

## REPORT No. 480

### THE AERODYNAMIC EFFECTS OF WING CUT-OUTS

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#### SUMMARY

*In connection with the interference program being conducted in the N.A.C.A. variable-density wind tunnel, an analysis was made of available material with the object of presenting a qualitative discussion of wing characteristics as affected by cut-outs and of determining means for their quantitative calculation.*

*The analysis indicated that extending a cut-out in the chord direction has much greater effect than extending it in the span direction. Unfairness in profile over the leading edge of the cut-out sections adversely affects the lift and induced drag as well as the profile drag.*

*Lifting-line airfoil theory can be successfully used to calculate the characteristics of a wing as affected by a cut-out when the section characteristics of the profiles along the span are known. It is useful, in such a problem, to employ the method of successive approximation for obtaining the span load distribution.*

*The information derived from the analysis was applied for illustration to the prediction of the characteristics of a wing with a center-section cut-out. The values thus obtained were found to agree fairly well with the test results of a model of the cut-out wing measured in the variable-density wind tunnel.*

#### INTRODUCTION

It is sometimes desirable to cut out portions of a wing, usually at the center. Such a change in plan form may, however, produce large changes in the characteristics of the wing. Therefore, information that would guide a designer in his choice of a cut-out and enable him to calculate the aerodynamic characteristics of cut-out wings should prove useful.

The information now available concerning wing cut-outs or applicable to the analysis of their effects is plentiful (references 1 to 7) but too disconnected and unorganized to be of the greatest possible usefulness. In connection with the interference program being conducted in the N.A.C.A. variable-density wind tunnel, an analysis was therefore made of existing material to determine the qualitative effects of the different features of wing cut-outs, and to obtain means of calculating wing characteristics as affected by them.

The characteristics of a cut-out wing of N.A.C.A. 0012 section were predicted from the information

derived for this report and compared with test results obtained for the purpose from a test of a model of the cut-out wing in the N.A.C.A. variable-density wind tunnel at a Reynolds Number of 3,160,000.

#### GENERAL EFFECTS OF WING CUT-OUTS

A monoplane wing of finite span experiences the least induced drag when the downwash is constant over the span, a condition occurring when the load distribution is elliptical. The constant downwash distribution also affords the highest maximum lift if the wing is untwisted and of the same profile throughout, because the sections along the span reach their lift peaks together. Departure from the elliptical-loading condition introduces a deformation in the downwash distribution that adversely affects the characteristics of the wing. The effects of a cut-out are due in a large measure to the change it produces in the span load distribution, resulting in what may be called "induced interference."

It is immediately evident that, for similar cut-outs, the deformation of the load distribution increases with cut-out area. Because of the induced interference, the adverse effects on the total lift and drag of the wing grow disproportionately to the sizes of the cut-outs. The total profile drag, however, tends to be reduced because of the reduction in area caused by a cut-out. At the lower lifts, this effect may be greater than the adverse effect on the drag due to the deformation of the load distribution.

For cut-outs of equal area, greater depth of cut-out along the chord produces the more severe deformation of the span load distribution and causes the greater interference. This effect can be noticed in two of the tests reported by Ackeret (reference 3) where the induced interference of a cut-out extending the full chord depth showed itself to be much greater than that of a second cut-out extending half the depth of the first but twice its width.

Unfairness in profile around the leading edge of the cutaway sections of a wing adversely affects the lifts of the sections involved and thus adds to the induced interference. At the trailing edge, however, unfairness in profile has negligible induced-interference effect, as is shown in tests by Ackeret.

The profile drag is naturally increased by any profile unfairness.

Unfairness of the plan form of a cut-out has little effect. Muttray (reference 4) tested two wings, each having a displaced rectangular center section, one forward, the other aft, and compared the results with those of the normal wing. No noticeable effect on lift or drag was found except near maximum lift where slightly earlier burbling occurred. Besides demonstrating the unimportance of plan-form fairness, those tests also show that whether the cut-out is at the leading or trailing edge is unimportant in regard to the effect on the interference, just as would be surmised from simple airfoil theory.

When the cut-out is in the form of a hole between the leading and trailing edges, the portion of the airfoil at the cut-out becomes, in profile, a tandem wing arrangement. Therefore, if airfoil profiles be retained along the cut-out, the induced interference at low lifts is no different, theoretically, than for a leading- or trailing-edge cut-out with the same profile arrangement and total chord distribution along the span. At high lift coefficients, however, another effect appears. The sections of the wing ahead of the hole, being in the added relative upwash of those to the rear, stall still earlier than they would otherwise. Conversely, the forward sections tend to maintain the air flow over the after sections, thus delaying their burble. Consequently, a hole cut-out near the trailing edge may be poorer with respect to maximum lift than an equivalent leading- or trailing-edge cut-out, and one near the leading edge may be better.

Because of the deformation impressed upon the span load distribution, a cut-out section experiences an upwash with relation to the rest of the wing and therefore tends to carry more load than its reduced chord would otherwise be called upon to support. This upwash, however, since it owes its existence to the deformation present in the loading curve, cannot be sufficient to make the cut-out section carry its full share of the load. The deformation in the span load distribution, and thereby the induced interference, may be eliminated, but only for one desired angle of attack of the wing, by adjusting the angles of the profiles across the cut-out. Below that angle of attack of the wing, a cut-out section will be carrying more than its designated share of the load, and above that angle, less.

The relative upwash previously mentioned causes a cut-out section to stall earlier than it would otherwise. The maximum lift of the wing suffers in consequence. Eliminating the induced interference for any one wing angle, by increasing the angles of incidence of the profiles across the cut-out, produces a similar result. This effect, however, may be avoided by employing higher-lift, later-stalling profiles along the cut-out.

Muttray (reference 4) tested some wing models provided with auxiliary airfoils before the cutaway sections which were formed of high-lift profiles set at increased angles of incidence. He found that most of the adverse effect on maximum lift was thus eliminated, and also most of the induced interference. The profile drag would, however, necessarily be increased by such an arrangement.

The effects of the various cut-out features on the pitching moment may be readily understood by considering the changes produced in the moments of the sections about the reference axis along the span. A front cut-out would tend to increase the diving moment and a rear cut-out to reduce it. Likewise an auxiliary airfoil before the center section would tend to decrease the diving moment. The total resultant moment of a cut-out wing with relation to any  $Y$  axis can be estimated by integrating the moments of the profiles about that axis across the span.

#### QUANTITATIVE CALCULATION OF WING CHARACTERISTICS AS AFFECTED BY CUT-OUTS

Lotz has attempted (reference 5) to provide a simple, easy, and rapid means of calculating the lift and induced drag of monoplane wings with cut-outs. The assumptions being crude, however, the application of this work would seem to be limited to approximating the lifts and induced drags of monoplane wings as affected by only the poorest type of cut-outs. Incidentally, as the paper now stands it contains an omission in the statement of the equation for the induced drag.

When the characteristics of any monoplane wing are to be calculated with some degree of precision, lifting-line airfoil theory is employed. The procedure consists of obtaining the span load distribution and its correlated downwash distribution by an application of the vortex theory and then, from a knowledge of the section characteristics and spatial arrangement of the profiles across the span, calculating the lift, induced-drag, profile-drag, and moment coefficients.

The Fourier series method of analysis as expounded by Glauert (reference 6, p. 138) is commonly relied on to obtain the characteristics of monoplane wings. However, when the span load distribution is deformed, as it is for a cut-out wing, the number of coefficients in the series required to define reasonably the load or downwash distribution increases rapidly and this method of applying airfoil theory becomes too involved for practical uses.

For problems in which the use of the Fourier series appears to be undesirable, the span loading and downwash distribution may be obtained by employing the method of successive approximation developed in reference 7. This method, stated briefly, is as follows: From consideration of the character of the wing, for any given angle of attack, some curve is drawn that is

thought to approximate the true span load distribution. The downwash for a number of stations along the span is then found from this assumed loading curve. The effective angles of attack at these stations are now obtained and, from the section characteristics of the profiles at those stations, the lift coefficient for each station is determined. A check span load distribution is thus derived, and from consideration of the new load distribution, together with the assumed curve from which it was obtained, a more nearly accurate span load distribution can be estimated. This process, continued through successive approximations until the check distribution agrees with the assumed curve from which it was derived, will arrive at an approximately true span load distribution curve with its correlated downwash distribution. In the derivation of the check distribution, the following equation for the downwash  $w_1$  at any station  $y_1$  may be employed (reference 6):

$$w_1 = \frac{1}{4\pi} \int_{-s}^s \frac{dK}{y_1 - y} dy$$

where  $K$  is the circulation around any profile along the span,  $y$  is its distance out from the center line along the span, and  $s$  is the length of the semispan. This equation can be put in a more convenient form for general use by substituting  $\frac{C_L}{2} cV$  for  $K$  where  $V$  is the free-stream velocity and  $c$  is the chord length at any station; thus:

$$\frac{w_1}{V} = \frac{c_r}{8\pi} \int_{-s}^s \frac{dC_L'}{y_1 - y} dy$$

where  $c_r$  is the reference chord and  $C_L'$  is therefore  $C_L \frac{c}{c_r}$ . The integration of this expression across the span can be performed graphically except for the region within some small distance  $\Delta$  to either side of the station  $y_1$  which cannot be thus evaluated because the integrand approaches infinity as  $y$  approaches  $y_1$ . The evaluation of the portion of the integral between the limits  $y_1 - \Delta$  and  $y_1 + \Delta$  may, however, be performed analytically by assuming the span-loading curve between those limits defined by the equation  $C_L' = A + By + Cy^2$  and expressing the constants  $A$ ,  $B$ , and  $C$  in terms of the slopes of the span load distribution curve at  $y = y_1 - \Delta$  and  $y = y_1 + \Delta$ . Then, the portion of the integral between the limits  $y = y_1 - \Delta$  and  $y = y_1 + \Delta$  becomes

$$\frac{c_r}{8\pi} \left[ \left( \frac{dC_L'}{dy} \right)_{y_1 - \Delta} - \left( \frac{dC_L'}{dy} \right)_{y_1 + \Delta} \right]$$

#### EXAMPLE OF THE PREDICTION OF CUT-OUT EFFECTS

In order to illustrate the application of the information presented in this report, the characteristics

of a wing with a cut-out were estimated from the considerations discussed and calculated for one angle by the method of successive approximation. The results predicted were then compared with the test results of a model of the wing. It is obviously necessary to calculate the characteristics for only one angle in the range preceding the burbling of the center section in order to be able to evaluate readily the characteristics for that whole range.

Test of cut-out wing model.—The wing employed for this example was a standard 5- by 30-inch duralumin airfoil model of N.A.C.A. 0012 profile (reference 8). It was prepared with a central-section cut-out patterned in plan form directly after the upper wing of the Vought Corsair, model 03U-1. (See figs. 1 and 2.) The chords of the profiles along the span were all in one plane and the same form of profile was maintained over the cut-out portion,

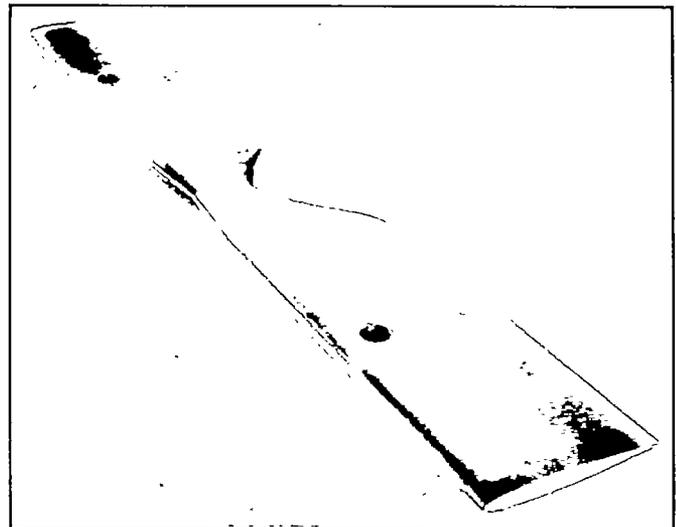


FIGURE 1.—Cut-out wing model.

which was faired to an N.A.C.A. 0015 profile at the center line for considerations of strength. As the characteristics of the N.A.C.A. 0015 are so very nearly the same as those of the N.A.C.A. 0012, it was assumed that N.A.C.A. 0012 profiles were kept over the entire span. This model was tested in the N.A.C.A. variable-density wind tunnel at a Reynolds Number of 3,160,000. A description of the variable-density wind tunnel and of the methods employed for testing is given in reference 9. The test was performed in the usual manner, except that two stings were employed to minimize set-up interference on the cut-out portion of the wing.

The test results are presented in figure 2 where  $C_L$ ,  $C_D$ ,  $L/D$ , and  $c_p$  curves are plotted against angle of attack  $\alpha$ . These curves are corrected for tunnel-wall effects and are compared with those of the normal rectangular N.A.C.A. 0012. Curves are also given of the effective-profile-drag coefficient,  $C_{D_e}$ , and the

moment coefficient about the original line of quarter-chord points,  $C_{m,c/4}$  plotted against  $C_L$ . The effective profile-drag coefficient  $C_{D_e}$  is the total drag coefficient  $C_D$  minus  $\frac{C_L^2 S}{\pi b^2}$ , the induced-drag coefficient for a wing of the same geometric aspect ratio but elliptically loaded. The effective profile drag, therefore, includes the additional induced drag due to the departure of the wing's span loading from the elliptical form and is thus a measure of the effect of wing deformation. The characteristics of the wing with the cut-out are given in figure 2 as based both on the original uncut-out plan form—thus including the total effects of the cut-out—

Likewise the burble should start earlier. The rear cut-out being considerably larger than the front one, the aerodynamic center of the wing should be shifted forward. Consideration of the test results presented in figure 2 checks these predictions.

Calculation of the aerodynamic characteristics of the cut-out wing.—Cut-outs may be compared and their effects estimated qualitatively as in the preceding discussion. However, once a cut-out is chosen it is desirable to calculate the characteristics of the cut-out wing. The characteristics of the wing in this example, based on the original plan form, were calculated for an angle of attack of  $8^\circ$  from zero lift, which would approx-

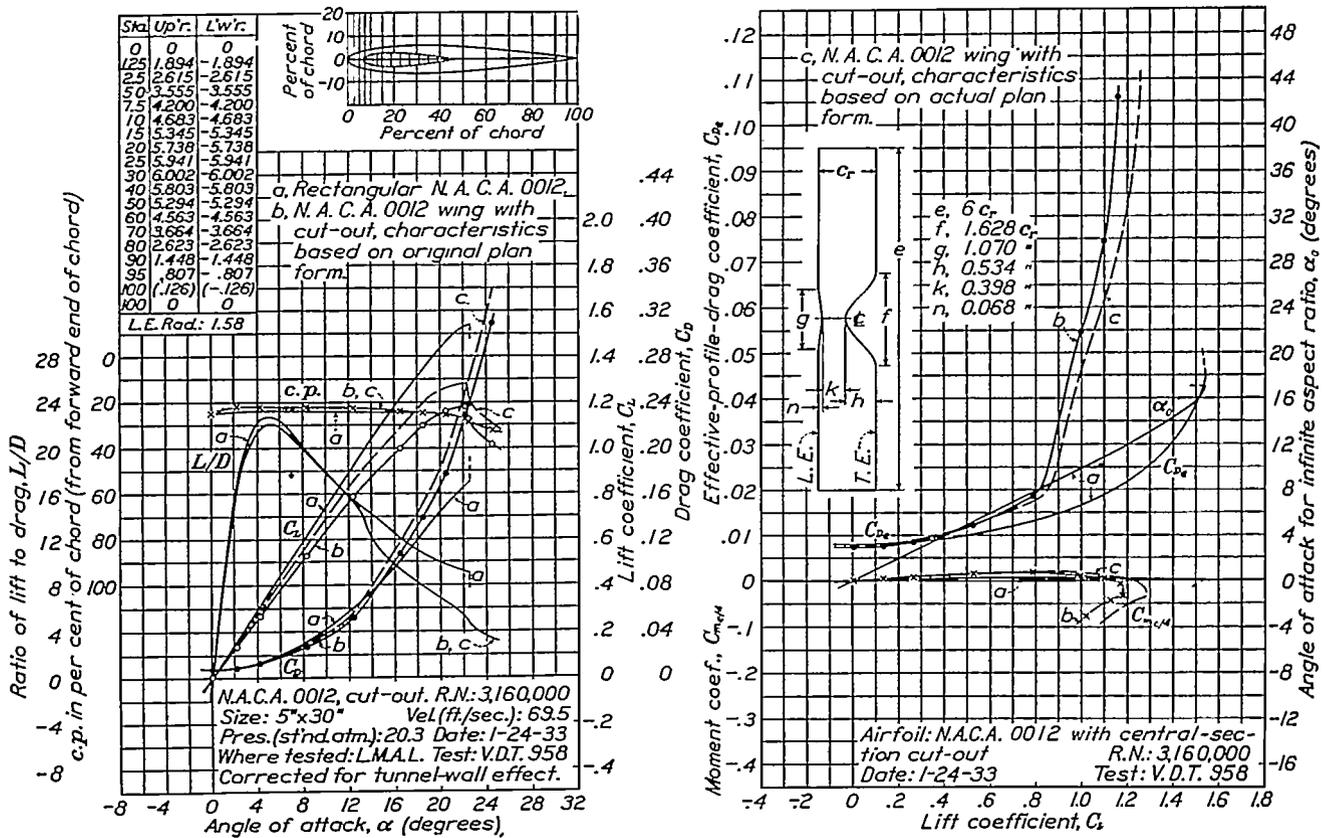


FIGURE 2.—Characteristics of a wing with central-section cut-out.

and on the actual plan form, thus presenting the true characteristics of such a wing.

Prediction of the effects of the cut-out.—From the qualitative discussion of the effects of wing cut-outs, the effects of the cut-out on the characteristics of this wing model can be predicted by consideration of its design. As the cut-out portion of the wing is fair, untwisted, and of the same profile, the total drag at zero lift may be expected to be reduced proportionately to area of cut-out. The induced drag would be expected to be greatly increased, and the adverse effects on the lift-curve slope and on maximum lift would be predicted to be greater than proportional to the size of the cut-out because of the induced interference.

imate the climbing attitude. In this calculation, the span load distribution and its correlated downwash distribution were obtained by the method of successive approximation. Figure 3 shows the successive steps undergone in arriving at the final acceptable span loading. Figure 3 also shows, for comparison, the downwash distribution obtained by using the Fourier series employing six coefficients. With this number of coefficients the distribution obtained probably cannot be relied upon to give a satisfactory approximation.

The values of  $C_L$  and  $C_{D_i}$  for the wing were obtained from the span load and the downwash distributions by graphical integration of the following equations:

$$C_L = \frac{c_r}{S} \int_{-s}^s C_L' dy$$

$$C_{D_i} = \frac{c_r}{S} \int_{-s}^s \frac{w}{V} C_L' dy$$

where  $S$  is the area on which the coefficients are based.  $C_{D_o}$  was similarly obtained from the downwash distribution and the section characteristics of the profiles (assumed to be all N.A.C.A. 0012) along the span.

$$C_{D_o} = \frac{c_r}{S} \int_{-s}^s C_{D_o}' dy$$

where  $C_{D_o}'$  is the profile-drag coefficient at any section multiplied by  $\frac{c}{c_r}$ . The pitching-moment coefficient for the wing about the original line of quarter-chord points,  $C_{m_{c/4}}$ , was calculated from the design of the cut-out wing, the downwash distribution, and the section characteristics of the profiles along the span by graphical integration of the following expression:

$$C_{m_{c/4}} = \frac{1}{S} \left[ \int_{-s}^s C_N' h dy + \int_{-s}^s C_{m_{c/4}}' c dy \right]$$

where  $C_N'$  is  $C_N \frac{c}{c_r}$  at any section along the span,  $h$  is the distance in the chord direction of the quarter-chord point of that section from the original line of quarter-chord points, and  $C_{m_{c/4}}'$  is  $C_{m_{c/4}} \frac{c}{c_r}$  at any section.

The characteristics of the cut-out wing based on the original uncut-out plan form as thus calculated for an angle of attack from zero lift,  $\alpha_a$ , of  $8^\circ$  were:  $C_L = 0.517$ ,  $C_{D_i} = 0.0182$ ,  $C_{D_o} = 0.0089$ ,  $C_D = 0.0271$ , and  $C_{m_{c/4}} = 0.011$ . These predicted values check well with the test results:  $C_L = 0.512$ ,  $C_D = 0.0259$ , and  $C_{m_{c/4}} = 0.014$ .

The induced-drag correction factor  $\sigma$  applying to the range below the burble of the center section, can be calculated from the induced-drag equation:

$$C_{D_i} = \frac{C_L^2}{\pi A} [1 + \sigma]$$

Using the values of  $C_L$  and  $C_{D_i}$  just calculated,  $\sigma$  based on the original plan form was computed to be 0.285. As  $\sigma$  equals zero for an elliptical wing, this cut-out rectangular plan form has the same effect as a 22 percent reduction in aspect ratio of the equivalent elliptical wing. The rectangular plan form itself for this instance is equivalent to but a 5-percent reduction. Similarly, the lift-curve slope can be obtained. The expression  $\frac{dC_L}{d\alpha}$  is approximately equal to  $\frac{C_L \alpha_a}{\alpha_a}$  for the range of lifts below the burble of the center section. Then, for

this example,  $\alpha_a$  being  $8^\circ$ ,  $\frac{dC_L}{d\alpha} = \frac{0.517}{8} = 0.065$  calculated as compared with 0.064 from the test results.

The point at which the center section stalls, causing the lift curve to depart suddenly from an approximately straight line and the drag curves to rise suddenly, can be calculated from the fact that the downwash distri-

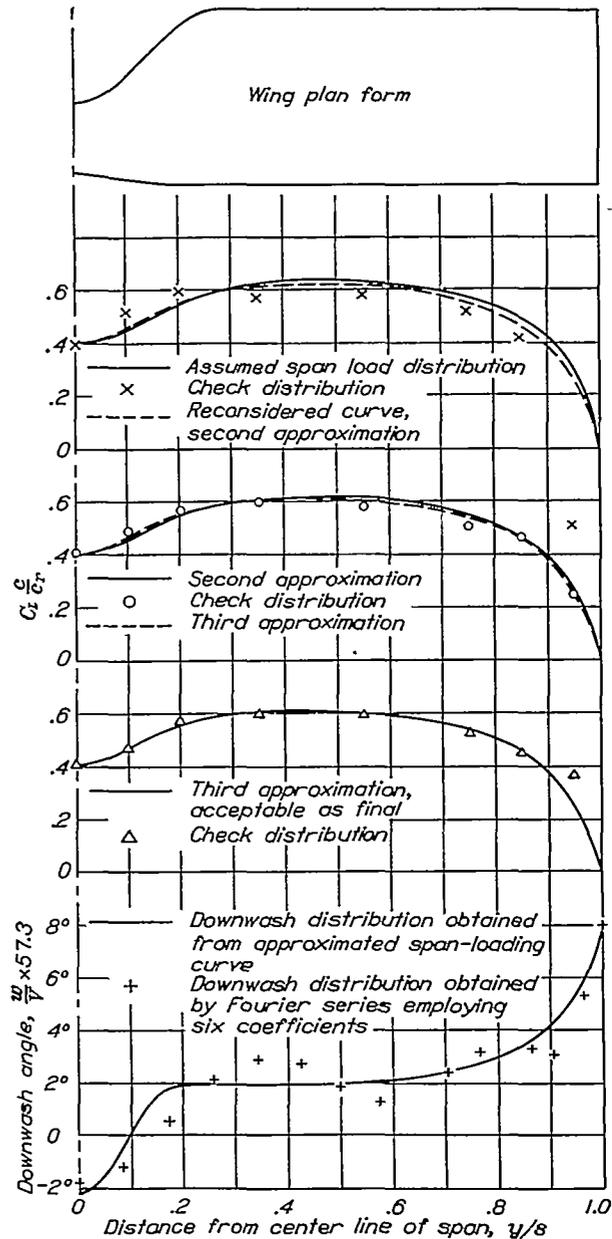


FIGURE 3.—Approximation of span load distribution for a wing with cut-out.  $\alpha_a = 8^\circ$ .

bution has the same proportionate shape throughout the range preceding the start of the burble. From figure 2 it is seen that the N.A.C.A. 0012 profile stalls at an  $\alpha_o$  of  $17^\circ$  from zero lift. Figure 3 shows that for an  $\alpha_a$  of  $8^\circ$  and a calculated  $C_L$  of 0.517, the effective angle of attack at the center is  $8^\circ$  plus an upwash of  $2.2^\circ$ , or  $10.2^\circ$ . The  $C_L$  of the wing at which the center section stalls can now be quickly found; for

$$\frac{C_L}{0.517} = \frac{17^\circ}{10.2^\circ} \text{ or } C_L = 0.862.$$

The corresponding  $\alpha_a$  will be  $\frac{C_L}{0.065}$  or  $13.3^\circ$ . These calculated results, however, must be considered approximate because the downwash distribution is no longer exactly proportionate when any section of the wing is acting above the straight portion of the lift curve. The agreement with test results is fairly close, as can be seen from figure 2, where the drag curves start to increase suddenly for a  $C_L$  between 0.8 and 0.9 and for an  $\alpha_a$  between  $12^\circ$  and  $14^\circ$ .

#### CONCLUSIONS

1. The adverse effects of a cut-out on wing characteristics are mainly due to the induced interference it produces. Extending a cut-out in the chord direction has a greater effect than extending it along the span, and unfairness in profile around the leading edge of the cut-out sections greatly increases the interference.

2. Lifting-line airfoil theory can be successfully employed to calculate the characteristics of a wing with a cut-out when the section characteristics of the profiles along the span are known. For such problems, the method of successive approximation for obtaining the span load distribution is considered satisfactory.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,  
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,  
LANGLEY FIELD, VA., November 4, 1933.

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