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AN ANALYTICAL AND EXPERIMENTAL STUDY OF THE EFFECT OF PERIODIC BLADE TWIST ON THE THRUST, TORQUE, AND FLAPPING MOTION OF AN AUTOGIRO ROTOR

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SUMMARY

An analysis is made of the influence on autogiro rotor characteristics of a periodic blade twist that varies with the azimuth position of the rotor blade and the results are compared with experimental data. The analysis expresses the influence of this type of twist upon the thrust, torque, and flapping motion of the rotor. The check against experimental data shows that the periodic twist has a pronounced influence on the flapping motion and that this influence is accurately predicted by the analysis. The influence of the twist upon the thrust and torque could be demonstrated only indirectly, but its importance is indicated.

INTRODUCTION

The resultant of the air forces and the mass reactions on an autogiro rotor blade produces a couple tending to twist the blade unless the chordwise center of gravity of the blade and the center of pressure of the air forces are coincident. This fact has been known for some time, but it has not been generally realized that, except in particular cases, the resultant twist is periodic and is a function of the angular position of the blade in azimuth. The periodic twist may be of a magnitude comparable with or even exceeding the pitch setting of the rotor, which demonstrates the necessity of including it as a factor in the analysis of autogiro-rotor characteristics.

It is the purpose of this paper to present an analysis of the periodic twist and to support the validity of the analysis by a comparison of predicted results with experimental information obtained from flight tests of a direct-control wingless autogiro. The scope of the paper will be limited to a study of the influence of a known periodic twist upon the thrust, flapping motion, and torque of a rotor; the effect of such a twist upon rotor vibrations and stability and the problem of predicting the twist will be treated in a subsequent report.

ANALYSIS

It has been experimentally shown in previously unpublished data that, except in special cases, the air forces acting on a rotor blade cause a twist of the blade and a consequent change in the blade pitch angle. The twist is not constant but is a function of blade azimuth angle because of the variation of the air forces on the blade with this angle. In the following analysis the basic equations expressing the rotor characteristics will be generalized to include the factor of periodic twist. The notation of reference 1 will be used throughout this paper except for minor changes; for convenience, the list of symbols is appended at the end of this section.

The following additional notation is used:

$$r=xR \tag{1}$$

$$dr=Rdx \tag{2}$$

The problem is now the solution of the equations for the autogiro rotor when the pitch angle θ has the form

$$\theta=\theta_0+x\theta_1+x\epsilon_0+x\epsilon_1 \cos \psi+x\eta_1 \sin \psi+x\epsilon_2 \cos 2\psi +x\eta_2 \sin 2\psi+ \ldots \tag{3}$$

where ϵ_n and η_n are coefficients descriptive of θ.

It will be noted that an assumption is here made concerning the distribution of the twist along the radius; equation (3) shows that a linear distribution, starting from zero at the blade hub, has been used. Actually the twist reaches zero just outboard of the vertical pin and is not, in general, absolutely linear from there to the tip; the assumption used, however, does not introduce a serious error and is considered justified for its simplicity.

It is obvious that the expressions of reference 1 for interference flow v, angle of attack α, blade flapping angle β, and the dynamic equation of flapping are unaltered; they are

$$v=\frac{C_T\Omega R}{2(\lambda^2+\mu^2)^{\frac{1}{2}}} \tag{4}$$

$$\tan \alpha=\frac{\lambda}{\mu}+\frac{C_T}{2\mu(\lambda^2+\mu^2)^{\frac{1}{2}}} \tag{5}$$

$$\beta=a_0-a_1 \cos \psi-b_1 \sin \psi-a_2 \cos 2\psi -b_2 \sin 2\psi- \ldots . \tag{6}$$

$$I_1\left(\frac{d^2\beta}{dt^2}+\Omega^2\beta\right)=M_T-M_W \tag{7}$$

Using the notation of equation (1), the velocity components at the rotor blade are:

$$\frac{U_T}{\Omega R}=u_T=x+\mu \sin \psi \tag{8}$$

$$\frac{U_P}{\Omega R}=u_P=\lambda+\frac{1}{2}\mu a_1+\left(-\mu a_0+xb_1+\frac{1}{2}\mu a_2\right) \cos \psi + \left(-xa_1+\frac{1}{2}\mu b_2\right)\sin \psi+\left(\frac{1}{2}\mu a_1+2xb_2\right)\cos 2\psi + \left(\frac{1}{2}\mu b_1-2xa_2\right)\sin 2\psi+\frac{1}{2}\mu a_2 \cos 3\psi+\frac{1}{2}\mu b_2 \sin 3\psi \tag{9}$$

Also

$$u_T^2=x^2+\frac{1}{2}\mu^2+2\mu x \sin \psi-\frac{1}{2}\mu^2 \cos 2\psi \tag{10}$$

$$\begin{aligned} u_Tu_P=&x\lambda+\frac{1}{4}\mu^2b_2+\left(-\mu x a_0+b_1\left[x^2+\frac{1}{4}\mu^2\right]-\frac{1}{2}\mu x a_2\right)\cos \psi \\ &+\left(\mu\lambda-a_1\left[x^2-\frac{1}{4}\mu^2\right]-\frac{1}{2}\mu x b_2\right)\sin\psi \\ &+(\mu x a_1+2x^2b_2)\cos 2\psi+\left(-\frac{1}{2}\mu^2a_0+\mu x b_1-2x^2a_2\right)\sin 2\psi \\ &+\left(-\frac{1}{4}\mu^2b_1+\frac{3}{2}\mu x a_2\right)\cos 3\psi+\left(\frac{1}{4}\mu^2a_1+\frac{3}{2}\mu x b_2\right)\sin 3\psi \\ &-\frac{1}{4}\mu^2b_2\cos 4\psi+\frac{1}{4}\mu^2a_2\sin 4\psi \end{aligned} \tag{11}$$

The rotor thrust is calculated upon the assumptions that the elemental force on the blade lies in a plane perpendicular to the blade-span axis and that the force depends only on the velocity in that plane; it is further assumed that u_P is small compared with u_T, so that the angle φ between u_T and the resultant velocity $(u_T^2+u_P^2)^{\frac{1}{2}}$ may be equated to its sine and tangent. As an approximate allowance for tip loss, it will be assumed that the thrust becomes zero at a radius $x=B$ where it is arbitrarily assumed that $B=1-\frac{c}{2R}$. Then

$$T=\frac{b}{2\pi}\int_0^{2\pi}d\psi\int_0^B\frac{1}{2}\rho c\Omega^2R^3u_T^2C_Ldx \tag{12}$$

It is further assumed that C_L is a linear function of the blade-element angle of attack α_r, which is, of course, accurate below the stall; then

$$C_L=a\alpha_r \tag{13}$$

$$\alpha_r=\theta+\varphi \tag{14}$$

Errors are introduced by the assumption that $C_L=a\alpha_r$; however, a graphical evaluation of the thrust made without this assumption and using a curve of C_L against α derived from wind-tunnel tests disclosed that the error so introduced was negligible. Another error in the thrust expression exists where u_T is negative; this error can be approximately nullified by the following correction. When u_T is negative, the normal expression for the blade-element angle of attack must be altered to

$$\alpha_r'=-\theta-\varphi \tag{15}$$

and equation (15) must be used in the part of the disk bounded by $x=-\mu \sin \psi$ and $x=0$ and by $\psi=\pi$ and $\psi=2\pi$. The expression for the thrust is now, after substituting for C_L, θ, and $\varphi=u_P/u_T$,

$$\begin{aligned} T=&\frac{1}{2}\rho ca\Omega^2R^3\frac{b}{2\pi}\int_0^{2\pi}d\psi\int_0^B u_T^2(\theta_0+x\theta_1+x\epsilon_0+x\epsilon_1 \cos \psi \\ &+x\eta_1 \sin \psi+x\epsilon_2 \cos 2\psi+x\eta_2 \sin 2\psi+\ \ldots\ )dx \\ &+\frac{1}{2}\rho ca\Omega^2R^3\frac{b}{2\pi}\int_0^{2\pi}d\psi\int_0^B u_Tu_Pdx \\ &-\frac{1}{2}\rho ca\Omega^2R^3\frac{b}{\pi}\int_\pi^{2\pi}d\psi\int_0^{-\mu\sin\psi} u_T^2(\theta_0+x\theta_1+x\epsilon_0+x\epsilon_1 \cos \psi \\ &+x\eta_1 \sin \psi+x\epsilon_2 \cos 2\psi+x\eta_2 \sin 2\psi+\ \ldots\ )dx \\ &-\frac{1}{2}\rho ca\Omega^2R^3\frac{b}{\pi}\int_\pi^{2\pi}d\psi\int_0^{-\mu\sin\psi} u_Tu_Pdx \end{aligned} \tag{16}$$

$$\begin{aligned} T=&\frac{1}{2}bc\rho a\Omega^2R^3\left\{\frac{1}{2}\lambda\left(B^2+\frac{1}{2}\mu^2\right)+\theta_0\left(\frac{1}{3}B^3+\frac{1}{2}\mu^2B-\frac{4}{9\pi}\mu^3\right)\right. \\ &+\theta_1\left(\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2-\frac{1}{32}\mu^4\right)+\frac{1}{8}\mu^3a_1+\frac{1}{4}\mu^2b_2B \\ &\left.+\epsilon_0\left(\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2-\frac{1}{32}\mu^4\right)+\frac{1}{3}\mu\eta_1B^3-\frac{1}{8}\mu^2\epsilon_2B^2\right\} \end{aligned} \tag{17}$$

It has already been shown (reference 1) that a_n and b_n are of the order μ^n; it can similarly be shown that ϵ_n and η_n are of the order μ^n.

The expression for thrust has been integrated upon the assumption, which experience has shown to be valid, that terms of higher order in μ than the fourth are negligible. This same assumption will be used throughout the remainder of this analysis.

The change in thrust caused by twist is

$$\Delta T=\frac{1}{2}bc\rho a\Omega^2R^3\left\{\epsilon_0\left(\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2-\frac{1}{32}\mu^4\right)+\frac{1}{3}\mu\eta_1B^3-\frac{1}{8}\mu^2\epsilon_2B^2\right\} \tag{18}$$

The thrust moment M_T is, from reference 1,

$$M_T=\int_0^B\frac{1}{2}\rho ca\Omega^2R^4\{\theta u_T^2+u_Tu_P\}xdx -2\left.\int_0^{-\mu \sin \psi}\frac{1}{2}\rho ca\Omega^2R^4\{\theta u_T^2+u_Tu_P\}xdx\right]_\pi^{2\pi} \tag{19}$$

where the second integral is added to thrust moment only in the interval $\psi=\pi$ to $\psi=2\pi$. The integrated value of M_T is

$$\begin{aligned}
\frac{M_T}{\frac{1}{2}\rho ca\Omega^2R^4}=&\left\{\frac{1}{3}\lambda B^3+0.080\mu^3\lambda+\frac{1}{4}\theta_0\left(B^4+\mu^2B^2-\frac{1}{8}\mu^4\right)\right.\\
&+\theta_1\left(\frac{1}{5}B^5+\frac{1}{6}\mu^2B^3\right)+\frac{1}{8}\mu^2b_2B^2+\epsilon_0\left(\frac{1}{5}B^5+\frac{1}{6}\mu^2B^3\right)\\
&\left.+\frac{1}{4}\mu\eta_1B^4-\frac{1}{12}\mu^2\epsilon_2B^3\right\}\\
&+\left\{\frac{1}{2}\mu\lambda B^2-\frac{1}{8}\mu^3\lambda+\frac{2}{3}\mu\theta_0B^3+0.053\mu^4\theta_0+\frac{1}{2}\mu\theta_1B^4\right.\\
&-a_1\left(\frac{1}{4}B^4-\frac{1}{8}\mu^2B^2\right)-\frac{1}{6}\mu b_2B^3+\frac{1}{2}\mu\epsilon_0B^4\\
&\left.+\eta_1\left(\frac{1}{5}B^5+\frac{1}{4}\mu^2B^3\right)-\frac{1}{4}\mu\epsilon_2B^4\right\}\sin\psi\\
&+\left\{-\frac{1}{3}\mu a_0B^3-0.035\mu^4a_0+b_1\left(\frac{1}{4}B^4+\frac{1}{8}\mu^2B^2\right)\right.\\
&\left.-\frac{1}{6}\mu a_2B^3+\epsilon_1\left(\frac{1}{5}B^5+\frac{1}{12}\mu^2B^3\right)+\frac{1}{4}\mu\eta_2B^4\right\}\cos\psi\\
&+\left\{-\frac{1}{4}\mu^2a_0B^2+\frac{1}{24}\mu^4a_0+\frac{1}{3}\mu b_1B^3-\frac{1}{2}a_2B^4+\frac{1}{4}\mu\epsilon_1B^4\right.\\
&\left.+\eta_2\left(\frac{1}{5}B^5+\frac{1}{6}\mu^2B^3\right)-\frac{1}{4}\mu\epsilon_3B^4\right\}\sin 2\psi\\
&+\left\{-0.053\mu^3\lambda-\frac{1}{4}\mu^2\theta_0B^2+\frac{1}{32}\mu^4\theta_0-\frac{1}{6}\mu^2\theta_1B^3\right.\\
&+\frac{1}{3}\mu a_1B^3+\frac{1}{2}b_2B^4-\frac{1}{6}\mu^2\epsilon_0B^3-\frac{1}{4}\mu\eta_1B^4\\
&\left.+\epsilon_2\left(\frac{1}{5}B^5+\frac{1}{6}\mu^2B^3\right)+\frac{1}{4}\mu\eta_3B^4\right\}\cos 2\psi \qquad (20)
\end{aligned}$$

Equation (7) can now be expanded into

$$I_1\Omega^2(a_0+3a_2\cos 2\psi+3b_2\sin 2\psi)=M_T-M_W \qquad (21)$$

The coefficients of the flapping angle β can now be obtained by substituting for M_T in equation (21) and equating the coefficients of similar trigonometric functions; then, letting $\gamma=c\rho aR^4/I_1$

$$\begin{aligned}
a_0=&\frac{1}{2}\gamma\left\{\frac{1}{3}\lambda B^3+0.080\mu^3\lambda+\frac{1}{4}\theta_0\left(B^4+\mu^2B^2-\frac{1}{8}\mu^4\right)\right.\\
&+\theta_1\left(\frac{1}{5}B^5+\frac{1}{6}\mu^2B^3\right)+\frac{1}{8}\mu^2b_2B^2+\epsilon_0\left(\frac{1}{5}B^5+\frac{1}{6}\mu^2B^3\right)\\
&\left.+\frac{1}{4}\mu\eta_1B^4-\frac{1}{12}\mu^2\epsilon_2B^3\right\}-\frac{M_W}{I_1\Omega^2} \qquad (22)
\end{aligned}$$

$$\begin{aligned}
a_1=&\frac{2\mu}{B^4-\frac{1}{2}\mu^2B^2}\left\{\lambda\left(B^2-\frac{1}{4}\mu^2\right)+\theta_0\left(\frac{4}{3}B^3+0.106\mu^3\right)+\theta_1B^4\right.\\
&\left.-\frac{1}{3}b_2B^3+\epsilon_0B^4+\frac{\eta_1}{\mu}\left(\frac{2}{5}B^5+\frac{1}{2}\mu^2B^3\right)-\frac{1}{2}\epsilon_2B^4\right\} \qquad (23)
\end{aligned}$$

$$\begin{aligned}
b_1=&\frac{4\mu}{B^4+\frac{1}{2}\mu^2B^2}\left\{a_0\left(\frac{1}{3}B^3+0.035\mu^3\right)+\frac{1}{6}a_2B^3\right.\\
&\left.-\frac{\epsilon_1}{\mu}\left(\frac{1}{5}B^5+\frac{1}{12}\mu^2B^3\right)-\frac{1}{4}\eta_2B^4\right\} \qquad (24)
\end{aligned}$$

The expressions for a_2 and b_2 must be expanded in powers of μ before solution is possible; it will be shown later that expansion to the order of μ^2 is sufficient to express the thrust and torque to the order of μ^4. To this order the equations for a_2 and b_2 are

$$\frac{6}{\gamma}b_2+\frac{1}{2}a_2B^4=\mu^2\left\{-\frac{1}{4}a_0B^2+\frac{1}{3}\frac{b_1}{\mu}B^3+\frac{1}{4}\frac{\epsilon_1}{\mu}B^4+\frac{1}{5}\frac{\eta_2}{\mu^2}B^5\right\} \qquad (25)$$

$$\begin{aligned}
\frac{6}{\gamma}a_2-\frac{1}{2}b_2B^4=&\mu^2\left\{-\frac{1}{4}\theta_0B^2-\frac{1}{6}\theta_1B^3+\frac{1}{3}\frac{a_1}{\mu}B^3-\frac{1}{6}\epsilon_0B^3\right.\\
&\left.-\frac{1}{4}\frac{\eta_1}{\mu}B^4+\frac{1}{5}\frac{\epsilon_2}{\mu^2}B^5\right\} \qquad (26)
\end{aligned}$$

Equations (25) and (26) may be solved for a_2 and b_2 after substituting for a_0, a_1, and b_1 to the appropriate order of μ; then

$$\begin{aligned}
a_2=&\frac{\mu^2\gamma}{\gamma^2B^8+144}\left\{\lambda B\left(16+\frac{7\gamma^2B^8}{108}\right)+\theta_0B^2\left(\frac{46}{3}+\frac{7\gamma^2B^8}{144}\right)\right.\\
&+\theta_1B^3\left(12+\frac{7\gamma^2B^8}{180}\right)+\epsilon_0B^3\left(12+\frac{7\gamma^2B^8}{180}\right)\\
&\left.-\frac{1}{30}\frac{\epsilon_1}{\mu}\gamma B^8+\frac{2}{5}\frac{\eta_1}{\mu}B^4+\frac{24}{5}\frac{\epsilon_2}{\mu^2}B^5+\frac{2}{5}\frac{\eta_2}{\mu^2}\gamma B^9\right\} \qquad (27)
\end{aligned}$$

$$\begin{aligned}
b_2=&\frac{-\mu^2\gamma^2}{\gamma^2B^8+144}\left\{\frac{5}{9}\lambda B^5+\frac{25}{36}\theta_0B^6+\frac{8}{15}\theta_1B^7+\frac{8}{15}\epsilon_0B^7\right.\\
&\left.+\frac{2}{5\gamma}\frac{\epsilon_1}{\mu}B^4+\frac{1}{30}\frac{\eta_1}{\mu}B^8+\frac{2}{5}\frac{\epsilon_2}{\mu^2}B^9-\frac{24}{5\gamma}\frac{\eta_2}{\mu^2}B^5\right\} \qquad (28)
\end{aligned}$$

The changes in the expressions for the blade-motion coefficients arising from the twist are:

$$\Delta a_0=\frac{1}{2}\gamma\left\{\epsilon_0\left(\frac{1}{5}B^5+\frac{1}{6}\mu^2B^3\right)+\frac{1}{4}\mu\eta_1B^4-\frac{1}{12}\mu^2\epsilon_2B^3\right\} \qquad (29)$$

$$\Delta a_1=\frac{2\mu}{B^4-\frac{1}{2}\mu^2B^2}\left\{\epsilon_0B^4+\frac{\eta_1}{\mu}\left(\frac{2}{5}B^5+\frac{1}{2}\mu^2B^3\right)-\frac{1}{2}\epsilon_2B^4\right\} \qquad (30)$$

$$\begin{aligned}
\Delta b_1=&\frac{4\mu}{B^4+\frac{1}{2}\mu^2B^2}\left\{\gamma\epsilon_0\left(\frac{1}{30}B^8+\frac{1}{36}\mu^2B^6\right)\right.\\
&\left.-\frac{\epsilon_1}{\mu}\left(\frac{1}{5}B^5+\frac{1}{12}\mu^2B^3\right)+\frac{1}{24}\mu\gamma\eta_1B^7-\frac{1}{4}\eta_2B^4\right\} \qquad (31)
\end{aligned}$$

$$\begin{aligned}
\Delta a_2=&\frac{\mu^2\gamma}{\gamma^2B^8+144}\left\{\epsilon_0B^3\left(12+\frac{7\gamma^2B^8}{180}\right)-\frac{1}{30}\frac{\epsilon_1}{\mu}\gamma B^8+\frac{2}{5}\frac{\eta_1}{\mu}B^4\right.\\
&\left.+\frac{24}{5}\frac{\epsilon_2}{\mu^2}B^5+\frac{2}{5}\frac{\eta_2}{\mu^2}\gamma B^9\right\} \qquad (32)
\end{aligned}$$

$$\begin{aligned}
\Delta b_2=&\frac{-\mu^2\gamma^2}{\gamma^2B^8+144}\left\{\frac{8}{15}\epsilon_0B^7+\frac{2}{5\gamma}\frac{\epsilon_1}{\mu}B^4+\frac{1}{30}\frac{\eta_1}{\mu}B^8\right.\\
&\left.+\frac{2}{5}\frac{\epsilon_2}{\mu^2}B^9-\frac{24}{5\gamma}\frac{\eta_2}{\mu^2}B^5\right\} \qquad (33)
\end{aligned}$$

The air torque on the rotor blade is the sum of the accelerating torque arising from the lift elements and the decelerating torque arising from the drag elements. It has been assumed that the lift elements are zero between $x=B$ and $x=1$; it is thought reasonable, however, to assume that the drag exists over the entire blade. An average value of the blade-element drag coefficient δ will be used; this value is assumed constant with respect to the angle of attack. The torque Q, which must be zero by hypothesis, since the rotor is in a state of steady rotation, is then

$$Q=0=\frac{b}{2\pi}\int_0^{2\pi}d\psi\int_0^B\frac{1}{2}\rho c\Omega^2R^4u_T^2\varphi C_Lxdx \\ -\frac{b}{2\pi}\int_0^{2\pi}d\psi\int_0^1\frac{1}{2}\rho c\Omega^2R^4u_T^2\delta xdx \qquad (34)$$

This expression after substitution for θ, C_L, and μ integrates into

$$\begin{aligned}0=&\lambda^2\left(\frac{1}{2}B^2-\frac{1}{4}\mu^2\right)+\lambda\left(\frac{1}{3}\theta_0B^3+\frac{2}{9\pi}\mu^3\theta_0+\frac{1}{4}\theta_1B^4+\frac{1}{32}\mu^4\theta_1\right)\\ &+\mu\lambda a_1\left(\frac{1}{2}B^2-\frac{3}{8}\mu^2\right)+a_0^2\left(\frac{1}{4}\mu^2B^2-\frac{1}{16}\mu^4\right)-\frac{1}{3}\mu a_0b_1B^3\\ &+a_1^2\left(\frac{1}{8}B^4+\frac{3}{16}\mu^2B^2\right)+b_1^2\left(\frac{1}{8}B^4+\frac{1}{16}\mu^2B^2\right)\\ &-a_2\left(\frac{1}{4}\mu^2a_0B^2+\frac{1}{6}\mu b_1B^3\right)+\frac{1}{2}a_2^2B^4\\ &+b_2\left(\frac{1}{8}\mu^2\theta_0B^2+\frac{1}{12}\mu^2\theta_1B^3+\frac{1}{6}\mu a_1B^3\right)+\frac{1}{2}b_2^2B^4\\ &-\frac{\delta}{4a}\left(1+\mu^2-\frac{1}{8}\mu^4\right)+\epsilon_0\left(\frac{1}{4}\lambda B^4+\frac{1}{32}\mu^4\lambda+\frac{1}{12}\mu^2b_2B^3\right)\\ &+\frac{1}{2}\epsilon_1\left(-\frac{1}{4}\mu a_0B^4+b_1\left[\frac{1}{5}B^5+\frac{1}{12}\mu^2B^3\right]-\frac{1}{8}\mu a_2B^4\right)\\ &+\frac{1}{2}\eta_1\left(\frac{1}{3}\mu\lambda B^3-a_1\left[\frac{1}{5}B^5-\frac{1}{12}\mu^2B^3\right]-\frac{1}{8}\mu b_2B^4\right)\\ &+\frac{1}{2}\epsilon_2\left(\frac{1}{4}\mu a_1B^4+\frac{2}{5}b_2B^5\right)\\ &+\frac{1}{2}\eta_2\left(-\frac{1}{6}\mu^2a_0B^3+\frac{1}{4}\mu b_1B^4-\frac{2}{5}a_2B^5\right)\end{aligned} \qquad (35)$$

Equation (35) is a quadratic in λ with the coefficients of the quadratic dependent upon μ, γ, B, θ_0, θ_1, and the ϵ and η coefficients describing the periodic twist. The evaluation of equation (35) requires the substitution of known values of ϵ and η in the expressions for a_n and b_n. The scope of this study includes only the prediction of the effect of a known twist upon rotor characteristics; consequently, the solution of equation (35) is possible when the drag term $\frac{\delta}{4a}\left(1+\mu^2-\frac{1}{8}\mu^4\right)$ is known.

Examination of equations (17) and (35) shows that the thrust and torque are expressed to the order μ^4 if a_2 and b_2 are expressed to the order μ^2, inasmuch as ϵ_n and η_n are of the order μ^n. This last condition has been analytically proved but will not be included in this paper; the analysis of the ϵ and η coefficients of twist will be the subject of another paper.

LIST OF SYMBOLS

R, blade radius.
b, number of blades.
c, blade chord, feet.
r, radius of blade element.
x, r/R.
θ_0, blade pitch angle at hub, radians.
θ_1, difference between hub and tip pitch angles, radians.
ϵ_n, coefficient of cos $n\psi$ in expression for θ, radians.
η_n, coefficient of sin $n\psi$ in expression for θ, radians.
θ, instantaneous pitch angle, radians.
δ, mean profile-drag coefficient of rotor-blade airfoil section.
ψ, blade azimuth angle measured from down wind in direction of rotation, radians.
v, rotor induced velocity.
Ω, rotor angular velocity, $d\psi/dt$, radians per second.
$\lambda\Omega R$, speed of axial flow through rotor.
$\mu\Omega R$, component of forward speed in plane of disk, equal to V cos α where V is forward speed, feet per second.
β, blade flapping angle, radians.
a_n, coefficient of cos $n\psi$ in expression for β, radians.
b_n, coefficient of sin $n\psi$ in expression for β, radians.
I_1, mass moment of inertia of rotor blade about horizontal hinge.
α, rotor angle of attack, radians.
M_T, thrust moment about horizontal hinge.
M_W, weight moment of blade about horizontal hinge.
$u_T\Omega R$, velocity component at blade element perpendicular to blade span and parallel to rotor disk.
$u_P\Omega R$, velocity component at blade element perpendicular to blade span and to $u_T\Omega R$.
T, rotor thrust.
$C_T=\frac{T}{\rho\Omega^2\pi R^4}$
Q, rotor torque.
$C_Q=\frac{Q}{\rho\Omega^2\pi R^5}$
a, slope of curve of lift coefficient against angle of attack of blade airfoil section, in radian measure.
$\varphi=\tan^{-1}\frac{u_P}{u_T}$
α_r, blade-element angle of attack, radians.
$\gamma=\frac{c\rho aR^4}{I_1}$, mass constant of rotor blade.
$B=1-\frac{c}{2R}$, factor allowing for tip losses.

EXPERIMENT

Flight tests were made of a Kellett KD-1 autogiro having the following characteristics:

Gross weight, W	2,100 pounds.
Rotor radius, R	20.0 feet.
Number of blades, b	3.
Blade chord, c	1.00 foot.
Blade weight, w_b	61.5 pounds.
Blade-weight moment, M_W	482 pound-feet.
Blade moment of inertia, I_1	175 slug-feet.^{2}
Rotor solidity, $\sigma=\frac{bc}{\pi R}$	0.0478.
Blade mass constant, (sea level) $\gamma=\frac{c\rho aR^4}{I_1}$	12.74.
Blade airfoil section	Göttingen 606.
Pitch setting, θ_0 ($\theta_1=0$)	0.0960 radian.
Airfoil section moment coefficient, $C_{m_{a.c.}}$ (about aerodynamic center)	—0.056.
Blade chordwise center-of-gravity location, c_T (aft of aerodynamic center)	0.038 foot.

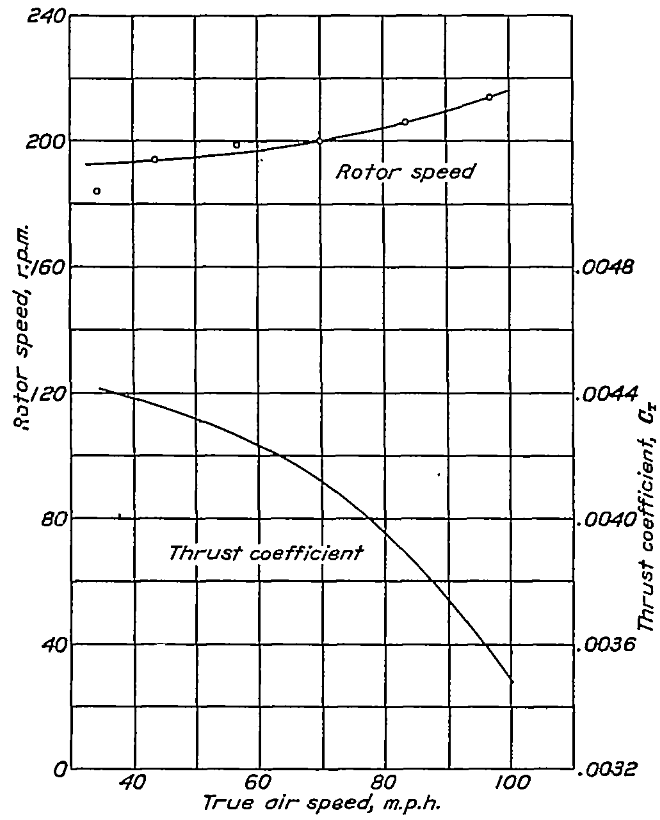

FIGURE 1.—Rotor speed and rotor thrust coefficient of KD-1 autogiro as measured in flight.

The flight tests included the measurement of rotor speed as a function of air speed (fig. 1) from which, since the autogiro had no fixed wing, the thrust coefficient could be calculated. Simultaneous measurements of the blade flapping angle and twist were made with a motion-picture camera mounted on and turning with the rotor hub. The data for twist are shown in figure 2. The flapping-angle data are represented by the experimental

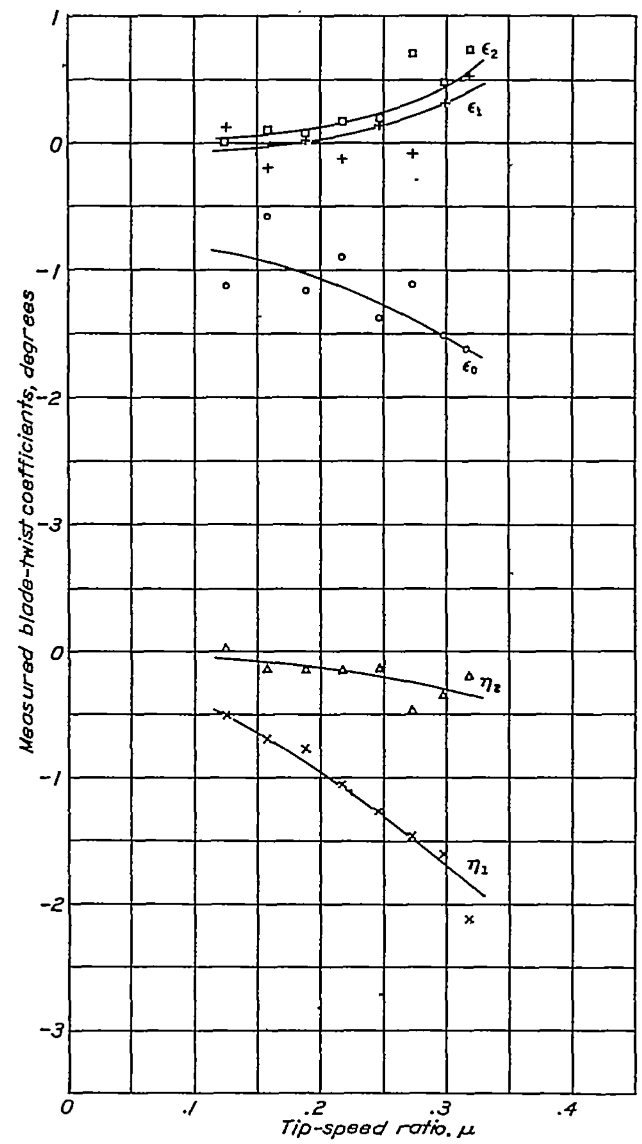

FIGURE 2.—Blade twist coefficients of KD-1 autogiro rotor as measured in flight.

points of figure 3. Both the flapping angle and the twist have been presented as the coefficients a_n, b_n, ϵ_n, and η_n of the expressions used in the previous section to represent β and θ.

The effect of periodic twist upon the thrust coefficient C_T was obtained by calculating the increment in C_T caused by the periodic twist and deducting the increment from the experimental value. The results are shown in figure 4.

In order to check the derived expressions for the effect of periodic twist upon the flapping motion, the inflow factor λ was calculated from the expression for the thrust (equation (17)) in which λ was the only unknown; the calculation was made using the experimental values of the periodic twist, and it was also made on the assumption that all ϵ and η coefficients except ϵ_0

were zero. The two results are shown in figure 5. These values of λ were then used in equations (22), (23), (24), (27), and (28) to calculate the blade flapping coefficients both with and without the effect of the periodic twist; these results are shown in figure 3, together with the measured values of the blade-flapping-angle coefficients. Since the measured values of the periodic twist were obtained from targets placed at 3/4 R, they

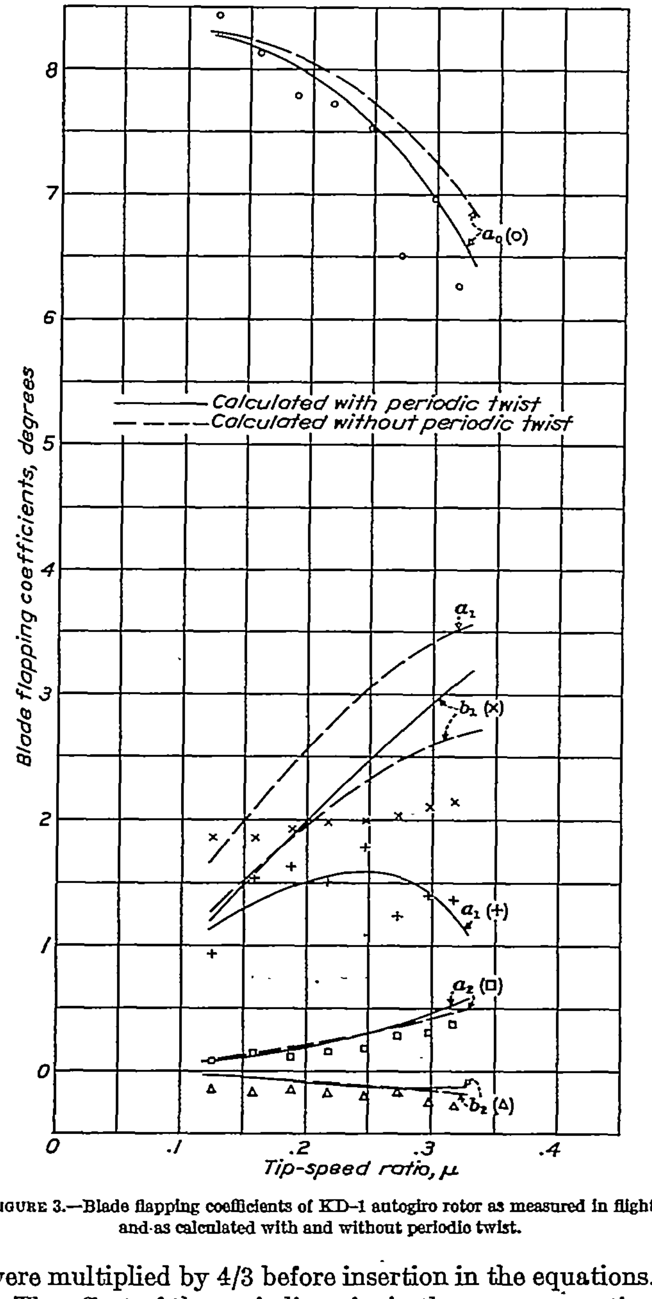

FIGURE 3.—Blade flapping coefficients of KD-1 autogiro rotor as measured in flight and as calculated with and without periodic twist.

were multiplied by 4/3 before insertion in the equations.

The effect of the periodic twist in the torque equation was estimated in the following manner: Known values of λ, a_n, b_n, ϵ_n, η_n, θ_0, and θ_1 were substituted in equation (35); the resultant expression was used to evaluate the remaining unknown term $\frac{\delta}{4a}\left(1+\mu^2-\frac{1}{8}\mu^4\right)$. The calculation was then repeated with the assumption that the periodic twist was zero and that λ rather than the δ term was unknown; the factors a_n, b_n, θ_0, θ_1, and ϵ_0 were assigned the same values in both calculations. The results of these calculations are shown in table I.

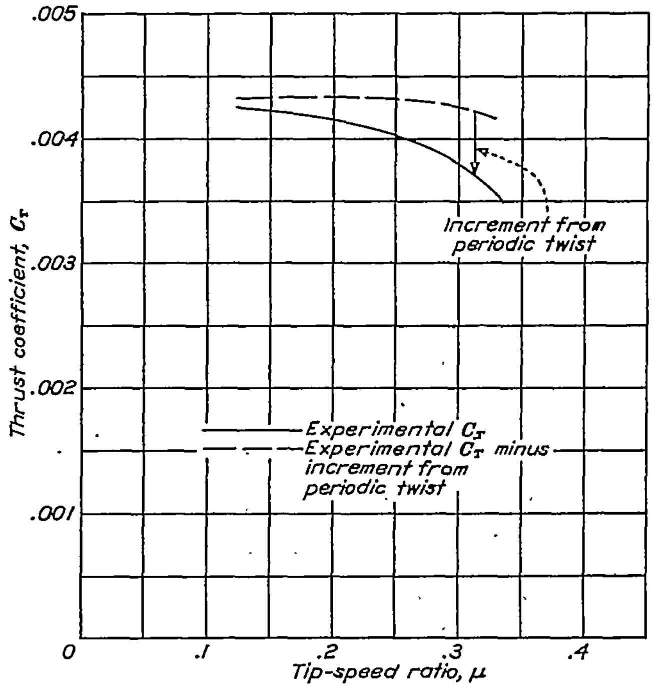

FIGURE 4.—Calculated periodic twist effect on thrust coefficient of KD-1 autogiro.

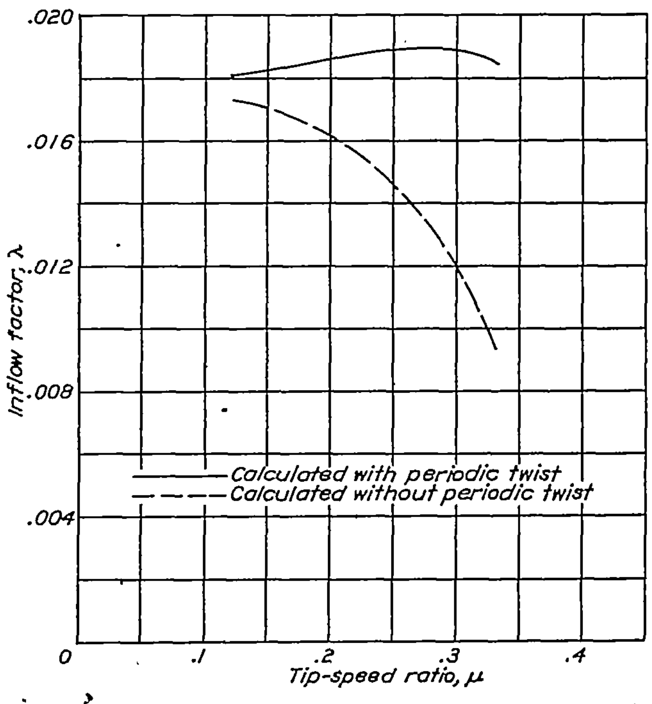

FIGURE 5.—Inflow factor λ of KD-1 autogiro as calculated from thrust coefficient with and without periodic twist effect.

DISCUSSION

The influence of periodic blade twist on the rotor characteristics is illustrated in figures, 3, 4, and 5. The data in figure 3 afford convincing proof not only of the validity of the twist analysis but also of the effect of periodic blade twist upon the rotor characteristics.

The data demonstrate that the type of twist developed in this rotor has a pronounced influence on the coning angle a_0 and on the flapping angle a_1. The influence on the lag angle b_1 and on the second harmonics a_2 and b_2 is considerably smaller. The agreement between the values of a_0 and a_1 calculated from the expressions including the periodic twist and the experimental values is quite good. The calculated values of the lag angle b_1 are in radical disagreement with experiment; this same condition was encountered in previous work (reference 1) and has been partially explained. In the reference it was shown that a variation of the rotor-induced velocity along the chord of the rotor disk had an appreciable effect upon the variation of b_1 with μ; an induced velocity increasing from the leading edge to the trailing edge increases b_1. Since this type of asymmetry exists (reference 2) and varies inversely in magnitude with μ, the evaluation of its influence upon b_1 would improve the qualitative agreement between the calculated and measured values.

Figure 4 illustrates the magnitude of the periodic-twist contribution to the thrust coefficient. A different result was obtained in figure 5 by showing the difference in the values of λ calculated when the periodic twist was considered and when it was neglected.

In table I the calculated influence of periodic twist upon the torque equation and upon the resultant value of λ is shown to be the least in magnitude of the effects studied. The effect is not, however, small enough to be neglected and would be an important factor if it were extrapolated to higher tip-speed ratios.

TABLE I.—EFFECT OF PERIODIC BLADE TWIST ON TORQUE EQUATION

μ	λ (experimental)	$\frac{\delta}{4a}\left(1+\mu^2-\frac{1}{8}\mu^4\right)$	λ (calculated without periodic twist)
0.15	0.0182	0.000700	0.0177
.20	.0186	.000794	.0175
.25	.0189	.000845	.0171
.30	.0189	.000800	.0161

CONCLUSIONS

1. The effect of periodic twist upon rotor-blade flapping coefficients is satisfactorily predicted by this analysis.

2. The influence of periodic twist upon rotor characteristics as calculated from and checked with available data is an important factor in rotor analysis and can be adequately evaluated by the methods presented.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *January 28, 1937.*

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