REPORT No. 749

PROPELLER CHARTS FOR THE DETERMINATION OF THE ROTATIONAL SPEED FOR THE MAXIMUM RATIO OF THE PROPULSIVE EFFICIENCY TO THE SPECIFIC FUEL CONSUMPTION

By David Biermann and Robert N. Conway

SUMMARY

A set of propeller operating efficiency charts, based on a coefficient from which the propeller rotational speed has been eliminated, is presented. These charts were prepared with data obtained from tests of full-size metal propellers in the NACA propeller-research tunnel. Working charts for nine propeller-body combinations are presented, including results from tests of dual-rotating propellers.

These charts are to be used in the calculation of the range and the endurance of airplanes equipped with constant-speed propellers in which, for given flight conditions, it is desired to determine the propeller revolution speed that gives the maximum ratio of the propulsive efficiency to the specific fuel consumption. The coefficient on which the charts are based may be written in the form of a thrust coefficient or a thrust-power coefficient.

A method of using the charts is outlined and sample computations for a typical airplane are included.

INTRODUCTION

In the calculation of the range and the endurance of airplanes equipped with constant-speed propellers, the thrust horsepower required for flight at a given gross weight, airspeed, and altitude is known. It is desired to determine, from the propeller-engine operating characteristics, the propeller rotational speed that gives the maximum ratio of the propulsive efficiency to the specific fuel consumption. The usual propeller coefficients are insufficient for use in such calculations because knowledge of both the engine power and the rotational speed is required and these quantities are unknown.

In order to facilitate the calculations, the Bureau of Aeronautics, Navy Department, requested the NACA to prepare working plots of a thrust coefficient that is independent of propeller revolution speed. The present report contains such working charts for nine propeller-body combinations and includes results obtained from tests of dual-rotating propellers. A method that can be used for the determination of the maximum value of the ratio of the propulsive efficiency to the specific fuel consumption is also included.

SYMBOLS

P	power of engine, foot-pounds per second
T_e	effective thrust ($T-\Delta D$), pounds
T	total propeller thrust
ΔD	increased drag of body due to propeller slipstream
W	weight, pounds
n	propeller rotational speed, rps
L/D	lift-drag ratio
V	airspeed, feet per second
D	propeller diameter, feet
ρ	mass density of air, slugs per cubic foot
ρ_0	mass density of air at sea level (0.002378)
σ	relative density of air (ρ/ρ_0)
q	dynamic pressure ($\frac{1}{2}\rho V^2$)
C	engine specific fuel consumption, pounds per brake horsepower-hour
Q	engine torque, foot-pounds
C_s	speed-power coefficient ($\sqrt[5]{\rho V^5/Pn^2}$)
V/nD	advance-diameter ratio
T_c	thrust coefficient $\left(\frac{T_e}{\rho D^2 V^2}\right)$
C_P'	power coefficient $\left(\frac{P}{\rho D^2 V^3}=P_c\frac{\pi}{8}\right)$
P_c	power disk-loading coefficient $\left(\frac{8P}{\rho\pi D^2 V^3}\right)$

DISCUSSION

In the calculation of the range and the endurance of airplanes equipped with constant-speed propellers it is desired to determine, from the engine-propeller operating characteristics, the propeller speed that gives the maximum ratio of the propulsive efficiency to the specific fuel consumption. The thrust horsepower required for a given flight at a given gross weight, airspeed, and altitude is already known.

The usual propeller coefficients of power, thrust, and propulsive efficiency that are defined as

$$C_P=\frac{P}{\rho n^3 D^5}$$

$$C_T=\frac{T_e}{\rho n^2 D^4}$$

$$\eta=\frac{C_T}{C_P}\frac{V}{nD}$$

are insufficient for this purpose because neither the engine power nor the engine speed is known.

In order to solve these special problems with the least work it is necessary to use plots of propeller coefficients that do not involve the rotational speed. Suitable forms of the coefficients may be obtained from the usual thrust or power coefficients as follows:

$$T_c=\frac{C_T}{(V/nD)^2}=\frac{T_e}{\rho D^2 V^2}$$

$$C_P'=\frac{C_P}{(V/nD)^3}=\frac{P}{\rho D^2 V^3}$$

Because the brake horsepower is unknown, the equation for C_P' should be multiplied by η to obtain a thrust-power coefficient:

$$\eta C_P'=\frac{\eta C_P}{(V/nD)^3}=\frac{\eta P}{\rho D^2 V^3}$$

or

$$\eta C_P'=\frac{T_e}{\rho D^2 V^2}=T_c$$

The two general forms of the coefficient are $\eta P/\rho D^2 V^3$ and $T_e/\rho D^2 V^2$, which may be converted into engineering units as follows:

$$\frac{\eta P}{\rho D^2 V^3}=\frac{73300\ thp}{\sigma D^2 (mph)^3}$$

and

$$\frac{T_e}{\rho D^2 V^2}=\frac{195.4\ T_e}{\sigma D^2 (mph)^2}$$

PROPELLER DATA

The propeller charts were prepared from data obtained in the NACA 20-foot propeller-research tunnel. The tests that provided the data were made with tractor propellers of approximately 10-foot diameter operating in conjunction with various representative body types. A list of the charts and the basic propeller data are given in the following table; principal model dimensions are given directly on the propeller charts.

Figure	Propeller drawing number	Total number of blades	Body condition	Approximate propeller diameter (ft)	Rotation	Blade-angle range at 0.75 R (deg)	Source of propeller data, reference number
	Hamilton Standard:						
1, 2	3155-6	3	Streamline body, with spinner, on symmetrical wing	10	Single	20 to 65	1
3, 4	6193A-3	3	Streamline body, with spinner	11.25	do	20 to 60	1
5, 6	3155-6	4	Streamline body, with spinner, on symmetrical wing	10	do	20 to 60	2
7, 8	3155-6	6	do	10	do	20 to 65	2
9, 10	3156-6, r. h. 3156-6, l. h.	4	do	10	Dual	20 to 60	2
11, 12	3156-6, r. h. 3156-6, l. h.	6	do	10	do	20 to 65	2
	Bureau of Aeronautics:						
13, 14	5868-9	3	Liquid-cooled engine nacelle, with spinner	10	Single	15 to 60	3
15, 17	5868-9	3	Radial engine nacelle	10	do	15 to 40	4
16, 18	5868-9	4	Liquid-cooled engine nacelle, with spinner	10	do	15 to 45	5

PROPELLER CHARTS

Two charts are provided for each of the propeller-body combinations. One chart contains curves of V/nD plotted against T_c between 0 and 0.10 with contour lines of constant η. The other chart contains curves of T_c between 0 and 0.40 and curves of η plotted against V/nD. The same quantities may be read on either type of plot but greater accuracy is obtained by the use of the first type for the range of T_c values that it covers. The second type of chart is included because it offers greater convenience in extending the range of T_c values.

USE OF CHARTS

It is possible that propeller charts of the type given in this report may have a number of uses that are not anticipated at the present time. Also, it is quite likely that different methods can be devised for using the charts in solving any specific problem. The chief object of these charts is, however, to furnish a means of determining the optimum propeller speed for given flight conditions to obtain the maximum ratio of the propulsive efficiency to the engine fuel consumption. A method of obtaining the optimum propeller speed is presented in the following outline.

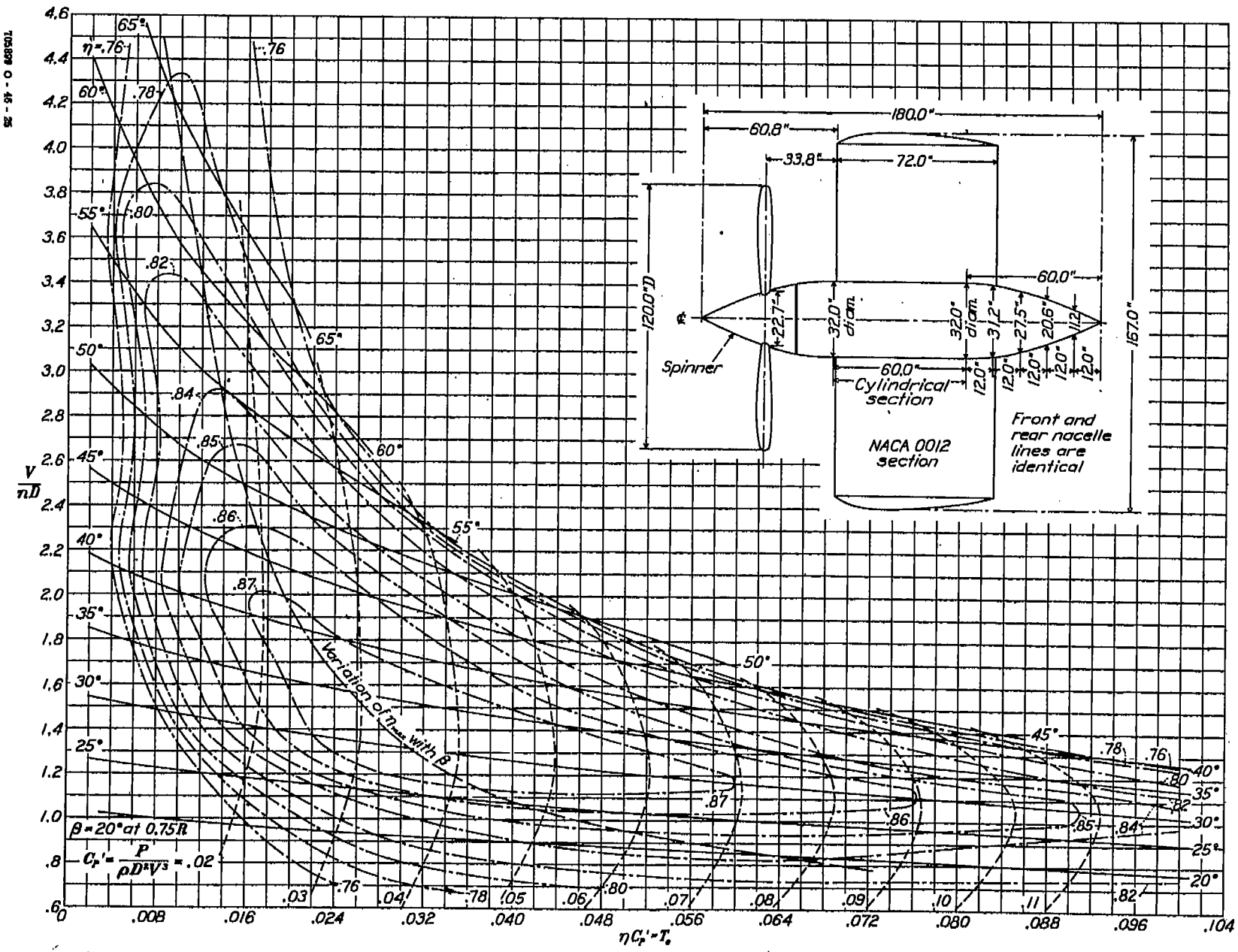

FIGURE 1.—Chart of V/nD against T_c for three-blade propeller 3155-6. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 1.

705839 O - 46 - 25

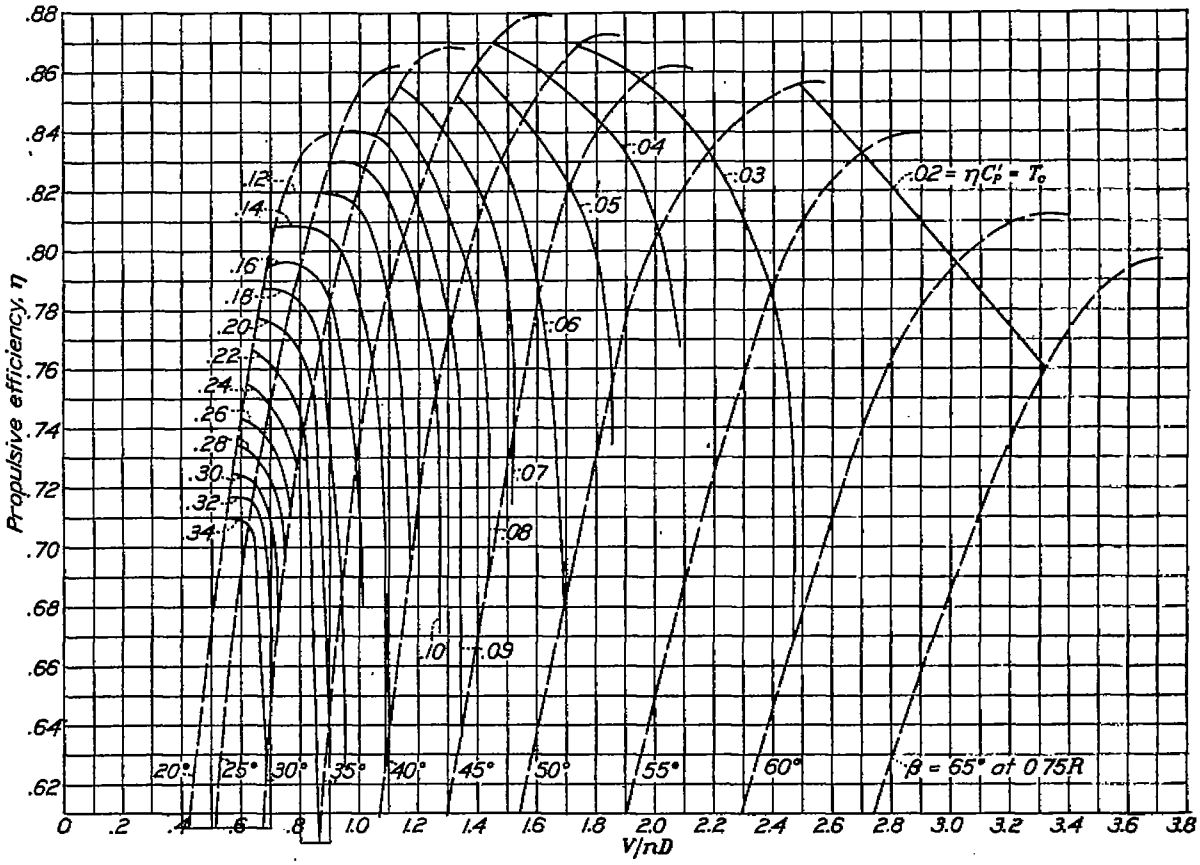

FIGURE 2.—Chart of η against V/nD for three-blade propeller 3155-6. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 1.

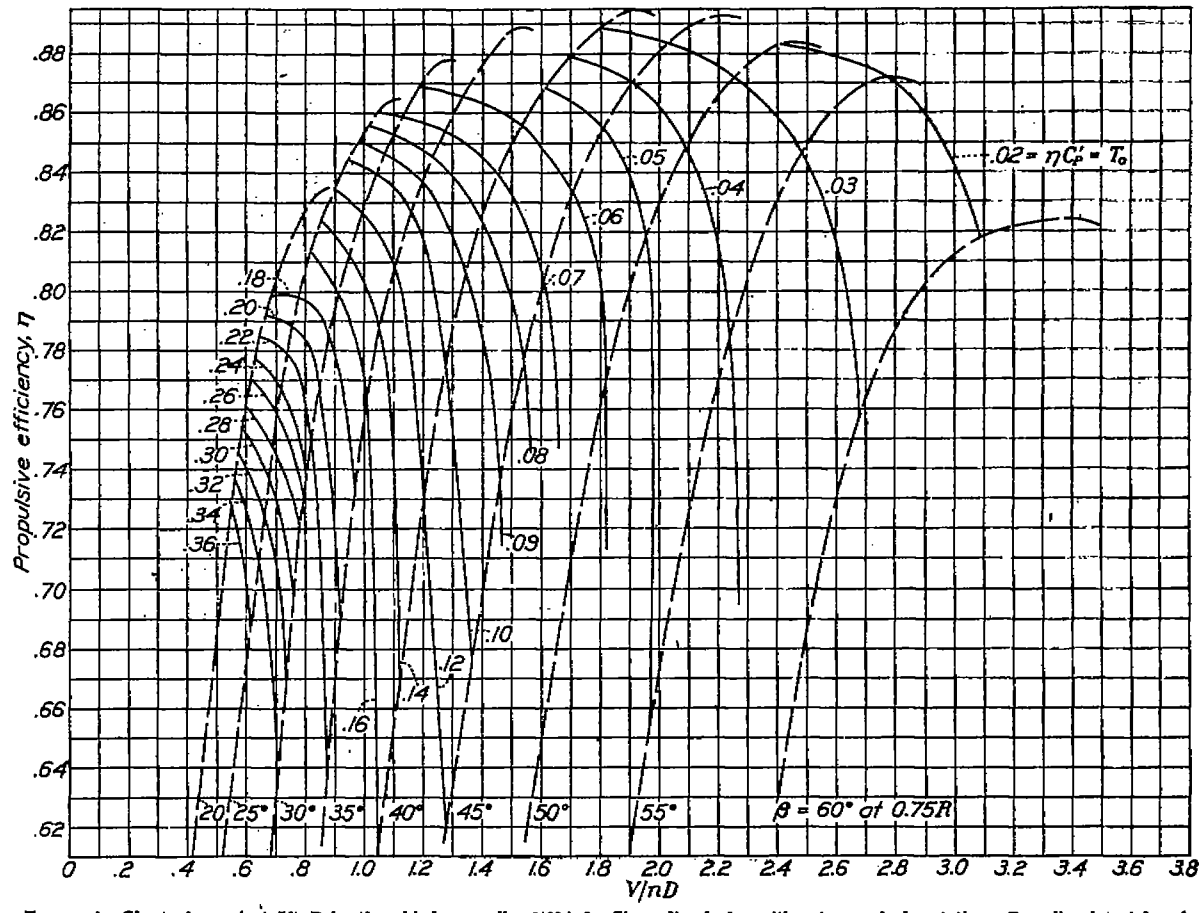

FIGURE 4.—Chart of η against V/nD for three-blade propeller 6193A-3. Streamline body, with spinner; single rotation. Propeller data taken from reference 1.

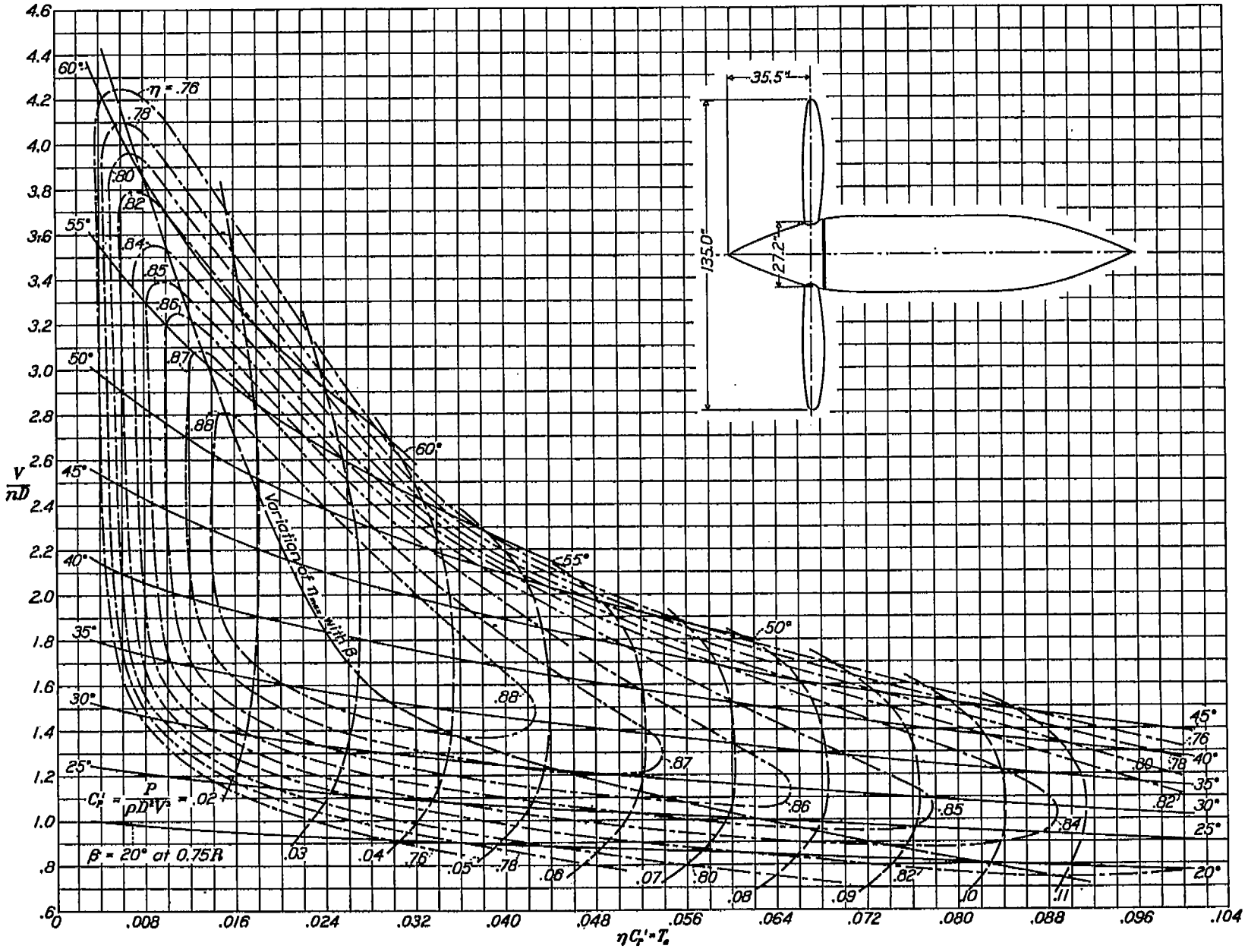

FIGURE 8.—Chart of V/nD against T_c for three-blade propeller 6193A-8. Streamline body, with spinner; single rotation. Propeller data taken from reference 1.

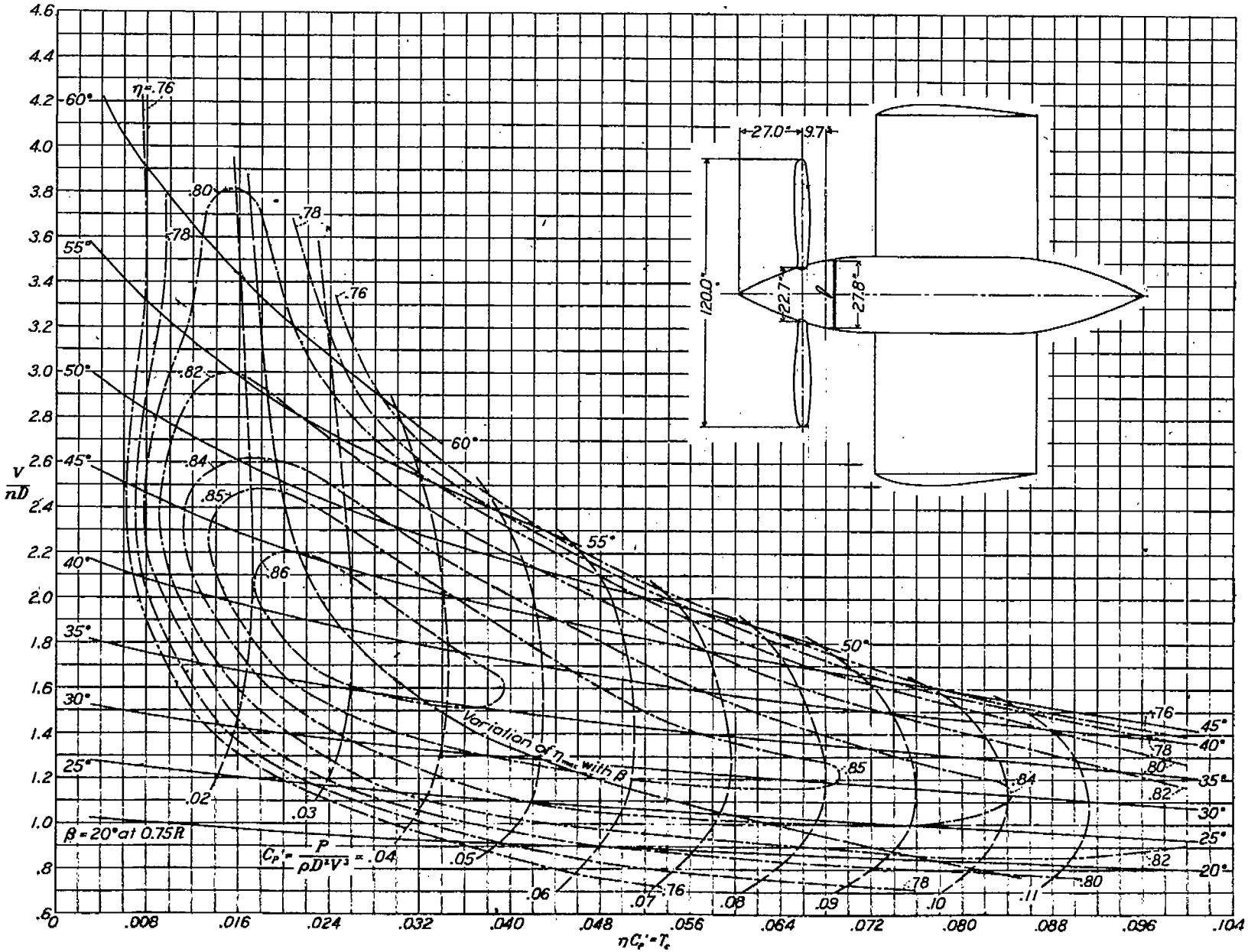

FIGURE 8.—Chart of V/nD against T_c for four-blade propeller 5155-6. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 2.

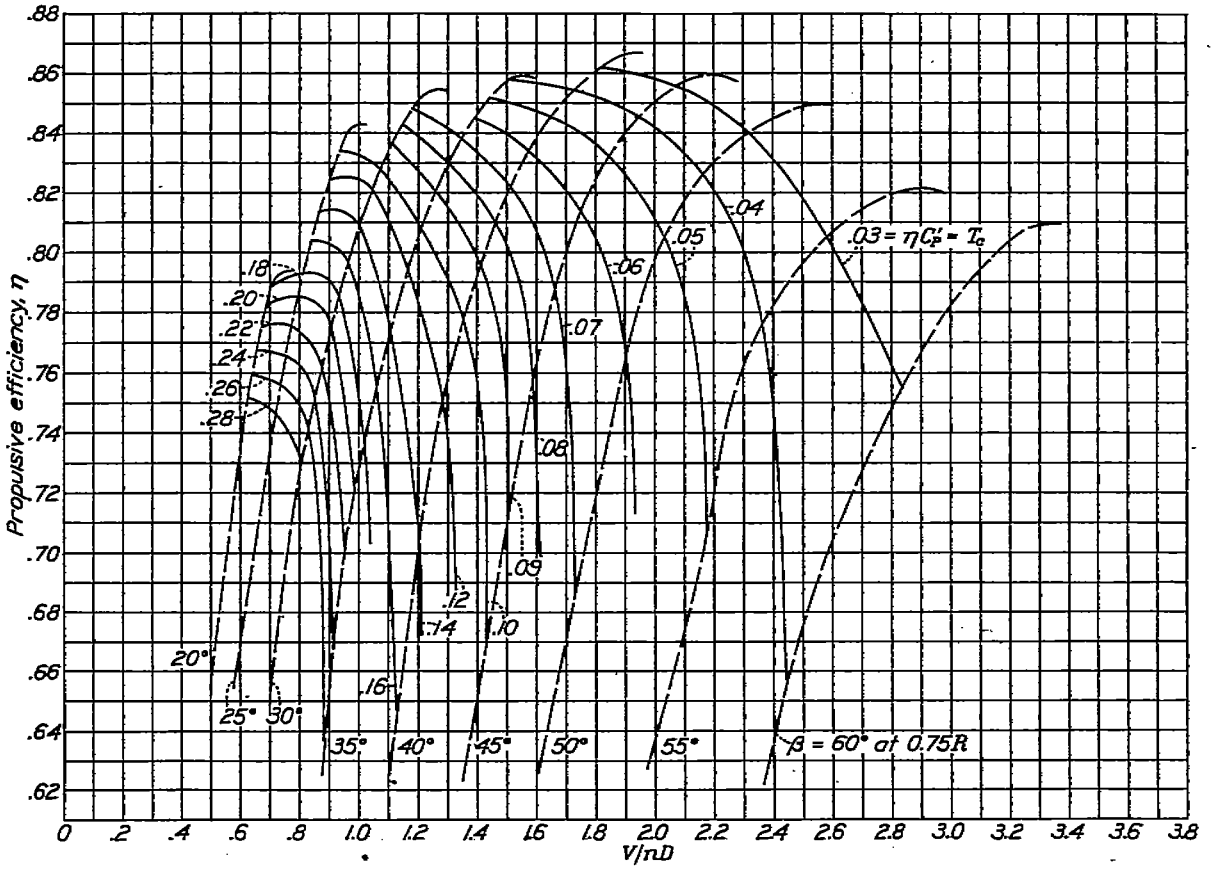

FIGURE 6.—Chart of η against V/nD for four-blade propeller 3155-6. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 2.

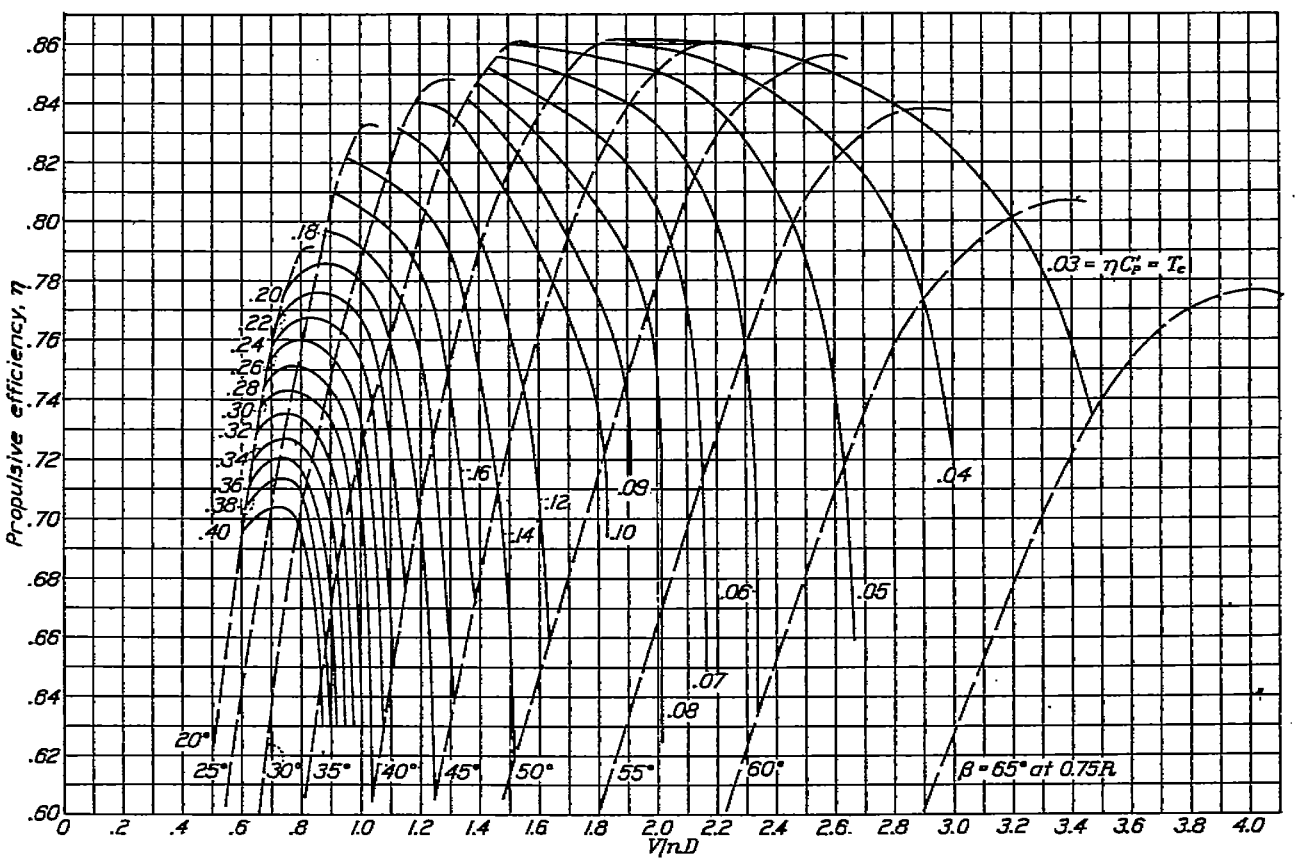

FIGURE 8.—Chart of η against V/nD for six-blade propeller 3155-6. Streamline body, with spinner, on symmetrical wing; single rotation. (The 50° efficiency curve was interpolated.) Propeller data taken from reference 2.

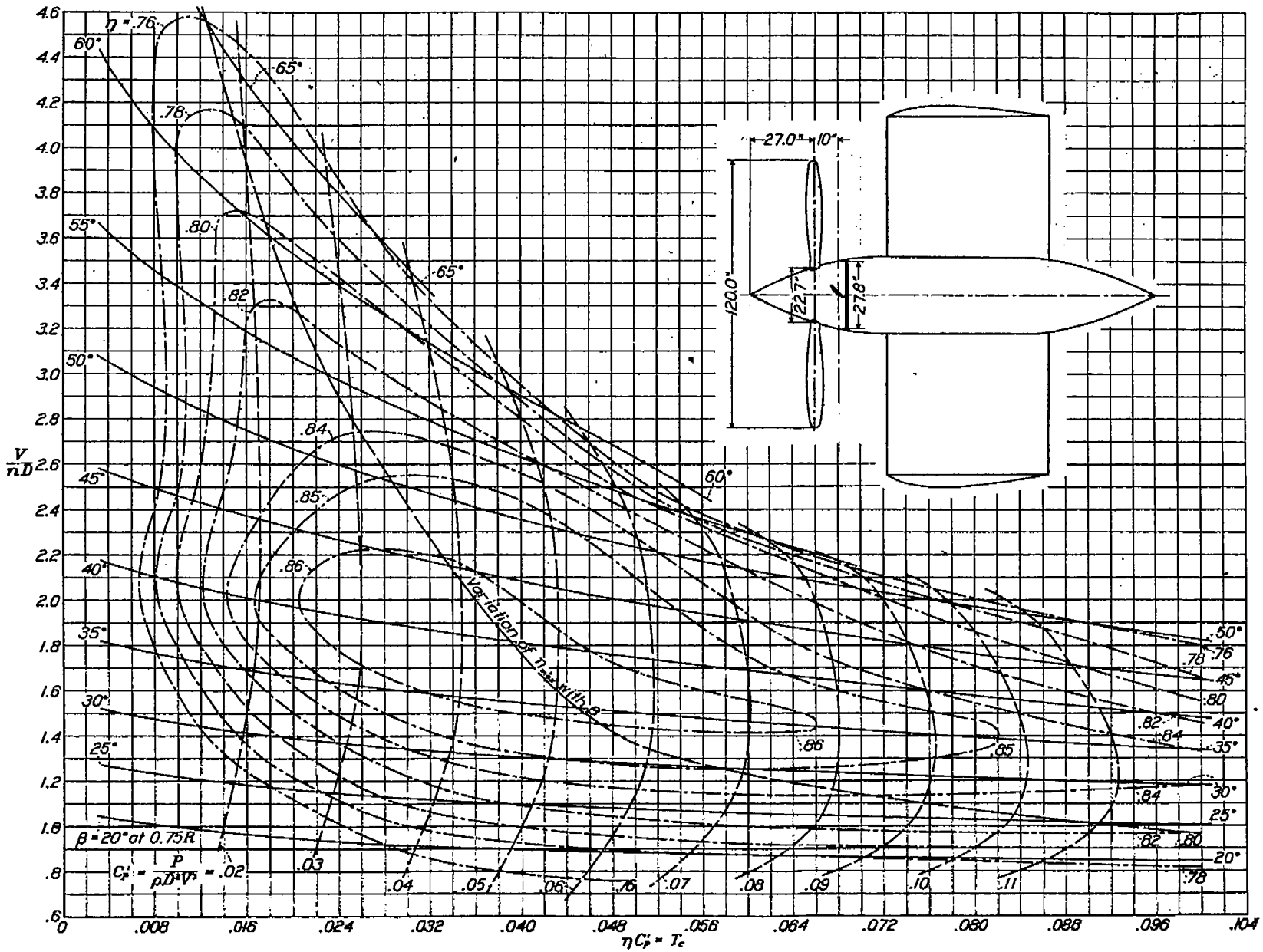

FIGURE 7.—Chart of V/nD against T_c for six-blade propeller 3155-6. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 2.

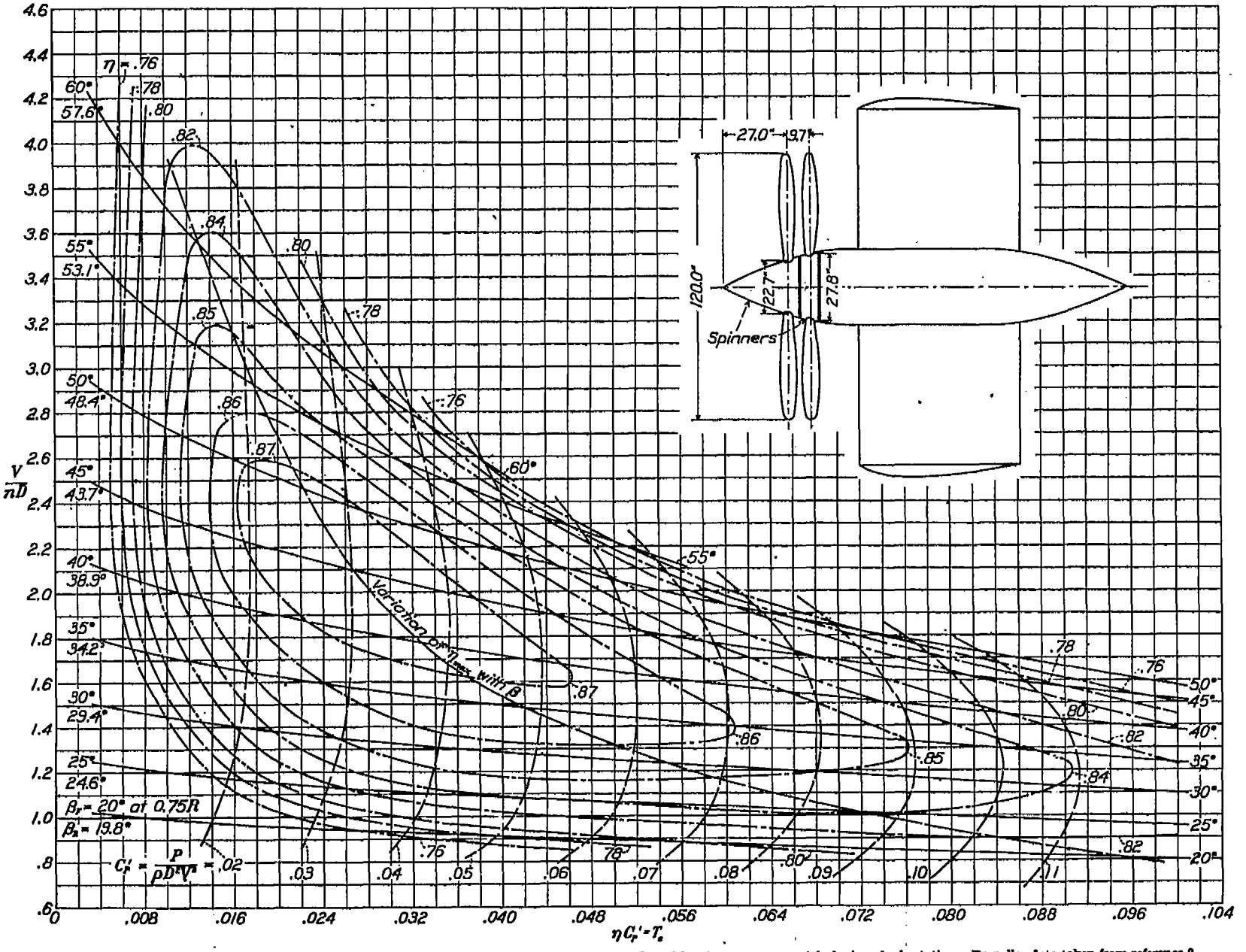

FIGURE 9.—Chart of V/nD against T_c for two two-blade propellers 3155-6 and 3156-6. Streamline body, with spinner, on symmetrical wing; dual rotation. Propeller data taken from reference 2.

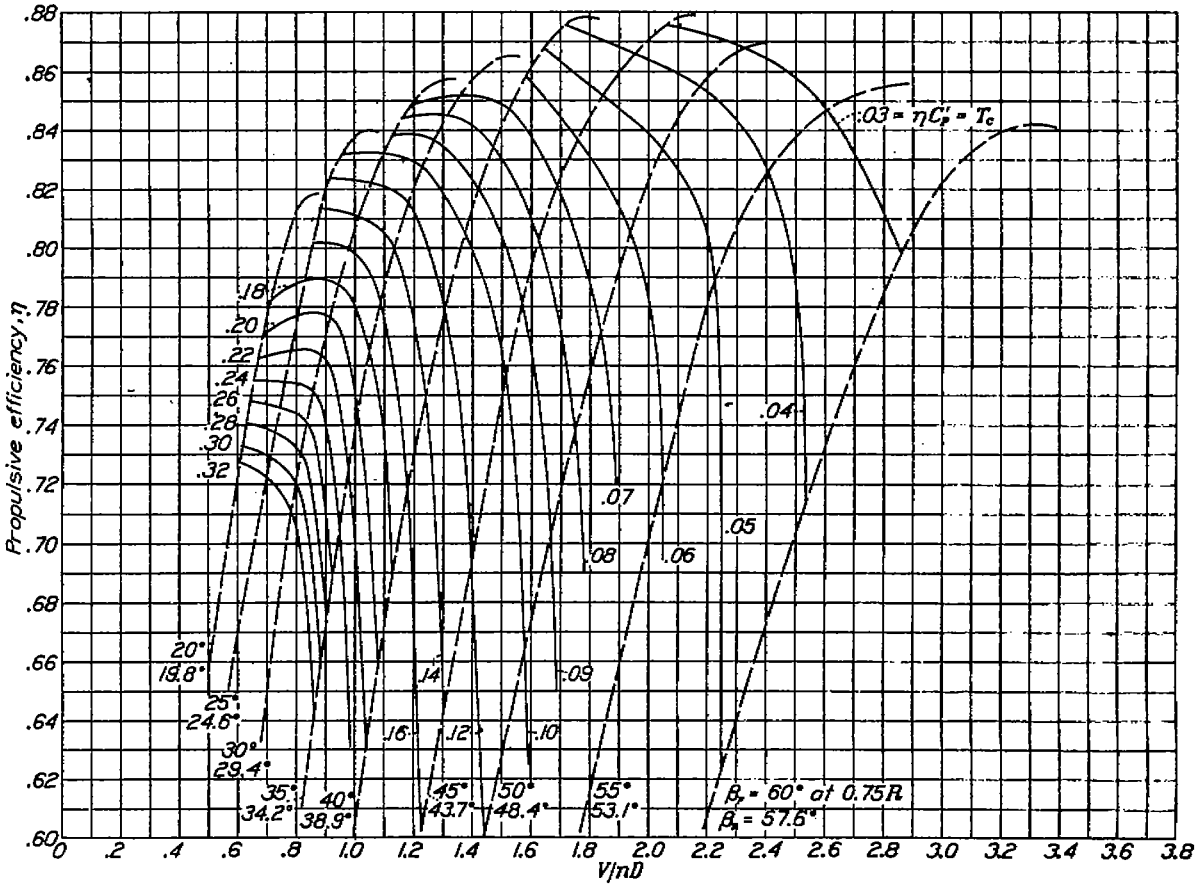

FIGURE 10.—Chart of η against V/nD for two two-blade propellers 3155-6 and 3156-6. Streamline body, with spinner, on symmetrical wing; dual rotation. Propeller data taken from reference 2.

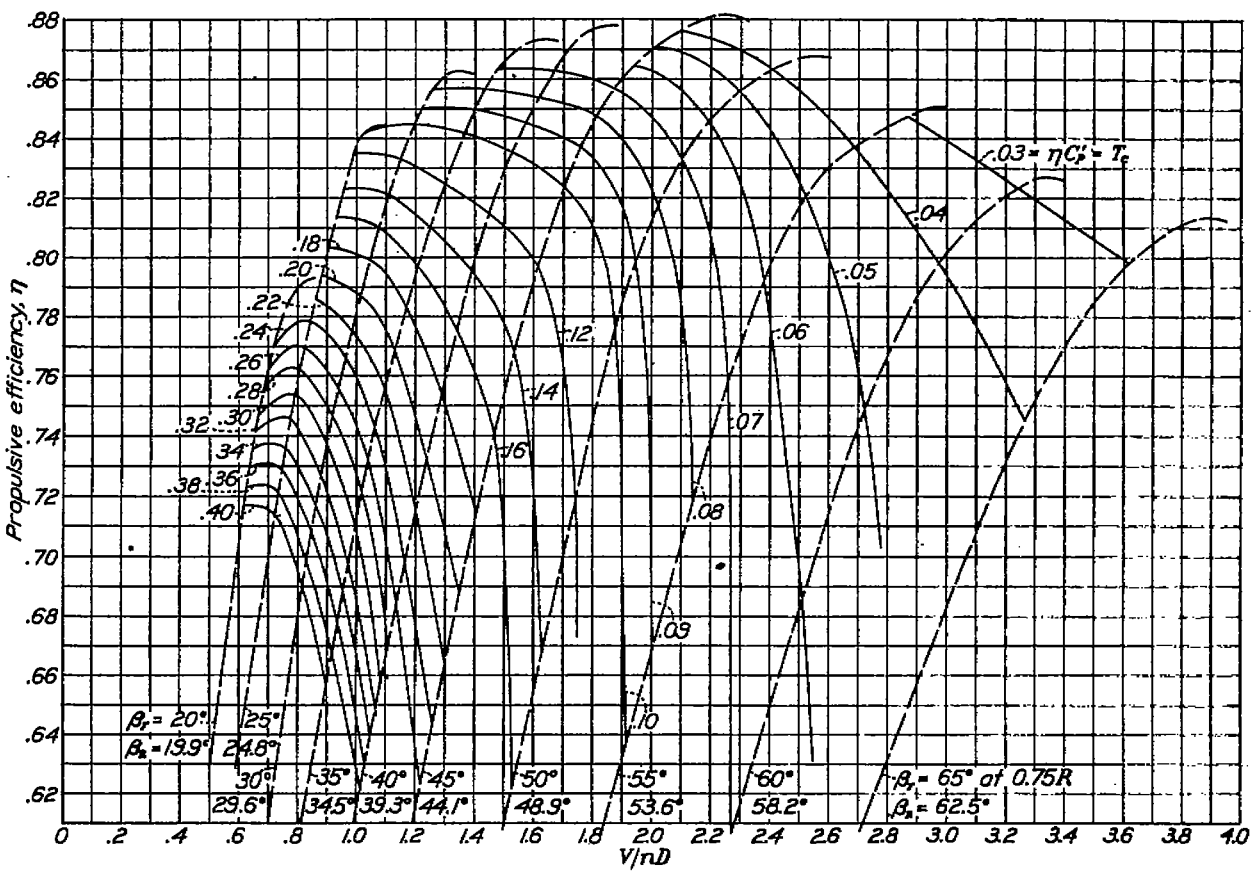

FIGURE 12.—Chart of η against V/nD for two three-blade propellers 3155-6 and 3156-6. Streamline body, with spinner, on symmetrical wing; dual rotation. Propeller data taken from reference 2.

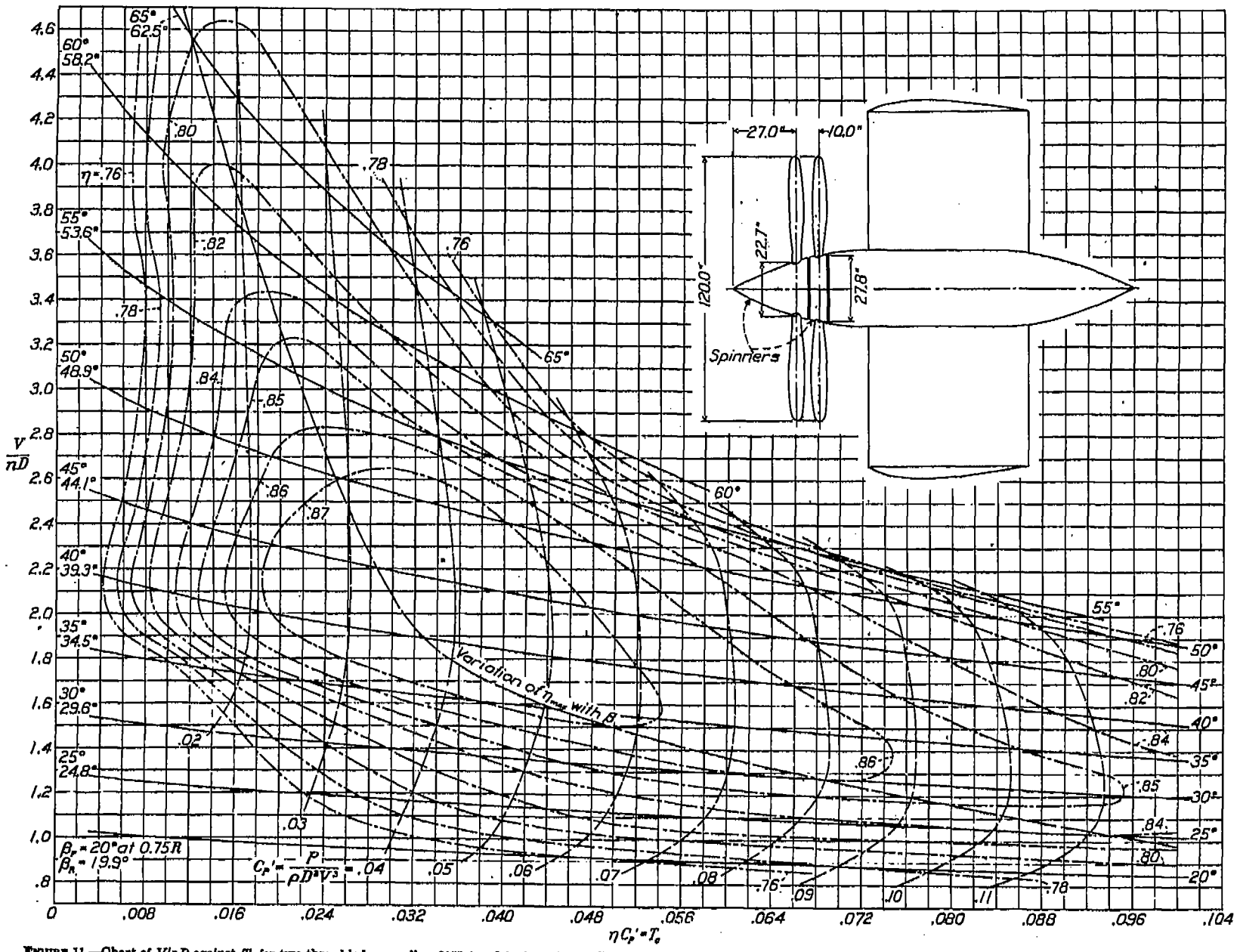

FIGURE 11.—Chart of V/nD against T_c for two three-blade propellers 3155-6 and 3156-6. Streamline body, with spinner, on symmetrical wing; dual rotation. Propeller data taken from reference 2.

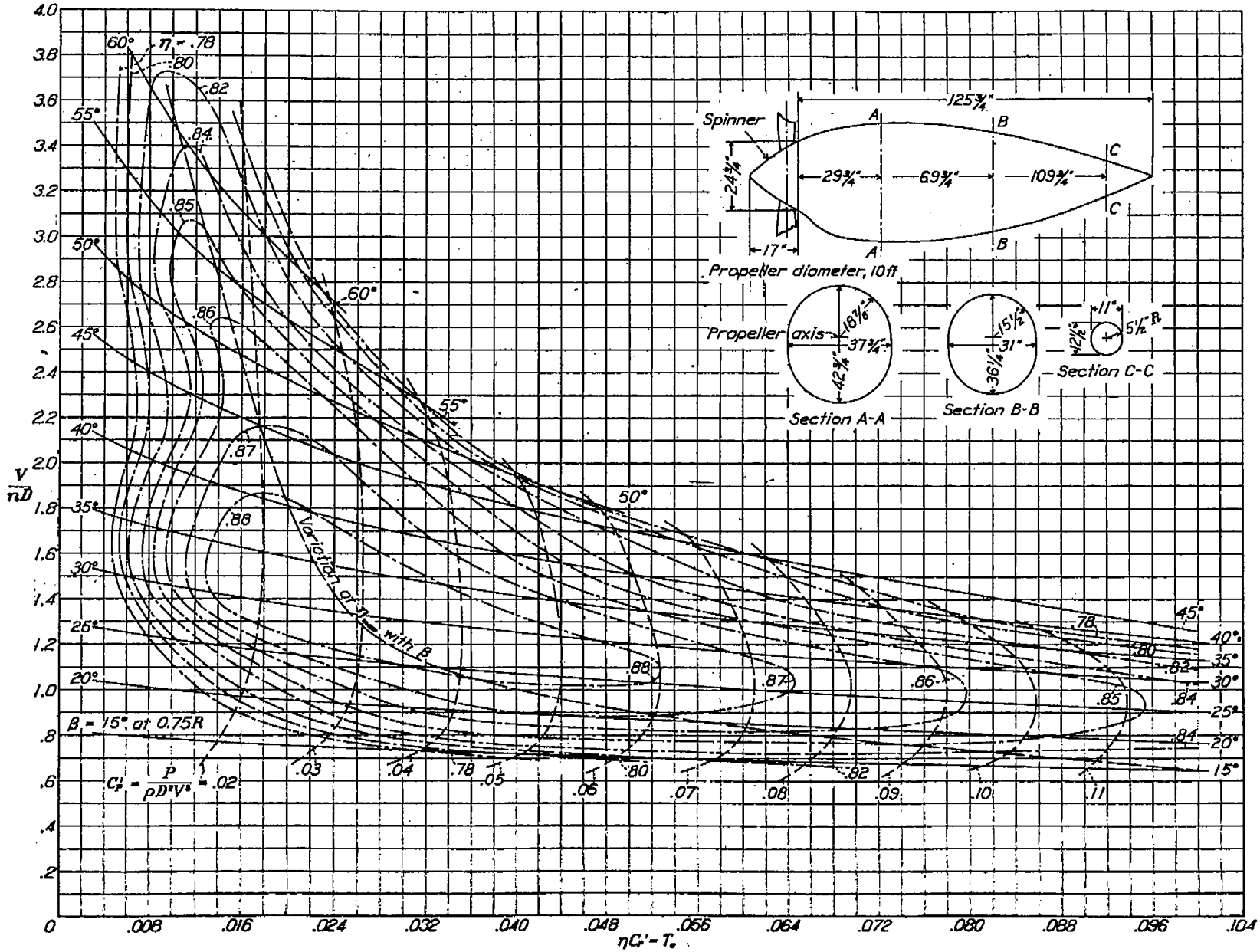

FIGURE 12.—Chart of V/nD against T_c for three-blade propeller 5868-9. Liquid-cooled engine nacelle, with spinner; single rotation. Propeller data taken from reference 3.

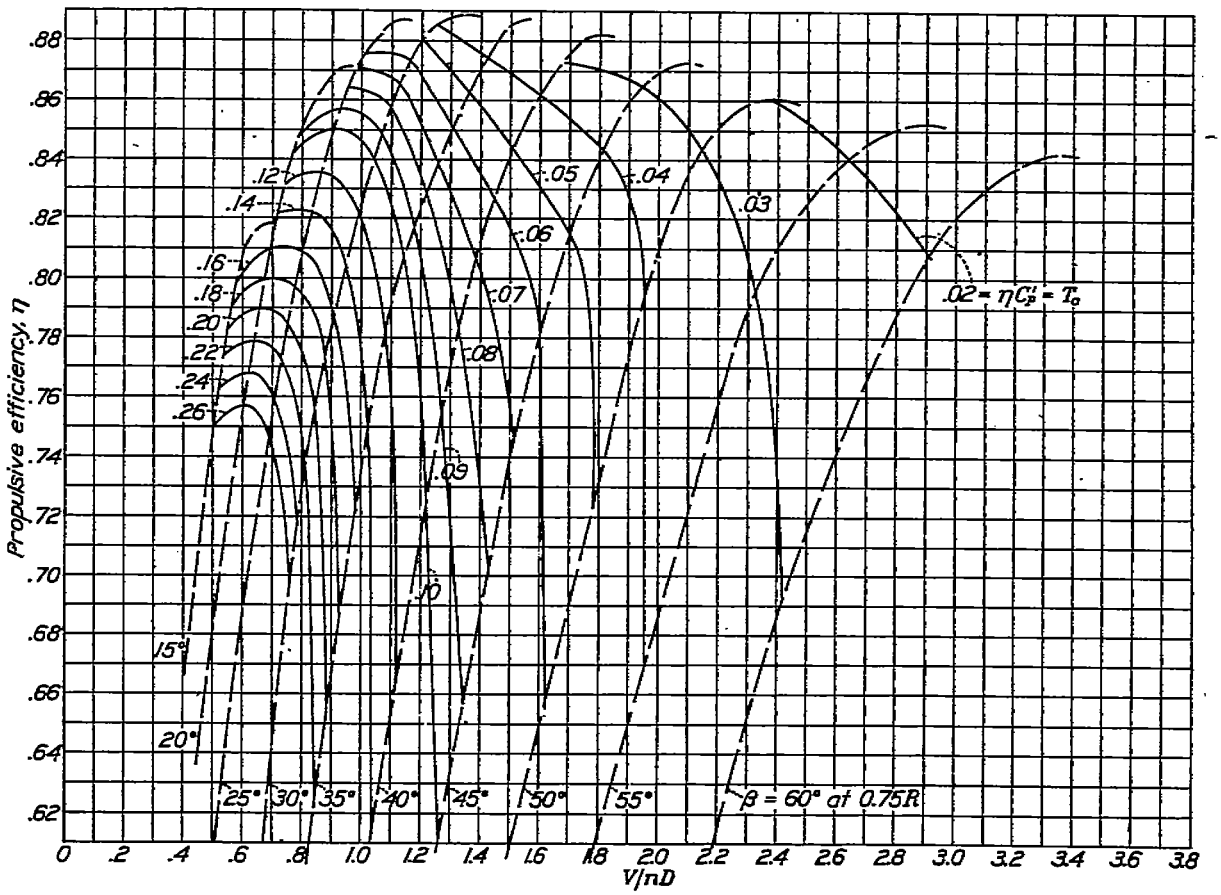

FIGURE 14.—Chart of η against V/nD for three-blade propeller 5868-9. Liquid-cooled engine nacelle, with spinner; single rotation. Propeller data taken from reference 3.

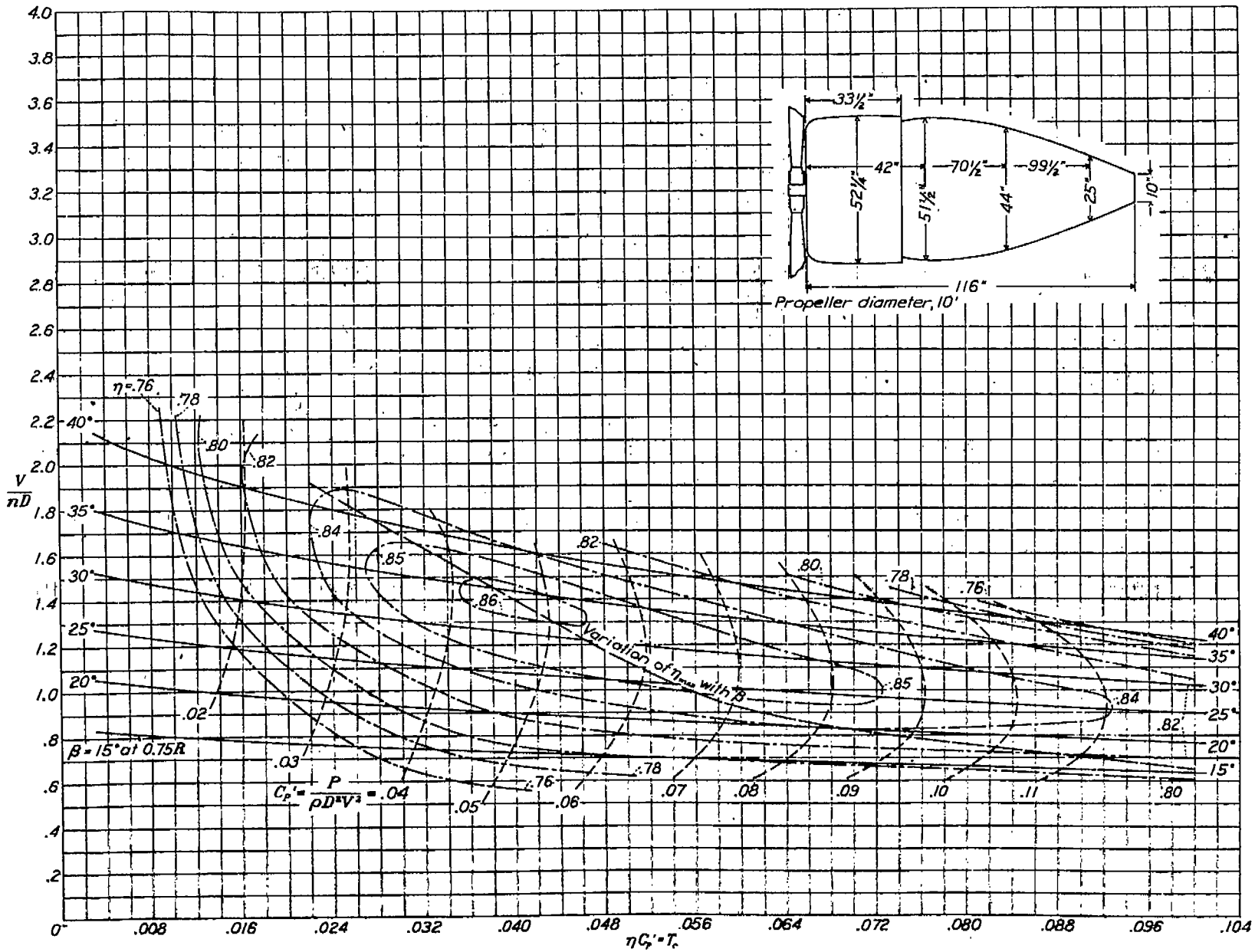

FIGURE 15.—Chart of V/nD against T_c for three-blade propeller 5868-9. Radial engine nacelle; single rotation. Propeller data taken from reference 4.

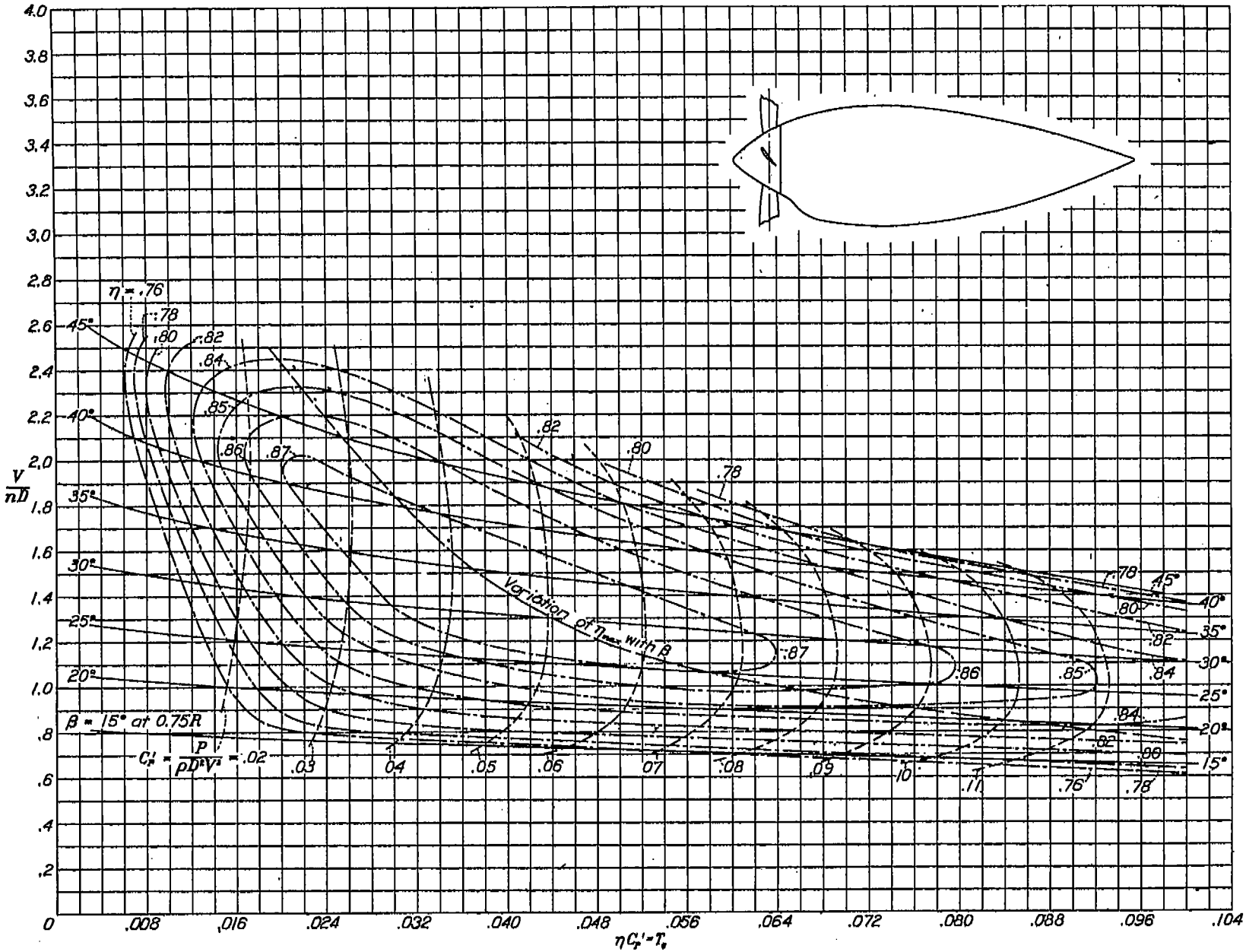

FIGURE 16.—Chart of V/nD against T_s for four-blade propeller 5868-9. Liquid-cooled engine nacelle, with spinner; single rotation. Data taken from reference 5.

Given: Aerodynamic characteristics of airplane and flight conditions:

(1) L/D for speed of flight and altitude (or thrust power for speed of flight and altitude)
(2) Weight, W
(3) Airspeed, V
(4) Diameter of propeller, D
(5) Density at altitude, ρ
(6) Engine gear ratio

Characteristics of engine, including:

(1) Brake horsepower for altitude of flight as a function of engine speed and manifold pressure
(2) Specific fuel consumption as a function of engine speed and manifold pressure or torque

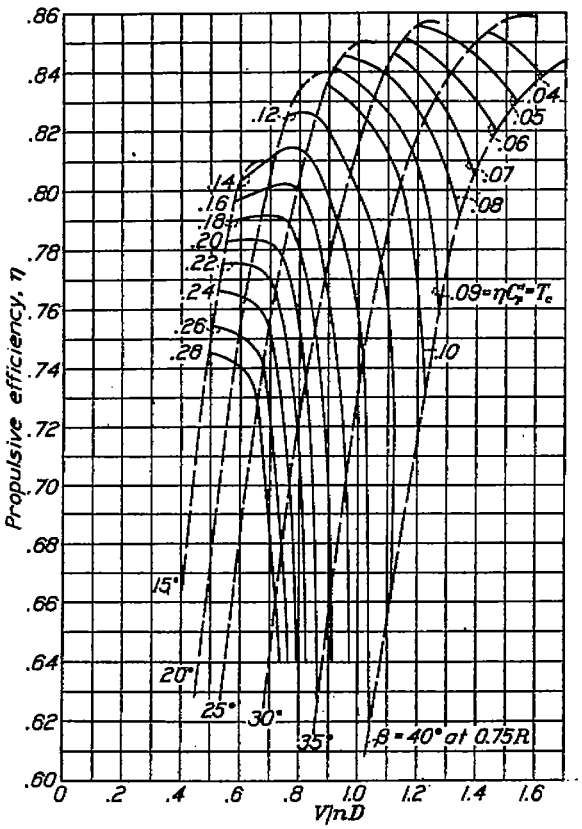

FIGURE 17.—Chart of η against V/nD for three-blade propeller 5868-9. Radial engine nacelle; single rotation. Propeller data taken from reference 4.

To obtain: Propeller speed to effect the maximum ratio of propulsive efficiency to specific fuel consumption

Solution based on thrust coefficient, T_c:

(1) Determine effective thrust T_e from L/D and W
(2) Solve for thrust coefficient

$$T_c=\frac{T_e}{\rho D^2 V^2}=\frac{195.4\ T_e}{\sigma D^2 (mph)^2}$$

(3) For several values of propeller speed n, solve for V/nD
(4) For several values of V/nD computed under step (3), read from propeller chart values of propulsive efficiency η
(5) For several values of n, compute propeller

$$\text{torque } Q=\frac{T_e V}{2\pi n \eta}$$

(6) For several propeller values of n, compute n/n_{rated} and Q/Q_{rated} (n_{rated} and Q_{rated} correspond to rated engine speed and torque values with gear ratio taken into account)

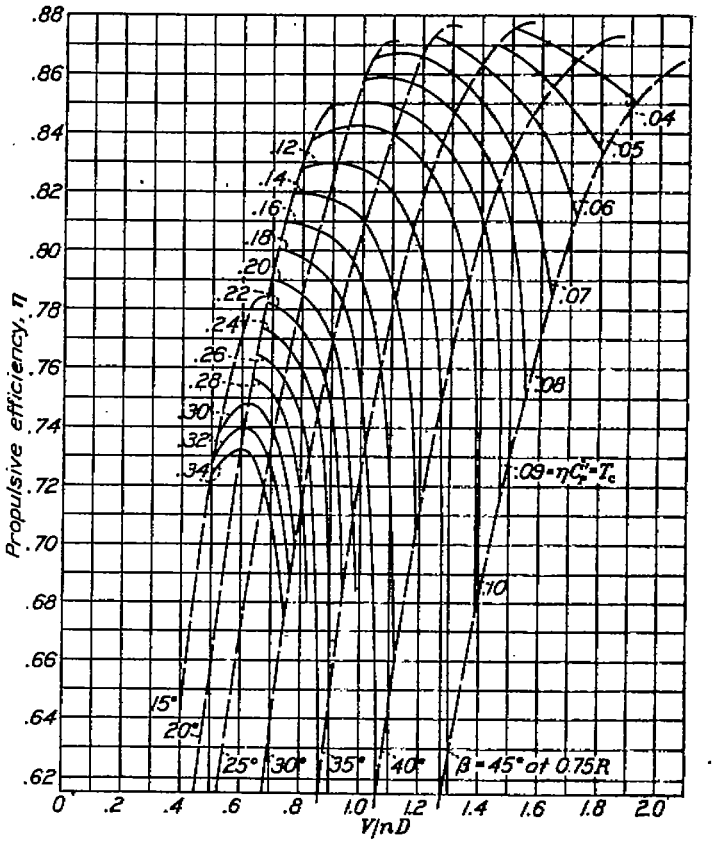

FIGURE 18.—Chart of η against V/nD for four-blade propeller 5868-9. Liquid-cooled engine nacelle, with spinner; single rotation. Propeller data taken from reference 5.

(7) From fuel-consumption data, determine values of engine specific fuel consumption C corresponding to engine speeds and torques computed under step (6)
(8) For several values of n, compute and plot values of η/C
(9) From curve obtained in step (8), determine n for maximum value of η/C

Solution based on thrust coefficient, $\eta C_P'$:

(1) Determine thrust power ηP or thrust horsepower from curves of power against speed of given data
(2) Solve for $\eta C_P'=\frac{\eta P}{\rho D^2 V^3}=\frac{73300\ thp}{\sigma D^2 (mph)^3}$

(3) The rest of the solution corresponds to steps in the solution based on thrust coefficient, except that $Q=\frac{\eta P}{2\pi n\eta}$

SAMPLE CALCULATION

A calculation for a sample airplane is herein performed to demonstrate the use of the charts. Values of n corresponding to the maximum value of η/C were determined for three flight speeds; the computations for only the condition of flight at maximum L/D of the airplane are included. Range computations have also been made for the three flight speeds based on a specific fuel load.

In addition, range computations were made for the same flight speeds but with the engine torque and speed

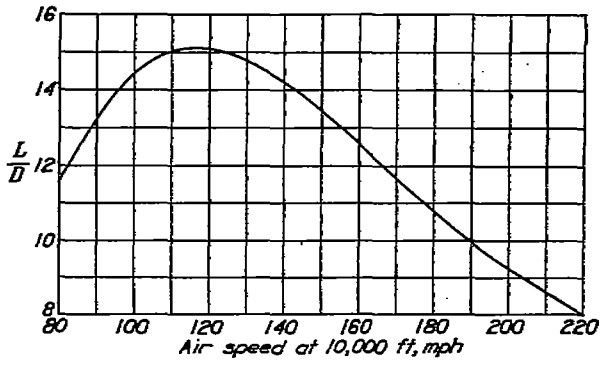

FIGURE 19.—Variation of L/D with airspeed for sample airplane.

adjusted according to the relation $Q\propto n^2$ to demonstrate the value of adjusting the engine speed to obtain the maximum engine-propeller operating efficiency. (The relation $Q\propto n^2$ corresponds to the operation with a fixed-pitch propeller.)

The two charts necessary for the calculations are included: One chart is a plot of L/D against airspeed for the sample airplane (fig. 19); the other chart is a fuel-consumption chart (fig. 20). The fuel-consumption chart was prepared from data taken during NACA tests of a 1340–H cylinder from a Pratt & Whitney 1340–S1H1–G engine.

Calculations were made for three conditions of flight, namely: maximum L/D, 50-percent power, and 75-percent power.

Given: Weight: gross=17,500 pounds; gas load=5930 pounds

Power plant: two 700-horsepower engines; engine speed=2380 rpm, geared 16:9

Propellers: two three-blade Hamilton Standard 3155–6 propellers; diameter=11 feet; propeller speed=1340 rpm

Maximum velocity: 210 miles per hour at 8100 feet

Flight altitude: 10,000 feet

Solution for flight at maximum L/D based on thrust coefficient T_c (calculations are shown for one point on fig. 21; other values are given in table I):

(1) Maximum L/D=15.1 (from fig. 19)
V for maximum L/D=116 miles per hour=170 feet per second

$$T_c=\frac{W}{L/D}=\frac{17500}{15.1}=1160 \text{ pounds or } 580 \text{ pounds per engine}$$

(2) $T_c=\frac{195.4\times580}{0.738\times121\times(1.346\times10^4)}=0.0945,$

(3) If it is assumed that n=20 rps,

$$V/nD=\frac{170}{20\times11}=0.773$$

(4) For V/nD=0.773 and T_c=0.0945, η=0.823 (fig. 1)

(5) $Q=\frac{580\times170}{2\pi\times20\times0.823}=953$ foot-pounds

(6) n_{rated}=22.33 rps,

$$Q_{rated}=\frac{550bhp_{rated}}{2\pi n_{rated}}=\frac{550\times700}{2\pi\times22.33}=2750 \text{ foot-pounds}$$

At n=20 rps,

$$n/n_{rated}=\frac{20}{22.33}=0.897$$

and Q/Q_{rated}=0.346

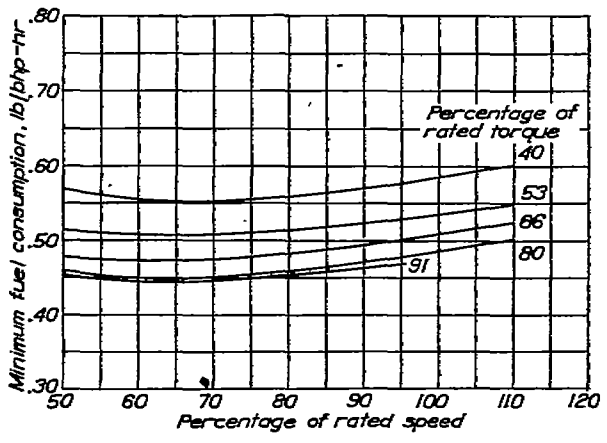

FIGURE 20.—Effect of engine speed on minimum fuel consumption of several torque values. The 1340–H cylinder.

(7) From figure 20, by extrapolation, at n/n_{rated}=0.897 and Q/Q_{rated}=0.346, C=0.600 pound per brake horsepower per hour

(8) $\eta/C=\frac{0.823}{0.600}=1.37$

(9) η/C is plotted against n in figure 21, together with the corresponding values of Q, C, and η, which have been added as a matter of interest. From this figure it is seen that the maximum value of η/C occurs at $n=13.7$

The results of similar computations for flight at 50-percent and 75-percent power are given in table I.

The range for the three flight speeds was calculated from the Breguet range formula:

$$\text{Range, miles}=863\,\frac{L}{D}\,\frac{\eta}{C}\log_{10}\frac{W_0}{W_e}$$

where W_0 is the gross weight and W_e is the gross weight less the gas load.

In the Breguet formula, L/D is assumed constant and the values of η/C are average during flight; whereas, the values of η/C as used in the problem are for the start of flight.

For maximum η/C at maximum L/D,

$$\text{Range, miles}=863\times15.1\times1.62\times0.181=3820$$

With the data from table I, the range at 50-percent power was computed as 3680 miles and at 75-percent power the range was 2940 miles. The variation of the range with the airspeed is given in figure 22. For comparison, figure 22 contains a plot of the range for the same airplane equipped with both controllable and fixed-pitch propellers, the fixed-pitch propeller being set for the condition of high-speed flight. Because the gain in range by the use of controllable-pitch propellers set for maximum engine-propeller efficiency was relatively small for the conditions assumed in the sample computations, it would appear that the optimum blade-angle setting is fairly close to the high-speed setting. The stating of such a generality is hazardous, however, chiefly because the fuel-consumption data herein presented may not apply to engines in general.

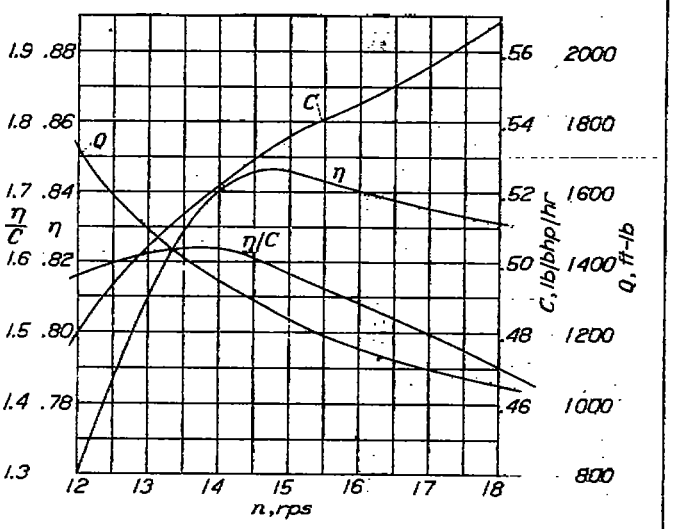

FIGURE 21.—Variation of engine-propeller operating characteristics for flight at maximum L/D of sample airplane.

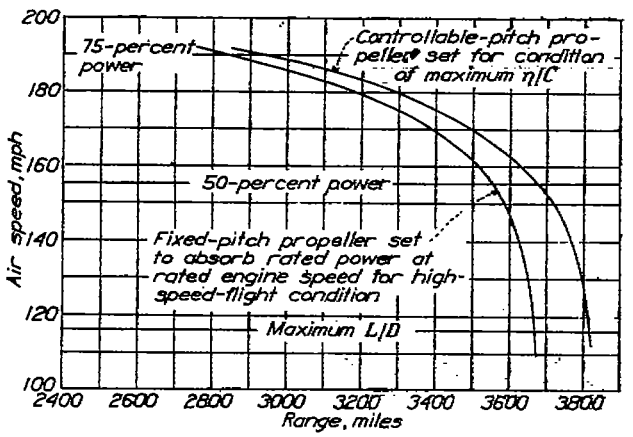

FIGURE 22.—Comparison of flight ranges for fixed- and controllable-pitch propellers on sample airplane at 10,000 feet. The foregoing comparisons were based on values of η/C for the start of flight, instead of on average values as stipulated in the Breguet formula.

TABLE I

CALCULATION OF η/C

Flight constants at start of flight	n (rps)	V/nD	η	$\frac{n}{n_{rated}}$	Q (ft-lb)	$\frac{Q}{Q_{rated}}$	C (lb/bhp-hr)	η/C
$L/D=15.1$	20	0.77	0.823	0.897	953	0.346	0.600	1.37
	19	.82	.828	.852	996	.362	.585	1.41
	18	.86	.831	.807	1050	.382	.570	1.45
V =116 mph	17	.91	.836	.763	1100	.400	.555	1.50
	16	.96	.840	.718	1170	.425	.545	1.54
T =1160 lb	15	1.03	.846	.672	1230	.448	.538	1.57
	14	1.10	.842	.628	1330	.484	.520	1.62
T_c =0.0945	13	1.19	.810	.583	1490	.542	.505	1.61
	12	1.29	.760	.538	1720	.625	.480	1.58
$L/D=13.0$	23	0.90	0.847	1.03	1250	0.455	0.570	1.49
	22	.94	.850	.987	1300	.473	.560	1.52
	21	.98	.854	.942	1360	.495	.545	1.57
V =155 mph	20	1.03	.860	.897	1420	.515	.530	1.62
	19	1.09	.867	.852	1480	.538	.517	1.68
T =1350 lb	18	1.15	.869	.807	1560	.567	.505	1.72
	17	1.21	.866	.763	1650	.602	.495	1.75
T_c =0.0615	16	1.29	.857	.718	1780	.646	.480	1.79
	15	1.38	.842	.672	1930	.702	.465	1.81
	14	1.48	.824	.628	2110	.767	.460	1.79
	13	1.59	.780	.583	2400	.872	.450	1.73
$L/D=10.0$	23	1.10	0.866	1.03	1940	0.705	0.507	1.71
	22	1.15	.871	.987	2020	.735	.497	1.75
	21	1.20	.872	.942	2120	.770	.485	1.80
V =190 mph	20	1.26	.871	.897	2220	.810	.470	1.85
	19	1.33	.865	.852	2350	.856	.462	1.87
T =1750 lb	18	1.40	.857	.807	2510	.913	.455	1.88
	17	1.49	.844	.763	2700	.982	.450	1.87
T_c =0.0530	16	1.58	.830	.718	------	------	------	------
	15	1.69	.800	.672	------	------	------	------

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *December 11, 1940.*

REFERENCES

1. Biermann, David, Hartman, Edwin P., and Pepper, Edward: Wind-Tunnel Tests of Propellers Equipped with Spinners, Cuffs, Airfoil and Round Shanks, and NACA 16-Series Sections. NACA Rep. to be published.
2. Biermann, David, and Hartman, Edwin P.: Wind-Tunnel Tests of Four- and Six-Blade Single- and Dual-Rotating Tractor Propellers. Rep. No. 747, NACA, 1942.
3. Biermann, David, and Hartman, Edwin P.: Tests of Two Full-Scale Propellers with Different Pitch Distributions, at Blade Angles up to 60°. Rep. No. 658, NACA, 1939.
4. Biermann, David, and Hartman, Edwin P.: Tests of Five Full-Scale Propellers in the Presence of a Radial and a Liquid-Cooled Engine Nacelle, Including Tests of Two Spinners. Rep. No. 642, NACA, 1938.
5. Hartman, Edwin P., and Biermann, David: The Aerodynamic Characteristics of Full-Scale Propellers Having 2, 3, and 4 Blades of Clark Y and R. A. F. 6 Airfoil Sections. Rep. No. 640, NACA, 1938.