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PRESSURE DISTRIBUTION AND SOME EFFECTS OF VISCOSITY ON SLENDER INCLINED BODIES OF REVOLUTION

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SUMMARY

In connection with a study of the flow around slender inclined bodies of revolution, a simplified solution for the pressure distribution on such a body has been developed. The solution should be suitable for bodies of high fineness ratio even at low supersonic speeds, provided the angle of attack is small. Comparison with low-speed experimental results indicates that the observed flow separation phenomena can be explained in the terms of the calculated pressure distributions together with the theory of oblique viscous flows.

INTRODUCTION

The longitudinal distribution of cross force on inclined bodies of revolution, which was primarily of interest to airship designers in the past, was solved simply and effectively by Max Munk (reference 1). Munk showed that the cross force f per unit length on an arbitrary body of revolution in a nonviscous fluid stream was given approximately by

$$f = q_o \frac{dS}{dx} \sin 2\alpha \tag{1}$$

where

q_o stream dynamic pressure

dS/dx change in body cross-sectional area with longitudinal distance along the body

α angle of inclination

Tsien (reference 2) investigated the cross force on slender bodies of revolution at moderate supersonic speeds – a problem of more interest at the present to missile and supersonic aircraft designers – and showed that, to the order of the first power of the angle of inclination, the reduced Munk formula

$$f = 2q_o \frac{dS}{dx} \alpha \tag{2}$$

was still applicable. This is not surprising when it is realized that the cross component of the flow field corresponds to a cross velocity

$$V_{y_o} = V_o \sin \alpha$$

where V_o is the stream velocity. Thus the cross component of velocity, and, hence, the cross Mach number will, for small angles of inclination, be a small subsonic value so that the cross flow will be essentially incompressible in character.

Although the cross-force problem in a nonviscous flow has been satisfactorily investigated, the problem of determining the incremental pressure distribution due to inclined flow has not received such complete attention. Kaplan (reference 3) treated, in a thorough manner, the flow about slender inclined bodies, but the solution, which is expressed in Legendre polynomials, unfortunately, is tedious to evaluate. Laitone (reference 4), by linearizing the equations of motion, obtained a solution for the pressure coefficient on slender bodies of revolution in inclined flow, but, as will be seen, the solution, due to linearization, is inadequate in the general case.

In this report, a simple incompressible potential flow method is developed for determining the incremental pressure distribution resulting from inclined flow on a slender body of revolution. The method, for the reasons discussed previously, should be applicable for subsonic and supersonic flows wherein the cross velocity is small compared to the speed of sound.

It is known that viscosity actually plays an important part in the cross-flow phenomena even at moderate angles of attack. It may be shown theoretically that, under certain restrictions, the viscous cross force will be given by the drag of the body in cross-wise motion at a velocity equal to the crosswise component of the stream velocity. As a simple rule, the total cross force on a very slender body may be formed by adding this viscous cross force to the potential forces determined by Munk's formula. For such bodies this rule gives predicted characteristics in good agreement with experiment. A comparison is made in this report between the experimental pressure distributions on an inclined, model-airship hull and those calculated by the nonviscous theory. The comparison is instructive in indicating the conditions under which the calculated characteristics will be obtained and in showing the manner in which viscosity effects are manifested.

NOTATION

C constant of integration

P surface pressure coefficient $[(p-p_o)/q_o]$

$P_{\alpha=0}$ surface pressure coefficient at zero angle of stream inclination $[(p_{\alpha=0}-p_o)/q_o]$

ΔP incremental pressure coefficient due to angle of inclination $[(p-p_{\alpha=0})/q_o]$

f cross force per unit length on an inclined body of revolution

L length of body

M_o free-stream Mach number

p pressure on body or in field of body, as indicated

p_o free-stream pressure

$p_{\alpha=0}$ surface pressure at zero angle of stream inclination

q_o free-stream dynamic pressure

r polar radius about body axis of revolution

R radius at any station of body of revolution

S cross-sectional area at any station of body of revolution

t time

V_o free-stream velocity

V_x axial velocity at any station of body surface

V_{x_o} axial component of free-stream velocity

V_{y_o} cross component of free-stream velocity

x axial coordinate

y ordinate in plane of inclination normal to axis of revolution

z ordinate normal to plane of inclination and to axis of revolution

α angle of inclination

θ polar angle about axis of revolution measured from the approach direction of the cross-stream velocity

η $\tan^{-1} \frac{dR}{dx}$

ρ fluid mass density

φ velocity potential

THEORY

Consider the flow over the body of revolution shown in figure 1 which is inclined at an angle α to the stream of velocity V_o. If the body is slender, the axial component velocity V_x at the body surface will not differ appreciably from the axial component V_{x_o} of the stream velocity. With this condition, it is clear that the flow may be treated by considering the two-dimensional flow in a plane parallel to the yz plane, which plane is moving downstream with the constant velocity V_{x_o}. In other words, the problem may be treated by determining the two-dimensional flow about a circular cylinder which is first growing (over the forebody) and then collapsing (over the afterbody) with time.

The velocity potential for the cross flow at any x station is given in polar coordinates as

$$\varphi = -V_{y_o} \left(r + \frac{R^2}{r}\right) \cos \theta \tag{3}$$

which in this moving reference plane is a function of time.

Bernoulli's equation for an incompressible flow which changes with time is

$$\frac{p}{\rho} = -\frac{\partial \varphi}{\partial t} - \frac{1}{2}\left[\left(\frac{\partial \varphi}{\partial r}\right)^2 + \left(\frac{\partial \varphi}{r \partial \theta}\right)^2\right] + C \tag{4}$$

Now from equation (3)

$$\frac{\partial\varphi}{\partial t} = -2V_{y_o}\left(\frac{R}{r}\right)\frac{dR}{dt}\cos\theta \tag{5}$$

but

$$\frac{dR}{dt} = \frac{dR}{dx}\frac{dx}{dt} = V_{x_o}\tan\eta \tag{6}$$

so that equation (5) becomes

$$\frac{\partial\varphi}{\partial t} = -2V_{y_o}V_{x_o}\tan\eta\left(\frac{R}{r}\right)\cos\theta \tag{7}$$

Also, by differentiation of equation (3),

$$\left.\begin{aligned}\frac{\partial\varphi}{\partial r} &= -V_{y_o}\cos\theta\left(1-\frac{R^2}{r^2}\right)\\ \frac{\partial\varphi}{r\partial\theta} &= V_{y_o}\sin\theta\left(1+\frac{R^2}{r^2}\right)\end{aligned}\right\} \tag{8}$$

so that equation (4) for the pressure at any point in the flow field becomes

$$\frac{p}{\rho} = 2V_{y_o}V_{x_o}\tan\eta\left(\frac{R}{r}\right)\cos\theta - \frac{V_{y_o}^2}{2}\left\{\cos^2\theta\left[1-\left(\frac{R}{r}\right)^2\right]^2 + \sin^2\theta\left[1+\left(\frac{R}{r}\right)^2\right]^2\right\} + C \tag{9}$$

for

$$r \rightarrow \infty,\ p \rightarrow p_o$$

so

$$C = \frac{p_o}{\rho} + \frac{V_{y_o}^2}{2}$$

and hence equation (9) for the pressure at the surface of the body becomes for $r = R$

$$\frac{p-p_o}{\rho} = 2V_{y_o}V_{x_o}\tan\eta\cos\theta + \frac{V_{y_o}^2}{2}\left(1-4\sin^2\theta\right) \tag{10}$$

and writing

$$V_{y_o} = V_o \sin \alpha$$

$$V_{x_o} = V_o \cos \alpha$$

the surface pressure in coefficient form becomes

$$P = \frac{p-p_o}{q_o} = 2 \tan \eta \cos \theta \sin 2\alpha + \left(1 - 4 \sin^2 \theta\right) \sin^2 \alpha \qquad (11)$$

For small angles of inclination

$$\sin 2\alpha = 2\alpha$$

$$\sin^2 \alpha = \alpha^2$$

then

$$P = \left(4 \tan \eta \cos \theta\right) \alpha + \left(1 - 4 \sin^2 \theta\right) \alpha^2 \qquad (12)$$

This equation applies to a body of revolution of such extreme length that, at zero inclination, the local pressure at any x station is essentially the stream static pressure p_o.

For bodies of lower fineness ratio at zero angle of inclination, the surface pressure at any station, designated $p_{\alpha=0}$, will differ slightly from the static pressure p_o but, if the fineness ratio is not too low, the pressure $p_{\alpha=0}$ will also closely approximate the pressure for some distance in this yz plane away from the body surface. Under this assumption that the pressure at the surface at zero inclination applies uniformly in the portion of the yz plane for which the major effects of the cross-flow distribution are felt, the change in pressure from p_o to $p_{\alpha=0}$ will be additive to, but will not otherwise influence, the cross-flow pressure distribution. Hence for any station on a body of high fineness ratio for which, at zero inclination, the pressure is $p_{\alpha=0}$, the pressure coefficient distribution at this same station under inclined flow conditions will be, from equation (11),

$$P = P_{\alpha=0} + \left(2 \tan \eta \cos \theta\right) \sin 2\alpha + \left(1 - 4 \sin^2 \theta\right) \sin^2 \alpha \qquad (13)$$

or, from equation (12) for small angles of inclination,[1]

$$P = P_{\alpha=0} + \left(4 \tan \eta \cos \theta\right) \alpha + \left(1 - 4 \sin^2 \theta\right) \alpha^2 \tag{14}$$

The cross force per unit length of the body is then found as

$$f = \int_0^{2\pi} p \cos \theta \, R \, d\theta = 2q_o \int_0^{\pi} PR \cos \theta \, d\theta + 2\int_0^{\pi} p_o \, R \cos \theta \, d\theta$$

and clearly

$$2\int_0^{\pi} p_o \, R \cos \theta \, d\theta \equiv 0$$

Substituting P from equation (11) gives

$$f = 2Rq_o \, P_{\alpha=0} \int_0^{\pi} \cos \theta \, d\theta + 4Rq_o \tan \eta \sin 2\alpha \int_0^{\pi} \cos^2 \theta \, d\theta +$$

$$2Rq_o \sin^2 \alpha \int_0^{\pi} (1 - 4 \sin^2 \theta) \cos \theta \, d\theta$$

The first and third integrals are zero while the second integral yields

$$f = 2\pi Rq_o \tan \eta \sin 2\alpha$$

and since

$$2\pi R \tan \eta = 2\pi R \frac{dR}{dx} = \frac{dS}{dx}$$

then

$$f = q_o \frac{dS}{dx} \sin 2\alpha$$

which is equation (1) derived by Munk for the cross force on slender airship hulls and, in the form,

$$f = 2q_o \frac{dS}{dx} \alpha$$

[1]Equation (14), for the case in which tangent η is constant, reduces to that derived by Busemann (reference 5) for the flow over an inclined cone.

that derived by Tsien for the cross force, to the order of the first power of the angle of inclination, for slender bodies at moderate supersonic speeds. This development shows that these equations for the cross force are also correct to the second power of α.

EXPERIMENTAL RESULTS AND DISCUSSION OF THE EFFECTS OF VISCOSITY

In reference 6, a thorough investigation at low speeds was made of the pressure distribution over a hull model of the rigid airship "Akron." Incremental pressure distributions due to inclination calculated by equation (13) for five stations along the hull at three angles of attack are compared with the experimental values in figures 2 to 6. In each of the figures is shown a sketch of the airship which indicates the station at which the incremental pressure distributions apply. This comparison represents a severe test of the theoretical method of this report since the method was developed on the assumption that the fineness ratio of the body is very large, while for the case considered the fineness ratio is only 5.9.

At the more forward stations (figs. 2 to 4), the agreement is seen to be essentially good[2] but some discrepancy – particularly at values of θ near 180° – is evident which increases with increasing distance from the bow. The discrepancy increases rapidly, proceeding to sections (figs. 5 and 6) downstream of the maximum diameter section until (fig. 6) the entire distribution is affected.

The disagreement that exists at the afterbody stations results from effects of viscosity not considered in the theory as will be seen from the following: R. T. Jones, in reference 8, showed that for laminar flow on an infinitely long, inclined right circular cylinder, the behavior of the component flow of a viscous fluid in planes normal to the axis of revolution was independent of the component flow parallel to this axis. That is, viewed along the cylinder, the flow of a viscous fluid about the cylinder would appear identical to the flow about a section of a right circular cylinder in a stream moving at the velocity $V_o \sin \alpha$. Hence, separation of the flow would occur in the yz plane as a result of the adverse pressure gradients that exist across the cylinder. Jones demonstrated that this behavior well explained the cross forces on inclined cylinders that were experimentally observed in reference 9. That such separation effects also occur on the inclined hull model of the "Akron" is also evident, particularly in figures 5 and 6.

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[2]At stations extremely close to the bow the method must be inaccurate as evident from the work of Upson and Klikoff (reference 7).

While the treatment of reference 8 explains qualitatively the observed behavior of the flow field about the hull model considered, it cannot be used quantitatively, in the general case, for at least three reasons: First, the theory of reference 8 was demonstrated only for laminar flows; the separation effects on bodies with turbulent flows may not be the same as indicated in that reference. Second, the forward sections of bodies of finite length will behave more nearly as a circular cylinder set in motion initially from rest. Thus, although the adverse gradients exist, the flow, as shown in reference 10, will not have had sufficient time to exhibit the usual separated flow characteristic of the steady-state flow across a circular cylinder. This, in part, explains the good agreement evident between the calculated and experimental values of figures 2 and 3. Third, the influence of the term

$$2 \tan \eta \cos \theta \sin 2\alpha$$

of equation (13) is to distort the typical circular cylinder pressure distribution, given by the term

$$(1 - 4 \sin^2 \theta) \sin^2 \alpha$$

to move the calculated position of minimum pressure away from the $\theta=90^\circ$ point and to change the magnitude of the pressure to be recovered. Over the forward stations of the body where $\tan \eta$ is positive, the position of minimum pressure lies between 90° and 180° and the pressure required to be recovered is small and even zero at the most forward stations. For the rearward station where $\tan \eta$ is negative, the minimum pressure lies between 0° and 90°, and the pressure recovery required is large and increases proceeding toward the stern. For the hull of the "Akron" model, the calculated line of minimum pressure along the hull is, for the angles of attack of 6°, 12°, and 18°, as shown in figure 7.[3] Since separation can only occur in an adverse gradient, it is clear that the line of separation will roughly follow the line of minimum pressures. Hence, again, the flow about forward stations will be, or will more nearly be, that calculated for a nonviscous fluid. Over the rearward stations the flow separation should tend to be even more pronounced than would occur on a right circular cylinder.

From the foregoing, it is evident that the potential flow solution for the pressures on inclined bodies can only be expected to hold over the forebody, and that over the afterbody the pressure distribution will be importantly influenced by the fluid viscosity.

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[3] It is of interest to note in this figure that even for small angles of inclination the lines of minimum pressure become oriented close to the direction of the axis of revolution, while at zero inclination it must, of course, be normal to this axis.

The equations of this report for the pressure distribution on an inclined body were developed on the assumption that the flow was incompressible. Nevertheless, as noted earlier, the equations should be applicable to supersonic flow when the appropriate values for $P_{\alpha=0}$ are used as long as the cross component of the Mach number $M_o \sin \alpha$ is not too large compared with the critical Mach number of a circular cylinder.

In reference 4, the pressure distribution was determined for bodies of high fineness ratio to the order of first power in α and the results agree with equation (14) of this report to this order. In light of the present solution of equation (14), it is clear that the solution of reference 4 is only correct when α is not only small but small by comparison with η, for, when α is of the same order as η, the term

$$(1 - 4 \sin^2 \theta) \alpha^2$$

assumes equal importance with the linear term. Hence, for the usual small cone angles used on supersonic bodies, the solution of reference 4 is seriously limited in the range of angles of inclination for which it will give sufficiently accurate results.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., Dec. 28, 1949.

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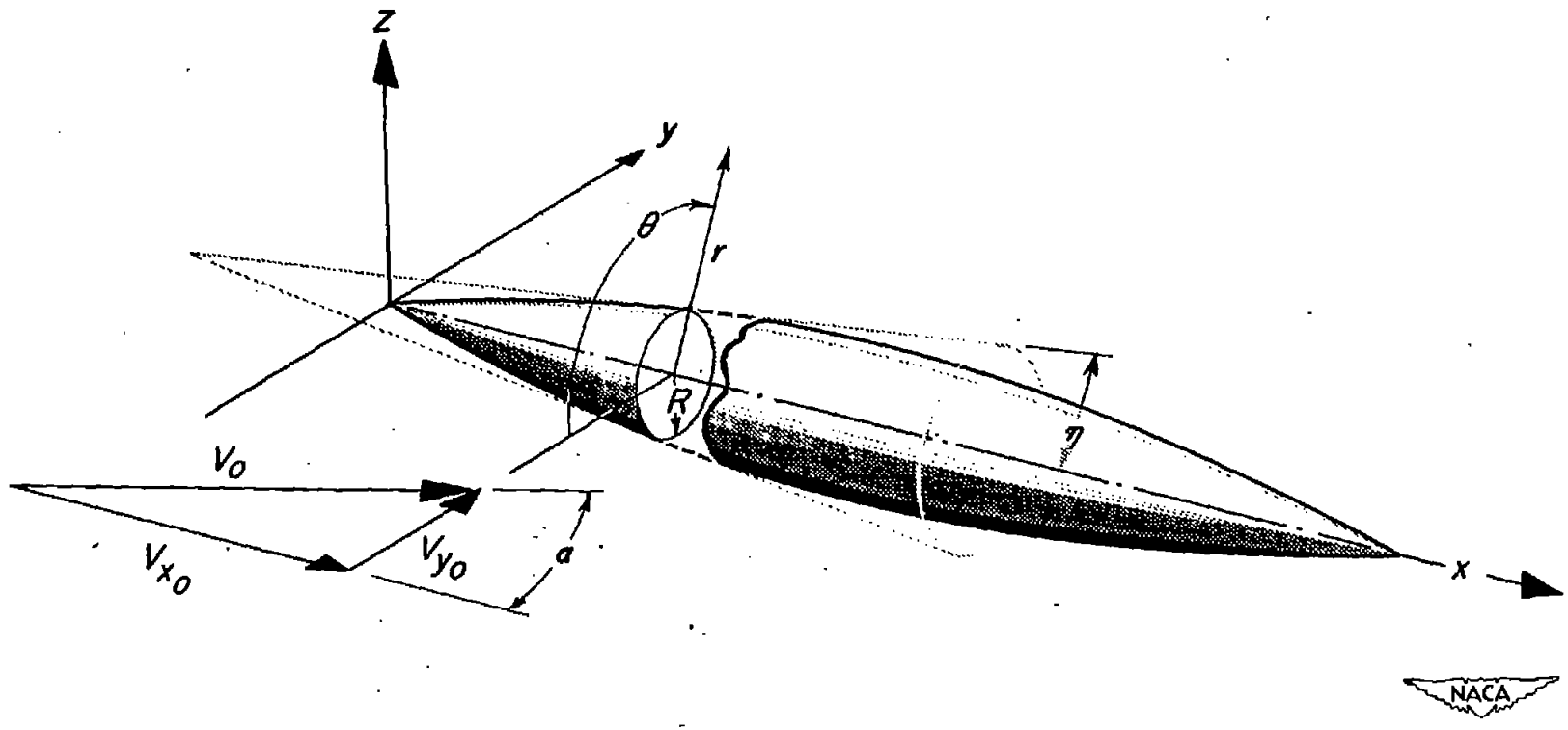

Figure 1.— Body of revolution in inclined flow field.

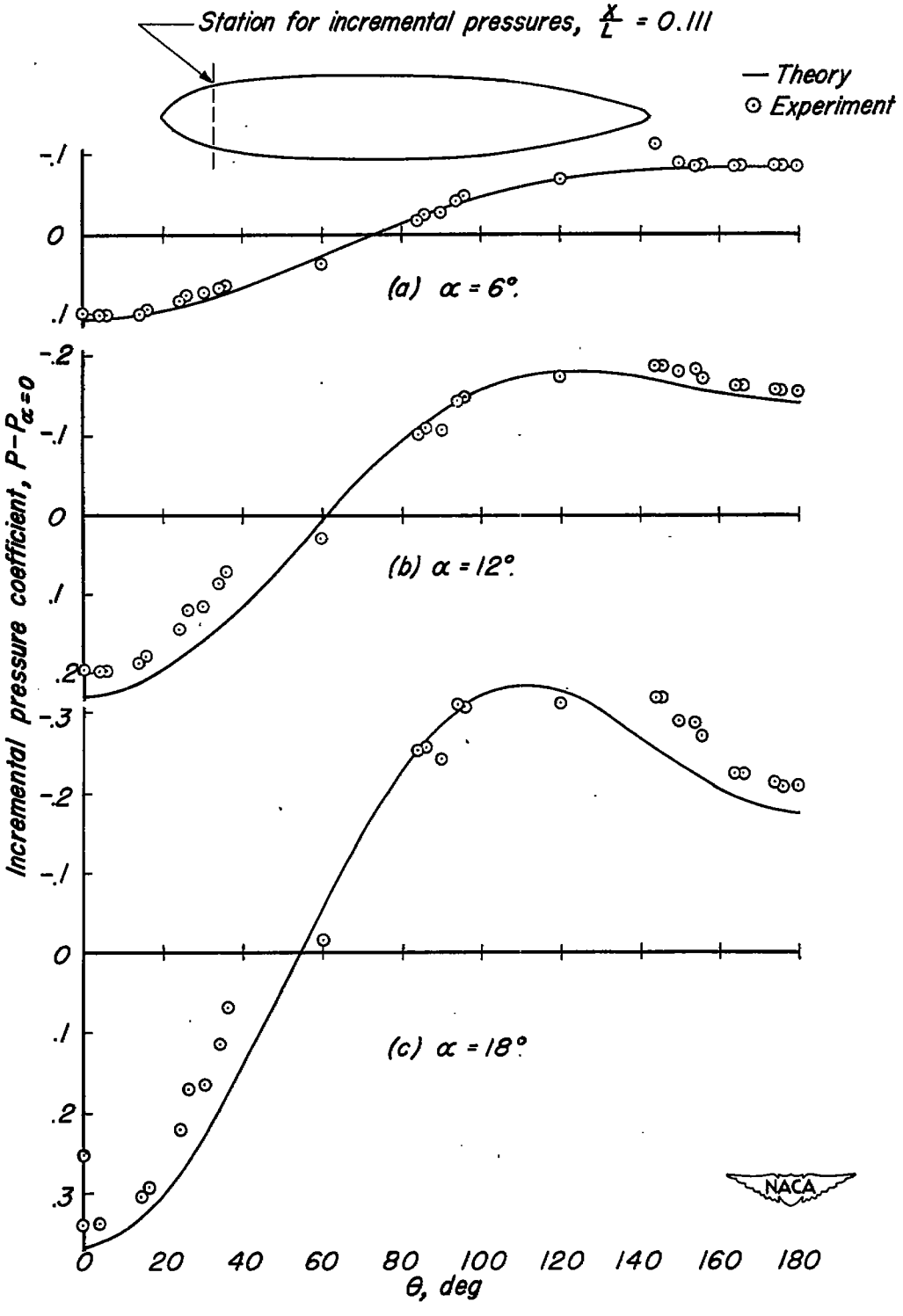

Figure 2.- Calculated and experimental pressure distribution on a model hull of the U.S.S. Akron at station $\frac{x}{L}$ = 0.111.

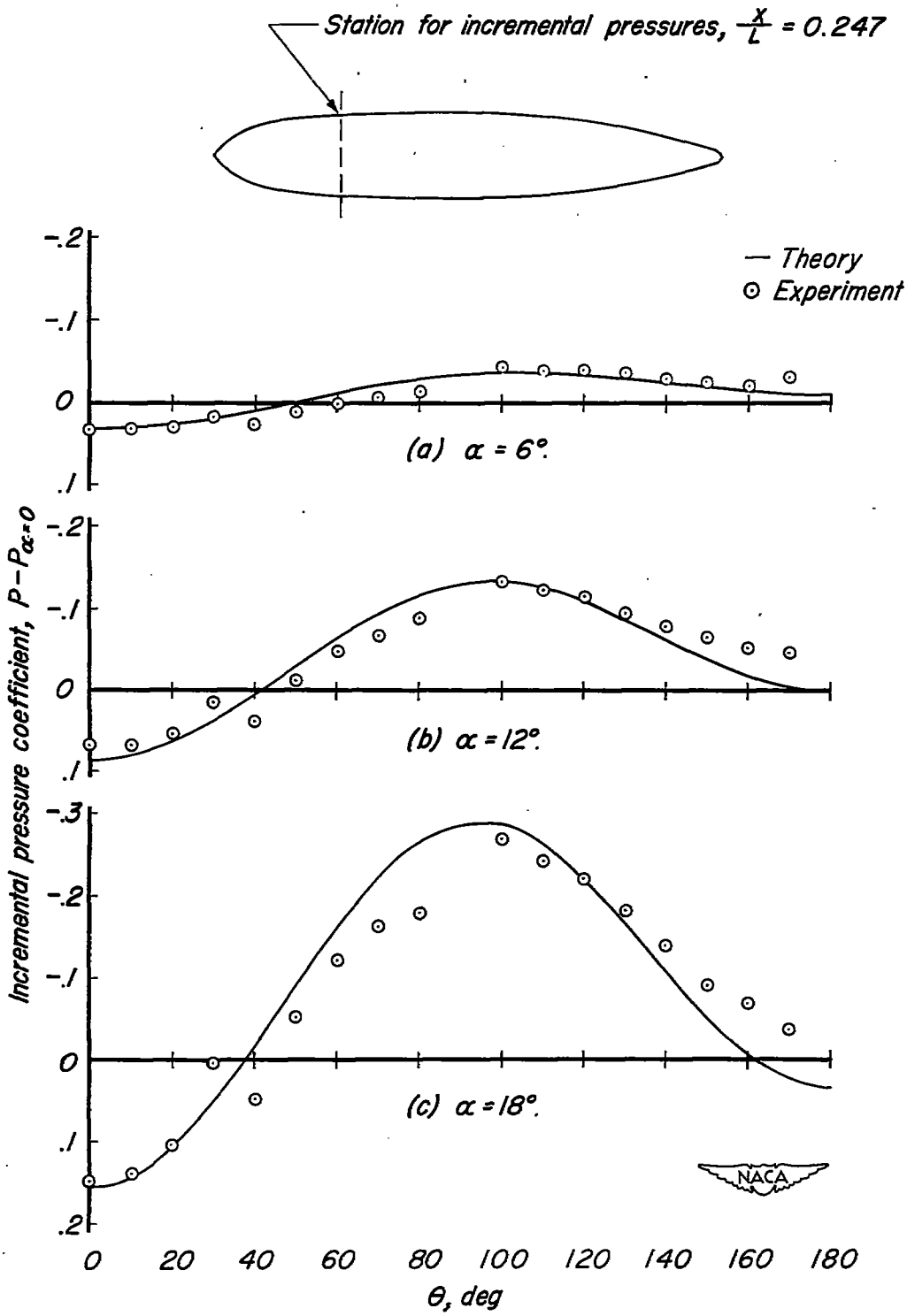

Figure 3.–Calculated and experimental pressure distribution on a model hull of the U.S.S. Akron at station $\frac{x}{L}$=0.247.

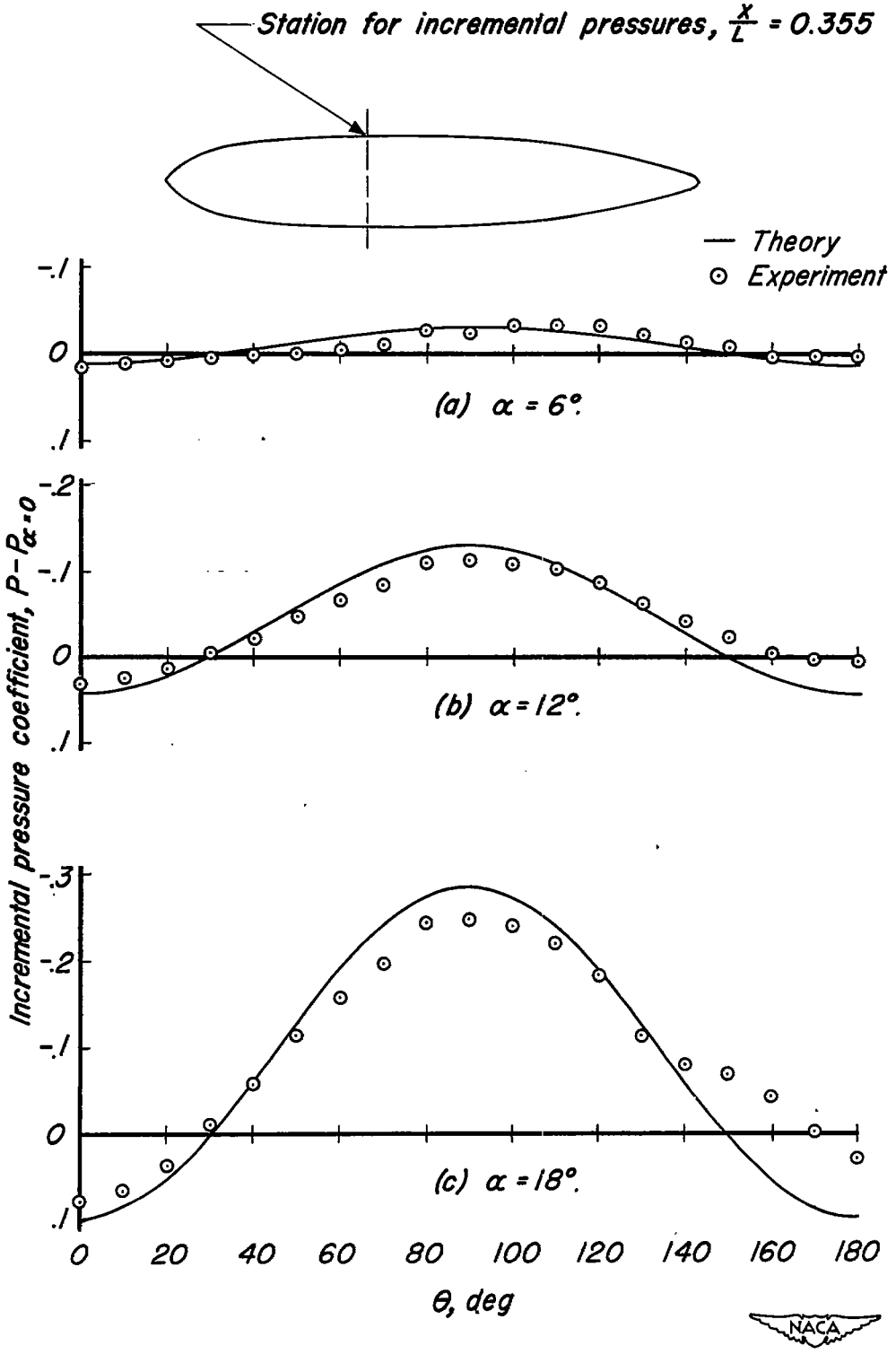

Figure 4.- Calculated and experimental pressure distribution on a model hull of the U.S.S. Akron at station $\frac{x}{l}$=0.355.

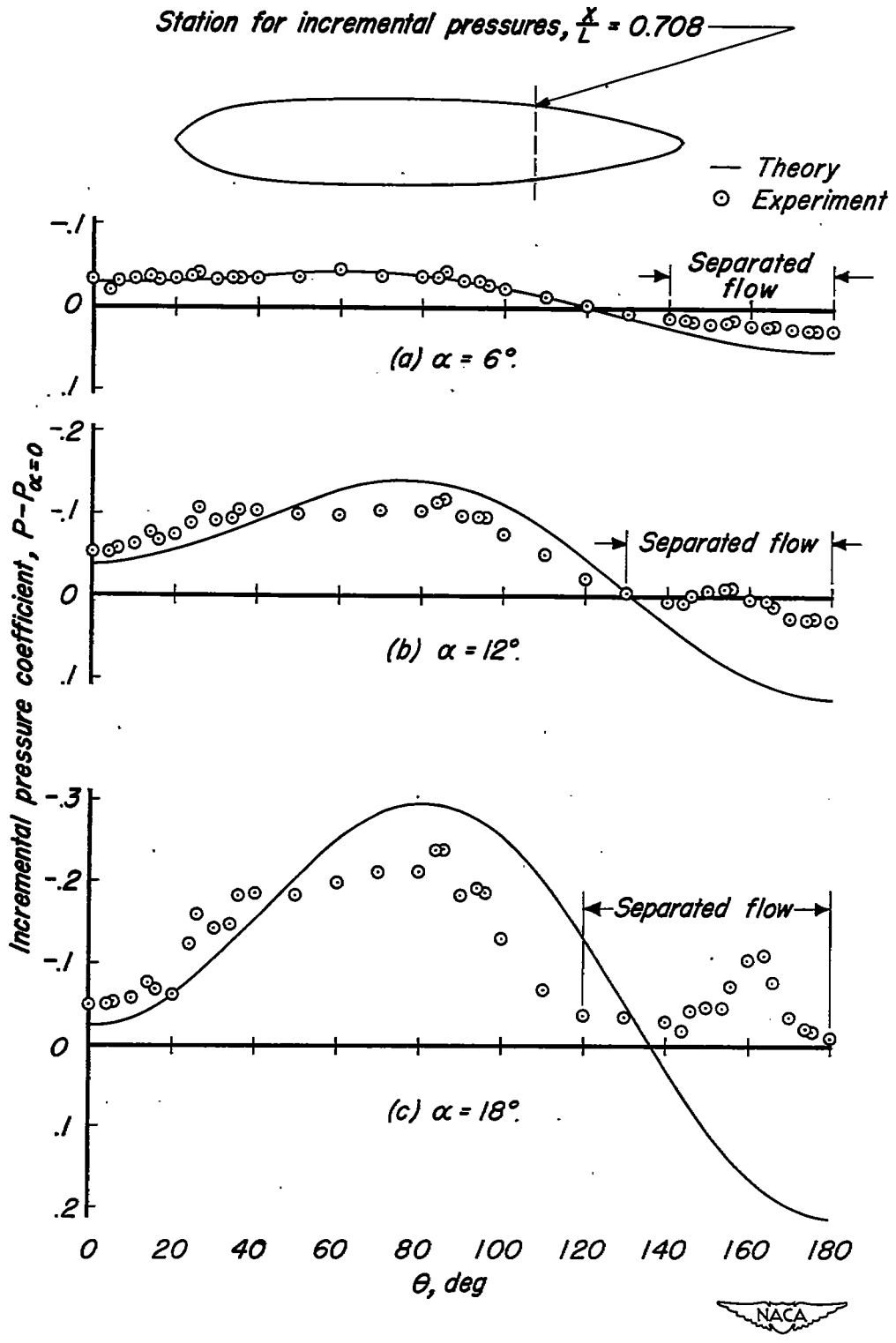

Figure 5.– Calculated and experimental pressure distribution on a model hull of the U.S.S. Akron at station $\frac{x}{L}$=0.708.

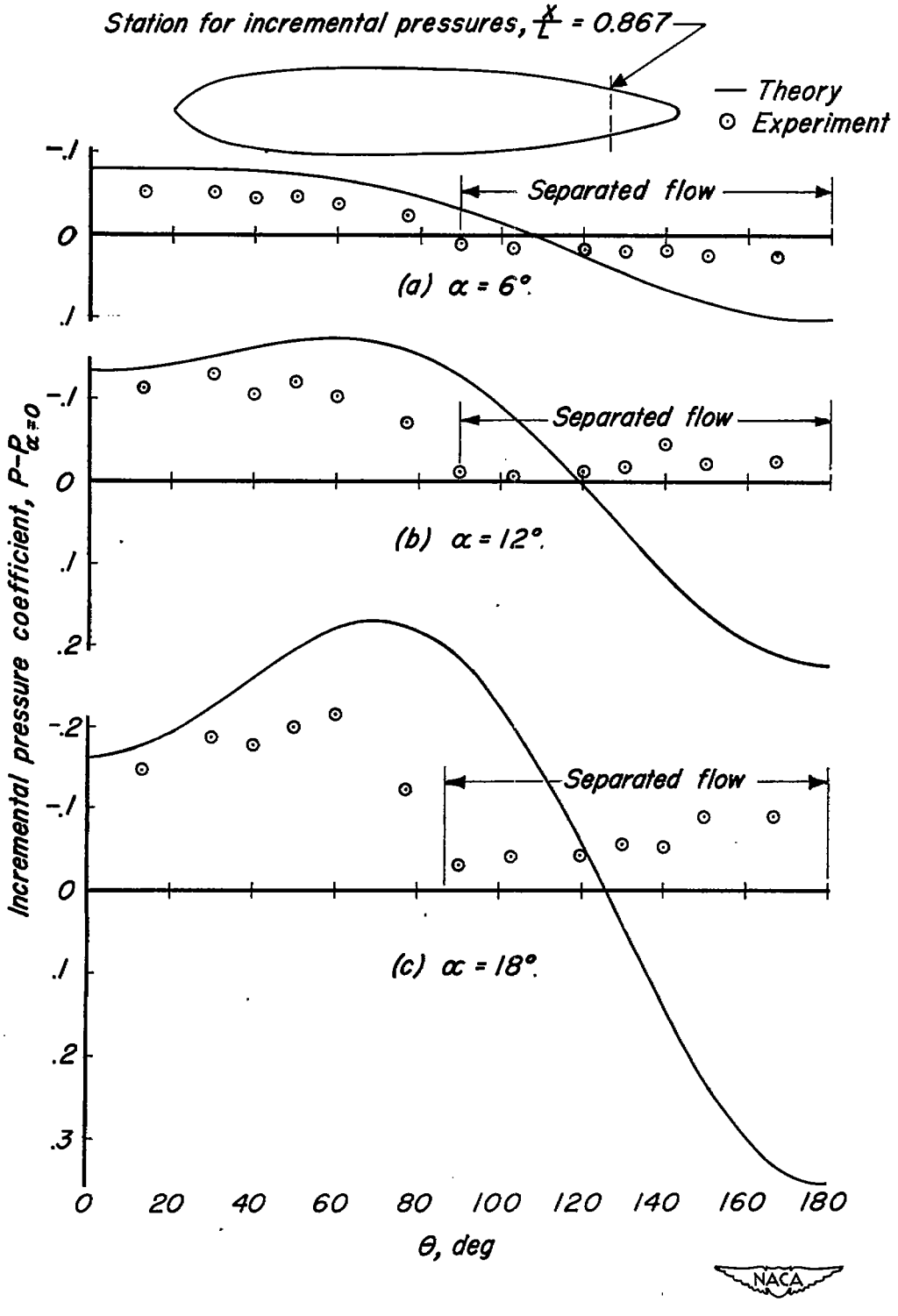

Figure 6.—Calculated and experimental pressure distribution on a model hull of the U.S.S. Akron at station $\frac{x}{L}$=0.867.

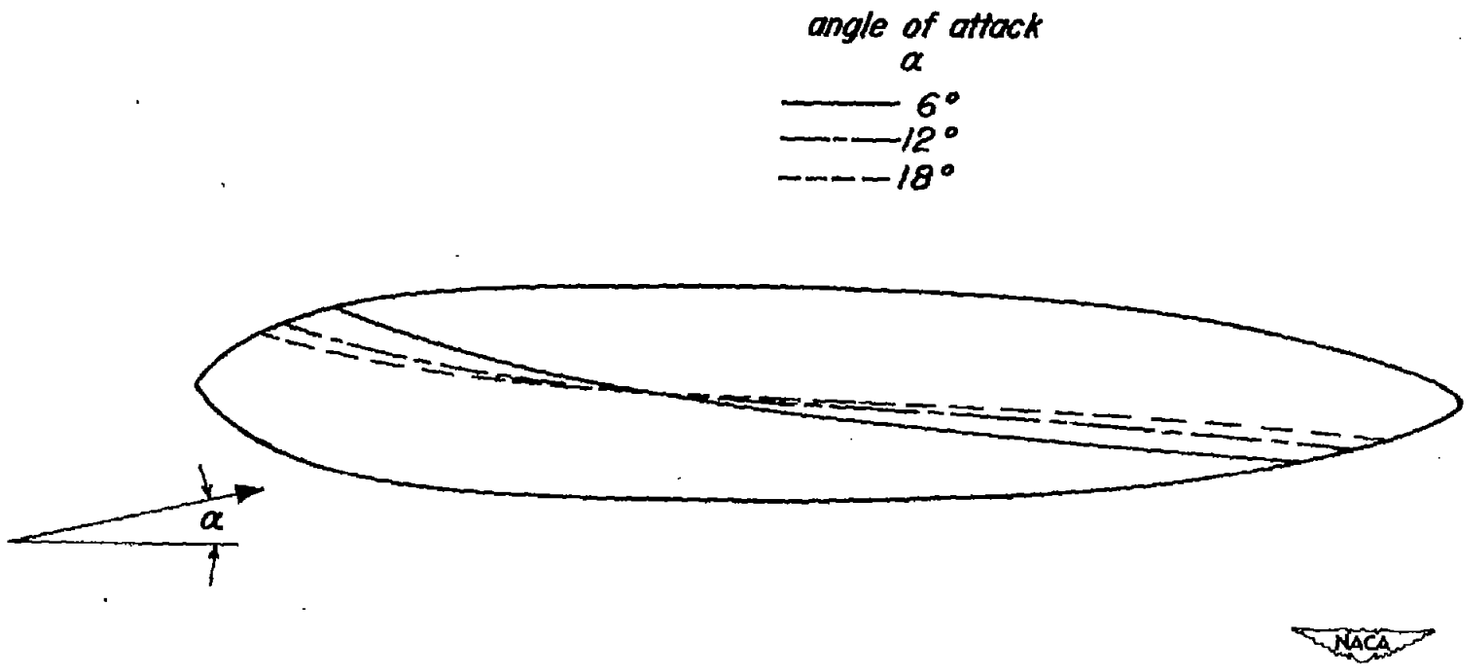

Figure 7.– Calculated lines of minimum pressures for a model hull of U.S.S. Akron at three angles of attack.

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