan velocity, there must be a thrust. Such a state of things is a direct consequence of the momentum theorem.

We will estimate the blowing effect of a blade screw by the quantity

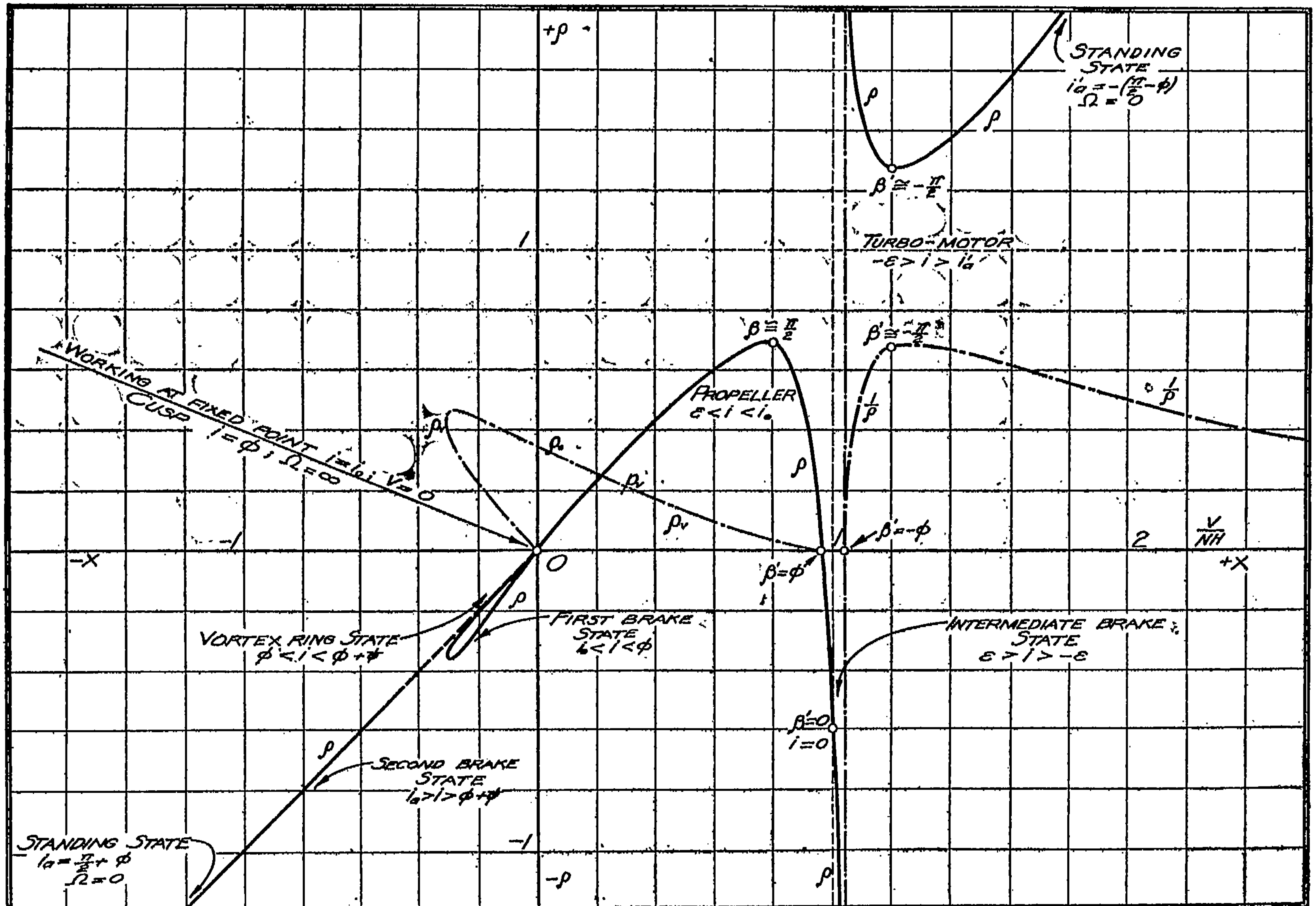
$$\rho_v=\frac{\tfrac{1}{2}\Delta Mv''^2}{\Omega\Delta C}=\frac{\tfrac{1}{2}\Delta Q2v\rho}{V\Delta Q}=\rho\,\frac{v}{V} \tag{79}$$

$$\rho_v=\rho\,\frac{az}{1-az}=\frac{az\ \mathrm{tg}\ (\beta'-\varphi)}{az(1+az)\mathrm{ctg}\ (\beta'-\varphi)-\mathrm{ctg}(i-\varphi)} \tag{80}$$

which we will call *fan efficiency*. In certain cases, when the blade screw is propulsive, for example, the fan efficiency represents in reality the *fan losses*, which we will in such cases designate by p_v. We shall take up this last question more in detail in the following. When a blade screw is working at a fixed point, the fan efficiency gives a valuation of the whole useful work produced by the screw, which exclusively consists in ventilation, or more generally in transfer of a fluid. In such cases we will designate the fan efficiency by ρ_o. Substituting in the relation (79) $az=1$, which corresponds to $V=0$, we get

$$\rho_o=\frac{\mathrm{tg}\ (\beta'-\varphi)}{2\ \mathrm{ctg}(\beta'-\varphi)-\mathrm{ctg}(i_o-\varphi)}=\frac{v_o}{r\Omega}\mathrm{tg}(\beta'-\varphi) \tag{81}$$

[1] For a better view of the general course of the specific function, some parts of it have been plotted on a larger scale in fig. B. In fig. 13 is given an exact drawing of the specific function in agreement with the foregoing diagrams. It must be noted that the extension of the different parts of the specific function curve depends upon the type of blade screw considered.



+ρ
-ρ
-X
+X
O
1
-1
2
V/NH
STANDING STATE
i'a = -(π/2 - φ)
Ω = 0
ρ
β' ≅ -π/2
TURBO-MOTOR
-ε > i > i'a
β ≅ π/2
PROPELLER
ε < i < i₀
1/ρ
WORKING AT FIXED POINT i = i₀ ; V = 0
CUSP i = φ ; Ω = ∞
ρ₀
ρᵥ
β = φ
β' = -φ
VORTEX RING STATE
φ < i < φ + ψ
FIRST BRAKE STATE
i₀ < i < φ
INTERMEDIATE BRAKE STATE
ε > i > -ε
β' = 0
i = 0
SECOND BRAKE STATE
ia > i > φ + ψ
STANDING STATE
ia = π/2 + φ
Ω = 0

FIG. A.

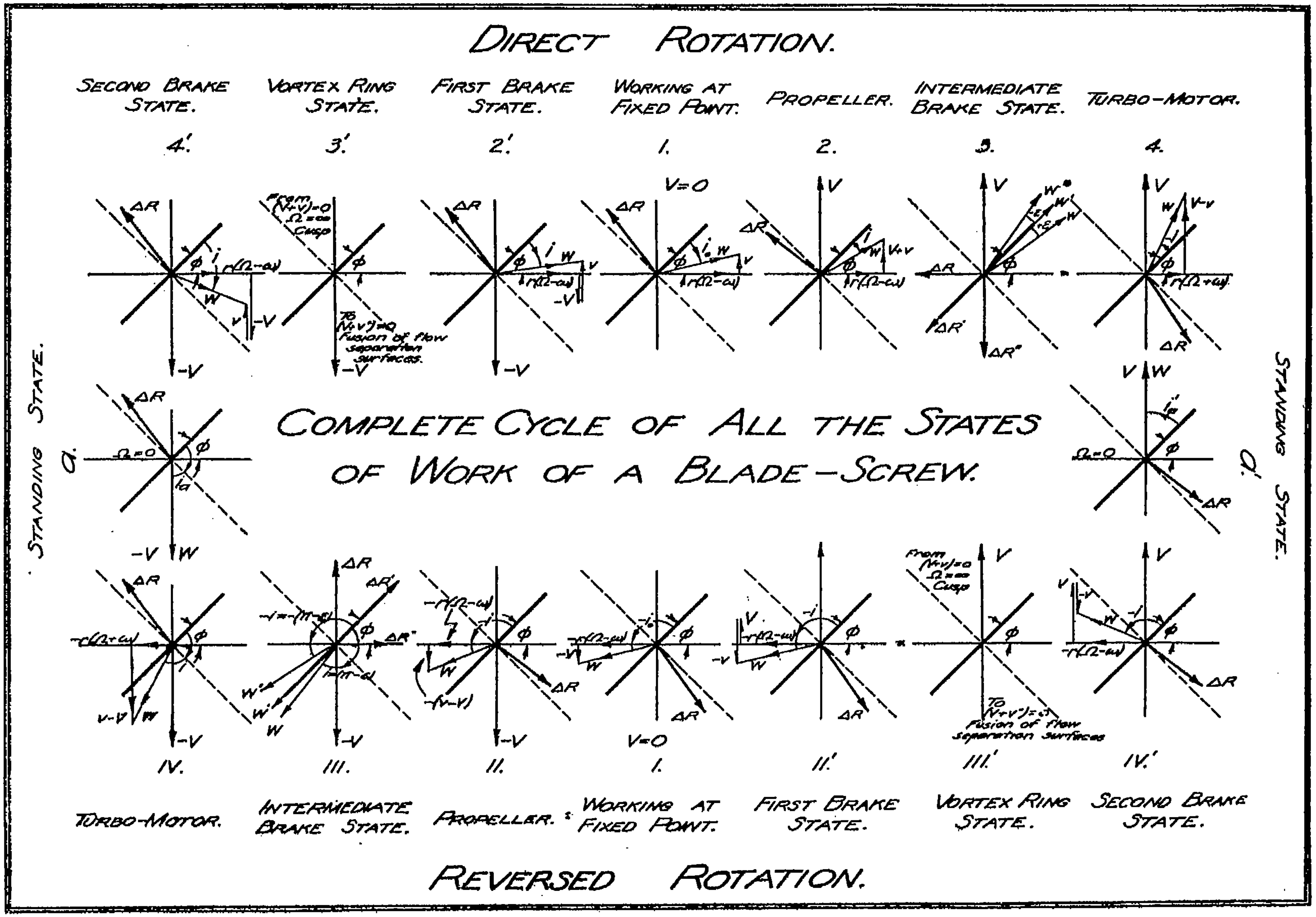

DIRECT ROTATION.
SECOND BRAKE STATE.
VORTEX RING STATE.
FIRST BRAKE STATE.
WORKING AT FIXED POINT.
PROPELLER.
INTERMEDIATE BRAKE STATE.
TURBO-MOTOR.
4.' 3.' 2.' 1. 2. 3. 4.
V=O
COMPLETE CYCLE OF ALL THE STATES OF WORK OF A BLADE-SCREW.
STANDING STATE.
O.
IV. III. II. I. II.' III.' IV.'
V=O
TURBO-MOTOR.
INTERMEDIATE BRAKE STATE.
PROPELLER.
WORKING AT FIXED POINT.
FIRST BRAKE STATE.
VORTEX RING STATE.
SECOND BRAKE STATE.
REVERSED ROTATION.

FIG. B.

The state of work of the blade screw at a fixed point is schematically represented in figure B, 1. On the curve of the specific function (see fig. A) the origin ($x=o$, $\rho=o$) is the representative point of the work at a fixed point. In this same figure A, I have represented the curve of the fan efficiency ρ_v as a function of the relative pitch.

Let us now allow the blade screw to take a translatory motion in the sense of its thrust. The blade screw will become an *helicoidal propeller*. The specific function will represent its efficiency. As the velocity V goes on increasing, the relative pitch, starting from zero value, will take positive values. The angle of attack i will go on decreasing; the function az will remain positive, but less than unity. As long as the angle of attack remains in the interval for which β' has values near to $\pi/2$, the efficiency ρ will be nearly equal to the relative pitch x, as directly follows from relation (63). But, when we reach the interval of values of the angles of attack i for which β' decreases rapidly, the specific function ρ, after having reached a maximum always less than unity, will rapidly decrease. This maximum of the specific function corresponds to the maximum of the propeller efficiency. The propulsive state of work of the blade screw will end when the specific function retakes the zero value, by the fact that the partial thrust ΔQ becomes equal to zero. At this moment $\beta'=\varphi$ and the angle of attack has the very small positive value ϵ. The function az is equal to zero. The relative pitch x has a value very near unity but a trifle less. In effect, from the relation (62) we directly find:

$$x=\frac{1-az}{\left[\frac{ak_t \sin(\beta'-\varphi)}{2\sin^2(i-\varphi)}(1+az)\frac{\cos(\beta'-\varphi)}{\sin(\beta'-\varphi)}-\operatorname{ctg}(i-\varphi)\right]\operatorname{tg}\varphi}$$

and substituting $\beta'=\varphi$; $az=o$; $i=\epsilon$ we get:

$$(82)\qquad x=\frac{1}{\left[\frac{a\,k_t}{2\sin^2(\epsilon-\varphi)}-\operatorname{ctg}(\epsilon-\varphi)\right]\operatorname{tg}\varphi}\cong\frac{\operatorname{tg}(\varphi-\epsilon)}{\operatorname{tg}\varphi}$$

It is thus seen that the propulsive state of work of the blade screw is included in the interval

$$(83)\qquad 0<x<1$$

in which

$$(84)\qquad 0<az<1;\ \epsilon<i<i.$$

$$(85)\qquad 0<\rho<1$$

The propulsive state of work of the blade screw is schematically represented in figure B, 2. It is easy to recognize on figure A that part of the specific function which corresponds to the efficiency of the propulsive screw. If the point $x=1$ on the axis of abscissae is adopted as origin, and the inverse sense of this axis taken as positive, the specific function will then represent the well-known curve of the propeller efficiency as a function of the slip $s=1-x$.

When the angle of attack decreases, starting from the value $i=\epsilon$, the relative pitch x will remain positive and will go on increasing; and from the propulsive state of work of the screw we will fall into a very short *intermediate state of brake work*, which will bring us asymptotically to the turbo-motor state of work of the blade screw. This intermediate state of brake work corresponds to very small variations of the angle of attack from $i=\epsilon$ to $i=-\epsilon$ (considering $\varphi=45°$), the angle β' varying from $\beta'=\varphi$ to $\beta'=-\varphi$ (see fig. B, 2 and 3). The value $x=1$ (for $i=0$ and $\beta'=0$ is thus included in this intermediate state (see fig. A). For $\beta'=-\varphi$ we have $\operatorname{ctg}(\beta'-\varphi)=0$, $\Delta C=0$, $\rho=\pm\infty$. The branch of the specific function corresponding to the intermediate brake state has an asymptote parallel to the axis of ordinates. The value of the relative pitch, abscissa of this asymptote, although greater than unity, is, however, near to unity, and its value is obtained by setting $i=\epsilon$ in the relation (62).

At the right-hand side of the asymptote just described is disposed the branch of the specific function which from positive infinity quickly reaches a minimum greater than unity—as directly follows from the equation (63)—and takes afterwards values increasing up to infinity, by a parabolic branch nearly rectilinear and bisecting the angles of the positive axes of coordinates, while for $\beta' \cong -\pi/2$ we have $x \cong \rho$ (see fig. A). This branch of the specific function corresponds to the *turbo-motor state* of work of the blade-screw, schematically represented in figure B, 4. In this interval the specific function is equal to the inverse of the efficiency ρ_T of the turbo-motor.

(86) $$\rho_T = \frac{1}{\rho}$$

The curve of the efficiency ρ_T is represented by dots in figure A. For the study of the turbo-motor state it is more convenient to consider the efficiency $\rho_T = 1/\rho$ as a function of $x_T = 1/x$. The curve of ρ_T will then be like the curve of the efficiency ρ of the propulsive screw. In the study of the turbo-motor state of work we will use these last variables. The turbo-motor state of work is ended by the stoppage of the blade screw (see fig. B, a'). This takes place when the torque of the resistance applied to the turbo-motor axis becomes equal to the turbo-motor torque. At this moment

(87) $$x=\infty;\ \rho=\infty;\ x_T=0;\ \rho_T=0$$

We thus see that the turbo-motor state is included in the interval:

(88) $$1<x<\infty;\ 0<x_T<1$$

in which

(89) $$0>az>-1;\ -\epsilon>i>-\left(\frac{\pi}{2}-\right)$$

(90) $$1<\rho<\infty;\ 0<\rho_T<1$$

If we now apply to the turbo-motor axis a power and oblige it to rotate in the inverse sense, the blade screw will be transformed into a hydraulic brake (see fig. B, IV′). To this last state of work, included in the interval

(91) $$-\infty<x<0$$

corresponds that part of the specific function curve which from negative infinity by a nearly rectilinear branch, bisecting the angle of the negative axes of coordinates, is directed toward the origin.

Let us now return to the screw working at a fixed point and oblige it to take a translatory motion in the sense inverse to its thrust. The blade-screw will produce a braking action (see fig. B, 2′). The relative pitch x and the specific function will take negative values whose absolute magnitude will at first increase; the curve of the specific function will nearly follow the bisectrix of the angle of the negative axes of coordinates, because β' has values near to $\pi/2$; but, as in this interval az is a function increasing up to infinity, ρ and x, after having reached a maximum in magnitude, will retake zero values. In fact, dividing the relation (62) by az we get:

$$x=\frac{\dfrac{1}{a^2z^2}-\dfrac{1}{a\,z}}{\operatorname{tg}\varphi\left[\left(\dfrac{1}{az}+\quad\right)\operatorname{ctg}(\beta'-\varphi)-\dfrac{1}{a^2z^2}\operatorname{ctg}(i-\varphi)\right]}$$

which expression, for $\alpha z = \infty$, is equal to zero. But after having reached zero values ρ and x retake negative values. We thus see that in this interval, the specific function describes a loop, reaches the origin by a cusp and by a parabolic branch nearly rectilinear and bisecting the angles of the negative axes of coordinates, goes to negative infinity (see fig. A). When the specific function describes the loop, we find ourselves in the *first brake state*, characterized by the formation of two surfaces of flow separation. (See fig. B, 2′.) The cusp corresponds to the *whirling phenomenon* mentioned in the foregoing, characterized by the disappearing of the brake action and the tendency of the blade-screw to take an infinite rotation. Afterwards the *vortex ring state* establishes itself, during which takes place the change of the sense of the fluid current crossing the blade-screw. (See fig. B, 3′.) The vortex ring state is ended by the fusion and disappearing of the surfaces of flow separation, after which a *second brake state* establishes itself. (See Fig. B, 4′.) If it is the screw that has a translatory motion, we have to do with a braking action as in the case of ship propellers. If it is a fluid current that is directed on the blade-screw we have to do with a hydraulic brake. The second brake state finishes by the stoppage of the screw with $\Omega = o$; $x = \rho = \pm\infty$. (See fig. B, a.) If we now continue to move the blade-screw in the same sense, or direct on the screw in the inverse sense a fluid current, and allow the screw to take a rotation in an inverse sense, we fall once more into the turbo-motor state, but only with a rotation in inverse sense. (See fig. B, IV.) The two stoppage states a' and a thus separate the states of work with rotation in one sense from the states of work with rotation in the inverse sense. The states of work of reversed rotation are represented in figure B. They constitute, as it were, a picture as reflected in a mirror of the states of work of direct rotation, and they close the complete cycle of all the states of work which a blade-screw can run through. In the case of the blade angle $\varphi = 45°$, the states of work of direct rotation are quantitatively identical with the states of work of reversed rotation. In the general case the states of work of direct rotation will be only qualitatively like the states of work of reversed rotation.

If we now look back to the foregoing discussion, the following picture appears: The complete cycle of the states of work which a blade screw can run through consists of seven states of direct rotation and seven states of reversed rotation, separated by the standing states.[1] The states of reversed rotation constitute, as it were, a reflected image of states of direct rotation. Figure B gives a schematical representation of the complete cycle of these states of work. The specific function unites into a continuous whole all this system of states of work of the blade screw. *The zero and infinite values of the specific function separate the different states of work one from the other. The maxima and minima of the specific function indicate the most favorable working conditions of the blade screw in the corresponding states.*

I shall finish this chapter by mentioning two very interesting cases of blade-screw working which at first glance may appear rather paradoxical.

Let us consider a blade screw with a constructive pitch equal to infinity, whose blades have their sides of different configuration. (See fig. 14.) It is evident that the rotation of such a screw at a fixed point will produce no thrust. But it is sufficient to communicate to such a screw a translation in one sense or in the other to get a thrust. The propulsive thrust will appear from the moment when the velocity W has such an incidence on the zero line that the fluid resistance ΔR will be disposed on the same side of the screw rotation plane as W. With the notations of figure 14 we will have a propulsive thrust as soon as the angle of attack has a value greater than the one which corresponds to

$$\beta' = \frac{\pi}{2} - \alpha$$

[1] Exactly speaking to these 16 states of work has to be added a 17th state; it is the one with $V=O$; $\Omega=O$ disposed between the two states of work at a fixed point with direct and reversed rotation. The complete cycle of states of work of a blade screw is thus a double cycle.

Let us consider again a screw with blades of different configuration on its sides, but with a constructive pitch equal to zero. (See fig. 15.) It is evident that such a screw disposed in a fluid current parallel to its axis will take no rotation. But it is sufficient to communicate to such a screw a rotation in one sense in order for the screw to remain rotating in that sense. The blade screw will become a turbo-motor from the moment when the angle of attack takes such values that W and ΔR are both disposed on the same side of the plane of the screw rotation. Those values of the angle of attack depend upon the disposition of the zero plane relative to the blade sections considered. The working of blade screws under this last condition is known under the name of *autorotation* [1] and has been observed by several experimenters.

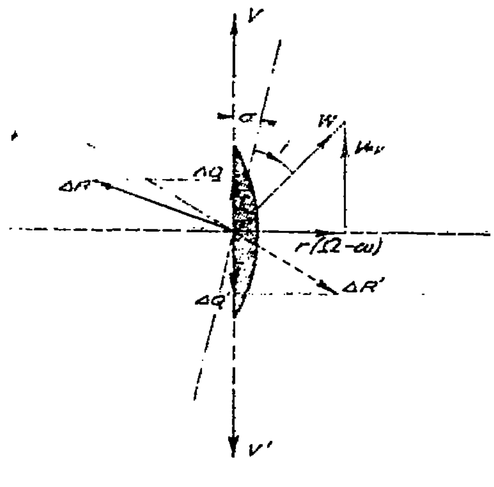


FIG. 14.

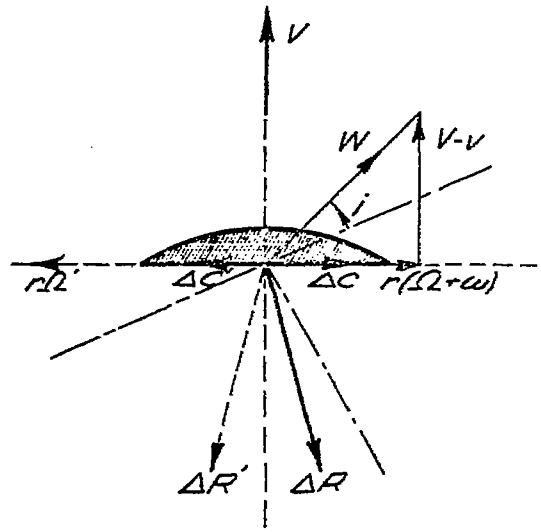


FIG. 15.

Our system of fundamental equations easily embraces these two cases of work of a blade screw and allows their complete quantitative study. These two cases of blade screw work are particularly fitted to show the great importance of the effective pitch. In the cases considered, the constructive pitches have values equal to zero and infinity, but the effective pitches have finite values, and there is nothing paradoxical in these cases.

After this general review of the phenomenon of working of a blade screw, we will pass to the special quantitative study of the different working states which can take place for a blade screw; we shall begin with those states of work which are the most important owing to their technical applications.

[1] See "La Technique Aeronautique," Tome I, No. 3, p. 108, 1910.

CHAPTER III.

THE STUDY OF THE PROPULSIVE SCREW.

For the study of propulsive screws or propellers it is more convenient to use the angle β_H which we will for simplicity designate by β in all this chapter. In the first chapter we have established the following system of formulæ:

$$az = \frac{v}{V+v} = \frac{aki\cos(\varphi+\beta)}{2\sin^2(\varphi-i)} \cong \frac{aki\cos(\varphi+\beta)}{2\sin^2(\varphi-i)}; \tag{92}$$

$$x = \frac{V}{NH} = \frac{(1-az)\operatorname{tg}(\varphi-i)}{\operatorname{tg}\varphi[1+az(1+az)\operatorname{tg}(\varphi+\beta)\operatorname{tg}(\varphi-i)]}; \tag{93}$$

$$\rho = \frac{V\Delta Q}{\Omega\Delta C} = \frac{x\operatorname{tg}\varphi}{\operatorname{tg}(\varphi+\beta)} = \frac{(1-az)\operatorname{tg}(\varphi-i)}{\operatorname{tg}(\varphi+\beta)[1+az(1+az)\operatorname{tg}(\varphi+\beta)\operatorname{tg}(\varphi-i)]}; \tag{94}$$

$$v = V\frac{az}{1-az} = r\Omega\frac{az}{1-az}\rho\operatorname{tg}(\varphi+\beta); \tag{95}$$

$$r\omega = \frac{az(1+az)}{1-az}V\operatorname{tg}(\varphi+\beta) = r\Omega\frac{az(1+az)}{1-az}\rho\operatorname{tg}^2(\varphi+\beta); \tag{96}$$

$$\Delta Q = 2\delta\Delta Sv(V+v) = q\delta\Delta SV^2; \tag{97}$$

$$q = \frac{2az}{(1-az)^2}; \tag{98}$$

$$\Delta C = \Delta Qr\operatorname{tg}(\varphi+\beta); \tag{99}$$

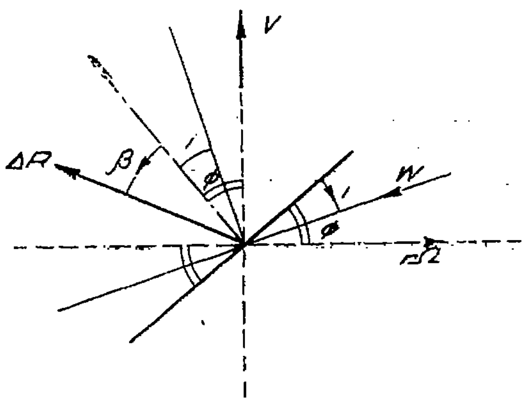


FIG. 16.

In the second chapter we have introduced the notion of fan efficiency

$$\rho_v = \frac{\frac{1}{2}\Delta Mv''^2}{\Omega\Delta C} = \rho\frac{az}{1-az} = \frac{az\operatorname{tg}(\varphi-i)}{\operatorname{tg}(\varphi+\beta)[1+az(1+az)\operatorname{tg}(\varphi+\beta)\operatorname{tg}(\varphi-i)]} \tag{100}$$

Introducing this quantity in the formulæ (95) and (96) we get

$$v = r\Omega\rho_v\operatorname{tg}(\varphi+\beta) \tag{101}$$

$$r\omega = (1+az)r\Omega\rho_v\operatorname{tg}^2(\varphi+\beta) \tag{102}$$

For the case of a propeller the fan efficiency ρ_v represents the fan losses p_v. We thus have

(103) $$p_v = \rho_v$$

For the propeller working at a fixed point we have $V=0$ and consequently

(104) $$az=1$$

as directly follows from relation (92). The condition (104) can also be written [1]

(105) $$a=\frac{nb}{2\pi r}=\frac{2\sin^2(\varphi-i_o)}{k_i\cos(\varphi+\beta_o)}\cong\frac{2\sin^2(\varphi-i_o)}{ki_o\cos(\varphi+\beta_o)}$$

This last relation defines the angle of attack of the blade element considered for the blade screw working at a fixed point. As this last relation does not contain the angular velocity Ω_o we are brought to the following important theorem, which gives the fundamental characteristic of the fixed point screw working:

THEOREM III.—*When a blade screw is working at a fixed point, the angles of attack of all the blade sections have constant values independent of the angular velocity of the screw rotation.*

We are thus brought to the conclusion that for a blade screw working at a fixed point all the quantities that are functions only of the angles of attack of the different blade sections keep constant values independent of the variations of the screw rotation.

In the second chapter it was also mentioned that for the fixed point screw working the fan efficiency gives the evaluation of the whole useful action produced by a blade screw, which consists in blowing, or, more generally, in transfer of a fluid. Substituting in the formulæ (100), (101), and (102) $az=1$ and replacing ρ_v by ρ_o we get:

(106) $$\rho_o=\frac{\operatorname{tg}(\varphi-i_o)}{\operatorname{tg}(\varphi+\beta_o)\,[1+2\operatorname{tg}(\varphi-i_o)\operatorname{tg}(\varphi+\beta_o)]};$$

(107) $$v_o=r\Omega_o\rho\operatorname{tg}(\varphi+\beta_o)$$

(108) $$r\omega_o=2r\Omega_o\rho_o\operatorname{tg}^2(\varphi+\beta_o)$$

We thus see that *the partial efficiency ρ_o of a blade screw at a fixed point has a constant value independent of the angular velocity Ω_o and that the slip and race velocities v_o and $r\omega_o$ are proportional to the angular velocity Ω_o.* The slip stream created by the blade-screw rotation at a fixed point remains thus similar to itself independent of the angular velocity of the screw. The configuration of the stream lines of the slip stream remains thus invariable relative to the screw axis; and it is only the velocities along these stream lines which vary proportionally to the angular velocity of the blade screw.

The values of the partial thrust and partial torque of a blade screw working at a fixed point are given by (see the relations (97) and (99)):

(109) $$\Delta Q_o=2\delta\Delta S v_o^2=2\delta\Delta S r^2\Omega_o^2\rho_o^2\operatorname{tg}^2(\varphi+\beta_o)$$

(110) $$\Delta C_o=2\delta r\Delta S r^2\Omega_o^2\rho_o^2\operatorname{tg}^3(\varphi+\beta_o)$$

and it is easy to see that we have

(111) $$\rho_o=\frac{\frac{1}{2}\Delta M v''^2}{\Omega_o\Delta C_o}=\frac{v_o\Delta Q_o}{\Omega_o\Delta C_o}=\frac{v_o}{r\Omega_o}\,\frac{1}{\operatorname{tg}(\varphi+\beta_o)}$$

[1] All the quantities relating to the work of a blade screw at a fixed point are marked by a sub zero.

From this last relation it directly follows that for the evaluation of the useful action of a blade screw at a fixed point, the slip velocity v_o plays the same rôle as the velocity V of a propulsive screw.

The expression (97) of the partial thrust becomes indeterminate for $V=0$ because we have $V^2. q=0$. It is why, when we have to follow the work of a propulsive screw up to the fixed point, it is more convenient to consider another form of the partial thrust which can be obtained from the expression (97), putting in evidence in it the angular velocity Ω. We have:

$$\Delta Q = q\delta\Delta S\, V^2 = q\delta\Delta S \frac{V^2}{r^2\Omega^2} r^2\Omega^2;$$

but as

$$\frac{V}{r\Omega} = x \operatorname{tg} \varphi$$

we have

$$q\frac{V^2}{r^2\Omega^2} = \frac{2az \operatorname{tg}^2 (\varphi - i)}{[1 + az\,(1 + az) \operatorname{tg} (\varphi + \beta) \operatorname{tg} (\varphi - i)]^2} \tag{112}$$

and on account of the relation (100)

$$q\frac{V^2}{r^2\Omega^2} = 2\frac{\rho_2^2}{az} \operatorname{tg}^2 (\varphi + \beta) \tag{113}$$

We thus finally get

$$\Delta Q = 2\delta\Delta S r^2\Omega^2 \frac{\rho_2^2}{az} \operatorname{tg}^2 (\varphi + \beta) \tag{114}$$

This last relation goes directly over into the expression (109) for $V=0$ ($az=1$; $i=i_o$).

Adopting the notation

$$q' = \frac{\rho_2^2}{az}\operatorname{tg}^2(\varphi + \beta) = \frac{az \operatorname{tg}^2(\varphi - i)}{[1 + az(1 + az) \operatorname{tg} (\varphi + \beta) \operatorname{tg} (\varphi - i)]^2} \tag{115}$$

we get

$$\Delta Q = 2q' \delta\Delta S r^2\Omega^2 \tag{116}$$

$$\Delta Q_o = 2q'_o \delta\Delta S r^2\Omega_o^2 \tag{117}$$

with

$$q'_o = \rho_o^2 \operatorname{tg}^2 (\varphi + \beta_o) = \frac{\operatorname{tg}^2 (\varphi - i_o)}{[1 + 2 \operatorname{tg} (\varphi + \beta_o) \operatorname{tg} (\varphi - i_o)]^2} \tag{118}$$

I will limit myself here to these brief general considerations concerning the work of a blade screw at a fixed point, which we will need for the following developments of this chapter, whose main subject is the propulsive screw. The working of a blade screw at a fixed point will be submitted by us to a separate detailed and complete study.

We shall begin the investigation of the propeller by the consideration of its losses. I divide these losses into three kinds:

I. The fan losses p_v.

II. The vortex losses p_t.

III. The resistance losses p_r.

The total losses will be the sum of the foregoing losses:

$$p = p_v + p_t + p_r \tag{119}$$

I call fan losses the ratio to the total power absorbed by the screw of the kinetic energy of the translatory motion of the fluid in the slip stream communicated to it by the screw. As has already been mentioned, the fan losses which correspond to the blade elements situated at a distance r from the screw axis are equal to:

$$p_v = \frac{\frac{1}{2}\Delta M v''^2}{\Omega \Delta C} = \rho \frac{v}{V} = \rho \frac{az}{1-az} = \frac{az \operatorname{tg}(\varphi - i)}{\operatorname{tg}(\varphi+\beta)[1+az(1+az)\operatorname{tg}(\varphi+\beta)\operatorname{tg}(\varphi-i)]} \tag{120}$$

I call vortex losses the ratio to the total power absorbed by the screw of the kinetic energy of the rotational motion of the fluid in the slip stream communicated to it by the screw. The vortex losses which correspond to the blade elements situated at a distance r from the screw axis are equal to

$$p_t = \frac{\frac{1}{2}\Delta I'' \omega''^2}{\Omega \Delta C} = \frac{\frac{1}{2}\Delta C . 2\omega}{\Omega \Delta C} = \frac{\omega}{\Omega} = \rho \frac{az(1+az)}{1-az} \operatorname{tg}^2(\varphi+\beta). \tag{121}$$

I call resistance losses the ratio to the total power absorbed by the screw of the power spent in the displacement of the blades themselves in the fluid. We shall obtain the resistance losses which correspond to the blade elements situated at a distance r from the screw axis by taking the difference between the total losses and the fan and vortex losses.

$$p_r = p - (p_v + p_t) \tag{122}$$

$$p_r = 1 - \rho - \rho \frac{az}{1-az} - \rho \frac{az(1+az)}{1-az} \operatorname{tg}^2(\varphi+\beta)$$

$$p_r = \rho \frac{\operatorname{tg}(\varphi+\beta) - \operatorname{tg}(\varphi - i)}{(1-az)\operatorname{tg}(\varphi - i)} \tag{123}$$

It is easy to see, as directly follows from the relations (113), (114) and (116), that all the quantities characterizing the screw working can be expressed as functions of the losses only. Let us for example calculate the load coefficient q as a function of the losses. From relation (113) we get directly:

$$az = \frac{p_v}{\rho + p_v} \tag{124}$$

and substituting this last value of az in the relation (98) we find:

$$q = 2\frac{p_v}{\rho}\left(1 + \frac{p_v}{\rho}\right) \tag{125}$$

This last relation shows us that a propeller of high efficiency must necessarily have a small load coefficient. For example, for $\rho = 0.8$ and $p_v \cong 0.08$ we have $p_v/\rho \cong 0.1$ and $q \cong 0.2$. It has always been experimentally noted that high efficiency propellers have values of the load coefficient near that obtained above.

Let us now examine the conditions of the maximum of the partial efficiency ρ of a blade element of a propeller. The maximum of the efficiency (see relation (94)) depends upon the course of the empirical functions β and k_i. But if we note that in the propulsive interval of the screw we have $\rho \cong x$ as long as $\beta \cong 0$, that is, for angles of attack $i > i'$ (1) and that afterwards β is a rapidly increasing function for $i < i'$, it is easy to see that the maximum of ρ takes place for values of β and i near $\beta = 0$ and $i = i'$. We will call optima angle of attack and designate by

(1) I designate by i' the angle of attack for which the fluid resistance ΔR is normal to the zero plane. See Note III at the end of this memoir.

$i_{op} \cong i'$, the angle of attack for which the partial efficiency ρ is a maximum. Under such conditions we can consider, as a first approximation,

$$\rho_{\max} \cong x \cong \frac{(1-az)\operatorname{tg}(\varphi - i')}{\operatorname{tg}\varphi[1+az(1+az)\operatorname{tg}\varphi\operatorname{tg}(\varphi-i')]} \tag{126}$$

with

$$az = \frac{ak_{t'}\cos\varphi}{2\sin^2(\varphi - i')}$$

We thus see that as a first approximation we have

$$p = 1 - \rho_{\max} = 1 - x = s$$

THEOREM IV.—*When a blade element of a propulsive screw is working under conditions near its maximum efficiency, its slip is nearly equal to its total losses, and its relative pitch is nearly equal to its efficiency.*

It is now easy to understand why in propeller practice only screws of low slip show high efficiency.

Let us now examine how $\rho_{\max}$ varies with the blade angle φ and the angle of attack i_{op}. We shall see in the following that high values of $\rho_{\max}$ are only possible for low values of the optima angle of attack. Under such conditions the function az will have a low value, of the order of a small number of hundredths; that is why, for a first orientation in the actual question, we can neglect az in the expression of $\rho_{\max}$ and thus admit

$$\rho_{\max} \cong \frac{\operatorname{tg}(\varphi - i_{op})}{\operatorname{tg}\varphi} \tag{127}$$

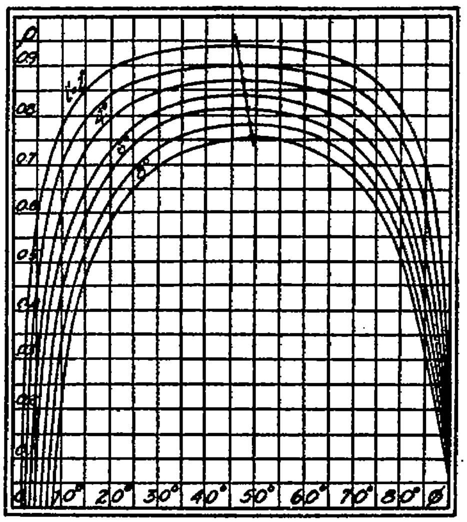


FIG. 17.

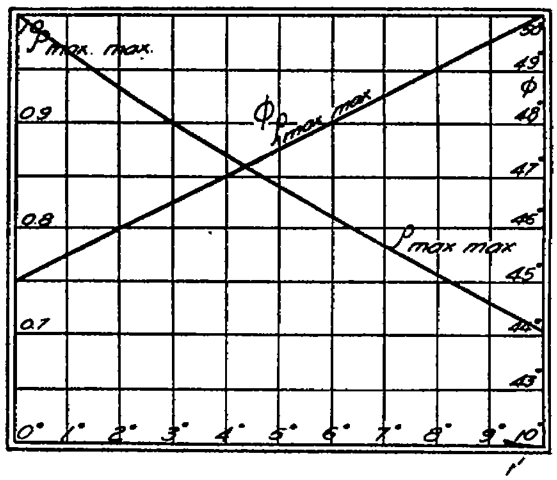


FIG. 18.

In figure 17 have been represented curves of the partial efficiency $\rho_{\max}$ as function of the effective blade angle φ for different values of the optima angle of attack i_{op}. It is easy to see from this diagram that the smaller the values of i_{op} the higher are the maximums of $\rho_{\max}$, and that the maximums of $\rho_{\max}$ occur for values of φ near 45°. In figure 18 are given the values of $\rho_{\max.\ \max.}$ as function of i_{op}, and there are also represented the corresponding values of φ. An examination of diagram 17 brings us to the following rule which must be used for the choosing of the profiles to be adopted for screw-blade sections.

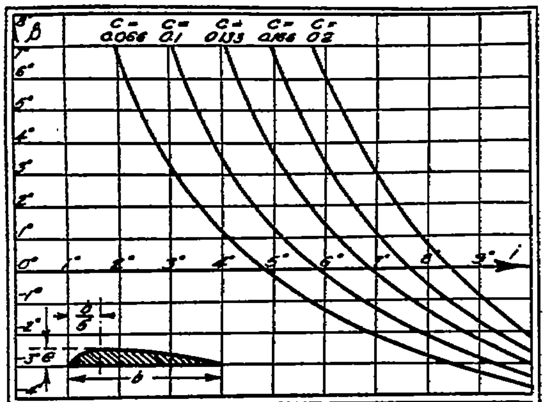
For the sections of screw blades there must be adopted profiles whose optima angles of attack are as small as possible.

This rule allows us to see directly the partial efficiency $\rho_{max.\ max.}$ as a first approximation that can be expected from a given profile. To give a general idea of the values which the optima angles of attack can have, in figure 19 is represented a series of β curves as functions of i, for the case of air screws, for plano-convex profiles whose ratios c of the thickness e to the breadth b are increasing.

(128) $$c=\frac{e}{b}$$

By aid of figure 19 was established figure 20, which gives for the profiles considered the angles i_{op} as functions of c. In the same figure is represented the curve of the angles γ of the zero lines of the profiles considered with the corresponding chords.[1]

FIG. 19.



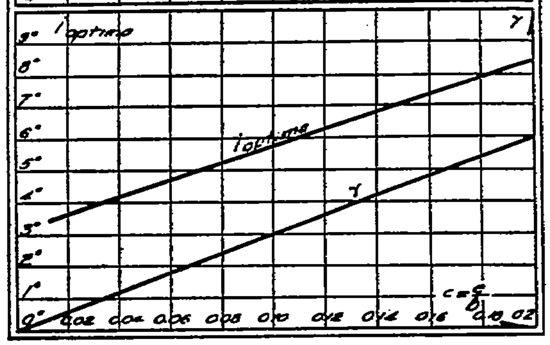


FIG. 20.

On account of the fact that the values of the ratio c go on necessarily decreasing from the boss to the tip of the blade, the optima angles of attack must also go on decreasing from boss to blade tip. It thus follows, according to diagram 17, that the blade elements, whose blade angles φ are a little smaller than 15° or larger than 75°, necessarily have small partial efficiencies ρ_{max}. According to the last, and on account of the relation $H=2\pi r \operatorname{tg} \varphi$ we can give ourselves a general idea of the limits between which must be included the portion of the blade which gives high partial efficiencies:

(129) $$\begin{cases} r_m \cong \dfrac{H}{2\pi r \operatorname{tg} 75^\circ} \cong 0,05\ H^{2} \\ r_m \cong \dfrac{H}{2\pi r \operatorname{tg} 15^\circ} \cong 0,6\ H \end{cases}$$

[1] These diagrams were established using the data furnished by G. Eiffel "Complements de la Premiere Edition de la Resistance de l'Air et l'Aviation," p. 15. As these data have been obtained at low velocity they are not of sufficient approximation to be used in propeller design. As is well known, the drag coefficients K_x decrease for large flow velocities under which the elements of propeller blades are generally working. There fore diagram 20 must give exaggerated values for the optima angles of attack.

[2] The demands of practice often require larger sizes to be adopted for the boss than are given by this relation.

These last relations bring us to the conclusion that *a propeller of high efficiency must have its diameter of the same order of magnitude as the effective pitch of its tip blade section.* This remark gives a solution to the question of the number of blades to be adopted for a screw.[3] For the preliminary design of a screw, the condition (126) fixes the effective pitch H of the blade section considered. We have

$$\rho \cong x = \frac{V}{NH} = \frac{\mu}{H}$$

The value of the effective pitch appears thus to depend upon the efficiency expected, and depends upon the power absorbed only so far as the ratio $c = e/b$ depends upon this power. It is the blade area A and the screw diameter D which depends upon this power. If the diameter is considered fixed by the relation

$$D = 2r_m \cong 1,2H$$

it will be sufficient, for a given power, to adopt as many blades of a length of the order of r_m, as will be necessary to absorb the whole power. The limit to the number of blades is given by the following considerations:

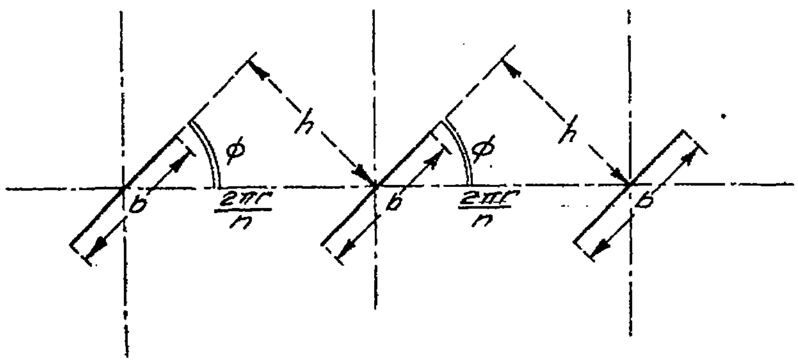


FIG. 21.

Let us cut the screw blades by a cylinder coaxial with the screw axis, and let us develop in the plane the sections obtained. We will thus get the general picture represented in figure 21, where we have designated by h the distances between the zero lines of the blade sections considered. By analogy to what we know about fluid resistance of systems of aerofoils, the blade interference will occur only from the moment when the ratio

$$\frac{b}{h} = \nu \tag{130}$$

becomes smaller than a certain limiting value to be fixed by experiment. Actually we do not possess any experimental indications of the limiting values for ν in the case of screw blades. For a first orientation in the question let us adopt

$$\nu \geq 1 \tag{131}$$

which will bring us to the conclusion that for

$$h \geq b \tag{132}$$

[3] In his air-screw investigations S. Drzewiecki (see "Helices Aeriennes," Paris, 1909), also reaches the conclusion that there exists a limit to be advantageously used for the length of screw blades, and that the number of blades to be adopted for a screw depends upon this limiting length.

an absence of screw-blade interference is to be expected. If we note that

$$h = \frac{2\pi r}{n} \operatorname{tg} \varphi \tag{133}$$

the condition of absence of screw-blade interference will take the form

$$\frac{nb}{2\pi r} = a \leq \sin \varphi \tag{134}$$

which means that *under the assumptions made for absence of screw-blade interference the breadth ratio must be smaller than the sine of the effective blade angle.* This last condition can also be written as follows:

$$nb \leq 2\pi r \sin \varphi \tag{135}$$

We will designate the product of the number n of blades by their breadth b at a certain distance from the screw axis by *total breadth.*

In the general case, without assuming the value of the coefficient ν, for the absence of screw-blade interference, we find the conditions [1]

$$h > \nu b \tag{136}$$

$$\frac{\nu n b}{2\pi r} = \nu a \leq \sin \varphi \tag{137}$$

$$nb \leq \frac{2\pi r}{\nu} \operatorname{tg} \varphi \tag{138}$$

When it is difficult to realize the condition (135), or, more generally, the condition (138), attempts will be made, however, to approach them as near as possible. But since, on the one hand, as is well known, the maximum breadth b_M of the blades must be smaller than a certain fraction of the screw diameter, and, on the other hand, the screw blades are working in a stream quite well limited, in all probability the values to be adopted for the maximum breadth b_M can be quite large. The limiting value which will be adopted for the total breadth nb_M and the maximum breadth b_M will fix the limiting number of blades.

Since for a screw of high efficiency there exist superior limits for the diameter D, the number of blades n and their maximum breadth b_M, the thrust power, which can be obtained from a propeller under given conditions, must also have a superior limit. If one tries to give to D, n, and b_M values higher than the limiting values, only the absorbed power—that is, the torque power—will be increased, but the rapid decrease of the efficiency will lower the thrust power developed by the propeller.

---

[1] Some elementary considerations allow us to establish for the limiting value h_0 of h, for absence of screw-blade interference the formula

$$h_0 \cong b \frac{k_y^2}{2k_x}$$

which gives a value of ν equal to

$$\nu \cong \frac{k_y^2}{2k_x}$$

This last formula gives a general idea of the value of the coefficient ν.

For the realization of screws of large power, one can go beyond the limit

$$\nu a \leq \sin \varphi$$

At the beginning we will have only a small decrease of efficiency, but will be able to increase the power used.

Making a summary of the foregoing discussion, the following rules can be formulated for high efficiency propellers.

I. *Each blade section must work under an angle of attack near the optima angle. For blade sections we must adopt such profiles that their optima angles of attack are as small as possible.*

II. *The screw diameter must be of the same order of magnitude as the effective pitch of the tip blade section.*[1]

III. *The total blade breadth in each blade section must not exceed a value fixed by the limiting value of the breadth ratio (condition (137)).*

IV. *The maximum blade breadth must not exceed a certain fraction of the diameter.*

V. *For given working conditions there exists a limiting value of the thrust power which a propeller can develop.*

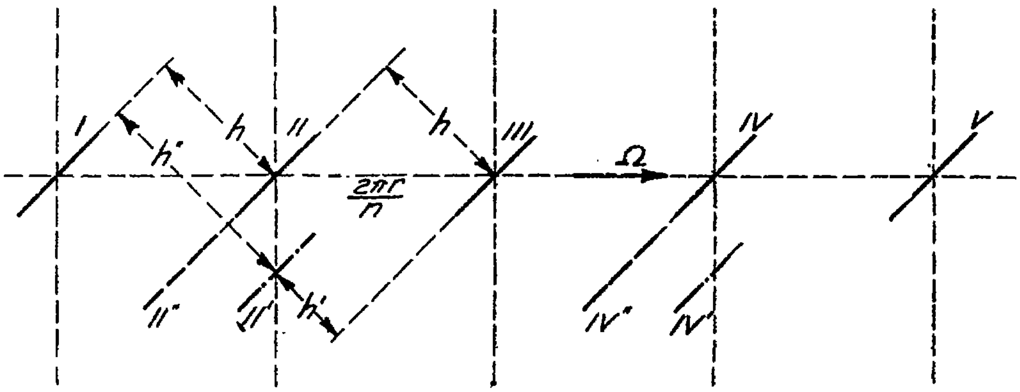


FIG. 22.

Some investigators have made the following experiments for the determination of the blade interference. They have first tested two identical screws separately, and afterwards have tested them coupled on the same axis. They have found that the efficiency of both screws working together was different from the efficiency of each screw working separately. Such an experiment does not prove at all the blade interference. As a matter of fact, two identical screws coupled on the same axis will first of all have a double breadth ratio compared to a single screw. But then, as directly follows from relation (93), the angles of attack of the different blade sections, for the same values of the relative pitch, will take other values, the breadth ratio having changed. The partial efficiencies will thus be modified and the total efficiency will therefore also be modified. Accordingly the modification of the efficiency of two coupled screws is first of all a consequence of the breadth ratio variation, as long as the conditions (135) or (138) remain satisfied. When we speak of blade interference, we shall always understand by this a modification of the values of the empirical functions k_t and β produced by the neighboring blades. It is only in the light of this remark that blade interference can be studied.

When two screws are coupled, the following circumstance can also take place. Let us consider on one hand a screw with $2n$ blades, and on the other hand two screws with n blades each, both coupled on the same axis. From the screw with $2n$ blades we can pass to the system of

[1] It is only for the blade sections for which the blade angles φ are near 45° that the partial efficiency will have the greatest values compatible with the corresponding optima angles of attack. But as $H=2\pi r \operatorname{tg} \varphi$, for $\varphi \cong 45°$ we have

$$r \cong \frac{H}{2\pi} \cong \frac{H}{6}$$

For a screw of high efficiency the effective pitch is of the same order of magnitude as the diameter. We thus see that it is the blade sections near the boss which will realize their maximum partial efficiency corresponding to the optima angles. But as ρ starts to vary slowly with φ, the partial efficiencies will generally first start increasing from boss to tip blade, and only afterwards, approaching the tip, will decrease. All the foregoing results from the fact that i and φ decrease from boss to blade tip.

two screws with n blades displacing, for example, the odd blades of the $2n$ blade screw along the screw axis. We will thus be brought to the picture of the figure 22, where I, II, III, IV, V represent the developed sections of a $2n$ blade-screw and I, II′, III, IV′, V represent the developed sections of two coupled n blade-screws. It is easy to see that in the first case the sections are disposed at the same distance h, and that in the second case the distances between the sections are on one hand increased up to h'' and on the other decreased up to h'. If we wish to maintain the distances between the blade sections considered in the case of two coupled screws, we must make the blades of the two screws approach in the sense inverse to their rotation according to the scheme I, II″, III, IV″, V. In figure 23a are represented two screws coupled according to the scheme I, II′, III, IV′, V and in figure 23b according to the scheme I, II″, III, IV″, V. This last remark explains the experiment with two coupled screws for a symmetrical and asymmetrical position of the last, made by G. Eiffel,[1] which showed a small increase of efficiency when the two screws were brought nearer one another in the inverse sense of their rotation. From the same experiment it follows that the interference of the screw blades is not large, because the results obtained for different dispositions of the screws do not show great differences. But the sum of the powers developed by each screw separately differs sensibly from the power developed by the two screws when coupled, which show the very sensible influence of the breadth ratio variation. When it is required to maintain for two coupled screws the equality of distances between the sections of different blades, it will be necessary to give to the blades of both screws or to the blades of one screw a special form not difficult to find.

All the foregoing relates to the study of the screw-blade elements, considered separately. We will now pass to the study of the screw-blade elements, considered together as a system. I shall begin by two general remarks.

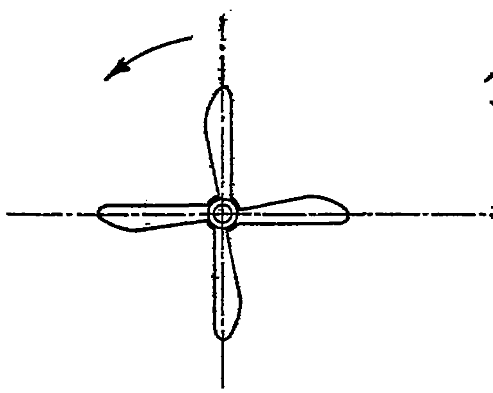


FIG. 23a.

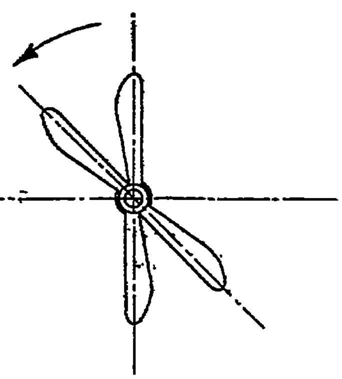


FIG. 23b.

Remark I.—Let us consider each blade of a screw divided into n elements. Let us designate, respectively, by ΔQ_1, ΔQ_2 ...... ΔQ_n; ΔC_1, ΔC_2 ...... ΔC_n; ρ_1, ρ_2 ...... ρ_n the partial thrusts, the partial torques, and the partial efficiencies of the blade elements equidistant from the screw axis. We have:

$$\rho_1 = \frac{V\Delta Q_1}{\Omega\Delta C_1}; \quad \rho_2 = \frac{V\Delta Q_2}{\Omega\Delta C_2} \;\cdots\cdots\; \rho_n = \frac{V\Delta Q_n}{\Omega\Delta C_n}.$$

Let us designate by η the total efficiency of the blade screw. We have:

$$\eta = \frac{V\Delta Q_1 + V\Delta Q_2 + \cdots\cdots + V\Delta Q_n}{\Omega\Delta C_1 + \Omega\Delta C_2 + \cdots\cdots + \Omega\Delta C_n}.$$

[1] See G. Eiffel, "Nouvelles recherches sur la resistance de l'air et l'aviation faites au laboratoire d'Auteuil," Paris, 1914, p. 345.

Let us now examine the correlation existing between the total efficiency η and the partial efficiencies $\rho_1, \rho_2 \ldots\ldots \rho_n$. For that purpose we shall use the following geometrical method. Let us consider the vectors

$$U_1, U_2 \ldots\ldots U_n$$

whose projections on the axis of abscissæ are equal, respectively, to

$$\Omega\Delta C_1, \Omega\Delta C_2 \ldots\ldots \Omega\Delta C_n$$

and on the axis of ordinates are equal to

$$V\Delta Q_1, V\Delta Q_2 \ldots\ldots V\Delta Q_n$$

Let us build, starting from the origin, the geometrical sum U of the vectors $U_1, U_2 \ldots\ldots U_n$ (see fig. 24)

$$U = U_1 + U_2 + \ldots\ldots + U_n$$

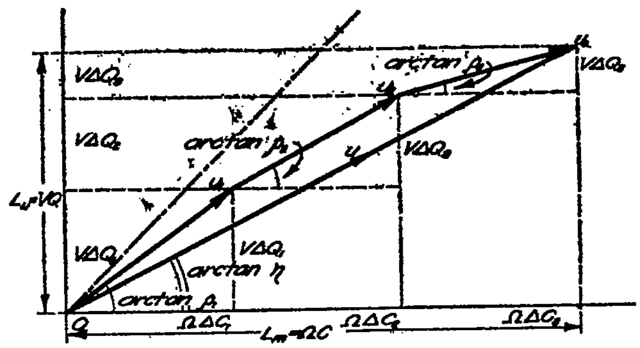


FIG. 24.

The tangent of the angle of inclination of each vector $U_1, U_2 \ldots\ldots U_n$ to the axis of abscissæ is just equal to the corresponding partial efficiency

$$\operatorname{tg}(U_1, X) = \rho_1 = \frac{V\Delta Q_1}{\Omega\Delta C_1};\ \operatorname{tg}(U_2, X) = \rho_2 = \frac{V\Delta Q_2}{\Omega\Delta C_2} \ldots\ldots \operatorname{tg}(U_n, X) = \rho_n = \frac{V\Delta Q_n}{\Omega\Delta C_n}$$

while the tangent of the angle of inclination of the vector U is equal to the total efficiency

$$\operatorname{tg}(U, X) = \eta = \frac{VQ}{\Omega C} = \frac{V(\Delta Q_1 + \Delta Q_2 + \ldots\ldots \Delta Q_n)}{\Omega(\Delta C_1 + \Delta C_2 + \ldots\ldots \Delta C_n)}$$

Q being the total thrust produced by the blade screw, C the total torque applied to its axis. The sides of the polygon $U_1, U_2 \ldots\ldots U_n$, U are necessarily making with the axis of abscissæ angles smaller than 45°. Considering thus the vector U as the geometrical sum of the vectors $U_1, U_2 \ldots\ldots U_n$, we see directly how the total efficiency is built up of the partial efficiencies. We can now see that the total efficiency η not only depends upon the values $\rho_1, \rho_2 \ldots\ldots \rho_n$ of the partial efficiencies, but depends also upon the partial powers $V\Delta Q_1, V\Delta Q_2 \ldots\ldots V\Delta Q_n$; $\Omega\Delta C_1, \Omega\Delta C_2 \ldots\ldots \Omega\Delta C_n$ because the total efficiency depends also upon the length of the vectors $U_1, U_2 \ldots\ldots U_n$ equal to

$$U_1 = \sqrt{\overline{V\Delta Q_1}^2 + \overline{\Omega\Delta C_1}^2}, \ldots\ldots U_n = \sqrt{\overline{V\Delta Q_n}^2 + \overline{\Omega\Delta C_n}^2}$$

In figure 25a is represented the case of a blade screw whose partial efficiencies decrease toward the blade tip; in figure 25b the case of partial efficiencies increasing toward the blade tip; and in figure 25c the case where the partial efficiencies first decrease and afterwards increase from boss to blade tip. In all the cases the total efficiency is increased when to the blade elements with higher partial efficiencies correspond larger partial powers. It follows from the foregoing that it is advantageous to give the greatest breadth to those parts of the blades where the partial efficiencies are highest.

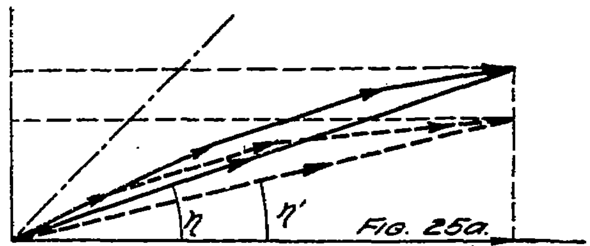


Fig. 25a.

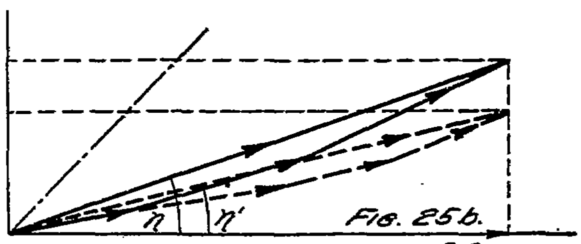


Fig. 25b.

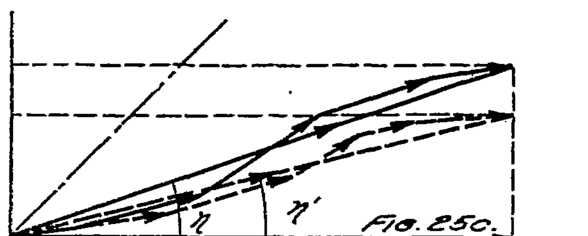


Fig. 25c.

Remark II.—Let us examine briefly the question of the effective pitch of the whole blade screw. When for a blade section the relative pitch becomes equal to unity, we have

$$H=\frac{V}{N}=\mu$$

and the knowledge of the advance which corresponds to $x=1$ gives the value of the effective pitch of the blade section considered. As we have seen, the value $x=1$ is disposed in the intermediate brake state which separates the propulsive state of screw work from the turbo-motor working state (see fig. A). Practically, this interval is very short; that is why as a first approximation we can consider $x=1$ either when in the propulsive state the partial thrust becomes equal to zero or when in the turbo-motor state the partial torque becomes equal to zero. By analogy with the conditions of work of a blade element, the value of the advance $\mu_1=V_1/N_1$ which corresponds for the whole screw either to $Q=0$ or $C=0$, defines the effective pitch H_h of the whole screw

$$H_h=\frac{V_1}{N_1}$$

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We will designate by x_h the value of the relative pitch which corresponds to the effective pitch of the whole blade screw.

$$x_h = \frac{V}{NH_h}$$

Let us now pass to the calculation of the thrust-power L_u developed by the propeller and the torque-power L_a absorbed by the propeller. We have

$$Q = \Sigma\Delta Q = \Sigma q\delta\Delta S\, V^2 = \delta V^2 \Sigma q \Delta S; \tag{139}$$

$$C = \Sigma\Delta C = \Sigma\frac{V}{\rho\Omega} q\delta\Delta S\, V^2 = \frac{\delta V^3}{\Omega}\Sigma\frac{q}{\rho}\Delta S; \tag{140}$$

Going from finite differences to differentials we get:

$$Q = 2\pi\delta V^2 \int qrdr = \pi\delta V^2 \int qd(r^2) = \pi\delta V^2 I_1 \tag{141}$$

$$C = \frac{2\pi\delta V^3}{\Omega}\int\frac{q}{\rho}rdr = \frac{\pi\delta V^3}{\Omega}\int\frac{q}{\rho}d(r^2) = \frac{\pi\delta V^3 I_2}{\Omega} \tag{142}$$

where we have introduced the notations

$$I_1 = \int qd(r^2);\ I_2 = \int\frac{q}{\rho}d(r^2) \tag{143}$$

We accordingly have

$$L_u = QV = \pi\delta V^3 \int qd(r^2) = \pi\delta V^3 I_1 \tag{144}$$

$$L_a = C\Omega = \pi\delta V^3 \int\frac{q}{\rho}d(r^2) = \pi\delta V^3 I_2 \tag{145}$$

$$\eta = \frac{\int qd(r^2)}{\int\frac{q}{\rho}d(r^2)} = \frac{I_1}{I_2} \tag{146}$$

The calculation of the thrust-power L_u, the torque-power L_a, and the total efficiency η is thus reduced to the quadrature of the two areas I_1 and I_2 limited by the curves of q and q/ρ plotted against r^2. The investigation of the conditions of maximum of the total efficiency is reduced to the determination of the maxima of the ratio I_1/I_2.

It must be noted that the integrals I_1 and I_2 are independent of fluid density. As a consequence, for different fluids the values of these integrals will depend upon the physical nature of the fluid only in the measure that the empirical coefficient k and the empirical function β depend upon fluid viscosity.

It is also easy to see that, for a given blade-screw whose working conditions are varying, the integrals I_1 and I_2 are functions of the advance μ only. In fact, for each blade element q and ρ are functions only of the corresponding angles of attack, the last being functions only of the relative pitch $x = V/NH$; in other words, functions only of the advance $\mu = V/N$, the effective pitches H of the different blade sections of a given screw having evidently invariable values. From the foregoing follows:

1. *The total efficiency η of a given screw is a function only of the advance μ.*

$$\eta=\frac{I_1(\mu)}{I_2(\mu)} \tag{147}$$

2. *The ratios*

$$\frac{Q}{V^2}=\pi\delta I_1(\mu);\ \frac{L_a}{V^3}=\pi\delta I_2(\mu) \tag{148}$$

are also functions only of the advance.

Let us compare the work of a propeller when advancing to its work at a fixed point. Starting from the relation (116) we get

$$Q=4\pi\delta\Omega^2\int q'r^3dr=\pi\delta\Omega^2\int q'd(r^4)=\pi\delta\Omega^2 I'_1, \tag{149}$$

and on account of the relation (99) we have

$$L_a=C\Omega={}^4/_5\pi\delta\Omega^3\int q'\operatorname{tg}(\varphi+\beta)d(r^5)={}^4/_5\pi\delta\Omega^3\int\frac{\sqrt{az}\cdot q'^{\,3/2}}{\rho_e^{\,2}}d(r^5)=\pi\delta\Omega^3 I'_2, \tag{150}$$

using the notations

$$\int q'd(r^4)=I'_1(\mu);\ {}^4/_5\int q'\operatorname{tg}(\varphi+\beta)d(r^5)=I'_2(\mu) \tag{151}$$

these last two integrals being functions only of the advance μ, as is easy to see. The integrals I'_1 and I'_2 are connected with the integrals I_1 and I_2 by the following relations:

$$I'_1=\frac{\mu^2}{4\pi^2}I_1;\ \ I'_2=\frac{\mu^3}{8\pi^3}I_2 \tag{152}$$

For the work of the screw at a fixed point the relations (149) and (150) go over into

$$Q_o=\pi\delta\Omega_o^2\int q'd(r^4)=\pi\delta\Omega_o^2 C_1 \tag{153}$$

$$L_o={}^4/_5\pi\delta\Omega_o^3\int q'_o\operatorname{tg}(\varphi+\beta_o)d(r^5)={}^4/_5\pi\delta\Omega_o^3\int\frac{q'_o{}^{3/2}}{\rho_o^{\,2}}d(r^5)=\pi\delta\Omega_o^3 C_2 \tag{154}$$

with the notations

$$\int q'_o d(r^4)=C_1;\ {}^4/_5\int q'_o\operatorname{tg}(\varphi+\beta_o)d(r^5)=C_2, \tag{155}$$

these last two integrals being, for a given screw, constant quantities independent of the angular velocity Ω_o. In fact, q'_o and $q'_o\operatorname{tg}(\varphi+\beta_o)$ are functions only of the angle of attack i_o, and the last is independent of Ω_o. The constants C_1 and C_2 are the limits, independent of the angular velocity Ω_o, toward which tend I'_1 and I'_2 when V tends toward zero. The two constants C_1 and C_2 thus appear as two fundamental characteristics of the dimensions of the blades of the propeller considered only. We thus see that the thrust Q_o and the power L_o are respectively proportional to the square and the cube of the number of turns of the propeller. The differences from these square and cube laws experimentally observed are due, as has already been mentioned in the introduction, on the one hand to the deformation of the blades, and on the other hand to the approximation of the velocity-square law for fluid resistance.[1] The calculation of the thrust Q_o and the power L_o of a screw at a fixed point is thus reduced to the

[1] See, for example, the experimental research of Ch. Maurain and A. Toussaint, Bulletin de l'Institut Aerotechnique de l'Université de Paris Fascicule III, 1913, where for all the screws tested the differences from the square and cube laws have been calculated. These differences are generally small.

quadrature of the two areas C_1 and C_2, respectively limited by the curves of q'_o and $q'_o \operatorname{tg}(\varphi+\beta_o)$ plotted against r_4 and r_5. Dividing (149) by (153) and (150) by (154), we get

$$\frac{Q}{Q_o}=\frac{\Omega^2}{\Omega_o^2}\frac{I'_1(\mu)}{C_1};\quad \frac{L_a}{L_o}=\frac{\Omega^3}{\Omega_o^3}\frac{I'_2(\mu)}{C_2} \tag{156}$$

For $\Omega=\Omega_o$ these last two ratios are functions only of the advance μ.[2] The expressions (149) and (150) show us that the thrust Q and the power L_a of a propeller can be written in the form.

$$Q=\delta N^2 I_1''(\mu);\quad L_a=\delta N^3 I_2''(\mu) \tag{157}$$

adopting the notations

$$4\pi^3 I_1'=I_1''-\pi\mu^2 I_1;\quad 8\pi^4 I_2'=I_2''-\pi\mu^2 I_2. \tag{158}$$

If we develop I_1'' and I_2'' in powers of μ, and take the first terms of the series obtained, we shall find the different approximate expressions which have been proposed by different authors for the representation of Q and L_a.

Let us now examine the different conditions which can be met in the quadrature of the integrals I_1 and I_2. We will consider that for the angles of attack values near the corresponding optima values are taken, so that we can admit the angle $\beta\cong 0$. Substituting in the relation (93) the values of az and H, respectively, equal to

$$az=\frac{aki\cos\varphi;}{2\sin^2(\varphi-i)};\quad H=2\pi r\operatorname{tg}\varphi$$

and on account of $\beta\cong 0$ and following $\rho\cong x$ we find:

$$\rho\operatorname{tg}\varphi=\frac{V}{r\Omega}=\frac{1-\frac{nb}{2\pi r}\frac{ki\cos\varphi}{2\sin^2(\varphi-i)}}{\frac{1}{\operatorname{tg}(\varphi-i)}+\frac{nb}{2\pi r}\frac{ki\cos\varphi}{2\sin^2(\varphi-i)}\left[1+\frac{nb}{2\pi r}\frac{ki\cos\varphi}{2\sin^2(\varphi-i)}\right]\operatorname{tg}\varphi} \tag{159}$$

These last equations constitute two relations between the seven quantities:

$$V,\ \Omega,\ r$$

$$\rho,\ b,\ \varphi,\ i.$$

For each blade section working under given conditions, the quantities of the first group are known quantities. The equations (159) thus connect with one another the four quantities of the second group. We thus see that from the four quantities ρ, b, φ, i, two of them can be arbitrarily chosen, or, more generally, for a given advance $\mu=V/N$, we can submit the four quantities ρ, b, φ, i to two supplementary conditions, adopting, however, for the angles of attack values near the optima values while we admit $\beta\cong 0$.

The simplest case for the quadrature of the integrals I_1 and I_2 is the one which corresponds to

$$q=\text{const};\quad \rho=\text{const}. \tag{160}$$

[2] Ch. Maurain and A. Toussaint, in their research just mentioned, for the representation of the results of their experiments, use the ratio Q/Q_o and L_a/L_o for $\Omega=\Omega_o$ as functions of the parameter V/ND, which is proportional to the advance $\mu=V/N$, while G. Eiffel in his experimental research on air screws uses for their representation the ratios Q/V^2D^4 and L_a/V^3D^5 as functions of the same parameter V/ND (compare with the relation (148)). These investigators came to these conclusions by way of considerations of similitude.

because under such conditions we have directly

(161) $$I_1 = \frac{qD^2}{4};\ I_2 = \frac{qD^4}{4\rho}$$

(162) $$\eta = \rho$$

expressions in which D is the blade-screw diameter. The condition q = const brings with it az = const, and thus the condition (92) shows us that we have

$$v = \text{const.}$$

The screws with constant load coefficients along the whole blade produce thus a slip stream with a uniform velocity in its cross section. That is why we will call such blade-screws *screws with uniform slip stream.*[1] The condition ρ = const obliges us to adopt for the partial efficiency such a value as can be realized for all the blade sections; the blade section with the lowest efficiency will thus fix the superior limit for the total efficiency. The screws with uniform slip stream will thus always have a reduced efficiency. The relations (160) have to be used for the calculation of the breadths b and the angles of attack i of all the blade sections of a screw with uniform slip stream.

Let us now liberate ourselves from the condition ρ = const and see how the total efficiency can be increased. It is easy to see that we have first of all to adopt for each blade section the optima angle of attack. If we now would like to maintain the slip stream uniformity, that is, q = const, the values of b, φ, and ρ will thus be fully fixed. But the screws of highest efficiency will be obtained when the breadth b is determined, not by the condition q = const, but directly by the condition of maximum of the total efficiency η. For the propellers of highest efficiency we have thus to seek for the law of variation of the breadth b along the blade which makes a maximum the integral ratio $I_1/I_2 = \eta$. The problem of the research of the most advantageous shape to be adopted for screw blades appears thus as a fully determined problem. Remark I of this chapter gives a first orientation in the last question. After these general considerations we will now pass to the detailed study of the question of design of propellers which have to work under given conditions.

THE PROBLEM OF PROPELLER DESIGN.

The design of a propeller which has to develop a given power and is destined to work with a given advance μ constitutes, as it were, a double problem. For the evaluation of the work of a blade screw we must know the exact dimensions of the blades. But the dimensions of the screw blades are fixed by the strength of the blades, which have to be able to resist the forces to which they are submitted. In the general case those forces can be exactly evaluated only when the dimensions of the blades are known. We are thus obliged, for the calculation of a screw, to adopt *a priori* its approximate dimensions, and by a series of calculations of the screw work and verification of its strength, to satisfy, by successive approximations, all the conditions of necessary strength and power demanded.

We shall in the following indicate a general method which will not only allow one to decide *a priori* upon the principal dimensions of a blade screw having to work under given conditions,

[1] It is evident that we have here only to do with uniformity of the slip velocity v. The race velocity has for its expression

$$r\omega = \frac{az\,(1+az)}{1-az} V \operatorname{tg}(\varphi+\beta)$$

and for v = const, which brings with it az = const, will be constant only if $tg\,(\varphi+\beta)$ = const. In the case of propellers the quantity $\operatorname{tg}(\varphi+\beta)$ is always variable along the blade. But we shall see in the following that fans and helicopter screws can be built with φ = const. In such a case we will have $\operatorname{tg}(\varphi+\beta)$ = const, owing to $\beta \cong 0$ and $r\omega$ will be constant when v = const.

but which will resolve, by simple reading on a diagram, the general problem of the screw selection. Let us thus consider to be known, as a first approximation, the blade dimensions of a screw which has to work with a given advance $\mu = V/N$, and for which we have to calculate the efficiency and the power it has to develop. For such a calculation, the quantities k_i and β, or K_x and K_y have to be known for all the blade sections of the screw considered, and also the angles γ of inclination of the zero lines to the chords of the different sections. These empirical quantities have to be determined from experiment performed at velocities of the same order of magnitude as the one under which are working the screw blade sections in their motion relative to the fluid, and in the same fluid as the one in which the screw considered will have to work. Actually we possess only very few data on the above-mentioned empirical functions at flow speeds occurring in blade-screw working. Especially for water we possess scarcely any data at all, the reason being that fluid resistance measurements in water are very troublesome. By analogy with experiments in air we can expect to get no more than a general idea of the order of magnitude of the quantities k_i, β and γ.

The experiments undertaken up to this time allow one to draw the conclusion that the lift coefficients K_y do not vary much with the velocity, but that the drag coefficients K_x sensibly diminish, which is an advantage for the blade-screw efficiency. The absence of sufficiently accurate data for the empirical functions k_i, β and γ is actually the only difficulty in the exact calculation of blade screws. In the question of propeller design we find ourselves actually in nearly the same condition as at the time when for the problems of strength of materials we did not possess sufficient data on the coefficients of resistance and the elasticity modulus. The author hopes that this lacuna will soon be helped by the use of a new method—which will be indicated in the following— based on the properties of the screw itself, which allows the measurement of the quantities k_i, β and γ in any kind of fluid, and in the exact working conditions of the screw. We will thus admit that the empirical functions k_i, β and γ have been evaluated by one or another method and consequently are known for all the blade sections of the screw considered.

Let us designate by $S(i)$ the system of the effective angles of attack under which are working the different sections of the blades of the screw considered. For a screw already built the system $S(i)$ has to be determined. For a new screw, to be built, the system $S(i)$ has to be chosen, and from its knowledge the effective pitches, or in other words the effective blade angles φ of the different blade sections have to be determined in such a way that the system of angles of attack $S(i)$ actually establishes itself when the advance reaches the given value. The angles of attack of the system $S(i)$ are always decreasing from boss to blade tip. The system of angles of attack $S(i)$ to be adopted depends upon the properties which we wish our screw to possess. If we wish to build a screw of high efficiency only, it is the system $S(i_{op})$ of the optima angles of attack which has to be adopted. But certain necessities of practice of blade-screw applications can demand some departure from the system $S(i_{op})$. In the following we shall come back in full detail to this important question.

The values of the effective blade angle φ and the effective angle of attack i, which for a given blade section correspond to one another are given by the relation (159) which may be written

$$\frac{V}{r\Omega} = \frac{(1-az)\,\mathrm{tg}\,(\varphi-i)}{1+az\,(1+az)\,\mathrm{tg}\,\varphi\,\mathrm{tg}\,(\varphi-i)} \tag{163}$$

with

$$az = a \cdot \frac{ki \cos \varphi}{2 \sin^2 (\varphi - i)} \tag{164}$$

in which we admit $\beta \cong 0$, that is, the system $S(i)$ to be close to the system $S(i_{op})$. The calculation of either φ or i from this last relation (163) is almost impossible by aid of actual algebraical methods; and yet the solution of this equation is necessary for the exact determination of these angles. That is why I have been led to seek for a nomographical solution, which, happily, can be given. I have made use of the method of parallel-tangential coordinates of M. d'Ocagne.

Let us first note, that for a propeller of good efficiency the quantity az is of the order of a small number of hundredths only. This is on account of the fact that the angles of attack i to be adopted are always small and that the coefficient k_i, of the same order of magnitude for air and for water, has also the value of some hundredths only. Although we adopt the formula (164) for az, it does not follow that the linear law for the coefficient k_i must necessarily be adopted; in each case we can consider the value of k taken from the relation $k=k_i/i$. But it must be noted that, for a given blade section profile and for the interval of the small values of the angles of attack which have to be used, the coefficient k is constant to a good approximation. After ascertaining that az is a small quantity, let us develop the relation (163) in series according to the increasing powers of az and neglect the terms of superior order. The error thus committed is out of consideration for the demands of practice of screw design. We thus find:

$$\frac{V}{r\Omega}\frac{1}{\operatorname{tg}(\varphi-i)}+az[1+\operatorname{tg}\varphi\operatorname{tg}(\varphi-i)]-1=0$$

and substituting for az its value (164) we get

$$\frac{V}{r\Omega}\frac{1}{\operatorname{tg}(\varphi-i)}+ak\frac{i\cos\varphi}{2\sin^2(\varphi-i)}\left[1+\operatorname{tg}\varphi\operatorname{tg}(\varphi-i)\right]-1=0 \tag{165}$$

and finally

$$\frac{V}{r\Omega}M+ak\,N-1=0 \tag{166}$$ [1]

using the notations

$$M=\frac{1}{\operatorname{tg}(\varphi-i)}$$

$$N=\frac{i\cos\varphi}{2\sin^2(\varphi-i)}\left[1+\operatorname{tg}\varphi\operatorname{tg}(\varphi-i)\right]=\frac{i\cos i}{\sin(\varphi-i)\sin 2(\varphi-i)} \tag{167}$$

On the other hand let us consider the equation

$$\frac{u}{l_1}\cdot l_1\frac{L-x}{2Ly}+\frac{v}{l_2}\cdot l_2\frac{L+x}{2Ly}-1=0 \tag{168}$$

which we refer to the system of the X and Y axes represented in figure 26. When in this equation (168) we consider u and v as parallel-tangential coordinates, it will represent the point (x, y) defined by the sheaf of straight lines (u, v); when we consider x and y as point coordinates, it will represent the straight lines (u, v) which the point (x, y) describes. $2L$ is the distance between the parallel axes of the u and v, counted along the abscissae axis; l_1 and l_2 two arbitrary numbers introduced for the convenience of the scale choosing. The angle between the

[1] If the angle β were not neglected, we would have found

$$\frac{V}{r\Omega}\frac{1}{\operatorname{tg}(\varphi-i)}+ak\frac{i\cos(i+\beta)}{\sin(\varphi-i)\sin 2(\varphi-i)}-1=0$$

But as the variations of the cosines of small angles are small, the error committed neglecting the small values of β is very small.

ordinates and abscissae axes is arbitrary. Let us establish a univocal and reciprocal correspondence between the terms of the equations (166) and (168) as follows:

$$\frac{V}{r\Omega}=\frac{u}{l_1};\ ak=\frac{v}{l_2}$$

$$M=l_1\frac{L-x}{2Ly};\ N=l_2\frac{L+x}{2Ly}$$

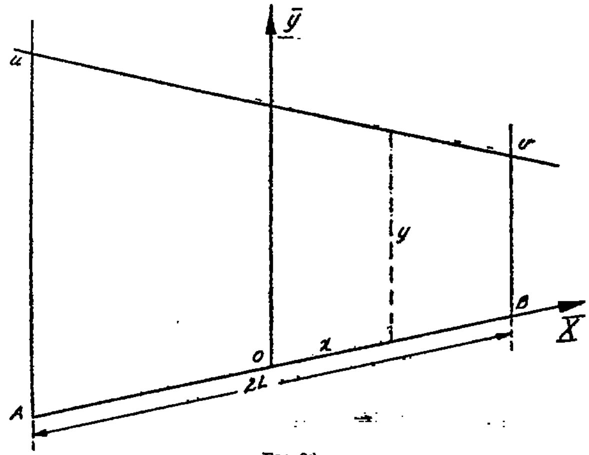

FIG. 26.

which corresponds to

(169)
$$u=l_1\frac{V}{r\Omega};\ v=l_2\cdot ak$$

(170)
$$x=L\frac{Nl_1-Ml_2}{Nl_1+Ml_2};\ y=\frac{l_1l_2}{Nl_1+Ml_2}$$

The equations (169) represent in parallel-tangential coordinates a system of straight lines; and the equations (170) represent in point coordinates two families of curves, with φ and i as parameters. Each straight line (169) which goes through the intersection of two of the curves (170), or all the curves (170) which intersect one another on one of the straight lines (167), defines a system of values of $V/r\Omega$, ak, φ, i which satisfies the equation (166). For the tracing of the nomogram which gives the solution of the equation (166) and which I call *the incidence nomogram* I have adopted the values

$$\text{tg}\,(x,y)=0.75;\ l_1=1;\ l_2=12.5$$

The incidence nomogram is joined to this memoir.[1] Its use is very simple; it is enough to join by a straight line two given values of $V/r\Omega$ and ak, in order to read on the curves which cut one another on this line the values of φ and i which correspond to each other.[2] Thus for $V/r\Omega=1$ and $ak=0.01$; for $i=5°$ we find $\varphi=52°$; for $\varphi=55°$ we find $i=7°$, and so on. The incidence nomogram gives thus the direct and complete solution of the finding of the effective blade-angle φ of a blade section when its effective angle of attack i is given, and of the finding of the effective angle of attack i under which a blade section is working when its effective blade angle φ is known.

[1] For all the details concerning this type of nomogram see "Traité de nomographie," by M. Maurice d'Ocagne, §§ 57 and 58, p. 125, and also § 121, p. 320.

[2] For its use it is good to cover the nomogram with a piece of tracing paper.

After we have found for all the blade sections of the screw considered the values of φ and i which correspond to one another, it will be easy to calculate the values of the function

$$az = \frac{nb}{2\pi r}\,\frac{k_t \cos\varphi}{2\sin^2(\varphi - i)}$$

for all the blade sections considered, while all the other quantities which figure in the expression of az are known for each section of a given screw blade. Knowing az we will calculate the values of the load coefficients

$$q = \frac{2az}{(1-az)^2}$$

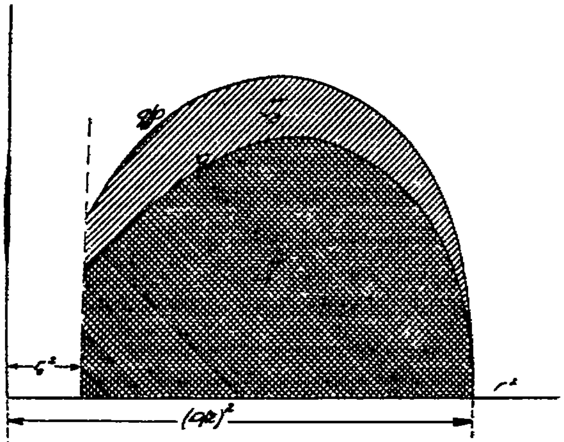

FIG. 27

for all the sections considered. We shall thus be able to plot point by point the curve of q against r^2 (see Fig. 27). The quadrature of the area limited by this curve will give the value of I_1. For the calculation of I_2 we shall have to determine the partial efficiency ρ of each section of the given blade, by aid of the formula

$$\rho = x\frac{\operatorname{tg}\varphi}{\operatorname{tg}(\varphi+\beta)} = \frac{V}{2\pi r N \operatorname{tg}(\varphi+\beta)} \tag{171}$$

in which it is necessary to take account of the values of the angle β, which has a sensible influence on the partial efficiency, especially when this angle is negative. The partial efficiencies ρ once calculated, the curve of q/ρ as a function of r^2 will be traced point by point, and its area will just be equal to I_2. Knowing I_1 and I_2 the values of the thrust power L_u and the torque power L_a will be directly found for the blade screw considered. We have

$$L_u = \pi\delta V^3 I_1;\quad L_a = \pi\delta V^3 I_2$$

And the value of the total efficiency η is equal to

$$\eta = \frac{I_1}{I_2}$$

The total thrust Q produced by the blade screw and the total torque C applied to its axis are equal to

$$Q = \frac{L_u}{V};\quad C = \frac{L_a}{\Omega}$$

For the purpose of rendering easier the calculation of az and q, I have made a second nomogram, also joined to this memoir, which I call *the load nomogram*, founded on a basis similar to that used for the incidence nomogram.

Let us take the decimal logarithms of the expression

$$az = ak\frac{i\cos\varphi}{2\sin^2(\varphi - i)} = ak\cdot M = cak\cdot\frac{M}{c} \tag{172}$$

in which we have put

$$M = \frac{i\cos\varphi}{2\sin^2(\varphi - i)}$$

and where c is an arbitrary quantity. We find:

$$\frac{\log az}{\log\frac{M}{c}} + \frac{\log\frac{1}{cak}}{\log\frac{M}{c}} - 1 = 0 \tag{173}$$

Let us establish a univocal and reciprocal correspondence between this equation and the equation

$$\frac{u}{l_1}\cdot l_1\frac{L-x}{2Ly} + \frac{v}{l_2}\cdot l_2\frac{L+x}{2Ly} - 1 = 0$$

as follows

$$\log az = \frac{u}{l_1};\ \log\frac{1}{cak} = \frac{v}{l_2}$$

$$\log\frac{M}{c} = \frac{2Ly}{l_1(L-x)} = \frac{2Ly}{l_2(L+x)}$$

which gives

$$u = l_1\log az;\ v = l_2\log\frac{1}{cak} \tag{174}$$

$$x = L\frac{l_1 - l_2}{l_1 + l_2};\ y = \frac{l_1 l_2}{l_1 + l_2}\log\frac{i\cos\varphi}{2c\sin^2(\varphi - i)} \tag{175}$$

For the tracing of the nomogram I have adopted: the angle between the axes of ordinates and abscissae equal to 90° (see fig. 26); $l_1 = l_2 = 1$ and $\log c = 1.5$, with $x = 0$. The second of the equations (174) represents a family of curves having i as parameter when φ is taken as abscissa and y as ordinate. Each point (i, φ) of these curves projected on the Y axis is situated on the straight line (u, v) corresponding to a system of values of az and ak, which with the foregoing values of i and φ satisfy the equation (172). The use of the nomogram follows from this last remark. Each straight line joining a point of the az scale to a point of the ak scale cuts the Y axis in such a point that the corresponding values of φ and i are situated at the intersection of the parallel to the x axis going through this point and the family of curves defined by the second of the equations (174).[1] Thus for $ak = 0.008$; $\varphi = 38°$; $i = 4°$ we shall find $az = 0.04$. As q is a function of az only, I have joined to the scale of az a functional scale which gives directly the corresponding values of q.[2]

It is to be noted that the incidence nomogram as well as the load nomogram are independent of the fluid mass density δ. These nomograms might thus be used for the computation of a screw in any fluid, the physical nature of the fluid will intervene only in the values to be adopted for the coefficient k.

[1] For all the details concerning this type of nomogram see "Traité de Nomographie," by G. d'Ocagne, pp. 145 and 324.

[2] The load nomogram, although established for $\beta = 0$, can be used with a practically sufficient approximation for values of β near zero.

Summing up the foregoing, for the design of a propeller, we have to proceed as follows: A certain number of sections, whose general configuration has to be fixed, are chosen on each blade. Practically from four to eight sections are sufficient. Having chosen the angles of attack under which we wish our blade sections to work, for the given advance $\mu = V/N$, the effective blade angles φ will be found by aid of the incidence nomogram. From these values we will be able to calculate the quantities,

$$\frac{H}{2\pi} = r \operatorname{tg}\varphi \tag{176}$$

for all the sections considered, and thus will be able to establish the propeller drawing[1] knowing also the corresponding values of the angles γ which the zero lines make with the chords of the blade sections. By aid of the load nomogram, q will be calculated and by aid of the formula (171) ρ will be calculated. Plotting the curves of q and q/ρ against r^2, by a quadrature of the area obtained, one can find I_1 and I_2 and thus L_u, L_a and η. The same method has to be followed for the verification of the power of a screw already built, only the order of finding φ and i is reversed.

The knowledge of the curves of q and q/ρ gives a complete picture of the contribution of each blade section to the work of the whole blade screw, and thus allows one to find in magnitude, as well as in sense, the load distribution along the blade, which has to be known for the computations of blade stresses.

I must finally remark that the neighborhood conditions can have an influence on the working of the propeller; that is why when such an influence is to be expected it is good to build the propeller with a small excess in diameter, whose progressive shortening when testing the propeller will easily allow us to bring the propeller to do exactly the required number of turns N at the speed V. The diameter thus obtained will be the one which, under the given neighborhood and working conditions, exactly corresponds to the disposable power L_a.

The author has designed many propellers by the method above described and has convinced himself of the entire availability of the foregoing method, not only for the design of propellers at their maximum efficiency, but also for the tracing of the total efficiency curve as function of the advance μ for a wide interval, including the maximum of the total efficiency. These computations have shown, as already mentioned, that the lift coefficients K_y are very slightly influenced by the value of the flow velocity, but that the drag coefficients K_x decrease with the increase of the flow velocity. The last follows from the fact that the values of K_x corresponding to low flow velocities when used for propeller calculations always lead to values of the total efficiency lower than those experimentally measured.[2]

After we have learned to calculate the power developed and absorbed by a propeller, we shall pass to the general discussion of the fundamental problem of the selection or adaptation of propellers.

THE THEORY OF THE UNIFORM SCREW FAMILIES.

I call *uniform family* a screw family whose blades can be made geometrically similar by a twisting of all the blade sections in such a way as to bring them all to have a zero pitch. All the screws of a uniform family thus have, at homologous distances from the screw axis, geometrically similar sections, but their pitches have different values. I divide the screws of each uniform family into *varieties*. Each variety is characterized by the fact that all the homologous blade sections are working under the same angles of attack, and thus each variety is defined by a given

[1] For all that concerns the blade-screw drawing see Note V at the end of this memoir.

[2] I will remember here that the expression (171) of the efficiency is fully independent of any hypothesis. Thus if the calculated value of the thrust, as above indicated, will correspond to the experimentally measured thrust, but the calculated efficiency will be found in disagreement with the experimentally observed, this can only mean that the values used for the angle β are not sufficiently accurate.

system $S(i)$ of angles of attack. Three principal varieties have to be considered. *The optima or maxima variety* is the one for which all the blade sections of the screws considered are working under the system $S(i_{op})$ of their optima angles of attack. *The minor variety* is the one for which all the blade sections are working under the system $S_1(i)$ of angles of attack, all smaller than the corresponding optima angles. *The major variety* is the one for which all the blade sections are working under the system $S_2(i)$ of angles of attack, all larger than the corresponding optima angles. Each blade screw can be considered as belonging to a certain uniform family. It is this last remark which makes the generality of the theory of uniform families.

To the screws of a given variety, characterized by a given system $S(i)$ of angles of attack, there corresponds for a given advance $\mu = V/N$, a system of effective blade angles φ and a system of effective pitches H which we have learned to calculate. The screws of a variety defined by a system $S(i)$ and having to work with different advances μ are not geometrically similar. Each screw of the optima variety, for the advance under which it has to work, will necessarily work at its maximum total efficiency, because for any other value of the advance, angles of attack different from the optima angles will establish themselves, all the partial efficiencies will thus be lowered, and the total efficiency will therefore be reduced. In the same way it will be easy to see, considering the curve of the total efficiency η as a funciton of the advance μ (see fig. 28), that a major screw for its advance will necessarily work on the left hand of the maximum efficiency and that a minor screw will work on the right hand of the maximum efficiency. The last follows directly when one remembers the law of variation of the angle of attack along the curve of the efficiency plotted against the relative pitch (see Chapter II).

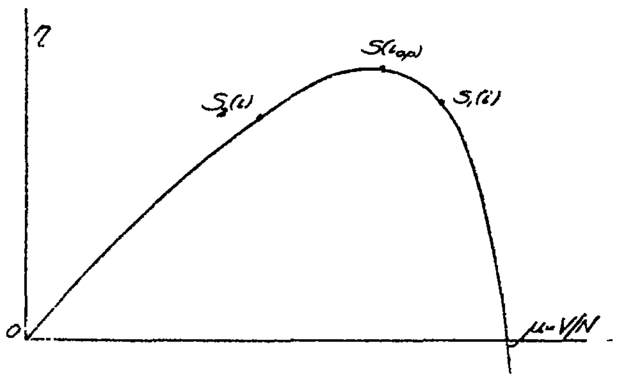

FIG. 28.

We shall now establish the fundamental relations which unite the screws of any one variety belonging to the same uniform family. Let us adopt for each screw of our variety a reference blade section, which can be, for example, the one situated on the third of the blade length counted from the tip. According to relation (159), page 60, we have

$$\frac{V}{r\Omega} = f(ak, \varphi, i) \tag{177}$$

with

$$\Omega = 2\pi N; \quad H = 2\pi r \operatorname{tg} \varphi$$

For all the blade sections of the screw considered the quantities r, ak, and i are known. The knowledge of the advance fixes by aid of the relation (177) the blade angle φ of each blade section; and, inversely, when the blade angle φ is known, the value of the advance under which the given screw is destined to work can be found. The knowledge of the blade angle of one blade section fixes thus the values of the blade angles of all the other sections, the system $S(i)$ being known. For a given screw of a given variety the blade angles of all the blade sections are functions of the blade angle of one of the sections. Let us now refer the relation (177) to the reference blade section. We thus can write

$$\frac{V}{r\Omega} = \frac{V}{ND} \cdot \frac{D}{2\pi r} = f(ak, \varphi, i)$$

or

$$\frac{V}{ND} = \xi = \frac{2\pi r}{D} f(ak, \varphi, i)$$

designating by ξ the ratio V/ND, which we will call the *relative advance*. For the reference blade section r/D is a constant, and as its blade angle φ is equal to

$$\operatorname{tg} \varphi = \frac{H}{2\pi r} = \frac{H}{D} \frac{D}{2\pi r}$$

we see that we necessarily have

(178) $$\frac{H}{D} = A(\xi)$$

For all the screws of a given variety of a uniform family the series of the ratios of the effective pitches of the reference blade sections to the corresponding diameters are functions of the relative advance ξ.

Let us designate by L'_a the power absorbed for a given value of the advance μ by the screw of our variety whose diameter is equal to unity. We have

(179) $$L'_a = \pi \delta V^3 \int \frac{q}{\rho} d(r^2) = \pi \delta V^3 I'_2$$

Let us first consider in our screw variety all the screws for which the ratio $V/r\Omega$ has the same value for all the homologous sections; that is, the screws whose diameters are proportional to the corresponding advances. For all these screws, for each homologous section, the quantities ak, φ, i will have the same values; the quantities q and ρ will thus also have the same values. Under such conditions the value of the integral I_2 will be proportional to the square of the diameter of the screw considered; that is, for any one of the screws considered we will have

$$L_a = \pi \delta V^3 D^2 \int \frac{q}{\rho} d(r^2) = \pi \delta V^3 D^2 I'_2$$

the integral I'_2 corresponding to the screw with the diameter equal to unity. Let us consider now in our variety all the screws of the same diameter, but for different values of the advance μ. The quantities q and ρ will be functions of the blade angles φ only, or, in other words, of the ratios H/r or H/D, the ratio D/r being constant for all the homologous sections. But H/D is a function of the relative advance ξ for all the screws of our variety; we thus will have

$$\frac{L_a}{\pi \delta V^3} = D^2 \phi(\xi)$$

or

$$\left(\frac{N}{V}\right)^2 \frac{L_a}{\pi \delta V^3} = \frac{D^2 N^2}{V^2} \phi(\xi) = B(\xi)$$

Varying first the diameters for $V/r\Omega = \text{const}$ and afterwards varying the advance μ for $D = \text{const}$ we will run through all the screws of our variety. We are thus brought to the following conclusion:

For all the screws of a given variety of a uniform family the series of the ratios $N^2 L_a/\pi \delta V^5$ is a function of the relative advance ξ.

By quite analogous reasoning it will be easy to see that the total efficiency η of all the screws of a given variety of a uniform family is also a function of the relative advance ξ only; that is

(181) $$\eta = C(\xi)$$

The functions $A(\xi)$, $B(\xi)$, $C(\xi)$ are characteristic for a given screw variety defined by a system $S(i)$. If we calculate a series of screws of a certain variety we will be able to trace point by point the curves

$$H/D = A(\xi);\ N^2 L_a/\pi\delta V^3 = B(\xi);\ \eta = C(\xi)$$

For each system $S(i)$ of angles of attack we will get a group of curves. We will arrive at a full picture of the properties of a uniform family when tracing a system of three groups at least, of curves $A(\xi)$, $B(\xi)$, and $C(\xi)$: A first group of curves, A_1, B_1, C_1, for a minor variety,

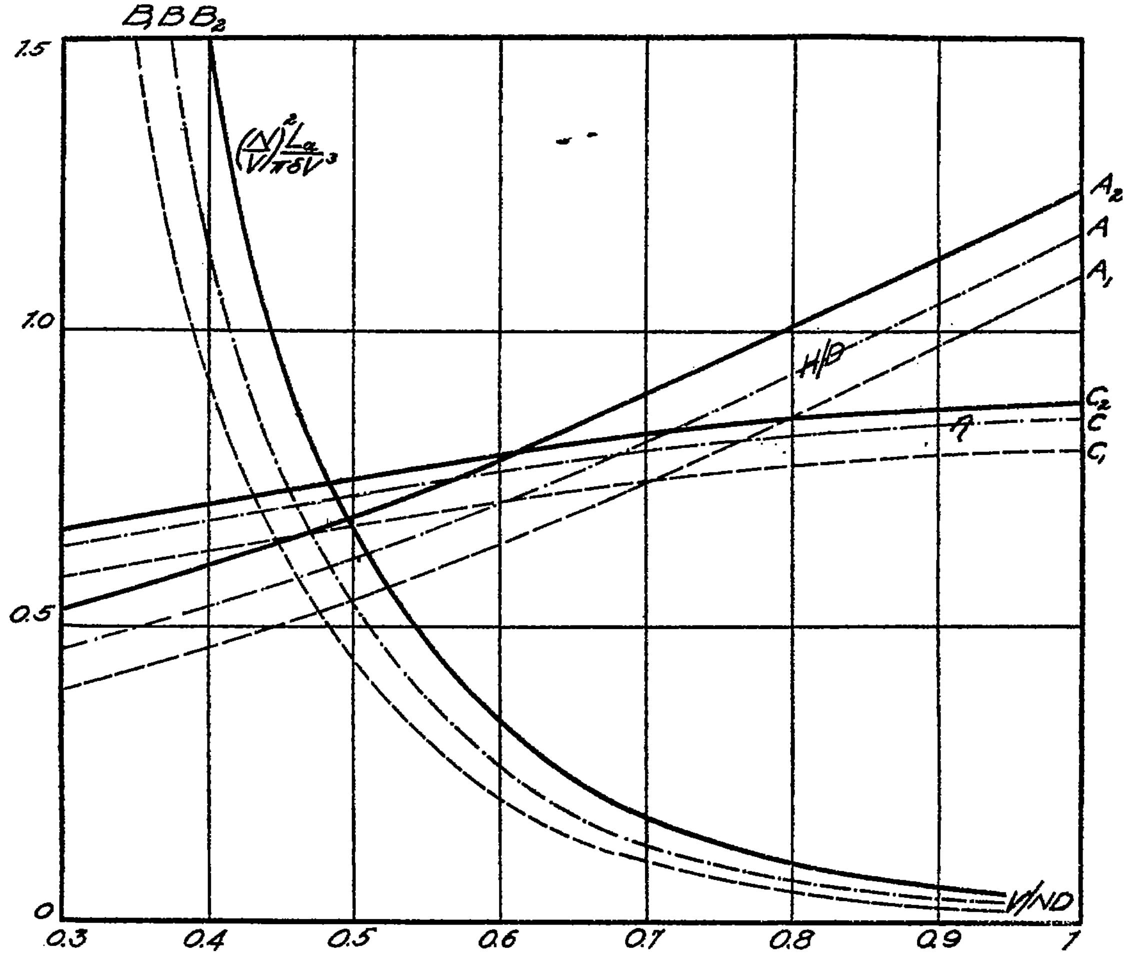

FIG. 29.

a second group of curves, A, B, C for an optima variety, and a third group of curves, A_2, B_2, C_2 for a major variety. Such a system of curves corresponding to a uniform family is represented in figure 29.[1] This system of curves gives the complete solution of the problem of the selection of a propeller. In fact, suppose we have to calculate a propeller to absorb a power L_a at the advance μ. The quantity $N^2 L_a/\pi\delta V^3$ will be calculated; and on the curves B_1, B, B_2, will be read three values of $\xi = V/ND$, from which will be deduced three diameters

[1] This figure corresponds to a uniform family of air-screw of the "Dorand" type with constant constructive pitch along the blade. See G. Eiffel, "Nouvelles Recherches sur la resistance de l'air et l'aviation faites au laboratoire d'Auteuil," atlas, Plate XXXIII. In figure 29 the curve H/D is referred to the constructive pitch.

D_1, D, D_2. To the three abscissae ξ will correspond on the curves A_1, A, A_2 three values H_1/D_1, H/D and H_2/D_2, and on the curves C_1, C, C_2 three values η_1, η, η_2. Knowing D_1, D, D_2 from the ratios H_1/D_1, H/D, H_2/D_2, we will find three values H_1, H, H_2 of the pitch. Plotting on a diagram D_1, D, D_2 as abscissae and H_1, H, H_2 as ordinates we will get by three points a curve on which, by interpolation and by extrapolation (not carried too far), we will be able to read all the system of pitches H and diameters D of all the screws of the uniform family considered which satisfy the conditions of given power L_a and advance $\mu = V/N$.[1] (See Fig. 30.)

We thus see that to the data L_a and $\mu = V/N$ there corresponds an infinity of propellers, among which we have to choose the most convenient for the application considered. The following considerations have to be taken into account. The efficiency of major screws goes on increasing in a certain interval when the advance increases, and the efficiency of a minor screw first increases when the advance decreases. Thus a propeller for a tug has to be a major screw in order to be able to give good efficiencies over a large scale of loads. The propeller of a trans-Atlantic ship has to be an optima screw, for the maxima ship speed and the number of revolutions of its engines. An airplane propeller has to be a minor screw, to be able to maintain a high efficiency when climbing. In practice we are often limited by the space disposable for the propeller. In such cases there will only be left to us to approach as near as possible the most favorable screw type.

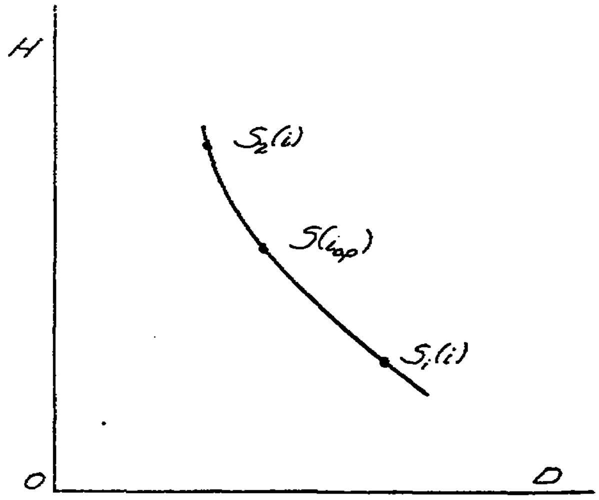

FIG. 30.

When we have to choose the propeller for a given application the great unknown of the problem is generally the head resistance or drag of the vehicle to which the propeller has to be adapted. This is why one must proceed as follows. We will determine either the minimum speed which we can expect from the vehicle and calculate for it a major screw, or the maximum speed expected from the vehicle and calculate for it a minor screw. The testing of the vehicle with such a *testing screw* will, with full certitude, indicate the speed which our vehicle can realize with the disposable power. If our first approximations to the speed of the vehicle should in the first testing appear to be far from the observed speed, a second testing screw would have to be used. Once having found the exact order of the vehicle speed magnitude compatible with its power, we will then have only to calculate the definitive screw which corresponds to the conditions L_a and $\mu = V/N$ and which this time will have to be a major, optima, or minor screw according to the screw application considered. Proceeding as indicated above, we will with full certitude find the very best screw which the case considered can admit.

The demands of the strength of the screw blades will fix the limits between which diagrams for screw selection can be established. It will usually be found necessary to establish a series of diagrams for increasing power intervals. A series of such diagrams gives the complete solution of the screw selection problem in its whole generality. For the important applications of blade screws it will be good, after having found the screw dimensions by aid of the screw

[1] In the general case we will have as many points of the H curves as functions of D, as groups of curves A, B, C have been traced.

selection diagram, to try by a series of calculations of the modified screw to improve its qualities. The author hopes that he will have the pleasure of seeing in the near future the spreading of the use of such screw selection diagrams for different uniform families, by aid of which will be eliminated all the difficulties of the delicate problem of selecting screws.

The screw selection diagram allows one also to judge of the influence of the variation of the number of revolutions on the efficiency of a propeller. It will be advantageous to use gearing only when the increase in efficiency is able to compensate the losses in the gears, if only the space disposable for the propeller or other conditions do not oblige us to use gears. It must be noted that, although the efficiency of a propeller increases generally when its number

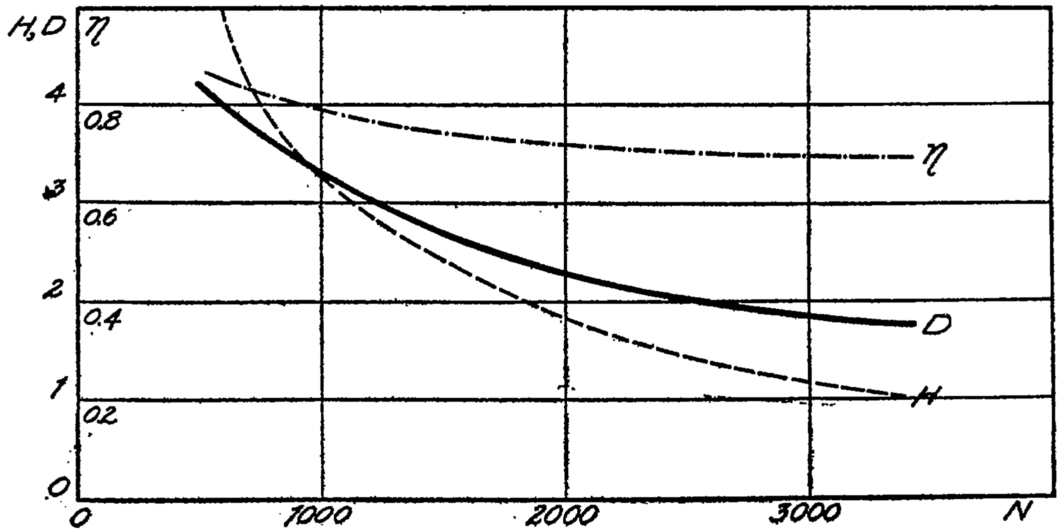

FIG. 31.

of revolutions is decreased, this increase, however, is not very large. Thus for the speeds of actual airplanes included between 100 km./hr. and 200 km./hr., and numbers of revolutions of the actual aviation engines included between 1,000 and 2,000 revolutions a minute, it is only exceptionally advantageous to use gears whose losses are generally greater than the gain in efficiency. The gearing up of a screw brings also with it an increase in size and consequently an increase in weight of the screw. On the diagram reproduced here (see fig. 31), by aid of the screw-selection diagram of figure 29, for a number of revolutions from 600 to 3,500 a minute, for an invariable power of 240 horsepower applied to the screw axis, there are calculated the efficiencies η, the diameters D and the pitches H of the whole series of corresponding minor screws. It is easy to see that the dimensions of the propellers go on increasing much more rapidly than the efficiency when the number of revolutions decreases.

Chapter IV.

NEW METHOD OF MEASURING THE COEFFICIENTS OF FLUID RESISTANCE BY AID OF THE PLANE RADIAL SCREW.

In the last chapter, for the screw working at a fixed point we have established the following system of formulæ:

$$a = \frac{nb}{2\pi r} = \frac{2 \sin^2 (\varphi - i_o)}{k_i \cos (\varphi + \beta_o)} \tag{105}$$

$$\rho_o = \frac{\operatorname{tg} (\varphi - i_o)}{\operatorname{tg} (\varphi + \beta_o)\left[1 + 2 \operatorname{tg} (\varphi - i_o) \operatorname{tg} (\varphi + \beta_o)\right]} \tag{106}$$

$$V_o = r\Omega_o \rho_o \operatorname{tg}(\varphi + \beta_o) \tag{107}$$

$$r\omega_o = 2r\Omega_o \rho_o \operatorname{tg}^2 (\varphi + \beta_o) \tag{108}$$

$$\Delta Q_o = 2\delta \Delta S v_o^2 = 2\delta \Delta S r^2 \Omega_o^2 \rho_o^2 \operatorname{tg}^2 (\varphi + \beta_o) \tag{109}$$

$$\Delta C_o = 2\delta r \Delta S r^2 \Omega_o^2 \rho_o^2 \operatorname{tg}^3 (\varphi + \beta_o) \tag{110}$$

$$\rho_o \Omega_o \Delta C_o = v_o \Delta Q_o \tag{111}$$

and we have shown that the angles of attack i_o of the different blade sections are constant, independent of the angular velocity of screw rotation, these invariable values of the angles of attack being given by the relation (105).

Let us consider a screw defined by the conditions

$$\left\{ \begin{array}{l} \varphi = \text{const} \\ a = \text{const} \\ k_i = \text{const} \end{array} \right. \tag{182}$$

all along each blade. From these conditions it follows that

$$i_o = \text{const}; \ \rho_o = \text{const}$$

along each blade.

The condition $\varphi = \text{const}$ expresses the fact that the screw blades are not twisted, having a constant blade angle.

The condition $a = \text{const}$ expresses the fact that each blade is limited by two radial straight lines.

The condition $k_i = \text{const}$ expresses the fact that all the blade sections are geometrically similar. The thickness of the blades must thus go on increasing from boss to tip proportionally to the distance from the screw axis.

I call *plane-radial screw* a screw whose blades satisfy the foregoing conditions (182).

It is easy to see that for such a screw the equations (109) and (110) can be directly integrated and we have

$$Q_o = \pi \delta \rho_o^2 \Omega_o^2 \operatorname{tg}^2 (\varphi + \beta_o) [r_2^4 - r_1^4] \tag{183}$$

$$C_o = {}^4/_5 \pi \delta \rho_o^2 \Omega_o^2 \operatorname{tg}^3 (\varphi + \beta_o) [r_2^5 - r_1^5] \tag{184}$$

r_2 being the screw radius at blade tip and r_1 the radius of the boss. The ratio of C_o to Q_o is equal to

(185) $$\frac{C_o}{Q_o} = {}^4/_5 \operatorname{tg}(\varphi + \beta_o) \frac{r_2^5 - r_1^5}{r_2^4 - r_1^4}$$

We will show that the empirical quantities k_t and β or K_x and K_y, which correspond to the blade section profile used for the blades of a plane-radial screw can be measuerd by testing the plane-radial screw at a fixed point and measuring its thrust Q_o and torque C_o.

In fact, knowing r_1, r_2, Q_o and C_o from relation (185) we find the value of $\operatorname{tg}(\varphi + \beta_o)$.

Knowing $\operatorname{tg}(\varphi + \beta_o)$ from relation (183) we find ρ_o.

Knowing $\operatorname{tg}(\varphi + \beta_o)$ and ρ_o from relation (106) we get $\operatorname{tg}(\varphi - i_o)$.

Knowing $\operatorname{tg}(\varphi + \beta_o)$ and $\operatorname{tg}(\varphi - i_o)$ the relation (105) gives the value of k_t.

Let us designate by ψ the constructive blade angle, that is, the angle between the chord of the blade section and the plane of screw rotation, by α the constructive angle of attack and by γ the angle between the chord and zero line of the blade section considered (see fig. 32). We have:

(186) $$\varphi = \psi + \gamma$$

and

(187) $$i_o = \alpha + \gamma$$

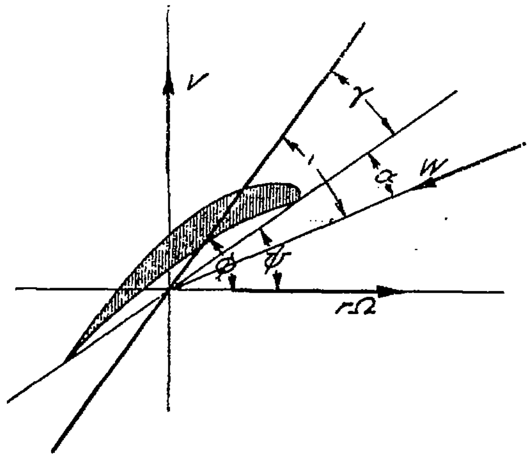

FIG. 32.

Suppose that, having measured Q_o and C_o we have found

(188) $$\varphi + \beta_o = \psi + \gamma + \beta_o = c_1$$

(189) $$\varphi - i_o = \psi + \gamma - i_o = \psi - \alpha = c_2$$

The angle ψ being known, we will have

(190) $$\alpha = \psi - c_2$$

Knowing thus for each value of ψ the value of k_t and the corresponding value of α, we shall be able, by a series of tests made with different values of ψ, to trace the curve of k_t as a function of

$$k_t = F(\alpha)$$

The intersection of this curve with the axis of abscissae will give us the value of γ, and we shall thus be able to calculate the values of β_o and i_o which correspond to each value of ψ.

(191) $$\beta_o = c_1 - \psi - \gamma$$

(192) $$i_o = c_2 - \psi - \gamma$$

We will thus obtain all the necessary data for the tracing of the curves of k_t and β as functions of i; or, if we prefer the curves of K_x and K_y, these can be directly deduced from those of k_t and β.

We thus see that for the measurement of k_t and β or K_x and K_y as functions of i, it is sufficient to take small plane-radial boards, whose cross sections have the profile to be studied, and to fix them to a boss permitting one to use them as blades under different values of the constructive blade angles ψ. Measuring by a series of tests Q_o and C_o for different values of ψ of such a plane radial screw, there will be found, as explained in the foregoing, first the series of corresponding values of k_t as functions of α; afterwards, γ having been determined, there will be found the series of corresponding values of k_t and β as functions of i.

I will not stop here to give the details of such experiments or to discuss the precautions to be observed for the exactitude of the measurements.

The importance of this experimental method consists first in its experimental simplicity, since we have only to make measurements upon a screw working at a fixed point; and, further, *it is the only method which allows measurements at the same great values of speeds of flow and in exactly the same blade-screw conditions as in actual use; and this in any fluid, water, air, etc.* The use of this method will without any doubt allow us to elucidate many questions of first importance about fluid resistance at high velocities and in different fluids.[1]

Although it is not my intention in this first memoir to treat the question of the screw working at a fixed point, to which a separate memoir will be devoted, I will, however, give a brief summary of the properties of the plane radial screw, which it will be interesting to note, and which will show in what measure the present screw theory can in reality treat any case of screw-working, including the case at a fixed point, which has always been considered, up to the present, as the most difficult to investigate.

GENERAL PROPERTIES OF THE PLANE-RADIAL SCREW WORKING AT A FIXED POINT.

When we have to do with a blade screw working at a fixed point, its efficiency ρ_o is the quantity which measures for the screw when advancing its fan losses. The losses of a screw at a fixed point thus reduce to the vortex losses p_t and resistance losses p_r. For each blade element we have

$$p_t = \frac{\omega_o}{\Omega_o} = 2\rho_o \operatorname{tg}^2(\varphi+\beta_o) \tag{193}$$

$$p_r = 1-\rho_o-p_t = 1-\rho_o\left[1+2\operatorname{tg}^2(\varphi+\beta_o)\right] \tag{194}$$

Let us examine briefly the conditions of maximum of ρ_o. If we note that ρ_o is an increasing function of i_o so far as β_o is nearly constant, and that β_o is a very rapidly increasing function of i for $i_o \leqq i'$, but that for $i_o > i'$ the variations of β_o are small, it will be easy to see that the maximum of ρ_o will occur for $i_o \cong i'$ and $\beta_o \cong 0$. The conditions of maximum of the efficiency of a blade screw at a fixed point are thus the same as for a blade screw when advancing. We have

$$\rho_o^{\max} = \frac{\operatorname{tg}(\varphi-i')}{\operatorname{tg}\varphi\left[1+2\operatorname{tg}\varphi\operatorname{tg}(\varphi-i')\right]} \tag{195}$$

In figure 33 have been traced by aid of this equation a series of curves of the efficiency $\rho_o^{\max}$ as functions of φ for different values of $i_o = i'$. It is easy to judge by aid of these curves about the maximum which the efficiency $\rho_o^{\max}$ can reach.

[1] According to an agreement which has been made between the author and the National Advisory Committee for Aeronautics, the author has permanently withdrawn from the United States Patent Office his pending patent on plane-radial screws, and has thus abandoned to the Public Domain the use of the plane-radial screw in the United States.

When, for a blade section of a given profile, values for φ and i_o have been adopted, by this fact the value of the breadth will be fixed. In this same figure 33, by aid of equation (105), a series of curves of the breadth ratio have been traced as functions of φ for different values of $i_o = i'$, and consequently $\beta_o = 0$. On the other hand, the limiting value of the breadth ratio is fixed by the relation (134) of Chapter III. The curves of the breadth ratio are thus limited from above, as a first approximation, by the curve of sin φ (see fig. 33).

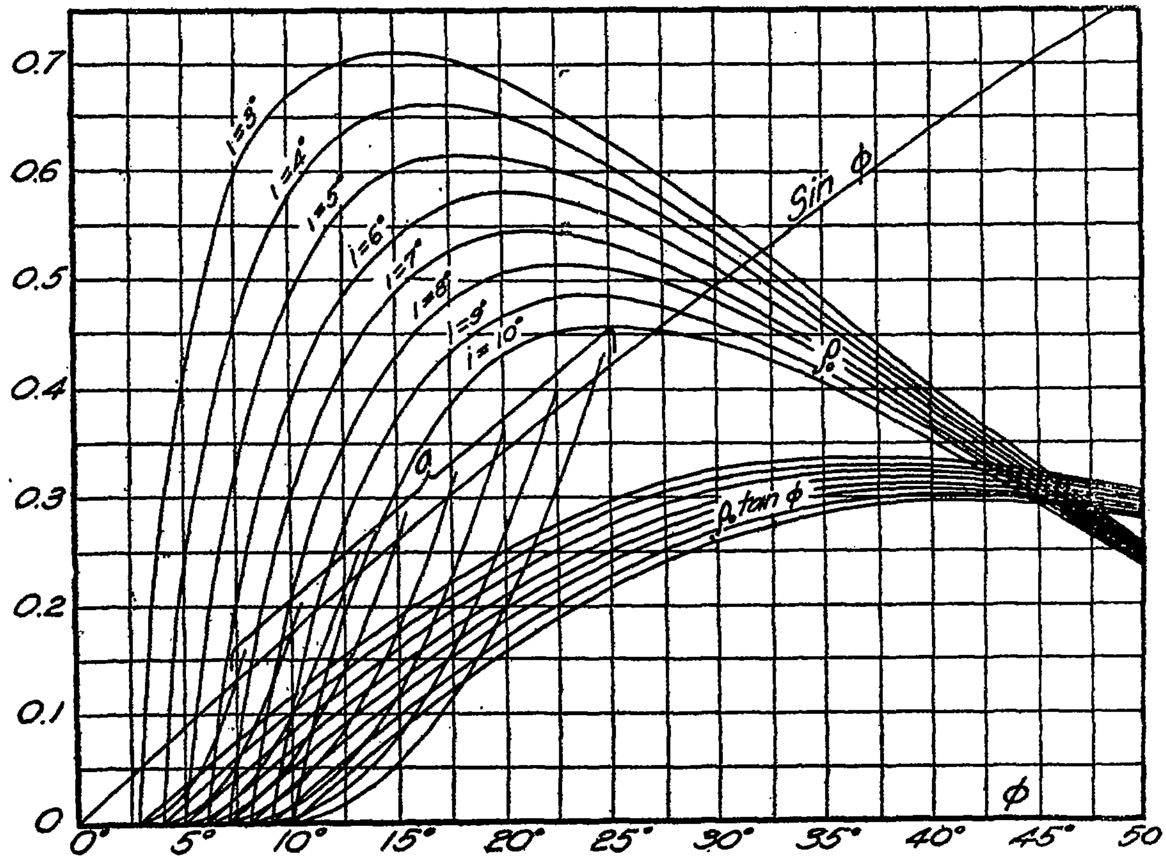

FIG. 33.

Finally, on this same figure 33, for the purpose of a quick calculation of the slip velocity

$$v_o = r\Omega_o\, \rho_o \operatorname{tg} \varphi,$$

has been traced a series of curves of ρ_o tg φ, as functions of φ for different values of $i_o = i'$, and thus $\beta_o \cong 0$.

It has to be noted that the curves of figure 33 are independent of the fluid density.

Let us in formulæ (183) and (184) put

$$r_1 = c\, r_2 = \frac{c\,D}{2} \tag{196}$$

D being the diameter of the screw considered, and substitute $\Omega_o = 2\pi N$. These formulæ can then be written:

$$Q_o = \tfrac{1}{4}\, \pi^3\, (1 - c^4)\, \delta \rho_o^{\,2} \operatorname{tg}^2 (\varphi + \beta_o)\, N^2 D^4 \tag{197}$$

$$\Omega_o C_o = L_o = {}^1/_5\, \pi^4\, (1 - c^5)\, \delta \rho_o^{\,2} \operatorname{tg}^3 (\varphi + \beta_o)\, N^3 D^5 \tag{198}$$

$$= {}^4/_5\, \pi\, \frac{1 - c^5}{1 - c^4}\, Q_o \operatorname{tg} (\varphi + \beta_o)\, ND \tag{199}$$

Let us give a quantitative evaluation of these last formulæ. According to figure 33 it will be easily seen that we will have good working conditions adopting:

$$\varphi = 15°;\ i_0 = 6°;\ a = 0,25;\ \rho_0 = 0,55$$

Introducing these last values in the formulæ (197), (198), and (199) and considering $\beta_0 = 0$ and $\delta = \frac{1}{8}$ (air density) and neglecting the high powers of c, we get

(200) $$Q_0 = 0,021\ N^2 D^4$$

(201) $$L_0 = 0,014\ N^3 D^5$$

(202) $$L_0 = 0,67\ Q_0 N D$$

From the last follows

(203) $$\frac{Q_0}{L_0} = 0,36 \sqrt[3]{\frac{D^2}{L_0}}$$

and

(204) $$N = 4,15 \sqrt[3]{\frac{L_0}{D^5}}$$

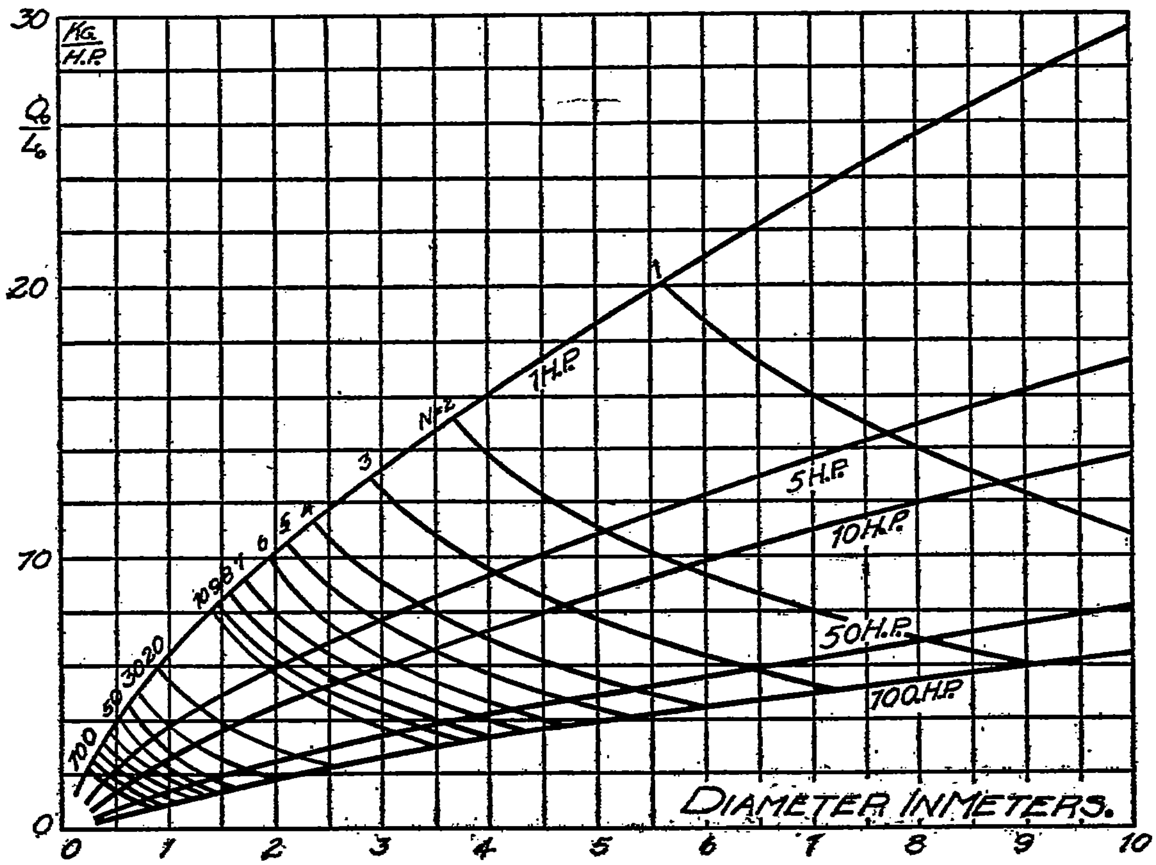

FIG. 34.

On figure 34 is represented according to the relation (203) a series of curves which give Q_0/L_0 in kilograms per horsepower for a plane-radial lifting screw, as function of its diameter D, and for different values of L_0. The power L_0 has been successively taken equal to 1, 5, 10, 50 and 100 horsepower. On the same figure, by aid of the relation (204) have been traced the curves corresponding to N=const for different values of N. We thus see that the thrust furnished by a lifting screw increases with the increase of its diameter, with the decrease of its number of revolutions and with the power. *It is thus the screws of small power and large diameter which turn slowly that will prove the best lifting screws.*

Let us calculate also, for the case of air-screws, their blowing capacity. Let us designate by U the number of cubic meters of air that a fan can blow in a second. We have

$$U=\int_{r_1}^{r_2} 2\pi r\,dr v_0$$

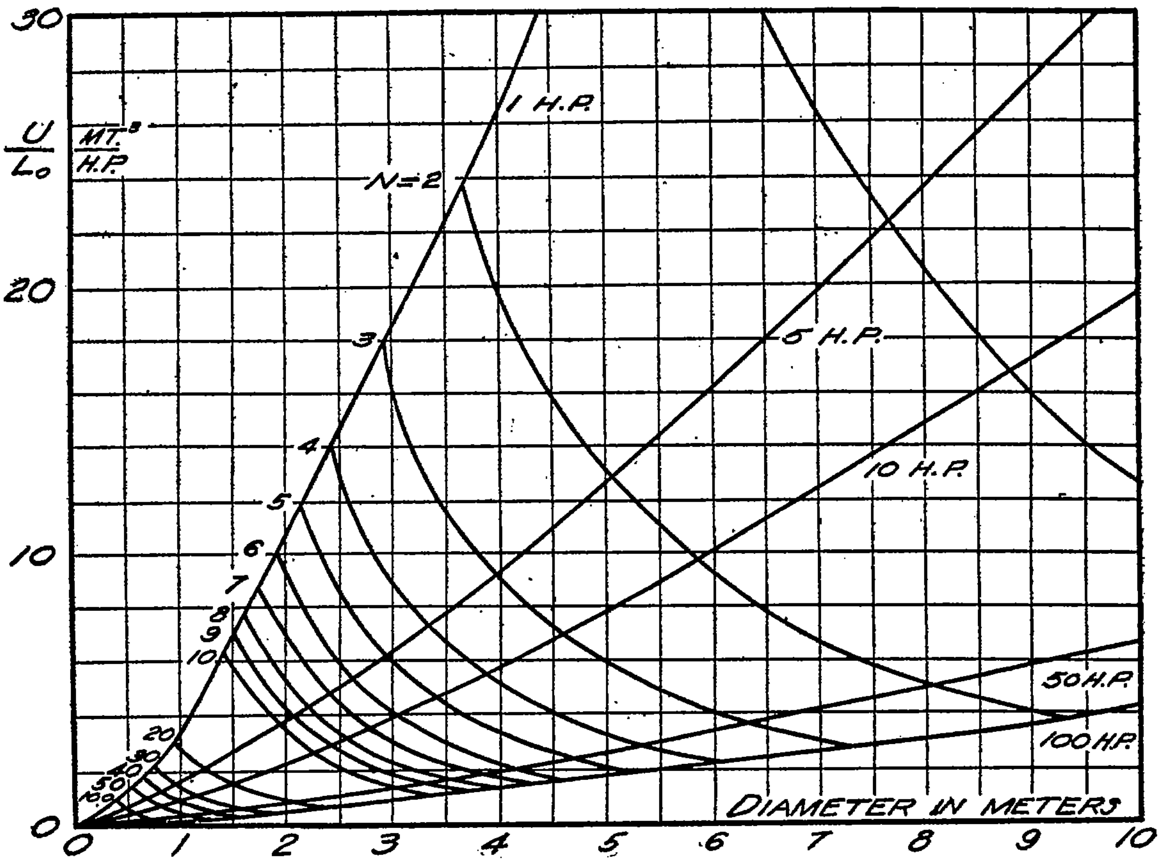

FIG. 35.

and substituting for v_0 its value we get:

$$U=2\pi\rho_0\Omega_0\,\mathrm{tg}\,(\varphi+\beta_0)\int_{r_1}^{r_2} r^2 dr;$$

and integrating we find

$$U=\frac{\pi^2}{6}(1-c^3)\,\rho_0\,\mathrm{tg}\,(\varphi+\beta_0)\,ND^3$$

and substituting finally the numerical values adopted above, we find

$$U=0{,}24\ ND^3$$

and

$$\frac{U}{L_0}=\sqrt[3]{\frac{D^4}{L_0^2}}$$

By aid of this last relation the curves of figure 35 have been traced and they indicate the value of U per horsepower as function of the diameter D for different powers. A second system of curves gives the corresponding numbers of revolutions. We see from these curves that *for fans, as well as for lifting screws, it is the fans of small power, large diameter and slow rotation that furnish the best blowing action.*

The plane-radial screw is thus able to furnish good lifting screws and good fans and its simplicity makes it specially fitted for many applications.

Thus, for example, (see figs. 34 and 35) with a plane-radial screw working 10 turns per second, that is, 600 turns per minute, we will be able to get:

$L_0=1.$	5	10	50	100 h.p.
$D\cong 1.4$	1.95	2.50	3.10	3.50 m.
$Q/L_0\cong 8$	6	5	4	3 kg./h.p.
$U/L_0\cong 6$	3.5	2.5	1.3	1 m.^{3}/h.p.
$Q_0=8$	30	50	200	300 kg.
$U=6$	17.5	25	65	100 m.^{3}

In the practical realization of plane-radial screws one will certainly not be obliged, since the screw is not used as a measuring instrument, to adopt the condition $k_t=$const, that is, the screw will be made with a thickness decreasing toward the blade tips. All the formulae established in the foregoing can be used as a first approximation. In a memoir which will follow the present I will treat in full detail the different kinds of lifting screws and fans and indicate the methods of their design.

George de Bothezat.

NOTE I.

THE THEOREMS OF MOMENTUM AND MOMENTS OF MOMENTUM IN THEIR APPLICATION TO THE STEADY MOTION OF FLUIDS.

The theorems of momentum, moments of momentum, and kinetic energy have been called *the three universal theorems of motion by Paul Appell*, in the sense that they can be applied to any mechanical system. The first two of these theorems are expressed by the following system of equations:[1]

$$\begin{cases} \frac{d}{dt}\Sigma\left(m\frac{dx}{dt}\right)=\Sigma\Sigma X_e \\ \frac{d}{dt}\Sigma\left(m\frac{dy}{dt}\right)=\Sigma\Sigma Y_e \\ \frac{d}{dt}\Sigma\left(m\frac{dz}{dt}\right)=\Sigma\Sigma Z_e \end{cases}$$

$$\begin{cases} \frac{d}{dt}\Sigma\left(x\cdot m\frac{dy}{dt}-y\cdot m\frac{dx}{dt}\right)=\Sigma\Sigma(xY_e-yX_e) \\ \frac{d}{dt}\Sigma\left(z\cdot m\frac{dx}{dt}-x\cdot m\frac{dz}{dt}\right)=\Sigma\Sigma(zX_e-xZ_e) \\ \frac{d}{dt}\Sigma\left(y\cdot m\frac{dz}{dt}-z\cdot m\frac{dy}{dt}\right)=\Sigma\Sigma(yZ_e-zY_e) \end{cases}$$

or, in vector notation:[2]

$$\frac{d}{dt}\Sigma\overline{mv}=\overline{F};\ \frac{d}{dt}\Sigma\overline{r\cdot mv}=\overline{R\cdot F} \tag{1}$$

$\overline{F}$ being the resultant of *all the exterior forces* applied to the system considered; $\Sigma\overline{mv}$ the geometrical sum of the linear moments of the system; $\Sigma\overline{r\cdot mv}$ the resultant moment of momentum of the system; $\overline{R\cdot F}$ the resultant moment of all the exterior forces; these last moments being taken relative to a point invariably connected to the absolute reference system.

Let us consider a fluid mass in a steady state of motion, and let us apply the above-mentioned theorems to the portion of a stream tube included between two of its orthogonal sections S_1 and S_2. Let us calculate the increment of the fluid momentum included between those sections. If, at the moment t the fluid occupies the portion of the tube between S_1 and S_2, at the moment $t+dt$ this same fluid mass will occupy the portion $S'_1\,S'_2$ of the stream tube (see Fig. 36). The motion being steady, the momentum of the fluid mass (III) common to both volumes $S_1\,S_2$ and $S'_1\,S'_2$ will remain invariable. The increment of the momentum of the fluid mass between S_1 and S_2 will thus be equal to the difference of the momentum of the fluid mass II and I. This last difference is equal to

$$\Sigma(\bar{v}_2\cdot m_2dt-\bar{v}_1\cdot m_1dt)=dt\Sigma_1(\overline{m_2v_2}-\overline{m_1v_1})$$

[1] See "Traité de mecanique rationelle," Tome II, 4me Édition, par M. Paul Appell, pp. 19 et 24.

[2] For vectors I use the notation $\overline{X}$—a horizontal line over the letter representing the vector; for the vector product the notation $\overline{A\cdot B}$—a horizontal line over both vectors of the product; for the scalar product the notation $\overline{A}\cdot\overline{B}$; that is, the same notation as for the algebraical product of two scalar quantities.

where m_1 represents the fluid mass which flows in a unit of time through an element ds of section S_1; v_1 the flow velocity at the same element, with similar notions for S_2. We thus have

$$\text{(2)} \qquad \frac{d}{dt}\Sigma\overline{mv} = \Sigma\,(\overline{m_2v_2} - \overline{m_1v_1})$$

By a fully analogous reasoning we find

$$\text{(3)} \qquad \frac{d}{dt}\Sigma\,\overline{r\cdot mv} = \Sigma\,(\overline{r_2 m v_2} - \overline{r_1\cdot mv_1})$$

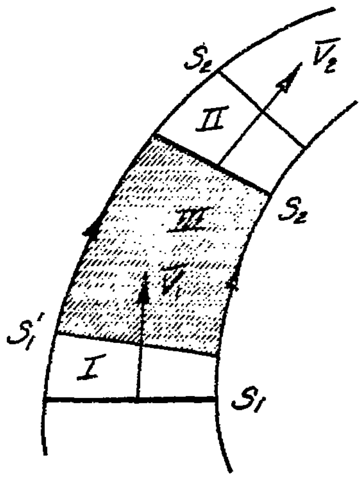

Fig 36.

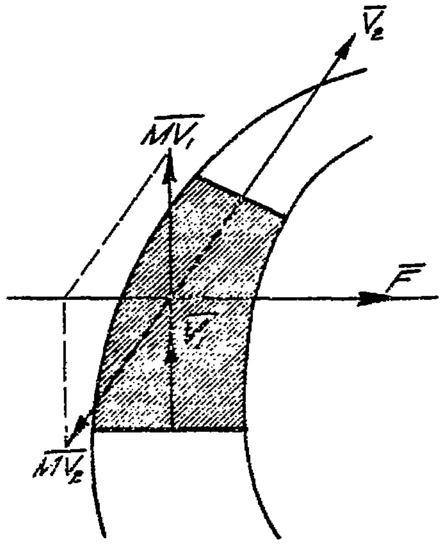

Fig. 37

Let M be the total fluid mass which flows in a unit of time through the stream tube considered, and let us define two vectors $\overline{V}_1$ and $\overline{V}_2$ by the following relations:

$$\text{(4)} \qquad \begin{cases} \overline{MV_1} = \Sigma_1\,\overline{m_1v_1} \\ \overline{R_1\cdot MV_1} = \Sigma_1\,\overline{r_1\cdot mv_1} \end{cases} \qquad \begin{cases} \overline{MV_2} = \Sigma\overline{m_2v_2} \\ \overline{R_2\cdot MV_2} = \Sigma\,\overline{r_2\cdot m_2v_2} \end{cases}$$

When, to a sufficient approximation, v_1 and v_2 can be considered as uniform in the whole section of S_1 and S_2 we shall have

$$\overline{V}_1 \cong v_1; \quad \overline{V}_2 \cong v_2$$

the vectors $\overline{V}_1$ and $\overline{V}_2$ being applied to the centers of mass of S_1 and S_2.

On account of the relations (2), (3), and (4) the equations (1) take the form

$$\overline{MV_2} - \overline{MV_1} = \overline{F}$$
$$\overline{R_2\cdot MV_2} - \overline{R_1\cdot MV_1} = \overline{R\cdot F}$$

or

$$\overline{F} + \overline{MV_1} - \overline{MV_2} = 0,$$
$$\overline{R\cdot F + R_1\cdot MV_1 - R_2\cdot MV_2} = 0,$$

and this brings us to the following theorem:

For a fluid mass in steady motion, the resultant wrench of all the exterior forces applied to a portion of a stream tube limited by two cross sections and of the resultant screw of the inflow momentum (the outflow momentum having to be taken in the reverse sense) is equal to zero. (See fig. 37[1]).

[1] It is supposed on this figure that the resultant screw of the exterior forces is reduced to a resultant force.

In the application of this theorem special attention must be paid to the importance of taking account of *all the exterior forces*. For a stream tube portion these exterior forces are not only *the normal pressures*, but also the *tangential stresses* exercised on its boundary surface.

As the preceding theorem can be applied to each stream tube, it will be easy to see that we can also apply it to any system of stream tubes. We are thus brought to the following theorem:

For a fluid mass in steady motion, the resultant of the resultant wrench of all the exterior forces applied to a portion of the fluid mass inclosed in any closed surface and of the resultant screw of the inflow momentum (the outflow momentum having to be taken in a reverse sense) is equal to zero.

When one or several bodies are plunged in the fluid mass contained in the closed surface considered, *the pressures of the bodies on the fluid* have to be considered as exterior forces for the fluid mass considered.

NOTE II.

GENERALIZATION OF BERNOUILLI'S THEOREM.

For the determination of the pressures in a fluid, we possess the Bernouilli theorem, which furnishes us the law of variation of pressure along a stream line and also along a vortex line. We also know that the Bernouilli theorem is applicable to the whole fluid, considering the Bernouilli constant as invariable, when the fluid motion is irrotational. But in the general case, when we go from one stream line to another, the Bernouilli constant changes its value. What is the law of variation of the Bernouilli constant in the whole fluid mass in the general case? It is the solution of this question that the present note gives. We so arrive at the general solution of the problem of the pressure distribution in a fluid mass.

Let us consider a fluid mass in a steady state of motion. Let us consider in this fluid mass the *stream line curves* and also two other families of fundamental curves: *the normal lines,* defined by the property that the tangent at each point to those curves coincides with the principal normal of the stream line through this point, and *the binormal lines,* defined by the property that the tangent at each point coincides with the binormal to the corresponding stream line. The stream lines, the normal lines, and the binormal lines form a system of triorthogonal curves.[1]

Let us consider a fluid element contained in the elementary parallelepiped, whose edges $d\tau$, $d\nu$, $d\beta$ are respectively directed along the stream lines, the normal lines, and the binormal lines. On these curves we choose the following positive senses: on the stream lines, the sense of the velocity of the fluid particles; on the normal lines, the sense toward the center of curvature of the corresponding stream lines; on the binormal lines, the positive sense is chosen in such a way that the trirectangular trihedral ($d\tau$, $d\nu$, $d\beta$) is positive.

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[1] These curves have for equations:

The stream lines

$$\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w}$$

The normal lines

$$\frac{dx}{Bw-Cv}=\frac{dy}{Cu-Aw}=\frac{dz}{Av-Bu}$$

The binormal lines

$$\frac{dx}{A}=\frac{dy}{B}=\frac{dz}{C}$$

In these equations u, v, w are the components of the velocity of the fluid and A, B, C the determinants of the matrix

$$\left\{\begin{matrix} u & v & w \\ \frac{du}{dt} & \frac{dv}{dt} & \frac{dw}{dt} \end{matrix}\right\}$$

For example

$$A=v\frac{dw}{dt}-w\frac{dv}{dt}$$

expressions in which $\frac{du}{dt}$, $\frac{dv}{dt}$, $\frac{dw}{d}$ are the total derivatives, for example

$$\frac{du}{dt}=u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial w}{\partial z}$$

the motion being steady.

Let us apply the d'Alembert principle to the fluid element $d\tau$, $d\nu$, $d\beta$ and let us consider, for the sake of simplicity, the fluid as incompressible and having no weight (see fig. 38). The resultant of the exterior pressure on the fluid element has for components:

$$-\frac{\partial p}{\partial \tau} d\tau\, d\nu\, d\beta \text{ along } d\tau$$

$$-\frac{\partial p}{\partial \nu} d\tau\, d\nu\, d\beta \text{ along } d\nu$$

$$-\frac{\partial p}{\partial \beta} d\tau\, d\nu\, d\beta \text{ along } d\beta$$

p being the pressure at the point considered.

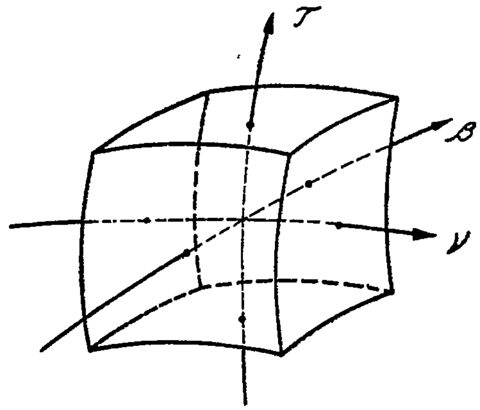


FIG. 38

The resultant of the forces of inertia applied to that element has for components

$$-\frac{dV}{dt} \delta\, d\tau\, d\nu\, d\beta \text{ along } d\tau$$

$$-\frac{V^2}{\rho} \delta\, d\tau\, d\nu\, d\beta \text{ along } d\nu$$

$$0 \text{ along } d\beta$$

δ being the density of the fluid at the point considered, V the velocity and ρ the radius of the principal curvature.

According to the d'Alembert principle we must have

$$\frac{\partial p}{\partial \tau} + \delta \frac{dV}{dt} = 0 \tag{1}$$

$$\frac{\partial p}{\partial \nu} + \delta \frac{V^2}{\rho} = 0 \tag{2}$$

$$\frac{\partial p}{\partial \beta} = 0 \tag{3}$$

This system of relations represents the equations of motion of the fluid referred to the tri-orthogonal curvilinear system of stream lines, normal lines, and binormal lines, which can be called the *natural curvilinear coordinates of the fluid*, or, shorter, the *natural coordinates of the fluid.*

I. The equation (1) brings us directly to the Bernouilli theoren; we have

$$\frac{\partial p}{\partial \tau}+\delta\frac{dV}{dt}=\frac{\partial p}{\partial \tau}+\delta\frac{dV}{d\tau}\frac{d\tau}{dt}=\partial p+\delta V\frac{dV}{d\tau}=0$$

and integrating along a stream line, we get

$$p+\frac{\delta V^2}{2}=C \tag{4}$$

a relation which constitutes the Bernouilli theorem, C being the Bernouilli constant.

II. The equation (2) gives us the distribution of pressure along the normal lines. Integrating this equation along a normal line, we get

$$p+\int\delta\frac{V^2}{\rho}d\nu=C \tag{5}$$

This last equation is susceptible of the following important transformation: [1]

Let us designate by ω_τ, ω_ν, ω_β the components of the vortex and by V_τ, V_ν, V_β the components of the resultant velocity V along the directions $d\tau$, $d\nu$, $d\beta$, at the point considered. We have

$$V_\tau=V;\ V_\nu=0;\ V_\beta=0$$

The relations between the double of the components of the vortex $\bar{\omega}$ and the components of the velocity are given by the determinants of the matrix

$$\left\{\begin{matrix}\frac{\partial}{\partial\tau} & \frac{\partial}{\partial\nu} & \frac{\partial}{\partial\beta}\\ V_\tau & V_\nu & V_\beta\end{matrix}\right\} \tag{6}$$

we thus have

$$2\omega_\beta=\frac{\partial V_\nu}{\partial\tau}-\frac{\partial V_\tau}{\partial\nu} \tag{7}$$

or

$$\frac{\partial V_\nu}{\partial\tau}-2\omega_\beta=\frac{\partial V_\tau}{\partial\nu}=\frac{\partial V}{\partial\nu} \tag{8}$$

[1] By aid of relation (5) the pressure distribution in a slip stream cross section as S'', for example, can be calculated. For such a calculation we can admit as a first approximation that the elements of the trajectories of the fluid particles in section S'' are elements of cylindrical helicoidal curves. Under such conditions in the section S'' the normal lines will be radial straight lines normal to the screw axis and we will have

$$d\nu=dr''$$

and

$$\rho=\frac{r''}{\cos^2\alpha};\ W''\cos\alpha=r''\omega''$$

α being the inclination of the elements of the helicoidal trajectories to the plane of screw rotation and W'' the velocity of the fluid particles in he section S''. Substituting these values in equation (5) from above we get:

$$p=p_0-\delta\int r''\omega''^2dr''$$

or

$$p=p_0-\frac{\delta}{2}\int\omega''^2d(r''^2)$$

where p_0 is the pressure on the boundary surface of the slip stream, δ the fluid density considered constant for reasons already mentioned. Knowing the law of distribution of ω'' in the section S'', the curve of $\delta\omega''^2/2$ will be traced as a function of r''^2. The quadrature of the surface limited by this curve will give the law of pressure distribution in a cross section. The general course of such a pressure distribution is represented on figure 2 of the first chapter. For a more accurate calculation it would be necessary to know the radii of principal curvature of the stream lines along a normal line.

On the other hand (see Fig. 39)

$$\frac{\partial V_\nu}{\partial \tau}=\frac{V d\theta}{\partial \tau}=\frac{V}{\rho} \tag{9}$$

$d\theta$ being the contingence angle. Substituting this last value of $\frac{\partial V_\nu}{\partial \tau}$ in the relation (8), for an integration along a normal line we get

$$d\nu=\frac{dV}{\frac{V}{\rho}-2\omega_\beta} \tag{10}$$

and substituting this last value of $d\nu$ in equation (5), we get

$$p+\int\frac{\delta V dV}{1-2\rho\frac{\omega_\beta}{V}}=C \tag{11}$$

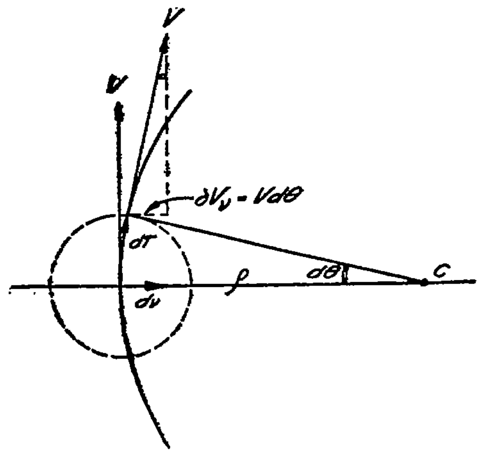


FIG. 39.

The integral of this last relation is susceptible of the following transformation:

$$\int\frac{\delta V dV}{1-2\rho\frac{\omega_\beta}{V}}=\int\frac{\delta V dV\left[1-\left(1-2\rho\frac{\omega_\beta}{V}\right)+\left(1-2\rho\frac{\omega_\beta}{V}\right)\right]}{1-2\rho\frac{\omega_\beta}{V}}$$

$$=\int\delta V dV-\int\frac{\delta V dV\cdot 2\rho\frac{\omega_\beta}{V}}{1-2\rho\frac{\omega_\beta}{V}}$$

$$=\frac{\delta V^2}{2}-\int\frac{\delta V dV}{\frac{V}{2\rho\omega_\beta}-1}$$

and equation (11) takes the form

$$p+\frac{\delta V^2}{2}=C+\int\frac{\delta V dV}{\frac{V}{2\rho\omega_\beta}-1} \tag{12}$$

which fixes the distribution of pressure along the normal lines.

We easily see that the last equation has the form of the Bernouilli equation, only the integra which figures in the second member determines the variation of the Bernouilli constant when we go from one stream line to another along a normal line.

If we put

(13)
$$\Delta C = \int \frac{\delta V dV}{\frac{V}{2\rho\omega_\beta} - 1}$$

equation (12) takes the form

(14)
$$p + \frac{\delta V^2}{2} = C + \Delta C$$

We now see it is sufficient that $\omega_\beta = 0$ along a normal line—which means that on the normal line considered the vortex $\bar{\omega}$ is disposed in the contingence plane—for the integral ΔC to be equal to zero and the Bernouilli constant to be invariable along this normal line. It is evident that ω_β is zero when $\bar{\omega} = 0$.

The integral ΔC can be written in the form

(15)
$$\Delta C = \int \frac{\delta dV}{\frac{1}{2\rho\omega_\beta} - \frac{1}{V}}$$

and is then susceptible of the following geometrical interpretation: The denominator of this integral represents the difference between the inverse of the speed which the fluid particle would have if rotating with the angular velocity $2\omega_\beta$ around the center of curvature of its instantaneous position and the inverse of the velocity V of the particle.

III. The integration of equation (3) along the binormal lines leads directly to the conclusion that along those lines

(16)
$$p = \text{const}$$

that is to say, *in the case of a non-heavy fluid, the binormal lines are isobars.* It will be easily seen *that in the case of a heavy fluid the distribution of pressure along the binormal lines will be the same as if the fluid were immobile.*

We also see that *for the case of irrotational motion of a fluid the binormal lines are also the ines of constant velocities*, the Bernouilli theorem being applicable to the whole fluid mass.

The system of relations (11), (12), and (13) fully determines in the general case the distribution of pressures in a fluid mass in motion. This system of relations leads us to the following consequences, which I will indicate in general outlines:

I. It is enough to know the distribution of pressure along a surface cutting all the binormal lines in order to know the distribution of pressure in the whole fluid mass.

This proposition is a direct consequence of the fact that the pressure is constant along a binormal line.

II. On both sides of a vortex layer, even thin, there can exist a difference of pressure which can be of sensible value.

To convince ourselves of such a possibility, it is sufficient to represent a vortex layer in which the quantity

$$V - 2\rho\omega_\beta$$

has a small value inside the layer, which can happen without V or ω_β having excessive values. Then, when traversing the layer along a normal line, the integral

$$\int \frac{\delta V^2 dV}{V - 2\rho\omega_\beta}$$

can have a large value and consequently, according to formula (11) the difference of pressure on the two sides of the layer will be sensible.

The conception of a thin vortex layer maintaining sensible differences of pressure allows one to understand the phenomena which take place in the slip stream created by the rotation of a propeller. Let us follow a stream line in the slip stream (see fig. 40). When we reach a point, such as B, disposed before the propeller, the pressure p must necessarily be lower than the exterior pressure p_0, because the velocity is all the time increasing as we approach the propeller and at points such as A_1 and A_2 we have pressures very close to the pressure p. But when we go through the plane of screw rotation, the pressure increases and at a point such as C disposed

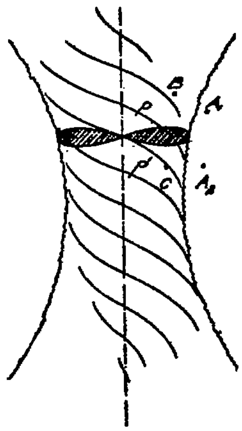


FIG. 40.

directly behind the propeller, the pressure p' is generally greater than the exterior pressure. It would be difficult to conceive the existence of a difference of pressures p' and p at points C and A_2, if it were not for the vortex layer, which consequently must constitute the surface of the slip stream created by the screw, and which we know capable of maintaining differences of pressure. Without the knowledge of the existence of the vortex layer forming the surface of the slip stream the pressure distribution around a propeller would be difficult to conceive. The exact configuration of the boundary surface of the slip stream demands further investigations.

NOTE III.

SHORT SUMMARY OF THE EMPIRICAL LAWS OF FLUID RESISTANCE OF AEROFOILS.

Let us consider a cylindrical surface generated by a rectilinear generatrix moving parallel to itself along a plane contour formed by two intersecting curve segments. Such surfaces are generally called *aerofoils* in aerodynamics. The orthogonal section of the aerofoil is called its *profile*. On figure 41 is represented in plane and profile such an aerofoil of rectangular perimeter of breadth b and span L.

Let us consider an aerofoil plunged in a fluid medium and moving in the last, normally to its generatrices, with a uniform and rectilinear velocity $\overline{W}$, or let us admit that a fluid current of uniform velocity $\overline{W}$ is directed in an inverse sense on the aerofoil maintained immobile. In both cases, on account of the principle of relativity of hydrodynamics, the aerofoil will be acted on by a resultant fluid resistance $\overline{R}$. When the aerofoil has a plane of symmetry normal to its generatrices and the flow velocity $\overline{W}$ is parallel to this plane, the fluid resistance $\overline{R}$ is then disposed in the plane of symmetry.

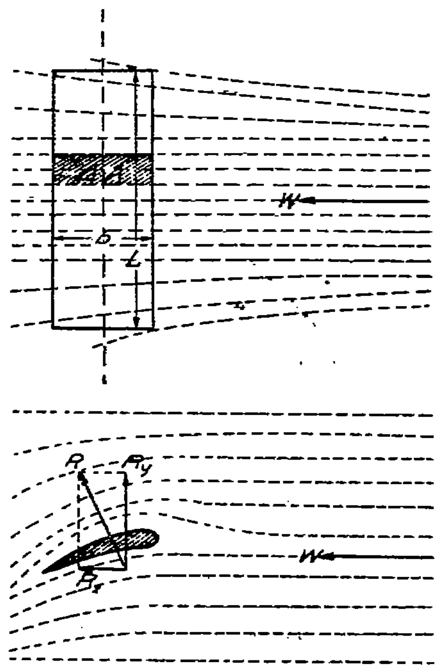


FIG. 41.

The fluid resistance $\overline{R}$ of an aerofoil obeys the following empirical laws:

1. R is proportional to the area A of the aerofoil.
2. R is proportional to the square of the velocity W.
3. R is a function of the orientation of the aerofoil relative to the flow velocity $\overline{W}$.
4. R is proportional to the fluid density δ.

These empirical laws are submitted to the following restrictions:

The proportionality of the resistance R to the area A holds true only for aerofoils of similar perimeters and of the same order of size. If we imagine a series of aerofoils of breadth b whose span L goes on increasing, it will be found that the ratio of R to A tends toward a certain limit when the aspect ratio L/b increases. Practically this limit is already reached for values of the aspect ratio near five or six. The existence of a limit for the ratio R/A depending upon the aspect ratio is due to the fact that the flow runs off, as it were, from aerofoil tips. But with increasing aspect ratio the tip influence rapidly decreases and the ratio R/A tends to its limiting value corresponding to an aerofoil with infinite aspect ratio. Thus, for a sufficient aspect ratio all area elements like ΔA will be in practically identical flow conditions and the total fluid resistance R can be considered as the sum of equal partial resistances ΔR due to each element ΔA (see fig. 41).

The proportionality of the fluid resistance R to the square of the velocity W is true only for variations of W in certain intervals. The component R_x of R along W is generally called *drag*, and the component R_y of R along the normal to W is called *lift*. It has already been observed by experiment that the ratio R_y/W^2 does not vary much with W, but that the ratio R_x/W^2 decreases with increasing W.

As for the dependence of resistance $\overline{R}$ upon the orientation of the aerofoil in the flow, the following facts are to be noted: There exist in general four orientations of the aerofoil in the flow for which the resistance $\overline{R}$ reduces to the drag R_x only. I call *zero plane*[1] the plane parallel to the aerofoil generatrices and containing $\overline{R}$ when $R_y=0$. It is by the orientation of the zero plane corresponding to the entering edge that we shall fix the orientation of the aerofoil relative to the flow. We will call *angle of attack* or *incidence* and will designate by i the acute angle between the velocity W and the zero plane adopted. We shall call *zero line* the trace of the zero plane in the plane of symmetry (see fig. 42).

At the present time it is customary to fix the orientation of aerofoils relative to the flow by the orientation of the chord of their profile. It must be noted that the notion of a chord is defined in geometry only in relation to arcs of curves and in reference to an aerofoil it needs a special definition. For a profile such as the one represented in figure 43a the chord instinctively adopted is the chord common to both arcs limiting the profile. But for a profile such as the one represented in figure 43b we can draw two such chords. Finally, for a profile such as the one of figure 43c any straight line crossing the profile could be with equal propriety adopted as the chord. We thus see that what is commonly called the chord of a profile is in reality a straight line arbitrarily chosen. When it concerns the experimental measurement of fluid resistance of aerofoils, the more convenient reference line has to be taken. But when we wish to proceed to the comparison of the results obtained, it is necessary to have a standard reference line whose definition is based on aerodynamical or hydrodynamical facts. The zero line introduced in the study of aerofoils by *Paul Painlevé* constitutes such a uniform basis for comparisons. The right understanding of the above explanation is of particular importance for the blade-screw theory. The conception of *effective pitch,* which is a direct consequence of the notion of zero line, at once clears up many misunderstandings, such as negative slip, for example.

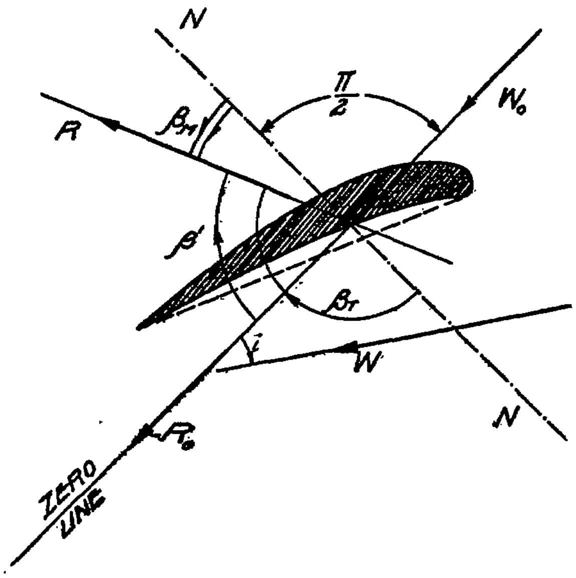


FIG. 42.

Finally, the fluid resistance is proportional to the fluid density δ. This resistance thus depends upon the temperature and pressure of the fluid considered.

We thus see that the fluid resistance R of an aerofoil can be expressed by the formula

(1) $$R=k_i\delta A W^2=K_i A W^2$$

with

$$k_i\delta=K_i$$

In this formula, k_i is an empirical function of the angle of attack which depends upon the perimeter and the aerofoil profile and upon the kind of fluid considered; A is the area of the aerofoil, or speaking more exactly, the area of the projection of the aerofoil on the zero plane; W is the flow velocity relative to the aerofoil. The formula itself is true only for a certain interval of velocity variation.

[1] The zero planes have been introduced into the study of laws of air resistance of aerofoils by *Paul Painlevé*, who called them "fictitious planes."

For small angles of attack we can, with a sufficient approximation, adopt

$$R = k\delta A W^2 i = K A W^2 i \tag{2}$$

with

$$k\delta = K$$

For most aerofoils moving in air the coefficient k has the mean value

$$k \cong \frac{1}{25} = 0{,}04 \tag{3}$$

the angle i being expressed in degrees, the area A in square meters, the velocity W in meters per second, and the resistance in kilograms. For mean conditions of temperature and pressure the coefficient K has, thus, for its mean value

$$K = k\delta = 0{,}04.0{,}125 = 0{,}005 = \frac{1}{200} \tag{4}$$

All the foregoing relates to the magnitude of the fluid resistance $\overline{R}$. As for the position and orientation of the fluid resistance $\overline{R}$ of aerofoils the following takes place:

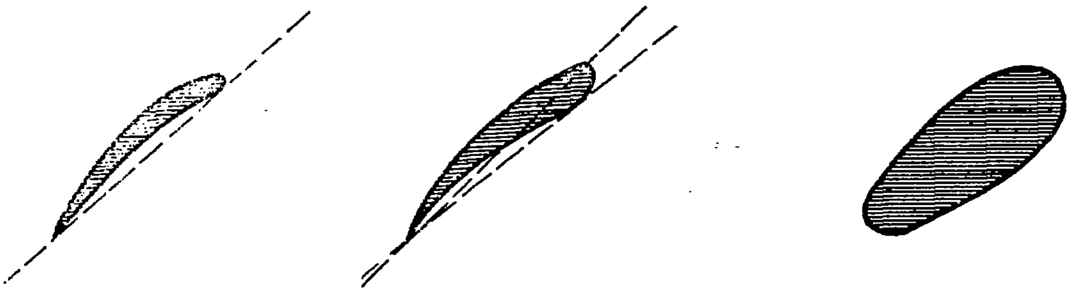


FIG. 43a. FIG. 43b. FIG. 43c.

We will fix the orientation of $\overline{R}$ by the angle this force makes either with the zero plane, or with the normal to the zero plane. The first of these angles will be designated by β', the second by β_H or β_T. The senses adopted as positive for these angles are represented in figure 42. These three angles are connected by the relations

$$\beta_H = \frac{\pi}{2} - \beta' = \pi - \beta_T \tag{5}$$

In cases where no confusion will be possible, we will simply write β instead of β_H or β_T.

We will fix the position of the resistance $\overline{R}$ relative to the aerofoil by the distance of its point of intersection with the zero plane counted from the projection of the entering edge on the zero plane and will call this point *center of pressure.*

For intervals of variation of the velocity W not too large, the angle β and the position of the center of pressure are independent of the value of W and are functions of the angle of attack i only.

When the center of pressure is defined as the intersection of the fluid resistance $\overline{R}$ and the *chord* of the profile, this last center of pressure moves into infinity for a certain value of the angle of attack. This takes place at the moment when $\overline{R}$ is parallel to the chord. Such a displacement of the center of pressure is due only to an inappropriate definition. When our definition of the center of pressure is adopted, this point tends toward a definite limit when the angle of attack tends toward zero.

In experimental aerodymanics it is customary to consider the fluid resistance $\bar{R}$ decomposed into its two components the drag R_x and the lift R_y. We have

$$R_x = R \sin (\beta_H + i) = K_i A W^2 \sin (\beta_H + i)$$
$$R_y = R \cos (\beta_H + i) = K_i A W^2 \cos (\beta_H + i)$$

When the angle β_H is a function of the angle of attack only, we can write

$$R_x = K_x A W^2 \tag{6}$$
$$R_y = K_y A W^2 \tag{7}$$

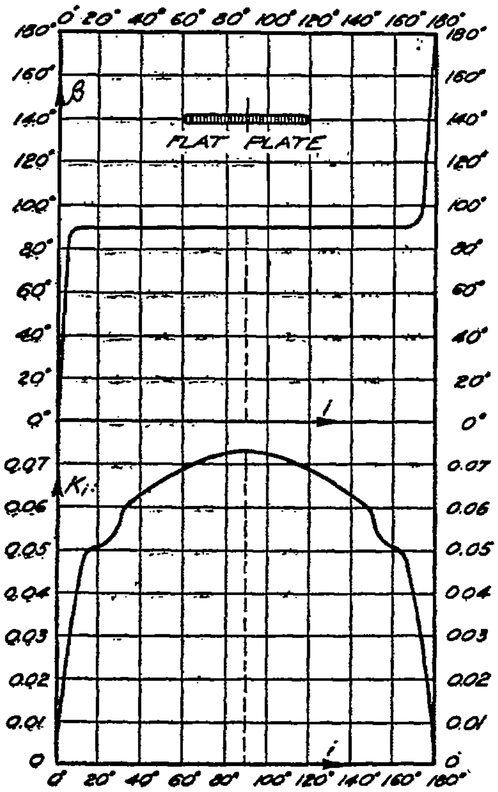


FIG. 44.

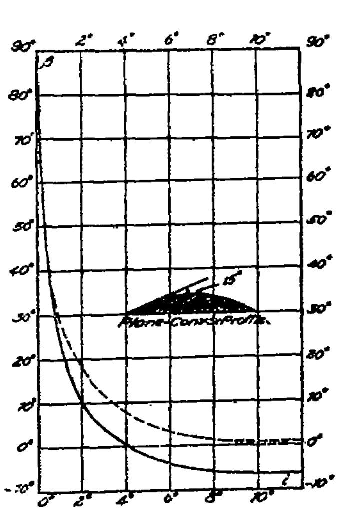


FIG. 45.

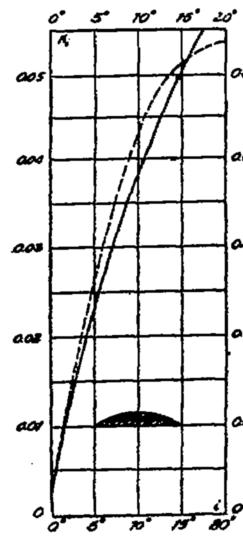


FIG. 46.

expressions in which K_x—called drag coefficient—and K_y—called lift coefficient—are functions of the angle of attack i only (for given conditions of temperature and pressure). The quantities K_i and β_H are connected with the coefficients K_x and K_y by the obvious relations:

$$\beta_H = \operatorname{arctg} \frac{K_x}{K_y} - i \tag{8}$$

$$K_i = \sqrt{K_x^2 + K_y^2} \tag{9}$$

The following figures give a general idea of the course of variation of the empirical functions K_i and β for the case of aerofoils moving in air.

Figure 44 gives the curves of K_i and β' as functions of the angle of attack i for a flat plate. The empirical function K_i follows very nearly a linear law for the interval of small values of the angle of attack. The ratio $\Delta K_i/\Delta i$ is larger for small values of i than for large values of the last.

For increasing values of the angle of attack, starting from zero degrees, β' at first increases very rapidly, but afterwards remains very nearly equal to 90° for $10° < i < 170°$.

In figure 45 is represented the empirical function β for a plano-convex profile. The line in dots in that figure corresponds to the flat plate. For a plano-convex profile the angle β rapidly reaches the value zero as i increases from zero; afterwards its variation is small. We will designate by i' the value of the angle of attack for which $\beta = 0$. This angle of attack i' is an important characteristic of a given profile in relation to the efficiency which can be expected from such a profile when used as a blade section. In figure 46 is represented the curve of K for a plano-convex profile; the curve in dots corresponds to the flat plate.

For most aerofoil profiles the empirical functions K_i and β have the same course of variation. In magnitude the fluid resistance $\bar{R}$ follows a nearly linear law for small values of the angle of attack; for larger values of the angle of attack the variations of R are more moderate. In orientation, for increasing values of the angle of attack, the line of action of the fluid resistance $\bar{R}$ very rapidly rises out of the zero plane and afterwards remains sensibly normal to the zero plane. This general character of variation of the fluid resistance in magnitude and orientation is of first importance for the properties of blade screws.

The fluid resistance $\bar{R}$ of an aerofoil is the consequence of very complicated hydrodynamical phenomena which take place in the fluid around the aerofoil and whose principal characteristics are:

A. Above the aerofoil we have a decrease of pressure; below, an increase. For most aerofoils the decrease of pressure above is greater than the increase below; so that the larger part of the fluid resistance is due to a suction on the upper surface of the aerofoil.

B. From the tips of an aerofoil run off vortices called tip vortices.

C. Behind the aerofoil the flow is generally not steady, but periodical. When measuring the fluid resistance of an aerofoil disposed in the wake of another the flow in the wake appears as deflected downwards.

For more details about all these questions, see the Author's "Introduction to the Study of the Laws of Air Resistance of Aerofoils."

NOTE IV.

GENERALIZATION AND GENERAL DISCUSSION OF KUTTA'S THEOREM ON CIRCULATION.

The circulation theorem discussed in the present note was first indicated for a particular case by W. M. Kutta.[1] Soon afterwards, Kutta [2] and Joukowski [3] recognized the generality of the theorem. This theorem is announced as follows:

When a rectilinear and uniform fluid current, having at infinity the velocity $\overline{V}$, flows normally to the generatrix of an infinite cylinder of any section, and when the circulation along the contour embracing the cylinder and situated in the plane of one of its orthogonal sections has a finite value I, the component R_y of the resultant pressure of the fluid on the cylinder, taken along the normal to the velocity and referred to the unit of length of the last is equal to the product of the velocity V, the circulation I and the density δ of the fluid; the sense from R_y to $\overline{V}$ is coincident with the sense of the circulation.

According to this theorem, the lift experienced per unit length of the cylinder is expressed by the following formula

$$R_y = \delta V I$$

We shall establish two fundamental and quite general relations from which the circulation theorem will appear as a particular case.

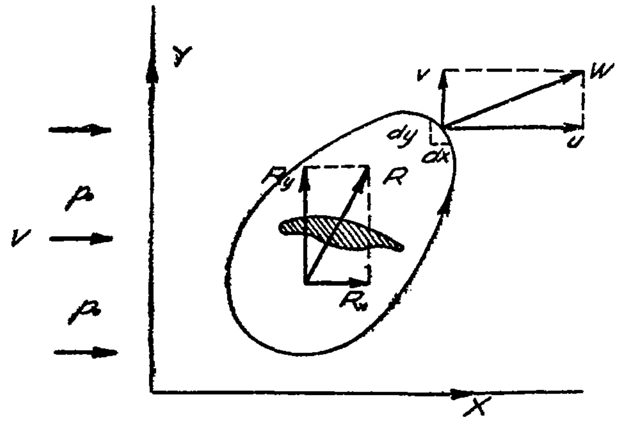


FIG. 47.

Let us embrace the infinite cylinder considered by any contour disposed in the plane of one of its orthogonal sections. Let W be the velocity of the fluid at the point (x, y) of the contour; u and v the components of the velocity W along the axis (see fig. 47); dx and dy the projections of one element of the contour on the axis. Let us designate by X and Y the components of the resultant force of all the exterior forces applied to the fluid contained in the contour considered, and let us apply the theorem of momentum to the motion of the portion of the fluid considered. We thus have

$$Y = \int v dm; \quad X = \int u dm \tag{1}$$

[1] *W. M. Kutta*, "Illustrirte Aeronautische Mitteilungen," 1902.

[2] *W. M. Kutta*, "Sitzungsberichte der Königlichen Bayerischen Academie der Wissenschaften," Munich, 1910 and 1911.

[3] *N. E. Joukowski*, "Geometrische Untersuchungen über die Kutta'sche Strömung," Moskow, 1910, 1911. See also his course, "Aerodynamique, Paris, 1916, p. 139.

the integrals being taken around the contour and dm representing the fluid mass which flows per unit of time through one element of the contour into the exterior space. Let us designate by ψ the current function. By the definition of that function, we have

(2) $$dm=\delta d\psi$$

and also

(3) $$d\psi=udy-vdx=\frac{\partial\psi}{\partial y}dy+\frac{\partial\psi}{\partial x}dx$$

with

(4) $$u=\frac{\partial\psi}{\partial y};\ v=-\frac{\partial\psi}{\partial x}$$

Substituting in the first of the equations, (1), the value of dm taken from equation (2) we get

(5) $$Y=\int\delta vd\psi=\int\delta v(udy-vdx)$$

or, identically,

$$Y=\int\delta\left[v(udy-vdx)+u^2dx-u^2dx\right]$$
$$=\int\delta u(udx+vdy)-\int\delta(u^2+v^2)dx$$

and remarking that

(6) $$udx+vdy=dI$$

is the flow dI along an element of our contour, we get

(7) $$Y=\int\delta udI-\int\delta W^2dx$$

and finally, integrating by parts the first term of the second member of that relation, we get

(8) $$Y=\left[\delta uI-\int\delta Idu\right]-\int\delta W^2dx$$

which relation holds for any contour and constitutes the first of the relations we wished to get.

Applying that relation (8) to a contour along which

$$v=0;\ u=V=\text{const}$$

we easily see that we have

(9) $$\int\delta Id\,u=0;\ \int\delta W^2dx=0$$

and consequently Y reduces to

(10) $$Y=\delta VI$$

I being the circulation around the contour in the direction of rotation of the X axis into the Y axis.

Following the same method with the second of the equations (1), we get

(11) $$X=\int\delta ud\psi=\int\delta u(udy-vdx)$$

$$=\int\delta(u^2dy-uvdx-v^2dy+v^2dy$$
$$=\int\delta(u^2+v^2)dy-\int\delta v(udx+vdy)$$

(12) $$X=\int\delta W^2dy-\int\delta vdI$$

(13) $$X=\int\delta W^2dy-\left[\int\delta vI-\int vIdv\right]$$

the last of these equations constitutes the second relation we wished.

Applying this last relation to a contour along which

$$v=0;\ u=V=\text{const}$$

we easily see that we have

$$X=0, \tag{14}$$

all three of the terms of the second member of the relation (13) being equal to zero.

Let us now stop to note the exact interpretation of the relations (10) and (14). As has been indicated, X and Y are the components of the resultant forces of all the exterior forces applied to the fluid volume contained in the contour considered. These forces are: first, the pressure of the cylinder on the fluid, which are equal and opposite to the pressures of the fluid on the cylinder; second, the exterior pressures on the contour. Let us consider a contour on which $v=0;\ u=V=\text{const}$, and which is limited in one sense by two stream lines sufficiently distant from the cylinder for them to be parallel to the X axis, and in the other sense by two lines perpendicular to these stream lines. Along the stream lines parallel to the X axis we can consider the Bernouilli constant as being effectively constant and in consequence the pressure p constant and equal to the exterior pressure p_o, the velocity V being constant. Under this condition the component along the Y axis of the exterior pressures on our contour will be zero, and Y will represent the reversed component of the pressures on our immersed cylinder. The expression (10) is consequently equal to the negative lift R_y of the fluid on our cylinder. But if we consider a stream line which flows near our cylinder, there must be some interior losses through viscosity along this stream line because each immersed body gives rise to drag. The Bernouilli constant along such a stream line must necessarily decrease, and when we reach the side of the contour parallel to the Y axis where the velocity V has already taken its original value, the pressure there will not take its original value p_o, the Bernouilli constant having decreased. The relation (14) consequently expresses the fact that the component along the X axis of the resultant of the exterior pressures on our contour is exactly equal to the drag, and this holds in the case when the sides of our contour are moved to infinity. In the last case, the exterior pressures tend to their limiting value p_o, but this is not reached, and the integral

$$\int p dy = R_x$$

always remains exactly equal to the drag. Kutta and Joukowski, who were the first to establish the relations (10) and (14), have limited themselves to the consideration of a perfect fluid. In that case, having no interior losses, the Bernouilli constant has an invariable value along any stream line, and relation (14) expresses then the fact that the drag of an immersed cylinder is zero. But it is absolutely unnecessary to limit ourselves to the perfect fluid, since the theorem of momentum, of which equations (10) and (14) are direct consequences, is applicable whatever the interior forces acting on the system considered are.

We are thus brought to the general conclusion that for any contour surrounding an immersed cylinder the following general relations must hold:

$$\int p dx - R_y = \int \delta v(udy - vdx) = \int \delta u dI - \int \delta W^2 dx = \left[\delta uI - \int \delta I du\right] - \int \delta W^2 dx \tag{15}$$

$$\int p dy - R_x = \int \delta u(udy - vdx) = \int \delta W^2 dy - \int \delta v dI = \int \delta W^2 dy - \left[\delta vI - \int vIdv\right] \tag{16}$$

which connect the lift and drag of the cylinder, referred to unit length of the last, with the flow around this cylinder. In the application of these formulae, three particular cases have to be distinguished:

I. The formulae are applied to the contour of the cylinder itself. The contour of the cylinder being a stream line through which we have no flow, we must have

$$R_y = \int p dx; \; R_x = \int p dy$$

which is the case considered in classical hydrodynamics.

II. The formulae are applied to a contour which consists of stream lines and normal lines (for the definition of these lines see Note II). In that case the integrals which figure in the second members of the relations (15) and (16) have to be calculated only along the normal lines.

III. The Kutta case

$$R_y = \delta VI; \; R_x = \int p dy$$

NOTE V.

THE GEOMETRY OF BLADE-SCREW DRAWING.

The tracing of the blade-screw drawing is based on some very convenient conventions, used in practice for a long time, which, however, as far as I know, have never been stated exactly.

For the tracing of a drawing of a blade-screw a *reference radius* has first to be chosen, and on this several *guiding points* are taken through which are drawn axes, which we will call *guiding axes,* parallel to the screw axis. Through the guiding axes are passed planes normal to the reference radius, which we will call *sectional planes.* The plane normal to the screw axis and containing the reference radius will be called the *plane of screw rotation,* and the plane containing the screw axis and the reference radius will be called the *meridional plane.* In principle the reference radius may be chosen arbitrarily—it is only necessary that the sectional planes cut the screw blade—but it is convenient for all the guiding axes to pierce the screw-blade as far as possible. As for the number of guiding points, it is sufficient practically to adopt from four to ten of them.

The drawing of a blade screw may be established either by plane blade sections or by cylindrical blade sections. The method adopted depends upon the process of screw manufacturing used. For some blade screws the difference between both methods of screw drawing is negligible. If it is a drawing by plane blade sections that we wish to have, it is the blade section by the sectional planes that has to be considered. If it is a drawing by cylindrical sections that we wish, we then have to pass cylindrical surfaces, having for axes the screw axis, going through the guiding points and tangent to the sectional planes, and to consider the sections of the blades by these cylindrical surfaces, developed in the sectional planes. All of the following relates to both methods of screw drawing.

Figure 48a gives a general view of a screw blade whose reference radius OR is supposed to go through the point of the blade farthest from the screw axis and is entirely contained in the lower side of the blade. For the sake of clearness in the drawing only the guiding points p_1 and p_2 are represented, through which are traced the guiding axes $a_1\ a_1'$ and $a_2\ a_2'$. The plane blade sections are designated by $s_1\ s_1'$ and $s_2\ s_2'$, and the cylindrical blade sections by $c_1\ c_1'$ and $c_2\ c_2'$. It is assumed for simplicity that the cylindrical blade sections developed in the sectional planes coincide with the plane blade sections. Let us extend the chords of the blade sections considered and take on the intersections of the sectional planes and the screw rotation plane lengths such as $p_1\ l_1$; $p_1\ l_1'$; $p_2\ l_2$; $p_2\ l_2'$ ...., respectively, equal to r_1, $2\pi r_1$, r_2, $2\pi r_2$ ......; the quantities r_1 and r_2 being the distances of the guiding points from the screw axis. The chord of each section will cut off, on the perpendiculars to the screw rotation plane through points such as l' and l, lengths respectively equal to H and $H/2\pi$ designating by H the constructive pitch of the sections considered.[1]

First generation of the screw drawing.—The sectional planes, containing the blade sections, are turned through an angle of 90° around the guiding axis in such a way that the heights $H_1/2\pi$, $H_2/2\pi$, etc., come on the screw axis. The sections $s_1\ s_1'$; $s_2\ s_2'$ .... will take the positions $t_1\ t_1'$; $t_2\ t_2'$ .... and will all be brought into the meridional plane. In figure

[1] When a guiding axis does not pierce the blade, it is the extension of the chord of a section that meets that axis; it is from this last point that the construction indicated above has to be made in order to find the pitch.

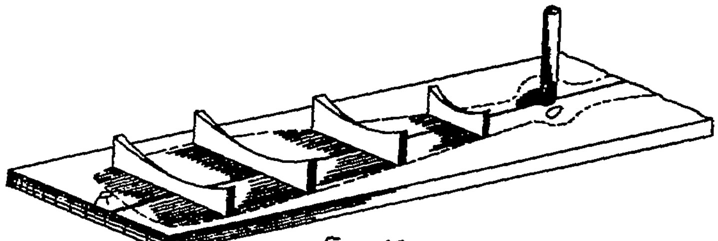


Fig. 48e

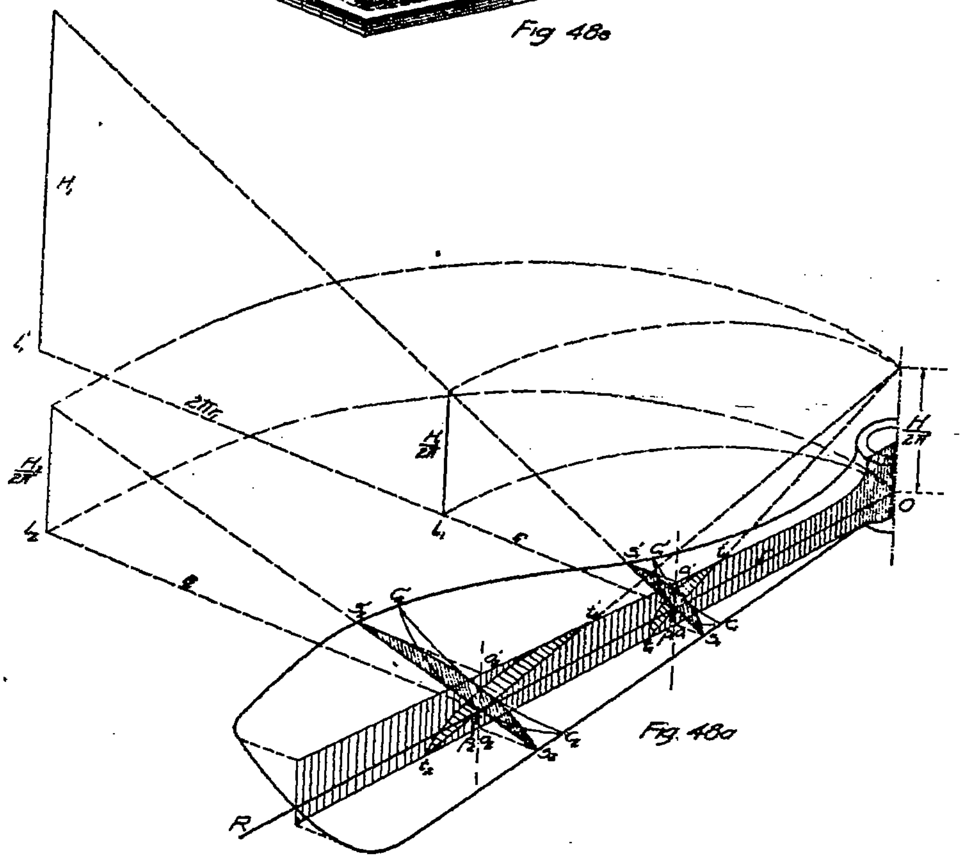


Fig. 48a

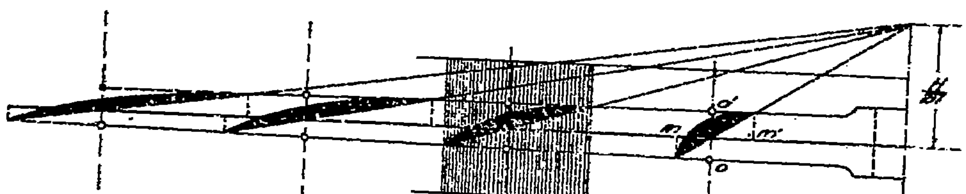


Fig. 48b

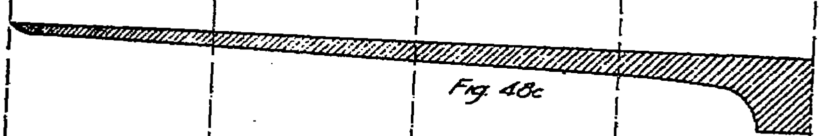


Fig. 48c

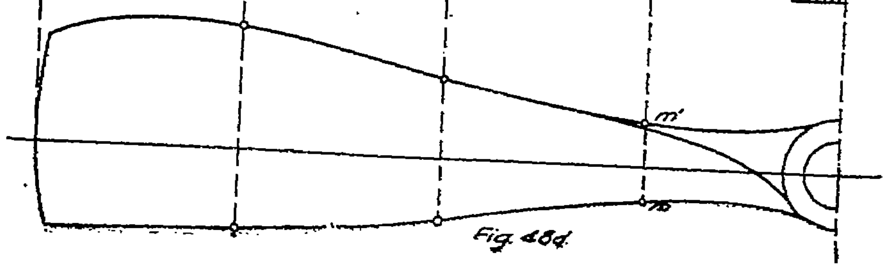


Fig. 48d

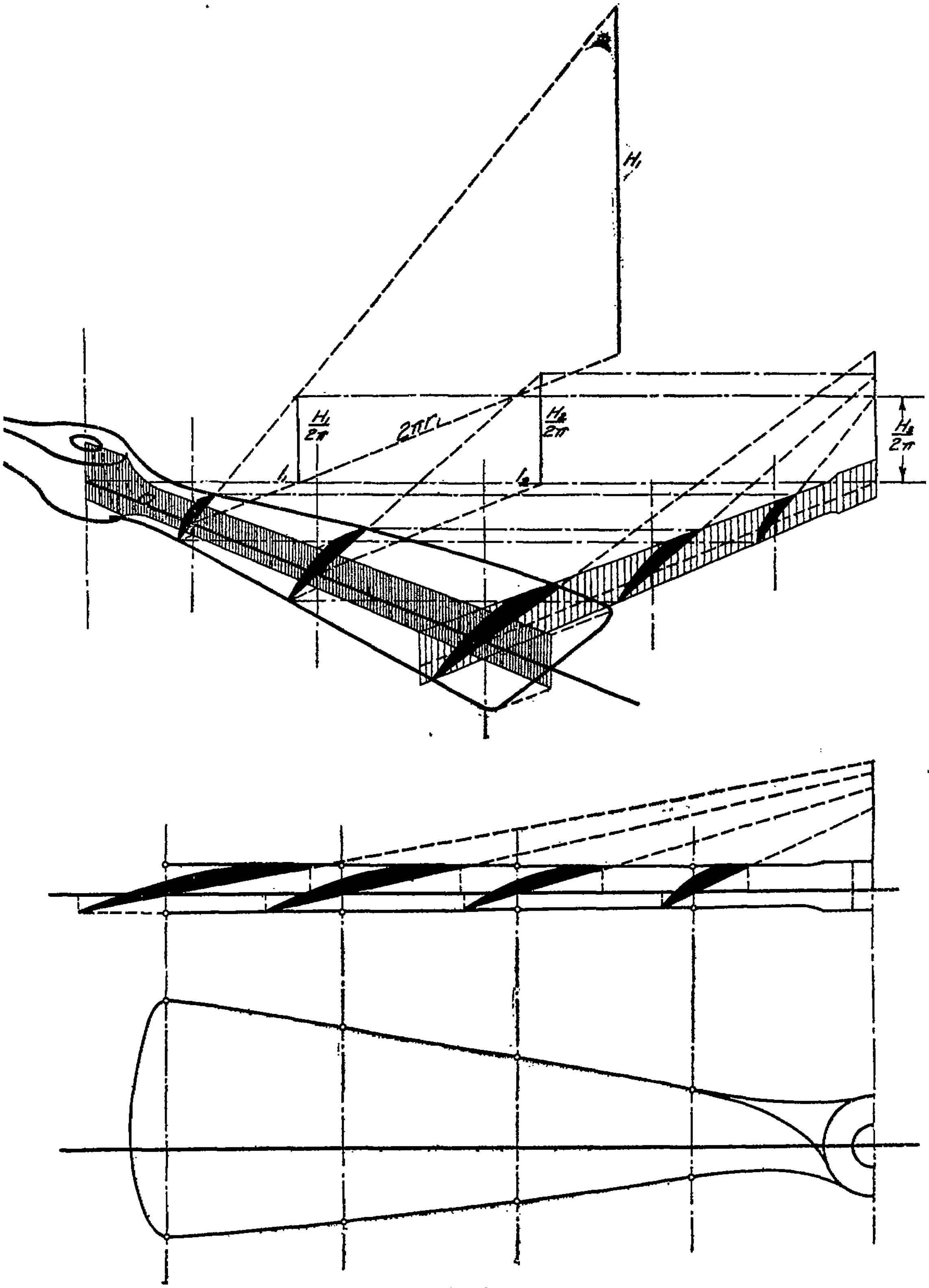
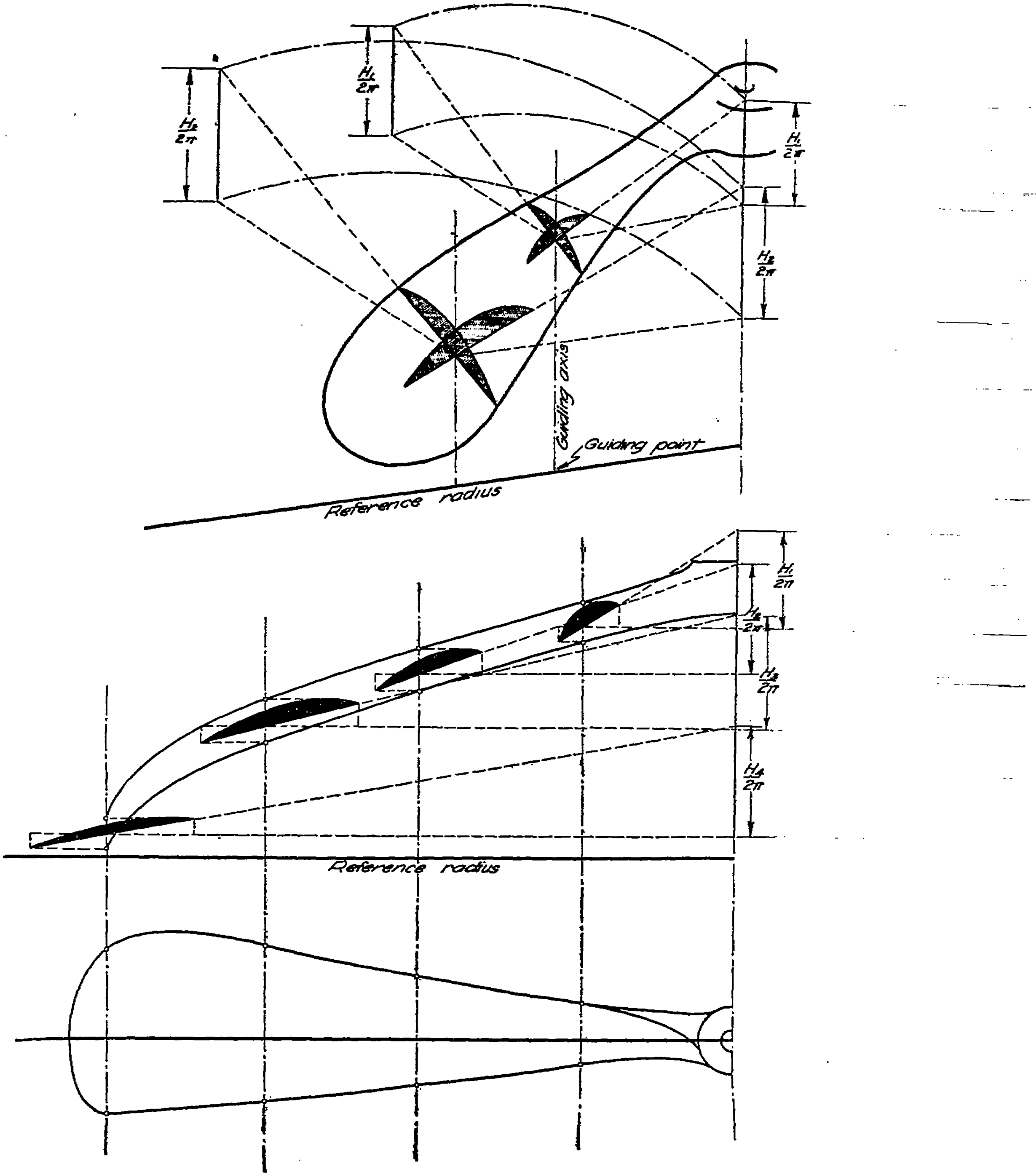


FIG. 49.


$\frac{H_2}{2\pi}$
$\frac{H_1}{2\pi}$
Guiding axis
Guiding point
Reference radius
$\frac{H_2}{2\pi}$
$\frac{H_3}{2\pi}$
$\frac{H_4}{2\pi}$
Reference radius

48a it has been assumed that we have to do with a blade screw of constant constructive pitch, and thus $H_1/2\pi = H_2/2\pi$ .... In such a way is obtained the screw drawing, represented on figure 48b, whose geometrical properties are evident. Thus when we go from the sections of the blades to the sections on the screw drawing the projections $a_1\ a_1'$; $a_2\ a_2'$ of the blades on the meridional plane remain unchanged, but the projections of these same blade sections of the screw on the screw rotation plane are turned through 90°. (See fig. 48a.) The screw drawing allows one to see at once all the blade dimensions. If we project, on the screw drawing, each section on the corresponding guiding axis we will get the projection of the blade on the meridional plane; if we project these same sections on the reference radius and turn these projections through 90° we will get the projection of the blade on the screw rotation plane. (See fig. 48d.)

The screw drawing is generally completed by conventional representation of the distribution of the maximum blade thickness along the blade. (See fig. 48c.)

By aid of the screw drawing, one can directly obtain the templates necessary for screw manufacturing. It is sufficient for that purpose to trace on the screw drawing two straight lines parallel to the reference radius. On figure 48c templates, one above and one below, have been traced. The templates corresponding to one blade face fixed normally to a board on which is traced the projection of the blade on the screw rotation plane will give a space picture of the blade face (See fig. 48e.) If we make use of cylindrical blade sections, the templates have first to be bent according to the corresponding radii.

In figure 49 is represented the general case of the screw drawing; the screw blade is assumed to have a general curved-down shape. All the details of this drawing are self-evident.

Second generation of the screw drawing.—Instead of rotating the sectional planes, we can bring them to coincide by a translation parallel to themselves, effected in such a way that the bases l_1, l_2 ...... of the height $H_1/2\pi$, $H_2/2\pi$ .... described in the screw rotation plane a straight line going through the screw axis and inclined at 45° to the reference radius (See fig. 50.) This construction, as well as the foregoing, gives directly the connection between the blade screw and the screw drawing. In figure 50 it has been assumed that the constructive pitches of the different sections increase from boss to blade tip.

NOTE VI.

SOME CRITICAL REMARKS ABOUT THE BLADE-SCREW INTEGRAL THEORY.

As has already been mentioned in the introduction to the present memoir, the general blade-screw theory can only be an integral theory. In the present note I will give the general outlines of the blade-screw integral theory. This will allow one to judge better the blade-screw differential theory developed in the actual memoir.

In its most general form the blade-screw problem can be stated as follows: Let us consider a blade screw rotating in a fluid with an angular velocity Ω around its axis and advancing with a speed V along that axis, and let us suppose, for one moment, our blade-screw problem to be fully solved; that is, let us assume that we know the exact distribution of the partial thrust ΔQ and the partial torque ΔC along the screw blades. Two sides of the problem have to be distinguished. First of all, knowing ΔQ and ΔC as functions of r we have to find the exact flow around the blade screw. This will be the *hydrodynamical part* of the problem. Afterwards, having found the flow and thus knowing exactly the stream running on the screw blades, we can seek for the dimensions and shape which have to be given to the blades, so that they realize the assumed system of partial thrust ΔQ and partial torque ΔC. This is the *technical part* of the problem. When the assumed system of ΔQ and ΔC lie in a practically possible range, and when we know the flow around the screw, it is always possible to give to its blades such size and shape that, for example, the assumed ΔQ will be realized, but the ΔC necessary to produce the assumed ΔQ can come out different frum the assumed values. All depends upon the losses which will take place. Under such conditions we will be brought to modify the first assumed system of ΔC, recalculate the flow and introduce changes in the size and shape of the blades, and so step by step approach nearer to the conditions of the problem. In this way, by a successive determination of the flow and calculation of the size and shape of the screw blades, redetermination of the flow and recalculation of the blades, we can reach an agreement between the hydrodynamical and technical parts of the blade-screw problem. The foregoing constitutes the most general statement of the blade-screw problem.

Let us consider the hydrodynamical part of the problem. We will make only two assumptions: We will neglect the periodicity of the velocities in the slip stream and neglect the interior losses between the sections S_0S_0, SS and $S'S'$, $S''S''$. These losses are very small in comparison with the other losses which occur in blade-screw working; and corrections for the periodicity of the slip stream velocities can always be introduced post factum. After the detailed explanations which were given in the first chapter of this memoir, I will allow myself to be very brief in the following statement of the general scheme of the most general blade-screw theory:

Condition of flow continuity in the slip stream

(1) $$\Delta M = \delta \Delta S(V+v) = \delta \Delta S''(V+v'')$$

From which follows

(2) $$\frac{\Delta S}{\Delta S''} = \frac{V+v''}{V+v} = \frac{rdr}{r''dr''}$$

We also have

(3) $$\Delta I = \Delta M r^2 = \delta \Delta S(V+v)r^2$$

(4) $$\Delta I = \Delta M r''^2 = \delta \Delta S(V+v)r''^2$$

and

(5) $$\frac{\Delta I}{\Delta I''} = \frac{r^2}{r''^2}$$

The theorems of momentum and moments of momentum applied to the slip stream lead to the relations

$$Q=\Sigma\Delta Mv''-\Sigma\Delta S''(p_0-p'')=F \tag{6}$$

$$C=\Sigma\Delta I''\omega''=\Sigma\Delta I\omega'=C_r \tag{7}$$

and also

$$\Delta Q=\Delta Mv''-\Delta S''(p_0-p'') \tag{8}$$

$$\Delta C=\Delta I''\omega''=\Delta I\omega' \tag{9}$$

so that

$$Q=\Sigma\Delta Q;\ C=\Sigma\Delta C \tag{10}$$

and we also have

$$\Delta p=\frac{\Delta Q}{\Delta S}=p'-p \tag{11}$$

In the limiting case the sections SS and $S'S'$ are considered to be very close together.

According to Note II, the pressure distribution in the section $S''S''$ is given by the relations

$$p''=p''_c+\delta\int_0^{r''}\omega''^2r''dr'' \tag{12}$$

$$p_0=p''_c+\delta\int_0^{R''}\omega''^2r''dr'' \tag{13}$$

$$p_0-p''_c=\delta\int_{r''}^{R''}\omega''^2r''dr'' \tag{14}$$

where p''_c is the pressure in the center of the section $S''S''$ and R'' the radius of that section.

Let us apply the Bernouilli theorem to one stream line of the slip stream. In the indraught the Bernouilli constant, which we will designate by B, has the value

$$B=p_0+\frac{\delta V^2}{2} \tag{15}$$

When we cross the area swept by the blades of the screw the Bernouilli constant undergoes an increase equal to

$$\Delta B=\Delta p+\frac{\delta r^2\omega'^2}{2} \tag{16}$$

so that in the outdraught the Bernouilli constant has for its value

$$B+\Delta B=p_0+\frac{\delta V^2}{2}+\frac{\Delta Q}{\Delta S}+\frac{\delta r^2\omega'^2}{2} \tag{17}$$

Consequently for the section $S''S''$ we must have

$$B+\Delta B=p_0+\frac{\delta V^2}{2}+\frac{\delta r^2\omega'^2}{2}+\delta(V+v)v''-\frac{\Delta S''}{\Delta S}(p_0-p'')=p''+\frac{\delta(V+v'')^2}{2}+\frac{\delta r''^2\omega''^2}{2} \tag{18}$$

or, after self-evident simplifications,

$$\delta v''\left(v-\frac{v''}{2}\right)=\frac{\delta r^2\omega'}{2}(\omega''-\omega')-(p_0-p'')\left(1-\frac{\Delta S''}{\Delta S}\right) \tag{19}$$

This last relation is *the fundamental characteristic equation* of the flow in the slip stream of a blade screw.

In the hydrodynamical part of the blade-screw problem the fundamental variable is r

$$0\leq r\leq\frac{D}{2}$$

The data given are the functions ΔQ and ΔC. The unknown functions, which have to be found as functions of r, are

$$r'',\ p'',\ \omega',\ \omega'',\ v,\ v'' \tag{20}$$

This makes six unknown functions, for the determination of which we have the six equations

$$\delta v''\left(v-\frac{v''}{2}\right)=\frac{\delta r^2\omega'}{2}(\omega''-\omega')-(p_o-p'')\left(1-\frac{\Delta S''}{\Delta S}\right) \tag{19}$$

$$p''=\delta\int_{r''}^{R''}\omega''^2 r''\,dr'' \tag{21}$$

$$\omega'=\frac{\Delta C}{\delta(V+v)\Delta S r^2} \tag{22}$$

$$\omega''=\omega'\frac{r^2}{r''^2} \tag{23}$$

$$\frac{\Delta Q}{\Delta S}=\Delta p=\delta(V+v)v''-\frac{\Delta S''}{\Delta S}(p_o-p'') \tag{24}$$

$$\frac{V+v''}{V+v}=\frac{r\,dr}{r''\,dr''} \tag{25}$$

with $\Delta S=2\pi r dr$ and $\Delta S''=2\pi r'' dr''$

The foregoing system constitutes the fundamental system of equations which fully determines the flow around the blade screw in the most general case. Owing to the integral relation (21), the solution of this system of equations can be found only by a method of successive approximations, and thus is very laborious. Under such conditions it is natural to seek for some assumptions, which being very close to the real conditions, could simplify the foregoing system of equations. For that purpose let us discuss the variation of the second member of the equation (19), which we will designate by G.

$$G=\frac{\delta r^2\omega'}{2}(\omega''-\omega')-(p_o-p'')\left(1-\frac{\Delta S''}{\Delta S}\right) \tag{26}$$

For the tips of the blades we have

$$r''=R'' \text{ and } p''=p_o$$

and consequently

$$G=\frac{\delta r^2\omega'}{2}(\omega''-\omega') \tag{27}$$

but as $\omega''>\omega'$, we will have

$$G>0 \text{ and } v''<2v \tag{28}$$

For the boss we have

$$r''\cong 0;\ p''=p''_c$$

and thus

$$G=-(p_o-p''_c)\left(1-\frac{\Delta S''}{\Delta S}\right) \tag{29}$$

and consequently

$$G<0 \text{ and } v''>2v \tag{30}$$

Thus G is positive at the tips and negative at the boss. Consequently, between tips and boss there must necessarily exist a blade section for which $G=0$ and consequently rigorously

$$v''=2v \tag{31}$$

On the other hand, it is easy to see that starting from the blades sections where $v''=2v$ the quantity G increases generally in magnitude to tips and to boss. But at tips and boss G is still a small quantity. In fact, at the tips $G=\frac{1}{2}\delta r^2\omega'(\omega''-\omega')$, but as ω' has generally a small value and the difference $(\omega''-\omega')$ is negligible, so far as the radial velocities can be neglected, G comes out to be small. At the boss $G=-(p_o-p''_c)\ (1-\Delta S''/\Delta S)$, but the pressure difference $(p_o-p''_c)$ being generally negligible, owing to the fact that ω'' is small, and $(1-\Delta S''/\Delta S)$ is in magnitude smaller than the unity, G at the boss is also small.

Thus it is at boss and tips that the difference between v'' and $2v$ reaches in magnitude its biggest values, but still here this difference is small.

We are thus brought to the conclusion that in the most general case the flow in the slip stream is such that very nearly

$$v''=2v \tag{31}$$

for the whole cross section of the slip stream. This last relation expresses the fact that the rotation of the fluid in the slip stream has only a very slight influence on the translatory motion of the same.

After we have convinced ourselves that the relation (31) holds, it is easy to see that to a good approximation the flow conditions come out to be similar in the section SS and $S''S''$. This fact is a direct consequence of the relation (31) for a blade screw working at a fixed point; in other cases for the similarity of flow conditions it is only necessary for v to be small relative to V or to have its variations small along the slip stream cross section, as has been shown in the first chapter of this memoir. We thus can consider, remaining still close to real conditions, that

$$\frac{\Delta S}{\Delta S''}=\frac{V+v''}{V+v}=\frac{rdr}{r''dr''}\cong\frac{r^2}{r''^2}=\frac{\Delta I}{\Delta I''}=\frac{\omega''}{\omega'} \tag{32}$$

and consequently

$$\frac{dr}{dr''}=\frac{r}{r''} \text{ or } r''=cr \tag{33}$$

where c is a constant.

It will now be easy to see from the relation (26) that the condition $G=0$ for any value of r between $r\cong 0$ to $r=D/2$ can only be satisfied with

$$\omega'=\omega'' \text{ and } p''=p_o \tag{34}$$

for the whole cross section of the slip stream.

We are thus naturally brought to the hypothesis made in the first chapter of this memoir.

To evaluate, however, the influence that the pressure difference in the section $S''S''$ can have on the blade-screw working, one can proceed as follows: As ω'' is generally a small quantity, let us neglect its variation along the slip stream cross section. Under such conditions, from the relation (14) we will find

$$p_o-p''=\delta\frac{\omega''^2(R''^2-r''^2)}{2}$$

or

$$p_o-p''=\delta\frac{r''^2\omega''^2}{2}\left(\frac{R''^2}{r''^2}-1\right)$$

and on account of the relations (32) and (33)

$$p_o-p''=\delta\frac{r^2\omega'^2}{2}\frac{\Delta S}{\Delta S''}\left(\frac{D^2}{4r^2}-1\right) \tag{35}$$

The equations (8) and (9) can thus be written

$$\Delta Q=\delta\Delta S(V+v)v''-\delta\Delta S\frac{r^2\omega'^2}{2}\left(\frac{D^2}{4r^2}-1\right) \tag{36}$$

$$\Delta C=\delta\Delta S(V+v)r^2\omega' \tag{37}$$

Comparing now, as in the first chapter, these last values of ΔQ and ΔC with those obtained by the direct consideration of the action of the stream on the screw blades, that is, bringing the hydrodynamical part of the blade-screw problem into agreement with the technical part, we find

$$\delta\Delta S(V+v)v''-\delta\Delta S\frac{r^2\omega'^2}{2}\left(\frac{D^2}{4r^2}-1\right)=nk_t\delta\Delta A\,W^2\cos(\varphi+\beta)$$

$$\delta\Delta S(V+v)r^2\omega'=nrk_t\delta\Delta A\,W^2\sin(\varphi+\beta)$$

Proceeding now with these equations exactly as was indicated in the first chapter, we will finally find

$$\frac{v}{V+v}-\frac{r^2\omega'^2}{4(V+v)^2}\left(\frac{D^2}{4r^2}-1\right)=az \tag{38}$$

$$\frac{r\omega'}{2(V+v)}=az\ \mathrm{tg}\ (\varphi+\beta)=\frac{r\omega''}{2(V+v'')}=\frac{r\omega''}{2(V+2v)} \tag{39}$$

These last equations are fully similar to the equations (46) and (47) of Chapter I, and only contain the complementary term

$$\frac{r^2\omega'^2}{4(V+v)^2}\left(\frac{D^2}{4r^2}-1\right)=a^2z^2\ \mathrm{tg}^2\ (\varphi+\beta)\cdot\left(\frac{D^2}{4r^2}-1\right)$$

which expresses the influence of the decrease of pressure in the section $S''S''$ produced by the rotation of the fluid in the slip stream. But as this complementary term appears to be of second order compared with $az\,\mathrm{tg}(\varphi+\beta)$, which is a very small quantity in most blade-screw applications, it comes out to be negligible.

We are thus brought to the conclusion that, from the most general standpoint, the only system of equations for the blade screw which can be reasonably adopted is the one established in the first chapter of this memoir.

The following has still to be noted. As far as the race velocity ω is concerned, exactly speaking we have

$$\omega=\frac{\omega'}{2} \tag{41}$$

In fact, by its definition $\omega\Delta C$ is the work communicated in a unit of time to the fluid by the blade element considered, in its rotational motion. And this work must be equal to the corresponding rotational kinetic energy of the fluid; that is, $\omega\Delta C=\frac{1}{2}\,\Delta I\omega'^2$ because the corresponding work $v\Delta Q$ of the thrust has its equivalent in the increase of pressure Δp produced, when crossing the area swept by the screw blades. But as $\Delta C=\Delta I\omega'$ we have

$$\omega\Delta C=\omega\omega'\Delta I=\tfrac{1}{2}\Delta I\omega'^2$$

and thus

$$\omega=\frac{\omega'}{2}$$

But when the radial velocities are neglected we have $\omega''\cong\omega'$, and consequently

$$\omega\cong\frac{\omega''}{2} \tag{42}$$

This gives for ω a slightly increased value.

When we neglect the decrease of pressure in the section $S''S''$, this brings with it a slightly increased value for ΔQ, but when simultaneously we use for ω the value (42) we obtain a certain correction of the neglect of the decrease of pressure, because a slightly greater value of ω decreases slightly the angle of attack i, and thus decreases slightly ΔQ. Under the hypothesis made the value (42) has also to be adopted for ω, in order to have a correct energy balance, because when the decrease of pressure in section $S''S''$ is neglected and thus the translational motion in the slip stream considered independent of its rotational motion we have

$$v\Delta Q=\tfrac{1}{2}\Delta Mv''^2$$

and consequently we have to take

$$\omega\Delta C=\tfrac{1}{2}\Delta I''\omega''^2$$

that is, $\omega''=2\omega$. All these last remarks concern only differences of second order.

The foregoing critical discussion of the blade-screw problem, from the most general standpoint, shows us the value of the system of equations established in the first chapter of this memoir. On the other hand, how convenient this system of equations is in its practical use follows in full evidence from the results obtained in the present memoir.

I will also remark that from a practical standpoint the square of az can be neglected in many of the formulae of the actual memoir.

I will finally give a numerical evaluation of the magnitude of the departure from unity that the ratio $2v/v''$ can reach.

Introducing, in a first approximation, the value (35) of the difference of pressure (p_0-p'') in the relation (19), we get

$$\delta v''\left(v-\frac{v''}{2}\right)=\frac{\delta r^2\omega'^2}{2}\left(\frac{\omega''}{\omega'}-1\right)-\frac{\delta r_1^2\omega'^2}{2}\left(\frac{\Delta S}{\Delta S''}-1\right)\left(\frac{D^2}{4r^2}-1\right)$$

and, taking into account the relations (32) and (33), we find

$$\frac{2v}{v''}-1=\frac{D^2\omega'^2}{v''^2}\cdot\frac{(1-c^2)}{c^2}\left(2\frac{r^2}{D^2}-0,25\right)=E\left(2\frac{r^2}{D^2}-0,25\right) \tag{43}$$

$$\text{with } E=\frac{D^2\omega'^2}{v''^2}\cdot\frac{(1-c^2)}{c^2}$$

To find the order of magnitude of the ratio $D^2\omega'^2/v''^2$ we will use equations (48) and (49) from Chapter I. Dividing (49) by (48), we find

$$\frac{D^2\omega'^2}{v''^2}\cong\frac{D^2\omega^2}{v^2}=4(1+az)^2\,\mathrm{tg}^2\,(\varphi+\beta)$$

Remembering that $H=2\pi r\,\mathrm{tg}\,\varphi$ and considering $\beta\cong 0$, we get

$$\frac{D^2\omega'^2}{v''^2}\cong\frac{4(1+az)^2H^2}{\pi^2D^2}$$

Concerning the ratio $(1-c^2)/c^2$, we have $(1-c^2)/c^2\cong 0,05$ when advancing and $(1-c^2)/c^2\cong 1$ at a fixed point, so that the quantity E comes out to be of the following order of magnitude: For a propulsive screw with $az\cong 0,1$; $H/D\cong 0,75$ and $(1-c^2)/c^2\cong 0,05$ we find

$$E\cong 0,01$$

For a helicopter or lifting screw we have $az=1$; $(1-c^2)/c^2\cong 1$ but necessarily H/D small. If we take $H/D\cong 0,3$ we find

$$E\cong 0,1$$

The quantity E reaches its greatest value for a propulsive screw working at a fixed point. With $az\cong 1$; $H/D\cong 0,75$; $(1-c^2)/c^2\cong 1$. We find

$$E\cong 0,8$$

Using the second of these last values we get

$$\frac{2v}{v''}\cong 1+0,1\left(2\frac{r^2}{D^2}-0,25\right)$$

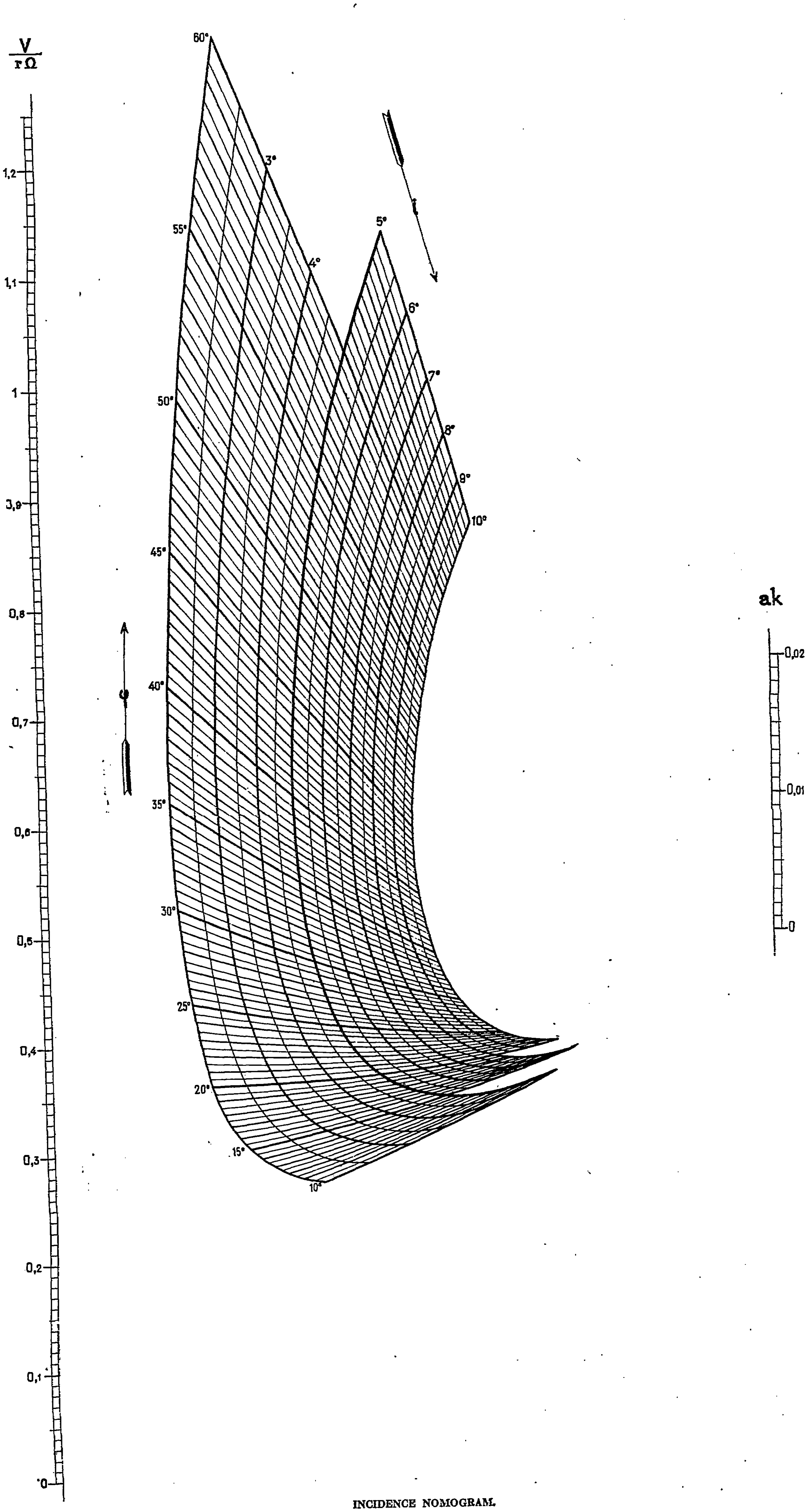
For $r=0$ and $r=D/2$ we find $2v/v''\cong 0,975$ and $2v/v''\cong 1,025$; for values of r between $r=0$ and $r=D/2$ the departure of $2v/v''$ from unity is still less; for $r/D\cong 0,354$ we have $2v/v''=1$.

The departure of $2v/v''$ from unity thus does not generally reach 3 p. c. (and this only at boss and blade tips), and is consequently fully negligible. For a propulsive screw when advancing with $E\cong 0,01$, it is absolutely negligible. Only for a propulsive screw working at a fixed point it may reach, at tips and boss only, 20 p. c., which is still negligible in a first approximation. We thus see in full evidence that the relation

$$v''=2v$$

although being in the general case only an approximate law, constitutes, however, a remarkable approximation.

GEORGE DE BOTHEZAT.



INCIDENCE NOMOGRAM.

To be used in connection with the propeller theory of George de Bothezat.

Copies of this Nomogram can be obtained from the National Advisory Committee for Aeronautics, Washington, D. C.

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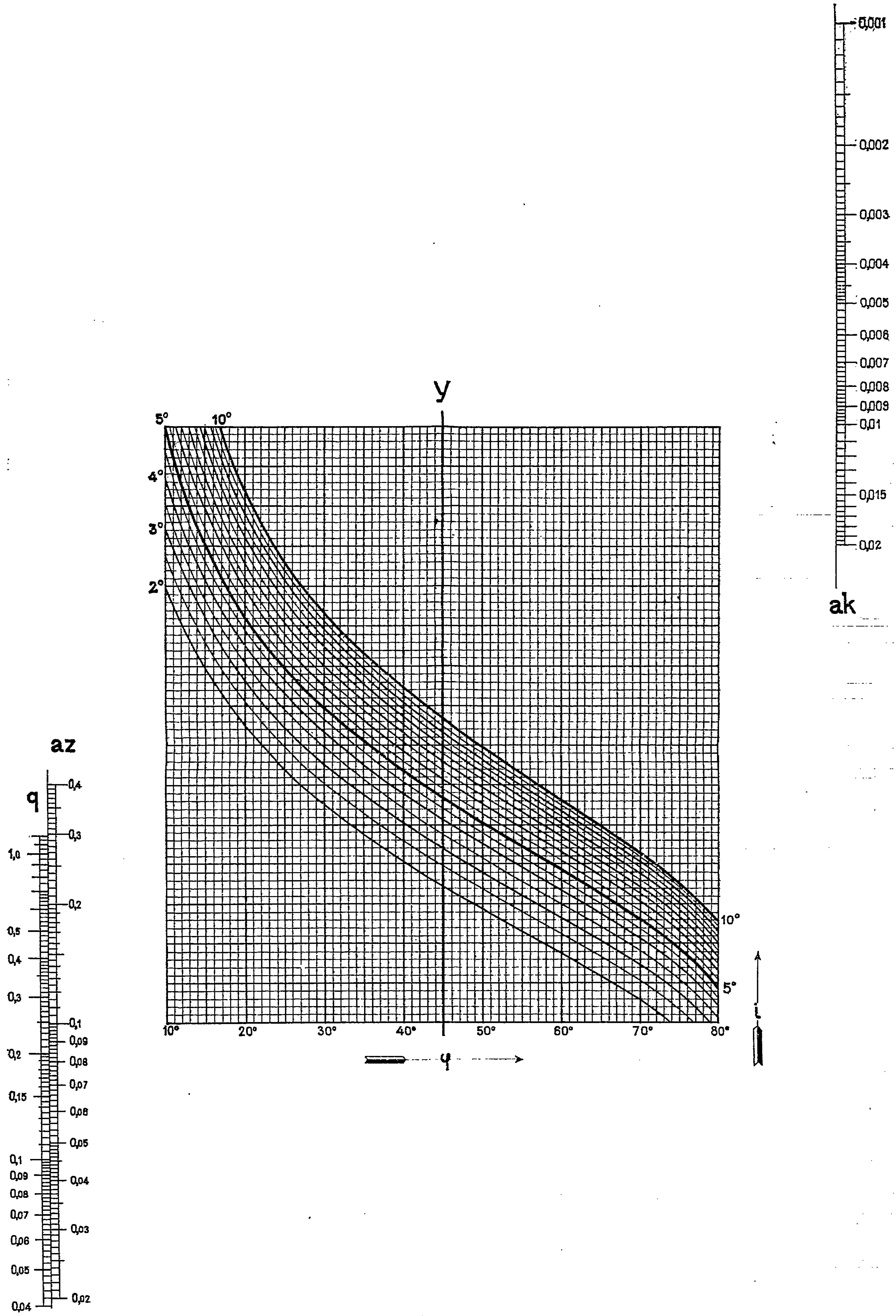

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