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THE EFFECT OF LATERAL CONTROLS IN PRODUCING MOTION OF AN AIRPLANE AS COMPUTED FROM WIND-TUNNEL DATA

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SUMMARY

An analytical study of the lateral controllability of an airplane has been made in which both the static rolling and yawing moments supplied by the controls and the reactions due to the inherent stability of the airplane have been taken into account. The investigation was undertaken partly for the purpose of coordinating the results of a long series of wind-tunnel investigations with phenomena observed in flight tests; for this reason a hypothetical average airplane, embodying the essential characteristics of both the wind-tunnel models and the full-size test airplanes, was assumed for the study.

Stability derivatives for the average airplane and for several of the actual flight-test airplanes were computed, and computations were made in an attempt to reproduce by the theory the conditions of several actual flight tests. Computations made of forced rolling and yawing motions of an F-22 airplane caused by a sudden deflection of the ailerons were found to agree well with actual measurements of these motions.

The conditions following instantaneous full deflections of the lateral control have been studied, and some attention has been devoted to the controlling of complete turn maneuvers. A portion of the work was devoted to a study of controllability at stalling angles, and the results of this application of theory were found to agree qualitatively with flight-testing experience.

The angle of bank produced in 1 second, φ_1, by a deflection of the rolling control may be taken as a relative measure of the control effectiveness. In the analysis of controllability below the stall, it was found that a simple measure of the rolling effectiveness of a control is given by the sum of a constant times the rolling moment and a constant times the yawing moment. Thus a relative weight or importance is given to the secondary yawing moment produced by the rolling control. It was concluded that the importance of such secondary moments can be minimized by alteration of the moments of inertia of the airplane. Increasing the yawing moment of inertia reduces the effectiveness of a given yawing control in producing either yawing or rolling motion. Changes of rolling moment of inertia have little direct effect on either the rolling or yawing motion produced by a given rolling-control moment.

The study of conditions above the stall indicated that satisfactory control could not be expected without some provision to maintain the damping in rolling and that a dangerous type of instability would arise if the damping were insufficient. The quantity $L_p N_r - L_r N_p < 0$ was found to give a good measure of this type of instability.

INTRODUCTION

For some time the N. A. C. A. has been conducting a program of research on lateral control for the specific purpose of obtaining information that would lead to improvement of control at the low speeds and high angles of attack above the stall, a region in which present conventional ailerons are known to be unsatisfactory. Several series of wind-tunnel investigations have been completed and an attempt has been made to compare a number of widely different lateral-control devices on the basis of what has been considered their primary function—the provision of rolling moment. Some of the secondary characteristics, such as the yawing moments given by the controls and their effect on the damping in rolling, were considered but only by comparing the various values separately. Flight

tests were then made with the devices that seemed to promise the best lateral control at the stall. Some of them did not perform as had been expected from the wind-tunnel tests (see reference 1), indicating that the first approximation, based largely on the rolling moments given by the devices, was an insufficient basis for comparison and that the complete interaction of the secondary factors must very likely be considered.

References 2 to 5 describe important work that has been done on the lateral control of airplanes in both normal and stalled flight. Reference 2 gives a general account of the problem of control of the stalled airplane; references 3 and 4 describe investigations of the lateral control and stability of different biplane types.

The present report contains the results of a study of control effectiveness made by means of computations that take into account the secondary factors including the yawing moments given by the controls, their effect on the damping in rolling, the other lateral-stability derivatives, and the moments of inertia of the airplane.

Two methods of computation are used. In the first, the rolling and yawing motions are computed step by step for the conditions following a sudden deflection of the lateral control; in the second method a complete turn is arbitrarily specified and the control moments and deflections necessary to perform the maneuver are found. The first method is used to compare the effectiveness below the stall of various lateral-control devices and to investigate primarily the effects of changed stability characteristics above the stall.

The results of calculations made for normal unstalled conditions are compared with measurements made in flight using different types of lateral-control devices. The effects of certain changes in the lateral-stability characteristics below the stall are also studied. The method used in the study of complete turn maneuvers has proved to be a very practical way of dealing with specific control problems. Here all the stability characteristics of the airplane are taken into account but the lengthy and tedious integration of the equations of motion is avoided by predetermining the actual movements of the airplane in the form of some desired maneuver and then finding the manipulation of the controls that would be necessary to execute the specified maneuver. The coordination of the rudder with different types of ailerons has been studied in this way.

MOTION FOLLOWING SUDDEN CONTROL APPLICATION

The method used for calculating the motion following a sudden application of the controls consists of a step-by-step integration. In most cases the control moments were assumed to be applied constantly throughout the motion.

Assumptions and symbols.—The assumptions usually made in the study of airplane stability were used here, including:

1. That the air forces and moments arising from displacements of the airplane, relative to its steady condition of flight, are proportional to the displacements or to their rates.

2. That the components of moment due to the different components of motion are additive (i. e., the rolling moment due to the combined rolling and sideslipping may be computed as though the rolling and sideslipping had occurred separately).

The axes used in specifying the moments, angular velocities, etc., are fixed in the airplane and therefore move relatively to the air and to the earth. The X axis passes through the center of gravity of the airplane in the plane of symmetry and is chosen to point directly into the line of the relative wind when the airplane is flying steadily. In other respects the axes form a conventional trihedral system, intersecting at the center of gravity of the airplane, the Z axis pointing downward in the plane of symmetry and the Y axis pointing along the direction of the right wing. The motions discussed are those of the moving axes relative to the undisturbed air with the exception of the angle of bank, which is measured relative to the horizontal.

The symbols used in the various formulas are defined as follows:

U_0, velocity along X axis in steady flight.
v, velocity of sideslip.
p, angular velocity in rolling.
r, angular velocity in yawing.
φ, angle of bank.

$\beta=\frac{v}{U_0}$, angle of sideslip.

δ, angle of control setting.
Y, component of force along Y axis.
L, rolling moment (about X axis).
N, yawing moment (about Z axis).

$L_p=\frac{\frac{\partial L}{\partial p}}{mk_X^2}$, rolling acceleration due to rolling.

$N_p=\frac{\frac{\partial N}{\partial p}}{mk_Z^2}$, yawing acceleration due to rolling.

etc.

$\delta L_\delta=\frac{C_l qSb}{mk_X^2}$, where C_l is the control rolling-moment coefficient.

$\delta N_\delta=\frac{C_n qSb}{mk_Z^2}$, where C_n is the control yawing-moment coefficient.

b, wing span.
c, wing chord.
S, wing area.
l, tail length (distance from c. g. to tail post).
mk_X^2, moment of inertia of airplane about X axis.
mk_Z^2, moment of inertia of airplane about Z axis.
Γ, dihedral angle.
Λ, sweepback angle.

Equations of motion.—The moments acting on the airplane during its maneuvers are considered to be divided into two main groups: (1) Those due to the deflected controls, and (2) those arising from the motions of the airplane. The motions are usually supposed to be started by the action of the controls alone but, at each succeeding instant, to be conditioned by factors that vary directly in magnitude with the motions or displacements relative to the air. The effects of the motions are described by quantities known as "resistance," or "stability," derivatives. The part of a rolling moment due to rolling motion is calculated by the expression $p\frac{\partial L}{\partial p}$; the partial rolling moment due to combined yawing and rolling is given by:

$$p\frac{\partial L}{\partial p}+r\frac{\partial L}{\partial r}$$

It will be found convenient to replace the actual moments by their corresponding angular accelerations, which are proportional to them. Since

$$\frac{\frac{\partial L}{\partial p}}{mk_X^2}=L_p$$

the component of rolling acceleration due to rolling motion is simply

$$pL_p$$

If the airplane is moving in all its degrees of lateral freedom with deflected controls, the total acceleration in rolling is expressed by

$$\frac{dp}{dt}=\delta L_\delta+pL_p+rL_r+\beta L_\beta \tag{1}$$

where δL_δ is the part of the acceleration due to the control. Likewise the sum of the components of yawing acceleration is

$$\frac{dr}{dt}=\delta N_\delta+pN_p+rN_r+\beta N_\beta \tag{2}$$

The equation for the angle of sideslip contains both the centrifugal effect due to turning and the effect of gravity,

$$\frac{d\beta}{dt}=\frac{g}{U_0}\sin\varphi-rU_0 \tag{3}$$

It is to be noted that, when the angle of sideslip β was computed, the component accelerations due to the sidewise air forces (i. e., terms containing Y) were neglected. The most important term here is Y_β; a rough estimate shows that its greatest probable effect would be negligible for the type of maneuver investigated.

Since the axes changes their orientation in the airplane with different lift coefficients, they will not be directly in line with the axes of the principal moments of inertia. The corrections are small, however, and have been neglected.

Integration of equations.—The equations show that in order to calculate the acceleration of the motion at any time, the velocities p, r, and the angle of sideslip β must be known. This knowledge is, of course, available only when all accelerations before the time in question are known; an integration is therefore necessary. This integration may be conveniently performed by dividing the time during which the motion occurs into very small steps and by assuming that the velocities remain constant over these small intervals. If a particular instant is denoted by the subscript n, the accelerations at this instant may be calculated by the formulas

$$\left.\begin{aligned}\left(\frac{dp}{dt}\right)_n&=(\delta L_\delta)_n+p_nL_p+r_nL_r+\beta_nL_\beta\\ \left(\frac{dr}{dt}\right)_n&=(\delta N_\delta)_n+p_nN_p+r_nN_r+\beta_nN_\beta\end{aligned}\right\} \tag{4}$$

If the preceding time instant is denoted by $n-1$, the accelerations at each succeeding instant may be calculated step by step, using the velocities computed from the previous instant. Thus:

$$\left.\begin{aligned}p_n&=\left(\frac{dp}{dt}\right)_{n-1}\times\Delta t+p_{n-1}\\ \varphi_n&=p_{n-1}\times\Delta t+\varphi_{n-1}\\ r_n&=\left(\frac{dr}{dt}\right)_{n-1}\times\Delta t+r_{n-1}\\ \beta_n&=\left(\frac{d\beta}{dt}\right)_{n-1}\times\Delta t+\beta_{n-1}\end{aligned}\right\} \tag{5}$$

The right-hand sides of these equations contain only quantities known from the preceding instant. At the start, $n=0$, all the velocities and angles are taken as zero, and the accelerations are caused by the control moments alone,

$$\left.\begin{aligned}\left(\frac{dp}{dt}\right)_0&=\delta L_\delta=\frac{C_lqSb}{mk_X^2}\\ \left(\frac{dr}{dt}\right)_0&=\delta N_\delta=\frac{C_nqSb}{mk_Z^2}\\ \left(\frac{d\beta}{dt}\right)_0&=0\end{aligned}\right\} \tag{6}$$

A typical example illustrating the step-by-step computation is given in table I.

Comparison of computed and measured motions.—The results of a number of flight tests of the F-22 airplane equipped with several widely different lateral-

control devices have been used as checks of the computations. These tests were conducted by gliding the airplane at various steady speeds and suddenly deflecting the aileron control to its full extent. Instrument records of the resulting rolling and yawing angular velocities were made as a measure of the effectiveness of the various controls. (See references 1 and 6.)

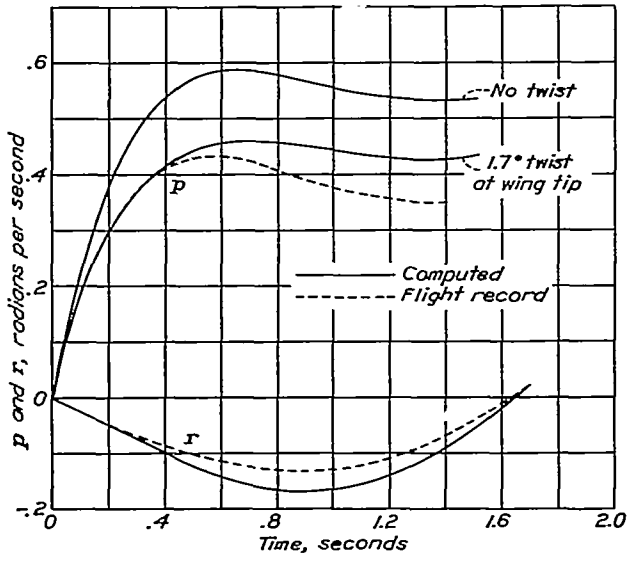


FIGURE 1.—Rolling and yawing motion of F-22 airplane with long narrow ailerons. Flaps up; C_L=1.0.

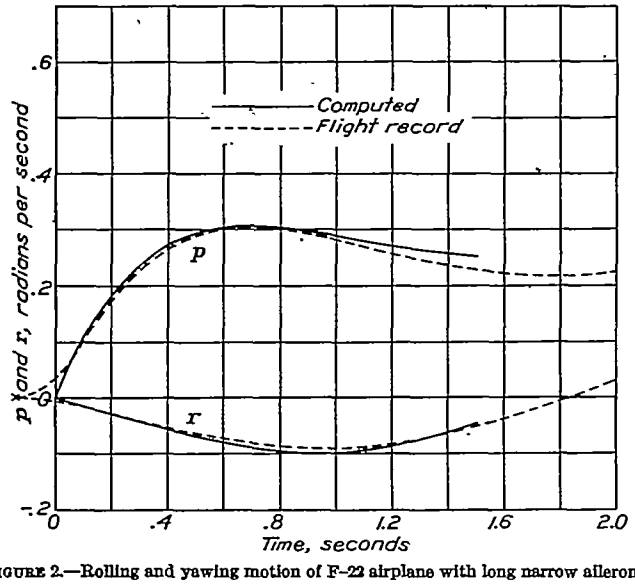


FIGURE 2.—Rolling and yawing motion of F-22 airplane with long narrow ailerons. Flaps down; C_L=1.75.

The procedure in these experiments simulated very closely the conditions assumed in the computations, although the flight records showed that about 0.15 second was actually required to accomplish the full deflection of the control, which was assumed to be instantaneous in the computations. In the comparisons included, this discrepancy was eliminated by appropriate shifts of the time scales.

The flight tests were intended to supplement a program of tests made in the 7- by 10-foot wind tunnel of a series of lateral-control devices (reference 7). The wind-tunnel program included experiments to determine several important lateral-stability characteristics as well as the static rolling and yawing moments produced by the control devices; the results of these experiments furnished the necessary basis for reproducing the conditions of the flights in the computations. The quantities needed in the computations, including the resistance derivatives, were determined from the known dimensions of the F-22 airplane by the methods given in appendix I.

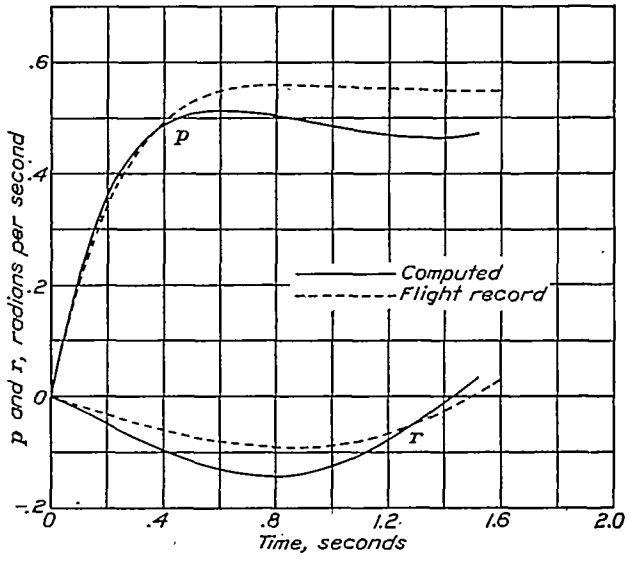


FIGURE 3.—Rolling and yawing motion of F-22 airplane with balanced short wide ailerons. Flaps up; C_L=1.10

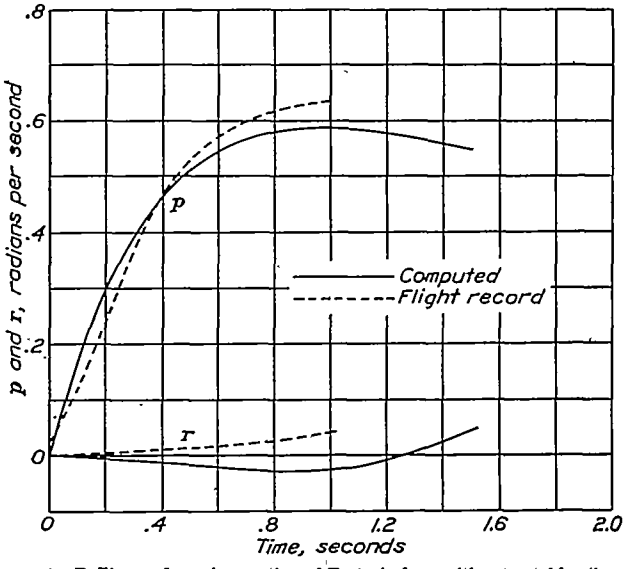


FIGURE 4.—Rolling and yawing motion of F-22 airplane with retractable ailerons. Flaps up; C_L=1.0.

When computed motions and flight records were first compared, it was found that in many cases the initial accelerations in roll predicted from the rolling moments obtained in the wind tunnel were larger than those shown by the motions recorded in flight. Thus, the full value of the rolling moment measured on the

models was apparently not realized in flight. Examination showed this lack of agreement to be especially apparent in the cases of devices that would be expected to exert the greatest twisting effect on the wings and, since appreciable twisting of the actual wings had been observed in full-scale wind-tunnel experiments, the discrepancy was attributed to this effect. Calculation showed that in the most extreme case (that of ordinary narrow-chord ailerons) a linearly distributed angle of twist reaching 1.7° at the wing tip would account for the observed difference and that the rolling-moment coefficient would be reduced from 0.056 to 0.043. In this case the flight test was made at a dynamic pressure of 9 pounds per square foot. With this first correction as a basis, a general correction formula was used in which the reduction in rolling moment was given as a proportion of the dynamic pressure and the change in section pitching-moment coefficient produced by deflecting the controls.

Figures 1 and 2 show the rolling and yawing motions of the F–22 equipped with long, narrow ailerons. This particular airplane was also equipped with flaps that retracted into the wing ahead of the ailerons. (See reference 6.) Figure 1 illustrates the effect attributed to twisting of the wings. The higher curve was obtained when a value of the rolling-moment coefficient based on a wind-tunnel test of a solid wooden model was used. The yawing angular velocity curves showed remarkably good agreement in these two cases, especially as regards the period of the oscillation of this motion.

The comparison of the yawing curves in figures 3 and 4 is not so favorable as in the former cases. In figure 3 it appears that the yawing-moment coefficient as computed from the wind-tunnel data was slightly greater than that recorded. In this case the control moment coefficients used in the computations were obtained from full-scale wind-tunnel tests of the actual airplane; hence no correction for wing twist was applied. The curves of figure 4 apply to a modified F–22 airplane equipped with retractable ailerons. It is possible that this control device, which is similar to a spoiler, has some effect on the yawing moment due to rolling. The disagreement in the yawing curves would seem to indicate that too large a negative value was assumed in the computations.

The curves of computed rolling motion show no consistent disagreements with the curves plotted from the flight measurements, the differences being of opposite sign in several cases. It seems probable that these comparisons represent the general accuracy obtainable either in the experiments or in the calculations.

COMPUTATIONS FOR AVERAGE AIRPLANE IN UNSTALLED FLIGHT

The results of the flight experiments with the F–22 airplane were not suitable for direct comparisons of the effectiveness of the various controls used because the airplane was modified considerably during the progress of the experiments (see references 1 and 6) so that different sets of stability derivatives and moments of inertia had to be used in the computations to represent the different individual tests. In order to secure data of more general significance and to make a more systematic investigation of control effectiveness than was possible in the flight experiments, it was thought desirable to make a series of computations based on a standard set of airplane characteristics, including standard resistance derivatives and moments of inertia. At the same time it was desired to retain the basic dimensions of the F–22 machine so that there would be at least a partial check with the flight-test work at all times.

Specifications of average airplane.—With these considerations in mind the specifications of an arbitrary standard airplane were devised. The weight and the wing area and span of the F–22 airplane were retained but, since other dimensions were obtained from statistical averages, the machine was called an "average airplane." These statistical averages were obtained by studying the specifications of a number of conventional airplanes of different sizes, weights, and types. Data from 20 to 40 airplanes were used for the determination of average values of the following characteristics:

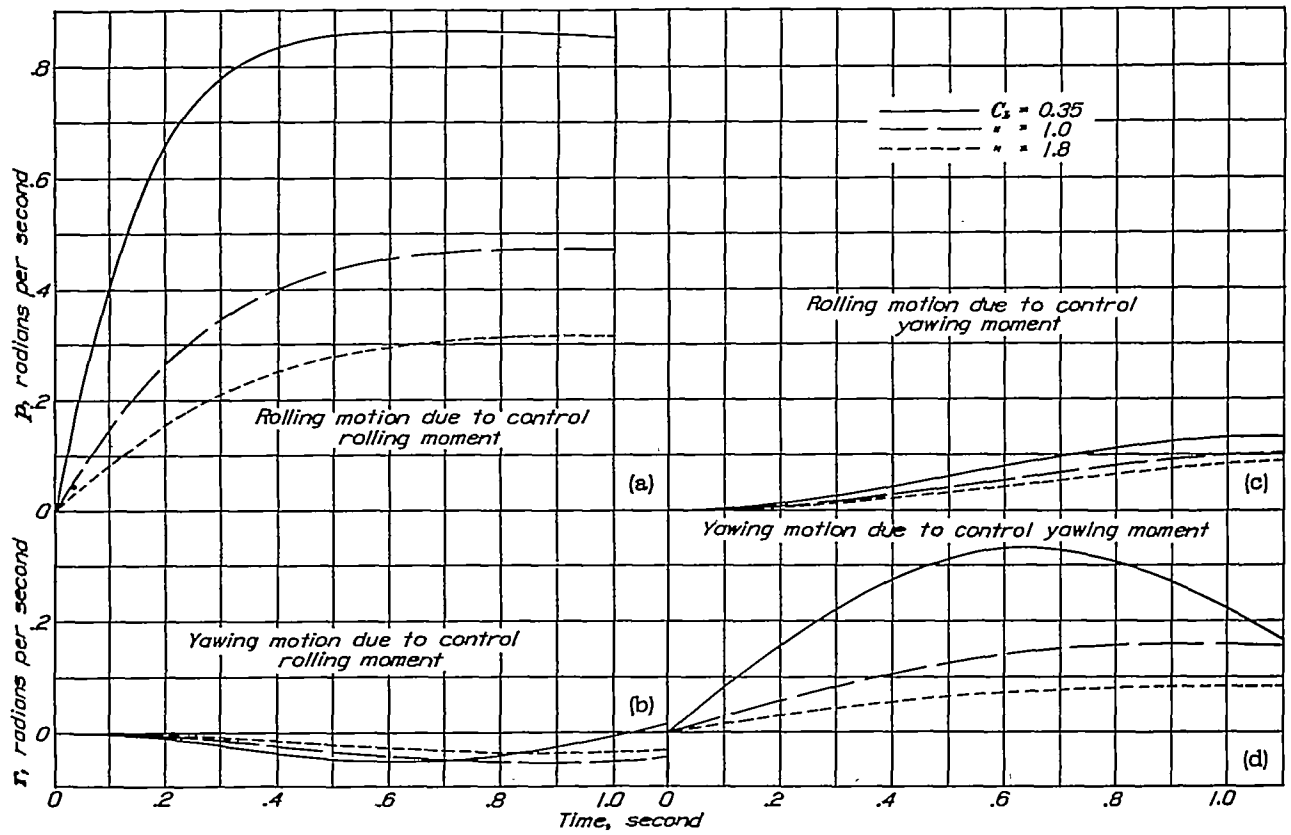
1. The ratio of the total fin and rudder area to the wing area.
2. The ratio of the tail length (i. e., distance from *c. g.* of airplane to the tail post) to the wing span.
3. The ratios of the radii of gyration in rolling and yawing to the wing span.

The moments of inertia were obtained from data listed in reference 8. That the characteristics thus obtained did not differ appreciably from those of the F–22 is shown by the following table:

	Range of characteristics of F–22 airplane used in flight tests	Characteristics of average airplane
Weight ..... pounds	1,500–1,650	1,600
Wing span ..... feet	30– 32.8	32
Wing area ..... square feet	161– 172	171
Area of fin and rudder ..... do	10.1	10.8
Tail length ..... feet	14.5	14.6
mk_x^2	696–1,554	1,216
mk_z^2	1,520–2,118	1,700

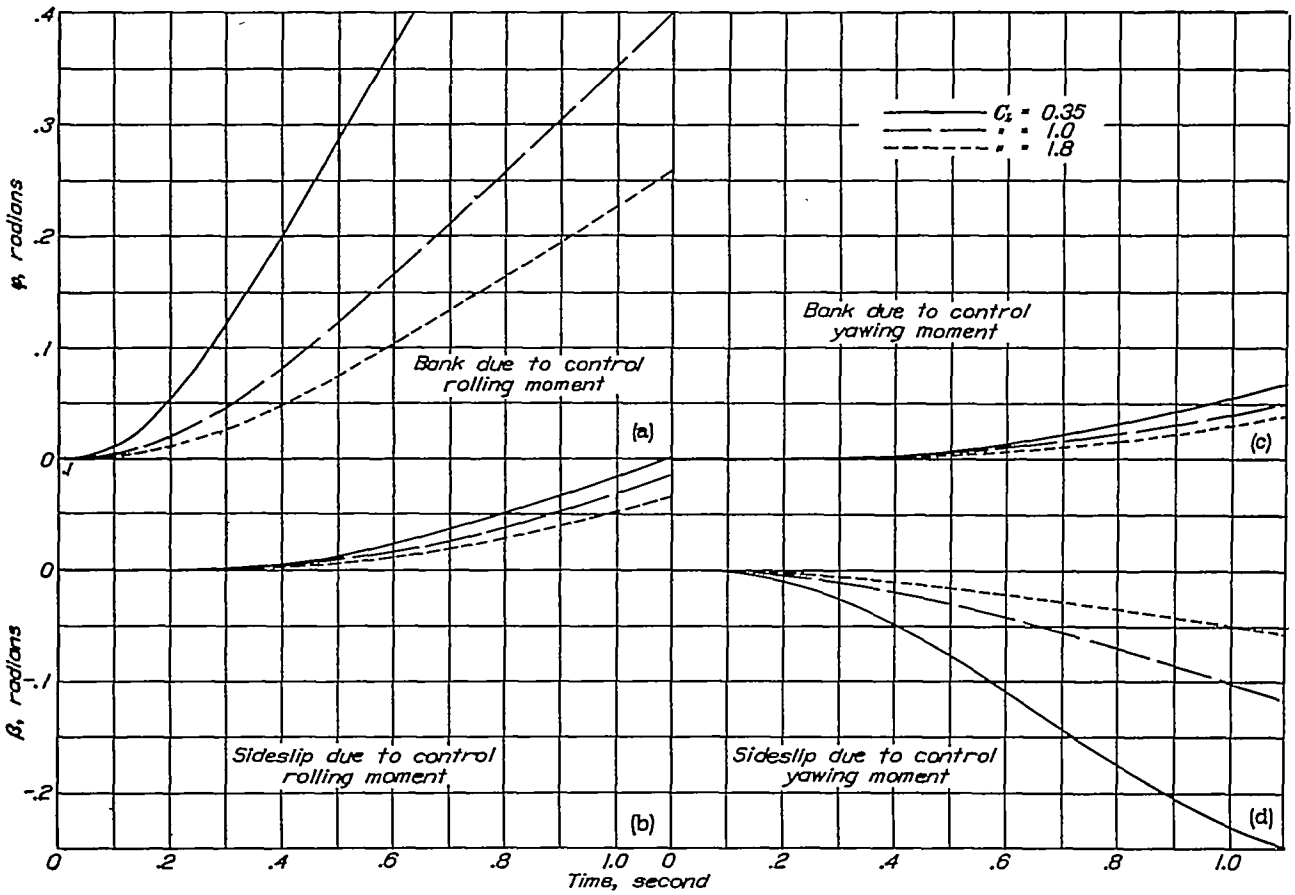
Computations based on a purely dimensionless average airplane were considered, but it was thought that the results would have a more concrete meaning if they were presented in terms of an airplane of particular size, especially since they could then be directly compared with the flight results.

Unstalled-flight computations.—Most of the lateral-control devices tested in the wind tunnel did not cause any change in the stability derivatives of the wings (spoiler devices are a notable exception). In such cases the sole effect of the control in producing motions can



(a) (b) C_l=0.04 C_n=0 (c) (d) C_l=0 C_n=0.01

FIGURE 5 (a, b, c, d).—Computed rolling and yawing motions of average airplane.



(a) (b) C_l=0.04 C_n=0 (c) (d) C_l=0 C_n=0.01

FIGURE 6 (a, b, c, d).—Computed angles of bank and sideslip of average airplane.

be attributed to the static rolling and yawing moments produced; consequently, a large class of devices could be investigated, in effect, by extending computations over a suitable range of combinations of static rolling and yawing moments.

On account of the linearity of the equations of motion it was possible to calculate the effects of yawing moments and rolling moments separately and later to add them in any desired proportion. Thus, at each of the three lift coefficients two computations were made, one to determine the motion due to a yawing moment and the other to determine the motion due to a rolling moment. The following table lists the values of the coefficients that were used:

C_L : 0.35; 1.0; and 1.8 (20 percent c split flaps, full span).

C_n : 0.01 and 0.

C_l : 0 and 0.04.

In these cases the dihedral angle assumed for the average airplane was 1°. Several additional computations were made to investigate the effect of variation of this factor, assuming angles of 5° and 9°.

Stability derivatives of average airplane.—The stability derivatives used in these computations were obtained by methods described in appendix I and are given in the following table; in the calculation the average airplane was assumed to have rounded-tip wings with 1° dihedral.

	L_p	L_r	L_β	N_p	N_r	N_β
C_L=0.35 Cruising speed U_0=150 feet per second	−5.44	1.11	−2.16	−0.207	−0.913	5.52
C_L=1.0 Gliding speed U_0=88.5 feet per second	−3.23	1.88	−1.11	−.301	−.663	2.04
C_L=1.8 Low speed (flaps) U_0=66 feet per second	−2.46	2.51	−1.66	−.310	−.977	1.46

Results of below-stall computations.—The results of the series of computations for the condition below the stall are shown in figures 5 to 9. Calculated examples of the complete motion are given in figures 5 and 6, which show the rates of rolling and yawing and the angles of bank and sideslip plotted against time for the different flight speeds.

It was thought that the amount of motion produced in 1 second would be a reasonable measure of the control effectiveness. As previously mentioned, the motion produced by a given yawing moment can be added to that produced by a given rolling moment to get the simultaneous effect of both. Thus the formulas for the motion produced in 1 second by any combination of rolling and yawing moments are

$$\left.\begin{aligned} p_1&=C_l\frac{\partial p_1}{\partial C_l}+C_n\frac{\partial p_1}{\partial C_n}\\ \varphi_1&=C_l\frac{\partial \varphi_1}{\partial C_l}+C_n\frac{\partial \varphi_1}{\partial C_n}\\ r_1&=C_l\frac{\partial r_1}{\partial C_l}+C_n\frac{\partial r_1}{\partial C_n}\\ \beta_1&=C_l\frac{\partial \beta_1}{\partial C_l}+C_n\frac{\partial \beta_1}{\partial C_n}\end{aligned}\right\} \tag{7}$$

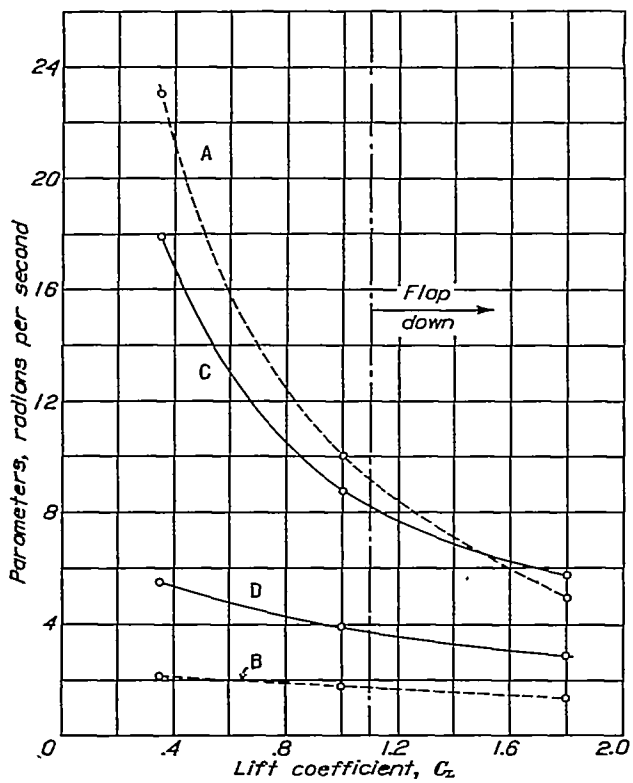
where $\frac{\partial p_1}{\partial C_n}$, $\frac{\partial \varphi_1}{\partial C_l}$, etc., are parameters that depend on the speed of flight and the stability characteristics of the airplane. These parameters are shown plotted against lift coefficient (as a measure of the flight speed) in figures 7 and 8 and represent the principal results of the series of computations for unstalled flight.

Discussion of below-stall computations.—The factors shown in figures 7 and 8 may be used to compare the effectiveness of various lateral-control devices on the basis of the motions and displacements they would produce on a 1,600-pound airplane of average stability characteristics. By showing the effect of secondary control moments in producing motion of the airplane, they give a measure of the relative weight to be assigned such secondary moments in comparing different devices. These factors will, of course, be somewhat different for airplanes of different stability characteristics and the relative effects of secondary control moments will be expected to be somewhat different also. The average airplane is simply a convenient yardstick in this respect.

If the factors given in figures 7 and 8 are used as absolute measures of the amount of motion produced in 1 second (aside from their use simply in comparing various control devices), a greater error will be committed in applying them to airplanes of different size than in applying them to airplanes of somewhat different stability characteristics. Reference 9 gives the necessary rules for correctly applying the present data to airplanes of any size or weight in which certain definite aspects of similarity are preserved. The theory requires that the airplanes be geometrically similar although they may have different densities. Practically, this requirement necessitates that the outward forms of the airplanes be similar and that the ratios of the radii of gyration about each axis to the wing span be the same. The motions of the different sized airplanes are compared at equal values of the lift coefficient. With equal values of the wing loading the angular velocities are inversely proportional to the spans: Thus,

$$\left.\begin{aligned}\frac{p'_1 b'}{2U_0}&=\frac{p_1 b}{2U_0}\\ \text{and}\qquad \frac{\varphi'_1 b'}{2U_0}&=\frac{\varphi_1 b}{2U_0}\end{aligned}\right\}\tag{8}$$

With similar airplanes of different wing loadings the state of motion existing at a given time for one will generally pertain to a different instant for another, which is also true of an airplane of the same size and loading but flying in air of different density. Given the motion of the average airplane at 1 second, the instant to which this state of motion (as indicated by



$-\frac{\partial\beta_1}{\partial C_n}$ curve A, $\frac{\partial\beta_1}{\partial C_l}$ curve B, $\frac{\partial\varphi_1}{\partial C_l}$ curve C, $\frac{\partial\varphi_1}{\partial C_n}$ curve D

$\varphi_1=C_l\frac{\partial\varphi_1}{\partial C_l}+C_n\frac{\partial\varphi_1}{\partial C_n}$, $\beta_1=C_l\frac{\partial\beta_1}{\partial C_l}+C_n\frac{\partial\beta_1}{\partial C_n}$

FIGURE 7.—Parameters for computing displacement of average airplane at end of 1 second with various combinations of rolling and yawing moments.

the value of $pb/2U_0$) pertains on a similar airplane may be found from:

$$t'=1\times\frac{S\frac{\rho}{2}\frac{U_0}{m}}{S'\frac{\rho'}{2}\frac{U_0'}{m'}}\tag{9}$$

Plots representing the motion of an airplane in nondimensional terms have as abscissa

$$\frac{S\frac{\rho}{2}U_0}{m}\times t$$

and as ordinate

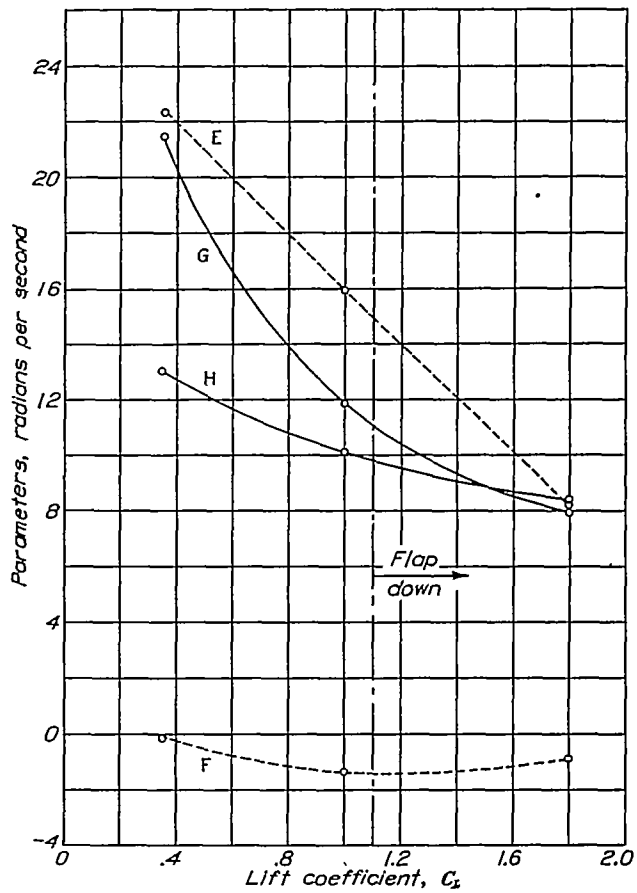
$$\frac{pb}{2U_0}$$

or

$$\frac{S\frac{\rho}{2}b}{\varphi\,2m}$$

etc.

In the case of the average airplane the influence of moderate dihedral on the lateral controllability below



$\frac{\partial r_1}{\partial C_n}$ curve E, $\frac{\partial r_1}{\partial C_l}$ curve F, $\frac{\partial p_1}{\partial C_l}$ curve G, $\frac{\partial p_1}{\partial C_n}$ curve H

$p_1=C_l\frac{\partial p_1}{\partial C_l}+C_n\frac{\partial p_1}{\partial C_n}$, $r_1=C_l\frac{\partial r_1}{\partial C_l}+C_n\frac{\partial r_1}{\partial C_n}$

FIGURE 8.—Parameters for computing motion of average airplane at end of 1 second with various combinations of rolling and yawing moments.

the stall was small, as is shown in figure 9. If, however, a large dihedral effect is combined with considerable adverse yawing tendency from the ailerons, the lateral control may become ineffective. This condition is most likely to occur at low speeds with flaps deflected because under these conditions the wings show their greatest tendency to roll when yawed (dihedral effect) and because the aileron yawing moment is usually greatest at high lift coefficient. Figure 9 shows that with a dihedral angle of 9° and an adverse yawing moment of one-fourth of the rolling moment, the aver-

age airplane actually reversed its normal roll, rolling against the ailerons less than 2 seconds after they were applied. The magnitude of the tendency for a given adverse yawing moment to render the lateral control ineffective depends to some extent on all the stability characteristics of the airplane but principally on the ratio of rolling to yawing moments in sideslip, i. e., on $\frac{dC_l/d\beta}{dC_n/d\beta}$. For the various cases depicted in figure 9 these ratios were:

Dihedral angle, degrees	$\frac{dC_l/d\beta}{dC_n/d\beta}$
1	0.8
5	1.2
9	2.4

Large values of this ratio decrease the aileron control effectiveness if the secondary yawing moments are adverse but will tend to increase it if they are favorable.

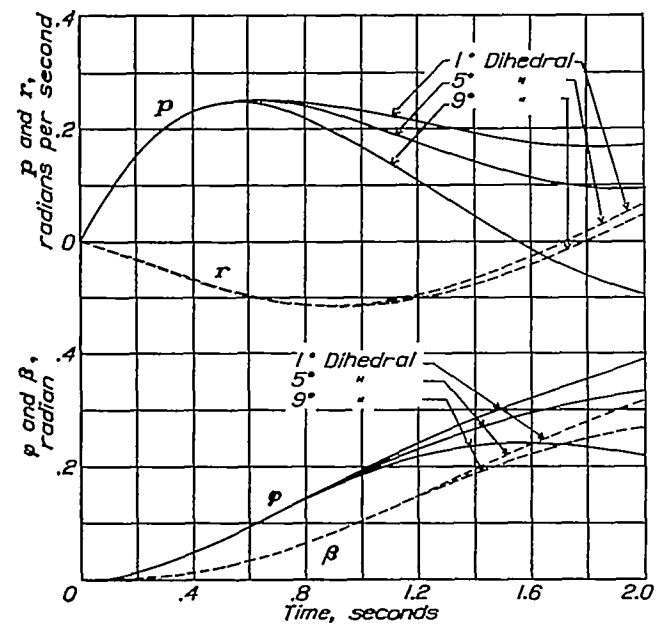


FIGURE 9.—Effect of dihedral combined with adverse yawing moment on rolling control. Flaps down; $C_L=1.3$; $C_l=0.04$; $C_n=-0.01$.

The curves of rolling motion given in figures 1 to 5 show that the rate of rolling rises quickly at the start on account of the relatively great rolling moment but soon becomes almost steady. This steady rate is attained in about 0.3 second at high speed and occurs when the air damping of the rolling motion is large enough to overcome the control moment. Obviously the lateral moment of inertia (mk_X^2) cannot have much influence on this portion of the curve since the airplane is not accelerating appreciably, and its effect will be shown mainly on the starting slope of the curve. (See fig. 10.) It may be seen that the area under the curve at, say, 1 second would not be appreciably affected by changes in this slope; hence the angle of bank reached in 1 second would not be much affected by the moment of inertia in rolling. This fact has been borne out by flight experiments made by the N. A. C. A. in which the test pilots were unable to detect with certainty the effects of changes in rolling moment of inertia of as high as 50 percent. (See also reference 10.)

The yawing-motion curves indicate a different phenomenon. Here the damping is relatively small and the effects of moment of inertia in yaw are fairly large. Thus it appears that the magnitude of the rudder moments should be accommodated to the airplane moment of inertia, while the principal consideration determining the rolling-control moments should be the air-damping factor.

Since the amount of yawing motion produced by a given yawing moment is primarily governed by the moment of inertia in this motion, it appears that the unfavorable influence of secondary aileron yawing moments could be effectively reduced by increasing this moment of inertia. Furthermore, since the direct effect of roll moment of inertia on the rolling motion is apparently slight, it is possible that increasing mk_Z^2 by distributing weight along the wing span would actually increase the aileron effectiveness if considerable adverse yawing moment were present.

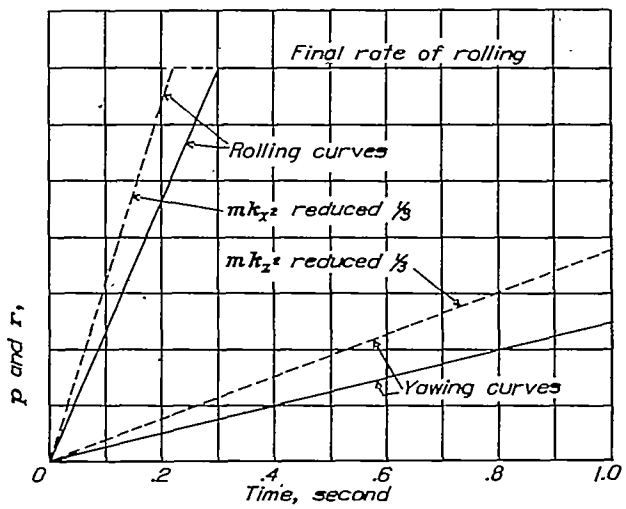


FIGURE 10.—Diagram illustrating effect of change of moments of inertia on rolling and yawing control.

COMPUTATIONS FOR STALLED FLIGHT

Experiments with lateral control at angles of attack above the stall having been made both in the flight and the wind-tunnel research projects, it was desired to extend the present investigation to cover this condition also. Accordingly, a study of the results of both series of tests was made with the object of determining whether the conditions encountered in practice could be reproduced in theory.

Unfortunately, the wind-tunnel experiments showed that no certain determination of the factors (resistance derivatives) involved in the motion of a stalled airplane was possible. On the other hand, the flight experiments indicated that these factors apparently had no definite values (according to their usual definition), inasmuch

as the action of the airplane could not be foretold from one experiment to the next. For example, the outcome of a simple aileron movement might in one instance be a roll in the direction urged by the control; whereas at another time, under practically the same conditions, the roll would be the reverse of that intended.

Stability derivatives above stalling angles.—The reasons for the apparently contradictory results of the flight tests may be found in the wind-tunnel measure-

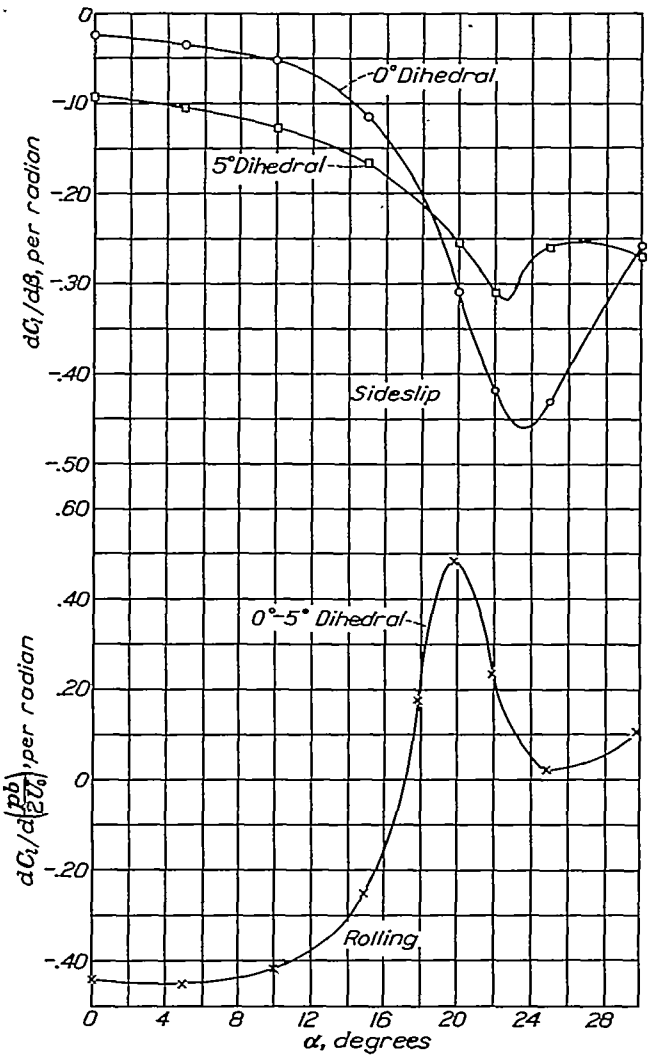


FIGURE 11.—Coefficients of rolling moment due to rolling and sideslip at various angles of attack. Rectangular Clark Y monoplane; data from tests in the 7- by 10-foot wind tunnel.

ments of the stability characteristics made at these high angles; these measurements show that motions of the wings may develop unstable moments, which could quickly overpower static rolling or yawing moments given by the controls. Figure 11 shows typical measurements of coefficients by which these quantities are determined. These curves show that the rolling moment due to sideslip of a straight wing increases enormously as the stall is approached, reaching values 7 or 8 times as great as those at medium angles below stalling. Under the same conditions the damping moment in rolling changes sign and becomes an "aiding" (autorotational) moment.

Obviously many of the assumptions of the method as used in investigating unstalled-flight phenomena are not true in the case of stalled flight. In particular, the assumed independence of small longitudinal and lateral motions, which is supported by both experience and reason for the ordinary-flight range, cannot be said to hold under these new conditions because the values of the derivatives change very rapidly with small changes of longitudinal attitude (angle of attack). The assumption that the components of a moment arising from different sources may be added together as though their causes occurred separately is apparently borne out only in the abstract sense of representing the average condition.

In spite of these limitations of the method, it was considered feasible to extend the computations to the condition of stalled flight in the study of the general conditions encountered in controlling such flight, although the results of the computations made for these conditions do not have the same significance as those made for conditions below the stall. The former results gave quantitative estimates of the amount of motion produced by given control moments; the extension of the computations to stalled flight will only illustrate the various phenomena that may result from the conditions predicted by the wind-tunnel experiments.

Experience in attempting controlled flight above the stall has shown that the possibility of controlling such flight depends as much on the natural stability characteristics of the airplane as on the possibility of securing adequate controlling moments. Because of this fact the present computations were made primarily to investigate the effects of changed stability characteristics (derivatives). Another important reason for choosing various combinations of stability derivatives is the fact that no very definite values can be assigned to them for a particular lift coefficient, as was possible in the unstalled-flight range.

For these reasons the investigation of controllability above the stall is necessarily presented in a manner different from that used in the cases of ordinary flight. The wind-tunnel measurements were studied to find the approximate variation of the resistance derivatives over a range of angles of attack definitely above the stall, chosen to include the region of most violent instability. The particular lift coefficient assumed was necessarily somewhat loosely defined ($C_L=1.2$); it was so taken to represent extreme stalling as well as intermediate conditions. The calculation of the stability derivatives at these angles is given in appendix I.

In the variation of the stability characteristics to take account of the range of possible conditions, the effects of the parts of the airplane other than the wings were not considered. The wing characteristics which

show the greatest variation in this region and which apparently have the greatest effect on the stability are:

1. The damping in rolling, L_p.
2. The rolling reaction due to sideslip, L_β.
3. The yawing reaction due to rolling, N_p.

Accordingly, three values for each of these were chosen, covering the range shown by the wind-tunnel data and representing two extremes and one mean condition. These values were designated a, b, and c and are listed in table II.

TABLE II.—VALUES OF STABILITY DERIVATIVES USED ABOVE STALL

Designation	L_p	L_β	N_p
a	−1.75	−7.4	−0.20
b	0	−11.4	.53
c	3.50	−14.4	1.06

In each case it will be noted that letter c denotes the most extreme condition likely to be encountered. Condition a may be fairly assumed to apply only to cases where some provision is made to prevent the wing tips from stalling, which may be accomplished by washout or twist or by means of some such device as tip slots. (See appendix I for determination of these derivatives.)

The computations were made to cover more than a dozen different combinations of these values of the derivatives in conjunction with a given fixed pair of control rolling and yawing moments. These arbitrary controlling moments were chosen to represent rolling- and yawing-moment coefficients somewhat greater than those obtained with ordinary ailerons but which might be attained in practice with rather large ailerons, especially those of the short, wide type described in reference 7, I. As in the previous computations (below stall), the sign of the standard yawing-moment coefficient was alternated, giving the effect of favorable and adverse, as well as zero, secondary yawing moment.

Range of investigation of stalled flight.—Since in these computations the plan was to study the *possibility of control* rather than to obtain any numerical measure of control effectiveness, the procedure of the computations was sometimes varied in such a way as to represent attempts of the control to check motions of the airplane as well as to start them. In some cases the motion was assumed to be due to some external cause and to exist at the start of the computations, while in other cases the initial setting of the control was reversed after a short interval in an attempt to check the motion it had already produced. The effects of both favorable and adverse yaw were tried in these cases.

Results of computations.—Figure 12 shows rolling motions resulting from suddenly applied and continuously maintained aileron deflections giving a rolling-moment coefficient of 0.04 and an adverse yawing-moment coefficient of −0.02. The different angular-velocity curves are the results of assuming different combinations of the stability derivatives listed in table II. In accordance with the plan of table II, the first letter in each symbol designation attached to the curves indicates the value of the damping factor L_p used; the second, the value of L_β; and the last, N_p.

These curves appear to represent the same erratic phenomena as were observed in the flight experiments. It will be noted that in some instances the direction of motion of the airplane after a short interval was the reverse of that urged by the rolling control, while in other instances it rolled with increasing acceleration in the direction urged. Either of these phenomena

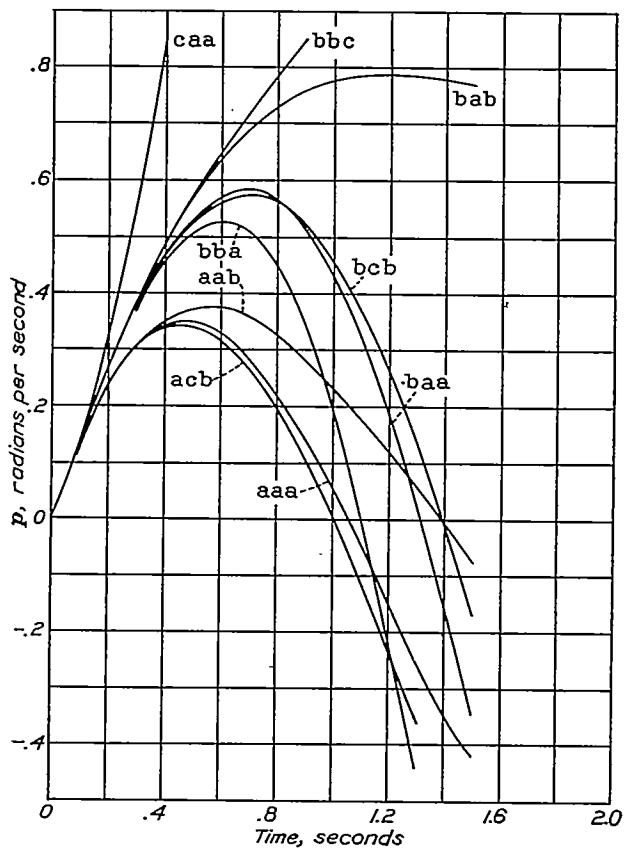


FIGURE 12.—Rolling motions resulting from application of adverse-yaw aileron control in stalled flight with different combinations of stability derivatives; C_l=0.04; C_n=−0.02.

occurred within the predictable range of the stability derivatives.

The effects of smaller control rolling and yawing moments may be visualized simply by reducing the scales of the motions. Thus in figure 12 the motions calculated for C_l=0.02 and C_n=−0.01 would be just half those plotted.

Figure 13 shows the results of attempts to check an initial disturbance in rolling with both favorable- and adverse-yaw ailerons. The failure of the adverse-yaw ailerons is due mainly to the yawed attitude they produce, although the actual yawing motion accounts for an appreciable effect. Figure 14 differs from figure 13 in that it includes also conditions in which the initial

motion countered by the ailerons was assumed to be due to the action of the control rather than to an external disturbance in rolling. Here the ailerons were called upon to check whatever yawing motion they had previously produced. In this case it will be noted that the favorable-yaw ailerons encountered difficulty because it was hard to recover from the initial motion they had produced.

Figure 15 shows the effect of a delay in attempting to recover from rolling and yawing motion. Because of the instability of the airplane, the motion could not be checked even though the yawing moment of the ailerons was favorable. Thus, for the particular case illustrated, a delay of 0.1 second in reversing the control changed the action from one in which the airplane followed the

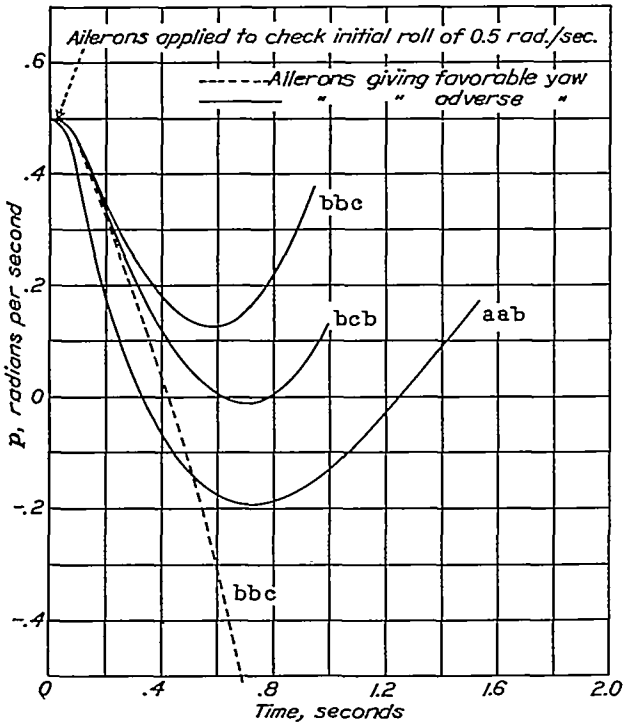


FIGURE 13.—Rate-of-rolling curves illustrating attempts to control initial motion in pure rolling. Stalled flight; $C_l=-0.04$; $C_n=\pm 0.02$.

control to one in which it continued to roll against it.

Discussion in terms of stability derivatives.—The motion of the average airplane in stalled flight is apparently governed more by its natural tendencies than by the applied control moments, a condition illustrated by the curves previously described which showed that the airplane developed tendencies that were uncontrollable in some instances. When using the step-by-step method, it was found convenient to tabulate each separate component of the rolling and yawing accelerations due to the stability factors as well as the components of motion. (See table I.) In this way a complete history of the contribution of each factor was obtained, thus enabling a study of the controllability in terms of the stability derivatives.

Undoubtedly the most important single factor contributing to the uncontrollable instability above the stall is the loss of the damping in rolling. Below the stall this damping is the most powerful constraint of the airplane, and the effects produced by its sudden drop to zero or to a negative value exert a great influence on the behavior of the machine. Apparently no airplane can be considered safely controllable above the stall if the autorotational tendencies observed in wind-tunnel tests of plain wings are retained.

During a roll maneuver in stalled flight there may be, in addition to the control moment, certain other factors that tend to accelerate the rolling. These factors arise because the rolling motion by itself usually tends to

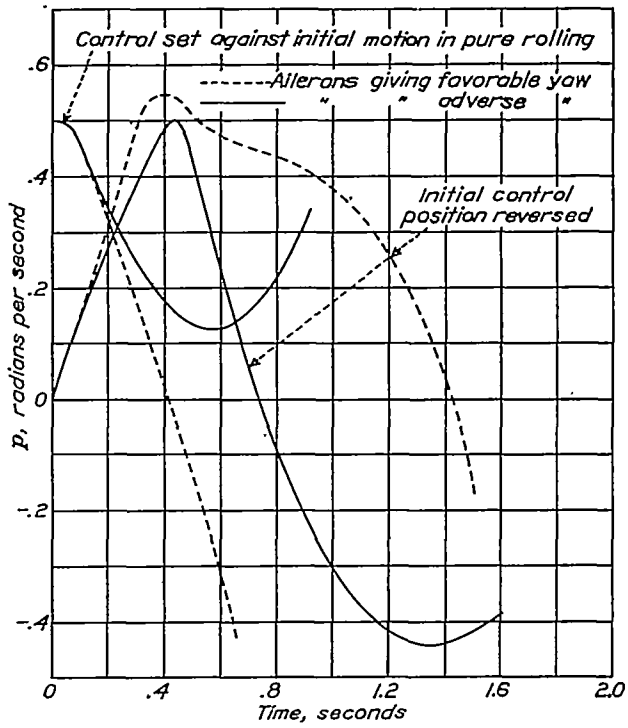


FIGURE 14.—Rate-of-rolling curves showing effect of favorable and adverse aileron yawing moments in attempting to control initial motion in pure rolling and in reversing initial control position. Case bbc; $C_l=0.04$; $C_n=\pm 0.02$.

induce a favorable yawing action above the stall. Thus when the right wing is dropping, its added drag causes a yaw to the right, retarding the wing tip and causing a loss of lift due to decreased speed and tending to aggravate the dropping of the wing. The factors that directly oppose these rolling and yawing motions by damping tend to check this sequence if they are present. The first two effects, which aid the angular motion indirectly, relate to L_r and N_p, proportional, respectively, to the rolling moment due to yawing and the yawing moment due to rolling. Evidently if these moments overcome the direct damping tendencies, the angular motion will tend to accelerate of its own accord or will diverge. Suppose for the moment that these opposing tendencies just balance each other, that is,

$$\left.\begin{aligned} pL_p+rL_r&=0\\ pN_p+rN_r&=0\end{aligned}\right\} \qquad (10)$$

Inasmuch as p and r are simultaneous, there will exist a relation between the derivatives that is independent of p and r; i. e.,

$$L_pN_r-L_rN_p=0 \tag{11}$$

If this sum is zero, L_r and N_p are sufficiently large to equilibrate the stabilizing damping terms; and, if it is negative, any combined rolling and yawing motion will tend to diverge with increasing acceleration even though the direct dampings are present. The relation between this criterion and the behavior of the airplane in lateral motions above the stall is shown in table III, which gives values for the cases shown in figures 12 to 16.

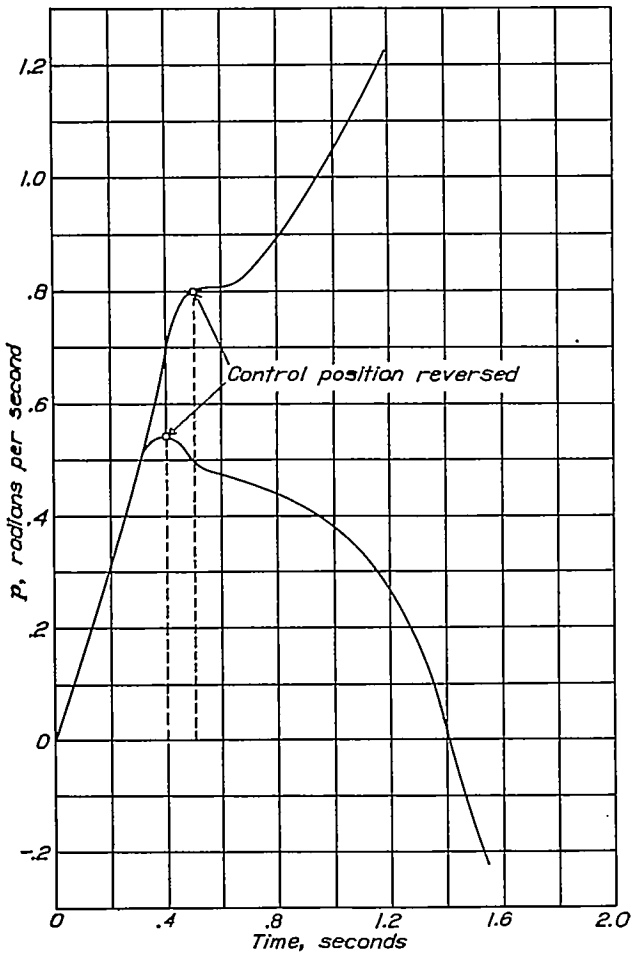


FIGURE 15.—Effect of delay in attempting to recover from motion started by ailerons with favorable yaw. Stalled flight. Case bbc; C_l=0.04; C_n=0.02.

It will be noted that the curves of figure 12 which indicate the greatest tendency toward continued rolling in the direction started correspond to the greatest negative values of $L_pN_r-L_rN_p$. In the curves shown, the rolling control was assumed to give an adverse yawing moment that served to oppose the tendency toward divergence indicated by negative values of this criterion. If a rolling moment with no secondary yawing moment had been assumed in these cases, each curve would have shown an increasing acceleration in rolling greater than that given by the control and according to the magnitude of the tendency exhibited by the value of the criterion, as shown in figure 16. After a definite interval this tendency would have exceeded the power of the controls, and recovery would have been impossible.

Below the stall this criterion appears to be in every case positive, indicating stability. Relatively large positive values indicate relatively great damping of combined rolling and yawing motion.

The foregoing considerations do not take account of any sideslipping effects. These considerations, when combined with the factors determining the sideslipping tendency, give a more complete idea of the controllability characteristics of the airplane at high angles of attack and in stalled flight.

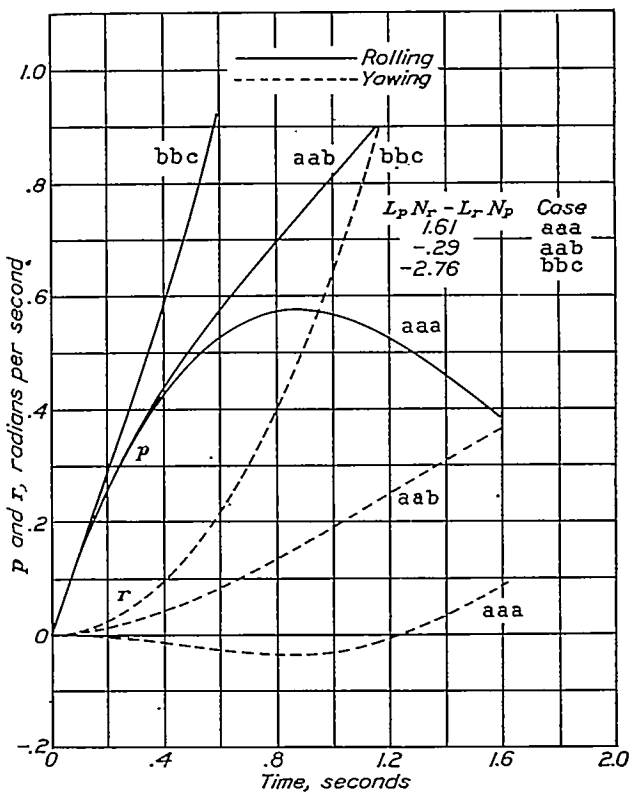


FIGURE 16.—Rolling and yawing motions resulting from application of rolling moment without secondary yawing moment showing effect of different degrees of damping; $L_pN_r-L_rN_p$; C_l=0.04, C_n=0.

It may be shown that the question of whether the airplane tends to sideslip inward or outward at the beginning of a rolling motion depends on the magnitude of N_p compared with g/U_0. As rolling commences from level flight the yawing tendency due to the rolling (which is usually positive above the stall) causes the downgoing wing to be dragged back, creating an outward sideslipping tendency. This tendency is opposed by the action of gravity when the plane is banked, tending to produce inward sideslip. The condition that the outward and inward accelerations cancel is that

$$rU_0=g\varphi \tag{12}$$

assuming sin φ equal to φ. The angular acceleration in yawing requisite to this condition is

$$\frac{dr}{dt}=\frac{g}{U_0}p \tag{13}$$

In a rolling disturbance the yawing angular acceleration will be due only to N_p, or:

$$\frac{dr}{dt}=pN_p \tag{14}$$

Hence the condition that a rolling disturbance from level flight result in neither outward nor inward sideslipping is that

$$N_p=\frac{g}{U_0} \tag{15}$$

and the relative magnitudes of these quantities may be taken as an indication of the resultant tendency.

The airplane may diverge in the combined rolling and yawing motion previously discussed without sideslipping although such will not generally be the case. Near stalling angles the magnitude of the dihedral effect of the wings increases enormously (especially if the actual dihedral angle is small) and, if the tendency of the airplane is to sideslip outward while rolling and turning, any divergence in the rolling and yawing motion (indicated by negative $L_pN_r-L_rN_p$) will be greatly aggravated. The question of whether the dihedral effect will increase the instability is determined by the sign of the quantity (N_p-g/U_0). The magnitude of the effect of the sideslipping tendency thus determined obviously depends on the stability derivatives in sideslip L_β and N_β or, more conveniently, on L_β/N_β. The values of N_p computed for the stalled-flight conditions b and c were considerably larger than g/U_0, indicating that the natural tendency would be toward outward sideslip during a lateral maneuver. In such cases N_β would exert a stabilizing influence, tending to straighten out the skid. The values of these sideslipping criterions for the cases shown in figures 12 to 16 are given in table III.

TABLE III.—CONTROLLABILITY CRITERIONS FOR CASES SHOWN IN FIGURES 12 TO 16

Designation (see fig. 12)	Damping of rolling L_p	Combined damping $L_pN_r-L_rN_p$	Sideslip indication N_p-g/U_0	Sideslip stability factor $-\frac{L_\beta}{N_\beta}$
	(1)	(2)	(3)	
aaa	−1.75	1.61	−0.60	2.73
aab	−1.75	−.29	.13	2.73
acb	−1.75	−.29	.13	3.76
baa	0	.52	−.60	2.73
bab	0	−1.38	.13	2.73
bba	0	.52	−.60	3.40
bcb	0	−1.38	.13	3.76
caa	3.50	−1.65	.60	2.73
bbc	0	−2.76	.66	3.40

[1] Positive values indicate instability.
[2] Negative values indicate instability.
[3] Positive values indicate outward sideslipping tendency.

Possible modifications of characteristics to improve stability above the stall.—Of the factors influencing the lateral controllability, the stability characteristics that depend on the moments developed by the wings appear to be most important, since it is to be expected that they will be changed most by stalling. In addition to the damping in rolling, the wing moment characteristics that show marked change at the stalling point and contribute to the instability are L_r, L_β, and N_p. The factor L_r, proportional to the rolling moment due to yawing, depends on the lift coefficient and on the spanwise distribution of lift. Obviously, the greater the lever arm of the supporting lift, the greater L_r will be and, since it is desired to make L_r smaller, tapering the wings or shortening their span should help. The factor L_β at normal angles of attack depends also on this spanwise lever arm of the lift and on the dihedral angle. At stalling angles different tip shapes have considerable effect and the relation between the dihedral angle and the rolling moment reverses, the greatest moment being shown by the straight wing with square or upturned tips. (See fig. 11.) Here also, shortening the span and tapering the wing should improve conditions. The use of a moderate dihedral angle appears desirable in the stalled condition. The other wing characteristic, N_p, would be favorably affected by shortening the span of the wings. Here its magnitude depends mainly on the rate of increase of the profile drag of the wings and on the effective arm of the increase. If no damping in rolling (L_p) is present, there will be no induced N_p but in this case the slope of the wing profile-drag curve is almost certain to be very great, more than accounting for the induced effect. (See appendix I.) Taper or washout of the wings should help this situation. The provision of damping in rolling calls for keeping the wing tips from stalling; this requirement is compatible with all the others mentioned except that for small L_r. The desirability of maintaining the damping, however, far outweighs this consideration.

In the consideration of modifications of wing design to improve the controllability at high angles, it is important to take account of the premature tip-stalling phenomena exhibited by tapered wings. As was pointed out in the previous discussion, reducing the lift and the slope of the drag curve near the tips would lead to improved conditions. If this improvement is effected simply by tapering the wings, however, the net result may be detrimental to controllability on account of the premature loss of roll damping due to the stalling of the tips. In the case of any wing with an extreme reduction of chord, the downwash distributes itself in such a manner as to tend to maintain a more uniform distribution of the actual lift, so that the lift coefficient, and hence the effective angle of attack, of the reduced-chord sections is greater than at other sections. Pressure-distribution tests show that the tip portions of a 5:1 tapered wing reach their maximum lift coefficients at angles as much as 5° below the stalling angle of the center portions of the wing. Thus,

tapering the wings cannot be expected to improve the controllability at low speeds unless the taper is accompanied by some washout, or unless other provision is made to prevent the tips from stalling.

It may be inferred from the foregoing discussions that the effects of high aspect ratio will be detrimental to controllability and stability above the stall. It is easily seen how the unstable tendencies of the wings would be more unfavorable to controllability if the wings were of large span. If the span is large in proportion to the lever arm of the rudder control, the wings may easily develop yawing tendencies that will completely overpower the rudder moments. Furthermore, since rapid yawing motion induces a rolling moment through L_r, it is important to provide a large damping in yawing as an indirect check on the rolling as well as on the yawing motions. Thus it appears that considerable tail length and fin area are desirable to increase both N_r and N_β. Inasmuch as there ordinarily exists a great disproportion between the dampings in rolling and yawing below the stall, it is probable that fairly large increases in N_r would be permissible without causing undesirable stiffness of the rudder control at high speeds. Increasing N_β by using larger vertical tail surfaces is especially desirable because in that way the available rudder control is increased. Data on conventional airplanes show that the rudders used produce the weakest of the three controlling moments; their maximum moment is often smaller than the secondary yawing moment of the ailerons, yet the rudder deals with the largest moment of inertia of the airplane and should be the most effective control in checking the unstable yawing tendencies of the wings (as, for instance, in spinning). It appears that considerable improvement in these characteristics could be effected by enlarging the fin surface of conventional machines. If the increased rudder control is found to be undesirable at high speed because of too great sensitiveness, a corresponding increase in N_r, the damping in yawing, should remedy this trouble and still further improve the controllability at high angles. Thus if the tail is made longer as the vertical surface is increased, the control characteristics at high speed should not be unfavorably affected. It appears unlikely, however, that such improvements could result in the retention of satisfactory control above the stall if the autorotational tendencies shown by ordinary wings in wind-tunnel experiments are developed.

TURN MANEUVERS

The foregoing computations were designed to represent the procedure employed in a particular type of flight test to compare the efficiency of various control devices purely on the basis of their independent action in producing roll. Another type of flight test, qualitative in nature, consisted of performing normal turn maneuvers with the airplane, using the device in conjunction with the other controls and observing the amount of coordination that was required.

The first type of computation together with the flight tests showed that the roll-producing effectiveness of some devices would be influenced by the occurrence of considerable incidental sideslipping, much of the apparent improvement due to favorable secondary yaw being obtained by the production of outward sideslipping.

Since it was not known in any quantitative way how the presence of this sideslipping tendency due to the secondary aileron moments would affect the controllability in making actual turn maneuvers, it was decided to make an analysis of these conditions, representing analytically as nearly as possible the second stage of the flight tests.

EXPLANATION OF METHOD OF COMPUTATIONS

In certain instances in the former computations a simple sort of controlled maneuver was used in which an initial deflection of the ailerons was reversed, representing an attempt to check a motion previously produced by them. (See fig. 13.) It was realized that an extension of this procedure could be applied to the present problem by means of step-by-step integrations of the motion due to any arbitrarily specified way of applying the controls. This adaptation of the former method would have required a knowledge of the control manipulations necessary to perform a normal turn, as well as lengthy step-by-step calculations. For these reasons it was considered more feasible to predetermine the actual motion of the airplane than to fix on an arbitrary way of applying the controls. Furthermore it seemed reasonable to presume that the pilot of an airplane would conform his use of the controls to suit a desired maneuver, rather than to prescribe beforehand his use of the control and accept whatever motion of the airplane followed. He would then judge the effectiveness of the control by the way it had to be used to obtain a desired result.

As the outcome of these considerations, the problem of investigating turn maneuvers presented itself in a way inverse to the previous problems. Here the motion of the airplane was given and the requisite use of the controls was sought. Previously the airplane motions had been determined from the controlling accelerations by integration, whereas here the accelerations incident to a given motion were to be determined; thus the process would simply be a differentiation.

Periodic or trigonometric functions of the time naturally suggested themselves for the representation of the angular velocities and displacements during a turn maneuver. By the use of trigonometric functions of the time, any conceivable maneuver of the airplane that begins and ends in level flight may be specified: that is, any given manner of varying the attitude or

angular velocity of the airplane during a given interval may be described by a formula such as

$$p, \text{ or } \varphi, \text{ or } \ldots \text{ etc.}$$
$$=A_1 \sin nt+A_2 \sin 2nt+A_3 \sin 3nt+\text{etc.} \qquad (16)$$

By a suitable choice of n the maneuver may be made to extend over as long or as short a time as desired.

In the present case it was intended that the airplane roll up to a moderate angle of bank, starting with the wings level, and check its rate of rolling so as to maintain this bank angle steadily, then roll back to the level condition after a definite time interval. Throughout this interval the airplane was to be yawing appropriately while banking and in the correct amount to prevent sideslipping during every part of the maneuver. Thus the turn was to be "perfect" in that no sideslip was permitted and the coordination of the lateral controls (ailerons and rudder) necessary to accomplish such a maneuver was to be studied.

A few trials in plotting cosine curves against time showed that the expression

$$\varphi=-A_1 \cos nt-\tfrac{1}{4} A_1 \cos 2nt+\text{constant} \qquad (17)$$

would represent a bank that assumed a steady angle at the midpoint of the maneuver, starting with zero at the time $t=0$ and becoming zero again at $t=\pi/n$. Arranging for the bank to become steady at the midpoint of the maneuver and choosing nt so as not to coincide with the natural period of the free motions of the airplane obviated the possibility of any reinforced oscillation phenomena during the maneuver. The form of the curve of bank angle against time plotted to this formula is shown in figure 17.

In order to attain the specified bank at every instant, a definite rate of rolling is required at all times, which is obviously found by differentiating the bank equation; thus

$$p=\frac{d\varphi}{dt}=nA_1 \sin nt+\frac{2n}{4}A_1 \sin 2nt \qquad (18)$$

In order for the airplane to turn without sideslipping, there must be a coordination between the banking and yawing at all times. The outward and inward accelerations must cancel, that is:

$$rU_0=g \sin \varphi \qquad (19)$$

(See equation (12).)

This equation enables the calculation of r from φ, assuming the condition that

$$r=\frac{g}{U_0} \sin \varphi \qquad (20)$$

is satisfied. The curve of yawing angular velocity plotted against time is thus very similar in shape to the bank-angle curve, reaching a steady value at its midpoint.

The specification of the angular velocities and angles of the airplane in the foregoing manner is analogous to the specification of constraints of the motion. The total accelerations necessary to constrain the airplane to the specified motions are calculated by differentiating the expressions for the angular velocities, p and r. (See equations (17) and (18).)

$$\frac{dp}{dt}=\frac{d^2\varphi}{dt^2}=n^2A_1 \cos nt+\frac{4n^2}{4}A_1 \cos 2nt \qquad (21)$$

and

$$\frac{dr}{dt}=\frac{g}{U_0}p \cos \varphi \qquad (22)$$

These accelerations are not furnished altogether by the controls but have components due to the air reactions on the moving airplane. The air reactions are calculated from the resistance derivatives and, when deducted from the total accelerations, give the components necessarily supplied by the deflected controls. Thus the acceleration supplied by the rolling control will be

$$\delta L_\delta=\frac{dp}{dt}-pL_p-rL_r \qquad (23)$$

If the application of rolling control is accompanied by a secondary (adverse or favorable) yawing moment, the rudder control will have to accommodate this moment as well as the residual acceleration of the yawing motion. This secondary yawing moment may be considered to be a function of the rolling moment and its acceleration written as $f(\delta L_\delta)$; then

$$\delta N_\delta=\frac{dr}{dt}-pN_p-rN_r-f(\delta L_\delta) \qquad (24)$$

Equation (24) gives the amount of rudder coordination necessary with a given aileron-control device. The rolling- and yawing-moment coefficients corresponding to these accelerations may be calculated by known means from the speed of flight and the airplane dimensions.

In the derivation of the equations for the turn maneuvers no account was taken of the pitching motion involved. Obviously if a banked airplane is turning without loss of altitude there will be a component of pitching involved in the motion. As was explained in the description of the step-by-step method of computation the pitching motion may be considered separately and independently of the lateral motions since the airplane is symmetrical about the plane in which pitching occurs. Presumably, the only ways in which pitching motions can influence the lateral motions are by a change of speed or attitude introducing changes in the lateral-stability derivatives or by gyroscopic couples. In the case of a prescribed turn maneuver the maximum gyroscopic couple may be estimated in advance and the relative importance of its effect may be foreseen. The other secondary influence may be partly accounted for

by assuming a certain increased speed throughout the turn. Either the air speed or the attitude will, in general, vary continuously throughout the turn if no altitude is lost or gained. For turns up to 30° angle of bank the change in stability derivatives thus produced will be slight and may be satisfactorily compen-

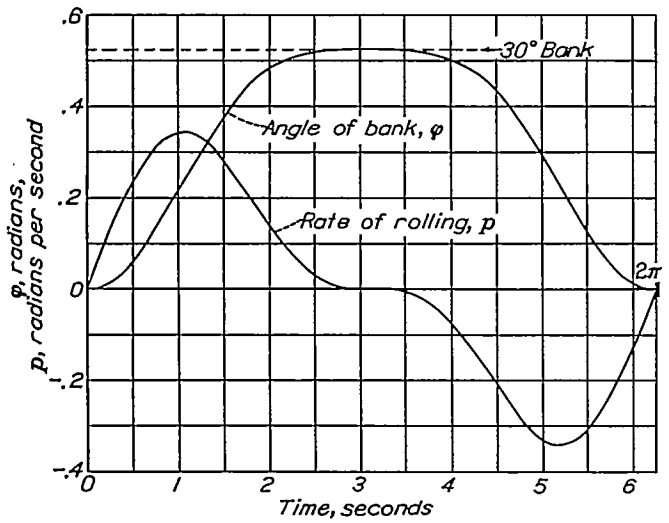


FIGURE 17.—Angle of bank and rate of rolling during specified turn maneuver. $\varphi=-0.262[\cos t+\frac{1}{4}\cos 2t]+0.327$; $p=d\varphi/dt$.

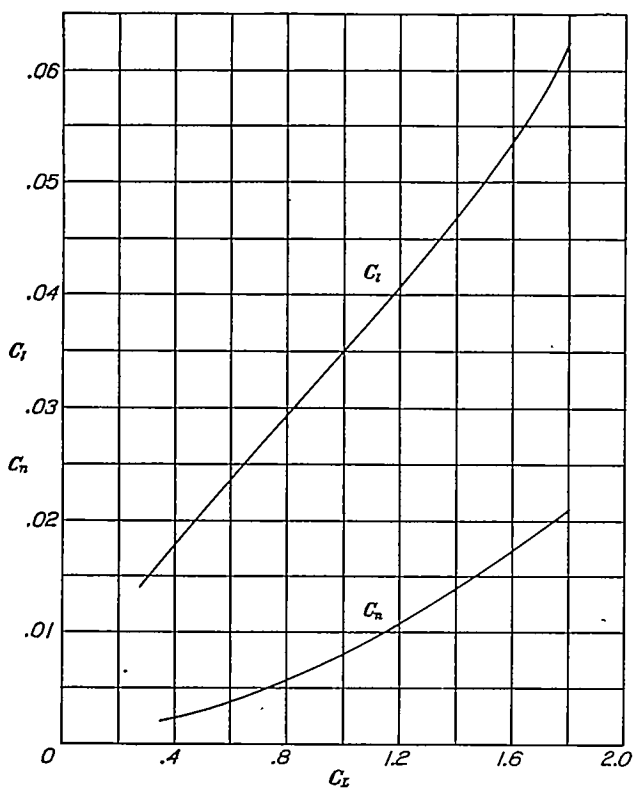


FIGURE 19.—Maximum control-moment coefficients required in performing a turn maneuver at various lift coefficients; average airplane (for 30° bank turn completed in 6.28 seconds).

sated by assuming an average value of the speed U somewhat greater than that for level flight. This speed may be calculated from the relation:

$$q=\frac{q_0}{\cos\varphi} \tag{25}$$

where q_0 is the dynamic pressure at steady-flight speed and φ is the angle of bank at which the airplane is assumed to lose or gain no altitude.

RESULTS AND DISCUSSION

The foregoing procedure was applied to the case of the average airplane performing 30° banked turns at various speeds. The time taken to complete the specified maneuver was chosen as approximately 6.28 (2π) seconds, since at the lowest speeds under consideration comparatively large rolling and yawing moments were required to execute the maneuver with this rapidity. Inasmuch as the angle-of-bank relation was held the

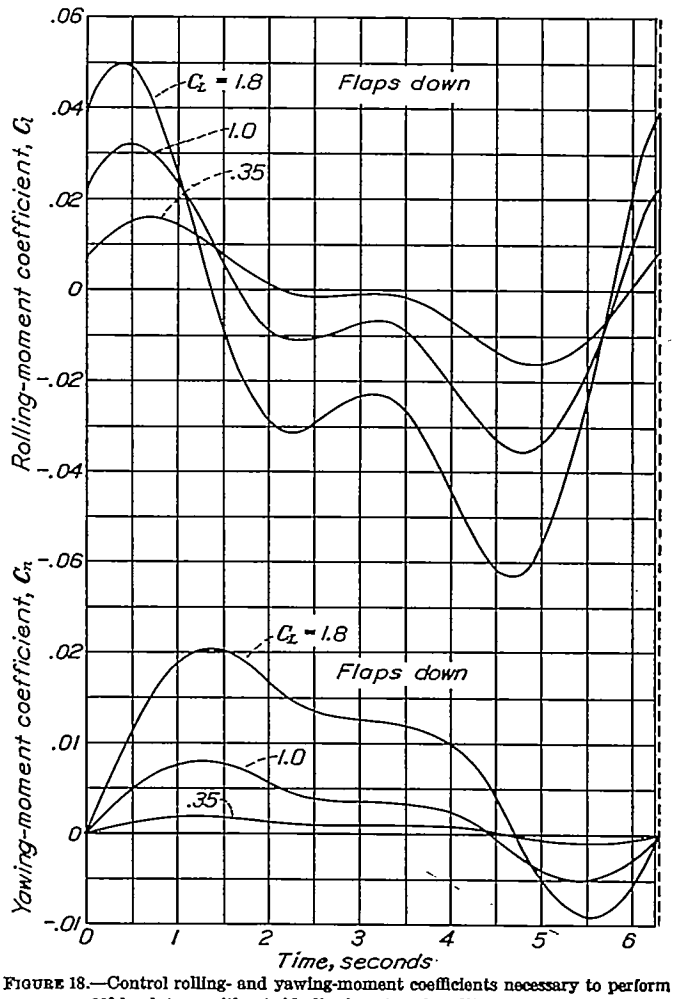


FIGURE 18.—Control rolling- and yawing-moment coefficients necessary to perform 30° bank turn without sideslipping at various lift coefficients.

same for all speeds, the rate of yawing was necessarily different and hence the actual angle of turn, or the changed heading of the airplane, was different for the different speeds. As in the previous computations, lift coefficients of 0.35, 1.0, and 1.8 were assumed, although the corresponding speeds were increased somewhat over those in the previous computations to account for the additional lift while turning, as previously explained. With the assumption of no loss of altitude at 30° bank, the speeds were increased by the factor

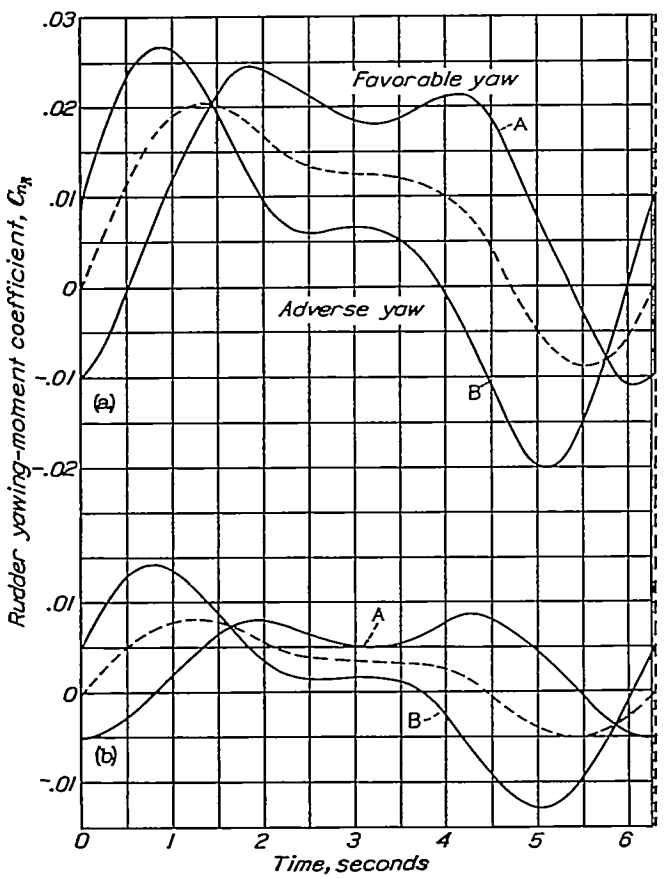
$$\sqrt{\frac{1}{0.866}}=1.074$$

The values of L_p, L_r, N_p, N_r, corresponding to the given lift coefficients, were also multiplied by this factor. (See appendix I.)

The curves of rolling motion and angle of bank calculated for these maneuvers are those shown in figure 17. The formula for the bank angle was

$$\varphi = -0.262\ (\cos t + 1/4 \cos 2t) + 0.327 \qquad (26)$$

reaching a maximum of 30° at π seconds. This formula determined the angular velocities and accelerations by the principles already demonstrated. Inasmuch as the turn reaches a steady rate at its midpoint, the whole



(a) Low speed, flaps down, C_L=1.8.
(b) Gliding speed, C_L=1.0.

FIGURE 20.—Rudder control-moment coefficients during 30° bank turn, showing effect of secondary aileron yawing moments on rudder control necessary to perform a turn maneuver without sideslipping. Assumed aileron yawing moment, ±¼ aileron yawing moment. C_{n_A} is secondary aileron yawing-moment coefficient.

C_{n_A}= ¼C_{l_A} curve A $\qquad$ C_{n_A}=−¼C_{l_A} curve B

maneuver may be presumed to be of any time extent by assuming a continuation of this steady point, which occurs at π seconds.

The results of a series of these computations showed principally the effect of flight speed on the degree of control deflection necessary to perform a given maneuver and the effect of favorable-yaw and of adverse-yaw ailerons on the amount of rudder control required. Figure 18 shows the rolling- and yawing-moment coefficients necessary to accomplish the maneuver at speeds corresponding to the three different lift coefficients. For the average airplane these were:

C_L	U
0.35	161 feet per second.
1.0	95 feet per second
1.8	71 feet per second (flaps deflected).

In this case no secondary aileron yawing moments were included and such moment coefficients would have to be added to or deducted from the yawing-moment curves. These computations showed that the maximum yawing moment necessary at the lowest speed was 10 times as great as that at high speed, while the maximum rolling-moment coefficient increased only 4 times under the same circumstances. Figure 19 illustrates this increase of coefficient necessary to perform the specified maneuver in the same time at the lower speeds.

Figure 20 shows the effects of favorable and adverse secondary aileron yawing moments on the rudder control necessary throughout the turn. Positive yawing moments indicate a setting of the rudder in a direction to aid the turning. It will be noted that the existence of any secondary aileron moment calls for a counteracting movement of the rudder applied simultaneously with the ailerons at the beginning of the turn. With no secondary aileron moments the curves show that the simultaneous initial deflection of both ailerons and rudder is not required, the turn being initiated by the ailerons alone with the rudder being applied after the start. In the case of favorable secondary yawing moments an initial setting of the rudder opposite to the direction of the turn is required, while on beginning the recovery the rudder has to be moved slightly in a direction that would normally tend to continue the turning. It appears that ailerons giving no secondary yawing moments of either sign would require the least rudder coordination in making turns without sideslipping.

CONCLUSIONS

1. The agreement of the computations with the results of flight tests verifies the usefulness of the method utilizing stability derivatives for the study of controllability both above and below the stall.

2. The angle of bank produced in 1 second, φ_1, by a full deflection of the lateral control may be taken as a relative measure of the control effectiveness. In the case of a conventional airplane this measure is given by a simple formula involving the static rolling and yawing moments produced by the control, namely:

$$\varphi_1 = \text{constant} \times C_l + \text{constant} \times C_n$$

3. The effect of secondary adverse yawing moments on the aileron control may be moderated by increasing the moment of inertia about the yaw axis, although it is to be expected that the power of the rudder will be correspondingly reduced. Increasing the moment of inertia about the roll axis should have little direct

influence on the lateral-control effectiveness with a given rolling-control moment.

4. The tendency for a given adverse yawing moment to render the lateral control ineffective becomes greater with increasing dihedral. In no case should the ratio of the control adverse yawing moment to the rolling moment be allowed to exceed (in absolute magnitude) either:

(a) The ratio of yawing to rolling moment acting on the airplane in sideslip; or

(b) The ratio of yawing to rolling moment acting on the airplane in yawing.

5. It appears that ailerons giving nearly zero yawing moment would require the least coordination of the rudder control in executing turn maneuvers without sideslip.

6. The study of conditions above the stall indicates that satisfactory control cannot be expected unless some provision is made to maintain the damping in rolling at these angles.

7. For control at high angles of attack it is important that the damping in both rolling and yawing be maintained above a definite minimum to avoid an uncontrollable form of instability arising from the interaction of these motions. The minimum damping is given by the condition that

$$L_p N_r > L_r N_p$$

This condition appears to be next in importance to direct damping in rolling.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *April 20, 1936.*

TABLE I.—STEP-BY-STEP COMPUTATION OF MOTION OF AVERAGE AIRPLANE

[$C_L=1.0$; $C_l=0.04$; $C_n=0$]

Time	$\left(\frac{dp}{dt}\right)_n$	p_n	φ_n	$\left(\frac{v'}{U_0}\right)_n$	$\left(\frac{dr}{dt}\right)_n$	r_n	β_n'	β_n	ΣL_i			ΣN_i		
	$\delta L_i+\Sigma L_i$ ($\delta L_i=1.68$)	$\left(\frac{dp}{dt}\right)_{n-1}\times\Delta t+p_{n-1}$	$p_{n-1}\times\Delta t+\varphi_{n-1}$	$\frac{32.2}{88.5}\times\Delta t\times\sin\varphi_{n-1}+\left(\frac{v'}{U_0}\right)_{n-1}$	$\delta_{n-1}+\Sigma N_i$ $\delta N_i=0$	$\left(\frac{dr}{dt}\right)_{n-1}\times\Delta t+r_{n-1}$	$-r_{n-1}\times\Delta t+\beta_{n-1}$	$\beta_n'+\left(\frac{v'}{U_0}\right)_n$	$p_nL_p+r_nL_r+\beta_nL_\beta$			$p_nN_p+r_nN_r+\beta_nN_\beta$		
									$p_n\times-3.23$	$r_n\times1.88$	$\beta_n\times-1.11$	$p_n\times-0.301$	$r_n\times-0.663$	$\beta_n\times2.04$
sec.														
0	1.680	0	0	0	0	0	0	0	0	0	0	0	0	0
.05	1.409	.084	0	0	−.025	0	0	0	−.271	0	0	−.025	0	0
.10	1.181	.154	.004	0	−.045	−.001	0	0	−.497	−.002	0	−.046	.001	0
.20	−.790	.272	.019	0	−.078	−.006	0	0	−.879	−.011	0	−.082	.004	0
.30	.518	.351	.046	.001	−.093	−.014	.001	.002	−1.134	−.026	−.002	−.106	.009	.004
.40	.329	.403	.081	.003	−.096	−.023	.002	.005	−1.302	−.043	−.006	−.121	.015	.010
.50	.199	.436	.121	.006	−.089	−.033	.004	.010	−1.408	−.062	−.011	−.131	.022	.020
.60	.109	.456	.165	.010	−.074	−.042	.007	.017	−1.473	−.079	−.019	−.137	.028	.035
.70	.050	.467	.211	.016	−.054	−.049	.011	.027	−1.508	−.092	−.030	−.141	.032	.055
.80	.009	.472	.257	.024	−.024	−.054	.016	.040	−1.525	−.102	−.044	−.142	.036	.082
.90	−.013	.473	.304	.033	.005	−.056	.021	.054	−1.528	−.105	−.060	−.142	.037	.110
1.00	−.027	.472	.351	.044	.039	−.055	.027	.071	−1.525	−.103	−.079	−.142	.036	.145

APPENDIX I

CALCULATION OF STABILITY DERIVATIVES [1]

In the report all moments and angular velocities are measured from axes fixed in the airplane along the directions perpendicular and parallel to the relative wind in the steady flight just previous to the maneuver computed.

In the computations of the stability derivatives as well as in the consideration of their modification by alteration of the design of an airplane, it is convenient to separate those governed by the wing characteristics from those depending mainly on the body and the tail.

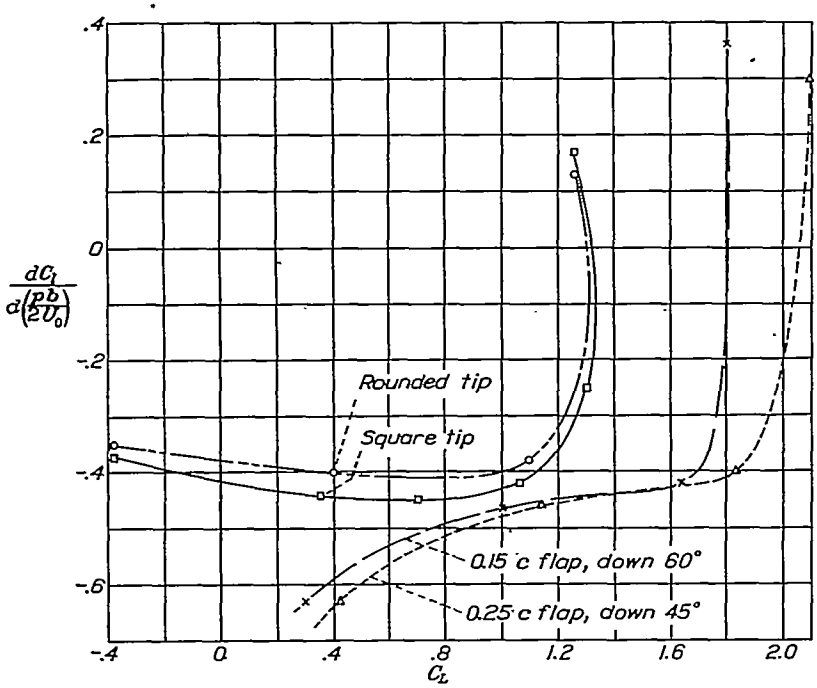


FIGURE 21.—Damping in rolling of rectangular wing. Aspect ratio, 6; 7- by 10-foot wind-tunnel measurements.

In the case of conventional airplanes, the derivatives that depend almost wholly on the wings are

$$L_p,\ L_r,\ L_\beta, \text{ and } N_p$$

The other two factors considered in the report, N_r and N_β, depend primarily on the disposition and area of the vertical tail and on the fuselage.

ROLLING ACCELERATION DUE TO ROLLING, L_p

The factor L_p may be determined from the results of tests of the damping in rolling of wings, such as the tests that have been made in the 7- by 10-foot wind tunnel. The test results are given in the form of the coefficient $\frac{dC_l}{d\left(\frac{pb}{2U_0}\right)}$ and are summarized in figure 21, which shows values of the coefficient measured on rectangular Clark Y wings of aspect ratio 6. The effects of deflected split flaps and tip rounding are also shown. Correction factors to convert these values to those for tapered wings and wings of different aspect ratio are given in figure 22. These correction factors are based on theoretical calculations of the load distribution on wings having a uniform twist which, in effect, reproduced the conditions encountered by a rolling wing as far as the rolling moment is concerned. The data for these corrections were deduced from calculations given in reference 11.

The actual damping moment of a full-size wing calculated from the coefficient is

$$L=p\times\frac{dC_l}{d\left(\frac{pb}{2U_0}\right)}S\frac{\rho}{2}U_0\frac{b^2}{2} \qquad (27)$$

The derivative, as used in the report, is obtained by dividing the coefficient of p by the moment of inertia of the airplane in rolling: i. e.,

$$L_p=\frac{dC_l}{d\left(\frac{pb}{2U_0}\right)}S\frac{\rho}{2}U_0\frac{b^2}{2mk_x^2} \qquad (28)$$

It will be readily appreciated that parts of the airplane other than the wings contribute only a negligible amount of this damping; for example, if the tail plane has an area 15 percent of that of the wing and a span of 25 percent b, its contribution will be less than

$$0.15\times(0.25)^2=0.019, \text{ or 2 percent}$$

of that due to the wing.

The rolling moment due to rolling of a biplane may be estimated by using its equivalent monoplane aspect ratio in figure 22.

For the damping of rolling above stalling angles, wind-tunnel tests show that there is no consistent linear relation between the damping moment and the rate of rolling even at very slow rates; hence there actually exists no definite L_p in the sense previously defined. Arbitrary values may be assumed to repre-

[1] The authors desire to acknowledge valuable aid received from Mr. C. H. Zimmerman of the laboratory staff in the preparation of this section.

sent roughly certain conditions, as was done in the described stalled-flight computations. In the case of wings with devices to prevent stalling at the tips, recourse must be had to wind-tunnel tests.

ROLLING ACCELERATION DUE TO YAWING, L_r

The rolling moment developed by a wing in circling flight may be easily calculated from the consideration that this motion brings about a difference in velocity along the span. If the yawing velocity is r and the spanwise distance from the reference origin (center of gravity) is y, this additional velocity will be ry. The lift on an element of the wing is proportional to the square of the whole velocity, or:

$$(U_0 \pm ry)^2 = U_0^2 \pm 2ryU_0 + (ry)^2 \tag{29}$$

The rolling moment produced by the change in lift on either side of the wing is directly proportional to r. A

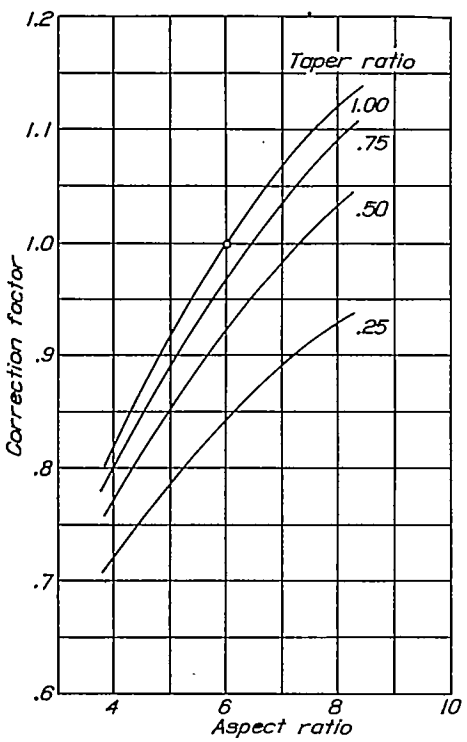


FIGURE 22.—Factors for correcting wind-tunnel values of $dC_l/d\left(\frac{pb}{2U_0}\right)$ for aspect ratio and taper.

simple integration shows the moment for a straight wing to be:

$$L = r\frac{b^2}{6}S\frac{\rho}{2}U_0C_L \tag{30}$$

if the lift is distributed uniformly along the span. Such a distribution is approximated in the case of a rectangular wing at stalling angles, hence the foregoing formula was used in the stalled-flight computations. Below the stall the actual distribution of lift on the wings in circling flight should be taken into account. This distribution is modified somewhat by the fact that the induction of the circular trail of vortices differs from the induction in straight flight. These phenomena have been treated by Glauert and Wieselsberger for the cases of rectangular and elliptical wings in circling flight and curves derived from their calculations are shown in figure 23. (See reference 12.) The derivative L_r is obtained from the coefficient by the formula

$$L_r = F_1C_LS\frac{\rho}{2}U_0\frac{b^2}{2mk_x^2} \tag{31}$$

It appears that the value of $\frac{1}{6}F_1$ previously calculated from simple integration as one-sixth should be more nearly one-eighth for aspect ratio 6, as indicated by the chart. Although no calculations have been made for tapered wings, it may be presumed that the interpolated curves given in figure 23 will apply with good approximation. The part of L_r due to the body and tail will be treated in a later paragraph.

ROLLING ACCELERATION DUE TO SIDESLIP, L_β

Measurements of the rolling moment due to sideslip have been made on a large number of wing models in the 7- by 10-foot wind tunnel. The results of these

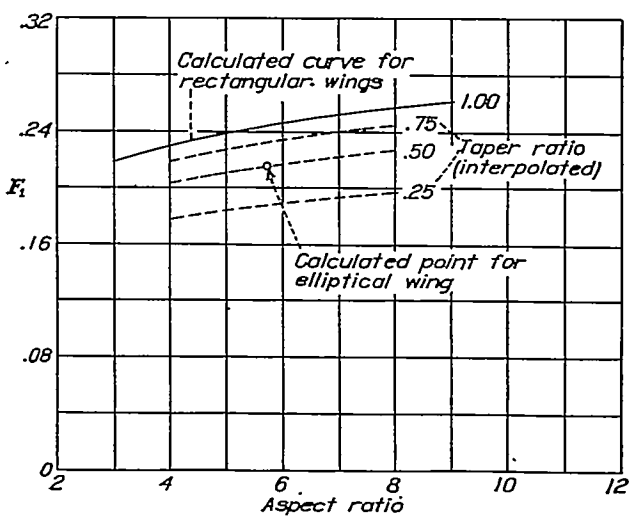


FIGURE 23.—Factors for calculating rolling moment in circling flight.

$dC_l/d\left(\frac{rb}{2U_0}\right) = F_1C_L$

tests are summarized in figure 24, which shows the influence of tip rounding and deflected split flaps on the dihedral effect of Clark Y wings without actual dihedral angle. Further tests made on wings with varying degrees of dihedral showed that the additional effect due to this angle was the same regardless of the tip shape or the lift coefficient of the wing (below the stall). Sweepback of the wings is known to have an effect similar to dihedral, although comparatively few tests have been made. Unlike the rolling moment due to dihedral angle, however, the rolling effect of sweepback appears to be approximately proportional to the lift coefficient, disappearing at zero lift as would be expected. Presumably, its effect may be added to the others as in the case of the dihedral. These considerations result in the following formula for the total rolling moment in sideslip

$$\frac{dC_l}{d\beta}=\left(\frac{dC_l}{d\beta}\right)_{\Gamma,\Lambda=0}+\Gamma\frac{\partial}{\partial\Gamma}\left(\frac{dC_l}{d\beta}\right)+\Lambda\frac{\partial}{\partial\Lambda}\left(\frac{dC_l}{d\beta}\right) \quad (32)$$

where Γ is dihedral angle and Λ is angle of sweepback.

An analysis of the available data indicates the following values for the parameters:

$$\left.\begin{aligned}&\frac{\partial\left(\frac{dC_l}{d\beta}\right)}{\partial\Gamma}=-0.012\\ &\text{(see reference 13),}\\ &\text{and}\\ &\frac{\partial\left(\frac{dC_l}{d\beta}\right)}{\partial\Lambda}=-0.0045C_L\end{aligned}\right\} \quad (33)$$

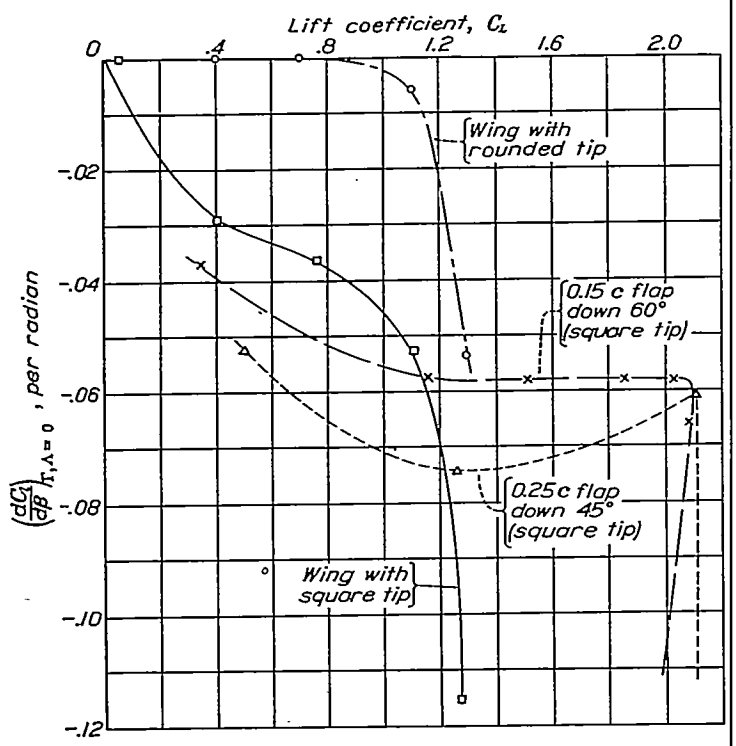


FIGURE 24.—Rolling moment due to sideslip. Untapered wings without dihedral; aspect ratio, 6; 7- by 10-foot wind-tunnel measurements.

where $dC_l/d\beta$ is in terms of radians and Γ and Λ are measured in degrees. The derivative L_β follows from the formula:

$$L_\beta=\frac{dC_l}{d\beta}Sq\frac{b}{mk_x^2} \quad (34)$$

Inasmuch as the wind-tunnel tests were of rectangular wings of aspect ratio 6, the formula (33) applies directly to them. Correction factors for calculating the rolling moment due to the dihedral of yawed wings of different aspect ratios and taper ratios are given in figure 25. These corrections were deduced from theoretical calculations made at the Laboratory (reference 11) on the span load distribution of wings having their right and left semispan portions set at different angles of attack and are somewhat different from those deduced previously for the damping in rolling.

Above stalling angles none of the given formulas or correction factors apply. In this region a straight wing shows a far greater rolling tendency when yawed than wings with either sweepback or dihedral. Adding either sweepback or dihedral tends to reduce this tendency and may on this account be desirable to a certain degree. Tests of wings with very large sweepback, such as are used on tailless airplanes, have been made in which the rolling moment due to yaw actually reversed its sign when the stall was reached.

YAWING ACCELERATION DUE TO ROLLING, N_p

It is assumed that the effect of a rolling motion of the wing can be replaced by a relative rolling motion

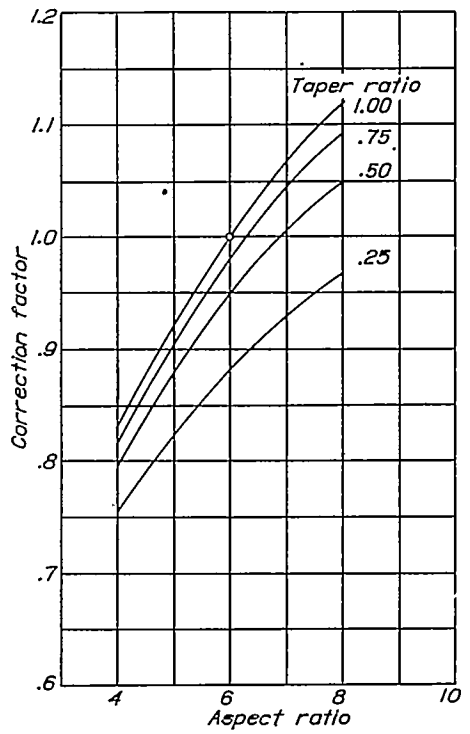


FIGURE 25.—Factors for correcting wind-tunnel value of $\partial\left(\frac{dC_l}{d\beta}\right)/\partial\Gamma$ for aspect ratio and taper.

of the air about the X axis of the airplane. Thus in positive rolling the relative air stream is rising toward the right wing tip and descending on the left. The lift vectors, being perpendicular to the relative wind at each point of the span, are inclined forward with respect to the Z axis on the right and backward on the left, resulting in a negative yawing moment for positive rolling of the wing. (This varying resolution of the lift vectors along the span is unimportant in computing the rolling moment due to rolling since the angle $pb/2U_0$ is small.)

In addition to the changed resolution of the lift vectors along the span, there is an increased drag on the downgoing wing that tends to reduce the negative yawing tendency. It should be noted that an asymmetrical change in the lift distribution, such as that caused by rolling, results in greater changes in the induced drag at various sections of the wing than would be produced by symmetrical lift changes. (See

reference 14.) Hence the uncorrected results of measurements made on the wing in direct lifting cannot be used in computing the rolling or yawing moments of a rolling wing.

Figure 26 shows the resolution of the lift at a point of the span y on the downgoing side of the wing. The air stream initially rising toward the section at the inclination py/U_0 is deflected somewhat by the resulting increased lift at that point so that the air meets the wing at the additional effective angle of attack, $\Delta\alpha_0=\frac{py}{U_0}-\Delta\frac{w}{U_0}$. This additional angle of attack may be found at each point of the span if the corresponding lift increment is known, since

$$\Delta\alpha_0=\frac{\Delta C_L}{\left(\frac{dC_L}{d\alpha}\right)_0} \tag{35}$$

where $\left(\frac{dC_L}{d\alpha}\right)_0$ is the slope of the lift curve for infinite aspect ratio. The lift vector on the wing in straight flight C_L is increased by the amount ΔC_L and inclined forward through the angle $\Delta\alpha_0$. If the usual assumptions regarding small angles are made, the total effect may be integrated along the span as

$$N=-2\frac{\rho}{2}U_0^2\int_0^{b/2}C_L\times\Delta\alpha_0\times c\times y\times dy \tag{36}$$

It will be noted that it is unnecessary to consider the resolution of the lift increments ΔC_L by the angles $\Delta\alpha_0$ since they are sensibly equal and opposite on either side of the wing and their yawing effects cancel, resulting simply in a bending moment about the midpoint. Replacing $\Delta\alpha_0$ by $\frac{\Delta C_L}{\left(\frac{dC_L}{d\alpha}\right)_0}$ and calculating the coefficient

$$C_n=-\frac{2}{Sb\left(\frac{dC_L}{d\alpha}\right)_0}\int_0^{b/2}C_L\times\Delta C_L\times c\times y\times dy \tag{37}$$

Since $-\frac{2}{Sb}\times\Delta C_L\times c\times y\times dy=dC_l$, an approximate expression of this formula is

$$\left.\begin{aligned} C_n&=\frac{C_L}{\left(\frac{dC_L}{d\alpha}\right)_0}C_l \\ \text{whence}\quad \frac{dC_n}{d\left(\frac{pb}{2U_0}\right)}&=\frac{C_L}{\left(\frac{dC_L}{d\alpha}\right)_0}\frac{dC_l}{d\left(\frac{pb}{2U_0}\right)}\end{aligned}\right\} \tag{38}$$

This approximation is based on the assumption of constant lift coefficient across the span and hence corresponds to an elliptical wing. The resolution of this yawing moment along the general wind direction results in:

$$\frac{dC_n}{d\left(\frac{pb}{2U_0}\right)}=\frac{dC_l}{d\left(\frac{pb}{2U_0}\right)}\left[\frac{C_L}{\left(\frac{dC_L}{d\alpha}\right)_0}-\frac{C_L}{\pi R}\right] \tag{39}$$

Reference 11 gives the lift and lift-increment distributions for both rectangular and tapered wings and these may be used in conjunction with the foregoing formulas if a more accurate theoretical value of N_p is desired.

A component of N_p due to the profile-drag effect may be estimated by a simple integration, assuming the slope of profile-drag coefficient with effective angle of attack to be constant across the span. Thus, if $\left(\frac{dC_D}{d\alpha}\right)_0$ is the slope of the drag curve for infinite aspect ratio at the lift coefficient in question

$$\Delta N=2\frac{\rho}{2}U_0^2\int_0^{b/2}\Delta\alpha_0\left(\frac{dC_D}{d\alpha}\right)_0 cydy \tag{40}$$

or, making the same substitutions as before, the coefficient giving the effect of profile drag is

$$\frac{dC_n}{d\left(\frac{pb}{2U_0}\right)}=-\frac{dC_{D_0}}{dC_L}\frac{dC_l}{d\left(\frac{pb}{2U_0}\right)} \tag{41}$$

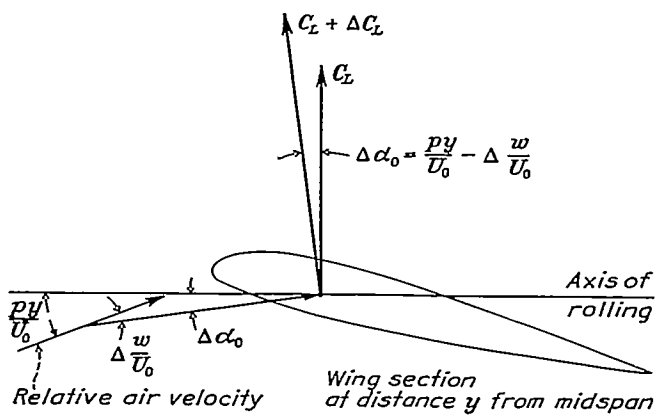


FIGURE 26.—Resolution of air velocity and lift at section of rolling wing.

where C_{D_0} is the profile-drag coefficient of the airfoil section. The final formula for N_p is

$$N_p=\frac{dC_n}{d\left(\frac{pb}{2U_0}\right)}S\frac{\rho}{2}U_0\frac{b^2}{2mk_Z^2} \tag{42}$$

where $\frac{dC_n}{d\left(\frac{pb}{2U_0}\right)}$ is the sum of the portions given by equations (39) and (41).

Above stalling angles the slope of the profile-drag coefficient with angle of attack reaches large values, and it is to be expected that N_p will change its sign. The foregoing theoretical formulas cannot be used at these angles, because the lift is no longer proportional to the angle of attack. A tentative formula for $\frac{dC_n}{d\left(\frac{pb}{2U_0}\right)}$ in the stalled condition is

$$\frac{dC_n}{d\left(\frac{pb}{2U_0}\right)}=\left[\left(\frac{dC_D}{d\alpha}\right)_0-C_L\right]\frac{4}{Sb^2}\int_0^{b/2}cy^2dy \tag{43}$$

or simply

$$\frac{dC_n}{d\left(\frac{pb}{2U_0}\right)}=\frac{1}{6}\left[\left(\frac{dC_D}{d\alpha}\right)_0-C_L\right] \tag{44}$$

for rectangular wings.

In the case of an airplane with a long fuselage, a certain increment of N_p at high angles of attack due to the effect of the body and fin must be considered, as will be explained later.

YAWING ACCELERATION DUE TO YAWING, N_r

Unlike the damping in rolling, the damping in yawing N_r, cannot be attributed to any single predominant factor. It is convenient, however, to consider it as primarily effected by the disposition and area of the vertical tail surface. Since only a few isolated experiments have been made for the determination of this derivative and since it is not known

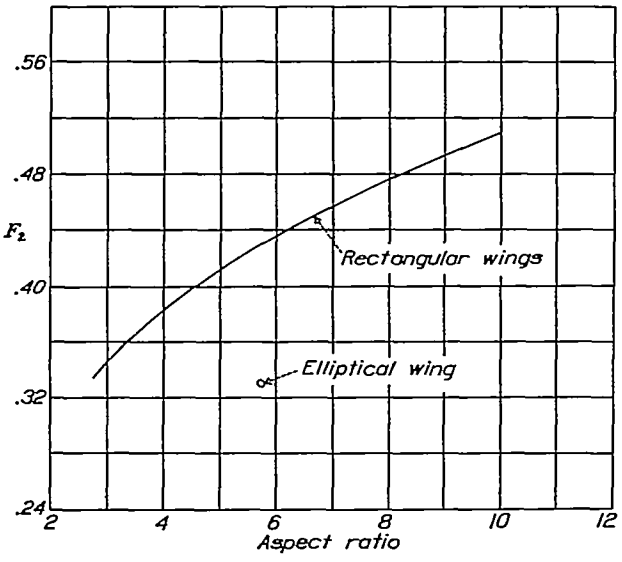


FIGURE 27.—Factors for calculating the yawing moment due to the induced-drag distribution in circling flight.

$dC_{n_i}/d\left(\frac{rb}{2U_0}\right)=-F_2C_{D_i}$

to what extent certain incalculable factors influence it, only a rough estimate of its value in any given case is possible.

The part of the damping of yawing due to the wings may be calculated from considerations similar to those employed in the determination of L_r. Here the changed drag distribution along the span in circling flight is to be considered and the resulting yawing moment found. The theoretical calculations of Glauert and Wieselsberger that were employed in the determination of L_r may be applied in this case as well. Here, however, it will be necessary to include the effect of profile drag of the wings and their attachments, since it is the actual magnitude of the drag that counts in determining N_r and not its rate of increase with angle of attack. On the assumption that the profile-drag coefficient is normally the same at all sections of the span, a simple integration (see L_r) gives the formula

$$\frac{dC_{n_0}}{d\left(\frac{rb}{2U_0}\right)}=-\frac{1}{3}C_{D_0} \tag{45}$$

for the part due to the profile drag of a rectangular wing. Figure 27 shows the results of the previously mentioned calculations, which were extended to the determination of the distribution of induced drag while circling. With the factor shown in the figure included, the formula for the total wing effect becomes

$$\frac{dC_n}{d\left(\frac{rb}{2U_0}\right)_w}=-F_2C_{D_i}-\frac{1}{3}C_{D_0} \tag{46}$$

where C_{D_i} is the induced-drag coefficient, i. e.,

$$C_{D_i}=\frac{C_L^2}{\pi R} \tag{47}$$

The part of N_r due to the vertical tail surfaces may be very simply calculated. The yawing angular velocity r about an axis through the center of gravity produces an effective sidewise velocity of the vertical tail equal to rl. Its change in angle of attack relative to the air stream is then rl/U_0. The yawing moment due to this effect is

$$N=\frac{rl}{U_0}\left(\frac{dC_y}{d\beta}\right)_f\frac{S_f}{S}S\frac{\rho}{2}U_0^2l \tag{48}$$

where $(dC_y/d\beta)_f$ is the slope of the normal-force coefficient of the fin against the sidewise angle of attack β and S_f is the area of the fin. An average value for $dC_y/d\beta$ is -2.2. Combining these factors and writing the expression in a form involving the span as the fundamental length results in

$$\frac{dC_n}{d\left(\frac{rb}{2U_0}\right)_f}=\frac{dC_y}{d\beta}\left(\frac{l}{b}\right)^2\frac{S_f}{S}\times 2 \tag{49}$$

Expressing the various factors thus calculated in the form of a single dimensionless coefficient, the formula for the total damping derivative in yawing becomes

$$N_r=\frac{dC_n}{d\left(\frac{rb}{2U_0}\right)}S\frac{\rho}{2}U_0\frac{b^2}{2mk_Z^2} \tag{50}$$

in which $\frac{dC_n}{d\left(\frac{rb}{2U_0}\right)}$ may be determined from an aerodynamic test of a complete model or may be estimated from the sum of several contributing factors.

It is not known how the body of the airplane influences its damping in yawing, although it is unlikely that its effect is as powerful as that of the vertical fin. In the case of the average airplane treated in this

report, an allowance equal to 60 percent of the fin effect was made for the fuselage and parts of the airplane other than the wings.

YAWING ACCELERATION DUE TO SIDESLIP, N_β

Measurements of the yawing moments in sideslip have been made on a large number of complete models in the course of routine wind-tunnel testing of military airplanes. A study of the results of these tests indicated that at low angles of attack the yawing moment may be estimated from the area and disposition of the vertical fin with a suitable allowance for the fuselage effect. Although airplane bodies when tested alone almost invariably show an unstable yawing tendency about the center of gravity, when tests of a complete model are made the results may show an additional stabilizing influence of the fuselage, possibly due to interference effects. At high lift coefficients the wings may exert considerable influence. The effect of the fuselage depends, of course, on its disposition with respect to the center of gravity and also on the nose shape. Models, especially those with uncowled radial engines, often show only 40 or 50 percent of the righting moment calculated for the fin and rudder alone.

The part of the yawing moment in yaw due to the vertical fin surface may be estimated by means of the data previously used for the calculation of N_r,

$$N_{\beta_f}=-\left(\frac{dC_y}{d\beta}\right)_f \frac{l}{b}\frac{S_f}{S}Sq\frac{b}{mk_z^2} \tag{51}$$

In cases of airplanes having wings set at a dihedral angle some provision must be made for an additional yawing moment in yaw that arises as a consequence of the setting of the wings. In straight flight, lift vectors drawn on each wing half, being inclined inward by the angle of dihedral, would intersect on the Z axis vertically above the center of gravity. These lift vectors remaining at the same time perpendicular to the leading-edge lines and to the relative wind direction do not intersect when the wing is yawed, giving rise to a couple. A simple approximation results in

$$C_n=-\frac{1}{5}\Gamma\beta C_L \tag{52}$$

Since this component of yawing moment is attributed to dihedral setting, it may be represented by

$$\frac{\partial}{\partial\Gamma}\frac{dC_n}{d\beta}=-\frac{1}{5}C_L \tag{53}$$

for calculation.

In addition to the simple dihedral effect, an induced yawing moment on the yawed wing must be considered as a secondary effect of the rolling moment. An approximate formula for this yawing moment derived from data given in reference 13 is

$$\left.\begin{aligned} C_n&=-\frac{1}{4}C_LC_l \\ \text{or}\qquad \frac{dC_n/d\beta}{dC_l/d\beta}&=-\frac{1}{4}C_L \end{aligned}\right\} \tag{54}$$

These formulas agree with the results of tests made in the 7- by 10-foot wind tunnel except near the region of zero lift. A formula for the total yawing-moment coefficient of the wings is

$$\left(\frac{dC_n}{d\beta}\right)_w=-C_L\left(0.0035\Gamma+\frac{1}{4}\frac{dC_l}{d\beta}\right) \tag{55}$$

where Γ is given in degrees.

CERTAIN CORRECTING TERMS AT HIGH ANGLES OF ATTACK

At high angles of attack the body of the airplane will be inclined appreciably to the reference axis about which the rolling moments are measured. The formulas given for the effects of the fin (and body) on the damping in yawing and yawing moment in yaw should for exactness have included the factor cos α, since the lever arm of the moment-producing effects will actually be shortened somewhat by the inclination. This correction is of no importance, however, and need not be considered. The same is true of the logical correction that should be applied to the wind-tunnel measurements of rolling moment in yaw, which were actually made about an axis pointing directly upstream and hence not quite in line with the axes considered in the report. The only correcting terms that are of sufficient magnitude to be considered here are those affecting L_r, L_β, and N_p and arising from the fact that the fin and body surfaces are disposed below the rolling axes. These terms are

$$\left.\begin{aligned} \Delta L_r&=N_r\sin\alpha\times\frac{k_z^2}{k_x^2} \\ \Delta L_\beta&=N_\beta\sin\alpha\times\frac{k_z^2}{k_x^2} \\ \Delta N_p&=N_r\sin\alpha \end{aligned}\right\} \tag{56}$$

Only the components of N_r and N_β attributed to the fuselage and vertical fin of the airplane should be used here.

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