

REPORT No. 654

GENERAL AIRPLANE PERFORMANCE

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SUMMARY

Equations have been developed for the analysis of the performance of the ideal airplane, leading to an approximate physical interpretation of the performance problem. The basic sea-level airplane parameters have been generalized to altitude parameters and a new parameter Ω_e has been introduced and physically interpreted.

The performance analysis for actual airplanes has been obtained in terms of the equivalent ideal airplane in order that the charts developed for use in practical calculations will for the most part apply to any type of engine-propeller combination and system of control, the only additional material required consisting of the actual engine and propeller curves for the propulsion unit.

Finally, a more exact method for the calculation of the climb characteristics for the constant-speed controllable propeller is presented in the appendix. This method replaces the approximate method, contained in the main body of the paper, in cases where the approximate method is not sufficiently accurate.

INTRODUCTION

Considerable interest has been evidenced in the past few years in the field of aircraft-performance analysis. With the increased use of the new types of engine and propeller equipment, many modifications of the existing performance-analysis methods have been introduced, which are designed for use in calculating the effect of these new developments on performance.

Oswald has investigated the problem and drawn up charts (reference 1) that can be used to calculate the performance of aircraft using unsupercharged engines and fixed-pitch propellers. Later he extended this analysis to include the case of supercharged engines (reference 2). White and Martin (reference 3) made a similar analysis for the case of constant-speed propellers with no supercharging, which can be extended to include supercharging.

In each of the foregoing analyses, special assumptions were made with regard to the variation of engine power with altitude and engine speed and the variation of propulsive efficiency with altitude and air speed. New engine and propeller developments that will cause changes in the characteristics of these units can, however, be expected to take place in the future, thus

necessitating a complete recalculation and revision of the charts to correspond to these changes. For this reason it was considered desirable to attack the problem in a more general manner, in order to obtain a method of performance calculation basically independent of the particular engine-propeller combination but readily adaptable to any type. In the following analysis, the attempt was made to eliminate from the basic relations any assumptions that were likely to require adjustment as a result of future design or experiment.

The author wishes to thank the men who have aided greatly in the development and preparation of the paper by contributing many excellent suggestions and criticisms.

The $\frac{d\eta}{dJ}$ against J charts presented in the appendix are due to Mr. Richard W. Palmer of the Vultee Aircraft Co., and the rest of the charts in this section were designed to conform to this suggestion. As mentioned in the body of the paper, Dr. Th. von Kármán and Dr. Clark B. Millikan are primarily responsible for the accurate physical interpretation of the three major parameters, Δ_e , Γ_e , and Ω_e . The author particularly wishes to thank Dr. Millikan for the considerable time and effort spent in the form of suggestions and criticisms throughout the preparation of the paper. Dr. Norton B. Moore and Dr. W. Bailey Oswald of the Douglas Aircraft Co. have also aided considerably in this regard. Various members of the wind-tunnel staff of the Daniel Guggenheim Aeronautics Laboratory contributed greatly in the technical preparation of the paper. In particular, Messrs. Frank J. Malina and W. T. Butterworth aided in the preparation of the numerous charts.

SUMMARY OF NOTATION

Subscripts:

σ , at altitude.

0, at sea level.

m , maximum.

c , at maximum rate of climb.

MP , at theoretical minimum power required.

V , velocity $\left\{ \begin{array}{l} \text{miles per hour in engineering equations} \\ \text{and charts.} \\ \text{feet per second in physical equations} \\ \text{and charts.} \end{array} \right.$

V_0 , maximum velocity in level flight at sea level.

V_σ , maximum velocity in level flight at altitude corresponding to σ .

V_{MP} , velocity at minimum power required for the ideal airplane.

V_c , velocity at maximum rate of climb.

V_{max} , maximum possible velocity at all altitudes.

V_t , minimum velocity in level flight for an ideal airplane with no parasite drag.

V_p , maximum velocity in level flight for an ideal airplane with no induced drag.

$$R_\sigma = \frac{V \text{ at altitude corresponding to } \sigma}{V_\sigma}$$

$$R_{\sigma_0} = \frac{V}{V_0}$$

$$R_{\sigma MP} = \frac{V_{MP}}{V_0} \text{ in section IV C.}$$

$$= \frac{V_{MP}}{V_\sigma} \text{ in all other sections.}$$

$$= \frac{V_c}{V_0} \text{ in section IV C.}$$

$$= \frac{V_c}{V_\sigma} \text{ in all other sections.}$$

$$R_R = \frac{V_c}{V_{MP}} \text{ both at same altitude.}$$

$$R_P = \frac{P}{P_0} = \frac{\text{t.hp.}}{\text{t.hp. (design)}}$$

$$R_{\sigma M} = \frac{V}{V_{MP}} \text{ both at same altitude.}$$

C , maximum rate of climb $\left\{ \begin{array}{l} \text{feet per minute in engi-} \\ \text{neering equations and} \\ \text{charts.} \\ \text{feet per second in physical} \\ \text{equations.} \end{array} \right.$

C_0 , maximum rate of climb of the ideal airplane with no drag.

h , altitude (feet).

H , absolute ceiling (feet).

h_m , altitude for V_{max} .

C_L , lift coefficient.

C_D , drag coefficient.

$$C_P = 5 \times 10^{10} \frac{\text{b. hp.}}{\sigma N^3 D^5}$$

P , thrust horsepower available.

P_c , thrust horsepower available at maximum rate of climb.

P_0 , thrust horsepower available at V_0 ($\sigma=1$).

b. hp., brake horsepower.

(t. hp.)_a, thrust horsepower available.

(t. hp.)_r, thrust horsepower required.

N , propeller revolutions per minute.

D , propeller diameter (feet).

$$J = \frac{88V}{ND}$$

$$J_c = J \text{ at } V_c.$$

$$J_{MP} = J \text{ at } V_{MP}.$$

ρ , mass density of air.

$\rho_0 = 0.002378$ (slugs per cu. ft.) = mass density of standard air at sea level.

$\sigma = \rho/\rho_0$, density ratio.

$\sigma_m = \sigma$ at altitude for V_{max} .

W , weight (lb.).

f , equivalent parasite area (sq. ft.).

e , airplane efficiency factor (reference 1).

k , Munk's span factor.

b_1 , largest individual span of wing cellule.

b , equivalent monoplane span = kb_1 .

$b_e = \sqrt{e(kb_1)^2}$, effective span.

η , propulsive efficiency.

Engineering parameters:

$$l_{\sigma e} = \frac{W}{\sigma e (kb_1)^2} = \frac{1}{\sigma} l_e, \text{ effective span loading (lb./sq. ft.).}$$

$$l_{\sigma p} = \frac{W}{\sigma f} = \frac{1}{\sigma} l_p, \text{ parasite loading (lb./sq. ft.).}$$

$$l_{\sigma t} = \frac{W}{t. \text{ hp.}} = l_t, \text{ thrust horsepower loading (lb./hp.).}$$

$$\Lambda_\sigma = \left(\frac{1}{\sigma}\right)^{2/3} \Lambda = \left(\frac{1}{\sigma}\right)^{2/3} \frac{l_e^{4/3}}{l_p^{1/3}}, \text{ major performance parameter.}$$

Physical parameters:

A , horsepower conversion factor (550 in American system).

$$\lambda_{\sigma e} = \frac{2}{\pi \rho_0} \frac{W}{\sigma e (kb_1)^2} = 267.7 l_{\sigma e}$$

$$\lambda_{\sigma p} = \frac{2}{\rho_0} \frac{W}{\sigma f} = 841.0 l_{\sigma p}$$

$$\lambda_{\sigma t} = \frac{1}{A \eta} \frac{W}{t. \text{ hp.}} = 0.001818 l_{\sigma t}$$

$$\Lambda_\sigma = \frac{\lambda_{\sigma e} \lambda_{\sigma t}^{4/3}}{\lambda_{\sigma p}^{1/3}} = \left(\frac{1}{\sigma}\right)^{2/3} \frac{\lambda_e \lambda_t^{4/3}}{\lambda_p^{1/3}} = \left(\frac{1}{\sigma}\right)^{2/3} \Lambda' = 0.006293 \Lambda'$$

$$\Omega_\sigma = \left(\frac{\lambda_{\sigma t}}{\lambda_{\sigma p}}\right)^{1/3} V_\sigma = \sigma^{1/3} \left(\frac{\lambda_t}{\lambda_p}\right)^{1/3} V_\sigma \text{ (} V_\sigma \text{ in ft./sec.).}$$

$$\Gamma_\sigma = \frac{\lambda_{\sigma e} \lambda_{\sigma t}}{V_\sigma} = \frac{1}{\sigma} \frac{\lambda_e \lambda_t}{V_\sigma} \text{ (} V_\sigma \text{ in ft./sec.).}$$

$$\Gamma_{\sigma MP} = \frac{\lambda_{\sigma e} \lambda_{\sigma t}}{V_{MP}} = \frac{1}{\sigma} \frac{\lambda_e \lambda_t}{V_{MP}} \text{ (} V_\sigma \text{ in ft./sec.).}$$

I. FUNDAMENTAL PERFORMANCE EQUATION

The fundamental performance equation may be written

$$W \frac{dh}{dt} = A \eta \text{ b. hp.} - \frac{\rho}{2} f V^2 - \frac{W^2}{\pi \frac{\rho}{2} e (kb_1)^2 V} \quad (1)$$

where dh/dt is the rate of climb.

Writing this equation in terms of the λ parameters (reference 1), there results

$$\frac{dh}{dt} = \frac{1}{\lambda_i} \left[\frac{P}{P_0} - \sigma R_{v_0}^3 \frac{\lambda_i}{\lambda_p} V_0^3 - \frac{1}{\sigma R_{v_0}} \frac{\lambda_s \lambda_i}{V_0} \right] \quad (2)$$

where the parameters are defined as

$$\lambda_p = \frac{2W}{\rho_0 f} = \frac{2}{\rho_0} l_p$$

$$\lambda_s = \frac{2}{\pi \rho_0} \frac{W}{e(kb_1)^2} = \frac{2}{\pi \rho_0} l_s$$

$$\lambda_i = \frac{1}{A} \frac{W}{(b. hp.)_{\sigma \eta_0}} = \frac{1}{A} l_i$$

and where

$$R_{v_0} = V/V_0$$

V , velocity under arbitrary conditions.

η_0 , propulsive efficiency at V_0 ($\sigma=1$).

In equation (2) the ratio P/P_0 represents the combined variation in engine output and propulsive efficiency from the design value P_0 .

II. METHODS OF PERFORMANCE ANALYSIS

There are two general methods of attack that may be employed in the calculation of airplane performance:

1. As suggested by Oswald, the sea-level performance may first be calculated and the performance at altitude may then be compared with that at sea level. In this case all the parameters used in the analysis will be "sea-level parameters" such as λ_p , λ_s , Γ , and Λ' . These parameters are used in Oswald's analysis.

2. The performance at altitude may be calculated directly without reference to the sea-level, or "reference-level," performance. In this case the parameters involved are somewhat altered and become what may be termed "altitude parameters."

Both methods are basically similar but take somewhat different forms insofar as the final formulas and charts are concerned. Each method has its own advantages, particularly as regards physical significance and facility for practical calculation, but the relative advantages vary with the particular problems to be solved. For this reason both methods will be herein presented, together with recommendations concerning the adaptability of each to specific types of problems that are encountered in practical performance calculation.

III. ANALYSIS INDEPENDENT OF THE SEA-LEVEL PERFORMANCE—ALTITUDE PARAMETERS

The second of the preceding methods will first be considered. As previously mentioned, Oswald's analysis has been developed on a sea-level basis; that is, the velocity and power ratios have been referred to the maximum speed and maximum power available in level flight at sea level, and all the parameters, λ_p , λ_s , Γ , and Λ' , with their corresponding engineering equivalents,

l_p , l_s , $\frac{l_i}{V_0}$, and Λ , are based on sea-level density and sea-level performance. This reference level is purely arbitrary and was chosen because of its convenience and because sea-level performance constitutes an important practical case in performance calculation. In many cases, however, it is desirable to determine the performance at some altitude other than sea level and, if no particular interest is attached to sea-level performance, it would be more convenient to calculate this performance directly without going through the intermediate step of the sea-level calculation. It is the object of this section to develop formulas and charts that will be useful for such calculations.

Since the analysis is to be based on altitude instead of on sea-level characteristics, it is only logical that altitude parameters should be introduced which will take the place of the sea-level parameters. The new parameters are, accordingly, defined as follows:

Physical parameters:

$$\left. \begin{aligned} \lambda_{p\sigma} &= \frac{2W}{\rho f} = \frac{1}{\sigma} \frac{2W}{\rho_0 f} = \frac{1}{\sigma} \lambda_p \\ \lambda_{s\sigma} &= \frac{2W}{\pi \rho e (kb_1)^2} = \frac{1}{\sigma} \frac{2W}{\pi \rho_0 e (kb_1)^2} = \frac{1}{\sigma} \lambda_s \\ \lambda_{i\sigma} &= \frac{W}{A \eta_0 (b. hp.)_{\sigma}} = \lambda_i \end{aligned} \right\} \quad (3)$$

Engineering parameters:

$$\left. \begin{aligned} l_{p\sigma} &= \frac{1}{\sigma} l_p \\ l_{s\sigma} &= \frac{1}{\sigma} l_s \\ l_{i\sigma} &= l_i \end{aligned} \right\} \quad (4)$$

It will be noted that the new parameters are obtained by merely replacing ρ_0 , the density at sea level, by ρ , the density at the altitude in question, wherever it occurs in the sea-level parameters.

In accordance with the previous analysis, it would be desirable to develop parameters similar in form to the sea-level parameters, Γ and Λ' , together with their associated engineering equivalents. Rewriting the general performance equation (2) in terms of the new altitude parameters

$$\frac{dh}{dt} = \frac{1}{\lambda_i} \left[\frac{P}{P_0} - \frac{\lambda_i}{\lambda_{p\sigma}} R_{v\sigma}^3 V_{\sigma}^3 - \frac{1}{R_{v\sigma}} \frac{\lambda_s \lambda_i}{V_{\sigma}} \right] \quad (5)$$

where V_{σ} has been defined as the maximum velocity in level flight at the altitude corresponding to σ and the new velocity ratio $R_{v\sigma}$ has been defined as

$$R_{v\sigma} = V/V_{\sigma}$$

Now, considering level flight with the thrust power equal to the design thrust power P_0 , i. e.,

$$\frac{dh}{dt} = 0, R_{v\sigma} = 1, \text{ and } \frac{P}{P_0} = 1$$

equation (5) becomes

$$1 = \frac{\lambda_t}{\lambda_{p\sigma}} V_\sigma^3 - \frac{\lambda_{s\sigma} \lambda_t}{V_\sigma} = 0 \quad (6)$$

Rearranging and multiplying by $\frac{\lambda_{s\sigma}^3 \lambda_t^3}{V_\sigma^3}$, equation (6) becomes

$$\frac{\lambda_{s\sigma}^3 \lambda_t^4}{\lambda_{p\sigma}} = \frac{\lambda_{s\sigma}^3 \lambda_t^3}{V_\sigma^3} \left[1 - \frac{\lambda_{s\sigma} \lambda_t}{V_\sigma} \right] \quad (7)$$

and finally, defining

$$\left. \begin{aligned} \Gamma_\sigma &= \frac{\lambda_{s\sigma} \lambda_t}{V_\sigma} = \frac{1}{\sigma} \frac{V_0}{V_\sigma} \frac{\lambda_s \lambda_t}{V_0} = \frac{1}{\sigma} \left(\frac{V_0}{V_\sigma} \right) \Gamma \\ \Lambda'_\sigma &= \frac{\lambda_{s\sigma} \lambda_t^{4/3}}{\lambda_{p\sigma}^{1/3}} = \left(\frac{1}{\sigma} \right)^{2/3} \frac{\lambda_s \lambda_t^{4/3}}{\lambda_p^{1/3}} = \left(\frac{1}{\sigma} \right)^{2/3} \Lambda' \end{aligned} \right\} \quad (8)$$

equation (6) becomes

$$\Lambda'_\sigma = \Gamma_\sigma (1 - \Gamma_\sigma)^{1/3} \quad (9)$$

which is identical in form with the equation obtained by Oswald (reference 1). In fact, substituting for sea-level conditions, equation (9) reduces identically to Oswald's equation. Thus, equation (9) is a generalization of Oswald's equation to include the level-flight-velocity performance at any altitude, and the same restrictions and physical interpretation apply to this equation at altitude as apply to the sea-level equation. It may be anticipated from this comparison that, if the altitude parameters and variables, $\lambda_{s\sigma}$, $\lambda_{p\sigma}$, Γ_σ , Λ'_σ , R_σ , and V_σ , are substituted for the sea-level parameters and variables, λ_s , λ_p , Γ , Λ' , R_{v_0} , and V_0 , in the equations for the various performance characteristics for sea level, the result will be the generalized equations for the performance at any altitude corresponding to the density ratio σ . This supposition will be verified in the subsequent analysis.

It will be found desirable for later analysis to develop another parameter that will aid both in the physical interpretation and in practical calculation. Accordingly, again rearranging equation (6) and multiplying by

$$\left(\frac{\lambda_t}{\lambda_{p\sigma}} \right)^{1/3} V_\sigma \quad \frac{\lambda_{s\sigma} \lambda_t^{4/3}}{\lambda_{p\sigma}^{1/3}} = \left(\frac{\lambda_t}{\lambda_{p\sigma}} \right)^{1/3} V_\sigma \left[1 - \frac{\lambda_t}{\lambda_{p\sigma}} V_\sigma^3 \right] \quad (10)$$

Finally defining

$$\Omega_\sigma = \left(\frac{\lambda_t}{\lambda_{p\sigma}} \right)^{1/3} V_\sigma = \sigma^{1/3} \left(\frac{V_\sigma}{V_0} \right) \left(\frac{\lambda_t}{\lambda_p} \right)^{1/3} V_0 = \sigma^{1/3} \left(\frac{V_\sigma}{V_0} \right) \Omega \quad (11)$$

equation (10) becomes

$$\Lambda'_\sigma = \Omega_\sigma (1 - \Omega_\sigma^3) \quad (12)$$

which is similar in form to equation (9).

Now returning to the general performance equation (5) and substituting for $\frac{\lambda_t}{\lambda_{p\sigma}} V_\sigma^3$ from equation (6)

$$\frac{dh}{dt} = \frac{1}{\lambda_t} \left[\frac{P}{P_0} - R_\sigma^3 + \left(R_\sigma^3 - \frac{1}{R_\sigma} \right) \Gamma_\sigma \right] \quad (13)$$

A second form of the equation may be obtained by substituting for $\frac{\lambda_{s\sigma} \lambda_t}{V_\sigma}$ instead of for $\frac{\lambda_t}{\lambda_{p\sigma}} V_\sigma^3$ from the same equation. This substitution will give

$$\frac{dh}{dt} = \frac{1}{\lambda_t} \left[\frac{P}{P_0} - \frac{1}{R_\sigma} + \left(\frac{1}{R_\sigma} - R_\sigma^3 \right) \Omega_\sigma^3 \right] \quad (14)$$

Both these equations materially simplify the calculation of airplane performance by reducing the number of airplane parameters to two, λ_t and Γ_σ in the case of equation (13) or λ_t and Ω_σ in the case of equation (14).

A. PHYSICAL SIGNIFICANCE OF THE PARAMETERS: Λ'_σ , Γ_σ , and Ω_σ

The physical significance of the dimensionless parameter Λ' introduced in Oswald's analysis has been discussed by Millikan (reference 1), and it has been found that the parameter is approximately proportional to the ratio of the minimum to the maximum speed at sea level for an ideal airplane. It will be desirable, however, to discuss these parameters more fully, particularly in view of their generalization to all altitudes and of the introduction of the new parameter Ω_σ .

For this purpose the ideal airplane, defined by Millikan (reference 1), will again be considered. This airplane, it will be noted, is one for which the thrust power is independent of speed and for which the phenomenon of burbling does not occur. Such an airplane would obey the performance equation for all values of the velocity and is one for which the thrust power is always equal to the design thrust power P_0 . These stipulations will be assumed to hold for all altitudes as well as for all velocities.

Three conditions of flight will be considered for this airplane:

1. Level flight with no induced drag (infinite span).

$$\frac{P}{P_0} = 1, \quad V = V_p, \quad \frac{dh}{dt} = 0, \quad \lambda_{s\sigma} = 0$$

2. Level flight with no parasite drag ($f=0$).

$$\frac{P}{P_0} = 1, \quad V = V_v, \quad \frac{dh}{dt} = 0, \quad \lambda_{p\sigma} = \infty$$

3. Maximum rate of climb with no parasite or induced drag,

$$\frac{P}{P_0} = 1, \quad \lambda_{s\sigma} = 0, \quad \lambda_{p\sigma} = \infty, \quad \frac{dh}{dt} = C_0$$

¹ The physical significance of these parameters was developed by Dr. C. B. Millikan after several discussions of the problem with Dr. Th. von Kármán and the author. The conclusions were first presented in Millikan's lectures in the fall of 1935.

Consider the first condition. Substituting the flight conditions in equation (5), equation (6) results. Solving for V_σ

$$V_\sigma = \left(\frac{\lambda_{p\sigma}}{\lambda_t} \right)^{1/3} \left(1 - \frac{\lambda_{s\sigma} \lambda_t}{V_\sigma} \right)^{1/3} \quad (15)$$

Substituting the conditions for zero induced drag, i. e., $\lambda_{s\sigma} = 0$ and $V_\sigma = V_p$, equation (15) becomes

$$V_p = \left(\frac{\lambda_{p\sigma}}{\lambda_t} \right)^{1/3} \quad (16)$$

and dividing into V_σ

$$\frac{V_\sigma}{V_p} = \left(\frac{\lambda_t}{\lambda_{p\sigma}} \right)^{1/3} V_\sigma = \Omega_\sigma \quad (17)$$

Now consider the second condition. Again substituting the flight conditions in equation (5), equation (6) results and, solving for V_σ , this time using the other term in equation (6),

$$V_\sigma = \frac{\lambda_{s\sigma} \lambda_t}{1 - \frac{\lambda_t}{\lambda_{p\sigma}} V_\sigma^3} \quad (18)$$

Substituting the conditions for zero parasite drag, i. e., $\lambda_{p\sigma} = \infty$ and $V_\sigma = V_t$, equation (18) becomes

$$V_t = \lambda_{s\sigma} \lambda_t \quad (19)$$

Dividing by V_σ

$$\frac{V_t}{V_\sigma} = \frac{\lambda_{s\sigma} \lambda_t}{V_\sigma} = \Gamma_\sigma \quad (20)$$

Finally, from equation (8),

$$\Lambda'_\sigma = \frac{\lambda_{s\sigma} \lambda_t^{4/3}}{\lambda_{p\sigma}^{1/3}} = \left(\frac{\lambda_{p\sigma}}{\lambda_t} \right)^{1/3} \quad (21)$$

so that, using equations (16) and (19),

$$\Lambda'_\sigma = \frac{V_t}{V_p} \quad (22)$$

and also

$$\Lambda'_\sigma = \Gamma_\sigma \Omega_\sigma \quad (23)$$

Finally, considering the third condition, equation (5) becomes

$$C_0 = \frac{1}{\lambda_t} \quad (24)$$

The three velocities, V_p , V_t , and C_0 , may be called the "design speeds" of the ideal airplane considered. The term V_σ represents the maximum speed of the airplane, V_p the maximum speed with no induced drag, V_t the minimum speed with no parasite drag, and C_0 the maximum rising speed with no induced or parasite drag, all at the design thrust power P_0 . The power curves and velocity relationships for a normal airplane with its associated ideal airplane are represented in figure 1. In the preceding analysis, all the dimensionless parameters have been obtained in terms of the design speeds. In particular, Λ'_σ is the ratio of the minimum

speed with no parasite drag to the maximum speed with no induced drag at the altitude corresponding to σ . In the analysis of reference 1, it was found that the parameter Λ' , to which Λ'_σ reduces at sea level, was

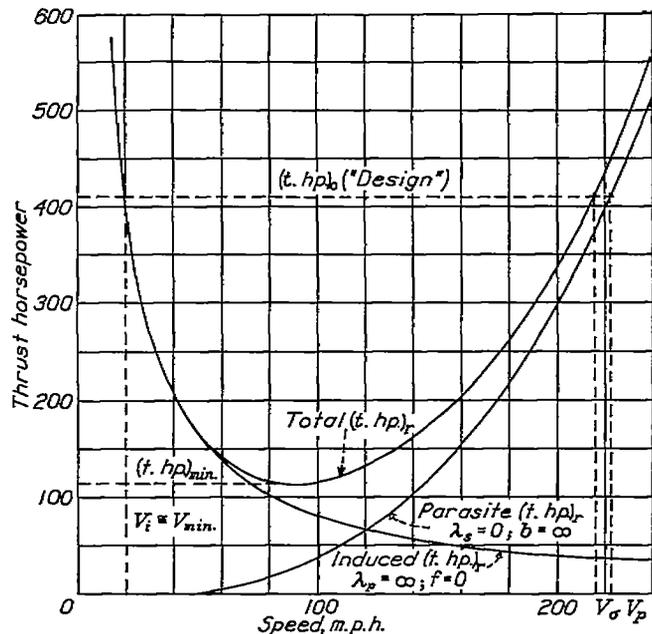


FIGURE 1.—Power characteristics for an ideal airplane with the following characteristics: $\lambda_{p\sigma} = 841,000$; $\lambda_{s\sigma} = 1,203$; $\lambda_t = 0.0844$; $\Delta_\sigma = 0.0823$; $W = 5,500$ lb.

approximately equal to the ratio of the minimum speed to the maximum speed of the ideal airplane, including both parasite and induced drag. Examination of figure 1 will show the extent of this approximation.

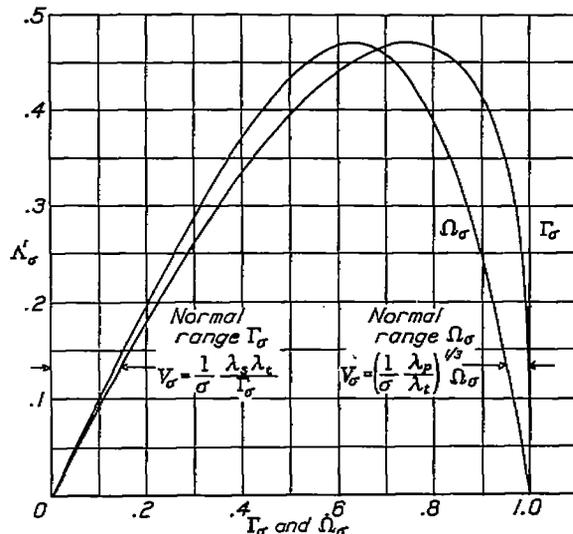


FIGURE 2.—The parameters Γ_σ and Ω_σ as functions of Λ'_σ .

Since all the parameters that appear in the general performance equation can now be represented by speed ratios, it is only logical that the entire equation may be represented by these same ratios. Thus, substitution in equation (5) from equations (16), (19), and (24) gives, since $V = R, V_\sigma$,

$$\frac{C}{C_0} = \frac{P}{P_0} - \left(\frac{V}{V_p} \right)^3 - \left(\frac{V_t}{V} \right) \quad (25)$$

which is an equation of speed and power ratios and is valid for any altitude. This is the most general form of the performance equation for an ideal airplane.

The relationships between the three parameters Λ'_σ , Γ_σ , and Ω_σ given by equations (9), (12), and (23), are plotted in figure 2 and show the maximum possible value of Δ'_σ , the significance of which will be explained later.

B. MAXIMUM VELOCITY AT ALTITUDE

Equations (9) and (12) give the relationship between the parameters and the maximum speed in level flight at the altitude corresponding to the density ratio σ . These

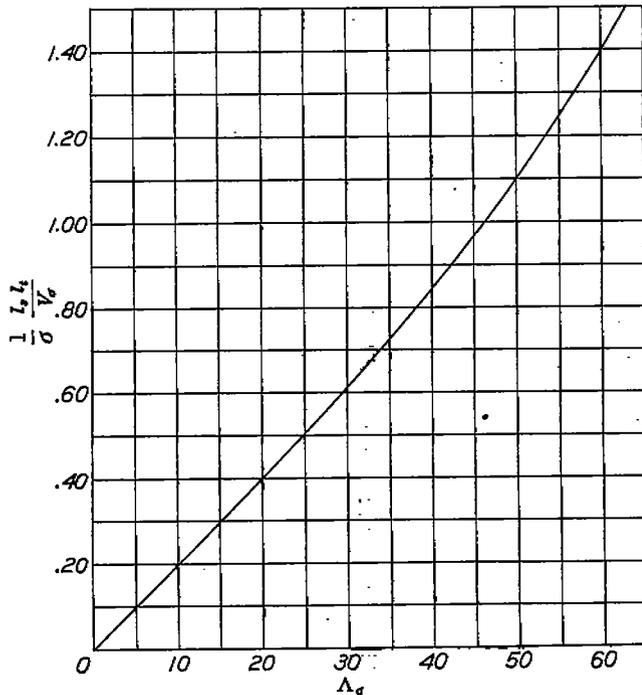


FIGURE 3.—The quantity $\frac{1}{\sigma} \frac{l_i l_t}{V_e}$ as a function of Λ_σ .

The maximum velocity in level flight at an altitude corresponding to σ is V_σ , where $V_\sigma = \frac{1}{\sigma} \frac{l_i l_t}{(\frac{1}{\sigma} \frac{l_i l_t}{V_e})}$

equations have been plotted in figures 3 and 4 in terms of the engineering parameters, where

$$\left. \begin{aligned} \lambda_{s\sigma} &= 267.7 \frac{W}{\sigma c (kb_1)^2} = 267.7 l_{s\sigma} \\ \lambda_{p\sigma} &= 841.0 \frac{W}{\sigma f} = 841.0 l_{p\sigma} \\ \lambda_t &= 0.001818 \frac{W}{\eta_0 b \text{ hp}_\sigma} = 0.001818 l_t \\ \Lambda'_\sigma &= 0.006293 \frac{l_{s\sigma} l_t^{4/3}}{l_{p\sigma}^{1/3}} = 0.006293 \Lambda_\sigma \end{aligned} \right\} (26)$$

as defined in equation (4). Although the two charts are basically the same, each has certain advantages in many problems that arise in performance calculation and flight-test reduction. In general, the type of performance chart presented later (see examples) is more useful because of the greater ease with which accuracy may be attained in normal calculations.

C. MAXIMUM RATE OF CLIMB

The maximum rate of climb is realized at the speed at which the excess of thrust power available over thrust power required for level flight is a maximum. Examination of figure 1 shows that, for constant thrust power available, this maximum occurs at the speed for minimum power required. Thus, by differentiating dh/dt with respect to R , and equating to zero, the speed for minimum power required may be obtained, assuming that the characteristics of the actual airplane follow those for the ideal airplane. This latter assumption is not correct in general, nor do normal propeller

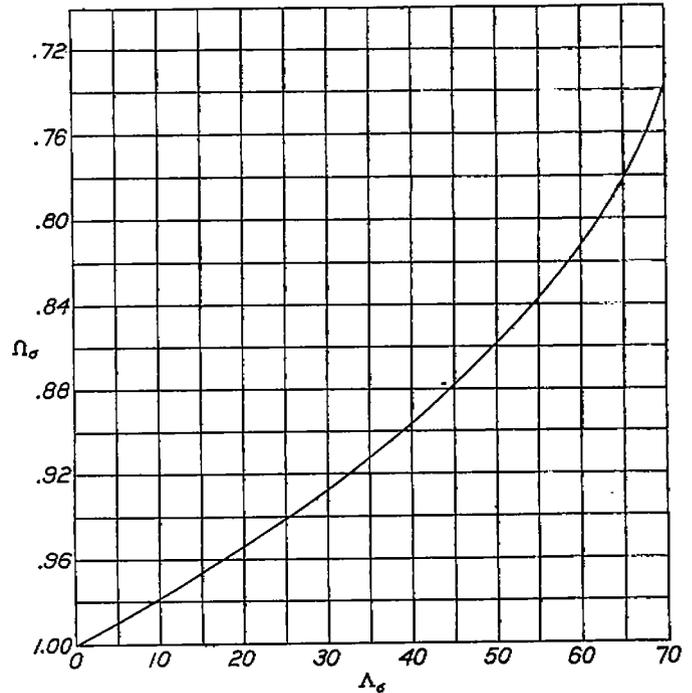


FIGURE 4.—The parameter Ω_σ as a function of Λ_σ .

The maximum velocity in level flight at an altitude corresponding to σ is V_σ , where $V_\sigma = 52.73 \left(\frac{1}{\sigma} \frac{l_i l_t}{V_e} \right)^{1/3} \Omega_\sigma$.

characteristics allow constant thrust power available independent of speed. It will be assumed for the moment, however, that the ideal characteristics hold, and a correction will later be made to allow for the effect that the actual airplane and propeller characteristics will have on the speed for the maximum rate of climb and on the rate itself.

Following the procedure outlined in the last paragraph, i. e., differentiating dh/dt with respect to R , in equation (13) and equating to zero, assuming $P=P_0$, obtain the resulting expression for the speed ratio for minimum power required, $R_{\sigma MP} = V_{MP}/V_\sigma$

$$R_{\sigma MP} = \left[\frac{\Gamma_\sigma}{3(1-\Gamma_\sigma)} \right]^{1/4} \quad (27)$$

By substituting this expression back into equation (13) and again assuming $P=P_0$, obtain the equation for the maximum rate of climb of the ideal airplane:

$$C\lambda_t = 1 - \frac{4}{(27)^{1/4} \Lambda'_\sigma{}^{3/4}} \quad (28)$$

or, in terms of engineering parameters and units,

$$C_{L_c} = 33,000 - 1,294\Lambda_c^{3/4} \quad (29)$$

Equations (27) and (29) are plotted in figures 5 and 6, respectively. In these figures the curves labeled Λ_c should be used. The other curves correspond to engineering equivalents of equations (53) and (54) developed in section IV C.

fore P/P_0 , for actual propeller-engine combinations does vary with speed in such a way that the speed for maximum rate of climb falls considerably above the speed for minimum power required for the corresponding ideal airplane, the theoretical power-required variations are still followed by the actual airplane at the speed for maximum rate of climb. This result is illustrated in figure 7. A large number of cases of

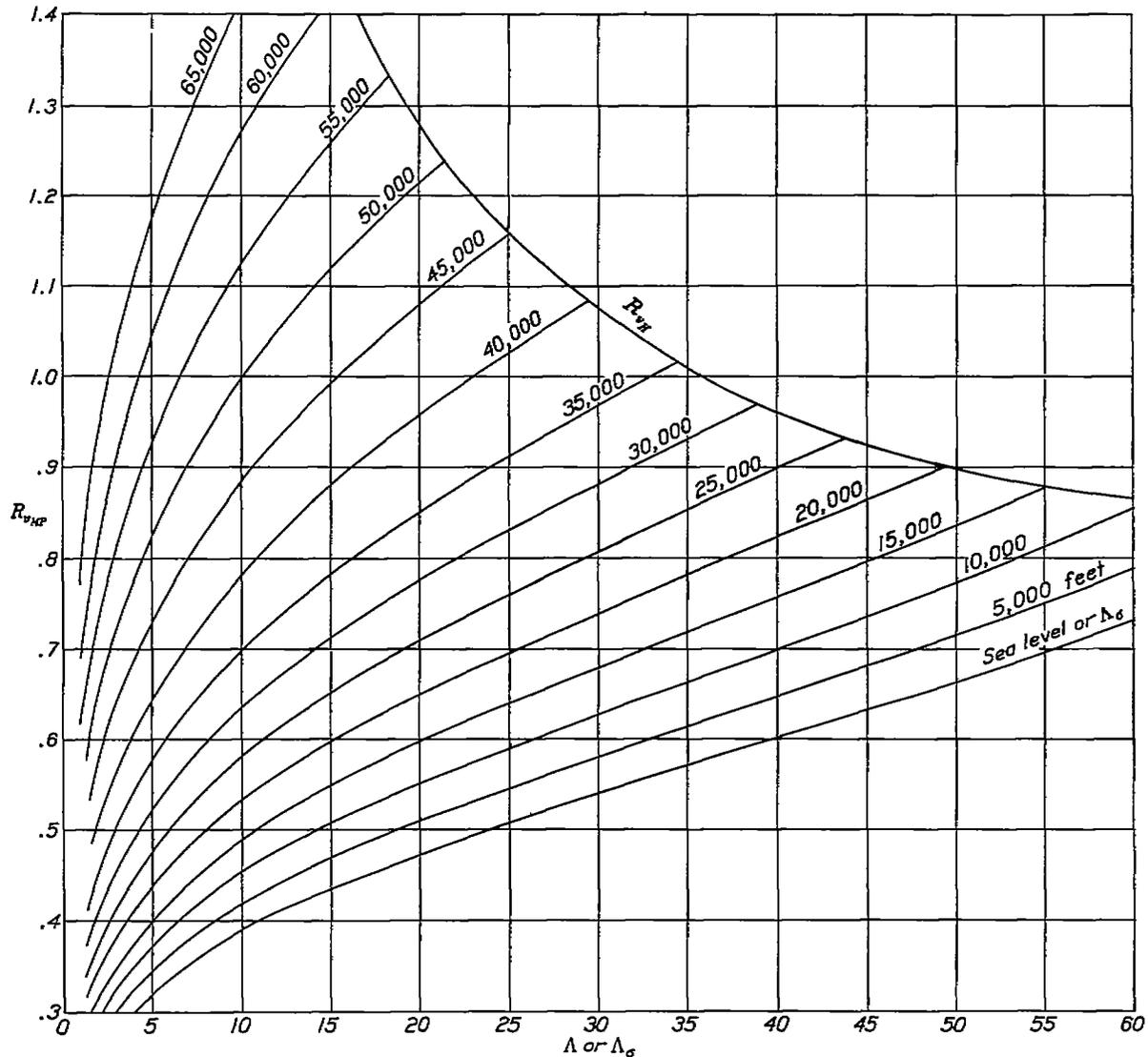


FIGURE 5.—The ratio $R_{MP} = V_{MP}/V_0$ as a function of Λ .

Now consider the case of the actual airplane with normal airplane characteristics. The solid curves in figure 7 present the thrust power required and available for a representative airplane. Equations (27), (28), and (29) would not, in general, be true for the actual airplane since the actual airplane does not, in general, follow the ideal airplane characteristics at the speed for minimum power of the ideal airplane. Therefore, even if the power ratio P/P_0 were independent of speed, the maximum rate of climb and the speed for maximum rate of climb would not be those given by the equations. However, since the thrust power available, and there-

actual modern airplanes has been investigated for which complete wind-tunnel data were available, and in every case the speed for maximum rate of climb lay well within the region where the theoretical power-required variations hold true.

It is convenient in the analysis of an actual airplane to consider the speed for minimum power of the corresponding ideal airplane. Then the speed for maximum rate of climb of the actual airplane will be determined in the form of a ratio between best climbing speed and the speed for minimum power of the ideal airplane. The speed for maximum rate of climb for the actual

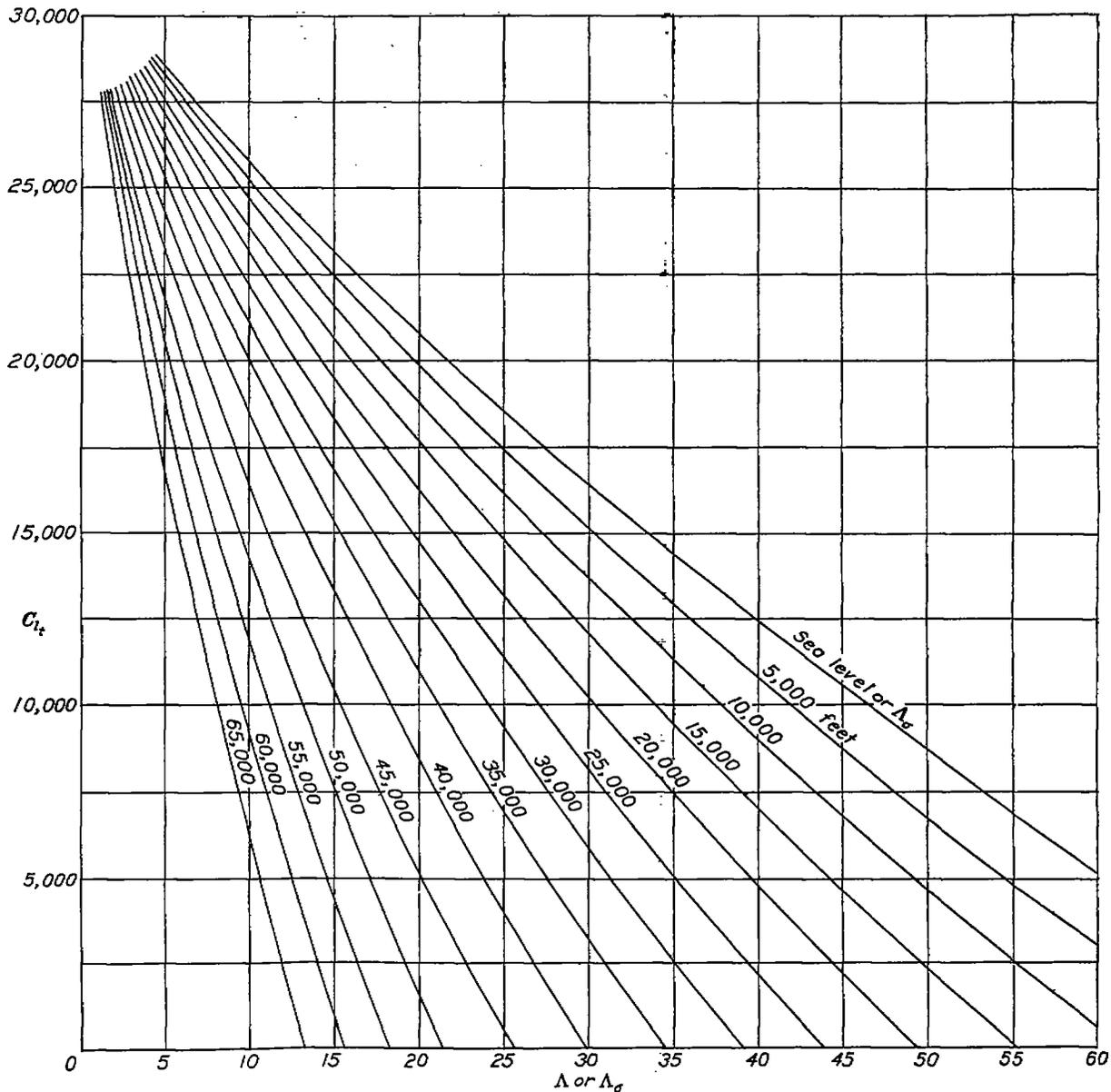


FIGURE 6.—The quantity C_{it} as a function of Λ .

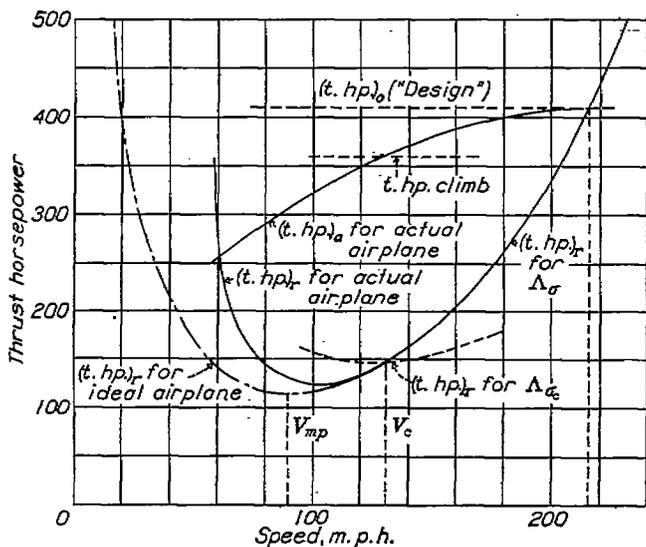


FIGURE 7.—Power characteristics for an actual airplane corresponding to Λ , and its associated ideal climbing airplane corresponding to Λ_{σ} .

airplane is indicated as V_c . The value of this speed will depend, for given thrust power-required characteristics, on the slope of the power-available curve, which in turn will depend upon the particular engine-propeller combination and the method of control.

The first problem is to determine V_c for the actual airplane. This value has been obtained for constant-pitch propellers from Oswald's analysis (reference 1) and for constant-speed propellers from reference 3, assuming best performance propellers selected for high-power cruising or high speed. The results are presented in the form of ratios of V_c/V_{MP} , called R_B , and are plotted in figure 8 as functions of Λ_{σ} .

In most practical cases of modern airplanes, the propellers are selected as a compromise between many factors including high speed and cruising speed at altitude, climb, partial-engine performance, and take-off. For this reason the results presented in figure 8 are not sufficiently general for calculating speed for

maximum rate of climb for many actual airplanes. In the appendix to the present paper is presented an analysis by which the speed for maximum climb can be determined for an arbitrary propulsive unit.

Now, in order to determine the maximum rate of climb from figure 6, it is necessary to find a value of Δ_σ , say Δ_{σ_c} , which has the same values of thrust power required and available at its V_{MP} as the actual airplane has at its V_c . The sinking speed for the actual air-

the second part of equation (32), and transferring to engineering parameters,

$$\frac{\Delta_{\sigma_c}}{\Delta_\sigma} = \left(\frac{R_R^4 + 3}{4R_R} \right)^{4/3} \left(\frac{l_{t_c}}{l_t} \right)^{4/3} \quad (34)$$

Now, if l_{t_c} is designated to be the thrust power loading of the ideal airplane with its thrust power equal to that of the actual airplane at maximum rate of climb, then Δ_{σ_c} and l_{t_c} designate an ideal airplane which, at its

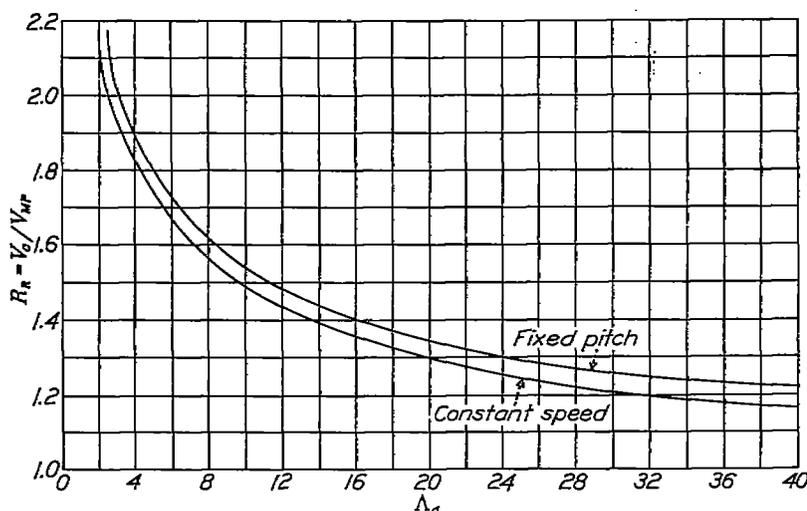


FIGURE 8.—The ratio $R_R = \frac{V_c}{V_{MP}}$ as a function of Δ_σ for fixed-pitch and constant-speed propellers. (For approximate method, see section III C.)

plane, i. e. $(\frac{t. hp.}{W})_r$, may be written, from equation (13)

$$\left(\frac{t. hp.}{W} \right)_r = \frac{1}{\lambda_i} \left[R_R^3 - \left(R_R^3 - \frac{1}{R_R} \right) \Gamma_\sigma \right] \quad (30)$$

Thus, for the speed for maximum rate of climb,

$$\left(\frac{t. hp.}{W} \right)_c = \frac{1}{\lambda_i} \left[R_R^3 R_{\sigma_{MP}}^3 - \left(R_R^3 R_{\sigma_{MP}}^3 - \frac{1}{R_R R_{\sigma_{MP}}} \right) \Gamma_\sigma \right] \quad (31)$$

Substituting for $R_{\sigma_{MP}}$ from equation (27)

$$\begin{aligned} \left(\frac{t. hp.}{W} \right)_c &= \frac{1}{\lambda_i} \frac{R_R^4 + 3}{3R_R} \Gamma_\sigma \left[\frac{3(1 - \Gamma_\sigma)}{\Gamma_\sigma} \right]^{1/4} \\ &= (3)^{1/4} \frac{R_R^4 + 3}{3R_R} \frac{\Delta'_{\sigma_c}{}^{3/4}}{\lambda_{t_c}} \end{aligned} \quad (32)$$

The sinking speed for a hypothetical ideal airplane, designated by Δ'_{σ_c} , Γ_{σ_c} , and λ_{t_c} , flying at its V_{MP} , will be given by setting $R_R = 1$ in equation (32). Thus

$$\left[\frac{t. hp.}{W} \right]_{MP} = \frac{4}{(27)^{1/4}} \frac{\Delta'_{\sigma_c}{}^{3/4}}{\lambda_{t_c}} \quad (33)$$

But it is desired to find a value of Δ'_{σ_c} , corresponding to an ideal airplane with λ_{t_c} , for which the thrust power required at minimum power is the same as that for the actual airplane, corresponding to Δ'_σ and λ_i , at maximum rate of climb. Thus, equating equation (33) and

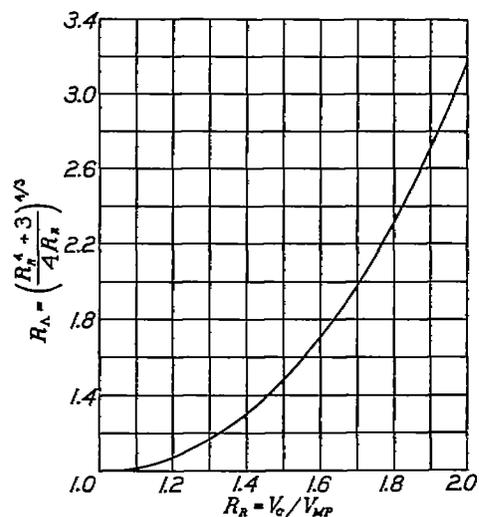


FIGURE 9.—The ratio R_A as a function of R_R .

speed for minimum power, will have the same rate of climb as the actual airplane at its speed for best climb. It is clear from the foregoing definition of l_{t_c} that

$$l_{t_c} = \left(\frac{P_0}{P_c} \right) l_t \quad (35)$$

and therefore that

$$\Delta_{\sigma_c} = \left(\frac{R_R^4 + 3}{4R_R} \right)^{4/3} \left(\frac{P_0}{P_c} \right)^{4/3} \Delta_\sigma \quad (36)$$

where P_c is the thrust horsepower available at the speed for maximum rate of climb of the actual airplane. Knowing R_R and P_c from previous calculation and substituting Δ_{σ_c} and l_{t_c} , determined by equations (35) and (36), into equation (29), the maximum rate of climb of the actual airplane may be determined. For the important case of constant-speed propellers, P_0/P_c reduces to the ratio η_0/η_c , which may be found for each individual case from charts giving the propeller characteristics. This method will be illustrated by an example at the end of the present paper. The expression $\left(\frac{R_R^4 + 3}{4R_R} \right)^{4/3}$, called R_A , is represented in figure 9 for convenience in calculation.

It is important to note that throughout the foregoing analysis none of the equations or charts, with the exception of figure 8, are dependent upon any empirical data. Thus the only modification that would be required to extend the analysis and charts to cover new types of propulsive systems or new propeller-blade designs

would be to construct new R_R against Λ_r curves in figure 8 to correspond to the new systems.

D. ABSOLUTE CEILING

The absolute ceiling occurs where the maximum rate of climb is zero. Thus, setting $C=0$ in equation (28), the value of Λ'_r for any airplane at its absolute ceiling will be

$$\Lambda'_r \sigma_H = \frac{3\sqrt{2}}{8} \quad (37)$$

and, in engineering units according to (26),

$$\Lambda \sigma_H = 75.075 \quad (38)$$

Solving for σ_H in equation (38),

$$\sigma_H = 0.001539 \Lambda^{3/2} \quad (39)$$

This equation is plotted in figure 10, σ_H being replaced by its corresponding altitude H .

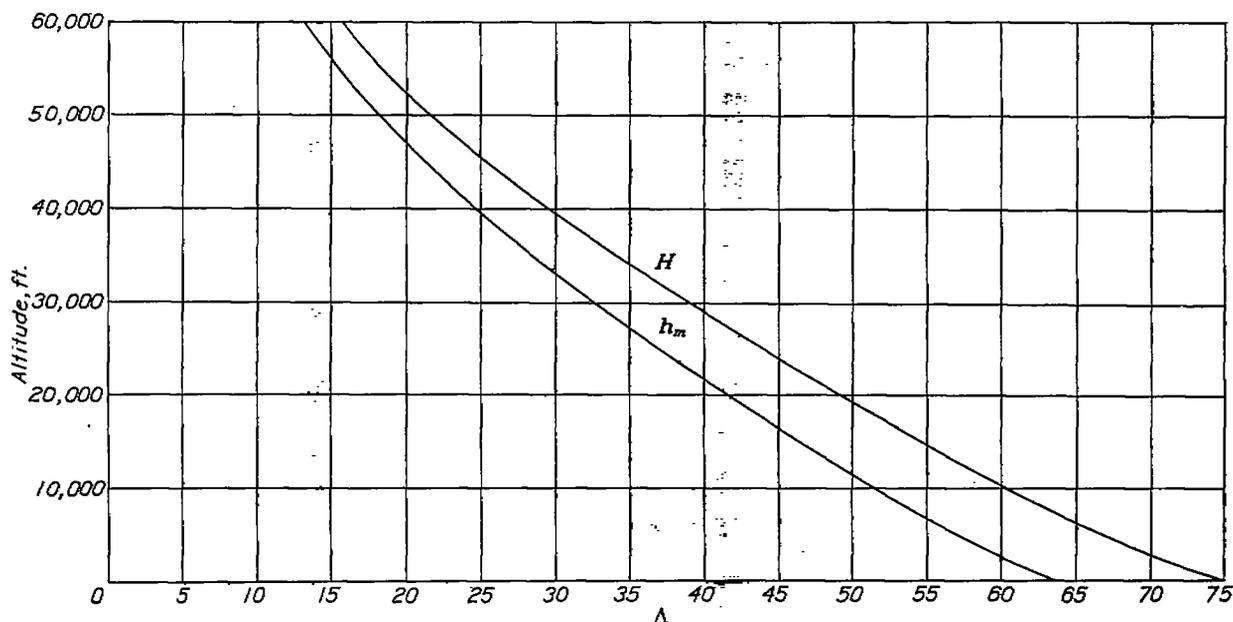


FIGURE 10.—The absolute ceiling H , and the altitude h_m corresponding to σ_m as functions of Λ .

The analysis in this particular section has little practical value but has rather interesting physical significance. It is shown that an airplane having a value of Λ_r greater than that given will be unable to maintain level flight at the altitude on which the particular value of Λ_r is based. This result is in agreement with the value obtained by Millikan for sea level (reference 1). The foregoing equations will be useful in the next section in developing the variation in performance due to variations in the parameters.

The only method that is satisfactory in general for the practical calculation of the absolute ceiling of an airplane is the method normally used for determining the ceiling from flight-test data; namely, calculating the maximum rates of climb at various altitudes at and above the critical altitude of the engine, plotting these rates of climb against density altitude, and extrapolating the curve so determined to zero rate of climb.

The altitude at which the rate of climb becomes zero is the absolute ceiling and at this altitude the relation between the parameters expressed by equation (39) is obeyed.

E. VARIATIONS IN PERFORMANCE DUE TO VARIATIONS IN THE PARAMETERS

It must be remembered that in all of the previous analysis, with the exception of the latter part of section III C, the basic assumption was that the power ratio P/P_0 remained constant and equal to 1.00. This assumption is obviously not correct, in general, and some provision must be made for taking into account the variation in P/P_0 . For this reason the variations of all the performance characteristics with thrust-power variations are investigated. At the same time the variations with respect to the other parameters are considered. Oswald (reference 1) makes a similar analysis, but the approximations that he makes would

introduce comparatively large errors in calculations for which the present analysis is to be extended. His approximations are sufficiently accurate for small values of Λ but introduce appreciable errors for large values of Λ_r such as are encountered in low-power cruising conditions and at high altitudes. Furthermore, in the present analysis, analytic expressions are obtained for all the variations extended to include all altitudes.

Taking the logarithmic derivatives of V , C , and σ_H in equations (9), (29), and (39), respectively, obtain the following equations, which are true for small variations in the parameters:

$$\frac{dV}{V} = -\left(\frac{1-\Gamma_r}{3-4\Gamma_r}\right) \frac{df}{f} \quad (40a)$$

$$\frac{dV}{V} = \left(\frac{1}{3-4\Gamma_r}\right) \frac{d(t. hp.)_a}{(t. hp.)_a} \quad (40b)$$

$$\frac{dV}{V} = -\left(\frac{2\Gamma_r}{3-4\Gamma_r}\right) \frac{dW}{W} \quad (40c)$$

$$\frac{dV}{V} = \left(\frac{2\Gamma_\sigma}{3-4\Gamma_\sigma} \right) \frac{db}{b} \quad (40d)$$

$$\frac{dV}{V} = \left(\frac{\Gamma_\sigma}{3-4\Gamma_\sigma} \right) \frac{de}{e} \quad (40e)$$

$$\frac{dV}{V} = - \left(\frac{1-2\Gamma_\sigma}{3-4\Gamma_\sigma} \right) \frac{d\sigma}{\sigma} \quad (40f)$$

$$\frac{dV}{V} = \left(\frac{1-2\Gamma_\sigma}{3-4\Gamma_\sigma} \right) \frac{dT}{T} \quad (40g)$$

$$\frac{dC}{C} = - \left(\frac{1}{4} \frac{m\Delta_\sigma^{3/4}}{1-m\Delta_\sigma^{3/4}} \right) \frac{df}{f} \quad (41a)$$

$$\frac{dC}{C} = \left(1 + \frac{m\Delta_\sigma^{3/4}}{1-m\Delta_\sigma^{3/4}} \right) \frac{d(t. hp.)_a}{(t. hp.)_a} \quad (41b)$$

$$\frac{dC}{C} = - \left(1 + \frac{3}{2} \frac{m\Delta_\sigma^{3/4}}{1-m\Delta_\sigma^{3/4}} \right) \frac{dW}{W} \quad (41c)$$

$$\frac{dC}{C} = \left(\frac{3}{2} \frac{m\Delta_\sigma^{3/4}}{1-m\Delta_\sigma^{3/4}} \right) \frac{db}{b} \quad (41d)$$

$$\frac{dC}{C} = \left(\frac{3}{4} \frac{m\Delta_\sigma^{3/4}}{1-m\Delta_\sigma^{3/4}} \right) \frac{de}{e} \quad (41e)$$

$$\frac{dC}{C} = \left(\frac{1}{2} \frac{m\Delta_\sigma^{3/4}}{1-m\Delta_\sigma^{3/4}} \right) \frac{d\sigma}{\sigma} \quad (41f)$$

$$\frac{dC}{C} = - \left(\frac{1}{2} \frac{m\Delta_\sigma^{3/4}}{1-m\Delta_\sigma^{3/4}} \right) \frac{dT}{T} \quad (41g)$$

where $m=0.03923$ (a dimensionless constant) and T is the absolute temperature, and

$$\frac{d\sigma_H}{\sigma_H} = \frac{1}{2} \frac{df}{f} \quad (42a)$$

$$\frac{d\sigma_H}{\sigma_H} = -2 \frac{d(t. hp.)_a}{(t. hp.)_a} \quad (42b)$$

$$\frac{d\sigma_H}{\sigma_H} = 3 \frac{dW}{W} \quad (42c)$$

$$\frac{d\sigma_H}{\sigma_H} = -3 \frac{db}{b} \quad (42d)$$

$$\frac{d\sigma_H}{\sigma_H} = -\frac{3}{2} \frac{de}{e} \quad (42e)$$

where $\frac{d\sigma_H}{\sigma_H}$ may be replaced by

$$\frac{dH}{H} = \left(\frac{\sigma_H}{H} \frac{dH}{d\sigma_H} \right) \frac{d\sigma_H}{\sigma_H} \quad (42f)$$

Values of $\frac{\sigma_H}{H} \frac{dH}{d\sigma_H}$ may be calculated from the equations of the standard atmosphere (reference 4), which may be obtained in the following form (for altitudes below the isothermal stratosphere):

$$\frac{\sigma_H}{H} \frac{dH}{d\sigma_H} = -0.235 \left(\frac{T_0}{a} \right) \left(\frac{1}{H-1} \right) = -0.235 \left(\frac{145373}{H} - 1 \right) \quad (42g)$$

and (for altitudes within the stratosphere)

$$\frac{\sigma_H}{H} \frac{dH}{d\sigma_H} = - \frac{\left(\frac{T_i}{T_0} \right) \rho_0}{g \rho_0} \frac{1}{H} = - \frac{20937}{H} \quad (42h)$$

The additional symbols in equations (42g) and (42h) are defined as:

- g , the gravitational constant=32.17 ft./sec.
- p_0 , the standard pressure at sea level=2,116 lb./sq. ft.
- a , the standard lapse rate of temperature in the troposphere=0.003566° F./ft.
- T_0 , the standard absolute temperature at sea level=518.4° F.
- T_i , the standard absolute temperature in the stratosphere=392.4° F.

It will be noticed in equations (40a-g) and (41a-g) that the variations of the velocity and maximum rate of climb here considered are not limited to the velocity and rate of climb at sea level, but apply as well at any altitude corresponding to σ . The variations depend only on the single parameter Δ_σ defined earlier.

Variations in velocity and maximum rate of climb are considered with respect to variations in density ratio and absolute temperature in addition to the usual parametric variations because of their usefulness in flight-test reduction. The variation equations are represented in chart form by figure 11.

All the foregoing variation formulas apply for small variations in the parameters. Many cases arise in performance problems in which it is necessary to calculate the effect of large changes of power on speed. For this purpose figure 12 has been constructed and gives the percentage change in V for 1 percent change in power for large changes in power. (See example C1.) This chart was calculated directly from equation (9).

F. MAXIMUM POSSIBLE VELOCITY FOR A GIVEN AIRPLANE

One of the interesting questions that arises in the field of aircraft performance is the problem as to what is the maximum possible velocity available at all altitudes for a given airplane with a given power. This maximum possible velocity can be obtained directly from equation (40f). The condition is that for a change in σ , dV/V must be zero. In other words,

$$\Gamma_\sigma = \frac{1}{2} \text{ or } \sigma_m = 2 \frac{\lambda_s \lambda_t}{V_{max}^2} \quad (43)$$

where σ_m is the density ratio at the altitude where the maximum velocity occurs. Substituting the value of Γ_σ from (43) into equation (9) and using (8) and (26), the equation for this density ratio becomes

$$\sigma_m = 4\Lambda^{1/3/2} = 0.001998 \Lambda^{3/2} \quad (44)$$

This equation has been plotted in figure 10. Using (8) in equation (9) and substituting the value of σ_m obtained in (43), the expression for the maximum velocity becomes

$$V_{max} = \frac{1}{2} \frac{\lambda_s \lambda_t}{\Lambda^{1/3/2}} \quad (45)$$

and, changing to engineering units,

$$V_{max} = 332.3 \frac{l_s l_t}{\Lambda^{3/2}} = 332.3 \frac{b_e^2}{W \sqrt{f}} \quad (46)$$

where $b_e = kb_1 \sqrt{e}$, the effective span.

The physical significance of this maximum possible velocity is apparent when the two factors that contribute to the total drag of the airplane are considered. The thrust power available has been assumed constant, independent of altitude. Thus the only variation in velocity with altitude must occur as a result of a variation in the drag. The drag coefficient has been

largely with regard to the importance of power and more especially of weight in the limit of the maximum velocity. The quantity $\sqrt{\frac{b_0^2}{f}}$ might be termed the "frontal" or "parasite" aspect ratio. The values of V_{max} and σ_m , of course, correspond to the condition for maximum lift/drag ratio of the airplane.

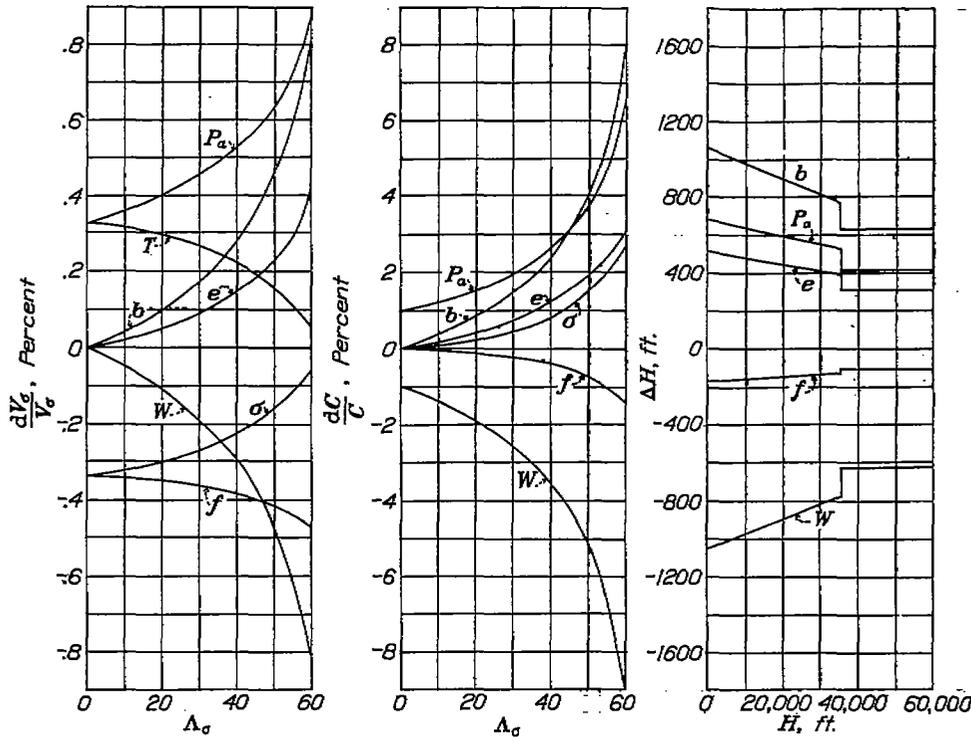


FIGURE 11.—Variation of performance with change of parameters. Charts show percentage change in performance due to a 1 percent change in parameter. V_0 , maximum velocity at altitude; C_0 , maximum rate of climb at altitude; H , absolute ceiling.

divided into two portions, that portion which is independent of C_L and that portion which varies with C_L . The drag forces corresponding to these coefficients can be written as follows:

$$D_1 = \frac{\rho_0 f}{2W} W V^2 \sigma = \frac{W}{\lambda_p} V^2 \sigma$$

and

$$D_2 = \frac{2W}{\pi \rho_0 e (kb_1)^2} \frac{W}{V^2 \sigma} = \lambda_s \frac{W}{V^2 \sigma}$$

where λ_p , λ_s , and W are constant for a given airplane. Now the altitude at which the maximum velocity is reached is that at which the total drag is a minimum for a given velocity; in other words, the altitude at which D_2 increases (with altitude) at the same rate as D_1 decreases, all for constant velocity. Thus for an ideal airplane, i. e., one for which $e=1.00$, this value is the altitude at which the rate of increase of induced drag is equal to the rate of decrease of parasite drag. Considered from this standpoint it is at once apparent that, as the span is increased other factors remaining constant, the altitude for maximum velocity is raised.

The form of the equation for the maximum possible velocity given in equation (46) is interesting, particu-

IV. PERFORMANCE ANALYSIS WITH SEA-LEVEL BASIS

In the previous sections an analysis has been presented that is independent of the sea-level performance of the airplane. There are many cases in practical performance calculation when it is desirable to obtain values of some performance characteristic, such as maximum speed, at a series of altitudes. For this purpose it is more convenient to calculate the performance at some reference level and then to find the performance at the other levels in terms of this reference-level performance. The choice of the reference level is purely arbitrary but, since sea-level characteristics constitute an important practical case, that altitude is chosen as the reference altitude.

After the reference altitude is chosen, the parameters are automatically determined. These parameters will be the sea-level parameters λ_s , λ_p , Λ' , Γ , and Ω . The maximum speed at sea level V_0 , can be calculated as a special case of section III B. The maximum speeds at all other altitudes may then be referred to that at sea level by the ratio $R_{v_0} = V/V_0$.

From section III B the relations between the sea-level parameters can be obtained. These will, of

course, be similar to the altitude relations. Thus, letting $\sigma=1.00$ in equations (6), (9), and (12),

$$\left. \begin{aligned} 1 - \frac{\lambda_t}{\lambda_p} V_0^3 - \frac{\lambda_a \lambda_t}{V_0} &= 0 \\ \Lambda' = \Gamma(1 - \Gamma)^{1/3} &= \Omega(1 - \Omega^3) \end{aligned} \right\} \quad (47)$$

based on the assumption of constant thrust power. This last equation has been represented graphically in the upper left-hand corner of figure 13, plotting against the engineering parameter Λ instead of the physical parameter Λ' . For rapid calculation of new values of Λ based on thrust horsepower available less than P_0 ,

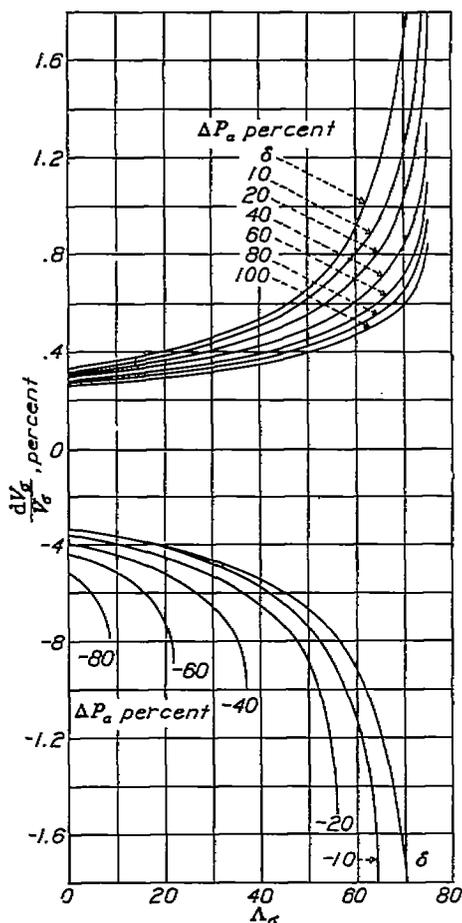


FIGURE 12.—Variation of maximum velocity at altitude with variation of thrust power available. Chart shows percentage change in velocity due to a 1-percent change in power for various percentage changes in power.

A. VELOCITY RATIO AT ALTITUDE

Using the sea-level parameters and the first equation in (47), equation (2) may be written

$$\frac{dh}{dt} = \frac{1}{\lambda_t} \left[\frac{P}{P_0} - \sigma R_{v_0}^3 + \left(\sigma R_{v_0}^3 - \frac{1}{\sigma R_{v_0}} \right) \Gamma \right] \quad (48)$$

Now, considering the condition of level flight at altitude with the design power P_0 , equation (48) becomes, omitting the subscript 0,

$$\Gamma = \sigma R_v \frac{1 - \sigma R_v^3}{1 - \sigma^2 R_v^4} \quad (49)$$

This equation gives the relationship

$$R_v = F(\sigma, \Gamma)$$

or, using equation (47),

$$R_v = F(\sigma, \Lambda')$$

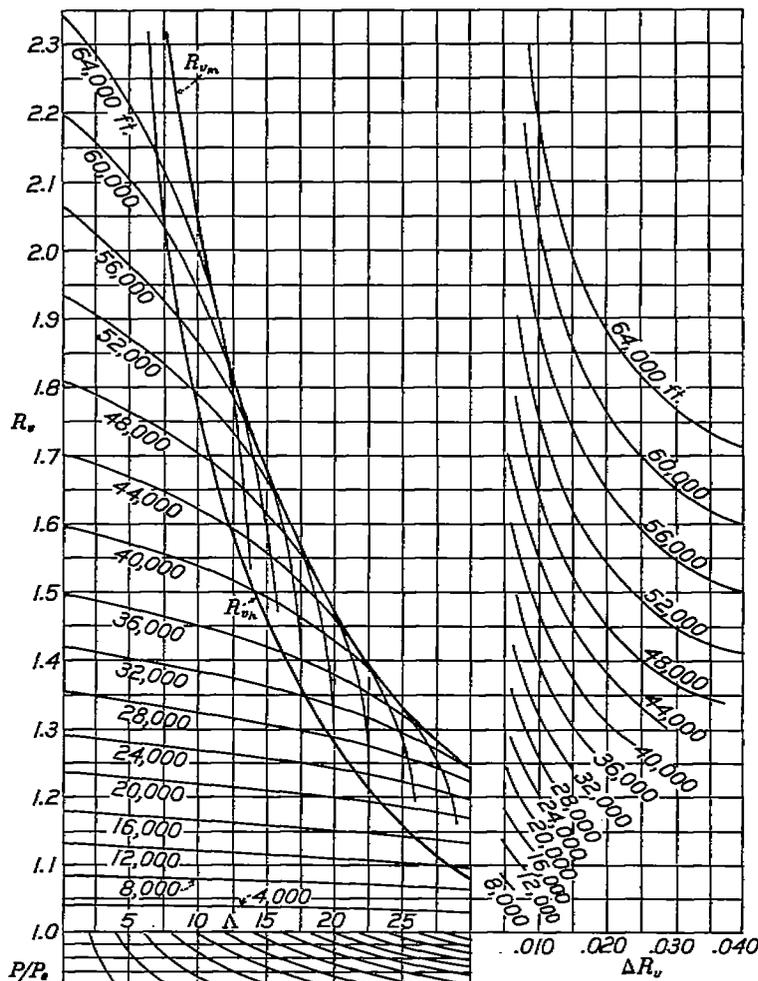


FIGURE 13.—The ratio $R_v = V_v/V_0$ as a function of Λ and altitude and ΔR_v as a function of R_v and altitude.

the chart at the lower left-hand corner of figure 13 is included. Entering the chart at $P/P_0=1.00$ with the value of Λ corresponding to P_0 and following down the curved lines to the desired value of P/P_0 , the value of

$$\Delta_1, \text{ defined by } \Delta_1 = \left(\frac{P_0}{P} \right)^{1/3} \Lambda \text{ may be obtained.}$$

B. VARIATION OF MAXIMUM VELOCITY AT ALTITUDE WITH SMALL CHANGES IN PROPULSIVE EFFICIENCY

In section III E charts were developed from which the variations in maximum speed due to variations in power could be computed. These charts were based upon the altitude parameter Λ_σ . When calculations are being made, using the sea-level parameters and charts, however, it is desirable to have more rapid and convenient methods available.

The effect of variations in power can be handled in either of two ways. A modified value of the thrust power loading based on the changed power can be calculated and used in the charts representing equation (9). If the changes in power are relatively large, this method is recommended. Figure 12 gives a chart for use in such cases since this method was exactly the one by which figure 12 was constructed. If the variations in power are small, as is generally the case when variations in propulsive efficiency are considered, a second method, the analysis of which follows, is more convenient.

The variation in propulsive efficiency may be represented by the ratio η/η_0 . If the brake horsepower is constant, then the variation in thrust horsepower is given by the ratio η/η_0 . The ratio P/P_0 then becomes η/η_0 . Introducing this ratio into equation (48) and considering the level-flight condition (again omitting the subscript 0), the following equation is obtained:

$$\eta = \eta_0 \left[\sigma R_s^3 - \left(\sigma R_s^3 - \frac{1}{\sigma R_s} \right) \Gamma \right] \quad (50)$$

Differentiating both sides with respect to η , solving for $dR_s/d\eta$, and substituting for Γ from equation (49),

$$\frac{dR_s}{d\eta} = \frac{R_s(1 - \sigma^2 R_s^4)}{\eta_0(4\sigma R_s^3 - 3\sigma^2 R_s^4 - 1)} \quad (51)$$

Thus, the change in R_s for a change in η of 0.01 is given by

$$\Delta R_s = \frac{R_s(1 - \sigma^2 R_s^4)}{\eta_0(4\sigma R_s^3 - 3\sigma^2 R_s^4 - 1)} \frac{1}{100} \quad (52)$$

Since the values of ΔR_s will always be small compared with R_s for most practical variations in η , an average value for η_0 can be used in equation (52) with a negligible error, and ΔR_s can be plotted in chart form as a function of R_s and σ . Such a plot is shown in the upper right-hand corner of figure 13, assuming η_0 to be 0.75.

C. MAXIMUM RATE OF CLIMB—SPEED FOR MAXIMUM RATE OF CLIMB

The speed ratio for minimum power required (maximum rate of climb for constant thrust power) is obtained by differentiating dh/dt with respect to R_s in equation (48) and equating to zero. When this operation is done, assuming $P=P_0$, the resulting expression for $R_{v_{MP}}$ is

$$R_{v_{MP}} = \left[\frac{1}{3\sigma^2} \frac{\Gamma}{1-\Gamma} \right]^{1/4} \quad (53)$$

By substituting this expression back into equation (48) and again setting $P=P_0$, obtain the equation for maximum rate of climb for constant thrust power:

$$C_{L_i} = 1 - 4 \left[\frac{\Lambda'^3}{27\sigma^2} \right]^{1/4} \quad (54)$$

Equations (53) and (54) in the sea-level analysis are directly parallel to equations (27) and (28) in the altitude analysis. In the sea-level analysis, $R_{v_{MP}}$ is referred to the maximum speed at sea level and the parameters are sea-level parameters; whereas, in the altitude analysis, $R_{v_{MP}}$ is referred to the maximum speed at altitude and the parameters are altitude parameters. Equations (53) and (54) are plotted in figures 5 and 6 along with equations (27) and (29) and, as will be seen

from the equations, these reduce to the same curve for the special case of sea level. In these figures the curves for the sea-level analysis are plotted against Λ while the curve in each figure for the altitude analysis is plotted against Λ' . The additional analysis in section III C (given by figs. 8 and 9) for correcting the speed for climb and rate of climb for actual thrust power variation applies to both analyses except that it must be remembered that $\left(\frac{1}{\sigma}\right)^{2/3} \Lambda$ must be used to enter figure 8 in both cases.

D. ABSOLUTE CEILING—SPEED AT ABSOLUTE CEILING

Setting $O=0$ in equation (54), the same equation (39) that was found before for absolute ceiling is obtained. Substituting this value of σ_H into equation (53), the equation for speed ratio at absolute ceiling (assuming constant thrust horsepower) becomes

$$R_{v_H} = \frac{\sqrt{3}}{4\sqrt{\Gamma(1-\Gamma)}} \quad (55)$$

This equation is of academic rather than practical interest, since it assumes that the airplane is flying at theoretical minimum power, which will usually be below the actual speed for best climb for the airplane.

E. MAXIMUM POSSIBLE VELOCITY FOR A GIVEN AIRPLANE

Examination of the curves in figure 13, which represent equation (49), will show that for a given airplane, i. e., for a given value of Λ , there is a maximum possible value of R_s that lies on the envelope of the constant-altitude curves. This value corresponds to the maximum possible velocity obtained in the altitude analysis, as given by equation (46).

It is of interest to obtain an analytical expression for the maximum speed ratio $R_{v_{max}}$. This expression is obtained by taking the partial derivative of equation (49) with respect to σ and setting this expression equal to zero. Then the resulting equation is solved simultaneously with equation (49), eliminating σ . This operation gives an equation for the maximum speed ratio:

$$R_{v_{max}} = \frac{1}{2\sqrt{\Gamma(1-\Gamma)}} = \frac{1}{\sqrt{\sigma_m(2-\sigma_m)}} \quad (56)$$

Calculating the ratio $R_{v_H}/R_{v_{max}}$, the following interesting result is obtained:

$$\frac{R_{v_H}}{R_{v_{max}}} = \frac{\sqrt{3}}{2} = 0.866 \quad (57)$$

Equations (55) and (56) are represented graphically in figure 13 along with the family of constant-altitude curves, using equation (47) to obtain the relation between Γ and Λ' and plotting against the engineering parameter Λ . Equations (39) and (44) are plotted in figure 10 in terms of altitude and Λ .

APPENDIX

SPEED FOR MAXIMUM RATE OF CLIMB FOR ARBITRARY PROPELLER

In section III C of the main paper there has been presented a means for obtaining the speed for maximum rate of climb at any altitude provided the propeller is selected for high speed at the same altitude. These values of the speed for maximum climb are then strictly true only at the critical altitude for high speed and are given in terms of the speed for minimum power at that altitude. Since the maximum excess power available for climb varies only slightly with speed near the best theoretical speed, the values of the speed for maximum climb will be approximately correct for propellers selected for cruising speed and for altitudes reasonably near the critical altitude.

One of the chief difficulties, however, with the method of section III is that the propellers are often selected as a compromise governed by best take-off, maximum rate of climb, best cruising speed at some relatively high altitude, propeller tip speed, and finally propeller clearance. Thus, for many cases, and particularly for high supercharging, such a determination of the speed for maximum climb will give a speed considerably below the true speed for best climb of the actual airplane so that an underestimate of the excess power will result with a consequent conservative estimate of the maximum rate of climb. For this reason it is desirable to develop a method for calculating this speed for any arbitrary propeller. It is the object of the following analysis to develop such a method.

The general performance equation has been written as in equation (5), letting $R_p = P/P_0$,

$$\frac{dh}{dt} = \frac{1}{\lambda_i} \left[R_p - \frac{\lambda_i}{\lambda_{pe}} V^3 - \frac{\lambda_{se} \lambda_i}{V} \right] \quad (A1)$$

To find the conditions for maximum rate of climb, differentiate with respect to V and equate to zero as before. Thus,

$$\frac{dR_p}{dV} - 3 \frac{\lambda_i}{\lambda_{pe}} V^2 + \frac{\lambda_{se} \lambda_i}{V^2} = 0 \quad (A2)$$

where V_c is the speed for maximum rate of climb. Now consider the case in which $dR_p/dV = 0$, which will correspond to the theoretical speed for minimum power as shown in the analysis in section III. Thus,

$$3 \frac{\lambda_i}{\lambda_{pe}} V_{MP}^2 = \frac{\lambda_{se} \lambda_i}{V_{MP}^2} \quad (A3)$$

Multiplying both sides by $\frac{\lambda_{se} \lambda_i}{V_{MP}^2}$ and defining

$$\Gamma_{\sigma MP} = \frac{\lambda_{se} \lambda_i}{V_{MP}^2} = \Gamma_{\sigma} \frac{V_{\sigma}}{V_{MP}} \quad (A4)$$

gives

$$\Gamma_{\sigma MP} = (3)^{1/4} \Lambda'_{\sigma}^{3/4} \quad (A5)$$

Replacing the physical by the engineering parameters, this equation is plotted in figure 14. Now defining

$R_{\sigma M} = \frac{V}{V_{MP}}$ and substituting for $\frac{\lambda_i}{\lambda_{pe}} V^3$ in equation (A1)

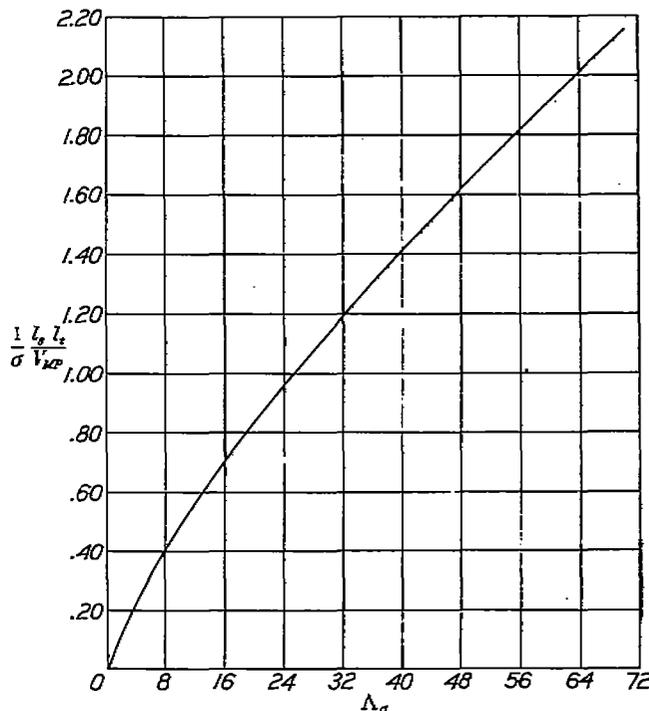


FIGURE 14.—The quantity $\frac{1}{\sigma} \frac{\lambda_i}{V_{MP}}$ as a function of Δ_{σ} . The velocity for minimum power required of the ideal airplane is V_{MP} , where $V_{MP} = \frac{1}{\sigma} \frac{\lambda_i}{V_{MP}}$

according to (A3) and (A4), obtain

$$\frac{dh}{dt} = \frac{1}{\lambda_i} \left[R_p - \left(\frac{R_{\sigma M}^3}{3} + \frac{1}{R_{\sigma M}} \right) \Gamma_{\sigma MP} \right] \quad (A6)$$

Also, making a similar substitution and defining $R_R = \frac{V_c}{V_{MP}}$, as in section III C of the main paper, equation (A2) becomes

$$\frac{dR_p}{dR_R} = \left(R_R^2 - \frac{1}{R_R^2} \right) \Gamma_{\sigma MP} \quad (A7)$$

Substituting for $\Gamma_{\sigma MP}$ in equation (A7) according to (A5) and converting to the engineering parameter Λ according to (26), obtain

$$\frac{dR_p}{dR_R} = 0.02935 \left(R_R^2 - \frac{1}{R_R^2} \right) \Lambda_{\sigma}^{3/4} \quad (A8)$$

CASE FOR CONSTANT SPEED PROPELLER

The foregoing analysis has been concerned with the power-required phase of the problem. The power available will now be considered. Propeller characteristics are normally available in terms of the dimensionless coefficients, C_P the power coefficient, and J the advance-diameter ratio, where

$$\left. \begin{aligned} C_P &= 5 \times 10^{10} \frac{\text{b. hp.}}{\sigma N^3 D^5} \\ J &= \frac{88V}{ND} \end{aligned} \right\} \quad (\text{A9})$$

- σ , relative air density.
- V , true speed of the airplane (m. p. h.).
- b. hp., engine brake horsepower.
- N , angular velocity of the propeller (r. p. m.).
- D , diameter of the propeller (ft.).

since both N and D are constant. Differentiating

$$dR_R = \frac{1}{J_{MP}} dJ \quad (\text{A13})$$

Thus, from (A11) and (A13)

$$\frac{dR_P}{dR_R} = \frac{J_{MP}}{\eta_0} \frac{d\eta}{dJ} \quad (\text{A14})$$

At the maximum rate of climb dR_P/dR_R , which is proportional to dP/dV , must be the same for the power-required as for the power-available characteristics. Thus, equating (A8) and (A14) and solving for $d\eta/dJ$,

$$\frac{d\eta}{dJ} = 0.02935 \frac{\eta_0}{J_{MP}} \left(R_R^2 - \frac{1}{R_R^2} \right) \Lambda_\sigma^{3/4} \quad (\text{A15})$$

which is the condition that must be satisfied for maximum rate of climb. It should be noted in (A15) that

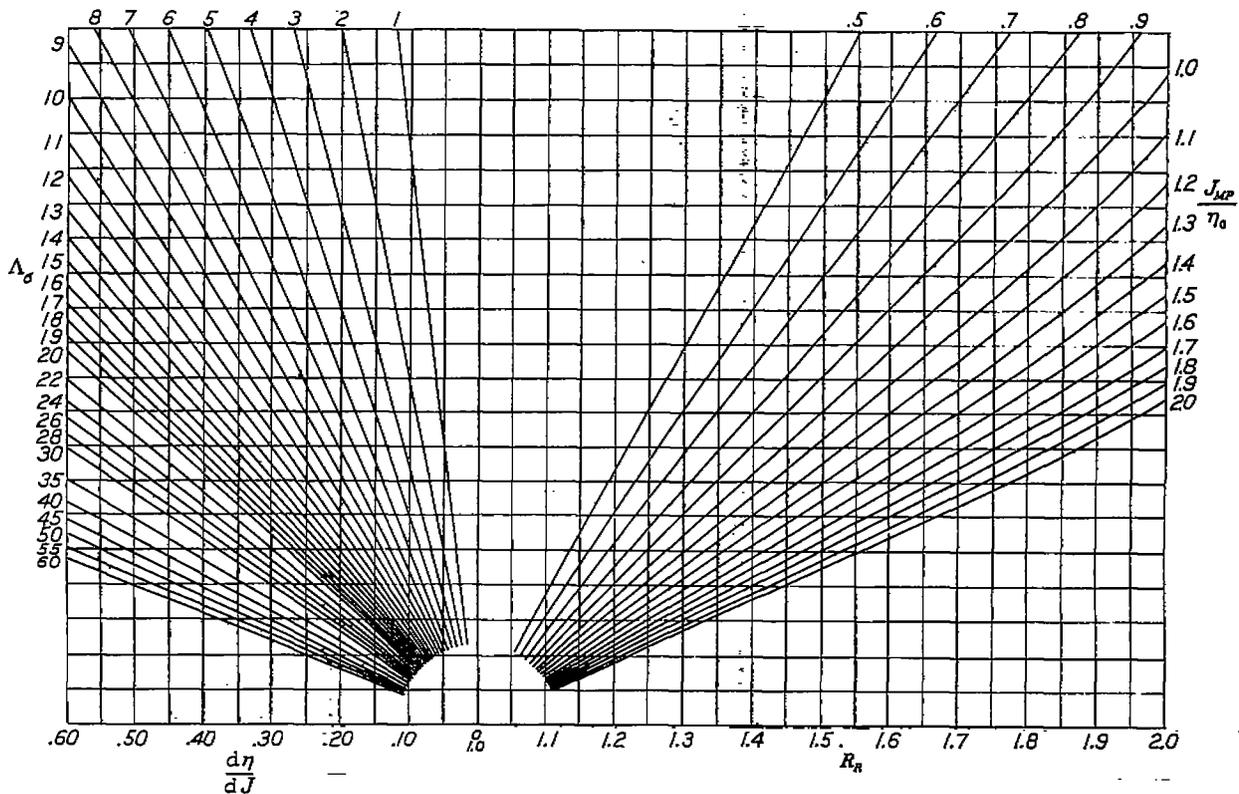


FIGURE 15.—Chart for calculating $d\eta/dJ$ as a function of J_{MP}/η_0 , Λ_σ , and R_R .

An equivalent expression for dR_P/dR_R for the propulsive unit must now be obtained.

For constant-speed-propeller control, the brake horsepower is constant and therefore

$$R_P = \frac{P}{P_0} = \frac{\eta}{\eta_0} \quad (\text{A10})$$

Differentiating

$$dR_P = \frac{1}{\eta_0} d\eta \quad (\text{A11})$$

Also, as previously defined

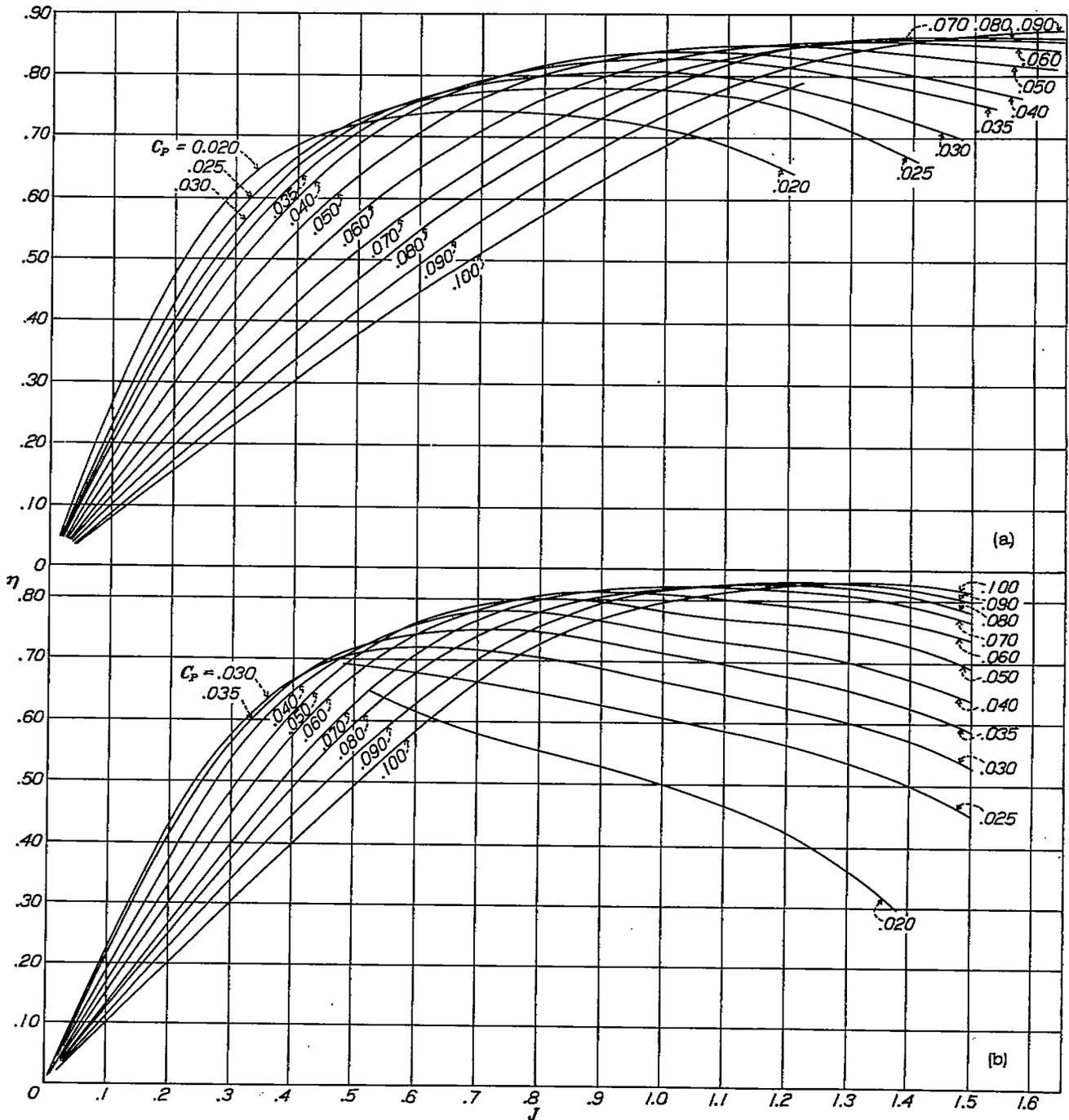
$$R_R = \frac{V}{V_{MP}} = \frac{J}{J_{MP}} \quad (\text{A12})$$

η_0 is multiplied by $\Lambda_\sigma^{3/4}$. By the definition of Λ_σ as given by (26)

$$\Lambda_\sigma^{3/4} = \frac{l_{s\sigma}^{3/4} l_t}{l_{p\sigma}^{1/4}} = \left(\frac{l_{s\sigma}^3}{l_{p\sigma}} \right)^{1/4} \frac{W}{\eta_0 \text{ b. hp.}}$$

Thus the product $\eta_0 \Lambda_\sigma^{3/4}$ is independent of the assumed value of η_0 as long as the same value is used in calculating Λ_σ . It is usually convenient to use a value of η_0 that has been used before in calculating Λ_σ , thus saving an additional calculation for Λ_σ .

For a given set of conditions defined for a given airplane, i. e., N , b. hp., η_0 , and Λ_σ , the entire right-hand



(a) Two-blade propeller characteristics.
 (b) Three-blade propeller characteristics.

FIGURE 16.—Plot of η against J for different values of C_p . (See reference 5.)

$$C_p = 5 \times 10^{-4} \frac{b, \text{hp.}}{\sigma N^3 D^5}; \quad J = \frac{V}{\pi D}$$

side of (A15) is determined except for the variable $R_R = V_c/V_{MP}$, since J_{MP} depends only upon the power-required characteristics of the airplane.

For rapid calculation figure 15 has been constructed from which $d\eta/dJ$ may be determined in terms of the constant parameters, J_{MP}/η_0 and $\Delta\sigma$, and the variable R_R . The left-hand side of the equation must now be defined from the propeller characteristics in terms of the same variable R_R .

For this purpose, the dimensionless parameters C_p

and J , defined by (A9), are chosen, and the propeller characteristics in terms of these parameters and the efficiency η are plotted. Such a plot is shown in figure 16, the data for which were taken from reference 5. As shown by the definition of C_p and J , C_p depends only upon the assumed characteristics of the airplane and is independent of the velocity V while

$$J = J_{MP} \frac{V}{V_{MP}}$$

which, at the speed for climb, becomes

$$J_c = J_{MP} \frac{V_c}{V_{MP}} = J_{MP} R_R \quad (A16)$$

Figure 17 has been constructed by graphical differentiation, plotting $d\eta/dJ$ against J for constant C_P . Thus, the propeller characteristics have been obtained in the form $\frac{d\eta}{dJ} = f_1(J, C_P)$ which, upon defining J_{MP} , becomes

$$\frac{d\eta}{dJ} = f_2(R_R, C_P)$$

Equation (A15) then becomes

$$f_2(R_R, C_P) = \frac{d\eta}{dJ} = 0.02935 \frac{\eta_0}{J_{MP}} \left(R_R^2 - \frac{1}{R_R^2} \right) \Lambda_\sigma^{3/4} \quad (A17)$$

Knowing Λ_σ , $\left(\frac{J_{MP}}{\eta_0} \right)$, and C_P , R_R may be calculated by iteration using figures 15 and 17(a) or 17(b). This method is shown clearly in the examples.

EXAMPLES

Two major examples have been illustrated. Example A is of a normal type and the calculations are standard. Example B was chosen for the specific purpose of illustrating a condition under which the approximate method of selecting R_R (figure 8) fails. As may be found, consulting figure 8, the approximate method would have given satisfactory results in example A. However, in example B, the approximate method is good at the altitude for which the propeller was chosen (30,000 feet) but becomes increasingly bad for the constant brake-horsepower case at other altitudes. It is a satisfactory approximation above the altitude mentioned, provided that the brake horsepower decreases in a normal manner above this altitude.

EXAMPLE A (SEE FIG. 18)

Given: A two-engine monoplane with constant-speed and three-blade propellers:

$W = 24,000$ lb.

$b = 95$ ft.

$e = 0.88$.

$f = 21$ sq. ft.

(b. hp.)_m = 850 hp. per engine at 2,100 r. p. m. at 8,000 ft. (assume constant (b. hp.)_m from sea level to 8,000 ft.).

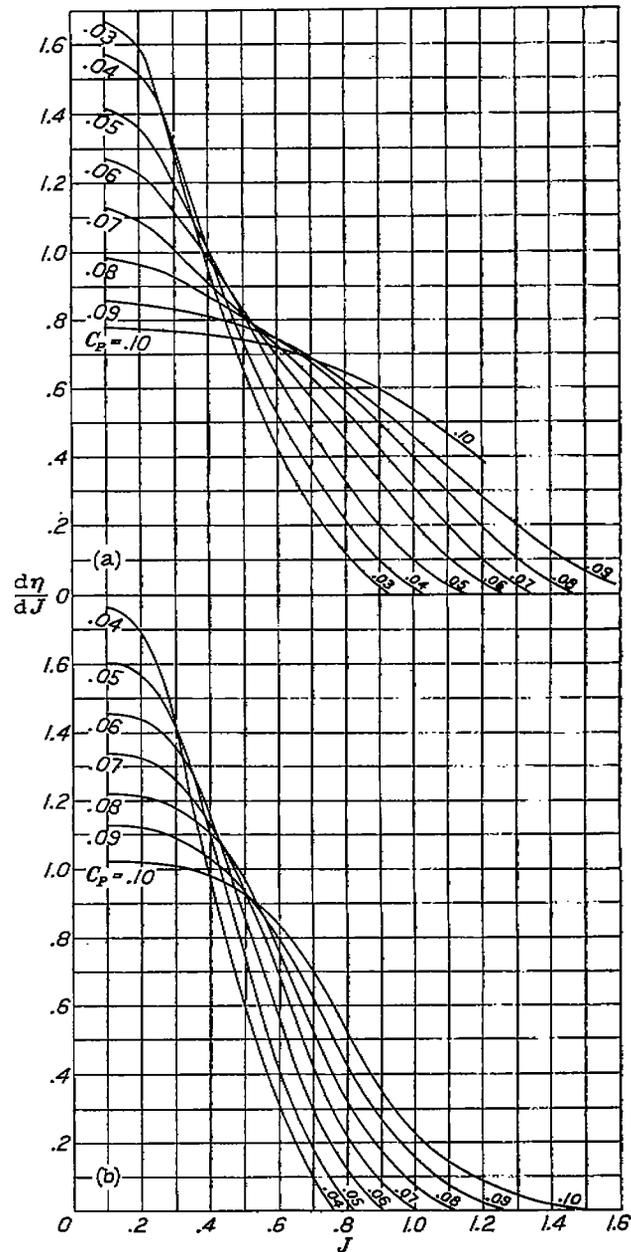
$k = 1.00$.

Select propeller for best performance at V_m with full throttle at 2,100 r. p. m. (geared 16:11) at 15,000 ft.

1. Calculate V_m at 15,000 feet.

$\eta_0 = 0.84$ assumed.

$\frac{1}{\sigma} = 1.590$ (fig. 19).



(a) Two-blade propeller (see fig. 16(a)).
(b) Three-blade propeller (see fig. 16(b)).
FIGURE 17.—Plot of $d\eta/dJ$ against J for different values of C_P .

$l_s = 8.02$
 $l_p = 1,142$ } equation (26).

(b. hp.)_m (at 15,000 ft.) = [(b. hp.)_m (at 8,000 ft.)] ×

$\left[\frac{(\text{b. hp.})_m \text{ (at 15,000 ft.)}}{(\text{b. hp.})_m \text{ (at sea level)}} \right] \left[\frac{(\text{b. hp.})_m \text{ (at sea level)}}{(\text{b. hp.})_m \text{ (at 8,000 ft.)}} \right] =$
649 hp. (fig. 20)

$l_0 = 22.0$
 $\Lambda_\sigma = 24.25$ } equation (26).

$\Omega_\sigma = 0.942$ (fig. 4).

$V_\sigma = 52.73 \left(\frac{1}{\sigma} \frac{l_p}{l_s} \right)^{1/3} \Omega_\sigma = 216$ m. p. h.

r. p. m. = $2,100 \frac{11}{16} = 1,442$.

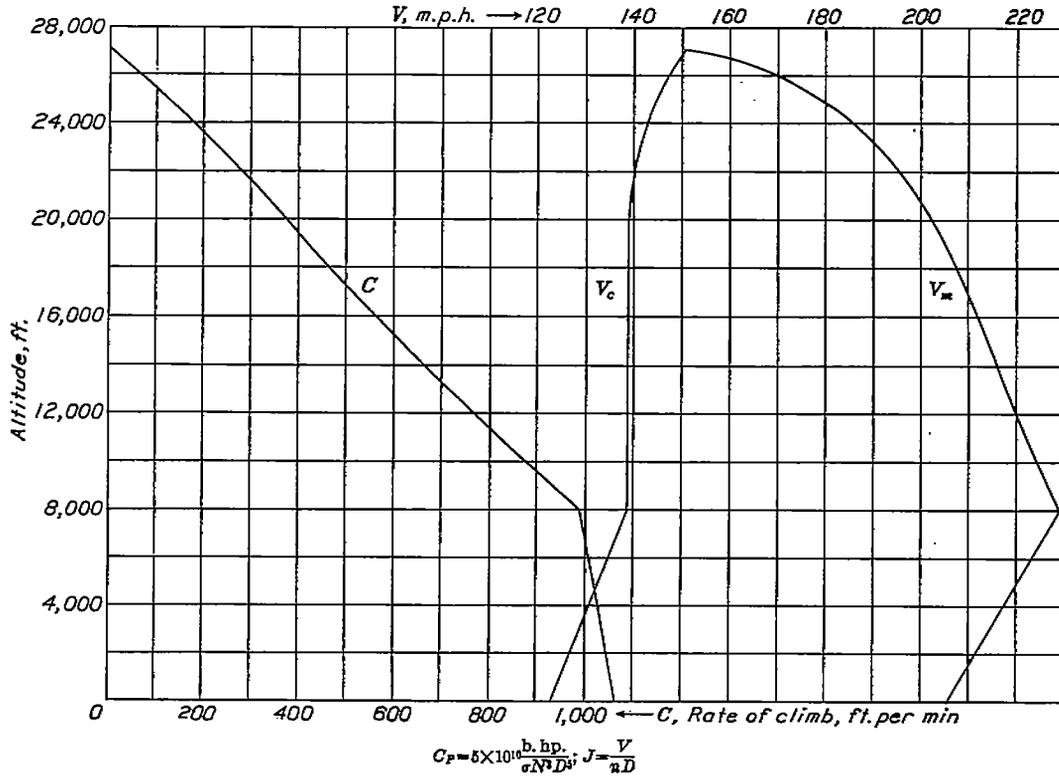


FIGURE 18.—Airplane performance summary chart for Example A.

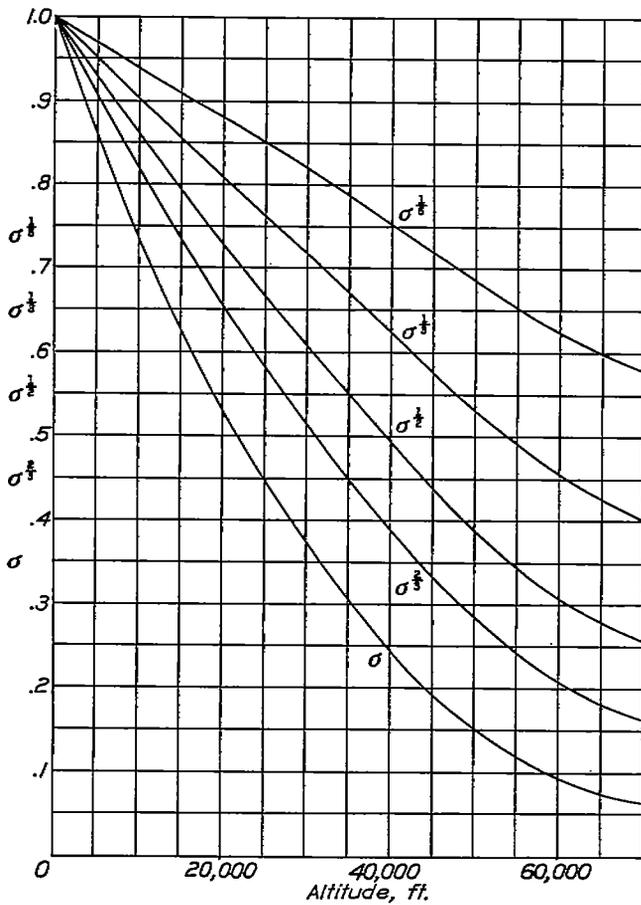


FIGURE 19.—Relative density as a function of altitude. $\sigma = \rho/\rho_0$; ρ , mass density of air; $\rho_0 = 0.002378$ slug/cu. ft. at sea level.

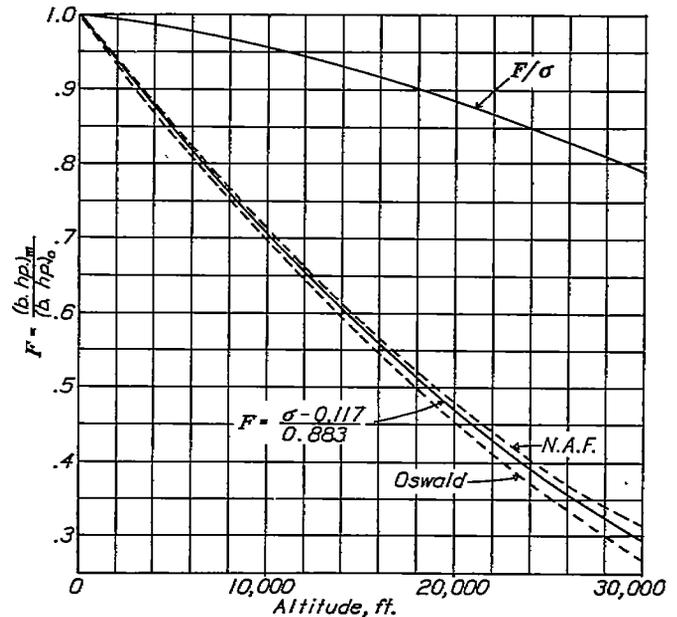


FIGURE 20.—Variation of full-throttle brake horsepower with altitude at constant r. p. m.

$$C_s = 0.638 \frac{\sigma^{1/5} V}{(b \text{ hp.})^{1/5} N^{2/5}} = 1.87.$$

$$J = 1.13 \text{ (fig. 7 of reference 5).}$$

$$D = \frac{88V}{NJ} = 11.65.$$

$$\eta = 0.825 \text{ (fig. 7 of reference 5).}$$

$$\frac{\Delta\eta}{\eta} = -1.8 \text{ percent.}$$

$$\frac{dV}{V} = 0.42 \text{ percent per percent change in power (fig. 11).}$$

$$V_{corr} = 216 (1 - 0.0042 \times 1.8) = 214 \text{ m. p. h.}$$

2. Calculate V_m at sea level, 8,000, 20,000, and 25,000 feet.

$$\eta_0 = 0.84 \text{ assumed.}$$

h (ft.)	$(b \text{ hp.})_m$	l_i	$\Delta\sigma$	$\Omega\sigma$	V_σ (m. p. h.)	C_P	J	η	$\frac{\Delta\eta}{\eta}$ (percent)	$\frac{dV}{V}$ (percent)	$V_{e,corr}$ (m. p. h.)
0	850	16.82	12.43	0.973	209	0.0659	1.09	0.81	-3.6	0.37	206
8,000	850	16.82	15.60	.965	230	.0842	1.20	.83	-1.2	.88	229
15,000	649	24.25	27.25	.909	205	.0762	1.07	.825	-1.8	.80	208
20,000	524	34.1	36.1	.889	186	.0724	.973	.805	-4.2	.74	178
25,000	418	34.20	54.7								

Fig. 20 (constant to 8,000 ft.).

$$l_i = \frac{W}{\eta_0 b \text{ hp.}}$$

$$\Delta\sigma = \left(\frac{1}{\sigma}\right)^{1/5} \frac{l_i \Delta l_i}{l_i^{1/5}}$$

Fig. 4.

$$V_\sigma = 52.73 \left(\frac{1}{\sigma}\right)^{1/5} \Omega\sigma$$

$$C_P = 6 \times 10^{-6} \frac{b \text{ hp.}}{\sigma^2 N^2 D^5}$$

$$J = \frac{88V}{ND}$$

Fig. 16(b).

$$\frac{\Delta\eta}{\eta} = \frac{\eta_{actual} - \eta_0}{\eta_0}$$

Fig. 11.

$$V_{e,corr} = V_\sigma \left[1 + \left(\frac{dV}{V}\right) \left(\frac{\Delta\eta}{\eta}\right) \right]$$

3. Calculate the maximum rate of climb at sea level, 8,000, 15,000, 20,000, and 25,000 feet.

$$l_i = 16.82$$

$$\eta_0 = 0.84 \text{ at sea level. Example A1}$$

$$(b \text{ hp.})_m = 850$$

h (ft.)	$(b \text{ hp.})_m$	$\Delta\sigma$	V_σ (m. p. h.)	R_{VMP}	V_{MP} (m. p. h.)	J_{MP}	$\frac{J_{MP}}{\eta_0}$	C_P	R_R	J_c	$\frac{d\eta}{dJ}$	R_R	V_c (m. p. h.)	R_A	η_0	$\Delta\sigma$	l_i	Cl_i	C (ft./min.)
0	850	12.43	209	0.414	86.5	0.453	0.539	0.0659	1.42	0.638	0.54	1.42	123	1.33	0.725	20.2	16.49	20,700	1,062
8,000	850	15.60	230	.442	101.5	.531	.634	.0842	1.37	.728	.46	1.37	136	1.26	.735	24.0	19.23	19,000	988
15,000	649	24.25	216	.503	106.5	.568	.675	.0762	1.28	.727	.50	1.28	139	1.14	.745	32.0	24.8	15,200	613
20,000	524	34.1	205	.573	118.0	.620	.739	.0724	1.18	.722	.53	1.18	139	1.06	.760	43.6	30.1	11,100	309
25,000	418	54.7	186	.693	129	.674	.802	.0724	1.12	.755	.53	1.12	145	1.02	.780	61.6	36.6	4,500	122

Example A2.

Examples A1 and A2.

Examples A1 and A2.

Fig. 5.

$$V_{MP} = V_\sigma R_{VMP}$$

$$J_{MP} = \frac{88V_{MP}}{ND}$$

$$C_P = 6 \times 10^{-6} \frac{b \text{ hp.}}{\sigma^2 N^2 D^5}$$

Assumed.

$$J_c = J_{MP} R_R$$

Fig. 17(b).

Fig. 16. Checks assumed value.

$$V_c = V_{MP} R_R$$

Fig. 9.

Fig. 16(b).

$$\Delta\sigma = \Delta\sigma R_A \left(\frac{2\sigma}{\eta_0}\right)^{1/5}$$

$$l_i = l_i \left(\frac{2\sigma}{\eta_0}\right) \left(\frac{b \text{ hp.}}{\sigma}\right) \text{ at sea level}$$

Fig. 6.

$$C = \frac{Cl_i}{l_i}$$

EXAMPLE B (SEE FIG. 21)

Given: A single-engine monoplane with a constant-speed and three-blade propeller:

$$W = 6,000 \text{ lb.}$$

$$b = 45 \text{ ft.}$$

$$e = 0.85 \text{ ft.}$$

$$f = 6 \text{ sq. ft.}$$

$(b \text{ hp.})_m = 700 \text{ hp. at 2,000 r. p. m. geared 4:3 at 30,000 ft. (assume constant (b. hp.)}_m \text{ from sea level to 30,000 ft.)}$

$$k = 1.00$$

Select propeller for best performance at V_m at 30,000 ft.

1. Calculate V_m at 30,000 feet.

$$\text{Assume } \eta_1 = 0.83$$

$$\left(\frac{1}{\sigma}\right)^{2/3} = 1.93 \text{ (fig. 19)}$$

$$l_s = \frac{W}{e(kb_1)^2} = 3.49$$

$$l_p = \frac{W}{f} = 1,000$$

$$l_{i1} = \frac{W}{\eta_1 b \text{ hp.}_m} = 10.33$$

$$\Lambda = \frac{l_s l_i^{4/3}}{l_p^{1/3}} = 7.85$$

Equation (26)

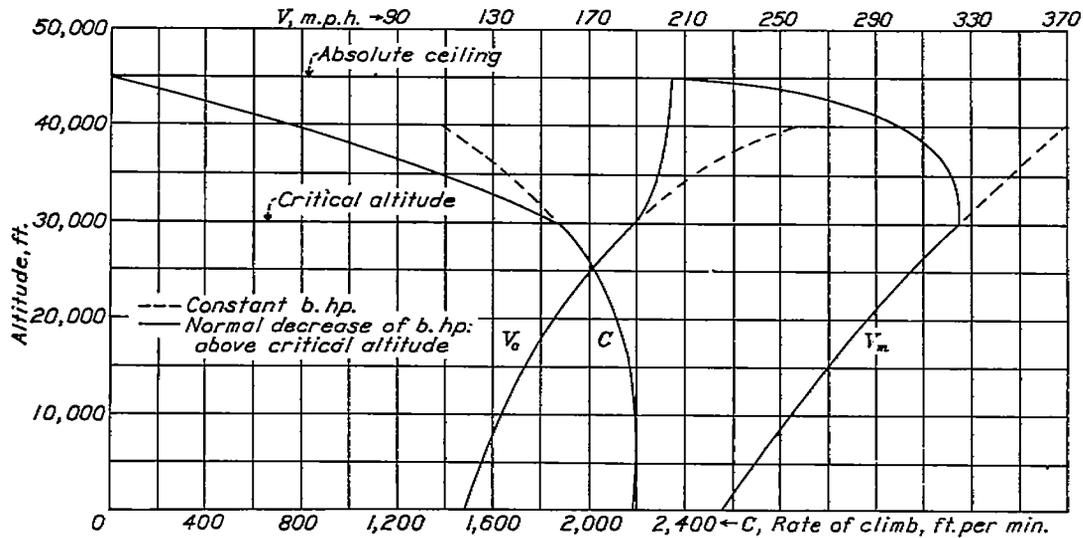


FIGURE 21.—Airplane performance summary chart for Example B.

$$\Lambda_\sigma = \left(\frac{1}{\sigma}\right)^{2/3} \Lambda = 15.15$$

Method (a)

$$\Omega_\sigma = 0.966 \quad (\text{fig. 4})$$

$$V_\sigma = 52.73 \left(\frac{1}{\sigma} \frac{l_i}{t_i}\right)^{1/3} \Omega_\sigma = 325 \text{ m. p. h.}$$

Method (b)

$$\frac{1}{\sigma} \frac{l_i}{t_i} = 0.2977$$

$$V_\sigma = \frac{1}{\sigma} \frac{l_i}{t_i} = 325 \text{ m. p. h.}$$

$$C_\sigma = 0.638 \frac{\sigma^{1/5} V}{(\text{b. hp.})^{1/5} N^{2/5}} = 2.47$$

$$\eta_1 = 0.83 \quad \text{Checks assumed value (fig. 7 of reference 5)}$$

$$J = \frac{88V}{ND} = 1.55 \quad (\text{fig. 7 of reference 5})$$

$$D = \frac{88V}{NJ} = 12.3 \text{ ft.}$$

2. Calculate V_m at sea level, 10,000, 20,000, and 40,000 feet, assuming constant brake horsepower.

Method (a):

$$\left. \begin{aligned} \Lambda &= 7.85 \\ \eta_1 &= 0.83 \end{aligned} \right\} \text{Example B1}$$

h (ft.)	$\left(\frac{1}{\sigma}\right)^{2/3}$	Λ_σ	Ω_σ	$\frac{1}{\sigma} \frac{l_i}{t_i}$	V_σ (m. p. h.)	C_P	J	η	$\frac{\Delta P}{P}$ (percent)	$\frac{dV}{V}$ (percent)	V (m. p. h.)
0	1.000	7.85	0.983	0.1515	238	0.0369	1.13	0.705	-15.1	0.36	226
10000	1.223	9.60	.979	.1861	262	.0502	1.25	.755	-9.0	.36	254
20000	1.522	11.95	.974	.2327	291	.0693	1.35	.795	-4.2	.37	286
30000	1.930	15.15	.966	.2977	325	.0953	1.55	.830	0		325
40000	2.538	20.83	.953	.405	369	.1500	1.75	.830	0		369

* By extrapolation.

Method (b):

$\Lambda = 7.85$
 $\eta_1 = 0.83$ } Example B1
 $V_0 = 238$ at sea level for $\eta_0 = 0.83$

h (ft.)	R_s	C_F	J	η	$\Delta\eta$ (percent)	ΔR_s	$R_{s,corr}$	V (m. p. h.)
0	1.000	0.0369	1.13	0.705	-12.5			225
10,000	1.102	.0802	1.26	.755	-7.5	0.0045	1.063	254
20,000	1.223	.0693	1.35	.795	-3.5	.0061	1.202	286
30,000	1.364	.0983	1.55	.880	0		1.364	326
40,000	1.551	.1600	1.76	.880	0		1.551	369

Fig. 13. $C_F = C_F$ (at sea level)
 $J = J$ (at sea level) R_s
 $\Delta\eta = \eta_{actual} - \eta_0$
 $R_{s,corr} = R_s(1 + \Delta\eta \times \Delta R_s)$
 $V = V_0 R_{s,corr}$

$$V_0 = \frac{V \text{ (at 30,000 ft.)}}{R_s \text{ (at 30,000 ft.)}} = \frac{326}{1.364} = 238 \text{ m. p. h.}$$

$$V = V_0 \left[1 + \left(\frac{\Delta P}{P} \right) \text{ (in percent)} \frac{dV}{V} \right] = 225 \text{ m. p. h.}$$

$\frac{dV}{V}$ from fig. 12.

3. Calculate V_m at 35,000, 40,000, and 42,000 feet, assuming an engine critical altitude of 30,000 feet.

$\Lambda = 7.85$
 $\eta_1 = 0.83$ } Example B1
 $(b. hp.)_0 = 700$

h (ft.)	$\left(\frac{1}{\sigma}\right)^{1/2}$	(b. hp.) _m	A_s	R_s	V_s (m. p. h.)	C_F	J	η	$\frac{\Delta P}{P}$ (percent)	$\frac{dV}{V}$ (percent)	V (m. p. h.)
35,000	2.195	605	20.9	0.951	325	0.1034	1.55	0.810	-2.4	0.41	322
40,000	2.568	446	36.6	.908	302	.0960	1.44	.820	-1.3	.51	300
42,000	2.726	357	47.2	.871	284	.0913	1.35	.825	-.6	.62	283

Fig. 19. $A_s = \left(\frac{1}{\sigma}\right)^{2/3} \left(\frac{b. hp.}{(b. hp.)_0}\right)^{1/3} \Lambda$
 Fig. 4. $V_s = 69.73 \left(\frac{1}{\sigma}\right)^{1/3} R_s$
 $C_F = 6 \times 10^{-5} \frac{b. hp.}{\sigma^{3/4} N D^2}$
 $J = \frac{88V}{ND}$
 $\frac{\Delta P}{P} = \frac{\eta_{actual} - \eta_1}{\eta_1}$
 $V = V_0 \left[1 + \left(\frac{\Delta P}{P} \right) \text{ (percent)} \times \frac{dV}{V} \right]$

4. Calculate the maximum rate of climb from sea level to 40,000 feet assuming constant brake horsepower.

(a) Calculate V_{MP}

Method (1):

$$\left. \begin{matrix} l_2 = 3.49 \\ l_1 = 10.33 \end{matrix} \right\} \text{Example B1}$$

h (ft.)	$\frac{1}{\sigma}$	Δ_r	$\frac{\frac{1}{\sigma} l_1}{V_{MP}}$	V_{MP} (m.p.h.)
0	1.000	7.85	0.415	87
10,000	1.363	9.60	.482	101
20,000	1.877	11.95	.569	119
30,000	2.675	15.15	.682	140
40,000	4.085	20.33	.847	174

Fig. 10.

Examples B1 and B2.

Fig. 14.

$$V_{MP} = \frac{\frac{1}{\sigma} l_1}{\frac{C}{V_{MP}}}$$

Method (2):

$$\Delta = 7.85$$

$V_0 = 258$ m. p. h. (corresponds to $\eta_0 = 0.83$). Example B2.

h (ft.)	R_{MP}	V_{MP} (m. p. h.)
0	0.364	87
10,000	.423	101
20,000	.483	119
30,000	.590	140
35,000	.650	156
40,000	.750	174
42,000	.767	183

Fig. 5.

$$V_{MP} = R_{MP} V_0$$

(b) Calculate the maximum rate of climb.

Method (1) (approximate method):

$$\left. \begin{matrix} l_1 = 10.33 \\ \eta_1 = 0.83 \end{matrix} \right\} \text{Example B1}$$

h (ft.)	Δ_r	R_R	V_{MP} (m. p. h.)	V_e (m. p. h.)	R_A	J_e	C_P	η_e	Δ_{r_e}	l_{1_e}	Cl_{1_e}	C (ft./min.)
0	7.85	1.57	87	137	1.64	0.655	0.0369	0.760	14.5	11.30	23,400	2,070
10,000	9.60	1.50	101	153	1.43	.725	.0502	.790	15.2	10.95	23,000	2,120
20,000	11.95	1.43	119	170	1.35	.811	.0693	.800	16.9	10.73	22,200	2,070
30,000	15.15	1.37	140	192	1.25	.915	.0953	.780	20.6	11.00	20,500	1,860
40,000	20.33	1.28	174	225	1.15	1.072	.1500	*.660	31.8	13.00	15,600	1,200

Examples B1 and B2.

Fig. 8.

Example B4.

$V_e = V_{MP} R_R$

Fig. 9.

$J_e = \frac{88 V_e}{ND}$

Example B2.

Fig. 16 (b).

$\Delta_{r_e} = \Delta_r R_A \left(\frac{\eta_1}{\eta_e} \right)^{1/2}$

$l_{1_e} = l_1 \left(\frac{\eta_1}{\eta_e} \right)$

Fig. 6.

$C = \frac{Cl_{1_e}}{l_{1_e}}$

* By extrapolation.

Method (2) (exact method):

$$\left. \begin{matrix} l_1 = 10.33 \\ \eta_1 = 0.83 \end{matrix} \right\} \text{Example B1}$$

h (ft.)	Δ_r	V_{MP} (m. p. h.)	J_{MP}	$\frac{J_{MP}}{\eta}$	C_P	R_R	J_e	$\frac{d\eta}{dJ}$	R_R	V_e (m. p. h.)	R_A	η_e	Δ_{r_e}	l_{1_e}	Cl_{1_e}	C (ft./min.)
0	7.85	87	0.415	0.500	0.0369	1.36	0.565	0.360	1.36	118	1.24	0.745	11.2	11.50	23,100	2,180
10,000	9.60	101	.481	.530	.0502	1.32	.635	.320	1.32	133	1.19	.775	12.5	11.05	24,400	2,200
20,000	11.95	119	.567	.684	.0693	1.31	.742	.315	1.31	156	1.17	.785	15.1	10.90	23,100	2,120
30,000	15.15	140	.667	.904	.0953	1.34	.893	.340	1.34	188	1.21	.770	20.2	11.14	20,700	1,860
40,000	20.33	174	.830	1.000	.1500	1.46	1.210	.460	1.46	254	1.40	.760	31.9	11.28	15,600	1,350

Examples B1 and B2.

Example B4.

$J_{MP} = \frac{88 V_{MP}}{ND}$

Example B2.

Assumed.

$J_e = J_{MP} R_R$

Fig. 17 (b).

Fig. 16. Checks assumed value.

$V_e = V_{MP} R_R$

Fig. 9.

Fig. 16 (b).

$\Delta_{r_e} = \Delta_r R_A \left(\frac{\eta_1}{\eta_e} \right)^{1/2}$

$l_{1_e} = l_1 \left(\frac{\eta_1}{\eta_e} \right)$

Fig. 6.

$C = \frac{Cl_{1_e}}{l_{1_e}}$

5. Calculate the maximum rate of climb at 35,000, 40,000 and 42,000 ft. assuming an engine critical altitude of 30,000 ft. as in Example B3.

Approximate method:

$$\left. \begin{aligned} l_{t_1} &= 10.33 \\ \eta_1 &= 0.83 \\ (b. hp.)_0 &= 700 \end{aligned} \right\} \text{Example B1}$$

h (ft.)	$(b. hp.)_m$	Λ_e	R_R	V_{MP} (m. p. h.)	V_c (m. p. h.)	R_Δ	J_e	C_P	η_e	Λ_{e_0}	l_{t_0}	Cl_{t_0}	C (ft./min.)
35,000	605	20.9	1.29	155	200	1.15	0.953	0.1034	0.770	26.6	12.90	17,800	1,390
40,000	445	36.6	1.13	174	205	1.05	.977	.0960	.800	40.3	16.82	12,300	732
42,000	387	47.2	1.16	183	212	1.04	1.010	.0918	.810	50.6	19.12	8,400	440
	Example B3.	Example B3.	Fig. 8.	Example B4 (a) (2).	$V_c = V_{MP} R_R$	Fig. 9.	$J_e = \frac{88V_c^2}{ND}$	Example B3.	Fig. 16 (b).	$\Lambda_{e_0} = \Lambda_e R_\Delta \left(\frac{\eta_1}{\eta_e}\right)^{1/3}$	$l_{t_0} = l_{t_1} \left(\frac{(b. hp.)_m \eta_1}{(b. hp.)_0 \eta_e}\right)^{1/3}$	Fig. 6.	$C = Cl_{t_0} / l_{t_0}$

Exact method:

$$\left. \begin{aligned} l_{t_1} &= 10.33 \\ \eta_1 &= 0.83 \\ (b. hp.)_0 &= 700 \end{aligned} \right\} \text{Example B1}$$

h (ft.)	$(b. hp.)_m$	Λ_e	V_{MP} (m.p.h.)	J_{MP}	$\frac{J_{MP}}{\eta_1}$	C_P	R_R	J_e	$\frac{d\eta}{dJ}$	R_R	V_c (m.p.h.)	R_Δ	η_e	Λ_{e_0}	l_{t_0}	Cl_{t_0}	C (ft./min.)
35,000	605	20.9	155	0.738	0.889	0.1034	1.28	0.943	0.325	1.28	199	1.14	0.770	26.3	12.90	18,000	1,395
40,000	445	36.6	174	.829	.999	.0960	1.16	.945	.260	1.16	202	1.04	.795	40.2	16.90	12,400	734
42,000	387	47.2	183	.872	1.061	.0913	1.11	.967	.200	1.11	203	1.02	.805	50.2	18.30	8,600	445
	Example B3.	Example B3.	Example B4 (a) (2).	$J_{MP} = \frac{88V_{MP}}{ND}$		Example B3.	Assumed.	$J_e = J_{MP} R_R$	Fig. 17 (b).	Fig. 15 (charts).	$V_c = V_{MP} R_R$	Fig. 9.	Fig. 16 (b).	$\Lambda_{e_0} = \Lambda_e R_\Delta \left(\frac{\eta_1}{\eta_e}\right)^{1/3}$	$l_{t_0} = l_{t_1} \left(\frac{(b. hp.)_m \eta_1}{(b. hp.)_0 \eta_e}\right)^{1/3}$	Fig. 6.	$C = Cl_{t_0} / l_{t_0}$

EXAMPLE C

Given: An airplane for which, for full power at 10,000 feet altitude:

$\Lambda_e = 15.0$

$V_c = 250$ m. p. h.

$W = 15,000$ lb.

1. Calculate the speed at the same altitude for 60 percent full power, assuming no change in propulsive efficiency.

$\frac{\Delta P}{P} = -40$ percent

$\frac{dV}{V} = 0.47$ percent per percent of $\frac{\Delta P}{P}$ (fig. 12)

$V_{corr} = 250 (1 - 0.0047 \times 40) = 203$ m. p. h.

2. Calculate the speed for a 1,000-pound increase in W .

$\frac{\Delta W}{W} = \frac{1000}{15000} = 6.7$ percent

$\frac{dV}{V} = 0.075$ percent (fig. 11)

$V = 250 (1 - 0.00075 \times 6.7) = 248$ m. p. h.

3. Calculate the speed for an overload of 7,500 pounds.

$W = 15,000$ lb., $\Lambda_e = 15.0$, $\Omega_e = 0.966$ (fig. 4).

$W = 22,500$ lb., $\Lambda_e = 15.0 \left(\frac{22500}{15000}\right)^2 = 33.8$,

$\Omega_e = 0.916$ (fig. 4).

V (for 22,500 lb.) = $250 \times \frac{0.916}{0.966} = 237$ m. p. h.

COMPOSITE CHART

The introduction of the altitude parameters into the performance analysis of the previous pages makes it possible to construct charts composed of single curves, in place of families of curves necessary in the case of the sea-level parameters, to represent the performance at all altitudes. This method results in a more compact form for all charts using the altitude parameters.

Accordingly, Mr. W. R. Sears of the California Institute of Technology has devised an arrangement of all of the curves, except for the propeller curves, in one compact, composite chart. This method avoids the necessity of cross reference from one chart to another in most performance calculations. Figure 22 shows the arrangement of the curves on this chart.

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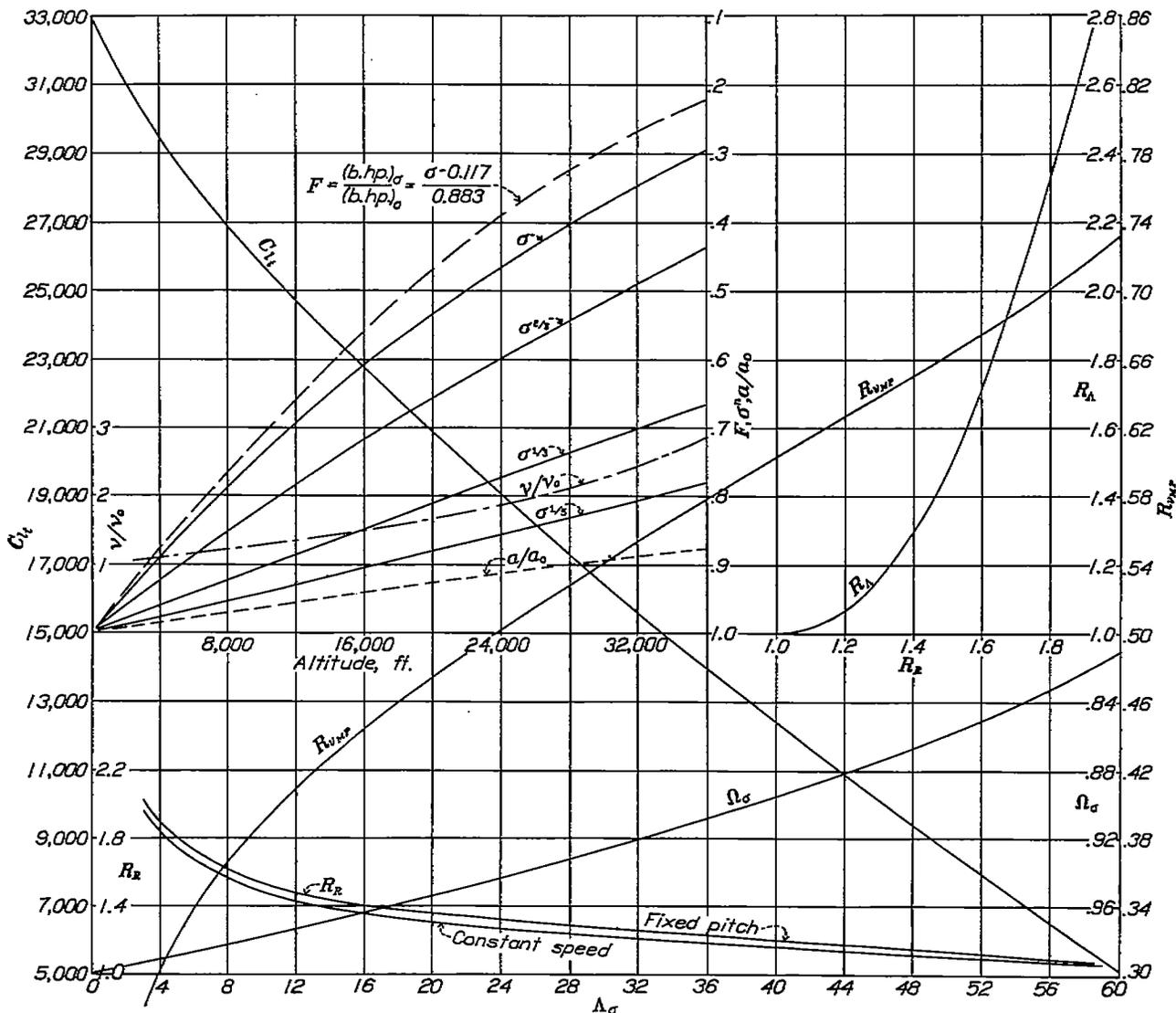


FIGURE 22.—Composite chart for the calculation of airplane performance.

ν kinematic viscosity at altitude
 ν_0 kinematic viscosity at sea level
 a speed of sound at altitude
 a_0 speed of sound at sea level