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TIME-VELOCITY-ALTITUDE RELATIONS FOR AN AIRPLANE DIVING IN A STANDARD ATMOSPHERE

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

ADVANCE RESTRICTED REPORT

TIME-VELOCITY-ALTITUDE RELATIONS FOR AN AIRPLANE DIVING IN A STANDARD ATMOSPHERE

By H. A. Pearson

SUMMARY

The present paper supersedes Technical Note No. 599 for determining the time-velocity-altitude relations for airplanes diving in a standard atmosphere and the Advance Confidential Report entitled "Time-Velocity-Altitude Relations for an Airplane Diving in a Standard Atmosphere." The charts of Technical Note No. 599 have been extended to include altitudes up to 36,000 feet and "nominal" terminal velocities up to 800 miles per hour. In addition, the present paper corrects an error in the factor that was used in both superseded papers for converting the original basic charts to the case of the inclined steady dive and gives a simple method for taking into account the effects of compressibility on the time-velocity-altitude relations. Two examples are included to illustrate the use of the charts.

INTRODUCTION

The velocity-altitude relations for airplanes in a dive have been treated by several writers. Diehl (reference 1) assumed a constant-density atmosphere. Wilson (reference 2) and Becker (reference 3), who have taken the variation of density into account though using different approaches, have given no method for determining the time to dive. Regardless of the manner in which the density is taken into account, the velocity-altitude equations become too lengthy and complicated for general use when a relatively quick answer may be desired. Charts showing the time-velocity-altitude relations have therefore been constructed.

Such a series of charts was given in reference 4 for determining the time-velocity-altitude relations for an

airplane diving in a standard atmosphere. When these charts were first prepared (1934), the range of initial altitudes extended to 32,000 feet and the range of airplane "nominal" terminal velocities extended to 550 miles per hour. These limits, for both the altitudes and the terminal velocities, were mainly determined by the performance of the airplanes available at that time, although the difficulty of including compressibility effects and altitudes above the tropopause was recognized.

At the present time, however, the nominal indicated terminal velocities of a large number of airplanes, that is, the terminal velocity based on a constant minimum profile-drag coefficient and standard sea-level conditions, are found to be considerably in excess of the speed of sound (763 mph) and altitudes above 32,000 feet are quite ordinary. The original charts are thus insufficient to cover the possible range even though the effects of compressibility on the profile-drag coefficient could be neglected.

The present paper extends the original charts to nominal terminal velocities above the speed of sound and to an altitude of 36,000 feet. The extension also includes a relatively simple method for determining the effect of compressibility on the time-velocity-altitude relations. The use of this method requires a knowledge of the nominal terminal velocity of the airplane and of a terminal Mach number for the airplane profile-drag coefficient.

In addition to the extensions made to the time-velocity charts of reference 4, the present paper corrects an error that was made in the factor of this reference, which was also used in reference 5, for converting the original basic charts to the case of the inclined steady dive. In order to make the present paper complete and independent of references 4 and 5, the necessary original derivations given therein are first repeated and the extensions and modifications are then introduced.

TIME-VELOCITY-ALTITUDE CHARTS WITH COMPRESSIBILITY EFFECTS NEGLECTED

The time-velocity-altitude charts, which are given in figures 1 to 14, cover a range of nominal terminal

velocities from 150 to 800 miles per hour in increments of 50 miles per hour. The starting altitudes vary from 8000 to 16,000 feet in intervals of 2000 feet and from 16,000 to 36,000 feet in intervals of 4000 feet. The terminal velocity U, by which each chart is designated, is the velocity that a body would have in an atmosphere of constant sea-level density and with a constant profile-drag coefficient such as would be obtained at relatively low speeds. The abscissas of the curves are the true, not the indicated, velocities.

In the establishment of the velocity-altitude curves (full curved lines in figs. 1 to 14), the type of equation developed in reference 2 was used with slight modifications to the constants to give better agreement with the recognized standard atmosphere of reference 6. These modifications consisted in replacing the factors 3 and 1200 occurring in the original equations of reference 2 by the factors 2.7 and 1254, respectively. The new equation is

$$V^2 = 2g\left(1 + \frac{2.7h}{64000}\right)^{\left(\frac{1254}{1.466U}\right)^2} \int_h^H \left(1 + \frac{2.7h}{64000}\right)^{-\left(\frac{1254}{1.466U}\right)^2} dh \qquad (1)$$

where

V true airspeed, feet per second

g acceleration of gravity (32.2 ft/sec^2)

h altitude, above sea level, feet

H starting altitude, above sea level, feet

U terminal velocity of airplane in air at standard sea-level density with constant profile-drag coefficient, miles per hour

The placing of the time network on the charts, shown by the full lines running diagonally across the various curves, was accomplished either by the use of the equations applying to a body falling in a vacuum or by the use of a step-by-step process of integration. Each of the foregoing methods had a particular region in which it was more easily applied than the other for the same degree of accuracy.

The acceleration of an airplane in a vertical dive, at any instant, can be given by

$$a = 32.2\left(1 - \frac{q}{q_u}\right) \tag{2}$$

where

a acceleration, feet per second per second

q dynamic pressure, pounds per square foot $\left(\frac{1}{2}\rho V^2\right)$

q_u dynamic pressure at the nominal terminal velocity, pounds per square foot

ρ mass density of air, slugs per cubic foot

Equation (2) indicates that, during the early part of the dive before appreciable velocity is gained, the acceleration differs only slightly from g; for this range of the charts, therefore, the vacuum formula is applicable within the plotting accuracy. The range within which the vacuum formula can be applied for determining the time increases with either the starting altitude or the terminal velocity. Most of the timing lines below 6 seconds were computed by using the vacuum formula and, in some cases, the range was extended to 8 seconds. At values above 8 seconds, the time lines were established by a step-by-step integration of the velocity-altitude curves.

The charts (figs. 1 to 14), although derived for a vertical dive starting from rest, may be used with various diving angles and starting velocities. If U is the terminal velocity of an airplane in a vertical dive, the terminal velocity in a dive in which the flight path makes a constant angle γ with the ground is

$$U\,(\sin\gamma)^{\frac{1}{2}}$$

The new nominal terminal velocity to be used in selecting the appropriate chart for determining the velocity-altitude relations in an inclined dive is therefore

$$U' = U\,(\sin\gamma)^{\frac{1}{2}} \tag{3}$$

Although the velocity-altitude relations for the inclined dive are correctly given by the new chart, the time-altitude relations that are obtained must be divided by the sine of the dive angle. The treatment of the inclined dive used in references 4 and 5 has been found to be in error.

Values of $U(\sin\gamma)^{\frac{1}{2}}$ are tabulated on each of the charts and it will be noted that, when these factors are used, interpolation between charts will almost always be necessary to obtain the necessary relations. The effect of an initial diving speed is taken into account by considering that the airplane started to dive from rest at a somewhat higher altitude.

The only errors in the charts that need be considered are those due to plotting and to the discrepancies that will occur in any step-by-step integration. The error in the time lines due to the last source is believed to be within 2 percent; the plotting error in the velocity-altitude curves is considered negligible.

In the application of these charts to diving airplanes, however, several uncontrollable sources of error will exist, namely:

1. Variation of atmosphere from standard

2. Manner of entry into the dive

3. Variation of the dive angle from the value assumed

4. Propeller effect on drag

5. Scale and compressibility effect on the airplane drag

The error due to the first source is believed to be negligible and need not be considered. The manner of entry into the dive, although relatively unimportant in the determination of the time-velocity-altitude relations for the longer dives, may become important in the shorter dives. The effect of using an erroneous dive angle is likely to cause appreciable errors in the determination of the time-velocity-altitude relations only when the dive angle is small. An erroneous evaluation of the propeller effect on the drag is another source of error that is likely to occur. In the selection of the proper chart, it is necessary that the propeller effect be taken into account.

The charts as given do not consider any variation of profile-drag coefficient with either Reynolds or Mach number; whereas it is known that, at certain critical speeds, large variations will occur because of compressibility effects. Before this region of rapid increase is reached, however, the profile-drag coefficient may be considered constant and the charts may be used up to this point to obtain the time-velocity-altitude relations. Because the point at which the drag coefficient increases cannot be established in advance, various dashed lines have been placed on figures 5 to 14 to represent a number of values of the Mach number M from 0.5 to 1.0. These lines were determined for the chart by the equation

$$V = 33.5M\sqrt{519 - 0.00356h} \qquad (4)$$

which expresses the relation between the velocity in miles per hour, the Mach number, and the altitude in a standard atmosphere. Values of M from 0.50 to 1.00 were first assumed, the points of the various curves that satisfied the foregoing relation were then obtained by a trial-and-error process, and all points for a given value of M were joined.

ESTIMATION OF COMPRESSIBILITY EFFECTS

In order to use the charts that are given (figs. 1 to 14), it is necessary to determine or to define a point in terms of the Mach number beyond which these charts should not be used. Obviously, no simple determination of this quantity is possible because the airplane is made up of a number of parts, each of which may have a different "critical" speed and a different rate of change of profile-drag coefficient in this critical region.

The rate of increase in the over-all airplane profile-drag coefficient beyond the point at which the first part reaches its critical speed may vary within rather wide limits, depending upon the shape and the relative size of the part and also upon whether a number of parts reach their critical speeds simultaneously or in succession.

In order to estimate the variation of the over-all profile-drag coefficient with Mach number from results obtained at speeds for which compressibility effects are

small, as in the ordinary wind-tunnel test, the increments of profile-drag coefficient of the various parts must be added to the initial over-all value, which is assumed to be reasonably constant. Then, inasmuch as the drag coefficient is generally based on wing area, it is necessary to correct the increments to the wing area by multiplying the drag-coefficient increments of each part by the ratio of the area on which the drag-coefficient increments are based to the wing area.

The variation with Mach number of profile-drag coefficient near zero lift for some well-known profiles is shown in figure 15; the variation with Mach number of drag coefficient for various cowlings on a typical wing nacelle is shown in figure 16. These data, which are reported in references 7 to 10, are typical of results obtained in the NACA high-speed tunnels. The drag coefficients given for the cowling-nacelle combination (fig. 16) are based on the frontal area, that is, on $\pi D^2/4$ where D is the largest diameter. Data are included for only the wing and the cowling because these surfaces appear to be the most critical factors in setting the final terminal velocity, although the control surfaces, the fuselage, the windshield, and protuberances will contribute to the over-all effect.

Figures 15(a) and 15(b) show that the thinner the section, the later the occurrence of the compressibility effects. Because tail surfaces usually employ thinner profiles than wings, compressibility effects are not likely to occur on tail surfaces operating near zero lift (tail lift coefficient <0.1) as is usually the case in the dive. Although they may show comparatively early compressibility effects, protuberances are not of particular importance in limiting the airplane dive speed unless numerous and comparatively large. Reference 11 gives the drag of two-dimensional cylinders of simple shapes such as might form protuberances. It must be remembered in using these data that important end effects are omitted; this statement also holds for wing tips where the air is not constrained to two-dimensional flow.

Inasmuch as the induced velocites are, in general, lower on streamline surfaces of revolution, such surfaces show later compressibility effects than cylinders of similar profile. On a practical fuselage, however, the windshield and the various junctures may be sources of local

compressibility effects that are difficult to evaluate. Data are given in reference 12 for determining the compressibility effects for a number of windshields on a typical fuselage.

TIME-VELOCITY-ALTITUDE RELATIONS IN THE COMPRESSIBILITY RANGE

As previously mentioned, the charts of figures 1 to 14 cannot be used beyond the point at which the profile-drag curves begin to deviate from a constant value without introducing some error in the relations sought. Because the rates of deviation may vary widely between airplanes, no single chart can be devised to represent the relations beyond the point of deviation. If an exact solution is required, it is therefore necessary to evaluate by a step-by-step process the equation

$$\frac{d^2h}{dt^2} = g\left(1 - \frac{C_{D_0}\rho V^2}{2\ W/S}\right) \tag{5}$$

where W is the weight in pounds and S is the wing area in square feet. This type of solution need not, however, be lengthy, as the steps may be taken at intervals as large as 1 second and still give accurate results.

An approximation to the time-velocity-altitude relations may be quickly obtained, however, by the following procedure, which employs the terminal Mach number that would exist under standard atmospheric conditions at 1000 feet and the norminal terminal velocity on which the previous charts were based. The method to be outlined assumes that the profile-drag coefficient is constant up to the time that the terminal Mach number is reached, at which point the drag coefficient immediately increases to the value necessary to satisfy the equation

$$W = C_{D_0}\frac{\rho}{2}V^2S \tag{6}$$

The rate of increase is obviously more gradual than the method implies and some error is therefore introduced.

This procedure, however, has been found to introduce only small percentage errors in the time-altitude relations and somewhat larger errors in the velocity-altitude relations.

The terminal Mach number for standard conditions at 1000 feet is determined by plotting the relation

$$M = 0.0264\sqrt{\frac{W/S}{C_{D_o}}} \tag{7}$$

in the same figure (fig. 17) as that giving the variation of airplane profile-drag coefficient C_{D_o} with Mach number. The abscissa of the intersection of these two curves represents the assumed terminal Mach number M_t that is required in the simplified method.

The results given by figures 5 to 14 are now used until the assumed terminal Mach number is reached and the values of the velocity and the altitude obtained at this point are denoted by V_t and H_t. Beyond this point the time-velocity-altitude relations are obtained from figure 18 by locating the values of V_t and H_t and interpolating between the appropriate Mach number curves to the altitude at which the results are required.

The time-velocity-altitude relations for an airplane diving in a standard atmosphere at a constant Mach number are represented in figure 18. The velocity-altitude lines of this figure were computed from the equation

$$V = M_t(763.2 - 0.0027h) \tag{8}$$

which is the binomial expansion of equation (4) with the first two terms retained. Integration of equation (8) gives the relation for the time in the form

$$\Delta t = \frac{247}{M_t} \log_e \left(\frac{763.2 - 0.0027h}{763.2 - 0.0027H}\right) \tag{9}$$

Equation (9) was employed to compute the time relations of figure 18. The time required in the new range is found

by noting the time at H_t and subtracting that value from the time read at any later altitude h.

EXAMPLES

1. In order to illustrate the use of the charts that are given, as well as to show how the results obtained with the simple method compare with the results of a step-by-step computation, the following examples are worked out for three hypothetical airplanes, A, B, and C, for which the common characteristics are:

Weight, pounds.	6000
Wing area, square feet.	240
Initial dive speed, miles per hour	0
Initial dive angle, degrees	90

The initial starting altitude for A is 20,000 feet; for B, 24,000 feet; and for C, 26,000 feet. The over-all C_{D_o} curves both have an initial value of $C_{D_{o_i}} = 0.0199$ but begin to vary at $M = 0.4$, as shown in figure 17. The nominal terminal velocities of these airplanes are computed as

$$V = \sqrt{\frac{2\ W/S}{\rho_o\ C_{D_{o_i}}}}$$

$$= \sqrt{\frac{2 \times 25}{0.00237 \times 0.0199}}$$

$$= 1028 \text{ feet per second, or}$$

$$= 700 \text{ miles per hour}$$

The values of M_t are seen to be 0.67, 0.75, and 0.77 for airplanes A, B, and C, respectively.

By use of these values of M_t in conjunction with figure 12 (U = 700 mph), the following values are obtained:

Airplane	H_t (ft)	V_t (fps)	Time to H_t (sec)
A	10,250	492	25.5
B	12,200	534	28.2
C	12,250	566	30.2

The foregoing values of H_t and V_t are located in figure 8 and a curve is interpolated to the ground, where the velocities are found to be 512, 557, and 593 miles per hour for airplanes A, B, and C, respectively. The time increments from the altitude H_t to the ground are found to be 14, 15.3, and 14.5 seconds, respectively. The comparison between the step-by-step and the chart methods is shown in figure 19, in which it may be noted that, even though wide variations in the rate of increase of profile-drag coefficient have been used, the errors in the velocity curves at any point are less than 3 percent and tend to be averaged over the whole range. The error in the time to reach a given altitude is much smaller than 3 percent because a constant difference of velocity as great as 15 miles per hour can be applied for over 20 seconds and still make a difference of less than 500 feet in altitude.

The procedure for taking an initial diving speed into account is also indicated in figure 12. If, therefore, airplane A had started to dive at a velocity of 150 miles per hour at 16,000 feet, the curve drawn from point D to point E would hold.

2. The following example is included to illustrate the use of the charts for an airplane in an inclined dive:

Given:

Nominal terminal velocity, U, miles per hour. . .	752
Starting altitude, H, feet	16,000
Initial dive speed, miles per hour	0
Initial dive angle, degrees	60
Terminal Mach number, M_t	0.75

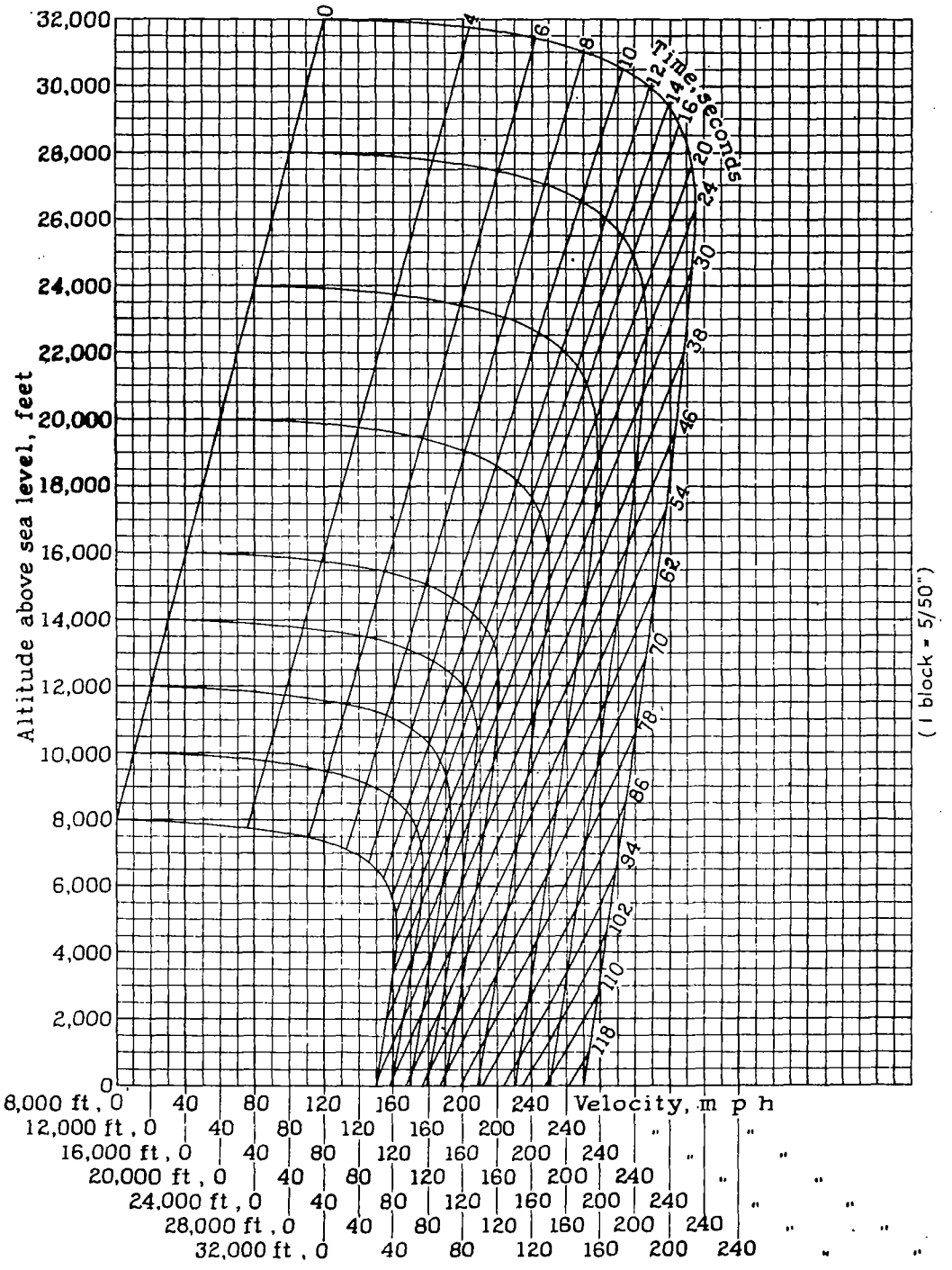

Figure 1. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 150 m p h

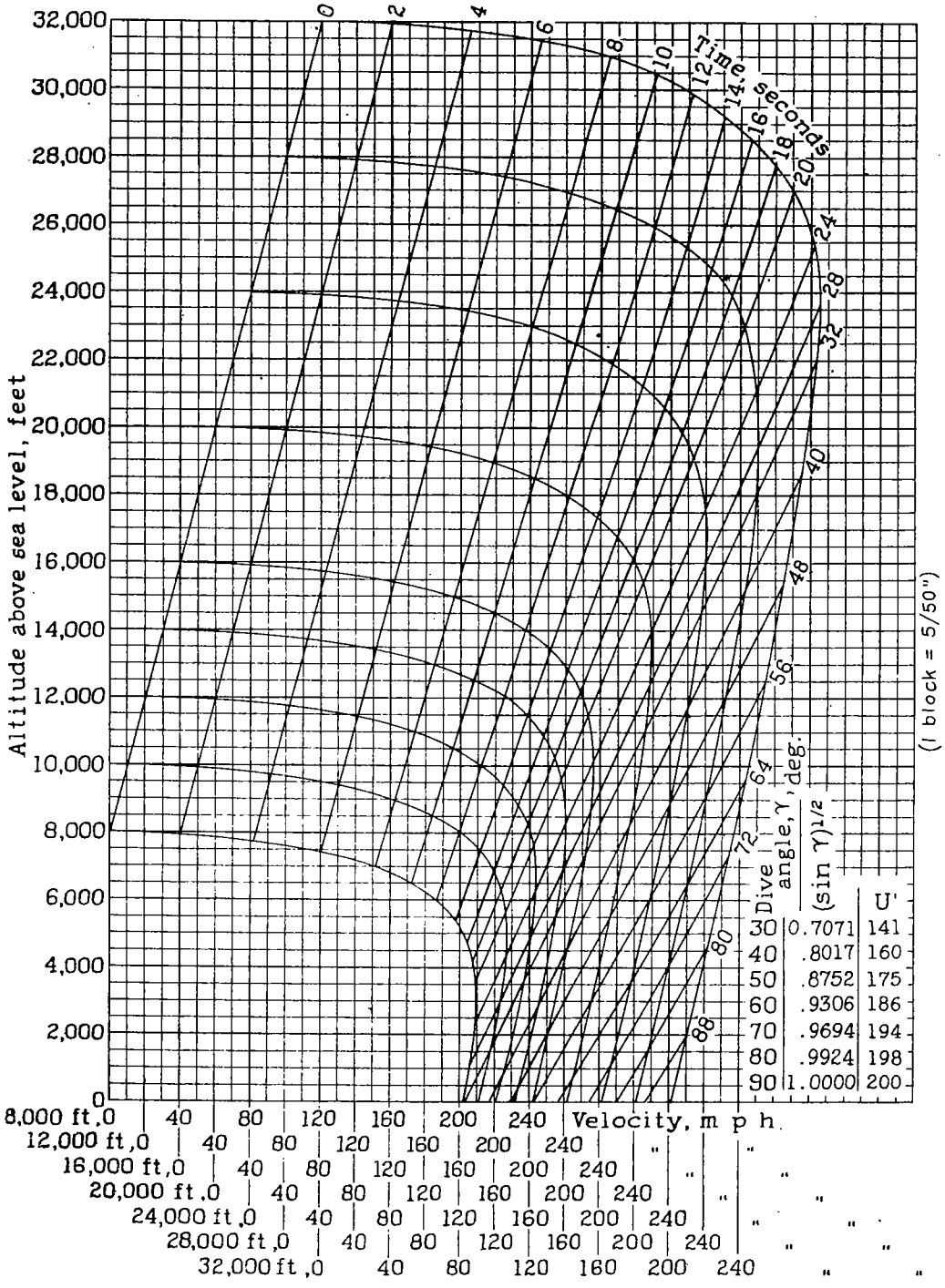

Figure 2. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 200 m p h

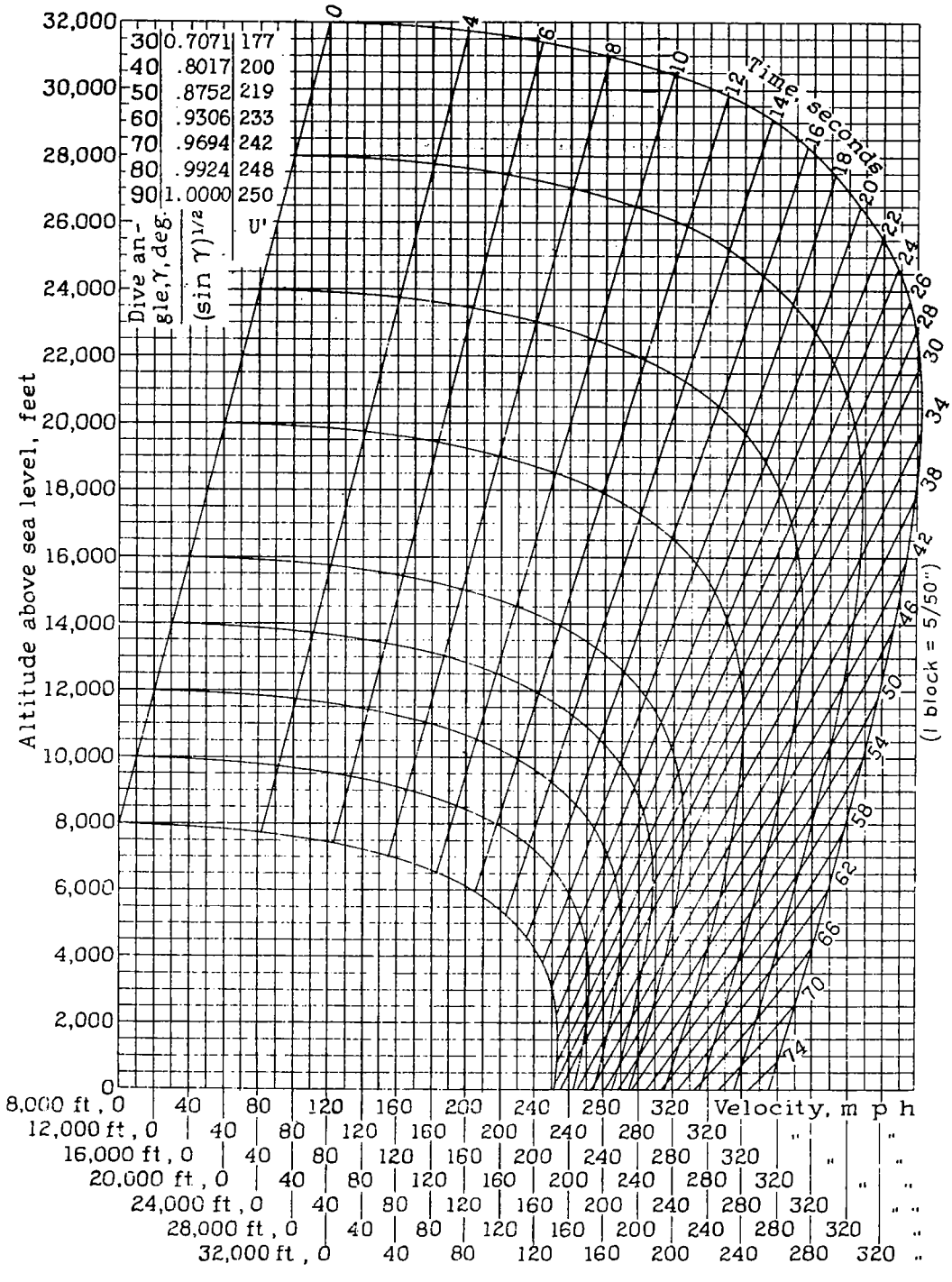

Figure 3. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere U = 250 m p h

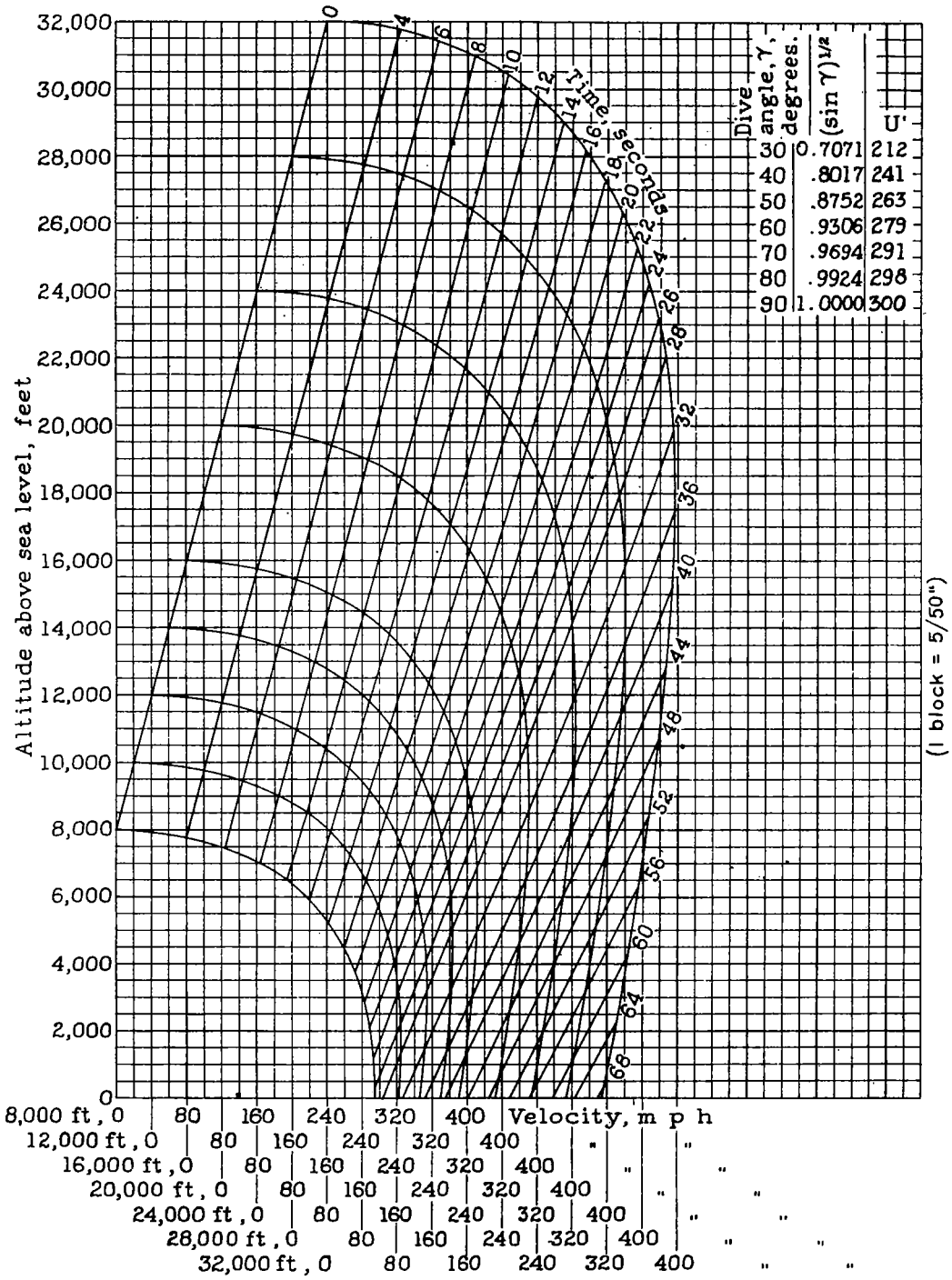

Figure 4. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 300 m p h

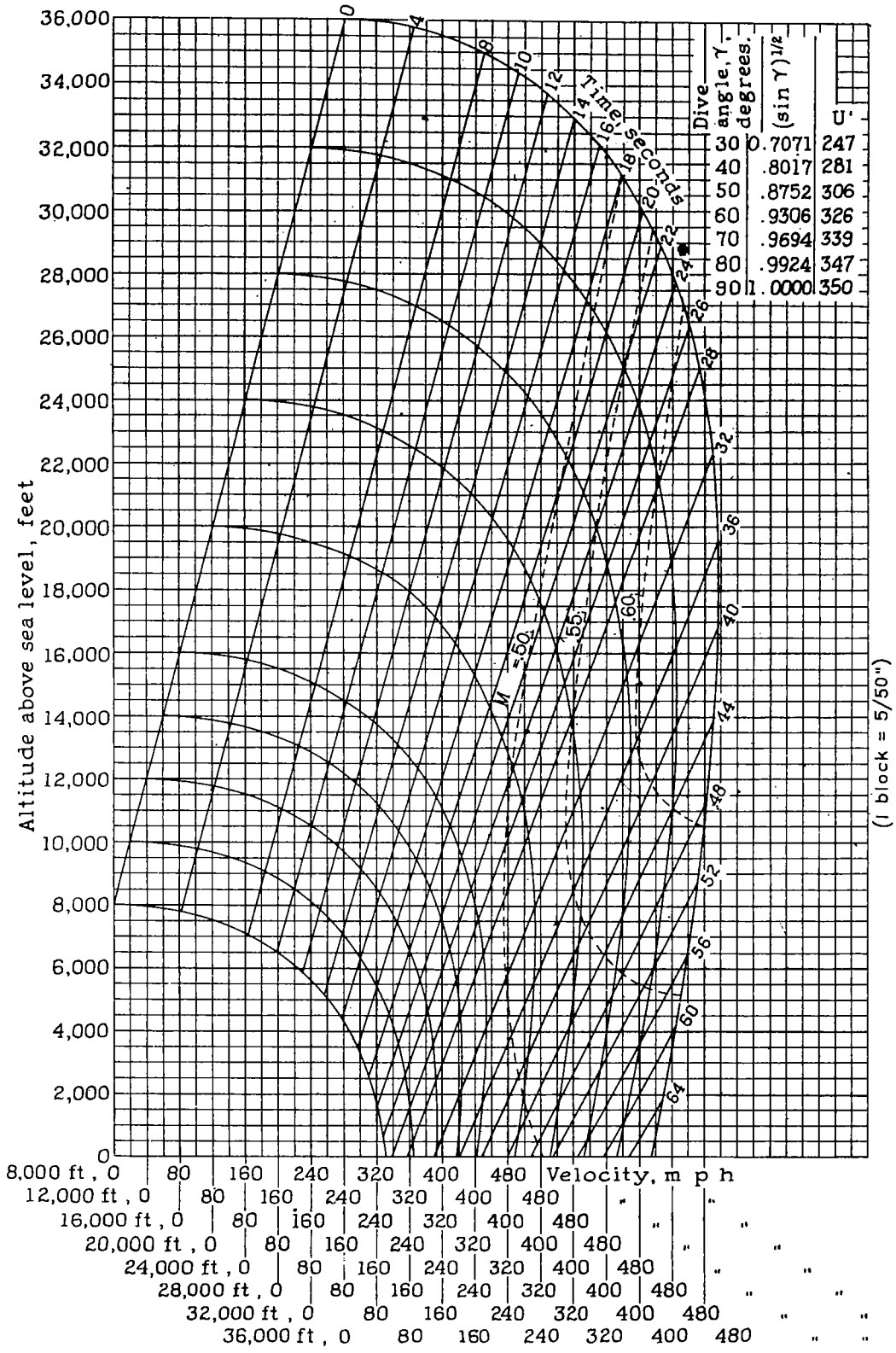

Figure 5. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 350 m p h

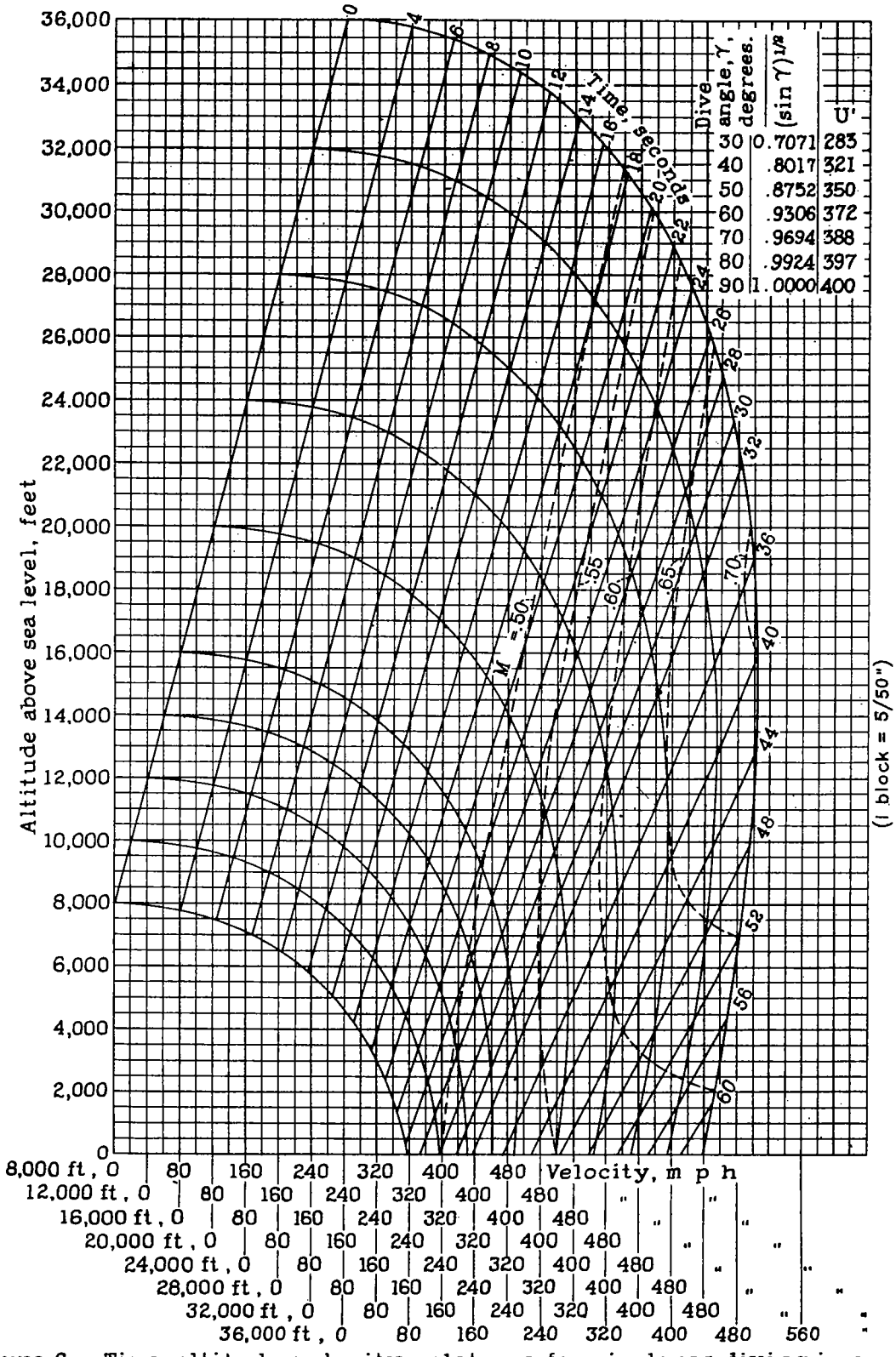

Figure 6. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 400 m p h

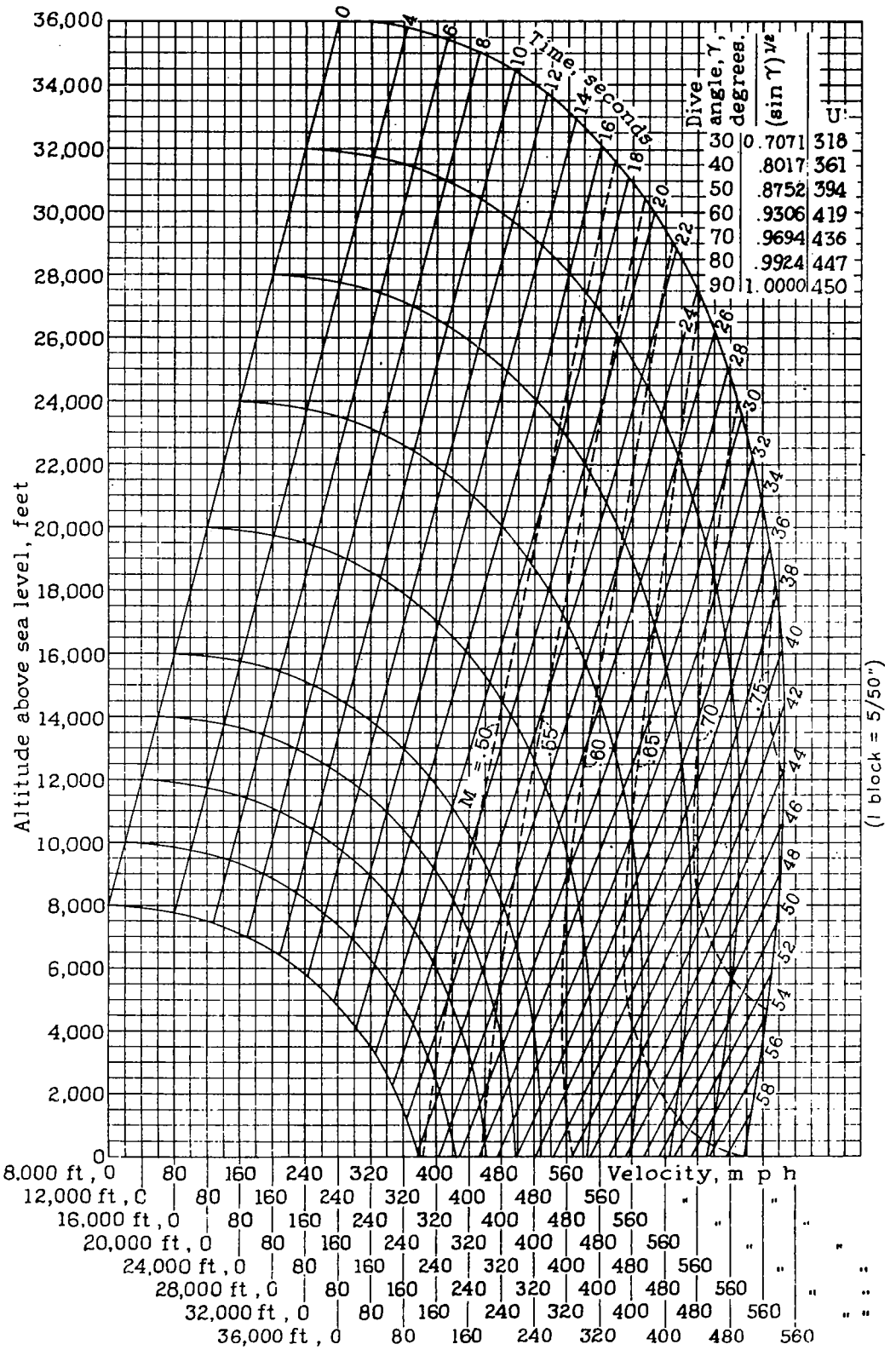

Figure 7. – Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 450 m p h

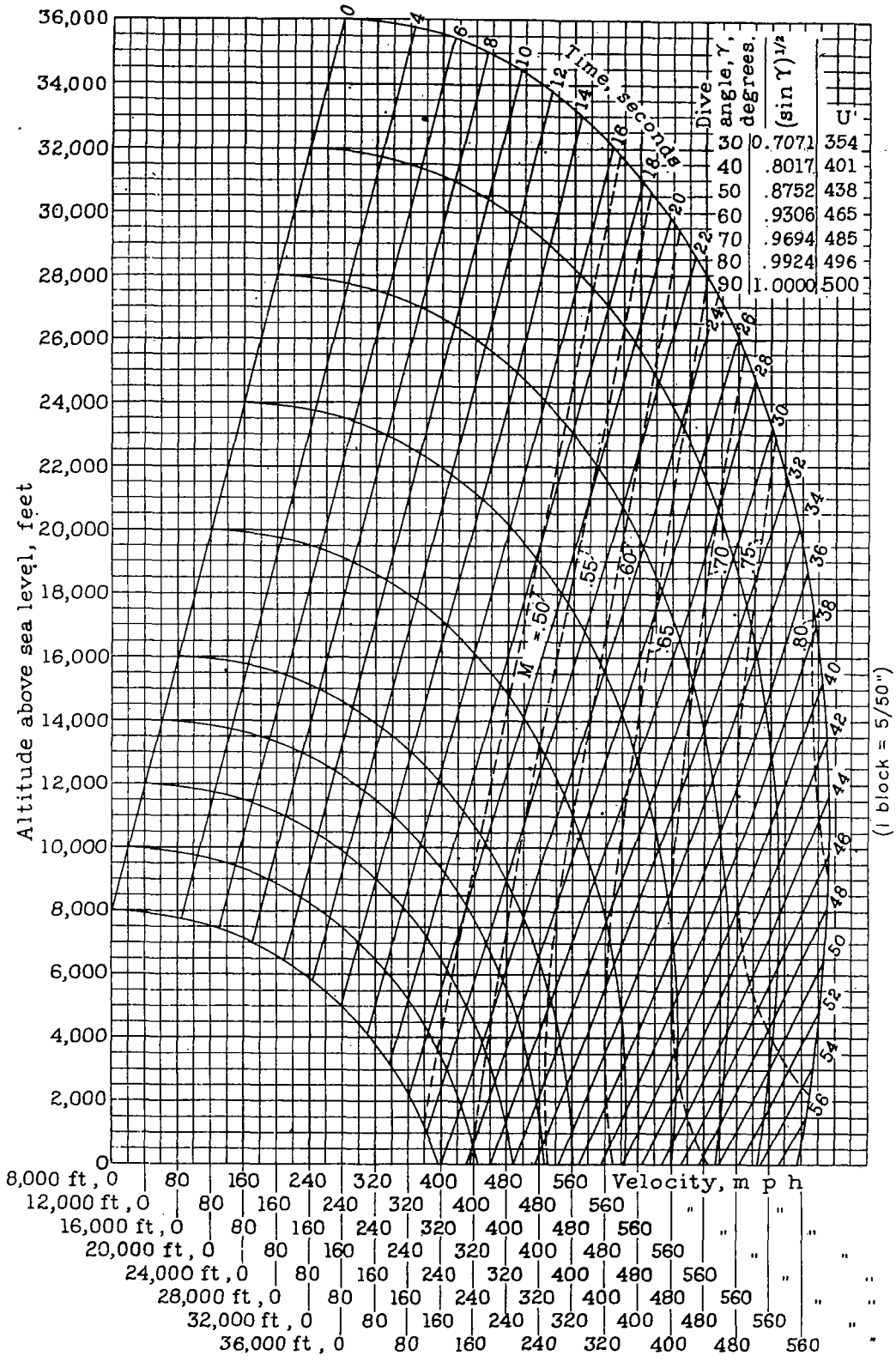

Figure 8. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 500 m p h

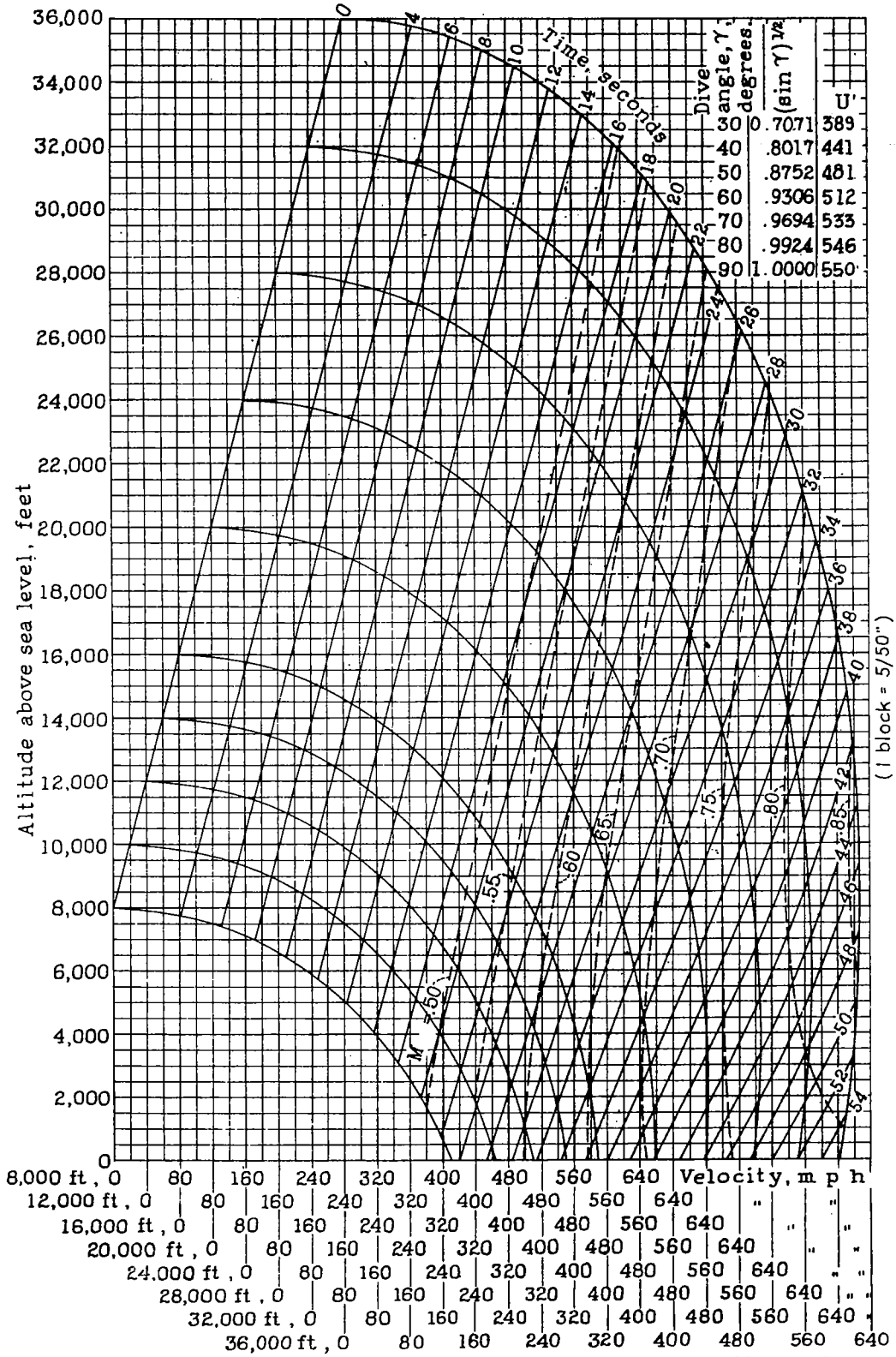

Figure 9. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 550 m p h

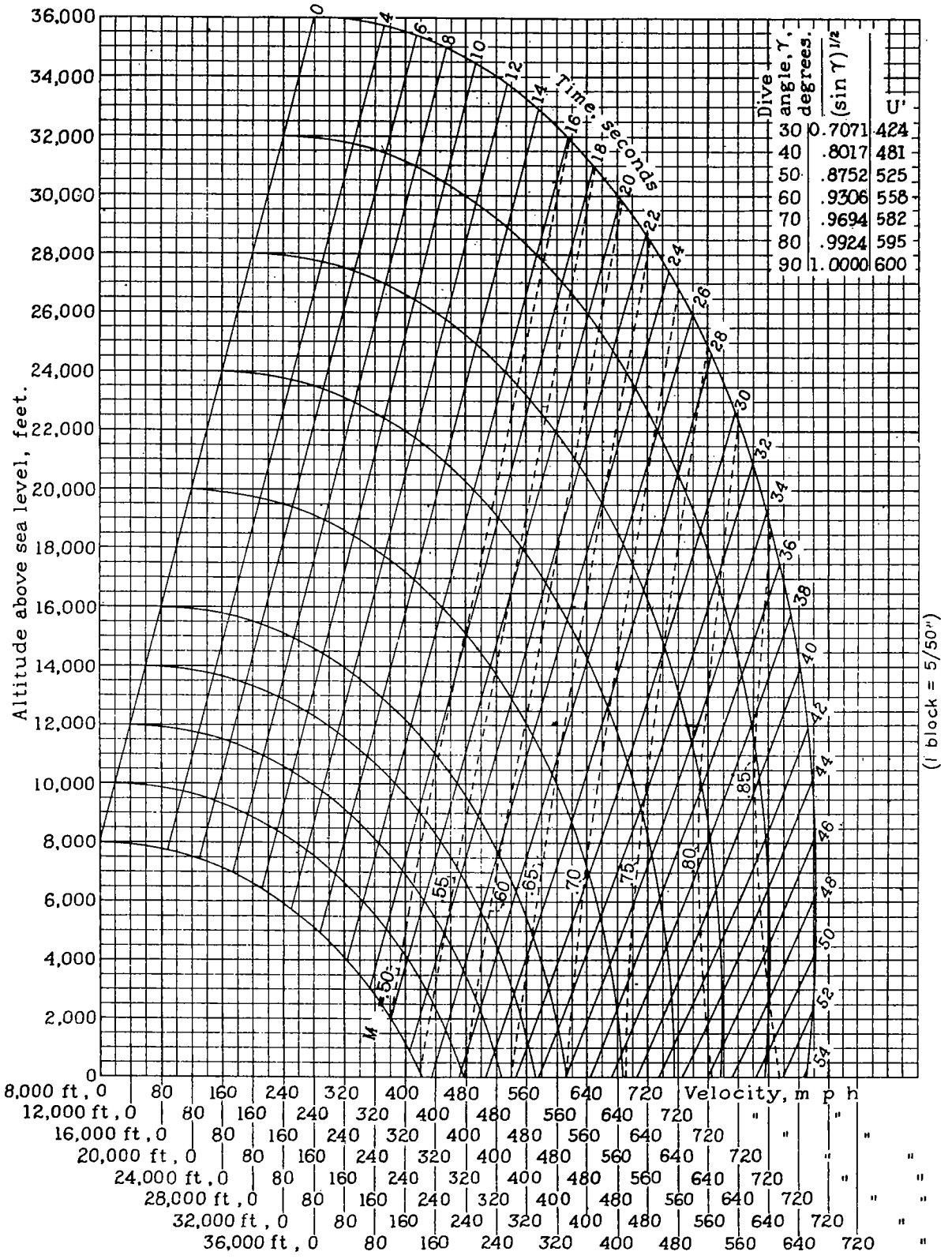

Figure 10.- Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 600 m p h

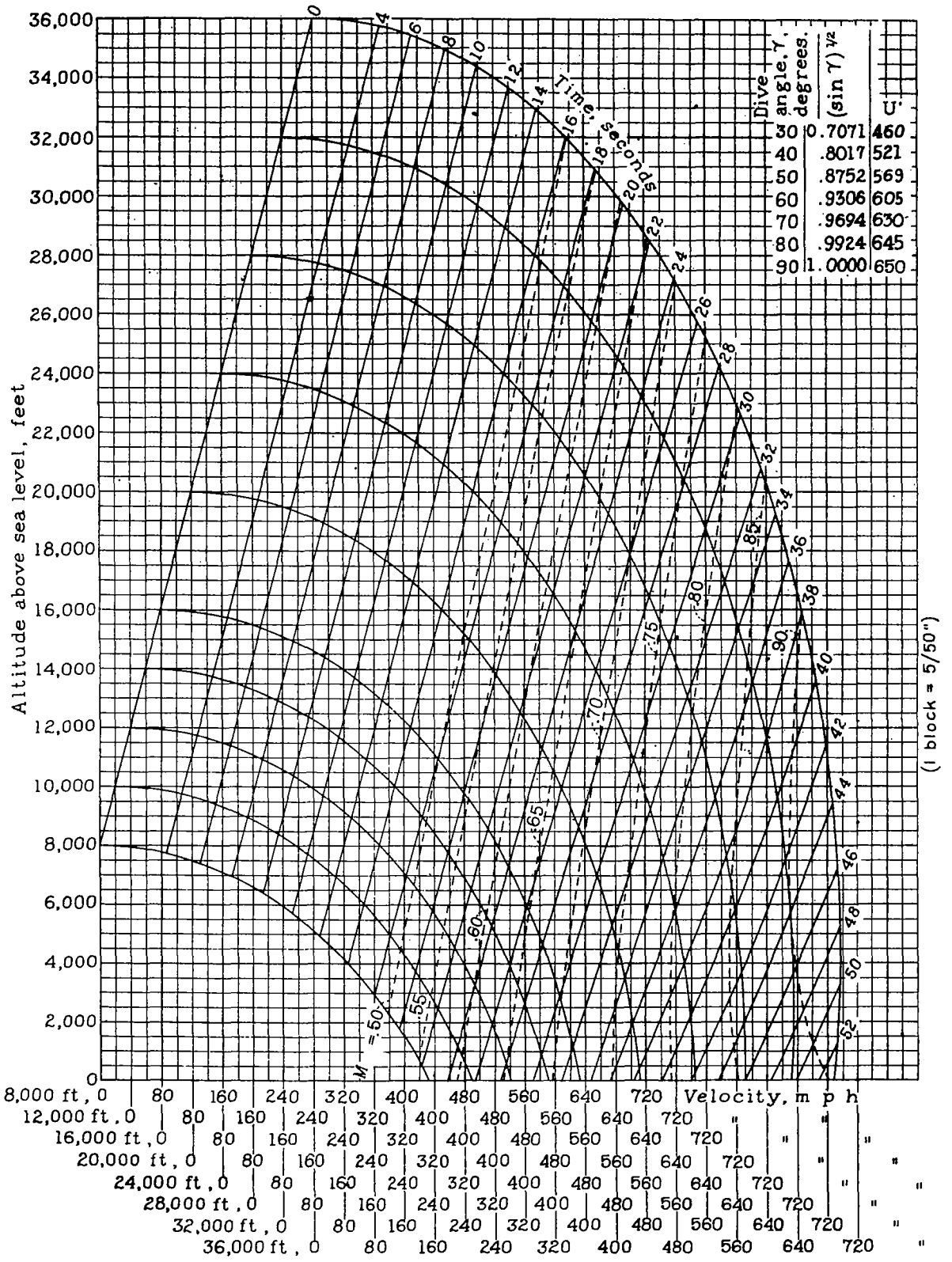

Figure 11.- Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 650 m p h

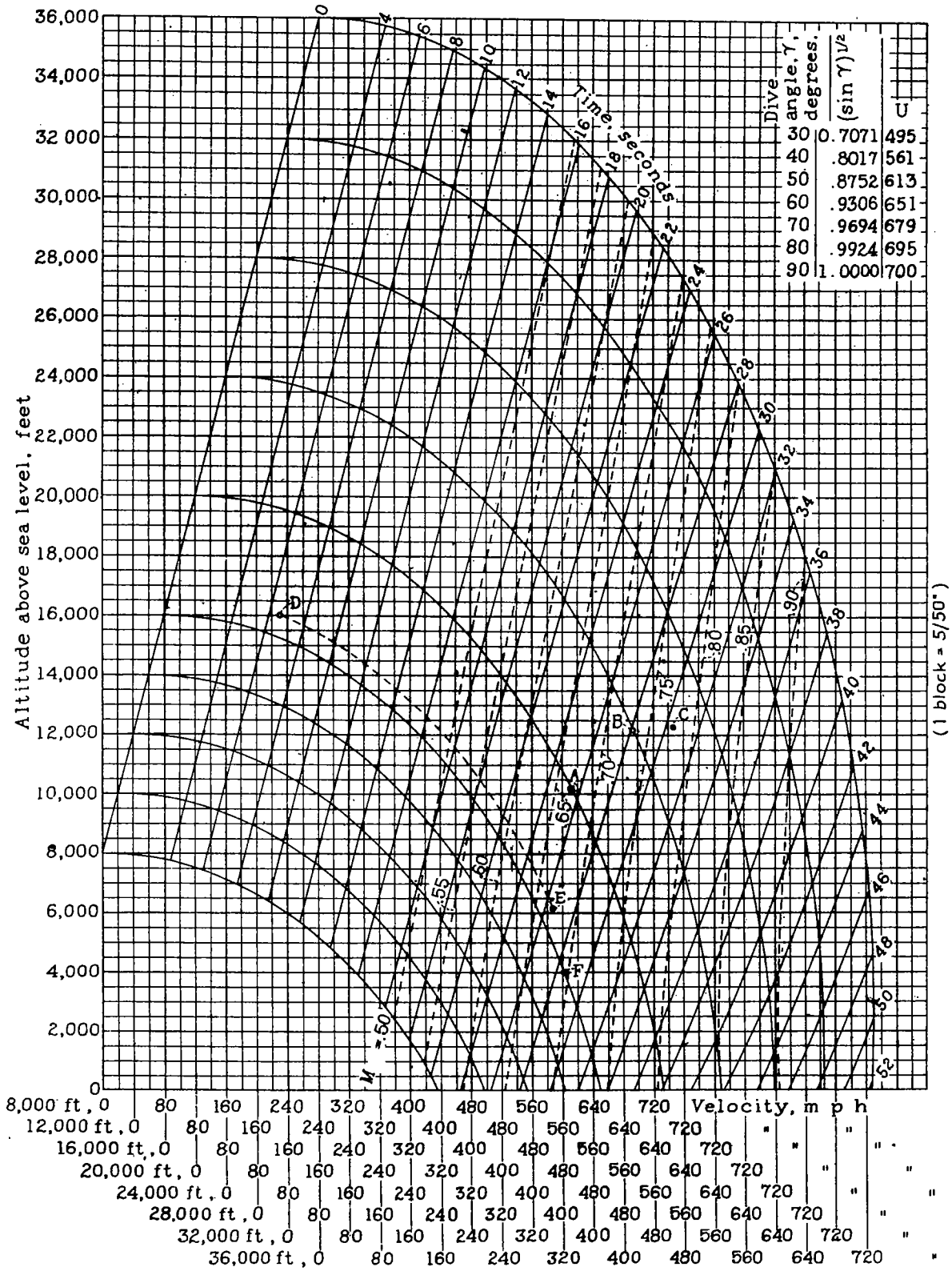

Figure 12.- Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 700 m p h

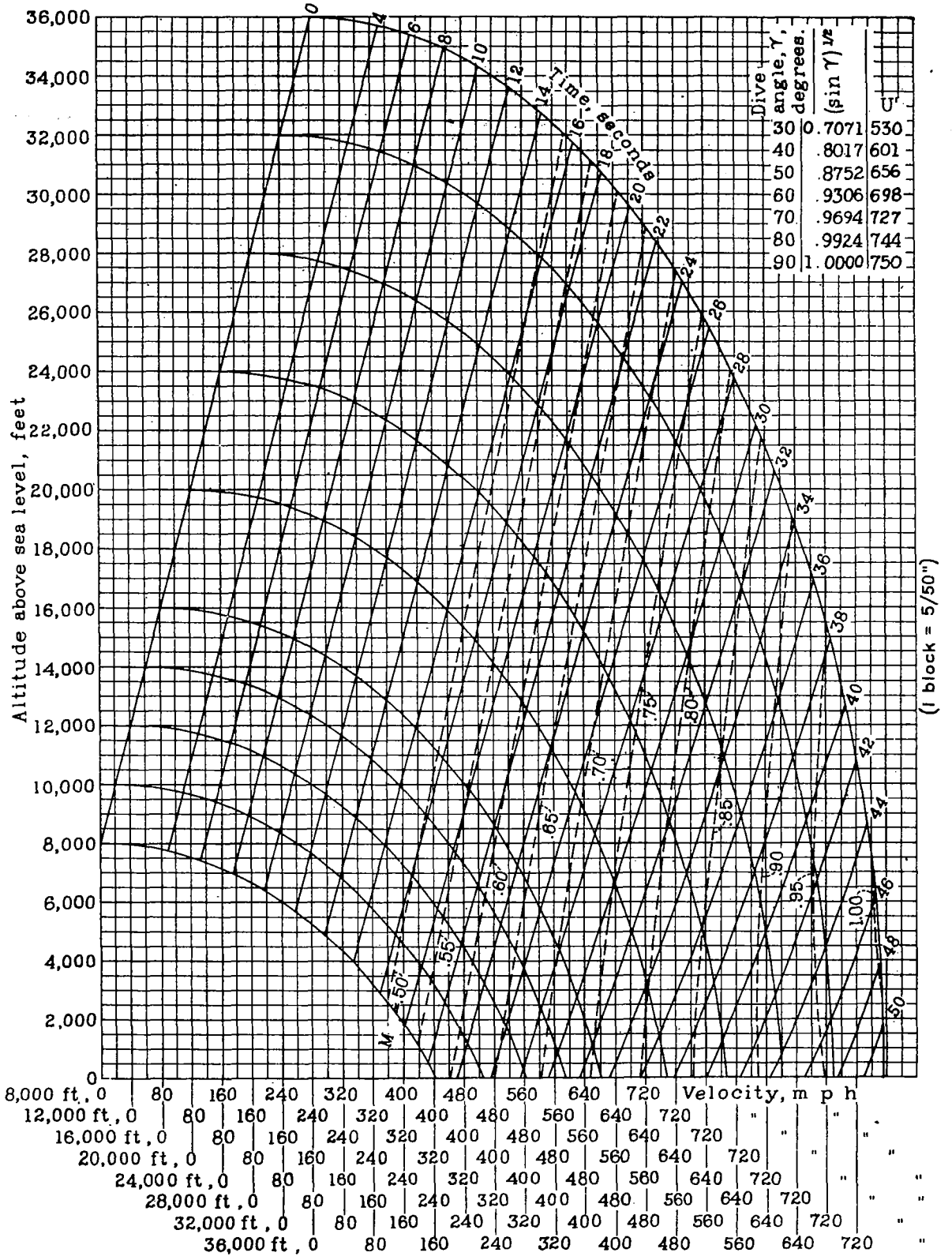

Figure 13.- Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 750 m p h

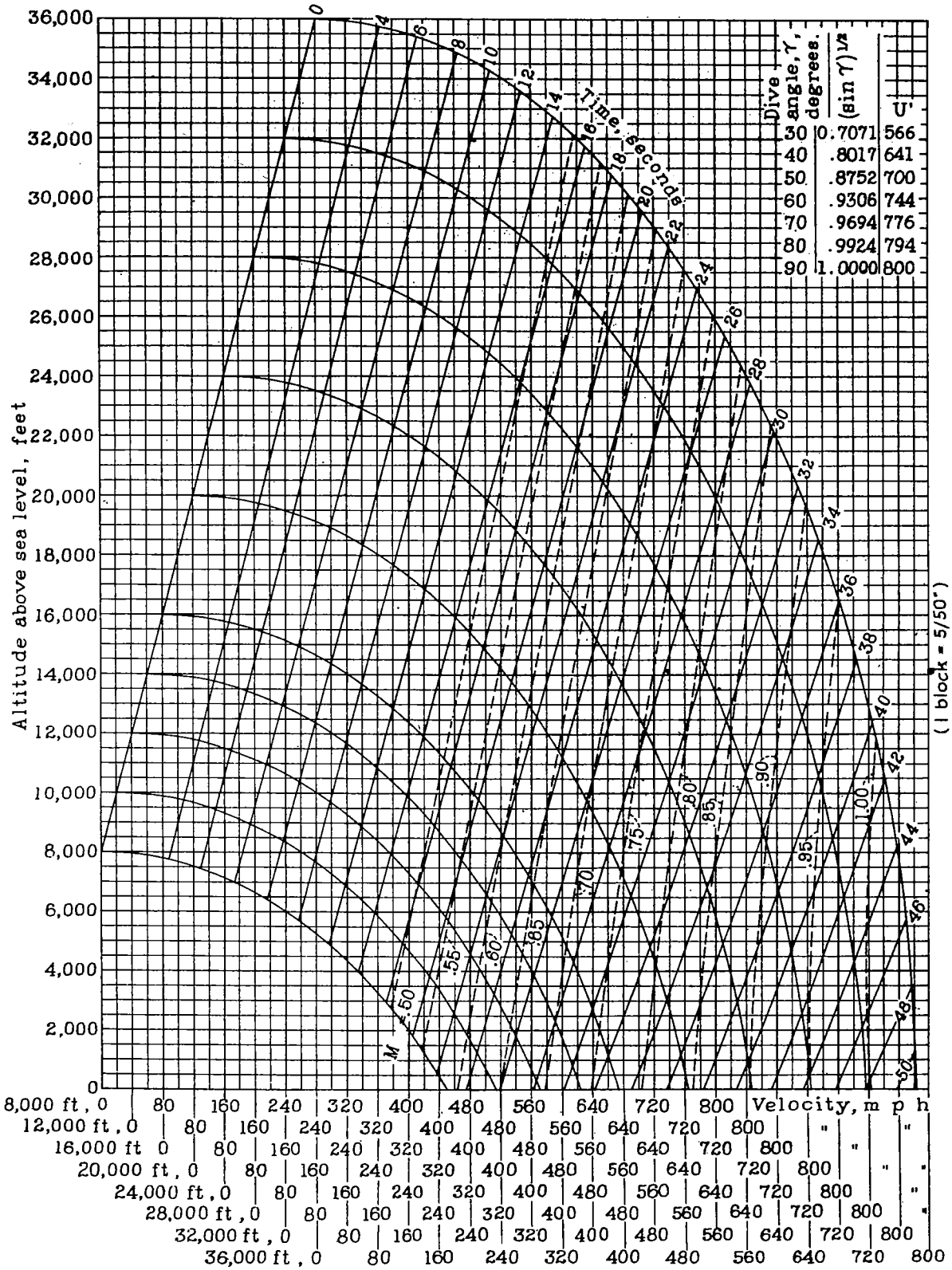

Figure 14.- Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 800 m p h

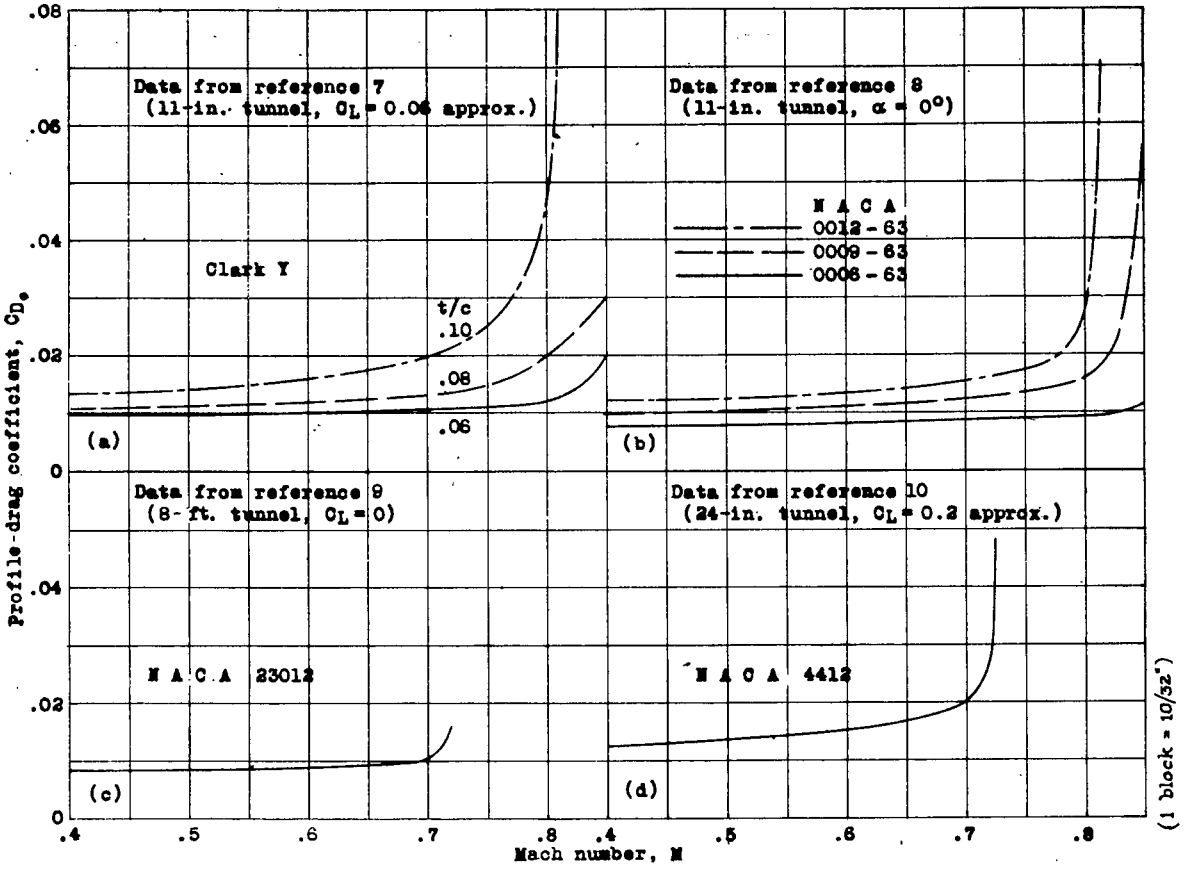

Figure 15.- Variation of profile-drag coefficient with Mach number for several wing profiles near zero lift, from high speed tunnel tests.

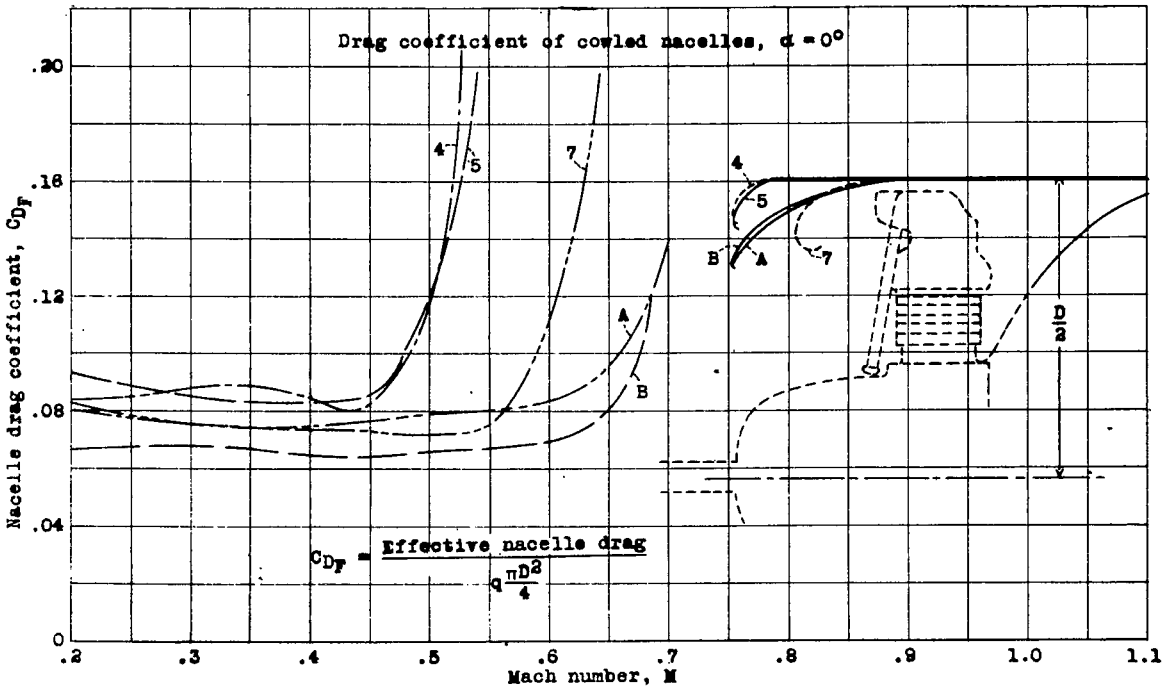

Figure 16.- Variation of drag coefficient for various cowlings.

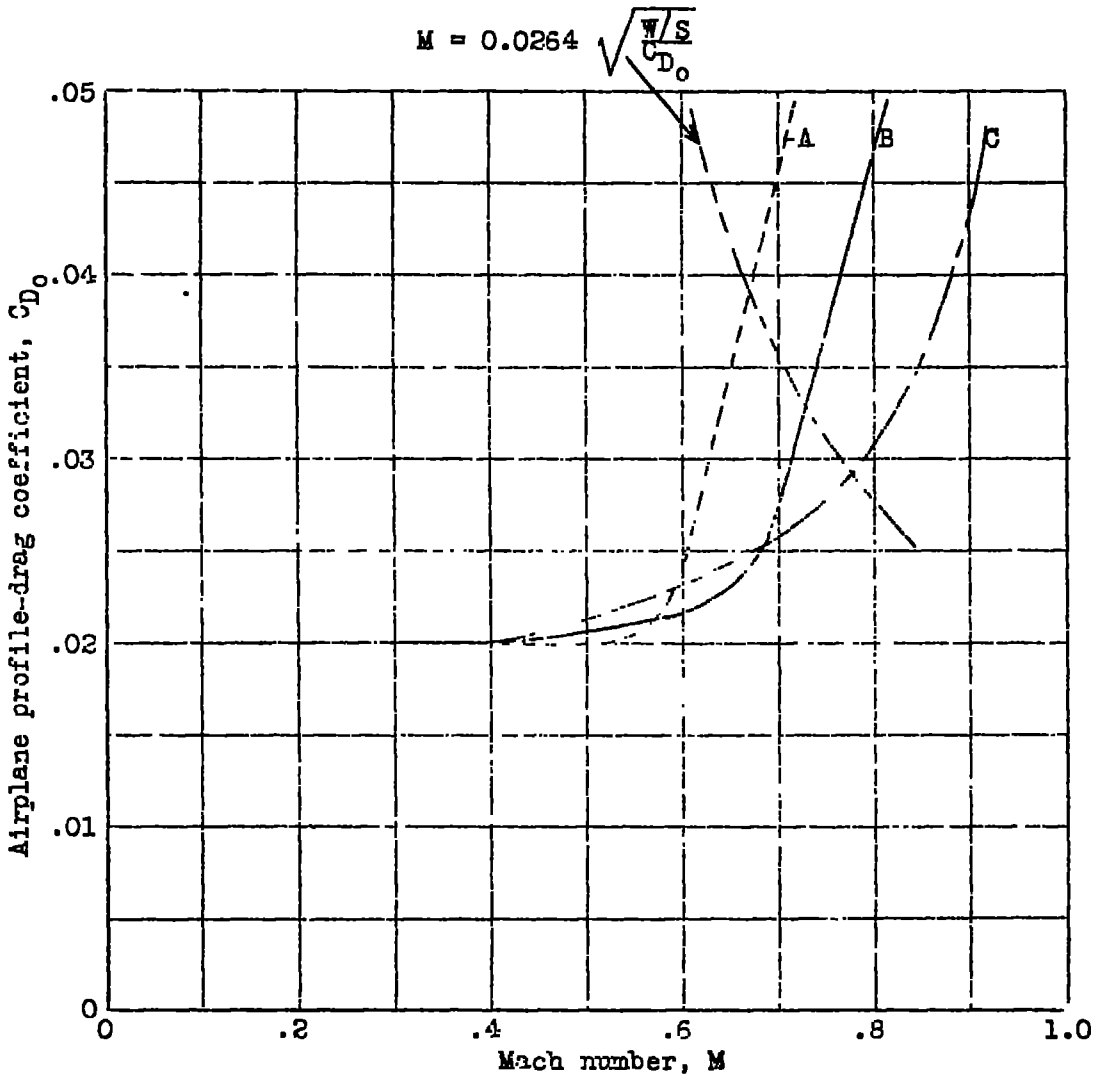

Figure 17.- Variation of profile-drag coefficient with Mach number for hypothetical airplanes, A,B, and C.

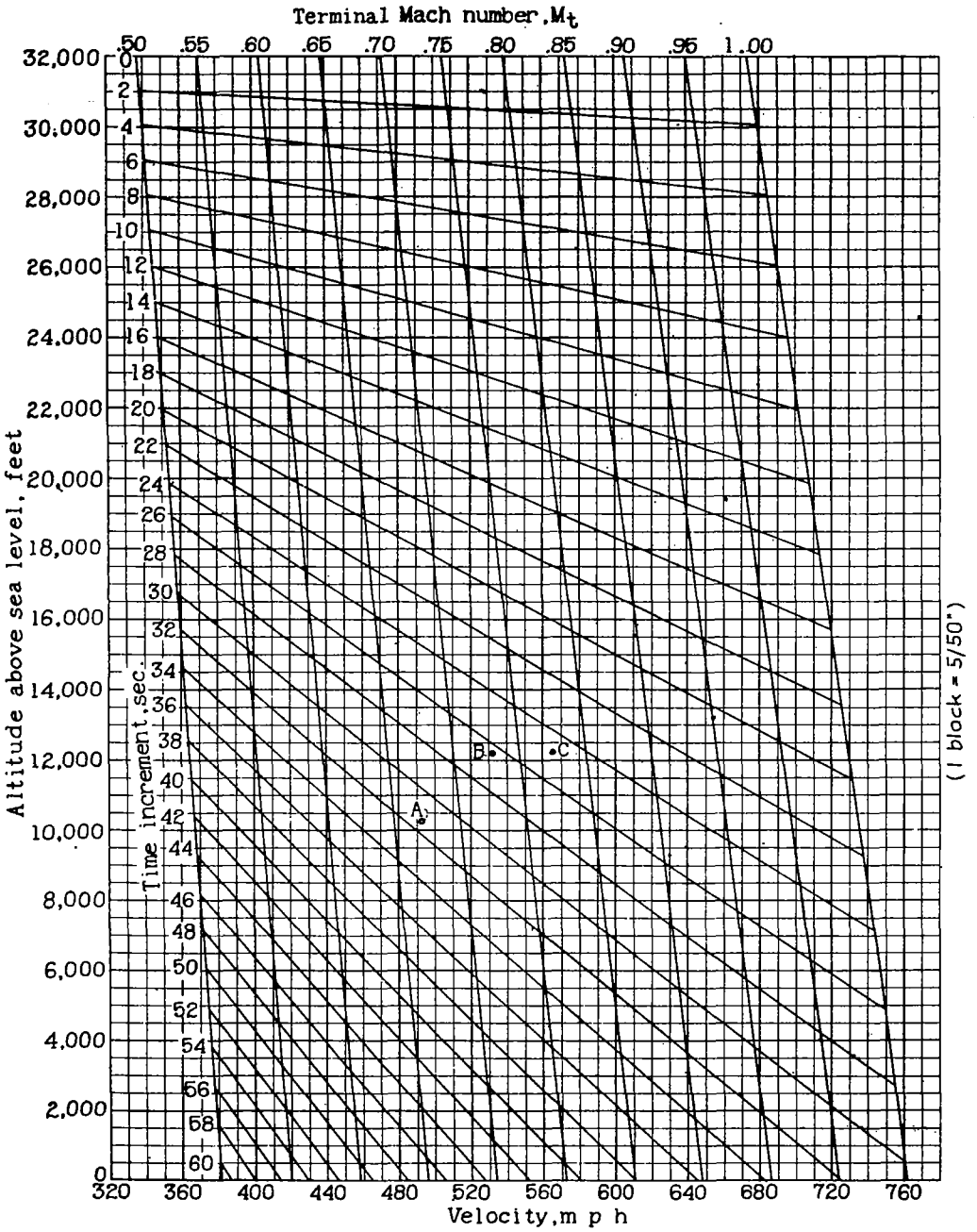

Figure 18.- Time-altitude-velocity relations for airplanes diving in a standard atmosphere at constant Mach number.

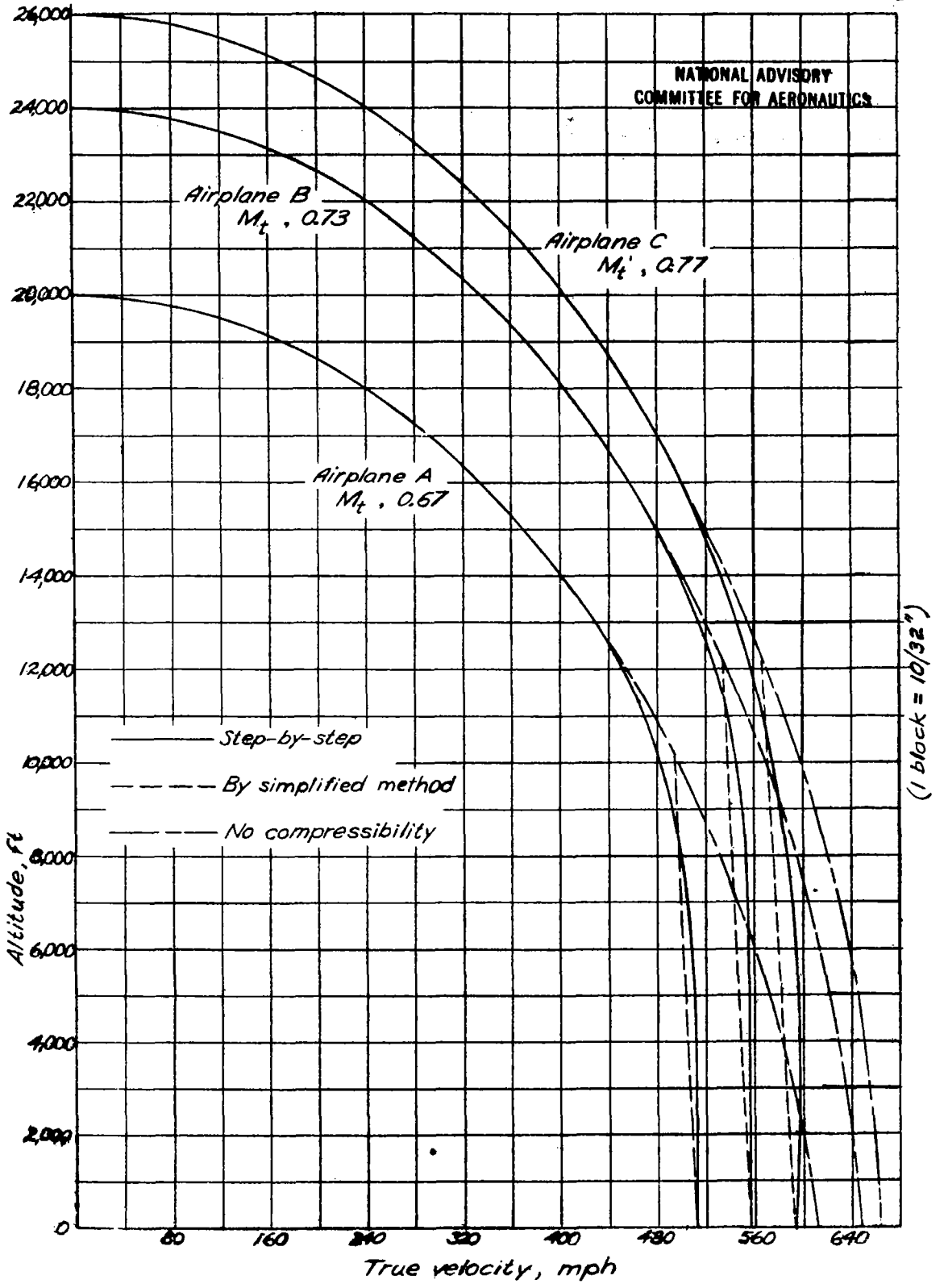

Figure 19.- Comparison between results from chart and step-by-step integration.

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