

REPORT 1030

INVESTIGATION OF SEPARATION OF THE TURBULENT BOUNDARY LAYER¹

By G. B. SCHUBAUER and P. S. KLEBANOFF

SUMMARY

An investigation was conducted on a turbulent boundary layer near a smooth surface with pressure gradients sufficient to cause flow separation. The Reynolds number was high, but the speeds were entirely within the incompressible flow range. The investigation consisted of measurements of mean flow, three components of turbulence intensity, turbulent shearing stress, and correlations between two fluctuation components at a point and between the same component at different points. The results are given in the form of tables and graphs. The discussion deals first with separation and then with the more fundamental question of basic concepts of turbulent flow.

INTRODUCTION

In 1944 an experimental investigation was begun at the National Bureau of Standards with the cooperation and financial assistance of the National Advisory Committee for Aeronautics to learn as much as possible about turbulent-boundary-layer separation. Considering that previous experimentation had been limited to mean speeds and pressures, it was decided that the best way to bring to light new information was to investigate the turbulence itself in relation to the mean properties of the layer. Since little was known about turbulent boundary layers in large adverse pressure gradients, the investigation was exploratory in nature and was pursued on the assumption that whatever kind of measurements that could be made on turbulence and turbulent processes would carry the investigation in the right direction.

The investigation was therefore long range, there being no natural stopping point as long as there remained unknowns and means for investigating them. The decision to stop came when it was decided that the more basic properties of turbulent motions, such as production, decay, and diffusion, which form the subject of modern theories, could better be investigated first without the effect of pressure gradient. The experimental work on separation was therefore halted after a certain fund of information had been obtained on turbulence intensity, turbulent shearing stress, correlation coefficients, and the scale of turbulent motions.

Use was made of the results from time to time as they could be made to serve a particular purpose. Certain of the results have appeared therefore in references 1 to 3. It is now felt that the results should be presented in their entirety for what they contribute to the separation problem and to the understanding of turbulent flow, even though they leave many questions unanswered.

The authors wish to acknowledge the active interest and support of Dr. H. L. Dryden during this investigation and the assistance given by Mr. William Squire in the taking of observations and the reduction of data.

SYMBOLS

x	distance along surface from forward stagnation point
y	distance normal to surface measured from surface
z	direction perpendicular to xy -plane
U	mean velocity in boundary layer
U_1	mean velocity just outside boundary layer
U_m	mean velocity just outside boundary layer at $x=17\frac{1}{2}$ feet, used as reference velocity
V	y -component of mean velocity in boundary layer
u, v, w	x -, y -, and z -components of turbulent-velocity fluctuations
u', v', w'	root-mean-square values of u, v , and w
ρ	density of air
ν	kinematic viscosity of air
p	pressure
q_1	free-stream dynamic pressure $\left(\frac{1}{2} \rho U_1^2\right)$
q_m	free-stream dynamic pressure at $x=17\frac{1}{2}$ feet $\left(\frac{1}{2} \rho U_m^2\right)$
τ	turbulent shearing stress $(-\rho \overline{uv})$
\overline{uv}	mean value of product of u and v
C_{τ_1}, C_{τ_m}	coefficients of turbulent shearing stress $\left(C_{\tau_1} = \tau / \frac{1}{2} \rho U_1^2, C_{\tau_m} = \tau / \frac{1}{2} \rho U_m^2\right)$
τ_o	skin friction
C_f	coefficient of skin friction $\left(\tau_o / \frac{1}{2} \rho U_1^2\right)$
δ	boundary-layer thickness
δ^*	boundary-layer displacement thickness $\left(\int_0^\infty \left(1 - \frac{U}{U_1}\right) dy\right)$
θ	boundary-layer momentum thickness $\left(\int_0^\infty \frac{U}{U_1} \left(1 - \frac{U}{U_1}\right) dy\right)$
H	boundary-layer shape parameter (δ^*/θ)
L_x, L_y	scales of turbulence

¹ Supersedes NACA TN 2133, "Investigation of Separation of the Turbulent Boundary Layer" by G. B. Schubauer and P. S. Klebanoff, 1950.

- R_y transverse correlation coefficient $(\overline{u_1 u_2} / \overline{u_1' u_2'})$,
where subscripts 1 and 2 refer to positions y_1
and y_2)
- R_x longitudinal correlation coefficient $(\overline{u_1 u_2} / \overline{u_1' u_2'})$,
where subscripts 1 and 2 refer to positions x_1
and x_2)

APPARATUS AND TEST PROCEDURE

The setup for the investigation was arranged with two things in mind: (1) The Reynolds number was to be as high as possible and (2) the boundary layer was to be thick enough to permit reasonably accurate measurements of all components of the turbulence intensity and shearing stress by hot-wire techniques which were known to be reliable. Since this required a large setup, the 10-foot open-air wind tunnel at the National Bureau of Standards was chosen, and a wall of airfoil-like section shown in figures 1 and 2 was constructed in the center of the test section. The wall was 10 feet high, extending from floor to ceiling, and was 27.9 feet long. It was constructed of $\frac{1}{4}$ -inch Transite on a wooden frame, and the surface on the working side was given a smooth finish by sanding and varnishing and, finally, waxing and polishing. The profile was chosen so that the adverse pressure gradient on the working side would be sufficient to cause separation and yet have sufficiently small curvature to make the pressure changes across the layer negligible.

Since the separation point was found to be very close to

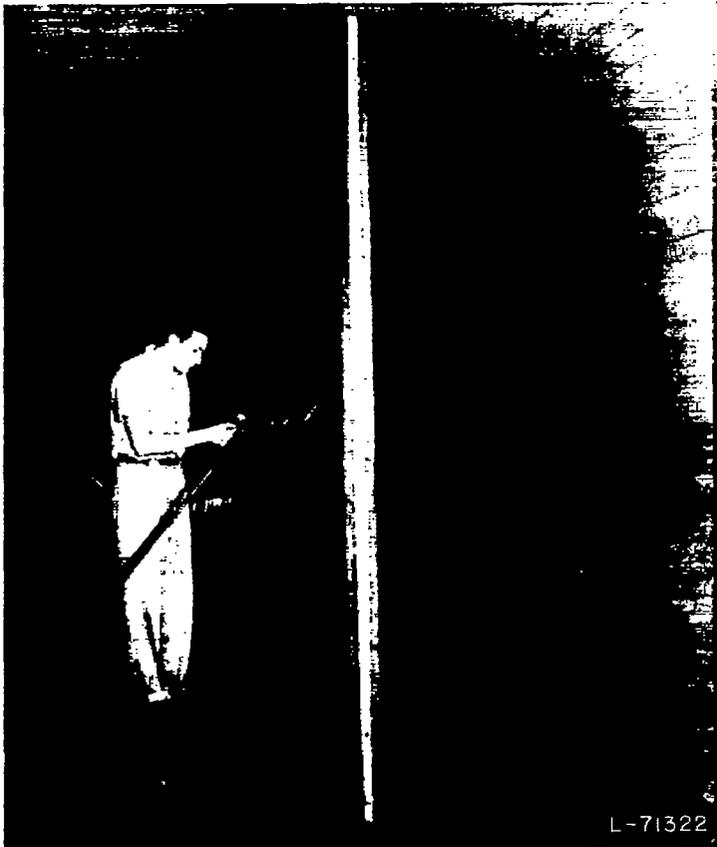


FIGURE 1.—Front view of "boundary-layer wall" in NBS 10-foot open-air wind tunnel.

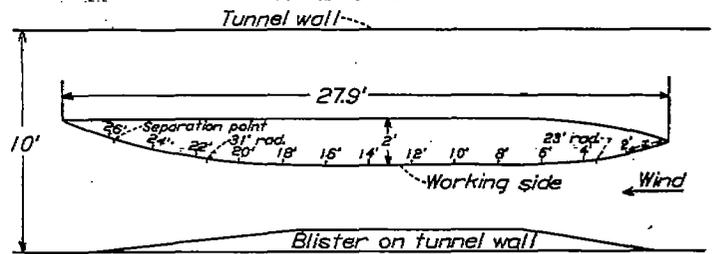


FIGURE 2.—Sectional drawing of "boundary-layer wall."

the trailing edge, a blister was constructed on the tunnel wall to move the separation point upstream to the location shown in figure 2. At the outset there was troublesome secondary flow from premature separation near the floor and, to a lesser degree, near the ceiling. A vent in the floor allowing air to enter the tunnel and blow away the accumulated dead air afforded a satisfactory remedy. The flow was then two-dimensional over the central portion of the wall from the leading edge to the separation point.

A steeply rising pressure, caused by the small radius of curvature of the leading edge and the induced angle of attack, produced transition about 2 inches from the leading edge. The boundary layer was therefore turbulent over practically the whole of the surface and, over the region of major interest, ranged in thickness from $2\frac{1}{4}$ inches at the $17\frac{1}{2}$ -foot position to 9 inches at the separation point. All measurements were made with a free-stream speed of about 160 feet per second at the $17\frac{1}{2}$ -foot position. The boundary-layer thickness at $17\frac{1}{2}$ feet was equivalent to that on a flat plate 14.3 feet long with fully turbulent layer and no pressure gradient, and the flat-plate Reynolds number corresponding to 160 feet per second was 14,300,000. The speed was always adjusted for changes in kinematic viscosity from day to day to maintain a fixed Reynolds number throughout the entire series of measurements. The turbulence of the free stream of the tunnel was about 0.5 percent.

All measurements were made at the midsection of the wall, where the flow most closely approximated two-dimensionality, and on the side labeled "Working side" in figure 2. While the measurements extended over a considerable period of time, there was no evidence from pressure and mean-velocity distributions that the geometry of the wall changed. There was, however, considerable scatter in the turbulence measurements from day to day, some of which was due to inherent inaccuracies associated with hot-wire measurements, and some of which may have been caused by actual changes in the flow. The results therefore do not lend themselves to a determination of differential changes in the x -direction with high accuracy. It was the intention to obtain results applicable to a smooth surface; therefore the surface was frequently polished and kept clean at all times. However, because of the texture of the Transite, the surface could not be given a mirrorlike finish equal to that of a metal surface.

Considerable emphasis was placed on the precise determination of the position of separation. A method was finally evolved whereby the line of separation and the direction of

the flow at the surface in the neighborhood of this line could be found. This consisted of pasting strips of white cloth on the surface with a starch solution. Small crystals of iodine were then stuck to the strips. Blue streaks on the starched cloth then showed the direction of the air flow. By this means separation could be located with an accuracy of ± 2 inches. Initially the line of separation was nowhere straight, but, after the removal of some of the reversed flow near the floor by the vent previously mentioned, the line was made straight for a distance of 2 feet in the center and was located 25.7 ± 0.2 feet from the leading edge.

The pressure distribution was measured with a static-pressure tube 0.04 inch in diameter, constructed according to the conventional design for such a tube. Mean dynamic pressure was obtained by adding a total-pressure tube of the same diameter but flattened on the end to form a nearly rectangular opening 0.012 inch wide.

The hot-wire equipment used in the investigation of turbulence has been fully described in reference 1, and it suffices here merely to call attention to the manner of operation and the performance of the equipment. The thick boundary layer made it possible to obtain essentially point measurements without having to construct hot-wire anemometers on a microscopic scale. The several types used are shown in figure 3. The $\frac{1}{16}$ -inch scale shows the high magnification of types A, B, C, and D. A complete holder is shown by E with the inch scale above. Heads of type A were used for measuring u' , those of type B or C were used for measuring turbulent shearing stresses, and those of type D were used for measuring v' and w' . In use the prongs pointed directly into the mean wind.

When the head of type C was used for measuring shearing stress, an observation of the mean-square signal from each of the wires was necessary. A similar pair of observations

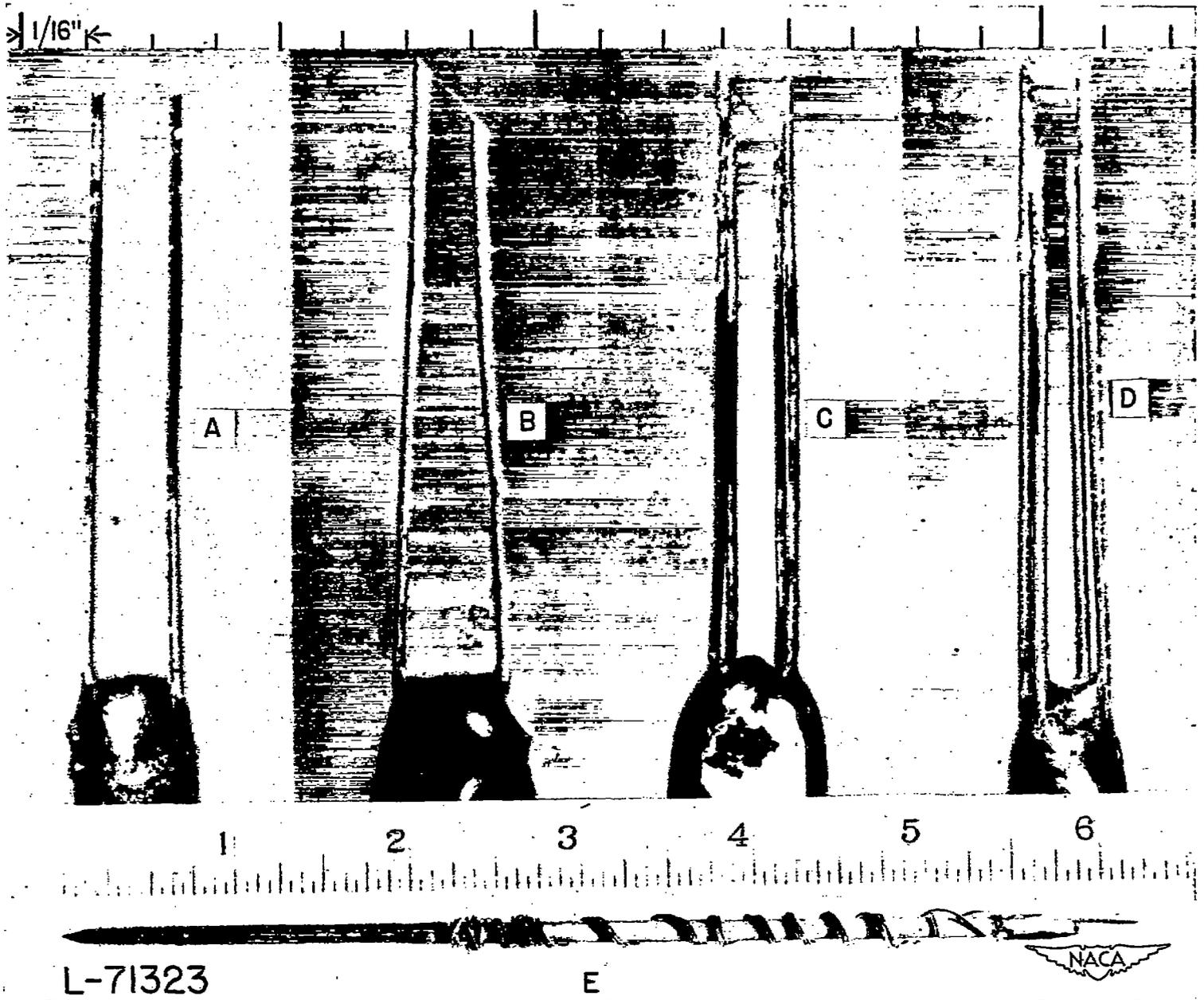


FIGURE 3.—Types of hot-wire anemometers and complete holder used in investigation

was necessary when using type B, but with only one wire the head had to be rotated through 180° . Since it was usually difficult to execute this rotation by remote control, most of the measurements of shearing stress were made with the head of type C.

The hot-wires themselves, shown at the tips of the prongs, were tungsten 0.00031 inch in diameter. Platinum wire could not be used because the air was taken into the tunnel from outdoors and platinum wires were broken by flying dirt particles. The diameter of 0.00031 inch was the smallest obtainable in tungsten at the time, and the length could not be reduced below about $\frac{1}{16}$ inch and still maintain the required sensitivity. In all cases the boundary-layer thickness was at least 25 times the wire length.

The uncompensated amplifier had a flat response from 2 to 5,000 cycles per second and an amplification decreasing above 5,000 cycles per second to about 50 percent at 10,000 cycles per second. The time constant of the wires ranged from 0.001 to 0.003 second, depending on operating conditions, and the over-all response of wire and amplifier could be made equal to that of the uncompensated amplifier by means of the adjustable compensation provided in the amplifier. However, with this relatively high time constant, the background noise level was high and had to be subtracted from the readings in order to obtain the true hot-wire signal.

The methods of determining u' , v' , w' , \overline{uv} , and $\overline{uw}/u'v'$ are fully described in reference 1. The determination of R_y and R_x involved the use of a pair of heads of type A, separated by known distances normal to the surface for R_y and along the tangent to the surface for R_x . The "sum-and-difference" method described in reference 4 was used, account being taken of the inequality of u' at the two wires and the differences in sensitivity.

The several measuring heads were mounted on various types of traversing equipment designed for convenience, rigidity, and a minimum of interference at the point where a measurement was being made.

TEST RESULTS

The results of the measurements are given in tables 1 to 8 and figures 4 to 14. Figures 15 to 22 repeat certain of the results to aid in the analysis.

The tabulation is made to present all of the detail contained in the measurements and to make the results readily available to any style of plotting that suits the reader's needs. Figures 4 to 14 are summary plots intended to show an over-all picture rather than detail.

PRESSURE DISTRIBUTION

The values given in table 1 and figure 4 were obtained from measurements of pressure with a small static-pressure tube placed $\frac{1}{4}$ inch from the surface at various positions along the midspan. The tube was also traversed in the y -direction, from which it was found that changes in pressure across the boundary layer were barely detectable in the region from $x=18$ to 23 feet and were not measurable elsewhere. The

pressure is therefore regarded as constant across the boundary layer, and all of the information on pressure gradient is given by the variation of q_1/q_m with x .

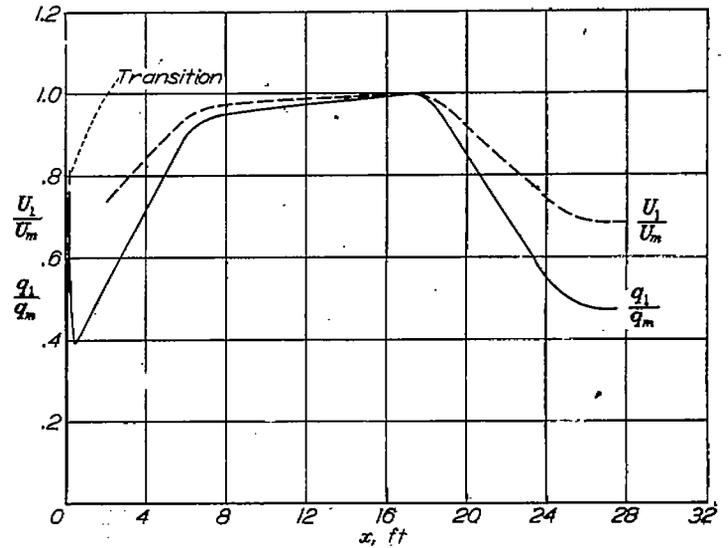


FIGURE 4.—Distribution of velocity and dynamic pressure just outside boundary layer.

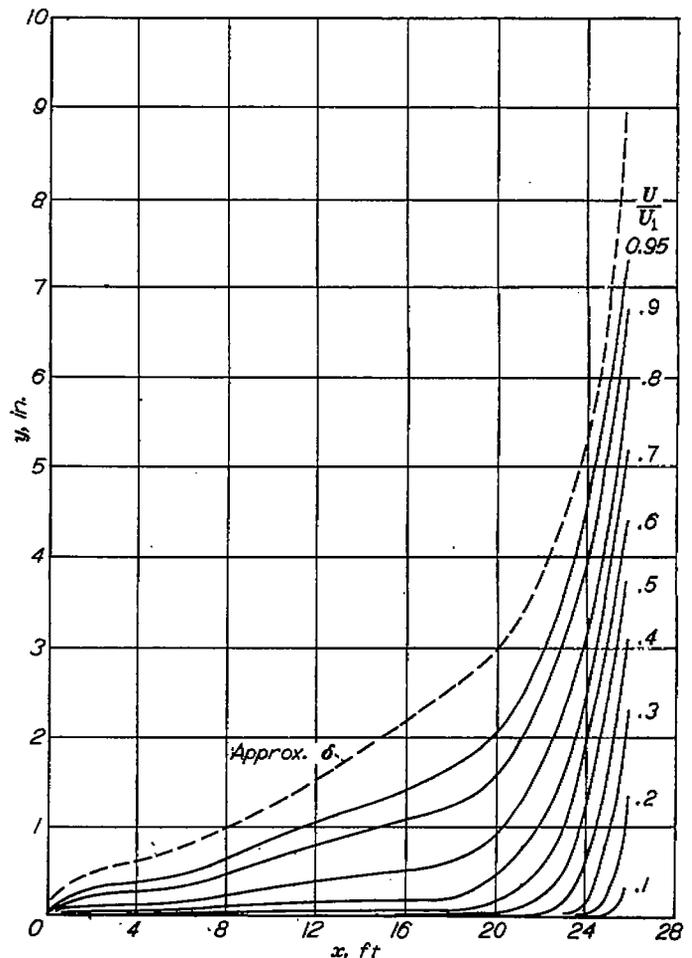


FIGURE 5.—Contour plot of mean velocities.

MEAN-VELOCITY DISTRIBUTION

Mean velocities were obtained from dynamic-pressure measurements made at various distances y . No correction was made for the effect of turbulence. The distributions of mean velocity are given in table 2 and summarized by the contour plot shown in figure 5. From these data were derived the values of δ^* , θ , and H given in table 3 and figure 6.

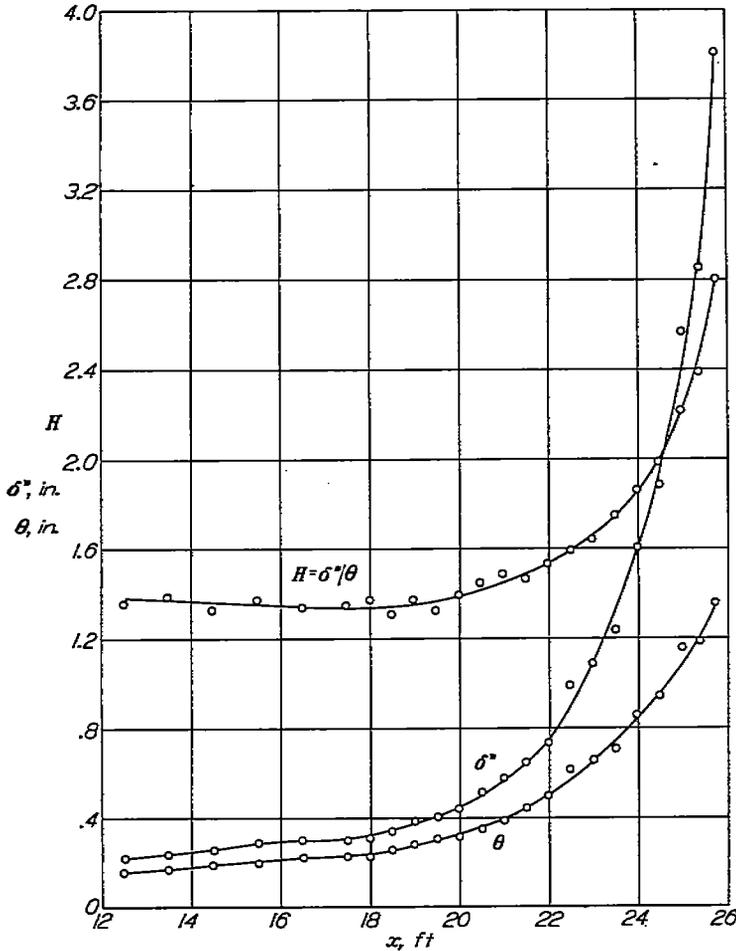


FIGURE 6.—Boundary-layer parameters.

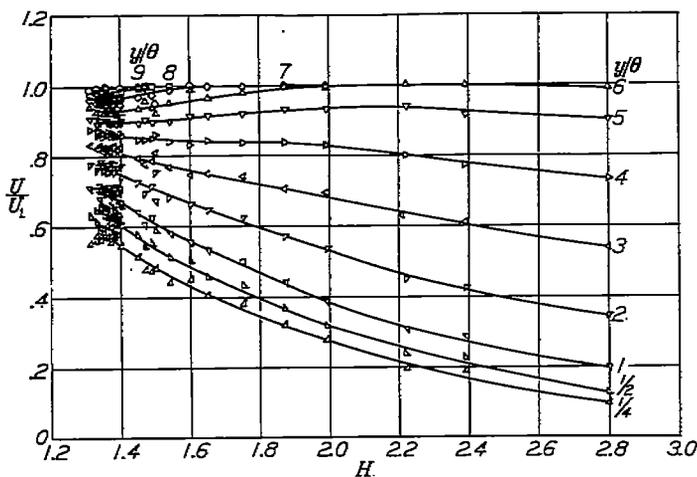


FIGURE 7.—Variation of U/U_1 with H for various values of y/θ .

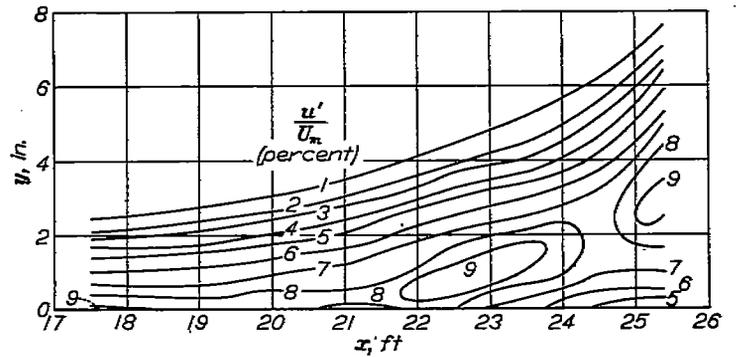


FIGURE 8.—Contour plot of u' .

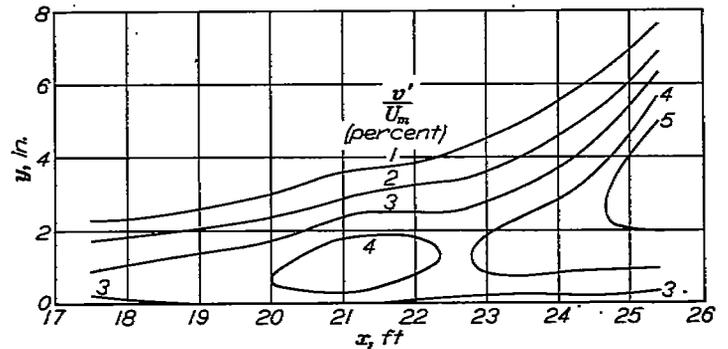


FIGURE 9.—Contour plot of v' .

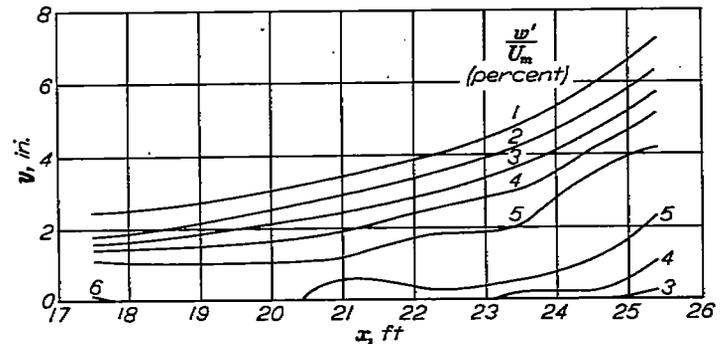


FIGURE 10.—Contour plot of w' .

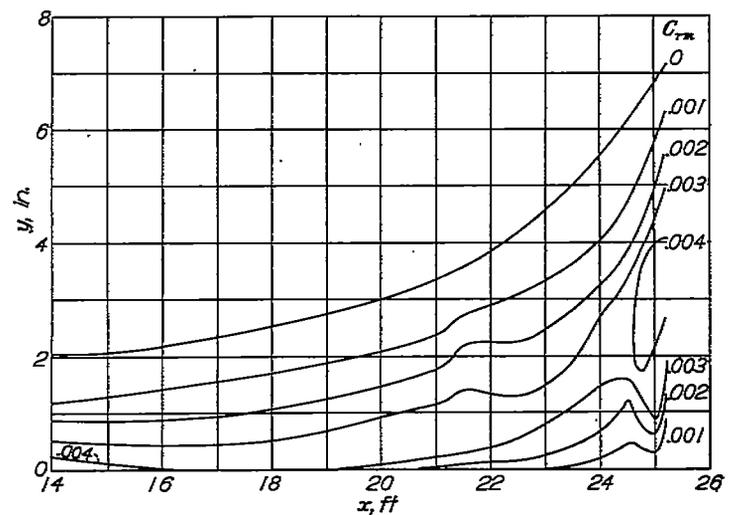


FIGURE 11.—Contour plot of coefficient of turbulent shearing stress.

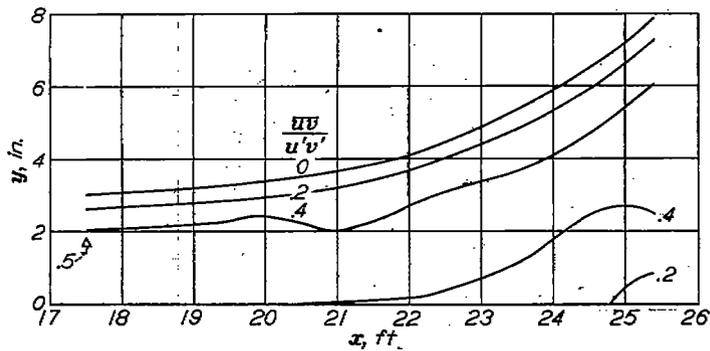
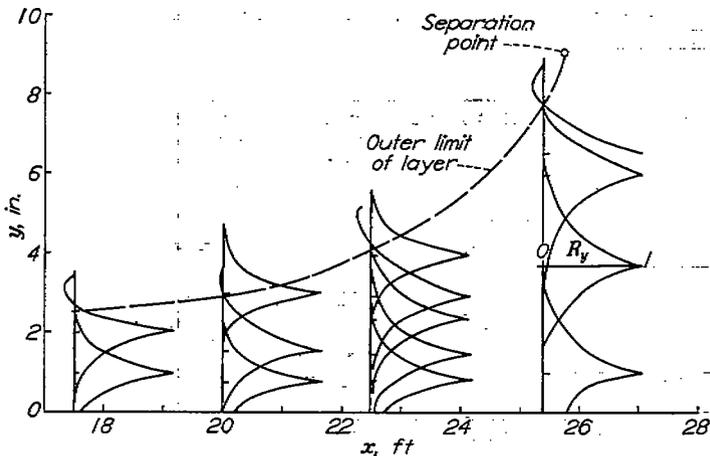
FIGURE 12.—Contour plot of $\bar{u}\bar{v}$ -correlation coefficient.

FIGURE 13.—Transverse correlation coefficient.

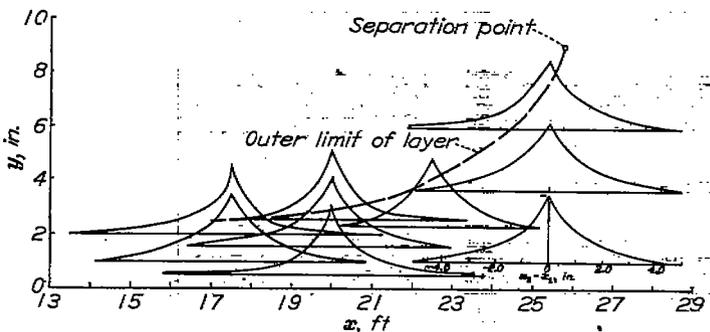


FIGURE 14.—Longitudinal correlation coefficient.

The distribution of mean velocity is plotted in figure 7 in the manner suggested by Von Doenhoff and Tetervin in reference 5. If H is a universal parameter specifying the boundary-layer profile, the curves of figure 7 should agree in all detail with those of figure 9 in reference 5. The agreement is good, although there are systematic differences slightly greater than the experimental dispersion.

TURBULENCE INTENSITIES

The turbulence intensities are given in table 4 in terms of u'/U_1 , v'/U_1 , and w'/U_1 . They are summarized in figures 8 to 10 in terms of u'/U_m , v'/U_m , and w'/U_m in order to show changes in the absolute magnitude of the fluctuations. As desired, u' , v' , and w' may be expressed in relation to any

of the mean velocities U , U_1 , or U_m by the aid of tables 1 and 2.

COEFFICIENT OF TURBULENT SHEARING STRESS AND $\bar{u}\bar{v}$ -CORRELATION COEFFICIENT

The directly observed quantity $\bar{u}\bar{v}$ has been expressed nondimensionally in terms of a coefficient of turbulent shearing stress

$$C_{r1} = \frac{2\bar{u}\bar{v}}{U_1^2}$$

$$C_{rm} = \frac{2\bar{u}\bar{v}}{U_m^2}$$

The choice of coefficients is arbitrary, and C_{r1} is tabulated in table 5 while contour plots for C_{rm} are given in figure 11. The choice of C_{rm} for the figure was made because it was desired to show an over-all picture of variations in τ independent of variations in mean velocity.

The values of the correlation coefficient $\bar{u}\bar{v}/u'v'$ are given in table 6 and figure 12.

CORRELATION COEFFICIENTS R_y AND R_x

The correlation coefficients R_y and R_x express the correlation between values of u at the same instant at two different points. This correlation between points separated by distances in the direction of the local normal to the surface is expressed by R_y , and the correlation between points separated by distances in the direction of the local tangent to the surface is expressed by R_x . These directions were normal and tangential to streamlines only when the local mean direction of the flow was tangential to the surface. Where the boundary layer was thickening rapidly, as near the separation point, the flow in the outer portion of the boundary layer had a greater radius of curvature than the surface and the direction was not tangential to the surface. In such regions, therefore, R_y and R_x do not conform strictly to the conventional definition of such coefficients.

Values of R_y are given in table 7 and values of R_x are given in table 8. Figures 13 and 14 show representative correlation curves in order to give an idea of the distances over which u is correlated compared with the boundary-layer thickness.

It will be noted that a correlation exists over much of the boundary-layer thickness. With the region near separation excluded, fluctuations at the center of the layer are related to those everywhere else in the same section. Under such conditions a small negative correlation is found between points in the layer and those outside, as shown in figure 13. Subsequent measurements in a boundary layer with approximately one-tenth of the free-stream turbulence have shown no effect of the free-stream turbulence on the magnitude of the negative correlation. An explanation of this negative correlation on the basis of continuity requirements is offered in reference 3.

From tables 7 and 8 one may calculate integral scales defined by

$$L_y = \int_0^{\infty} R_y dy$$

$$L_x = \int_0^{\infty} R_x dx$$

These are not given here because it is felt that the qualitative concept of scale obtained from figures 13 and 14 conveys about as much physical significance to scale as is possible at present.

DISCUSSION OF RESULTS

MECHANICS OF SEPARATION

The separation point is defined as the point where the flow next to the surface no longer continues to advance farther in the downstream direction. This results from a failure of the medium to have sufficient energy to advance farther into a region of rising pressure. Certain characteristics of the mean flow serve as a guide to the imminence of separation. For example, the shape factor H can be expected to have a value greater than 2. In the present experiment H was found to have the value 2.7 at the separation point, comparing well with the value of 2.6 given in reference 5.

The empirical guides, however, give little insight into the physical factors involved. Separation is a natural consequence of the loss of energy in the boundary layer, and the burden of explanation rests rather with the question as to why separation does not occur at all times at a pressure minimum. At the surface the kinetic energy of the flow is everywhere vanishingly small. At a pressure minimum the potential energy is a minimum, and the air at the surface, having a vanishing amount of kinetic energy to draw upon, could never advance beyond a pressure minimum without receiving energy from the flow farther out. The necessary transfer is effected by the shearing stresses.

It is a well-known fact that viscous shearing stresses are so small that laminar flow can advance but a little distance beyond a pressure minimum. In contrast with this, turbulent shearing stresses can prevent separation entirely if the rate of increase of pressure is not too great. This emphasizes an important fact; namely, that when separation has not occurred, or has been delayed to distances well beyond the pressure minimum, as in the present experiment, viscous stresses play an insignificant role in the prevention or delay of separation.

Turbulent shearing stresses also determine the magnitude of shearing stresses in the laminar sublayer by forcing there a high rate of shear. This, in fact, gives boundary-layer profiles the appearance of near slip flow at the surface. Thus, turbulent stresses dominate all parts of the boundary layer. Viscous effects in the laminar sublayer and elsewhere still play an important role in determining the existing state of the turbulence. However, in dealing with the effects of turbulence, and not with the origin of turbulence, effects of viscosity can be neglected.

At the high Reynolds numbers of the present experiment the laminar sublayer was extremely thin and was never approached in any of the measurements. At the 17½-foot position at 0.1 inch from the surface the turbulent shearing stress was 190 times the viscous shearing stress. Considering the low order of magnitude of the viscous stresses compared with that of the turbulent stresses, the equations of motion may be closely approximated by including only the Reynolds stresses, and may be written

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{\partial \bar{u}^2}{\partial x} - \frac{\partial \bar{u}v}{\partial y} \quad (1)$$

$$U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - \frac{\partial \bar{u}v}{\partial x} - \frac{\partial \bar{v}^2}{\partial y} \quad (2)$$

While all terms in equations (1) and (2) have been measured, they have not been measured with sufficient accuracy to test the adequacy of the equations. The relative importance of the terms involving Reynolds stresses depends on location in the boundary layer. The normal stresses $\rho \bar{u}^2$ and $\rho \bar{v}^2$ are pressures and their gradients make merely small contributions to $\partial p/\partial x$ and $\partial p/\partial y$. Among the Reynolds stresses the shearing stress is the more important quantity and, accordingly, attention is devoted to it.

It is easy to see qualitatively on physical grounds how the shearing stress must be distributed across the boundary layer. The shearing stress is always in such a direction that fluid layers farther out pull on layers farther in. When the pressure is either constant or falling, all pull is ultimately exerted on the surface. Therefore the shearing stress must be at least as high at the surface as it is elsewhere, and it would be expected to be a maximum there, as it must fall to zero outside the boundary layer. When the pressure is rising, part of the pull must be exerted on the fluid near the surface that has insufficient energy of its own to advance to regions of higher pressure. In other words, the fluid in such layers must be pulled upon harder than it pulls upon the layer next nearer the surface. This means that the shearing stress must have a maximum away from the surface in regions of adverse pressure gradient.

Representative observed distributions are shown in figure 15. It will be seen that the maximum shear stress develops first near the surface and move progressively outward. The region between the surface and the maximum is receiving energy from the region beyond the maximum, the rate per unit volume at each point being $U \frac{\partial \tau}{\partial y}$. Thus the fall in the shearing stress toward the surface, producing a positive slope, is evidence that the shearing stress is acting to prevent separation. It is clear then that a falling to zero, as for example the curve at $x=25.4$ feet, is not the cause of separation. It is rather an indication that the velocity gradient is vanishing at the surface. This means that the velocity in the vicinity of the surface is vanishing and that a condition is developing in which no energy can be

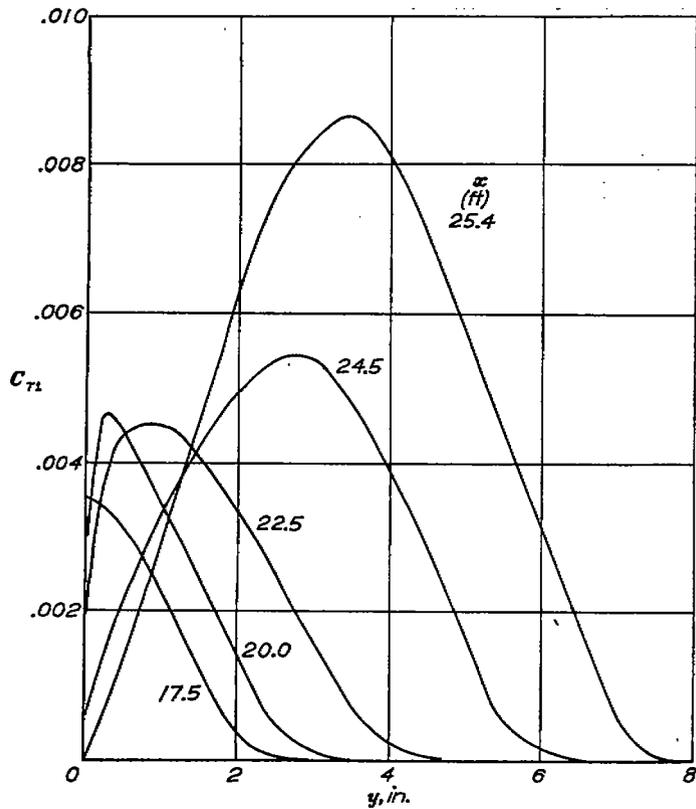


FIGURE 15.—Distribution of coefficient of turbulent shearing stress across boundary layer.

received. When this condition is fulfilled, the fluid can move no farther and separation has occurred.

The initial slope of the curves in figure 15 is given by equation (1), which becomes, when $y=0$:

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y} \quad (3)$$

A theory of the distribution of shearing stress based on the inner boundary conditions $\partial^2 \tau / \partial y^2 = 0$ and equation (3) and on the outer boundary conditions $\tau = 0$ and $\partial \tau / \partial y = 0$ at $y = \delta$ has been given by Fediaevsky (reference 6). The agreement between Fediaevsky's theory and experimental values from the present investigation was fair at the 17½-foot position and excellent at the 25-foot position, but elsewhere was poor. Two examples of the agreement are given in figure 16. The Fediaevsky theory, which defines merely how the curves shall begin and end, either loses control over the middle portion or ignores other controlling factors.

Since equation (3) specifies the initial slope, it is an aid in finding the skin friction by the method of extrapolating the distribution curves to $y=0$. The values found in this way are given in figure 17. As would be expected, the skin friction falls to zero at the separation point. The lack of agreement with values calculated by the Squire-Young formula (reference 7) is to be expected, as this formula does not include the effect of pressure gradient.

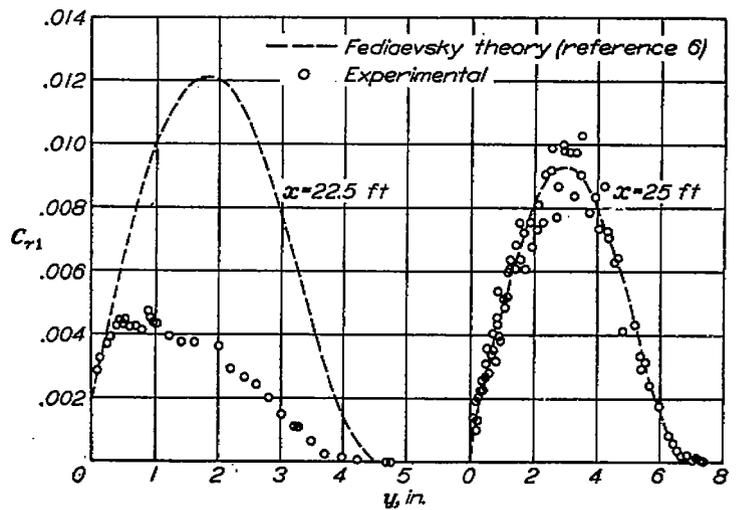


FIGURE 16.—Experimental values for coefficient of turbulent shearing stress compared with curves from Fediaevsky theory.

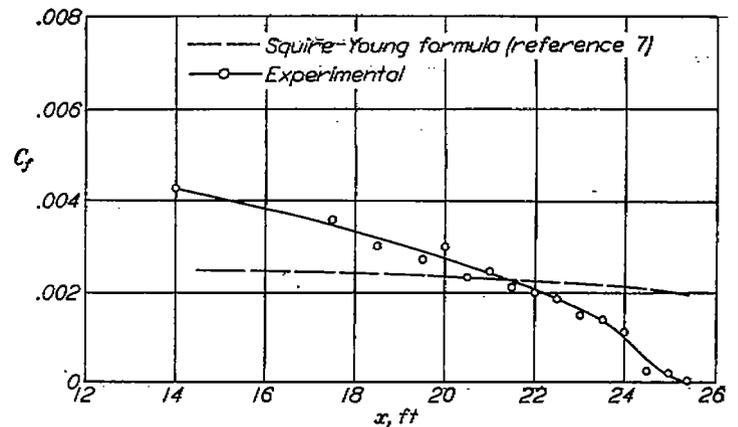


FIGURE 17.—Experimental values for coefficient of skin friction compared with values calculated with Squire-Young formula.

The foregoing discussion has simply described the shearing stress in the light of the present experiment and pointed out the role of shearing stress as an energy-transferring agent. While these phenomena are characteristic in every adverse pressure gradient, the form of the shearing stress and also the velocity profiles will be different for different pressure distributions. The present experiment gives merely one example.

ORIGIN OF TURBULENCE AND TURBULENT SHEARING STRESS

The discussion of origin of turbulence and turbulent shearing stress will be based on concepts that have superseded the older mixing-length theories. Unfortunately, experiments have not kept pace with ideas and the concepts have not yet been fully verified.

In recent years definite ideas have taken shape regarding the decay of turbulence. These stem from an observation made by Dryden (reference 8), namely, that the rates of decay of different frequency components in isotropic tur-

bulence require that the higher-frequency components gain energy at the expense of the lower-frequency components. It has now become generally accepted that decay involves a transfer of energy from larger eddies to smaller eddies by Reynolds stresses when the Reynolds number characteristic of the eddies is sufficiently high. This idea forms the physical basis for modern theories of isotropic turbulence (for example, references 9 to 15).

Information about turbulent flow points more and more to the conclusion that the concept is basic and may be carried over to shear flow. (See, for example, Batchelor's discussion of Kolmogoroff's theory, reference 9, and Townsend's discussion, reference 16.) The general idea may be expressed as follows: The highest Reynolds number is associated with the mean flow, and here the mean Reynolds stresses transfer energy to the flow system comprising the next smaller spatial pattern, for example, the largest eddies. This second system involves other Reynolds stresses which in turn transfer energy to smaller systems and so on through a spectrum of turbulence until the Reynolds number gets so low that the dissipation is completed by the action of viscosity alone. The evolution of heat by the action of viscosity is small for the larger systems and gets progressively greater as the systems get smaller and smaller, with a weighting depending on some Reynolds number characterizing the whole system, say, a Reynolds number based on the outside velocity and the boundary-layer thickness. The higher the Reynolds number the more is the action of viscosity confined to the high-frequency end of the spectrum. Thus at sufficiently high Reynolds numbers the action of viscosity is not only removed from the mean flow but also from all but the smaller-scale components of the turbulence. An exception must, of course, be made for the laminar sub-layer, and the likelihood that this is a valid picture increases with distance from the surface.

These ideas then might be regarded as describing a tentative model of a turbulent boundary layer to be examined in the light of experiment. The model is, of course, conceived only in general outline and should not be assumed the same for all conditions.

The rate of removal of kinetic energy per unit volume from the mean flow by Reynolds stresses is given by:

$$\rho \left[\bar{u}^2 \frac{\partial U}{\partial x} + \bar{v}^2 \frac{\partial V}{\partial y} + \bar{u}v \left(\frac{\partial V}{\partial x} + \frac{\partial U}{\partial y} \right) \right] \quad (4)$$

This energy goes directly into the production of turbulence.

The term $\bar{u}v \frac{\partial U}{\partial y}$ will generally outweigh the others, but in order to see the relative magnitudes near separation the terms in expression (4) were calculated for the 24.5-foot position. The term $\bar{u}v \frac{\partial V}{\partial x}$ was found to be negligible. The other terms within the brackets together with their sum are

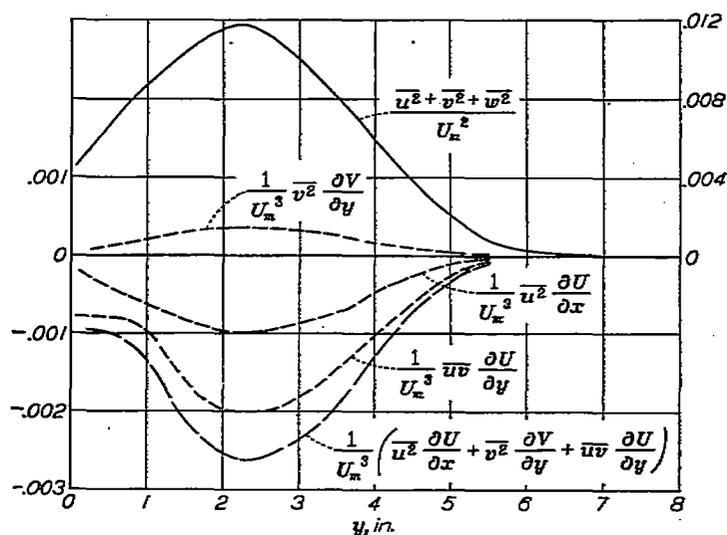


FIGURE 18.—Rate of production of turbulence and mean energy of turbulence at $x=24.5$ feet. Right ordinate scale to be used only for top curve.

shown in figure 18 divided by U_m^3 . It is seen that the term $\bar{u}v \frac{\partial U}{\partial y}$ is still the largest and therefore remains the most important contributor to turbulence.

The distribution of turbulence energy is also given in figure 18. This shows a maximum energy content where the rate of production is the greatest; otherwise the comparison has no particular significance. Such coincidence is not required and is not found farther upstream. Data are not available for establishing the balance between production, diffusion, convection, and dissipation of turbulence energy.

It is clear that the turbulence exists because of the Reynolds stresses, and it is self-evident that the normal stresses $\rho \bar{u}^2$ and $\rho \bar{v}^2$ exist because of the turbulence, but the source of the shearing stress $\rho \bar{u}v$ is not apparent without further examination.

Since

$$\tau = -\rho \bar{u}v = -\rho \frac{\bar{u}v}{u'v'} u'v' \quad (5)$$

where $\bar{u}v/u'v'$ is the correlation coefficient, it is seen that τ depends on the correlation and intensity of u and v . If a flow is turbulent without a gradient in mean velocity, there can be no mean shearing stress and therefore no mean correlation between u and v . It is apparent then that a gradient is necessary to produce a correlation, and one might expect to find $\bar{u}v/u'v'$ proportional to dU/dy . From figure 12 it appears that $\bar{u}v/u'v'$ shows too little variation across the boundary layer to be proportional to the local value of the mean-velocity gradient. To apply a more direct test, $\bar{u}v/u'v'$ was plotted in figure 19 against the mean local gradient. Obviously $\bar{u}v/u'v'$ cannot be regarded as proportional to $(\theta/U_1)(dU/dy)$, and, what is more, it becomes independent of the local gradient for a wide range of values of $(\theta/U_1)(dU/dy)$.

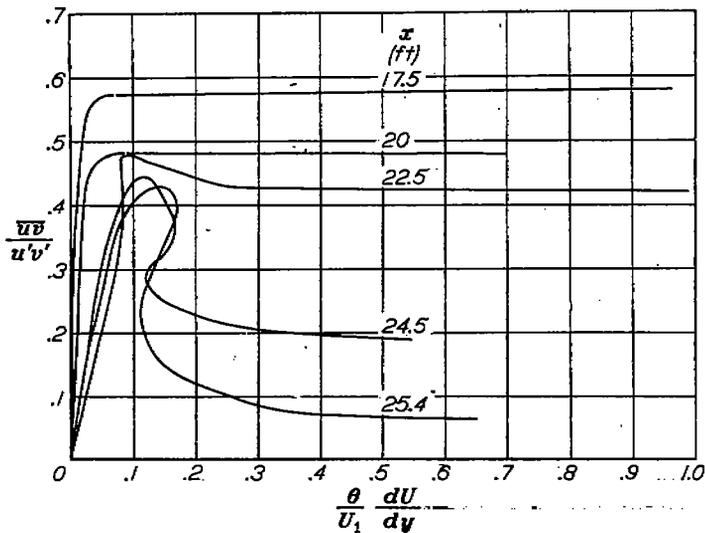


FIGURE 19.—Relation between \overline{uv} -correlation coefficient and local mean-velocity gradient.

Assuming the correctness of the concept of transfer of energy from larger to smaller flow regimes, it is seen that energy flows into turbulence mainly by way of the largest eddies, and it is then mainly these that account for the average shearing stress. Returning to figure 13, it is seen by the curves of R_v that the turbulent motions are correlated over much of the boundary-layer thickness up to the position $X=23$ feet and are still correlated over a considerable portion of the thickness at larger values of x . The extent of the R_v -correlation is roughly a measure of the extent of the largest eddies. This means that the correlation coefficient $\overline{w}/u'v'$ arises from those components of the turbulence that extend over much of the boundary-layer thickness, and the correlation between u - and v -components of such a motion would be expected to depend on the mean-velocity gradient as a whole rather than upon the local gradient at any one point. Large mean gradients exist near the surface without producing correspondingly large correlation coefficients in the same locality, and it appears that the correlations here are very likely fixed by some over-all effect. If an over-all velocity gradient is represented at each position by U_1/U_m divided by δ , and this is used as the independent variable in figure 20 to cross-plot values of $\overline{uv}/u'v'$ taken from the flat portion of the curves in figure 19, a definite proportionality between these two quantities is found. This bears out the foregoing argument.

Figure 21 was originally prepared to test one of the equations of state in Nevzgljadov's theory (reference 17), which expresses the shearing stress as proportional to the turbulent energy per unit volume and the mean-velocity gradient. The theory is not supported by the results for the same reason as that mentioned in connection with figure 19. In fact, shearing stress per unit energy is much like the correlation coefficient and would be two-thirds of $\overline{uv}/u'v'$ if u' , v' , and w' were all equal. The similarity between figures 19 and 21 is therefore not surprising. The hairpin loops in the curves in these two figures apparently result from the distribution

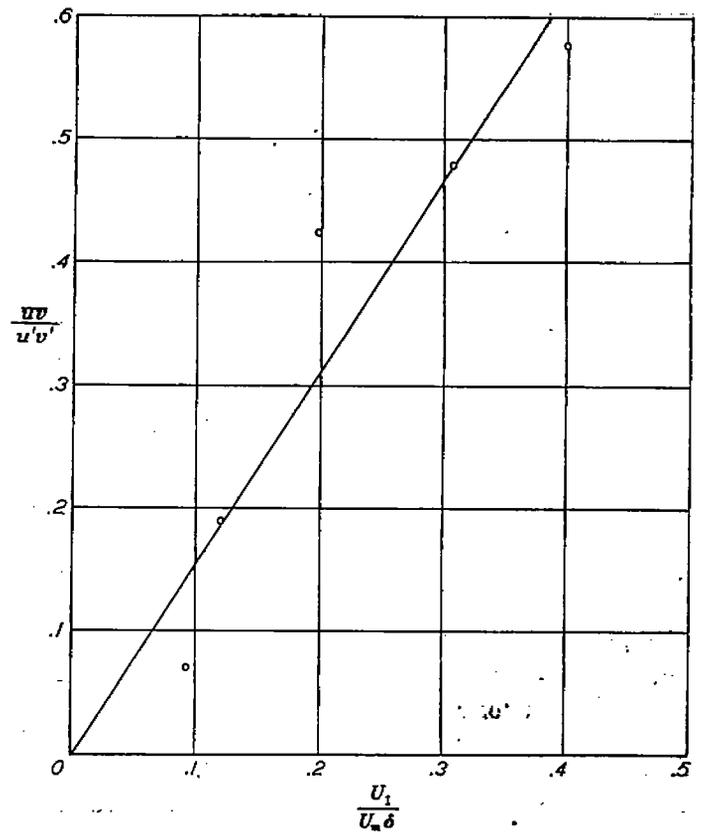


FIGURE 20.—Relation between \overline{uv} -correlation coefficient near surface and general mean-velocity gradient.

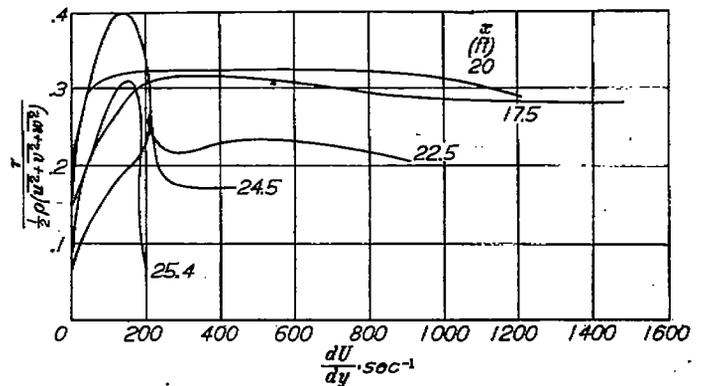


FIGURE 21.—Relation between shearing stress per unit energy of turbulence and local mean-velocity gradient.

of shearing stress imposed by the adverse pressure gradient.

Figure 22 emphasizes the great difference between turbulent shear flow and laminar shear flow. In laminar flow the shearing stress is directly proportional to the local velocity gradient. In turbulent flow, shown in figure 22, the shearing stress may rise abruptly for scarcely any change in the local velocity gradient and again fall with increasing velocity gradient. This illustrates the difficulty of adopting the concepts of viscous flow in turbulent flow. The difference probably arises because turbulent phenomena, unlike molecular phenomena, are on a scale of space and velocity of the same order as that of the mean flow.

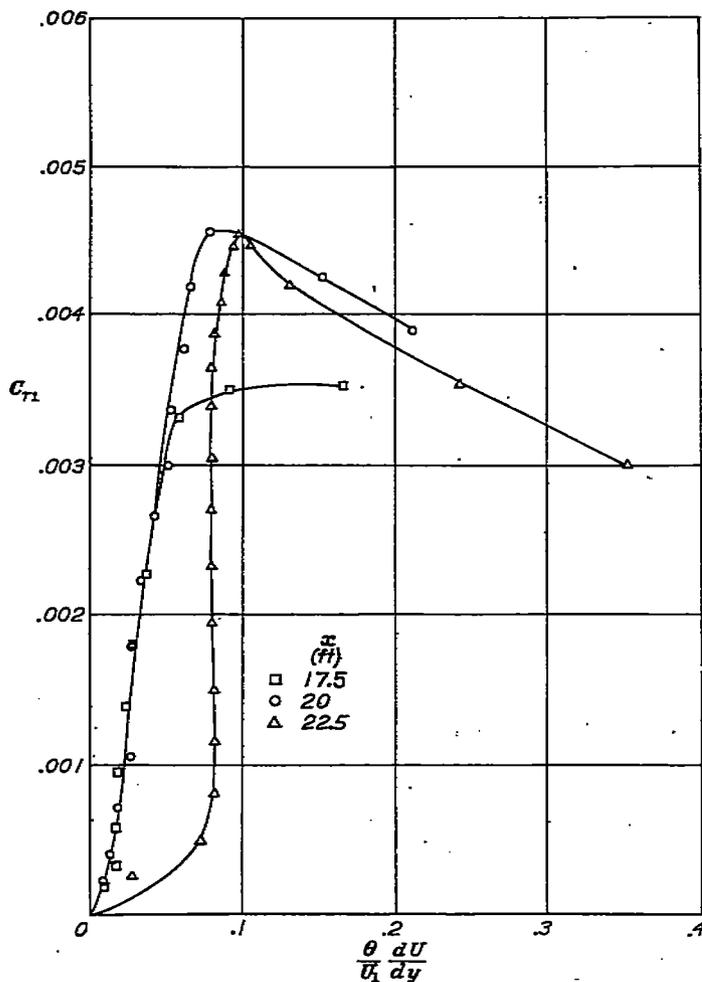


FIGURE 22.—Relation between coefficient of shearing stress and local mean-velocity gradient.

The tentative picture in the form of the model previously described is still speculative and probably oversimplified. It has, however, received support in the present experiment, perhaps as much as could be expected from over-all measurements embracing the entire frequency spectrum. Observations of these same quantities as a function of frequency would be much more informative, but unfortunately the experimental conditions in an open-air wind tunnel discouraged work of this sort. Other types of hot-wire measurements, such as those described by Townsend in reference 16, would be of as great value in probing for the true picture of a turbulent boundary layer as they were in bringing to light phenomena in the turbulent wake of a cylinder.

The present model is but an extension of the concepts required to explain the spectrum and decay of isotropic turbulence. However, in going from the relative simplicity of isotropic turbulence to boundary-layer turbulence many new factors are introduced. Distance from transition point, pressure gradient, curvature, and surface roughness doubtless affect details and may have profound influences. It must be left to future experiments and theory to fill in the gaps, and when this has been done perhaps the data given herein will have more meaning than they have at

present. It is with this thought in mind that the data are given in tables, in which form they are the more readily available for new uses.

CONCLUDING REMARKS

Certain measured characteristics of a separating turbulent boundary layer have been presented. The average characteristics are mean velocity, pressure, and the derived parameters, displacement thickness, momentum thickness, and shape factor. The turbulent characteristics comprise intensities, shearing stresses, transverse correlations, longitudinal correlations, and correlations between two fluctuation components at a point.

The results have been discussed, first, in connection with what they reveal about separation and, second, in connection with what they reveal about the nature of turbulent boundary layers. The modern concept of energy transfer through a spectrum was extended to the turbulent boundary layer. The resulting model of a turbulent boundary layer was supported by the results. This together with the support from theory and experiment in isotropic turbulence makes it appear that the model may be a very useful one for guiding future experiments.

It is seen that the investigation of separation of the turbulent boundary layer had to go beyond the mere investigation of separation. The real problem is the understanding of the mechanics of turbulent shear flow under the action of pressure gradient. The solution of this problem depends on the understanding of the mechanics of turbulence, and in this only rudimentary beginnings have been made.

NATIONAL BUREAU OF STANDARDS,
WASHINGTON, D. C., June 1, 1949.

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TABLE 4.—TURBULENCE INTENSITIES

x=17.5 ft						x=20.0 ft					
y (in.)	w'/U ₁	y (in.)	v'/U ₁	y (in.)	w'/U ₁	y (in.)	w'/U ₁	y (in.)	v'/U ₁	y (in.)	w'/U ₁
0.05	0.088	0.06	0.029	0.10	0.058	0.05	0.091	0.11	0.047	0.06	0.056
.10	.092	.11	.029	.15	.058	.10	.088	.16	.045	.11	.056
.15	.090	.16	.030	.20	.062	.10	.088	.21	.044	.16	.056
.20	.085	.21	.031	.25	.064	.15	.086	.20	.048	.21	.059
.25	.082	.26	.031	.30	.069	.20	.086	.31	.049	.26	.057
.30	.082	.31	.031	.35	.061	.25	.086	.30	.047	.31	.061
.35	.081	.36	.031	.40	.059	.30	.083	.41	.045	.36	.058
.45	.074	.46	.031	.50	.058	.40	.087	.51	.048	.46	.059
.55	.073	.56	.031	.60	.059	.50	.086	.61	.049	.56	.057
.65	.069	.66	.030	.70	.059	.60	.088	.71	.047	.66	.058
.75	.067	.76	.031	.80	.055	.70	.083	.81	.046	.76	.058
.85	.066	.86	.030	.90	.055	.80	.089	.91	.046	.86	.059
.95	.064	.96	.029	1.00	.052	.90	.080	1.01	.047	.96	.054
1.05	.062	1.06	.029	1.10	.049	1.00	.077	1.11	.045	1.06	.054
1.25	.055	1.26	.028	1.30	.043	1.20	.069	1.31	.040	1.26	.051
1.45	.047	1.46	.023	1.50	.035	1.40	.063	1.51	.038	1.46	.047
1.65	.041	1.66	.021	1.70	.029	1.65	.062	1.71	.037	1.66	.042
1.85	.033	1.86	.017	1.90	.016	1.75	.061	1.91	.034	1.86	.039
2.05	.022	2.06	.013	2.10	.015	1.95	.064	2.11	.031	2.06	.036
2.25	.016	2.26	.010	2.30	.012	2.14	.043	2.31	.028	2.26	.029
2.45	.0095	2.46	.0080	2.50	.0094	2.24	.036	2.51	.024	2.46	.023
2.65	.0072	2.61	.0073	2.70	.0088	2.44	.028	2.71	.019	2.66	.018
2.85	.0061	2.81	.0059			2.58	.020	2.91	.015	2.86	.014
		3.01	.0032			2.88	.013	3.11	.011	3.06	.011
		3.21	.0045			3.03	.010	3.31	.0090	3.26	.0091
		3.41	.0042			3.23	.0080	3.51	.0079	3.46	.0079
		3.61	.0042			3.43	.0060	3.71	.0068	3.66	.0071
		3.81	.0042			3.63	.0060	3.91	.0068	3.86	.0068
								4.11	.0062	4.06	.0062
								4.31	.0062	4.26	.0065
								4.51	.0062		

x=21.0 ft						x=22.5 ft					
y (in.)	w'/U ₁	y (in.)	v'/U ₁	y (in.)	w'/U ₁	y (in.)	w'/U ₁	y (in.)	v'/U ₁	y (in.)	w'/U ₁
0.01	0.082	0.05	0.039	0.06	0.051	0.05	0.089	0.06	0.034	0.10	0.056
.06	.091	.10	.042	.11	.054	.10	.103	.11	.037	.15	.058
.11	.095	.15	.044	.16	.055	.15	.108	.16	.038	.20	.061
.16	.097	.20	.046	.21	.057	.20	.108	.21	.040	.25	.062
.21	.095	.25	.048	.26	.058	.25	.113	.26	.041	.30	.063
.26	.096	.30	.048	.31	.058	.30	.114	.31	.042	.35	.060
.31	.095	.35	.049	.36	.058	.35	.114	.40	.042	.40	.064
.41	.092	.45	.053	.46	.061	.40	.116	.50	.043	.50	.063
.51	.088	.55	.052	.56	.061	.45	.117	.60	.042	.60	.068
.61	.084	.65	.053	.66	.062	.55	.117	.70	.044	.70	.070
.71	.089	.75	.054	.76	.062	.65	.114	.80	.045	.80	.071
.81	.084	.85	.052	.86	.058	.75	.116	.90	.044	.90	.072
.91	.084	.95	.054	.96	.058	.85	.117	1.00	.044	1.00	.070
1.01	.079	1.05	.053	1.06	.059	.95	.119	1.10	.045	1.10	.070
1.21	.077	1.25	.048	1.26	.050	1.05	.103	1.30	.045	1.30	.065
1.41	.073	1.45	.048	1.46	.055	1.25	1.09	1.45	.042	1.50	.063
1.61	.064	1.65	.048	1.66	.049	1.45	1.09	1.60	.042	1.70	.062
1.81	.063	1.85	.030	1.86	.047	1.65	1.00	1.80	.042	1.90	.065
2.01	.060	2.05	.042	2.06	.043	1.85	.097	2.00	.043	2.10	.058
2.21	.052	2.25	.042	2.26	.038	2.05	.089	2.20	.040	2.35	.056
2.41	.045	2.45	.032	2.46	.033	2.25	.083	2.40	.037	2.60	.051
2.61	.041	2.65	.028	2.66	.028	2.45	.078	2.60	.035	2.85	.044
2.81	.032	2.85	.024	2.86	.023	2.65	.075	2.80	.033	3.10	.038
3.01	.025	3.05	.022	3.06	.018	2.85	.063	3.00	.030	3.40	.032
3.21	.018	3.25	.017	3.26	.014	3.05	.059	3.20	.026	3.70	.024
3.41	.012	3.45	.014	3.46	.011	3.30	.052	3.40	.023	4.00	.017
3.91	.0049	3.95	.011	3.96	.0091	3.55	.040	3.80	.021	4.30	.012
4.01	.0045	3.95	.0092	3.96	.0071	3.80	.029	3.80	.018	4.60	.0095
4.21	.0043	4.05	.0078	4.06	.0069	4.05	.021	4.00	.014	4.90	.0076
4.41	.0040	4.25	.0078	4.26	.0061	4.30	.015	4.20	.012	5.20	.0067
				4.46	.0062	4.55	.012	4.40	.0095	5.50	.0067
						4.80	.010	4.60	.0076		
						5.05	.0091	4.80	.0064		
						5.30	.0060	5.00	.0055		
						5.55	.0089	5.05	.0051		
								5.25	.0049		
								5.45	.0043		
								5.65	.0043		

TABLE 4.—TURBULENCE INTENSITIES—Concluded

x=23.5 ft						x=24.5 ft					
y (in.)	w'/U ₁	y (in.)	v'/U ₁	y (in.)	w'/U ₁	y (in.)	w'/U ₁	y (in.)	v'/U ₁	y (in.)	w'/U ₁
0.04	0.090	0.18	0.037	0.08	0.049	0.04	0.061	0.35	0.045	0.10	0.056
.09	.083	.26	.044	.13	.054	.09	.065	.40	.047	.14	.054
.14	.088	.31	.043	.18	.054	.14	.074	.45	.048	.19	.058
.19	.088	.36	.042	.23	.056	.19	.071	.50	.046	.29	.054
.24	.094	.46	.045	.28	.058	.24	.074	.55	.049	.39	.058
.29	.091	.56	.049	.33	.060	.29	.076	.65	.051	.49	.064
.34	.093	.66	.051	.48	.063	.34	.077	.75	.052	.59	.063
.41	.097	.76	.052	.58	.063	.44	.063	.85	.052	.69	.061
.54	.099	.86	.056	.68	.066	.54	.084	.95	.057	.79	.063
.64	.089	.96	.053	.78	.064	.64	.085	1.05	.054	.89	.063
.74	.104	1.06	.052	.88	.067	.74	.086	1.15	.059	.99	.062
.84	.103	1.16	.054	.98	.071	.84	.085	1.25	.057	.99	.062
.94	.104	1.36	.055	1.08	.073	.94	.077	1.29	.059	1.09	.069
1.04	.115	1.56	.053	1.28	.069	1.04	.097	1.35	.057	1.29	.070
1.24	.112	1.76	.055	1.48	.067	1.24	.098	1.49	.063	1.49	.070
1.44	.127	1.96	.053	1.68	.067	1.44	.098	1.55	.063	1.69	.075
1.64	.130	2.16	.053	1.88	.065	1.64	.098	1.69	.062	1.89	.075
1.84	.101	2.41	.056	2.08	.064	1.84	.099	1.89	.058	2.09	.077
2.04	.102	2.66	.048	2.33	.060	2.04	.105	2.09	.063	2.34	.079
2.22	.091	2.91	.047	2.58	.058	2.29	.112	2.34	.063	2.59	.077
2.54	.083	3.16	.039	2.83	.054	2.54	.108	2.59	.064	2.84	.075
2.72	.078	3.46	.036	3.08	.050	2.79	.097	2.84	.063	3.09	.074
3.04	.072	3.76	.034	3.33	.042	3.04	.097	3.09	.059	3.39	.065
3.24	.069	4.06	.026	3.68	.037	3.24	.091	3.39	.054	3.69	.061
3.64	.061	4.36	.022	3.93	.029	3.64	.086	3.69	.051	3.99	.056
3.94	.050	4.66	.017	4.28	.023	3.94	.081	3.99	.046	4.29	.052
4.24	.039	4.96	.014	4.58	.016	4.24	.066	4.29	.042	4.59	.043
4.64	.028	5.26	.010	4.88	.011	4.54	.057	4.59	.039	4.89	.036
4.94	.017	5.56	.0077	5.18	.0080	4.84	.046	4.89	.032	5.19	.029
5.14	.011	5.86	.0073	5.48	.0072	5.14	.042	5.19	.026	5.49	.020
5.44	.0080	6.16	.0065	5.78	.0060	5.44	.031	5.49	.022	5.79	.015
5.74	.0068	6.46	.0059	6.08	.0060	5.74	.022	5.79	.017	6.09	.011
6.04	.0063	6.76	.0056	-----	-----	6.04	.014	6.09	.013	6.39	.0079
6.34	.0062	-----	-----	-----	-----	6.34	.011	6.39	.011	6.69	.0068
-----	-----	-----	-----	-----	-----	6.64	.0060	6.69	.0062	6.99	.0063
-----	-----	-----	-----	-----	-----	6.94	.0067	6.99	.0060	7.29	.0068
-----	-----	-----	-----	-----	-----	7.24	.0066	7.29	.0078	-----	-----
-----	-----	-----	-----	-----	-----	-----	-----	7.59	.0071	-----	-----
-----	-----	-----	-----	-----	-----	-----	-----	7.89	.0064	-----	-----
-----	-----	-----	-----	-----	-----	-----	-----	8.19	.0064	-----	-----

x=25.4 ft					
y (in.)	w'/U ₁	y (in.)	v'/U ₁	y (in.)	w'/U ₁
0.05	0.057	0.13	0.034	0.08	0.038
.15	.062	.13	.036	.18	.042
.30	.072	.18	.036	.28	.042
.40	.081	.16	.036	.38	.045
.50	.083	.22	.039	.48	.047
.60	.091	.42	.046	.58	.049
.70	.091	.32	.045	.68	.048
.80	.089	.62	.049	.78	.053
.90	.099	.72	.050	.88	.057
1.05	.101	.82	.055	.98	.055
1.25	.106	.92	.057	1.08	.054
1.45	.095	1.02	.056	1.28	.059
1.65	.108	1.12	.053	1.48	.060
1.85	.117	1.32	.065	1.68	.060
2.05	.122	1.52	.064	1.88	.072
2.30	.126	1.72	.065	2.08	.068
2.55	.131	1.92	.071	2.33	.069
2.80	.129	2.12	.069	2.58	.074
3.05	.127	2.37	.072	2.83	.073
3.30	.129	2.62	.073	3.08	.076
3.55	.132	2.87	.083	3.33	.075
3.80	.121	3.12	.081	3.63	.074
4.05	.119	3.42	.079	3.96	.073
4.35	.114	3.72	.082	4.28	.070
4.65	.105	4.02	.081	4.58	.066
4.95	.102	4.32	.079	4.88	.059
5.25	.094	4.62	.081	5.18	.057
5.55	.087	4.92	.073	5.48	.052
5.85	.078	5.22	.068	5.78	.049
6.15	.065	5.52	.057	6.08	.039
6.45	.059	5.82	.055	6.38	.035
6.75	.045	6.12	.046	6.68	.027
7.05	.034	6.42	.041	6.98	.022
7.35	.023	6.72	.035	7.28	.015
7.65	.016	7.02	.026	7.58	.011
7.95	.012	7.32	.020	7.88	.0092
8.25	.010	7.62	.015	8.18	.0079
8.55	.0082	7.92	.011	8.30	.0079
8.85	.0078	8.22	.0098	8.63	.0079
-----	-----	8.39	.0098	-----	-----

TABLE 5.—COEFFICIENT OF TURBULENT SHEARING STRESS

x=14 ft		x=17.5 ft		x=17.5 ft		x=18.5 ft		x=19.5 ft		x=20.0 ft	
y (in.)	C _{r1}	y (in.)	C _{r1}	y (in.)	C _{r1}	y (in.)	C _{r1}	y (in.)	C _{r1}	y (in.)	C _{r1}
0.16	0.0040	0.09	0.0037	0.08	0.0034	0.08	0.0041	0.06	0.0029	0.08	0.0037
.21	.0039	.15	.0037	.13	.0033	.20	.0040	.13	.0031	.14	.0036
.26	.0037	.24	.0031	.18	.0033	.32	.0033	.30	.0039	.19	.0016
.31	.0040	.36	.0032	.23	.0030	.42	.0023	.36	.0032	.24	.0045
.36	.0037	.50	.0031	.28	.0033	.52	.0033	.53	.0033	.29	.0047
.41	.0035	.67	.0027	.33	.0037	.77	.0026	.61	.0024	.34	.0045
.46	.0034	.86	.0024	.38	.0034	.91	.0028	.77	.0024	.39	.0046
.51	.0035	1.00	.0021	.48	.0032	1.04	.0022	.90	.002	.49	.0045
.56	.0031	1.16	.0018	.53	.0030	1.21	.002	1.09	.0021	.59	.0041
.66	.0029	1.34	.0020	.63	.0030	1.40	.0016	1.29	.0016	.69	.0043
.76	.0025	1.52	.0011	.78	.0025	1.56	.0014	1.49	.0016	.79	.0039
.86	.0024	1.69	.0007	.88	.0025	1.86	.00094	1.71	.0013	.89	.0037
.96	.0020	1.87	.00054	.96	.0022	2.04	.00045	1.89	.00094	.99	.0034
1.06	.0018	2.01	.00028	1.06	.0023	2.25	.00024	2.09	.00096	1.19	.0031
1.16	.0014	2.14	.00016	1.28	.0019	2.54	.00004	2.22	.00045	1.39	.0027
1.36	.00087	2.27	.00009	1.48	.0014	2.84	0	2.42	.00031	1.59	.0022
1.56	.00026	2.54	.00002	1.68	.0010			2.65	.00008	1.79	.0018
1.76	.00006	2.74	.00001	1.88	.00053			2.89	.00002	1.99	.0016
1.96	.00002			2.08	.00025			3.09	0	2.19	.0011
2.06	.00001			2.28	.00011					2.39	.00083
2.26	0			2.48	.00005					2.59	.00043
				2.68	.00003					2.61	.00027
				2.88	.00002					2.81	.00022
				3.08	.00001					2.81	.00013
				3.28	0					3.01	.00007
										3.21	.00002
										3.41	0

x=20.5 ft		x=21.0 ft		x=21.0 ft		x=21.5 ft		x=22.0 ft		x=22.5 ft	
y (in.)	C _{r1}										
0.26	0.0030	0.08	0.0031	0.08	0.0040	0.10	0.0037	0.07	0.0028	0.08	0.0028
.30	.0031	.13	.0041	.13	.0049	.16	.0041	.12	.0032	.13	.0033
.35	.0031	.18	.0044	.18	.0048	.19	.0040	.17	.0035	.18	.0034
.40	.0032	.23	.0047	.23	.0053	.20	.0048	.22	.0041	.23	.0037
.45	.0032	.28	.0032	.28	.0032	.24	.0046	.27	.0041	.28	.0039
.55	.0029	.33	.0040	.33	.0054	.25	.0045	.32	.0037	.33	.0039
.65	.0029	.38	.0042	.38	.0054	.29	.0043	.37	.0040	.38	.0043
.75	.0029	.43	.0044	.43	.0052	.34	.0047	.42	.0042	.43	.0045
.85	.0029	.48	.0047	.43	.0049	.30	.0044	.47	.0045	.48	.0043
.95	.0031	.53	.0046	.48	.0052	.44	.0050	.52	.0047	.53	.0045
1.05	.0031	.58	.0045	.53	.0051	.49	.0052	.57	.0046	.58	.0043
1.15	.0029	.68	.0049	.63	.0047	.54	.0056	.67	.0044	.68	.0043
1.25	.0028	.78	.0047	.73	.0052	.64	.0052	.77	.0050	.78	.0041
1.45	.0024	.88	.0041	1.03	.0050	.74	.0045	.87	.0048	.88	.0047
1.65	.0021	1.08	.0044	1.18	.0049	.84	.0050	.97	.0052	.91	.0046
1.85	.0018	1.28	.0032	1.23	.0043	.94	.0045	1.07	.0051	1.06	.0043
2.05	.0014	1.48	.0031	1.33	.0042	1.04	.0045	1.27	.0044	1.11	.0044
2.25	.0012	1.68	.0029	1.53	.0036	1.24	.0051	1.47	.0044	1.11	.0044
2.45	.00084	1.88	.0022	1.73	.0033	1.44	.0040	1.67	.0045	1.21	.0040
2.70	.00045	2.08	.0021	1.93	.0029	1.64	.0042	1.87	.0039	1.41	.0038
2.95	.00025	2.38	.0014	2.13	.0022	1.84	.0034	2.07	.0032	1.61	.0038
3.20	.00013	2.58	.00032	2.33	.0020	2.04	.0032	2.32	.0030	1.81	.0043
3.45	.00006	2.73	.00043	2.53	.0017	2.24	.0028	2.67	.0023	2.01	.0036
3.70	.00003	3.08	.00031	2.78	.0013	2.44	.0025	3.02	.0016	2.21	.0030
3.95	.00002	3.38	.00012	3.03	.00076	2.64	.0019	3.07	.0011	2.41	.0027
4.26	.00002	3.53	.00004	3.28	.00042	2.84	.0013	3.26	.00072	2.61	.0024
		3.88	.00002	3.58	.00012	3.04	.00090	3.32	.00072	2.81	.0020
		4.03	.00002	3.83	.00018	3.24	.00045	3.51	.00038	3.01	.0016
		4.33	0	3.83	.00012	3.34	.00038	3.76	.00018	3.23	.0012
						3.45	.00033	4.01	.00009	3.27	.0012
						3.54	.00021	4.14	.00001	3.43	.00070
						3.74	.00009	4.26	.00006	3.73	.00031
						3.84	.00005	4.39	.00001	3.98	.00016
						3.94	.00007			4.23	.00003
						4.04	.00001			4.48	.00002
						4.29	.00002			4.69	0
						4.44	.00001			4.74	.00001

TABLE 5.—COEFFICIENT OF TURBULENT SHEARING STRESS—Concluded

x=23.0 ft		x=23.5 ft		x=24.0 ft		x=24.5 ft		x=24.5 ft	
y (in.)	C _τ								
0.06	0.0039	0.13	0.0026	0.05	0.0017	0.12	0.00079	0.10	0.0022
.13	.0026	.18	.0028	.10	.0026	.16	.0011	.15	.0021
.18	.0030	.23	.0029	.15	.0034	.17	.0016	.20	.0018
.23	.0032	.28	.0034	.20	.0031	.22	.0013	.25	.0015
.28	.0029	.33	.0035	.25	.0027	.27	.0017	.30	.0017
.33	.0033	.38	.0033	.30	.0040	.32	.0022	.35	.0018
.38	.0035	.43	.0035	.35	.0029	.37	.0020	.40	.0022
.46	.0040	.53	.0040	.40	.0039	.42	.0019	.50	.0020
.56	.0038	.63	.0046	.45	.0034	.47	.0022	.60	.0024
.68	.0042	.70	.0041	.50	.0039	.57	.0025	.70	.0027
.73	.0047	.73	.0044	.55	.0041	.67	.0020	.80	.0024
.88	.0047	.80	.0046	.65	.0045	.68	.0023	.90	.0023
.92	.0041	.83	.0035	.75	.0046	.77	.0031	1.00	.0031
.98	.0047	.90	.0043	.85	.0047	.78	.0021	1.10	.0034
1.02	.0048	1.00	.0042	.95	.0047	.88	.0027	1.30	.0035
1.22	.0047	1.20	.0047	1.05	.0048	1.08	.0031	1.50	.0047
1.42	.0051	1.40	.0049	1.25	.0048	1.28	.0031	1.70	.0038
1.62	.0044	1.60	.0052	1.45	.0061	1.48	.0043	1.90	.0050
1.82	.0046	1.80	.0053	1.65	.0057	1.68	.0045	2.10	.0057
2.02	.0045	2.00	.0049	1.85	.0072	1.88	.0048	2.30	.0050
2.22	.0045	2.06	.0037	2.05	.0065	2.08	.0047	2.50	.0052
2.42	.0037	2.20	.0043	2.25	.0065	2.22	.0048	2.70	.0053
2.62	.0031	2.26	.0039	2.50	.0047	2.28	.0047	2.90	.0061
2.82	.0026	2.46	.0040	2.75	.0035	2.42	.0051	3.10	.0066
3.02	.0022	2.66	.0035	3.00	.0047	2.62	.0048	3.40	.0045
3.27	.0019	2.86	.0031	3.25	.0040	2.82	.0052	3.70	.0051
3.52	.0014	3.11	.0026	3.50	.0032	3.12	.0051	4.00	.0042
3.77	.00088	3.36	.0023	3.81	.0038	3.42	.0053	4.30	.0031
4.02	.00044	3.61	.0019	3.86	.0025	3.72	.0062	4.60	.0040
4.27	.00023	3.86	.0014	4.16	.0017	4.02	.0038	4.90	.0019
4.52	.00010	4.11	.00093	4.46	.00087	4.32	.0032	5.20	.00060
4.70	.00005	4.36	.00043	4.76	.00037	4.60	.0023	5.40	.00061
4.77	.00004	4.61	.00021	5.06	.00022	4.82	.0021	5.67	.00029
4.96	.00002	4.86	.00014	5.36	.00013	4.90	.0023	5.77	.00027
5.20	.00001	5.11	.00005	5.62	0	5.20	.0015	5.97	.00010
5.45	.00001	5.18	0	5.82	0	5.60	.00079	6.11	.00004
-----	-----	-----	.0001	-----	-----	5.80	.00042	6.27	.00004
-----	-----	-----	-----	-----	-----	6.10	.00011	6.49	.0001
-----	-----	-----	-----	-----	-----	-----	-----	6.79	0

x=25.0 ft		x=25.0 ft		x=25.0 ft		x=25.4 ft	
y (in.)	C _τ						
0.10	0.0011	0.17	0.0010	0.06	0.00077	0.18	0.00050
.15	.0020	.22	.0013	.11	.0014	.28	.00086
.20	.0024	.27	.0020	.21	.0019	.38	.00064
.25	.0021	.37	.0026	.31	.0023	.48	.0011
.30	.0023	.40	.0023	.31	.0021	.58	.00088
.35	.0025	.47	.0026	.41	.0025	.68	.0013
.40	.0036	.50	.0031	.51	.0036	.78	.0019
.45	.0037	.60	.0028	.61	.0034	.88	.0022
.60	.0039	.70	.0035	.71	.0041	.98	.0031
.60	.0032	.85	.0033	.83	.0044	1.08	.0031
.70	.0031	.90	.0040	1.03	.0051	1.18	.0043
.80	.0043	.95	.0038	1.23	.0061	1.48	.0044
.80	.0042	1.08	.0049	1.43	.0085	1.58	.0049
1.00	.0063	1.13	.0060	1.51	.0076	1.68	.0040
1.21	.0065	1.13	.0032	1.71	.0073	1.88	.0059
1.31	.0070	1.28	.0064	1.91	.0076	2.08	.0067
1.40	.0071	1.43	.0061	2.11	.0074	2.28	.0071
1.51	.0069	1.68	.0064	2.31	.0076	2.48	.0077
1.71	.0077	1.73	.0061	2.51	.0082	2.68	.0083
1.91	.0082	1.93	.0068	2.71	.0077	2.88	.0071
1.95	.0087	2.13	.0081	2.91	.0100	3.08	.0071
2.04	.0072	2.33	.0091	3.11	.0098	3.28	.0084
2.18	.0091	2.53	.0099	3.31	.0096	3.48	.0086
2.23	.0087	2.73	.0087	3.51	.0104	3.68	.0088
2.53	.0097	2.93	.0099	3.71	.0085	3.88	.0078
2.63	.0102	3.23	.0084	3.96	.0084	4.13	.0072
3.13	.0102	3.43	.0091	4.07	.0074	4.38	.0077
3.43	.0090	3.73	.0079	4.23	.0067	4.62	.0063
3.65	.0060	4.03	.0061	4.32	.0072	4.88	.0065
3.73	.0078	4.33	.0071	4.67	.0062	5.18	.0054
3.95	.0054	4.63	.0064	4.82	.0041	5.48	.0040
4.25	.0046	4.93	.0056	5.12	.0040	5.78	.0037
4.65	.0041	5.23	.0044	5.42	.0029	6.08	.0030
4.95	.0033	5.37	.0035	5.72	.0020	6.38	.0018
5.16	.0031	5.63	.0032	6.12	.0012	6.68	.0012
5.45	.0022	5.67	.0025	6.42	.00082	6.72	.0012
5.75	.0011	5.97	.0018	6.72	.00021	6.98	.00066
6.02	.00066	6.27	.00082	7.02	.00010	7.02	.00043
6.05	.00076	6.57	.00038	7.32	0	7.23	.00017
6.36	.00007	6.87	.00023	-----	-----	7.52	.00009
-----	-----	7.17	.00018	-----	-----	7.72	.00002
-----	-----	7.32	.00007	-----	-----	-----	-----
-----	-----	7.89	.00005	-----	-----	-----	-----

TABLE 6.— \overline{uv} -CORRELATION COEFFICIENT

x=17.5 ft		x=20.0 ft		x=21.0 ft	
y (in.)	$\overline{uv}/u'v'$	y (in.)	$\overline{uv}/u'v'$	y (in.)	$\overline{uv}/u'v'$
0.10	0.59	0.10	0.45	0.10	0.37
.25	.57	.25	.50	.25	.42
.50	.57	.50	.49	.50	.46
.75	.55	.75	.49	.75	.47
1.00	.55	1.00	.48	1.00	.45
1.25	.55	1.25	.47	1.25	.45
1.50	.53	1.50	.47	1.50	.43
1.75	.48	1.75	.46	1.75	.42
2.00	.42	2.00	.44	2.00	.41
2.25	.32	2.25	.42	2.25	.39
2.50	.21	2.50	.40	2.50	.37
2.75	.14	2.75	.35	2.75	.31
3.00	.04	3.00	.19	3.00	.27
---	---	3.25	.02	3.25	.18
---	---	---	---	3.50	.07

x=22.5 ft		x=23.5 ft		x=24.5 ft		x=25.4 ft	
y (in.)	$\overline{uv}/u'v'$						
0.10	0.42	0.25	0.39	0.25	0.22	0.25	0.11
.25	.44	.50	.39	.50	.25	.50	.15
.50	.45	1.00	.40	1.00	.30	1.00	.24
.75	.47	1.50	.48	1.50	.34	1.50	.31
1.00	.47	2.00	.44	2.00	.37	2.00	.36
1.25	.48	2.50	.42	2.50	.41	2.50	.38
1.50	.48	3.00	.41	3.00	.43	3.00	.40
1.75	.48	3.50	.40	3.50	.44	3.50	.41
2.00	.47	4.00	.36	4.00	.45	4.00	.42
2.25	.47	4.50	.30	4.50	.42	4.50	.42
2.50	.45	5.00	.21	5.00	.38	5.00	.42
2.75	.45	5.50	0	5.50	.30	5.50	.43
3.00	.42	---	---	6.00	.19	6.00	.42
3.25	.39	---	---	6.50	0	6.50	.36
3.50	.34	---	---	---	---	7.00	.26
3.75	.26	---	---	---	---	7.50	.18
4.00	.21	---	---	---	---	8.00	0
4.25	.12	---	---	---	---	---	---
4.50	.06	---	---	---	---	---	---
4.75	0	---	---	---	---	---	---

TABLE 7.—TRANSVERSE CORRELATION COEFFICIENT

x=17.5 ft				x=20.0 ft									
y ₁ =0.98 in.		y ₁ =2.02 in.		y ₁ =0.76 in.		y ₁ =1.01 in.		y ₁ =1.53 in.		y ₁ =2.03 in.		y ₁ =2.06 in.	
y ₂ -y ₁ (in.)	R _{xy}	y ₂ -y ₁ (in.)	R _{xy}	y ₂ -y ₁ (in.)	R _{xy}	y ₂ -y ₁ (in.)	R _{xy}	y ₂ -y ₁ (in.)	R _{xy}	y ₂ -y ₁ (in.)	R _{xy}	y ₂ -y ₁ (in.)	R _{xy}
0.01	0.96	0.01	0.92	0.03	0.94	0.04	0.93	0.02	0.93	0.02	0.96	0.05	0.91
.07	.84	.02	.89	.06	.78	.09	.82	.07	.85	.07	.88	.10	.82
.12	.73	.07	.76	.13	.70	.14	.74	.12	.79	.12	.79	.15	.70
.22	.69	.11	.73	.18	.60	.19	.63	.17	.72	.17	.75	.20	.56
.32	.49	.17	.51	.23	.53	.24	.56	.22	.65	.22	.69	.26	.48
.42	.41	.21	.54	.33	.42	.34	.45	.32	.55	.32	.55	.30	.39
.62	.22	.27	.29	.43	.33	.44	.35	.42	.47	.42	.46	.35	.37
.82	.12	.31	.35	.53	.27	.54	.33	.52	.36	.52	.34	.40	.33
1.02	.068	.41	.21	.73	.16	.64	.23	.62	.32	.62	.30	.45	.26
1.22	.027	.47	.13	.93	.10	.74	.21	.72	.37	.62	.15	.60	.24
1.42	0	.61	.12	1.13	.054	.84	.18	.82	.21	.82	.032	.55	.24
1.62	0	.67	0	1.33	.023	.94	.11	.92	.15	1.02	0	.55	.18
1.82	.97	.81	-.040	1.43	0	1.04	.091	1.02	.10	1.02	.96	.75	.14
2.02	.89	.87	-.097	1.63	0	1.14	.105	1.22	.042	1.02	-.06	.78	.088
2.22	.80	1.01	-.066	1.83	.93	1.24	.088	1.42	.040	1.12	-.13	.66	.083
2.42	.64	1.07	0	2.03	.88	1.34	.078	1.62	-.020	1.22	-.22	.56	1.15
2.62	.53	1.21	-.098	2.23	.78	1.44	0	1.82	0	1.32	-.38	.45	1.073
2.82	.38	1.37	0	2.43	.68	1.54	.93	2.02	0	1.42	-.48	.32	1.75
3.02	.30	1.41	0	2.63	.51	1.64	-.05	2.22	.90	1.52	-.58	.24	0
3.22	.40	1.45	-.95	2.83	.37	1.74	-.10	2.42	-.02	1.62	-.68	.14	.89
3.42	.24	1.49	-.89	3.03	.34	1.84	-.15	2.62	.75	1.72	-.78	.08	.80
3.62	.19	1.53	-.79	3.23	.26	1.94	-.20	2.82	.67	1.82	-.88	.18	.65
3.82	.16	1.57	-.62	3.43	.17	2.04	-.25	3.02	.53	1.92	-.98	.28	.47
4.02	.10	1.61	-.52	3.63	.15	2.14	-.30	3.22	.42	2.02	-1.08	.38	.34
4.22	.058	1.65	-.39	3.83	.13	2.24	-.35	3.42	.33	2.12	-1.18	.48	.18
4.42	---	1.69	-.26	4.03	---	2.34	-.40	3.62	.24	2.22	-1.28	.58	.12
4.62	---	1.73	-.21	4.23	---	2.44	-.45	3.82	.17	2.32	-1.38	.68	.052
4.82	---	1.77	-.082	4.43	---	2.54	-.50	4.02	.16	2.42	-1.48	.78	.033
5.02	---	1.81	-.076	4.63	---	2.64	-.55	4.22	.11	2.52	-1.58	.88	0
5.22	---	1.85	-.028	4.83	---	2.74	-.60	4.42	.10	2.62	-1.68	.98	---
5.42	---	1.89	0	5.03	---	2.84	-.65	4.62	.044	2.72	-1.78	1.08	---
5.62	---	1.93	---	5.23	---	2.94	-.70	4.82	.011	2.82	-1.88	1.18	---
5.82	---	1.97	---	5.43	---	3.04	-.75	5.02	0	2.92	-1.98	1.28	---
6.02	---	2.01	---	5.63	---	3.14	-.80	5.22	---	3.02	-2.08	1.38	---
6.22	---	2.05	---	5.83	---	3.24	-.85	5.42	---	3.12	-2.18	1.48	---
6.42	---	2.09	---	6.03	---	3.34	-.90	5.62	---	3.22	-2.28	1.58	---
6.62	---	2.13	---	6.23	---	3.44	-.95	5.82	---	3.32	-2.38	1.68	---
6.82	---	2.17	---	6.43	---	3.54	-.98	6.02	---	3.42	-2.48	1.78	---
7.02	---	2.21	---	6.63	---	3.64	---	6.22	---	3.52	-2.58	1.88	---
7.22	---	2.25	---	6.83	---	3.74	---	6.42	---	3.62	-2.68	1.98	---
7.42	---	2.29	---	7.03	---	3.84	---	6.62	---	3.72	-2.78	2.08	---
7.62	---	2.33	---	7.23	---	3.94	---	6.82	---	3.82	-2.88	2.18	---
7.82	---	2.37	---	7.43	---	4.04	---	7.02	---	3.92	-2.98	2.28	---
8.02	---	2.41	---	7.63	---	4.14	---	7.22	---	4.02	-3.08	2.38	---
8.22	---	2.45	---	7.83	---	4.24	---	7.42	---	4.12	-3.18	2.48	---
8.42	---	2.49	---	8.03	---	4.34	---	7.62	---	4.22	-3.28	2.58	---
8.62	---	2.53	---	8.23	---	4.44	---	7.82	---	4.32	-3.38	2.68	---
8.82	---	2.57	---	8.43	---	4.54	---	8.02	---	4.42	-3.48	2.78	---
9.02	---	2.61	---	8.63	---	4.64	---	8.22	---	4.52	-3.58	2.88	---
9.22	---	2.65	---	8.83	---	4.74	---	8.42	---	4.62	-3.68	2.98	---
9.42	---	2.69	---	9.03	---	4.84	---	8.62	---	4.72	-3.78	3.08	---
9.62	---	2.73	---	9.23	---	4.94	---	8.82	---	4.82	-3.88	3.18	---
9.82	---	2.77	---	9.43	---	5.04	---	9.02	---	4.92	-3.98	3.28	---
10.02	---	2.81	---	9.63	---	5.14	---	9.22	---	5.02	-4.08	3.38	---
10.22	---	2.85	---	9.83	---	5.24	---	9.42	---	5.12	-4.18	3.48	---
10.42	---	2.89	---	10.03	---	5.34	---	9.62	---	5.22	-4.28	3.58	---
10.62	---	2.93	---	10.23	---	5.44	---	9.82	---	5.32	-4.38	3.68	---
10.82	---	2.97	---	10.43	---	5.54	---	10.02	---	5.42	-4.48	3.78	---
11.02	---	3.01	---	10.63	---	5.64	---	10.22	---	5.52	-4.58	3.88	---
11.22	---	3.05	---	10.83	---	5.74	---	10.42	---	5.62	-4.68	3.98	---
11.42	---	3.09	---	11.03	---	5.84	---	10.62	---	5.72	-4.78	4.08	---
11.62	---	3.13	---	11.23	---	5.94	---	10.82	---	5.82	-4.88	4.18	---
11.82	---	3.17	---	11.43	---	6.04	---	11.02	---	5.92	-4.98	4.28	---
12.02	---	3.21	---	11.63	---	6.14	---	11.22	---	6.02	-5.08	4.38	---
12.22	---	3.25	---	11.83	---	6.24	---	11.42	---	6.12	-5.18	4.48	---
12.42	---	3.29	---	12.03	---	6.34	---	11.62	---	6.22	-5.28	4.58	---
12.62	---	3.33	---	12.23	---	6.44	---	11.82	---	6.32	-5.38	4.68	---
12.82	---	3.37	---	12.43	---	6.54	---	12.02	---	6.42	-5.48	4.78	---
13.02	---	3.41	---	12.63	---	6.64	---	12.22	---	6.52	-5.58	4.88	---
13.22	---	3.45	---	12.83	---	6.74	---	12.42	---	6.62	-5.68	4.98	---
13.42	---	3.49	---	13.03	---	6.84	---	12.62	---	6.72	-5.78	5.08	---
13.62	---	3.53	---	13.23	---	6.94	---	12.82	---	6.82	-5.88	5.18	---
13.82	---	3.57	---	13.43	---	7.04	---	13.02	---	6.92	-5.98	5.28	---
14.02	---	3.61	---	13.63	---	7.14	---	13.22	---	7.02	-6.08	5.38	---
14.22	---	3.65	---	13.83	---	7.24	---	13.42	---	7.12	-6.18	5.48	---
14.42	---	3.69	---	14.03	---	7.34	---	13.62	---	7.22	-6.28	5.58	---
14.62	---	3.73	---	14.23	---	7.44	---	13.82	---	7.32	-6.38	5.68	---
14.82	---	3.77	---	14.43	---	7.54	---	14.02	---	7.42	-6.48	5.78	---
15.02	---	3.81	---	14.63	---	7.64	---	14.22	---	7.52	-6.58	5.88	---
15.22	---	3.85	---	14.83	---	7.74	---	14.42	---	7.62	-6.68	5.98	---
15.42	---	3.89	---	15.03	---	7.84	---	14.62	---	7.72	-6.78	6.08	---
15.62	---	3.93	---	15.23	---	7.94	---	14.82	---	7.82	-6.88	6.18	---
15.82	---	3.97	---	15.43	---	8.04	---	15.02	---	7.92	-6.98	6.28	---
16.02	---	4.01	---	15.63	---	8.14							

TABLE 7.—TRANSVERSE CORRELATION COEFFICIENT—Concluded

x=22.5 ft.										x=23.5 ft.							
y ₁ =0.85 in.		y ₁ =1.45 in.		y ₁ =2.32 in.		y ₁ =2.90 in.		y ₁ =3.92 in.		y ₁ =0.84 in.		y ₁ =1.54 in.		y ₁ =3.05 in.		y ₁ =3.71 in.	
y ₂ -y ₁ (in.)	R _r																
0.02	0.95	0.01	0.96	0.02	0.96	0.02	0.96	0.01	0.97	0.03	0.95	0.02	0.96	0.02	0.97	0.01	0.96
.05	.87	.06	.89	.07	.84	.04	.91	.06	.87	.07	.83	.06	.88	.05	.88	.06	.85
.10	.76	.11	.76	.12	.79	.09	.85	.11	.83	.12	.76	.11	.74	.10	.80	.13	.79
.15	.68	.16	.69	.17	.71	.14	.78	.21	.69	.22	.64	.21	.63	.20	.67	.22	.65
.20	.65	.21	.63	.22	.65	.24	.68	.31	.65	.32	.62	.31	.54	.30	.56	.33	.62
.30	.49	.31	.58	.32	.56	.34	.60	.41	.57	.42	.42	.41	.41	.40	.48	.43	.50
.40	.42	.41	.46	.42	.49	.44	.52	.51	.22	.52	.37	.61	.34	.60	.30	.53	.41
.50	.33	.51	.37	.52	.32	.54	.35	.61	.12	.52	.28	.81	.23	.80	.23	.63	.29
.60	.26	.61	.28	.62	.23	.64	.21	1.01	.055	.62	.22	1.01	.13	1.00	.19	.63	.18
.80	.17	.81	.19	1.02	.15	1.04	.068	1.21	.097	1.03	.14	1.21	.10	1.20	.78	1.03	.079
1.00	.11	1.01	.14	1.22	.094	1.24	.028	1.41	.061	1.22	.12	1.41	.084	1.40	.067	1.23	.053
1.20	.093	1.21	.12	1.42	.11	1.44	-.076	1.61	0	1.42	.085	1.71	.048	1.60	.024	1.43	0
1.40	.067	1.41	.063	1.62	0	1.64	-.051	1.81	.068	1.62	.045	2.01	.026	1.80	0	1.63	-.076
1.60	.042	1.61	.045	1.80	0	1.84	-.14	2.01	.051	1.82	.049	2.31	.029	2.00	0	1.83	-.16
1.80	.030	1.81	.051	-.02	.94	2.04	-.059	2.41	0	2.02	.015	2.61	0	-.02	.98	2.03	-.11
2.00	.033	2.01	.028	-.08	.80	-.04	.92	-.08	.92	2.22	.016	2.91	0	-.04	.89	2.23	0
2.20	0	2.21	0	-.13	.71	-.07	.83	-.08	.86	2.32	.017	-.02	.95	-.09	.81	-.02	.97
2.40	0	2.51	0	-.23	.60	-.12	.82	-.13	.76	2.62	.018	-.04	.85	-.14	.77	-.06	.84
2.60	0	-.06	.92	-.33	.51	-.22	.68	-.23	.60	2.82	0	-.06	.78	-.24	.67	-.11	.76
-.02	.91	-.10	.79	-.43	.45	-.32	.55	-.33	.61	-.03	.97	-.14	.70	-.34	.59	-.21	.72
-.14	.72	-.15	.73	-.63	.28	-.42	.46	-.43	.39	-.07	.87	-.23	.62	-.44	.52	-.31	.59
-.34	.44	-.26	.62	-.83	.18	-.52	.36	-.53	.29	-.11	.82	-.32	.53	-.64	.34	-.41	.50
-.50	.21	-.35	.49	-1.03	.12	-.62	.28	-.63	.23	-.18	.59	-.41	.43	-.84	.26	-.61	.36
-.80	.16	-.41	.43	-1.07	.15	-.82	.20	-.83	.12	-.41	.36	-.60	.27	-1.04	.19	-.81	.28
		-.45	.40	-1.27	.066	-1.02	.15	-1.08	.097	-.55	.26	-.79	.23	-1.24	.19	-1.01	.17
		-.71	.25	-1.47	.040	-1.22	.052	-1.23	.071	-.73	.19	-.94	.18	-1.44	.14	-1.21	.14
		-.91	.14	-1.67	.042	-1.42	.050	-1.43	.068	-.82	.18	-1.14	.11	-1.64	.088	-1.41	.12
		-1.11	.082	-1.87	.025	-1.62	.038	-1.63	0			-1.34	.12	-1.84	.091	-1.61	.081
		-1.31	.051			-1.82	.019	-1.83	0			-1.44	.069	-2.04	.035	-1.81	.076
		-1.41	.051			-1.97	0					-1.53	.032	-2.24	.024	-2.01	.062
														-2.44	.023	-2.81	.036
														-2.74	0	-2.61	0
														-2.92	0		

x=24.5 ft										x=25.4 ft							
y ₁ =0.81 in.		y ₁ =1.73 in.		y ₁ =3.00 in.		y ₁ =4.01 in.		y ₁ =5.02 in.		y ₁ =0.98 in.		y ₁ =3.66 in.		y ₁ =5.93 in.		y ₁ =6.47 in.	
y ₂ -y ₁ (in.)	R _r																
0.03	0.90	0.02	0.95	0.03	0.90	0.02	0.95	0.02	0.97	0.01	0.97	0.02	0.95	0.02	0.96	0.01	0.96
.07	.78	.07	.86	.04	.90	.06	.92	.06	.84	.09	.84	.08	.90	.05	.95	.06	.83
.12	.67	.12	.75	.09	.82	.08	.85	.13	.79	.14	.75	.13	.84	.10	.86	.13	.79
.22	.54	.22	.61	.14	.78	.13	.79	.18	.72	.24	.69	.23	.81	.15	.81	.23	.71
.32	.44	.32	.52	.24	.64	.23	.69	.26	.65	.34	.55	.33	.64	.20	.75	.33	.60
.42	.37	.42	.42	.34	.51	.33	.57	.33	.56	.44	.47	.43	.53	.25	.73	.43	.45
.62	.23	.62	.28	.44	.42	.43	.43	.46	.44	.39	.63	.45	.35	.64	.63	.31	.60
.82	.16	.82	.24	.64	.32	.63	.36	.63	.24	.84	.35	.83	.45	.58	.83	.65	.65
1.02	.12	1.02	.15	.84	.20	.83	.24	.83	.15	1.04	.25	1.03	.26	.65	.41	1.03	.051
1.22	.091	1.32	.11	1.04	.17	1.03	.19	1.03	0	1.24	.18	1.23	.20	.85	.35	1.23	0
1.42	.045	1.62	.045	1.24	.13	1.23	.13	1.13	-.04	1.44	.16	1.43	.17	1.05	.17	1.43	-.070
1.72	.025	1.92	.022	1.64	.075	1.43	.075	1.33	-.09	1.64	.076	1.63	.13	1.25	.074	1.63	-.135
2.02	.015	2.22	0	1.84	.041	1.63	0	1.53	-.10	1.84	.037	1.83	.11	1.45	0	1.83	-.095
2.32	0	2.52	0	2.14	.026	1.83	0	1.83	-.055	2.04	.022	2.03	.060	1.65	0	2.03	-.060
2.62	0	-.02	.97	2.44	0	-.06	.92	2.03	-.13	2.24	0	2.23	.045	-.01	.97	2.23	0
-.04	.93	-.09	.79	2.74	0	-.11	.86	2.23	0	2.44	0	2.43	.025	-.05	.89	2.43	0
-.10	.75	-.14	.72	-.02	.66	-.21	.75	-.01	.69	-.01	.99	2.63	0	-.10	.84	-.01	-.01
-.15	.69	-.24	.62	-.07	.88	-.31	.64	-.05	.88	-.02	.97	2.83	0	-.20	.73	-.02	-.02
-.20	.63	-.34	.55	-.12	.81	-.41	.57	-.10	.81	-.05	.89	-.02	.98	-.30	.58	-.03	-.03
-.31	.41	-.44	.49	-.22	.69	-.61	.42	-.20	.71	-.10	.78	-.06	.89	-.40	.54	-.04	-.04
-.51	.33	-.54	.42	-.32	.59	-.81	.32	-.30	.59	-.16	.61	-.11	.79	-.60	.42	-.05	-.05
-.69	.23	-.64	.34	-.42	.46	-1.01	.25	-.40	.62	-.20	.71	-.21	.68	-.80	.31	-.06	-.06
-.74	.15	-.81	.24	-.52	.39	-1.21	.22	-.50	.44	-.26	.68	-.31	.61	-1.00	.20	-.07	-.07
-.80	.21	-.84	.22	-.62	.32	-1.41	.15	-.60	.40	-.30	.44	-.41	.53	-1.20	.23	-.08	-.08
		-1.01	.16	-.82	.27	-1.61	.11	-.80	.30	-.35	.37	-.61	.41	-1.40	.16	-.09	-.09
		-1.41	.10	-1.02	.17	-1.81	.065	-1.00	.22	-.40	.49	-.81	.33	-1.60	.13	-.10	-.10
		-1.61	.040	-1.22	.11	-2.01	.025	-1.20	.17	-.45	.33	-1.01	.28	-1.80	.079	-.11	-.11
		-1.74	0	-1.42	.079	-2.21	.025	-1.40	.12	-.58	.36	-1.21	.22	-2.00	.054	-.12	-.12
				-1.62	.055	-2.41	.015	-1.60	.069	-.68	.34	-1.41	.15	-2.20	.028	-.13	-.13
				-1.82	.015	-2.61	0	-1.80	.068	-.78	.23	-1.61	.095	-2.40	0	-.14	-.14
				-2.02	.025			-2.00	.053	-.88	.33	-1.81	0	-2.60	.042	-.15	-.15
				-2.09	.038			-2.30	0	-.97	.35	-2.01	0	-2.80	.017	-.16	-.16
				-2.22	.020			-2.50	0					-3.00	0	-.17	-.17
				-2.69	0											-.18	-.18
				-2.71	0											-.19	-.19

TABLE 8.—LONGITUDINAL CORRELATION COEFFICIENT

$x_1=17.5$ ft				$x_1=20.0$ ft					
$y=0.97$ in.		$y=2.01$ in.		$y=0.47$ in.		$y=1.55$ in.		$y=2.56$ in.	
x_2-x_1 (in.)	R_x								
0.07	0.96	0.08	0.88	0.08	0.89	0.07	0.89	0.05	0.95
.15	.84	.15	.83	.06	.85	.12	.83	.10	.88
.31	.78	.30	.72	.15	.79	.22	.76	.20	.82
.51	.70	.50	.68	.23	.71	.32	.69	.30	.85
.81	.63	.80	.44	.33	.70	.42	.63	.40	.80
1.11	.49	1.10	.29	.53	.59	.52	.56	.50	.44
1.51	.34	1.50	.17	.72	.48	.72	.45	1.20	.23
2.01	.23	2.00	.13	1.01	.38	1.12	.42	1.50	.18
2.51	.14	2.50	.10	1.33	.32	1.32	.36	2.00	.094
3.01	.12	3.00	.080	1.68	.27	1.68	.34	2.50	.077
3.51	.075	3.50	.049	2.03	.20	1.84	.25	3.00	.038
4.01	.038	4.00	0	2.43	.17	2.12	.22	3.50	.046
4.51	0	4.50	.015	2.83	.12	2.52	.11	4.00	.022
5.01	0	5.00	.035	3.23	.085	3.02	.14	4.51	0
— .08	.97	5.50	0	3.83	.054	3.52	.071	5.00	0
— .08	.92	— .08	.81	4.33	.053	4.02	.028	— .04	.91
— .13	.88	— .08	.80	4.33	.060	4.52	0	— .06	.92
— .28	.74	— .28	.61	5.83	0	— .02	.97	— .11	.88
— .48	.68	— .48	.49	— .02	.98	— .05	.97	— .21	.77
— .78	.59	— .78	.44	— .06	.86	— .10	.95	— .41	.60
— 1.08	.49	— 1.08	.32	— .14	.82	— .20	.82	— .61	.51
— 1.48	.35	— 1.48	.23	— .29	.71	— .40	.71	— .91	.34
— 1.98	.23	— 1.98	.20	— .49	.60	— .60	.62	— 1.21	.24
— 2.48	.22	— 2.48	.15	— .79	.44	— .90	.51	— 1.51	.21
— 2.98	.14	— 2.98	.12	— 1.09	.33	— 1.20	.37	— 2.01	.12
— 3.48	.10	— 3.48	.13	— 1.49	.22	— 1.50	.32	— 2.51	.065
— 3.98	.043	— 3.98	.059	— 1.99	.15	— 2.00	.24	— 3.01	.022
— 4.44	.022	— 4.48	.045	— 2.49	.11	— 2.50	.15	— 3.51	0
— 4.98	0	— 4.98	.025	— 2.99	.059	— 3.00	.13	— 4.01	0
-----	-----	-----	.026	— 3.49	.056	— 3.50	.10	-----	-----
-----	-----	-----	-----	— 3.99	.026	— 3.94	.10	-----	-----
-----	-----	-----	-----	— 4.69	.049	— 4.34	.071	-----	-----
-----	-----	-----	-----	— 5.19	.016	— 4.84	.028	-----	-----
-----	-----	-----	-----	— 5.69	0	— 5.34	0	-----	-----
-----	-----	-----	-----	— 6.19	0	-----	-----	-----	-----

$x_1=22.5$ ft		$x_1=24.5$ ft		$x_1=25.4$ ft					
$y=2.32$ in.		$y=3.01$ in.		$y=0.98$ in.		$y=3.66$ in.		$y=5.96$ in.	
x_2-x_1 (in.)	R_x								
0.08	0.94	0.08	0.96	0.08	0.94	0.02	0.97	0.08	0.97
.08	.91	.08	.91	.10	.92	.07	.96	.09	.94
.16	.87	.15	.86	.17	.86	.14	.89	.16	.92
.31	.78	.30	.78	.32	.75	.29	.78	.31	.76
.51	.65	.50	.67	.52	.60	.49	.68	.51	.62
.81	.52	.85	.58	.82	.54	.79	.56	.81	.42
1.11	.38	1.10	.36	1.12	.48	1.09	.40	1.11	.50
1.51	.28	1.50	.33	1.52	.32	1.48	.37	1.51	.34
2.01	.20	2.00	.29	2.02	.26	1.99	.22	2.01	.26
2.51	.12	2.50	.18	2.52	.18	2.49	.17	2.51	.16
3.01	.040	3.00	.10	3.02	.19	2.99	.15	3.01	.10
3.51	.019	3.50	.055	3.52	.077	3.49	.11	3.51	.076
4.01	0	4.00	0	4.02	.034	3.99	.047	4.01	.039
— .08	.97	— .08	.97	4.52	0	4.49	0	4.51	0
— .07	.95	— .09	.94	— .03	.93	4.99	0	5.02	0
— .14	.91	— .17	.84	— .08	.92	— .08	.96	— .03	.99
— .29	.79	— .32	.78	— .28	.81	— .08	.94	— .07	.99
— .49	.65	— .52	.65	— .48	.67	— .15	.90	— .15	.92
— .79	.51	— .82	.55	— .78	.47	— .30	.82	— .30	.83
— 1.09	.38	— 1.12	.39	— 1.08	.45	— .50	.69	— .50	.75
— 1.49	.26	— 1.52	.28	— 1.48	.36	— .80	.60	— .80	.60
— 1.99	.13	— 2.02	.18	— 1.98	.26	— 1.10	.49	— 1.10	.51
— 2.49	.10	— 2.52	.13	— 2.48	.21	— 1.50	.39	— 1.50	.36
— 2.99	.055	— 3.02	.085	— 2.98	.16	— 2.00	.26	— 2.00	.23
— 3.49	0	— 3.52	.038	— 3.48	.10	— 2.50	.15	— 2.50	.15
-----	-----	— 4.02	.019	— 3.98	.15	— 3.00	.14	— 3.00	.10
-----	-----	— 4.52	0	— 4.48	.038	— 3.50	.11	— 3.50	.084
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