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TECHNICAL NOTE

No. 1034

BENDING-TORSION FLUTTER CALCULATIONS MODIFIED BY SUBSONIC COMPRESSIBILITY CORRECTIONS

By I. E. Garrick

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Langley Field, Va.

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SUMMARY

A number of calculations of bending-torsion wing flutter are made at two Mach numbers, $M = 0$ (incompressible case) and $M = 0.7$, and the results are compared. The air forces employed for the case of $M = 0.7$ are based on Frazer's recalculation of Possio's results, which are derived on the assumption of small disturbances to the main flow. For ordinary wings of normal density and of low bending frequency in comparison with torsion frequency, the compressibility correction to the flutter speed appears to be of the order of a few percent; whereas, the correction to the flutter speed for high-density wing sections, such as propeller sections, and to the wing-divergence speed in general may be based on a rule using the $(1 - M^2)^{1/4}$ factor and, for $M = 0.7$, represents a decrease of the order of 17 percent.

INTRODUCTION

The question of the influence of the compressible properties of a gas on wing flutter is, of course, directly tied to the primary problem of determining the air forces and moments on oscillating airfoils moving at high forward speeds. This problem has been attacked by Possio (reference 1), along lines indicated by Prandtl, by a procedure utilizing the pressure or acceleration potential and the method of linearization of the equation satisfied by the acceleration potential for small disturbances to the main flow. A review and summary of Possio's work with certain simplifications have been given by Frazer (reference 2). Frazer and Skan (reference 3) listed improved numerical tables of Possio's results and made some numerical applications to the flutter problem.

It appeared worth while to perform additional calculations along similar lines utilizing the notations and parameters more familiar in this country. The present report has for its limited objective the reporting of the results of a number of pertinent calculations on bending-torsion flutter for a stream Mach number $M = 0.7$ and the comparison of these results with those given in reference 4 for the incompressible case based on the Theodorsen theory (reference 5).

The numerical accuracy of the results of Possio's theory and method deteriorates as M approaches unity and as the frequency increases. It has been estimated that the theory is not safely applicable much beyond $M = 0.7$, nor at $M = 0.7$ for values of the reduced frequency k much beyond 1. Thus, the transonic or supersonic ranges of speeds are not considered in the present paper. The purely supersonic case for small disturbances is also tractable and flutter calculations for this case are being prepared.

PROCEDURE

In the idealized case of a wing with two degrees of freedom, wing bending and wing torsion, and based on two-dimensional air forces, the determinantal equation yielding the flutter condition may be written in the form (references 4 and 5)

$$\begin{vmatrix} \overline{R}_{a\alpha} + iI_{a\alpha} & R_{ah} + iI_{ah} \\ R_{c\alpha} + iI_{c\alpha} & \overline{R}_{ch} + iI_{ch} \end{vmatrix} = 0$$

The two real equations contained in the complex determinant may yield in any given problem the two unknowns, flutter speed and flutter frequency.

Expressions for the R's and I's in the incompressible case are listed following the definitions of the various symbols in the appendix, and the evaluation of the terms is facilitated by the use of table I. In the compressible case the R's and I's are expressed in terms of the notation of Frazer and Skan (reference 3), and table II contains values for $M = 0.7$.

Instead of a direct solution of the flutter speed and flutter frequency for a case in which the wing structural parameters are given, it is more convenient to solve for the parameter ω_h/ω_α (the ratio of frequency in bending to frequency in torsion) which belongs to the border-line case of flutter for a definite chosen value of the parameter $1/k$. The elimination of X from the two equations contained in the determinant yields a quadratic equation in $(\omega_h/\omega_\alpha)^2$ from which ω_h/ω_α may be found and, subsequently, X may be evaluated; and X, together with the given value of $1/k$, determines the flutter speed and frequency. It is convenient first to perform the calculations for the compressible case with $M = 0.7$ and finally, in order to furnish the desired numerical comparisons, to perform the calculations for the incompressible case $(M = 0)$ utilizing the given structural parameters and the derived values of the frequency ratio ω_h/ω_α.

RESULTS AND DISCUSSION

The main numerical results are summarized in table III and are shown plotted in figures 1 to 4. (No tabular values are included for fig. 4.) The ordinate in the figures is the flutter-speed coefficient $v/b\omega_\alpha$ where $b\omega_\alpha$ represents a convenient reference speed. The abscissa is the frequency ratio ω_h/ω_α.

The parameter κ may be considered to determine the wing density at a given altitude; thus, $\kappa = 0.025$ represents the highest wing density used and $\kappa = 0.2$ the smallest. Alternatively, the change in κ may be interpreted to represent a change in altitude for a given wing, and a change from $\kappa = 0.10$ to $\kappa = 0.05$ may be considered to represent an altitude change from sea level to an altitude at which ρ equals one half the density at sea level, or approximately 18,000 feet. The normal drop in sonic speed with altitude should be taken into account in interpreting $M = \frac{V}{c}$ (see reference 3, fig. 2). The examples treated may be further classified by values of the parameter a representing positions of the torsional elastic axis; thus, $a = -0.4$, $a = -0.2$, and $a = 0$ represent, respectively, elastic axes at 30 percent, 40 percent, and 50 percent chord from the leading edge.

Also $x_\alpha = 0.2$ represents a position of the center of gravity 10 percent of the chord behind the elastic axis.

The figures show flutter curves calculated for the two values ($M = 0$ and $M = 0.7$) representing respectively a low-speed or incompressible case and a high-speed or compressible case. For the usual circumstance of low values of the frequency ratio ω_h/ω_α, the effect of compressibility on the flutter speed is seen to be relatively small; for the lower wing densities the effect is a small increase, whereas for the highest wing density used the effect is a small detrimental one.

For the divergence speed (frequency $\omega \longrightarrow 0$) the formulas of the static case are applicable and the slope of the lift curve increases according to the Glauert-Prandtl rule, which yields the approximate formula

$$\frac{v}{b\omega_\alpha} \approx \left(1 - M^2\right)^{\frac{1}{4}} \sqrt{\frac{r_\alpha^2}{\kappa} \frac{1/2}{\frac{1}{2} + a}}$$

For very heavy wings ($\kappa \longrightarrow 0$) the values of $1/k$ for flutter approach ∞; that is, the low frequency or the static case is approached and the $\left(1 - M^2\right)^{1/4}$ rule given in reference 4 appears applicable. The empirical formula for the flutter speed of reference 4 (p. 17), which is valid for high wing density and low values of ω_h/ω_α, may be modified to read

$$\frac{v_f}{b\omega_\alpha} \approx \left(1 - M^2\right)^{\frac{1}{4}} \sqrt{\frac{r_\alpha^2}{\kappa} \frac{1/2}{\frac{1}{2} + a + x_\alpha}}$$

Since the slope of the lift curve does not increase in accordance with the Glauert-Prandtl formula beyond a certain value of $M < 1$, the formula is clearly inapplicable beyond a certain value of M. This value may be taken roughly to be in the order of $M = 0.7$ to 0.75. For $M = 0.7$, the formula indicates a decrease in the flutter speed of approximately 17 percent.

The effects of internal damping and of the modes of vibration have been omitted in the calculations. Inclusion of these effects would tend to reduce further the differences between the numerical results for the compressible and incompressible cases. This statement is borne out by the results of reference 3 for a tapered wing.

Calculations for a wing with an aileron cannot be made by the method of reference 1 without very extensive and difficult computations. It may also be remarked that the numerical tables of references 1 and 3 show the need of additional extensions and recalculations.

CONCLUDING REMARK

The main conclusion to be derived from study of the numerical flutter calculations is that the effect of compressibility on the flutter speed (wing bending - wing torsion, no aileron) for subsonic speeds with no shocks, although complicated, is relatively small in the usual cases and, for a Mach number of 0.7 can be allowed for by corrections of small order to the incompressible-case results.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., December 18, 1945

APPENDIX

SYMBOLS AND NOTATION

b — half-chord used as reference unit length

a — coordinate of axis of rotation (torsional axis) measured from midchord (reference 4)

ρ — air density

κ — ratio of mass of cylinder of air of diameter equal to chord of wing to mass of wing, both taken for equal length along span; this ratio may be expressed as $\kappa = 0.24\left(\frac{b^2}{W}\right)\left(\frac{\rho}{\rho_0}\right)$, where W is weight in pounds per foot span, b is in feet, and ρ/ρ_0 is ratio of actual air density to standard density at sea level

x_a — location of center of gravity of airfoil measured from a (reference 4)

r_a — radius of gyration of airfoil referred to a (reference 4)

ω_a — natural angular frequency of torsional vibrations around a in vacuum

ω_h — natural angular frequency of wing in bending in vacuum

v — speed of forward motion

c — speed of sound in undisturbed medium

M — Mach number (v/c)

v_f — flutter or critical speed

ω — angular frequency of wing vibrations

k — reduced frequency $\left(\frac{b\omega}{v}\right)$

F and G functions of k (see references 4 and 5)

$\lambda = 2k$

In each of the following formulas the incompressible-case formula is listed first in the notation of references 4 and 5 followed by the conversion of the formula to make use of the notation and numerical tables of reference 3. (See tables I and II.)

$$\overline{R}_{a\alpha} = \Omega_\alpha X - \frac{r_\alpha{}^2}{\kappa} - \left(\frac{1}{8} + a^2\right) + \left(\frac{1}{4} - a^2\right)\frac{2G}{k} - \left(\frac{1}{2} + a\right)\frac{2F}{k^2}$$

$$= \Omega_\alpha X - \frac{r_\alpha{}^2}{\kappa} + \frac{4}{k^2}\left[M_3 - \frac{1}{2}\left(\frac{1}{2} + a\right)\left(M_1 + Z_3 - \frac{a}{2} Z_1\right)\right] + \frac{1}{k^2} M_1$$

$$R_{ah} = -\frac{x_\alpha}{\kappa} + a + \left(\frac{1}{2} + a\right)\frac{2G}{k}$$

$$= -\frac{x_\alpha}{\kappa} + \frac{2}{k^2}\left[M_1 - \frac{1}{2}\left(\frac{1}{2} + a\right) Z_1\right]$$

$$R_{c\alpha} = -\frac{x_\alpha}{\kappa} + a - \left(\frac{1}{2} - a\right)\frac{2G}{k} + \frac{2F}{k^2}$$

$$= -\frac{x_\alpha}{\kappa} + \frac{2}{k^2}\left(Z_3 - \frac{a}{2} Z_1\right)$$

$$\overline{R}_{ch} = \Omega_h X - \frac{1}{\kappa} - \left(1 + \frac{2G}{k}\right)$$

$$= \Omega_h X - \frac{1}{\kappa} + \frac{1}{k^2} Z_1$$

$$I_{a\alpha} = \frac{1}{k}\left[-\left(\frac{1}{2} + a\right)\frac{2G}{k} - \left(\frac{1}{4} - a^2\right) 2F + \frac{1}{2} - a\right]$$

$$= \frac{4}{k^2}\left[M_4 - \frac{1}{2}\left(\frac{1}{2} + a\right)\left(M_2 + Z_4 - \frac{a}{2} Z_2\right)\right] + \frac{1}{k^2} M_2$$

$$I_{ah} = \frac{1}{k}\left[-\left(\frac{1}{2} + a\right) 2F\right]$$

$$= \frac{2}{k^2}\left[M_2 - \frac{1}{2}\left(\frac{1}{2} + a\right) Z_2\right]$$

$$I_{ca} = \frac{1}{k}\left[\frac{2G}{k} + \left(\frac{1}{2} - a\right) 2F + 1\right]$$

$$= \frac{2}{k^2}\left(Z_4 - \frac{a}{2} Z_2\right)$$

$$I_{ch} = \frac{1}{k} 2F$$

$$= \frac{1}{k^2} Z_2$$

where

$$\Omega_a X = \frac{r_a^2}{\kappa}\left(\frac{\omega_a}{\omega}\right)^2$$

$$\Omega_h X = \frac{1}{\kappa}\left(\frac{\omega_h}{\omega}\right)^2$$

$$\Omega_h = \left(\frac{\omega_h}{\omega_a}\right)^2 \frac{1}{r_a^2}$$

$$X = \frac{r_a^2}{\kappa}\left(\frac{\omega_a}{\omega}\right)^2$$

The functions tabulated for convenience in tables I and II are the primed quantities given in the following definitions:

$$\overline{R}_{a\alpha} = R_{a\alpha}' + \Omega_\alpha X - \frac{r_\alpha^2}{\kappa}$$

$$R_{ah} = R_{ah}' - \frac{x_\alpha}{\kappa}$$

$$R_{c\alpha} = R_{c\alpha}' - \frac{x_\alpha}{\kappa}$$

$$\overline{R}_{ch} = R_{ch}' + \Omega_h X - \frac{1}{\kappa}$$

$$I_{a\alpha} = \frac{1}{k} I_{a\alpha}'$$

$$I_{ah} = \frac{1}{k} I_{ah}'$$

$$I_{c\alpha} = \frac{1}{k} I_{c\alpha}'$$

$$I_{ch} = \frac{1}{k} I_{ch}'$$

REFERENCES

1. Possio, Camillo: L'Azione aerodinamica sul profilo oscillante in un fluido compressibile a velocita iposonora. L'Aerotecnica, vol. XVIII, fasc. 4, April 1938, pp. 441-458. (Available as British Air Ministry Translation No. 830.)

2. Frazer, R. A.: Possio's Derivative Theory for an Infinite Aerofoil Moving at Sub-Sonic Speeds. 4932, 0.205 (Rev.), British A.R.C., Jan. 23, 1941.

3. Frazer, R. A., and Skan, Sylvia W.: Influence of Compressibility on the Flexural-Torsional Flutter of Tapered Cantilever Wings. 5916, 0.274 (Rev.), British A.R.C., June 30, 1942.

4. Theodorsen, Theodore, and Garrick, I. E.: Mechanism of Flutter - A Theoretical and Experimental Investigation of the Flutter Problem. NACA Rep. No. 685, 1940.

5. Theodorsen, Theodore: General Theory of Aerodynamic Instability and the Mechanism of Flutter. NACA Rep. No. 496, 1935.

TABLE I

THE R' AND I' FUNCTIONS FOR M = 0

$\lambda = 2k$	$1/k$	a	R_{aa}'	R_{ah}'	R_{ca}'	R_{ch}'	I_{aa}'	I_{ah}'	I_{ca}'	I_{ch}'
0.2	10	-0.5	-0.37500	-0.50000	169.34600	2.446	1.00000	0	-0.78200	1.66400
		-.4	-17.23514	-.74460	169.10140		1.09484	-.16640	-.94840	
		-.2	-50.80866	-1.23380	168.61220		1.38436	-.49920	-1.28120	
		0	-84.18650	-1.72300	168.12300		1.80700	-.83200	-1.61400	
0.4	5	-0.5	-0.37500	-0.50000	37.76600	0.886	1.00000	0	0.56920	1.45520
		-.4	-4.09274	-.58860	37.67740		.95763	-.14552	.42368	
		-.2	-11.47506	-.76580	37.50020		.96021	-.43656	.13264	
		0	-18.78650	-.94300	37.32300		1.07920	-.72760	-.15840	
0.6	3.33333	-0.5	-0.37500	-0.50000	15.47300	0.195	1.00000	0	1.13500	1.33000
		-.4	-1.87035	-.51950	15.45350		.89980	-.13300	1.00200	
		-.2	-4.84935	-.55850	15.41450		.77920	.39900	.73600	
		0	-7.81275	-.59750	15.37550		.76500	.66500	.47000	
0.8	2.5	-0.5	-0.37500	-0.50000	8.13750	-0.175	1.00000	0	1.42500	1.25000
		-.4	-1.14050	-.48250	8.15500		.87000	-.12500	1.30000	
		-.2	-2.68200	-.44750	8.19000		.68500	-.37500	1.05000	
		0	-4.23750	-.41250	8.22500		.60000	-.62500	.80000	
1.0	2.0	-0.5	-0.37500	-0.50000	4.88600	-0.3972	1.00000	0	1.59300	1.19580
		-.4	-.81757	-.46028	4.92572		.85266	-.11958	1.47342	
		-.2	-1.72655	-.38084	5.00516		.62972	-.35874	1.23426	
		0	-2.66730	-.30140	5.08460		.50245	-.59790	.99510	
2.0	1.0	-0.5	-0.37500	-0.50000	0.77940	-0.79940	1.00000	0	1.87820	1.07880
		-.4	-.41093	-.42006	.85934		.82297	-.10788	1.77032	
		-.2	-.53077	-.26018	1.01922		.53363	-.32364	1.55456	
		0	-.71455	-.10030	1.17910		.33060	-.53940	1.33880	
3.0	0.66667	-0.5	-0.37500	-0.50000	0.06123	-0.90187	1.00000	0	1.94387	1.04200
		-.4	-.34014	-.40981	.15142		.81603	-.10420	1.83967	
		-.2	-.32454	-.22944	-.33179		.51062	-.31260	1.63127	
		0	-.38108	-.04907	.51217		.28857	-.52100	1.42287	
4.0	0.5	-0.5	-0.37500	-0.50000	-0.18580	-0.94230	1.00000	0	1.96830	1.02600
		-.4	-.31584	-.40577	-.09157		.81343	-.10260	1.86570	
		-.2	-.25407	-.21731	.09689		.50185	-.30780	1.66050	
		0	-.26768	-.02885	.28535		.27235	-.51300	1.45530	
5.0	0.4	-0.5	-0.37500	-0.50000	-0.29936	-0.96216	1.00000	0	1.97956	1.01740
		-.4	-.30469	-.40378	-.20314		.81221	-.10174	1.87782	
		-.2	-.22179	-.21135	-.01071		.49770	-.30522	1.67434	
		0	-.21585	-.01892	.18170		.26457	-.50870	1.47086	

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TABLE II

THE R' AND I' FUNCTIONS FOR M = 0.7

$\lambda = 2k$	$1/k$	a	R_{aa}'	R_{ah}'	R_{ca}'	R_{ch}'	I_{aa}'	I_{ah}'	I_{ca}'	I_{ch}'
0.2	10	-0.5	2.00100	-1.06200	201.38200	5.88300	1.79150	0.02700	-3.96780	1.94450
		-.4	-17.97100	-1.65000	200.79400		2.20510	-.16740	-4.16220	
		-.2	-57.56700	-2.82600	199.61600		3.14870	-.55640	-4.55120	
		0	-96.68700	-4.00400	198.44000		4.24790	-.94520	-4.94000	
0.4	5	-0.5	0.89050	-0.93100	43.13200	2.36400	1.69465	0.05730	-0.83100	1.59400
		-.4	-3.30550	-1.16750	42.89550		1.78805	-.10210	-.99040	
		-.2	-11.55750	-1.64000	42.42300		2.07025	-.42090	-1.30920	
		0	-19.61850	-2.11300	41.95000		2.48005	-.73970	-1.62800	
0.6	3.33333	-0.5	0.54089	-0.86667	17.97800	1.10256	1.66314	0.08893	0.24407	1.44200
		-.4	-1.15911	-.97689	17.86778		1.64420	-.05527	.09987	
		-.2	-4.49333	-1.19733	17.64711		1.69300	-.34367	-.18853	
		0	-7.73909	-1.41800	17.42663		1.85713	-.63207	-.47693	
0.8	2.5	-0.5	0.41231	-0.82537	9.92187	0.51887	1.65403	0.12605	0.72055	1.37550
		-.4	-.49219	-.87725	9.87000		1.58313	-.01150	.58300	
		-.2	-2.26994	-.98100	9.76625		1.52383	-.28660	.30790	
		0	-4.00619	-1.08475	9.66250		1.57463	-.56170	.03280	
1.0	2.0	-0.5	0.40712	-0.80880	6.47680	0.21436	1.69760	0.17920	0.96912	1.36360
		-.4	-.15752	-.83024	6.45536		1.59640	.04284	.83276	
		-.2	-1.27400	-.87312	6.41248		1.47584	-.22988	.56004	
		0	-2.37320	-.91600	6.36960		1.46432	-.50260	.28732	
2.0	1.0	-0.5	0.51805	-0.56310	1.92266	-0.08949	1.56510	0.53940	0.99680	1.45000
		-.4	.38121	-.55416	1.93160		1.42598	.39440	.85180	
		-.2	.10213	-.53626	1.94950		1.23474	.10440	.56180	
		0	-.18411	-.51836	1.96740		1.15950	-.18560	.27180	
3.0	0.66667	-0.5	0.19253	-0.39111	0.85401	-0.17465	1.23707	0.65280	1.03113	1.54067
		-.4	.14449	-.37364	.87148		1.08408	.49873	.87706	
		-.2	.03793	-.33872	.90641		.87056	.19060	.56893	
		0	-.08258	-.30379	.94135		.78027	-.11753	.26080	
4.0	0.5	-0.5	0.08080	-0.28700	0.48699	-0.16302	1.08573	0.77185	0.99870	1.65300
		-.4	.05917	-.27070	.50329		.92521	.60655	.83340	
		-.2	.00613	-.23809	.53589		.70333	.27595	.50280	
		0	-.05995	-.20549	.56850		.61371	-.05465	.17220	
5.0	0.4	-0.5	0.12866	-0.14550	0.32638	-0.09733	1.00880	0.97440	0.82501	1.69640
		-.4	.10960	-.13577	.33611		.84582	.80476	.65537	
		-.2	.06564	-.11631	.35557		.62165	.46548	.31609	
		0	.01389	-.09684	.37504		.53320	.12620	-.02319	

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TABLE III

NUMERICAL RESULTS

$\left[r_\alpha^2 = 0.25;\ x_\alpha = 0.2\right]$

k	M = 0.7				M = 0			
	1/k	ω_h/ω_α	ω/ω_α	$v/b\omega_\alpha$	1/k	ω_h/ω_α	ω/ω_α	$v/b\omega_\alpha$
a = -0.4								
0.200	1.250	1.000	1.080	1.350	0.83	1.000	1.16	0.97
	2.000	.590	.736	1.472	1.63	.590	.82	1.35
	2.500	.436	.623	1.559	2.05	.436	.72	1.48
	3.333	.234	.507	1.691	2.62	.234	.62	1.62
	3.850	0	.450	1.740	2.99	0	.57	1.72
0.100	1.400	1.000	1.070	1.500	----	-----	----	-----
	2.000	.653	.773	1.547	1.79	0.653	0.86	1.54
	2.500	.519	.666	1.666	2.23	.519	.77	1.71
	3.333	.361	.554	1.847	2.85	.361	.67	1.90
	4.990	0	.430	2.130	3.60	0	.56	2.16
0.050	1.700	1.100	1.150	1.950	1.13	1.100	1.27	1.44
	2.000	.818	.901	1.803	1.77	.818	1.00	1.77
	2.000	2.002	1.970	3.941	----	-----	----	-----
	2.500	.690	.757	1.893	2.20	.690	.89	1.95
	2.500	6.106	5.785	14.462	----	-----	----	-----
	3.333	.494	.627	2.091	3.11	.494	.73	2.28
	5.000	.292	.490	2.448	4.30	.292	.61	2.63
	6.490	0	.410	2.630	5.40	0	.54	2.90
0.025	2.500	0.889	0.943	2.358	2.30	0.889	1.04	2.39
	2.500	1.297	1.295	3.237	1.81	1.297	1.42	2.56
	3.333	.638	.724	2.412	3.34	.638	.81	2.72
	3.333	2.270	2.120	7.068	----	-----	----	-----
	5.000	.435	.560	2.799	4.80	.435	.66	3.19
	9.860	0	.350	3.420	7.84	0	.51	4.00
a = -0.2								
0.200	1.030	1.000	1.050	1.080	0.53	1.000	1.17	0.62
	2.000	.446	.622	1.243	1.49	.446	.77	1.14
	2.000	3.242	3.107	6.215	----	-----	----	-----
	2.500	.269	.515	1.288	1.81	.269	.68	1.23
	2.500	4.791	4.545	11.363	----	-----	----	-----
	2.980	0	.440	1.310	2.09	0	.61	1.28
0.100	1.350	1.000	1.030	1.390	0.65	1.000	1.21	.79
	2.000	.574	.698	1.396	1.62	.574	.84	1.36
	2.000	2.365	2.251	4.501	----	-----	----	-----
	2.500	.435	.595	1.487	1.99	.435	.74	1.48
	2.500	3.922	3.649	9.122	----	-----	----	-----
	3.333	.253	.480	1.599	2.49	.253	.64	1.61
	4.040	0	.410	1.650	2.89	0	.58	1.68
0.050	1.700	1.000	1.000	1.690	1.22	1.000	1.09	1.33
	2.000	.755	.828	1.657	1.67	.755	.96	1.60
	2.000	1.347	1.323	2.647	1.25	1.347	1.44	1.79
	2.500	.594	.695	1.737	2.20	.594	.82	1.80
	2.500	1.728	1.637	4.094	----	-----	----	-----
	3.333	.431	.567	1.890	2.86	.431	.70	2.00
	3.333	2.641	2.408	8.027	----	-----	----	-----
	5.000	.204	.423	2.115	3.83	.204	.58	2.21
	5.700	0	.370	2.130	4.26	0	.54	2.29
0.025	2.500	0.838	0.875	2.189	2.27	0.838	0.97	2.20
	2.500	1.050	1.047	2.619	1.95	1.050	1.15	2.24
	3.333	.588	.666	2.221	3.24	.588	.77	2.49
	3.333	1.459	1.364	4.547	----	-----	----	-----
	5.000	.368	.503	2.517	4.65	.368	.61	2.83
	5.000	2.279	2.010	10.048	----	-----	----	-----
	8.430	0	.340	2.870	6.46	0	.49	3.17

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TABLE III - Concluded

NUMERICAL RESULTS - Concluded

$\left[r_a^2 = 0.25;\ \ x_a = 0.2\right]$

κ	M = 0.7				M = 0			
	$1/k$	ω_h/ω_a	ω/ω_a	$v/b\omega_a$	$1/k$	ω_h/ω_a	ω/ω_a	$v/b\omega_a$
				a = 0				
0.200	1.150	1.000	1.020	1.170	0.12	1.000	1.16	0.14
	1.500	.600	.710	1.060	.85	.600	.90	.77
	2.000	.343	.543	1.086	1.23	.343	.77	.94
	2.000	1.466	1.416	2.832	----	-----	----	-----
	2.500	.126	.444	1.110	1.45	.126	.70	1.01
	2.500	1.518	1.458	3.644	----	-----	----	-----
	3.333	1.570	1.500	5.000	----	-----	----	-----
	5.000	1.590	1.512	7.560	----	-----	----	-----
	2.610	0	.430	1.110	1.48	0	.69	1.02
0.100	1.350	1.000	1.010	1.360	----	-----	----	-----
	1.500	.800	.850	1.270	----	-----	----	-----
	2.000	.515	.637	1.275	1.43	0.515	0.83	1.18
	2.000	1.416	1.352	2.703	----	-----	----	-----
	2.500	.373	.538	1.344	1.76	.373	.74	1.29
	2.500	1.561	1.466	3.664	----	-----	----	-----
	3.333	.174	.424	1.414	2.17	.174	.64	1.40
	3.333	1.752	1.617	5.391	----	-----	----	-----
	5.000	1.943	1.764	8.822	----	-----	----	-----
	3.690	0	.380	1.420	2.38	0	.60	1.43
0.050	2.000	0.700	0.768	1.536	1.68	0.700	0.90	1.51
	2.000	1.092	1.070	2.141	1.43	1.092	1.21	1.73
	2.500	.549	.642	1.605	2.13	.549	.78	1.66
	2.500	1.268	1.204	3.009	2.60	1.268	1.15	2.99
	3.333	.385	.517	1.724	2.74	.385	.66	1.81
	3.333	1.475	1.361	4.536	----	-----	----	-----
	5.000	.147	.373	1.867	3.60	.147	.55	1.96
	5.000	1.757	1.576	7.879	----	-----	----	-----
	5.430	0	.340	1.870	3.91	0	.51	1.99
0.025	2.500	0.797	0.819	2.047	2.36	0.797	0.90	2.12
	2.500	.909	.906	2.264	2.31	.909	.97	2.24
	3.333	.548	.618	2.059	3.21	.548	.71	2.28
	3.333	1.174	1.101	3.669	----	-----	----	-----
	5.000	.343	.458	2.288	4.44	.343	.57	2.52
	5.000	1.426	1.285	6.424	----	-----	----	-----
	7.940	0	.310	2.450	6.57	0	.42	2.76

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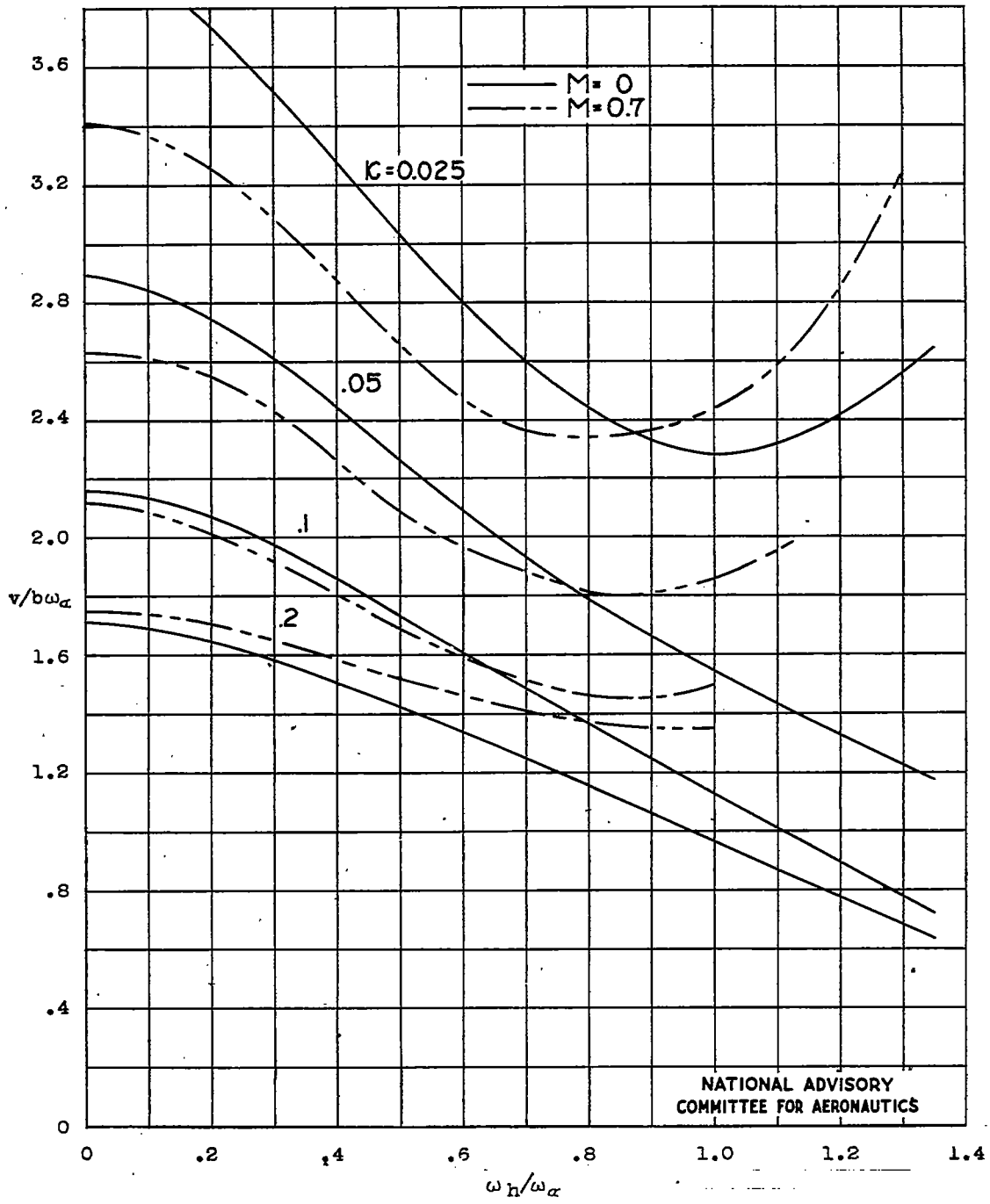

Figure 1.- The flutter coefficient $v/b\omega_\alpha$ against frequency ratio ω_h/ω_α for various values of K for $M = 0$ and $M = 0.7$. Elastic axis at 30 percent chord; $r_\alpha^2 = 0.25$; $a = -0.4$; $x_\alpha = 0.2$.

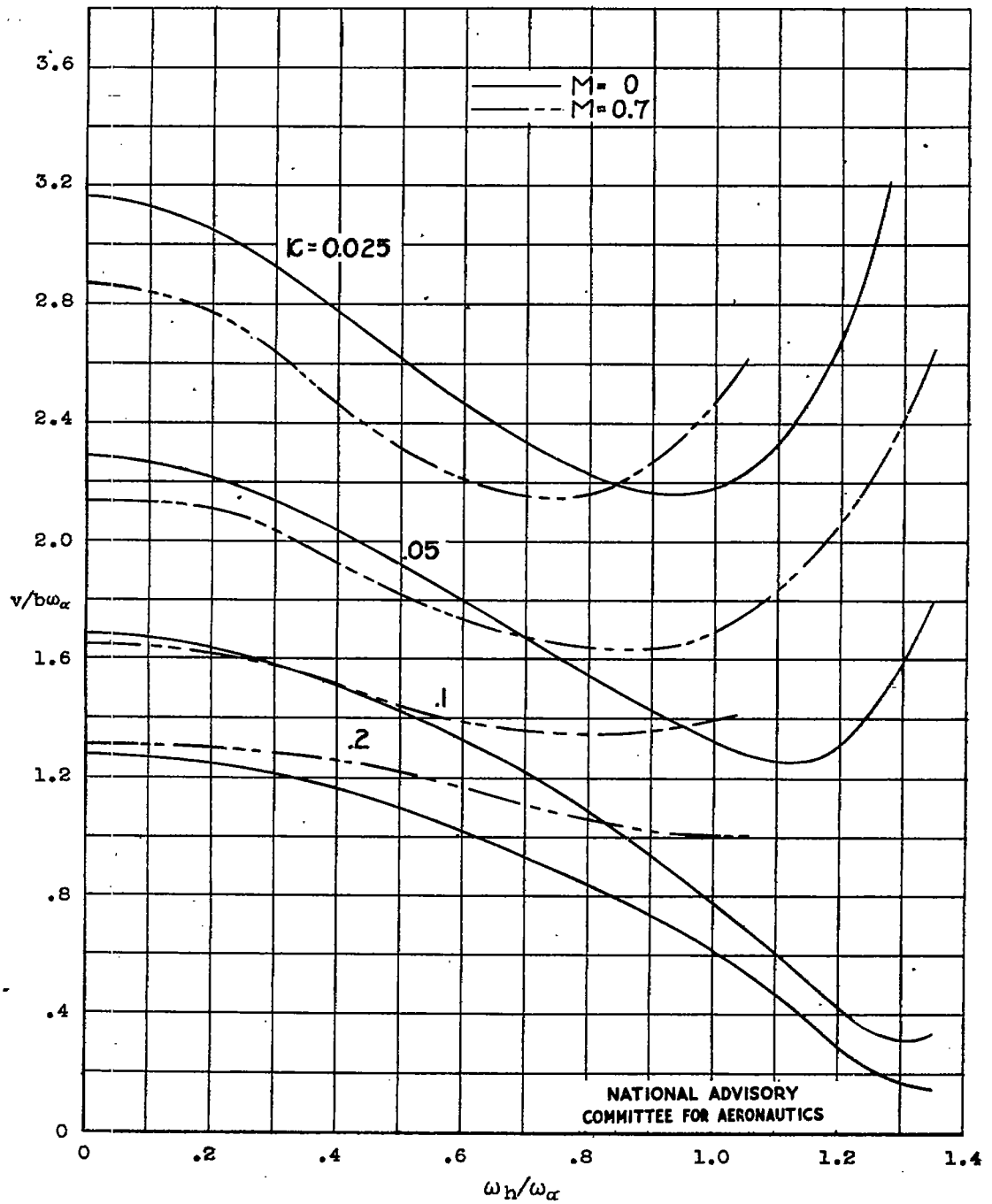

Figure 2.- The flutter coefficient $v/b\omega_\alpha$ against frequency ratio ω_h/ω_α for various values of K for $M = 0$ and $M = 0.7$. Elastic axis at 40 percent chord; $r_\alpha^2 = 0.25$; $a = -0.2$; $x_\alpha = 0.2$.

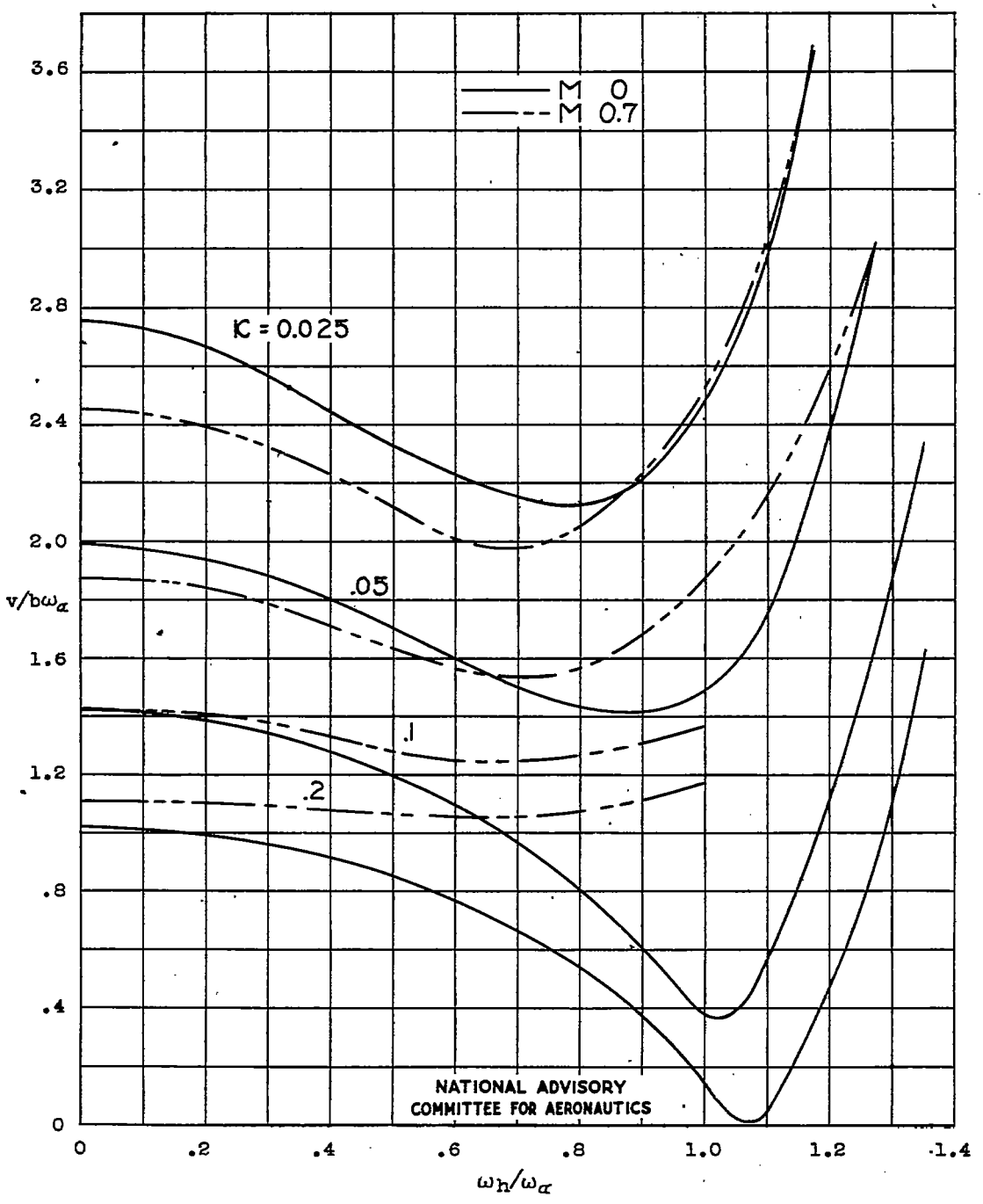

Figure 3.- The flutter coefficient $v/b\omega_\alpha$ against frequency ratio ω_h/ω_α for various values of K for $M = 0$ and $M = 0.7$. Elastic axis at 50 percent chord; $r_\alpha^2 = 0.25$; $a = 0$; $x_\alpha = 0.2$.

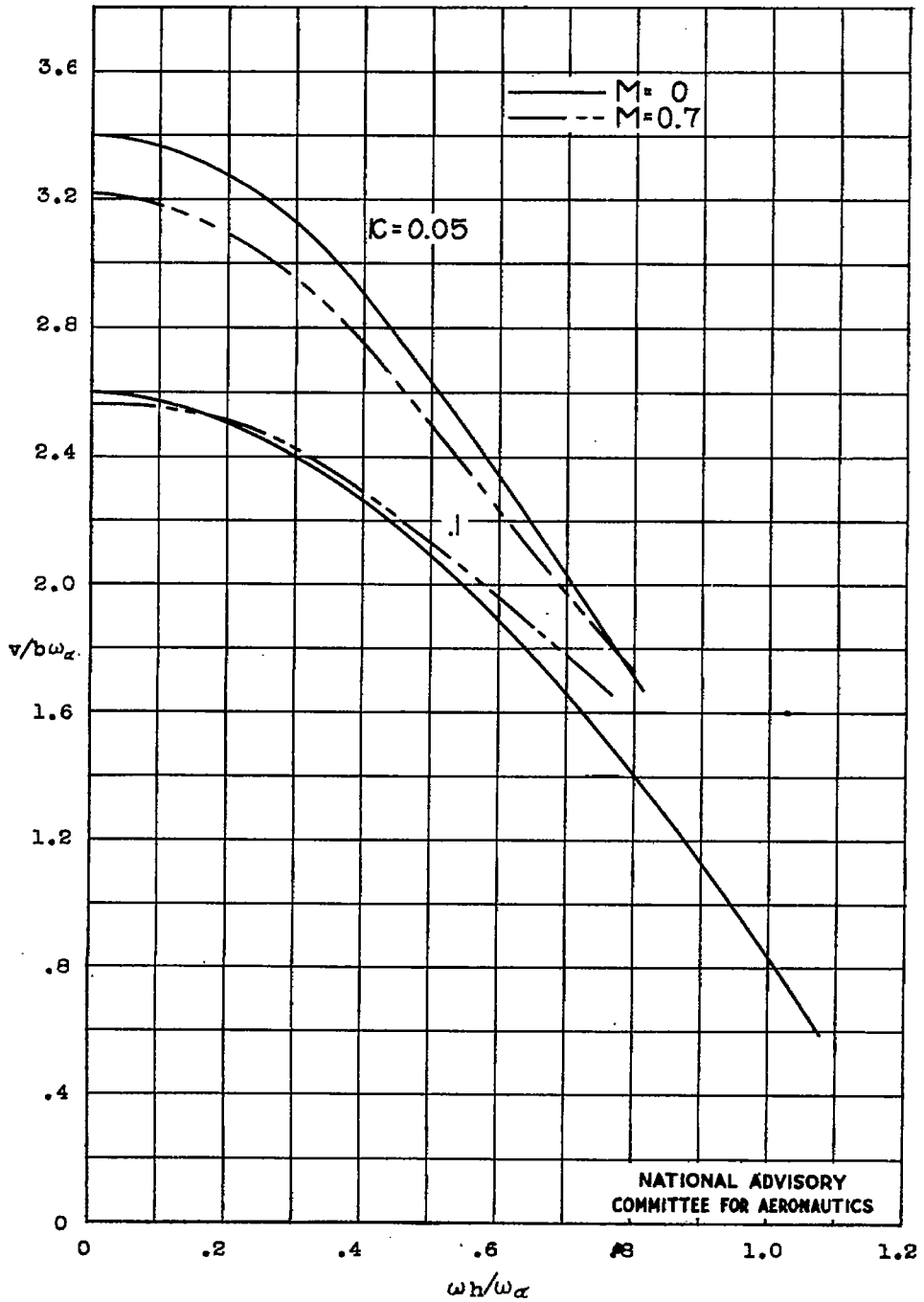

Figure 4.- The flutter coefficient $v/b\omega_\alpha$ against frequency ratio ω_h/ω_α for two values of K for $M = 0$ and $M = 0.7$. Elastic axis at 30 percent chord; $r_\alpha^2 = 0.25$; $a = -0.4$; $x_\alpha = 0.1$.