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THE PROBLEM OF THE HELICOPTER.

by

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The idea of using a propeller rotating about a vertical shaft to give lift and to sustain a weight by balancing it directly with the thrust is almost as old as the screw propeller itself, and the elements of the theory of the action of a lifting propeller have been understood for at least fifteen years. Unfortunately, however, the printed discussions of this theory are almost all in French, German, or Italian, and those which are available in English are mostly contained in advanced treatises which are not likely to fall into the hands of the casual student. A vast number of helicopters have been invented, many have been built, and a very, very few have been successful up to the point of raising themselves from the ground. The possible advantages of the helicopter are obvious, a machine which can rise and descend vertically, and which requires no large space over which to run before taking off and after descending, manifestly being more useful, other things being equal, than the present type of airplane. It is regrettable that the inventors of direct-lift aircraft have, in many instances, seen only these possible gains and have failed to consider fully the problem which they have to meet or to familiarize themselves with the fundamental theory on which the action of every helicopter must be based. It is felt, therefore, that a broad survey of the problem will be of use in making clear the nature of some of the obstacles which have prevented any helicopter from reaching the stage of practical usefulness as yet and may lead to a saving of some of the time and money which are constantly being squandered on attempts to demonstrate anew facts which are already perfectly well understood without in the least striking at the root of the problem.

The cruces of the helicopter question are the securing of the necessary lift to rise from the ground, the assurance of a safe descent after complete failure of the engines, the securing of stability and controllability, and the maintenance of a reasonably high forward speed in the horizontal plane; and each of these points will be discussed in turn. Manifestly, until the first problems are solved satisfactorily the others do not rise at all, and the discussion will therefore be started with the question fundamental to all others, the question of the thrust which can be secured from a direct-lifting propeller and of the specifications to which the design of the propeller must conform in order that this thrust may be a maximum.

THE THEORY OF THE DIRECT-LIFTING SCREW PROPELLER.

The characteristics of propellers can be expressed in several different ways, but all of these except one involve the speed of advance, which is zero in the case of the helicopter. The only formulae which can be used in investigating the performance of the direct-lift machine are then,

$$T = T_c \frac{\rho}{g} N^2 D^4 \qquad (1)$$

$$Q = Q_c \frac{\rho}{g} N^2 D^5 \qquad (2)$$

$$P = P_c \frac{\rho}{g} N^3 D^5 \qquad (3)$$

where T is the thrust, Q the torque, and P the power, and T_c, P_c, and Q_c are experimentally determined coefficients, functions of V/ND alone and therefore independent of peripheral speed when applied to a helicopter.

Dividing (1) by (3) to find the thrust per horsepower, which is always the factor of primary interest,

$$\frac{T}{P} = \frac{T_c}{P_c} \times \frac{1}{ND} \qquad (4)$$

The thrust per horsepower is therefore inversely proportional to the peripheral speed. It follows that an increase in the power applied to a given propeller causes the thrust to increase in a smaller ratio than the power, as the increase of power increases the peripheral speed and this causes a decrease in the thrust per unit power. (3) may be written,

$$P = P_c \frac{\rho}{g} (ND)^3 D^2 \qquad (5)$$

If P and P_c are assumed to remain constant, ND, which is proportional to the peripheral speed, varies inversely as $D^{2/3}$. It is therefore possible, by making the diameter of the propeller large enough, to reduce ND below any designated value, and so to increase the thrust per horsepower without limit.

Since the thrust per horsepower is inversely proportional to ND, the product of thrust per horsepower and ND is a fundamental characteristic of any given type of propeller for helicopter use. This product is non-dimensional, or, rather, it would be if power were expressed in ft. lbs. or kg. m. per sec., and is equal to the ratio of T_c to P_c. The mean value of the product for the propellers tested at the request of the National Advisory Committee for Aeronautics at

Leland Stanford Junior University,* the units being lbs. per H.P. and ft. per sec., was 819 for propellers having a pitch-diameter ratio of 1.1, 984 when that ratio was reduced to 0.9, 1124 for 0.7, and 1318 for 0.5. These propellers were all two-bladed. In some experiments conducted at the National Physical Laboratory in 1917 a maximum of 1750 was obtained with a two-bladed propeller especially designed for helicopter work, the blades having a constant angle; and it is probable that this value cannot be very much exceeded.

Solving (4) for ND and substituting the value obtained in (5), the expression for power consumed becomes,

$$P = P_c \frac{\rho}{g} \cdot \left(\frac{T_c}{P_c}\right)^3 \cdot \left(\frac{P}{T}\right)^3 \cdot D^2$$

The product of the first three factors is a constant for any family of geometrically similar peopellers, assuming them always to work under the same atmospheric conditions, and the product of P_c and

$$\left(\frac{T_c}{P_c}\right)^3$$

can therefore be used as another fundamental characteristic of the type of propeller. Denoting this product by K, and solving for diameter,

$$D = \sqrt{\frac{P \times \left(\frac{T}{P}\right)^3}{K \times \frac{\rho}{g}}} = \frac{T^{3/2}}{P \times \sqrt{K \times \frac{\rho}{g}}}.$$

Solving similarly for N,

$$N = \sqrt{\frac{\left(\frac{T_c}{P_c}\right)^5 \times P_c \times \frac{\rho}{g}}{\left(\frac{T}{P}\right)^5 \times P}} = \sqrt{\frac{K' \times \frac{\rho}{g}}{\left(\frac{T}{P}\right)^5 \times P}}$$

where K' is equal to $P_c \times \left(\frac{T_c}{P_c}\right)^5$. Since it is always desirable to make D as small as possible and N as large as possible, other things being equal, in order that the helicopter may occupy a minimum of space and in order that the gear reduction ratio from the engine shall not be any larger than necessary, the best propeller for helicopter use will be that one which has the largest values of K and K'. The mean values of these coefficients for the propellers of several pitch-diameter ratios which have been tested at Stanford are tabulated below, together with the values for several propellers of different numbers of blades which were designed especially for helicopter use and tested at the National Physical Laboratory.

*Experimental Research on Air Propellers, II, by William F. Durand and E. P. Lesley: Report No. 30, National Advisory Committee for Aeronautics: Washington, 1919.

Propellers	K	$K'/10^8$
Stanford, P/D = 1.1, average	113,600	761
" P/D = 0.9, average	173,000	1673
" P/D = 0.7, average	191,100	2418
" P/D - 0.5, average	217,400	3773
N.P.L. Type A, 2-bladed	130,000	2734
" " A, 3-bladed	158,000	2511
" " A, 4-bladed	146,000	1610
" " B, 4-bladed	144,500	1401
" " C, 4-bladed	167,000	2822

It is clear that the propellers having a constant geometrical pitch of one-half the diameter are, rather strangely, distinctly superior to those designed especially for helicopter use. Since the question of helicopter design has received only the slightest attention, no wind tunnel experiments except those tabulated above having been run in recent years, there is no doubt that propellers better suited for use with direct-lift machines than any that are now available can be devised. As a basis for computation K may be taken as 250,000 and K' as 44×10^{10}. A table can then be constructed showing the diameter and r.p.m. necessary to secure various lifts per horsepower with different engine powers. Such a table is given on the next page. In applying the table, the power taken should of course be the power used on a single propeller. For example, if a 400 H.P. engine drives two propellers the necessary diameter of a single propeller will be found in the column headed 200.

Propeller Diameters in Feet. (R.p.m. in parentheses.)

Thrust per H.P. Lbs.	Horse Power 20	40	60	100	150	200	300	400	600	800	1000
5	(7740) 2.06	(5490) 2.90	(4470) 3.56	(3460) 4.60	(2830) 5.63	(2450) 6.50	(2000) 7.95	(1730) 9.19	(1420) 11.2	(1220) 13.0	(1100) 14.5
10	(1370) 5.80	(966) 8.21	(786) 10.1	(612) 13.0	(500) 15.9	(432) 18.4	(353) 22.5	(306) 26.0	(250) 31.8	(216) 36.8	(193) 41.1
15	(495) 10.7	(351) 15.1	(286) 18.5	(221) 23.9	(181) 29.2	(157) 33.8	(128) 41.3	(111) 47.6	(90.6) 58.5	(78.6) 67.5	(70.2) 75.5
20	(242.) 16.4	(171.) 23.2	(140.) 28.4	(108.) 36.7	(88.8) 44.9	(76.8) 51.9	(62.4) 63.6	(54.2) 73.4	(44.3) 90.0	(38.2) 104.	(34.3) 116.
30	(87.6) 30.2	(61.8) 42.7	(50.7) 52.3	(39.2) 67.5	(32.1) 82.6	(29.3) 90.5	(22.6) 117.	(19.6) 135.	(16.1) 165.	(13.9) 191.	(12.4) 214.
40	(42.9) 46.4	(30.2) 65.6	(24.7) 80.4	(19.1) 104.	(15.6) 127.	(13.5) 147.	(11.0) 180.	(9.5) 208.	(7.8) 254.	(6.8) 294.	(6.1) 328.)
50	(24.5) 60.5	(17.3) 91.1	(14.2) 112.	(11.0) 145.	(8.9) 178.	(7.8) 205.	(6.3) 252.	(5.5) 290.	(4.5) 356.	(3.9) 411.	(3.5) 459.

It is not correct to speak of the lifting power of a helicopter as its efficiency, as is often done, since a helicopter screw which is merely sustaining a load in the air is not doing any useful work. Only when ascending is useful work done, and only under that condition is it proper to speak of propulsive efficiency. The helicopter experiments at the National Physical Laboratory were extended to cover ascending and descending flight, and it was found that the thrust per H.P. is almost independent of vertical velocity over a wide range. This is particularly true of descent. For example, a helicopter designed to barely sustain 30 lbs. per H.P. (ND = 44) could ascend with a vertical velocity of 800 ft. per min., if the load were reduced to 22 lbs. per rated H.P. and if the power were kept constant. The r.p.m., however, would be greater during ascent than during level flight, and it would be necessary, in order to keep the engine from racing with full throttle, to use either a variable-pitch propeller or a variable-speed transmission. If no such mechanism were used, and if the r.p.m. were held constant, the load would have to be reduced to 16 lbs. per H.P., instead of only 22, to permit the attainment of the climbing speed specified above. If the throttle were left wide open and the motor permitted to race until its torque was fully balanced by the resisting torque of the propeller no reduction in load would be required, except that there would have to be a very slight initial excess of power to produce a vertical acceleration and start the upward motion. Once started, it would continue of its own accord. It would not be possible to ascend at much more than 800 ft. per min. with a propeller of fixed pitch. By varying the pitch and reducing the load to about one-half what it would be possible to sustain (say 15 lbs. per H.P. in the problem just discussed) it probably would be possible to climb 1,800 ft. per min. or better, although there are not enough experimental data to

make it possible to speak with certainty on this point.

It is usually assumed that propellers designed primarily to work under static conditions should have the blade sections all set at the same angle to a plane perpendicular to the propeller axis. This would be correct if there were no indraught, and it is also correct indraught existing, if the indraught velocity at every point is directly proportional to the distance from the propeller axis. It can be shown by a combination of the momentum and blade-element theories of propeller action that this condition is realized when the blade has the same sectional form and angle of attack at all points and when the blade width is directly proportional to the distance from the hub (i.e., when the blade has the form of the sector of a circle). Such a propeller is of course impossible to build, as it would have no strength near the hub. It is probable, therefore, that in actual practice the indraught velocity near the hub is always considerably larger, in proportion to the radius, than that farther out along the blade, and the angle of setting of the elements near the hub should therefore be a little larger than that of those in the neighborhood of the tip. In other words, the propeller blades should have a little warp of the same sort as that which is given to the blades of propellers intended for driving airplanes. The warp of the blades of lifting screws should, however, be much smaller than that of propulsive screws. The British experiments already mentioned dealt with helicopters the blades of which had no warp, and the angles of the blades were varied during the tests with a view to finding the most efficient disposition. It was found that the ratio of T_c to P_c was largest for a blade angle of $5\frac{1}{2}^{\circ}$ for the 2-bladed propeller and $7\frac{1}{2}^{\circ}$ for the 4-bladed one. The difference is accounted for by the larger indraught velocity of the multi-bladed screw. The product K was largest for an angle of 9° for the 2-bladed propeller and 11° for the 4-bladed. The best angle to adopt would ordinarily be about half-way between that of maximum thrust per horsepower and that of maximum K.

THE SAFETY OF HELICOPTERS IN FORCED DESCENTS.

The gravest charge brought against the helicopter is its lack of means of making a safe descent when the engine has stopped. This charge is frequently answered by the inventors and promoters of the direct-lift machines with the statement that the blade area of the propellers acts as a parachute to prevent the velocity of descent from rising to a dangerous value, but a moment's consideration will show the fallacy of this. A parachute of the usual type carries a load of not more than 0.25 lbs. per sq.ft. of projected area, yet it lands at a velocity much too high to be safe for a helicopter. In order to prevent damage by excessively rapid deceleration the vertical velocity at landing should be kept below 8 ft. per second, any larger velocity requiring the provision of shock-absorbers of considerable size and complexity. However, the limiting safe velocity may be taken, to be generous, as 16 ft. per second. The resistance of a flat plate normal to the wind at a speed of 16 ft. per second is 0.38 lb. per sq.ft., and this would accordingly be the limiting safe loading of the propeller blades, considered as a parachute. Since the area of the propeller blades is never likely to be more than 40% of the propeller

disc area, the loading calculated on the basis of the whole propeller disc would have to be kept down to 0.15 lbs. per sq.ft. To carry a load of 2000 lbs., and have the helicopter descend safely on the parachute principle after an engine stoppage, it would therefore be necessary to have a total propeller disc area of 13,300 sq.ft., which corresponds to two propellers each 92 ft. in diameter. This is manifestly too large to be considered if it is by any means possible to do better.

To have the propeller blades give a true parachute effect, it would be necessary that the propellers be locked after the engine stopped to keep them from spinning around, acting as windmills. A possible alternative method is to leave the propellers free, permitting them to spin. The direction of rotation when acting as a windmill would be opposite to the direction in which the propellers are driven by engine power, and the leading edges of the sections would therefore be what are normally the trailing edges. The propeller would operate very inefficiently under this condition, and the lift resisting the descent would therefore be small. Besides, even if there were a marked advantage to be gained from this reverse rotation as compared with the case in which the propeller is held stationary, that advantage would be of no avail when an engine stoppage occurred near the ground, as it would take some time for the force tending to reverse the direction of rotation to overcome the inertia of the rotating parts, and the propeller would have to pass through all the intermediate stages of decelerating forward rotation, remaining at rest, and accelerating reverse rotation before the full effect of the spinning of the blades would be realized. If the machine were initially so low as to strike the ground during this transition stage it would be no better off, so far as limiting speed of fall is concerned, than if the propeller had been locked.

The remaining possibility is to provide means of changing the angles of setting of the blades, and to set them, as soon as the engine stops, at such a position that they permit the propeller to spin around, impelled by the upward pressure of the air against the blades, while maintaining the same direction of rotation as that in which it is driven by the engine. It is obvious that this mode of operation is superior to the one just mentioned, and a detailed analysis of the resistance which the propeller offers to descent when working as a windmill will therefore be made. A great deal depends on the frictional resistance, which has the effect of partial braking, and two assumptions as to this will be made in turn. In the first case, it will be assumed that a clutch is provided to permit the pilot to disconnect the propeller from the engine entirely, and that, the shaft being mounted on ball or roller bearings, the frictional torque can be entirely neglected. Under this condition, the rate of rotation of the propeller will be such that the mean line of action of the reaction on the blades is parallel to the shaft of the propeller.

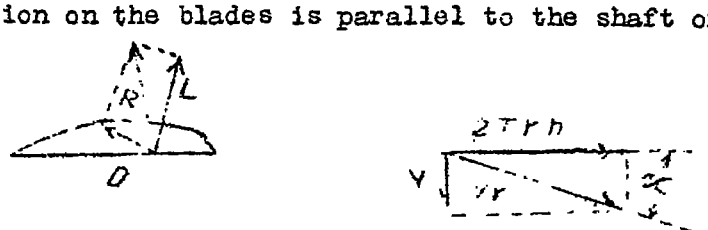


Fig. 1.

Considering any single element, the resultant and component velocities and forces are shown in Fig. 1. V_r is the vector representing, in magnitude and direction, the resultant velocity. Since there is to be no torque in either direction the equation of equilibrium may be written

$$L \sin \alpha = D \cos$$

and it follows from this that,

$$\frac{D}{L} = \tan \alpha \text{ and } \frac{V}{2\pi r n} = \frac{D}{L}$$

Since it is desired to secure the largest possible lift from each element of the propeller, the blades should be set at that angle which will give the largest mean value to the product of the lift coefficient and the square of the resultant velocity. Since V is fixed by the conditions of safe landing, this product may be written,

$$L_c \times V_r^2 = L_c \times (2\pi r n)^2 \times \sec^2 \alpha = L_c \times V^2 \times \left(\frac{L}{D}\right)^2 \times \left[1 + \left(\frac{D}{L}\right)^2\right]$$

The term in brackets is always so nearly equal to one that it may be disregarded, and the critical function is therefore the product of the lift coefficient by the square of the L/D ratio. The function has its maximum value when the angle of attack is approximately 6° for most representative wing sections. Since the mean arc tan D/L for the whole blade of a propeller at this angle of attack would be in the neighborhood of $4^\circ.5$ the chord of the blade should be set at about $1^\circ.5$ to the plane perpendicular to the axis. If L/D has a mean value of 12.5, which corresponds to the assumption just made with regard to the arc tan D/L, the mean peripheral speed for a vertical velocity of 8 ft. per sec., which has already been shown to be the maximum safe landing speed, would be 100 ft. per sec. Assuming this to correspond to a section lying two-thirds of the way out along the blade, the peripheral speed at the blade tips would be 150 ft. per sec. and ND would be 48. When the propeller is being driven by the engine the angle of attack of the sections would normally be from 4° to 5°, and the lift coefficient would therefore be about 12% smaller than when the machine is descending without power and with the pitch reduced so that the propeller acts as a windmill. The peripheral speed for the same upward force would therefore be about 6% greater in the case with power than in that without, and the normal ND would be 51 in the first case for a propeller capable of carrying the weight of the machine during descent without allowing the velocity to rise above 8 ft. per sec. This corresponds to a lift of 26 lbs. per H.P., and it is therefore unsafe to design a helicopter so that it will not be able to sustain normally its full weight at the rate of at least 26 lbs. per H.P., as one which had less lifting capacity than that would have a higher normal value of ND than 51, and would fall with excessive rapidity when the power was cut off (it is assumed in giving these figures than the most efficacious type of propeller available is employed.) The real criterion is that ND shall not exceed

51 under normal conditions, and the load per H.P. for which the helicopter should be designed to insure safe descent would vary somewhat as between different types of propellers).

The second case that has to be considered is that in which there is no means of breaking the connection between the engine and propeller, and in which the propeller is therefore burdened with the task of cranking the engine against its friction during the descent. It will be assumed that the total friction in the engine and transmission is 20% of the brake horsepower, and also (as an initial assumption the propriety of which can be checked at a later stage of the work) that a 150 H.P. engine is used to drive a propeller 240 ft. in diameter at 5.4 r.p.m. The horsepower required to turn the engine over against friction would then be 30, and the torque applied at the propeller, rotating 5.4 r.p.m., would be 29,170 lbs. ft. Taking the mean effective radius of the propeller, as before, as being two-thirds of the maximum radius, the force which it would be necessary to apply to produce this torque would be 365 lbs. Since ND for the propeller just specified is 21.6 the thrust would be 61 lbs. per H.P. and the total thrust 9150 lbs. The ratio of torque force to thrust would then be .04. Writing the equations for the elements of these forces and for their ratio,

$$dT = (L_c \cos\alpha + D_c \sin\alpha) \times V_r^2 \times dA = V_r^2 \times dA \times L_c \times \cos\alpha$$

$$\times (1 + \tan\alpha \tan\gamma),$$

where $\gamma = \text{arc tan } \frac{D}{L}$

$$dQ = (L_c \sin\alpha - D_c \cos\alpha) \times V_r^2 \times dA = V_r^2 \times dA \times L_c \times \cos\alpha$$

$$\times (\tan\alpha - \tan\gamma).$$

$$\frac{dQ}{dT} = \frac{\tan\alpha - \tan\gamma}{1 + \tan\alpha \tan\gamma} = \tan(\alpha - \gamma)$$

Then $\tan(\alpha - \gamma) = .04 \qquad \alpha - \gamma = 2^\circ.3$

and, since the mean value of γ would be very nearly $4^\circ.5$,

$$\alpha = 6^\circ.8$$

and

$$\tan\alpha = \frac{V}{2\pi r n} = .119.$$

Allowing V, as in the first case, to have a maximum value of 8 ft. per second, the limiting mean peripheral speed would be 67.2 ft. per second, corresponding to values of 101 ft. per second for the tip speed and 32 ft. per second for ND. The minimum load capacity for which a helicopter should be designed if it is to have variable pitch propellers but no means of disconnecting the propellers from the engine is therefore 42 lbs. per H.P. For a 150 H.P. engine on each screw this

would require a propeller 138 ft. in diameter turning 14 r.p.m. The initial assumption as to propeller size was therefore rather wide of the truth, but this has no effect on the ultimate result. The torque for a given power is inversely proportional to the r.p.m. of the propeller, the torque force for a given torque is inversely proportional to the propeller diameter, and the torque force per horsepower therefore varies inversely as ND. Since the thrust also varies inversely as ND, the ratio between the two components of the reaction is quite independent of the initial assumption as to the propeller size and speed, and the problem could be treated in a perfectly general way without making any such assumption, but it simplifies the work a little to insert some consistent set of figures.

In order to keep the propeller rotating in the original direction and at the maximum effectiveness when it has to turn the engine and transmission over, the mean chord of the blades would have to be set at -1° to the plane perpendicular to the axis, instead of at $+1.5$ as in the case where there is no frictional torque to contend with.

HORIZONTAL TRAVEL.

The most important question remaining to be discussed has to do with the possibility of progressing at a satisfactory rate in a horizontal plane. To be sure, it has sometimes been proposed to use captive helicopters to do the work for which observation balloons are now used, but their use for that purpose would never be very extensive under any conditions, and the helicopter can never be considered a practical possibility unless it is capable of making headway against ordinary strong winds.

There are two methods frequently suggested for securing a propelling force in a helicopter. The most commonly proposed, and the one which is likely to be most successful, entails the inclination of the axis so that the thrust may have a horizontal component. The second, which will not be discussed in detail in this report, depends on the use of subsidiary propellers with horizontal axes for propulsion, the transmission being of such a type as to permit of the distribution of the engine power between the sustentative and propulsive screws in any desired proportion. This scheme entails considerable structural difficulties, the requisite being practically two transmissions of infinitely variable gear ratio driven from a single engine and remaining continuously in engagement while the ratio is being changed.

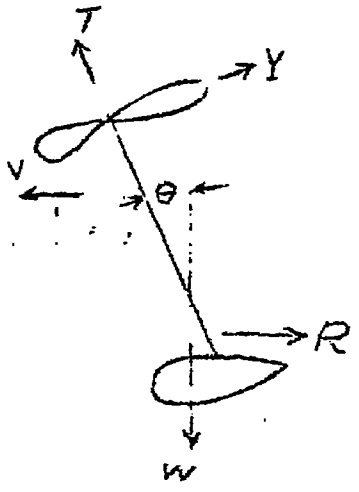


Fig 2

The forces acting on a helicopter with inclined axis are shown in Fig. 2. The method of throwing the machine into the inclined position need not be considered at present, nor need the moments which tend to increase the inclination or to restore the helicopter to a vertical position, as these will be taken up separately in a later section of the report in connection with the general problems of stability and control. The equations of equilibrium for steady horizontal travel are

$$T \cos \theta + Y \sin \theta = W$$

$$T \sin \theta - Y \cos \theta = R$$

The angle of inclination which the machine assumes depends on the characteristic of the propeller and on the structural resistance. It can best be approximated by examining the conditions under which a propeller would work when exposed to a wind at right angles to its axis (i.e., at 90° yaw). Some tests under this condition were made by Riabouchinsky at Koutchino in 1906, and the results are summarized by M. See in "Les Lois Experimentales des Helices Aeriennes," but the results are so surprising in some respects that it is difficult to give them entire credence, especially as the experiments were performed in the very early days of aerodynamical research, when methods of measurement were rather crude. If a propeller is presented to a wind of velocity at 90° yaw, the velocity with which each blade meets the air varies between $2\pi r n + V$ and $2\pi r n - V$. The angle of attack also varies somewhat during the revolution, as the indraught must be nearly constant. The angle of attack is obviously largest when the speed is largest. Neglecting for the moment the variation in angle of attack, and designating the maximum and minimum speed of the blades by v_1 and v_2, respectively, it is seen that the ratio of T to Y at the instant when the line of the blades is perpendicular to the direction of the wind, and so when the effect of the yaw is a maximum, is

$$\frac{T}{Y} = \left(\frac{V_1^2 + V_2^2}{V_1^2 - V_2^2} \right) \cdot \frac{L \cos \alpha - \sin \alpha}{D \cos \alpha + \sin \alpha} = \frac{V_1^2 + V_2^2}{V_1^2 - V_2^2}$$

$$\times \cot (\alpha + \gamma).$$

This is approximately equal to

$$6 \times \frac{V_1^2 + V_2^2}{V_1^2 - V_2^2}.$$

For high speeds of advance v_2^2 is negligible by comparison with v_1^2, and the ratio of T to Y therefore is probably in the neighborhood of 6. If it be assumed that this ratio is sustained unchanged when the axis is slightly inclined it appears that an inclination of about 10° would be necessary in order that the resultant of T and Y might be vertical when the helicopter was advancing rapidly. This, however, is not sufficient, as R is yet to be overcome. The total resistance

of fuselage, propeller shafts, and structure (not including the propellers) at 60 m.p.h. should not exceed one-twentieth of the weight of the helicopter, and the additional tilt necessary to overcome the resistance at this speed would therefore be 3°, making a total of 13°. The angle of yaw is then 77°.

Unfortunately there are no data available for such angles of yaw as this. Riabouchinsky's experiments cover (not entirely satisfactorily in the light of modern practice) the case of 90° yaw, and the only other experiments which have been published are some made by the N.P.L. on propulsive screws at angles of yaw up to 25°. The working conditions of propellers at small angles of yaw and those at 90° are entirely different, and it is difficult to interpolate between two sets of experiments so diverse as those just mentioned. At 90° yaw, as has just been pointed out, the angle of attack is almost independent of the rate of advance, and the thrust increases steadily as the speed of advance increases. When there is little or no yaw, on the other hand, the rate of advance has an important and direct effect on the angles of attack of the blade elements and the thrust falls off rapidly with increasing speed. In order that a helicopter may be suitable for use in all ordinary weather conditions it must be capable of maintaining a forward speed nearly or quite equal to the tip speed of the propellers. This is quite hopeless if the thrust is to fall off rapidly as the speed increases, and not even the provision of a variable pitch propeller would make it possible to secure sustentation and propulsion at high speeds under such conditions. It is possible, however, that when the inclination of the axis from the vertical is only ten or fifteen degrees the rate of change of the angle of attack with changing speed will be small enough so that high speeds can be attained. This is a point which can most easily be settled by wind tunnel tests on propellers at angles of yaw ranging from 60° to 90°, and such tests should be undertaken as soon as possible.

The dissymmetry between the blades when the helicopter is advancing, resulting in one or more blades carrying more than their share of the load at any given instant, makes trouble structurally, both because of the increased maximum stresses in the propeller blades and because of the large bending moment produced in the propeller shaft and frame of the helicopter. This bending moment can best be taken by placing one propeller above another and keeping them as close together as possible. The bending moments induced by the two propellers will then be opposite and will neutralize each other except in the section of shaft between the propellers. The placing of one propeller in the slipstream of the other may make a little trouble, at times, in the equalization of torque, but it should be possible to overcome any difficulty of that sort by proper adjustment of the blade angles.

If two-bladed propellers were used there would be likely to be some trouble with vibration of the structure when advancing horizontally if the propellers were not perfectly synchronized, as the total moment and force due to each propeller would vary during the revolution. Y, for example, as already noted, would be at a maximum when the line of the blade axes was perpendicular to the line of motion

of the helicopter, and would be almost zero when those two lines were parallel. By using propellers with four or more blades all these difficulties can be avoided.

STABILITY AND CONTROL OF THE HELICOPTER.

The stability of the helicopter is dependent on the fin action of the propeller and of any surfaces which may be exposed in the slip-stream. As long as the machine is neither ascending or descending, the primary effect of any inclination of the axis from the vertical is to produce a horizontal component of the thrust. This causes side-slipping, which, in turn, causes the propeller and any fin surface to be subjected to a lateral force. If the center of fin surface is above the center of gravity the lateral force gives a righting moment. Control can be secured by adjustable surfaces placed above the C.G. if the damping out of oscillations as soon as they are started is the only consideration. The dynamical stability of helicopters, or the rapidity with which oscillations are damped out when once started, has been thoroughly investigated by Professor H. Bateman in a report soon to be published by the National Advisory Committee for Aeronautics.

When the helicopter is moving the conditions are materially altered. When moving horizontally the forces are as shown in Fig. 2, the axis being inclined, and there is a moment, due to Y, tending to return the helicopter to a vertical attitude. A fin surface above the C.G. then has little or no controlling effect, as the force on it is always in the same direction as Y and R. A small moment tending to hold the helicopter in its inclined position can be secured by setting the control surface above the C.G. nearly horizontal, but this would be very ineffective if the horizontal translational velocity were much more than the slip-stream velocity. By placing a fin surface low down, on the other hand, any desired measure of control can be secured, but only with the accompaniment of some structural disadvantages. Such a surface would be set horizontally when it was desired to hover motionless, and would be inclined at an angle to the horizontal in order to go ahead. Once forward motion was started, the surface could be set vertical and this position would correspond to the maximum moment about the C.G., to the maximum inclination of the propeller axis for equilibrium, and so to the maximum forward speed. As already mentioned, there are constructional difficulties in the way of placing a control surface far below the center of gravity, most of the weight being concentrated in a car which should be as close to the ground as possible to save landing gear weight and resistance. It may be possible to arrange the control surface in two parts, one above and one below the C.G., and to provide means of folding up, just before touching the ground, the framework which carries the latter, since the high control surface is sufficient during vertical descent.

During ascent and descent the stability is much the same as when stationary, except that any inclination now changes the angle at which the propeller meets the air, and a lateral force is therefore set up at once, before the helicopter has moved laterally out of its vertical path; In the case of ascent this force tends to increase

the deviation from the vertical, in the case of descent to decrease it (always assuming the propeller to be above the center of gravity). To secure stability during a climb a large fin surface placed far below the C.G. of the machine would be necessary. Such a fin surface would operate rather inefficiently, as the inclination of the axis produces a change in direction of the slip-stream which would partially counterbalance the effect of the presentation of the fin surface at an angle to the relative wind due to the upward motion of the helicopter. It would be advantageous, from the standpoint of stability when rapidly ascending, to have the fin and control surfaces outside the slip-stream, and this might be possible to arrange in those helicopters which have two propellers in parallel and rotating in opposite directions. Part of the fin surface could then be placed between the two slip-streams. It would not be safe to put it all there, as there would then be no control when poised motionless. In short, there is no single disposition of fin surface which satisfies all requirements, but it is absolutely essential, if a helicopter is to travel horizontally, that there be enough fin surface low down, to bring the center of lateral resistance well below the center of gravity and that the inclination of this surface be variable under the control of the pilot.

APPENDIX

To

Theory of the Helicopter.

Some additional experiments on the thrust and power consumption of propellers working under static conditions have recently been carried out by Messrs. Lesley and Snyder at the Stanford University wind tunnel. A systematic investigation of the effect of varying pitch-diameter ratio, the tests covering a family of otherwise similar propellers with pitch-diameter ratios ranging from 0.1 to 1.3, showed that the largest thrust per horse power for a given peripheral speed was obtained with a pitch of .32 times the diameter. The maximum value of K corresponded to a ratio of .6, and the maximum of K' to one of .5. In a similar set of tests on propellers with unwarped blades set at various angles the highest thrust per H.P. was obtained with an angle of 6°, the best value of K with 15°, and the largest K' with 12°. In none of these tests were the values for any of the coefficients larger than those already reported. A propeller designed by R. Jacuzzi, especially for helicopter use, had K equal to 122,000 and K' 3880×10^{8}. The latter figure is close to a record, but the former is rather poor as compared with the best of the constant pitch propellers.

During the winter of 1919-20 standing thrust and power tests for air propellers were conducted at the Stanford University Aerodynamic Laboratory by Mr. Howard O. Snyder, a graduate student in Mechanical Engineering.

In these tests one form of two blade propeller only was tried. This was the narrow curved and tapering form with uniform geometrical pitch and non-cambered driving face designated as P_1 F_2 A_1 S_1 in Reports No. 14 and 30, National Advisory Committee for Aeronautics.

In addition to re-testing the propellers that had been already tried three additional pitch-diameter ratios, .1, .3 and 1.3 were investigated, making in all 7 propellers varying in pitch ratio from .1 to 1.3 by increments of .2. The results of these tests, reduced to coefficients of the form used by Mr. Warner, are shown in the accompanying Fig. 1.

In the curves as shown T_c and P_c are non-dimensional. $\frac{T_c}{P_c}$ is multiplied by 550 in order to make it comparable to the coefficient used by Mr. Warner, in which thrust is expressed in pounds and power in horse power instead of foot pounds per second.

The coefficients K and K^1 were derived in the same manner as Mr. Warner's.

As may be seen a somewhat higher value of T_c was realized for the .3 pitch ratio propeller than for the one of .5 pitch ratio. However, the coefficients K and K^1 are both considerably less for the propeller of smaller pitch so that to realize the same lift with the equal power a larger propeller running at a slower speed would be required, making on the whole the .5 pitch ratio superior.

Besides the foregoing, tests were made on a flat or non-warped blade propeller of the same contour , area, and section as F_2 A_1 S_1. The blades were fitted into a spherical hub provided with means for adjusting them to various angles. The results of these tests are shown in the accompanying figure 2.

These recent experiments indicate that, regarding 550 $\frac{T_c}{P_c}$ as a measure of efficiency, practically the same may be realized from the non-warped blade as from one of uniform geometrical pitch. However, as Mr. Warner has pointed out, it is not enough to attain a high value for 550 $\frac{T_c}{P_c}$. It is also necessary, in order to keep the diameter reasonably small and the rate of revolutions high, to secure large values of the coefficients K and K^1.

Tests at Stanford University on a two blade propeller 6 ft. in diameter and about 1 ft. nominal pitch, designed for helicopter use by R. Jacuzzi of Berkeley, California, in 1918, determined the following coefficients:

$$T_c = .0382$$

$$P_c = .0118$$

$$\frac{550 \quad T_c}{P_c} = 1785$$

$$K = 122000$$

$$K^1/10^8 = 3880$$

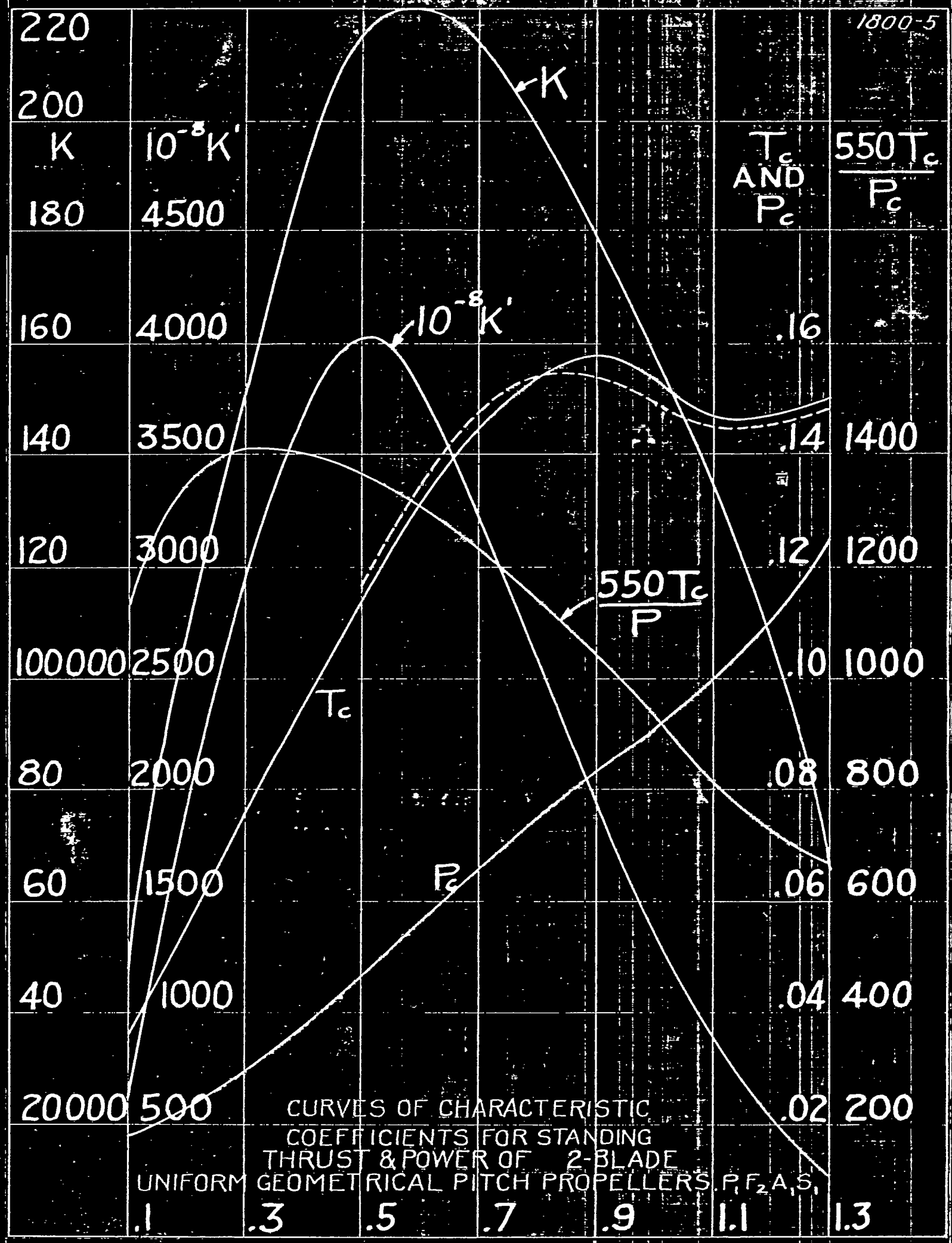
Although for this propeller $\frac{550\ T_c}{P_c}$ is larger than for any other tested in the Stanford Laboratory, K and K^1 are relatively small.

To realize with this propeller a lift of 30 lbs. per horse power at sea level air density with 100 horse power input, a diameter of nearly 97 feet, and about 37 revolutions per minute would be required, whereas with the .5 pitch ratio blade the same lift and power input could be secured with a propeller 72.5 feet in diameter running at 37.5 r.p.m.

The form of the T_c curve for uniform pitch propellers between pitch ratios of .7 and 1.3 is somewhat surprising. Repeated tests have determined its substantial accuracy however , the dotted line showing the results of investigations on a similar series of propellers of different blade contour and section but of approximately the same area.

W. F. Durand.

Stanford University, April 3, 1920.



CURVES OF CHARACTERISTIC COEFFICIENTS FOR STANDING THRUST & POWER OF 2-BLADE UNIFORM GEOMETRICAL PITCH PROPELLERS. $P_1 F_2 A_1 S_1$

FIG. 1.

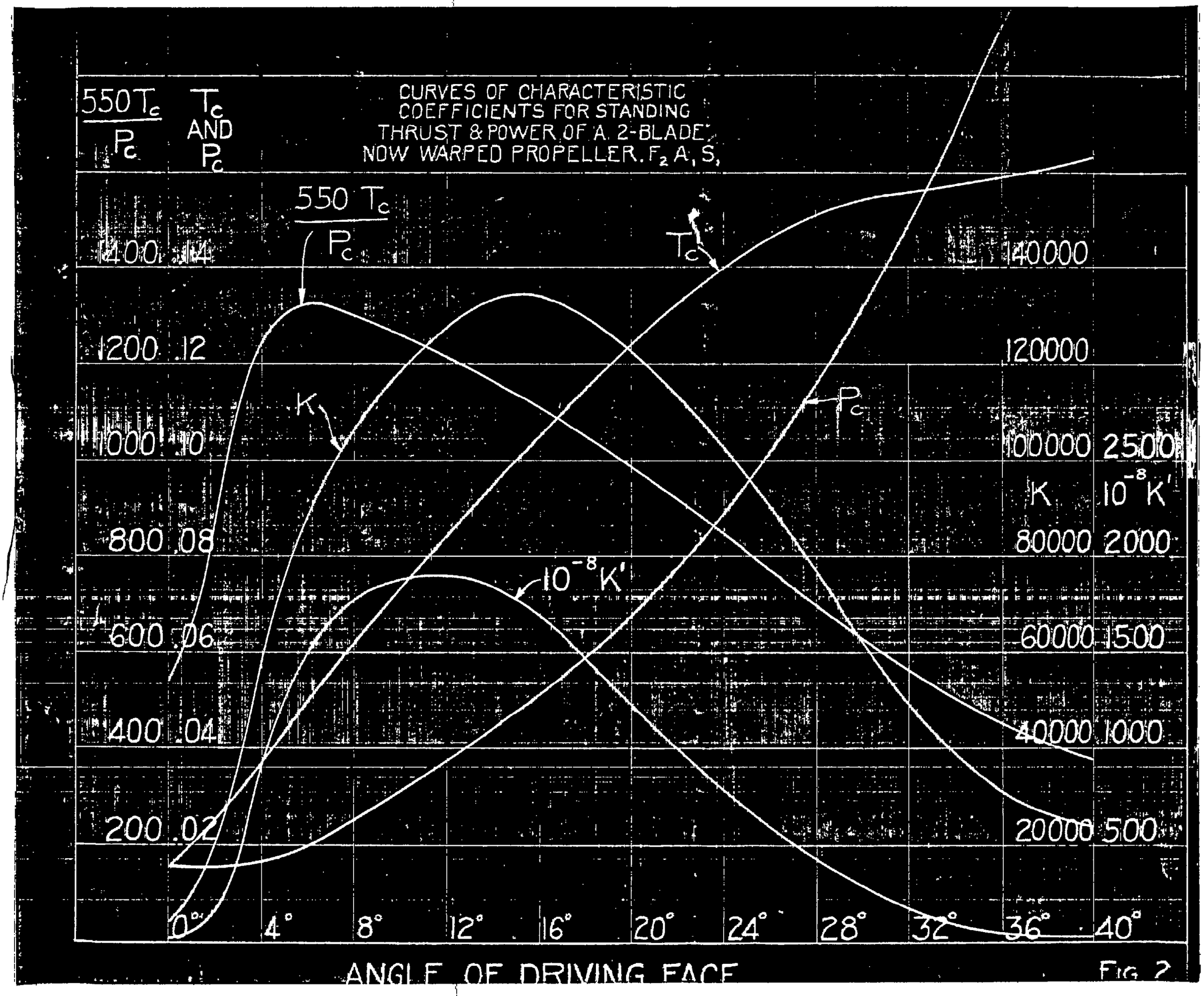

550 Tc / Pc
Tc AND Pc
CURVES OF CHARACTERISTIC COEFFICIENTS FOR STANDING THRUST & POWER OF A 2-BLADE NOW WARPED PROPELLER. F2 A, S,
550 Tc / Pc
Tc
Pc
K
10^-8 K'
1400 .14
1200 .12
1000 .10
800 .08
600 .06
400 .04
200 .02
140000
120000
100000 2500
80000 2000
60000 1500
40000 1000
20000 500
0° 4° 8° 12° 16° 20° 24° 28° 32° 36° 40°
ANGLE OF DRIVING FACE

Fig 2