# REPORT No. 50.

## CALCULATION OF LOW-PRESSURE INDICATOR DIAGRAMS.

By E. C. Kemble.

### INTRODUCTION.

It is well known that the recent remarkable advances in the design of high-speed internal-combustion engines have been made almost entirely by cut and try methods. Increased valve areas, increased compression ratios, better intake manifold design, better carburetion, etc., have enabled us to attain mean effective pressures that a few years ago were not dreamed of. Yet there is at present no accepted way of determining theoretically the relation between valve areas, stroke volume, speed, and volumetric efficiency. The question of the proper relative areas of inlet and exhaust valves is still moot. There is no way of fixing the proper valve timing for any given case, except the way of experiment, and there is little information available regarding the effect of improper valve settings on the volumetric efficiencies and pumping losses. In short, the designer dispenses almost entirely with theory.

This condition of affairs is obviously due to the multiplicity of the factors which affect the performance of internal-combustion engines. The large number of such factors not only makes theorizing extremely difficult, but it makes it exceedingly hard to interpret test results with any degree of certainty, so that we have little that is reliable in the way of data for checking up a theory once formulated. The increasing number of well-equipped experimental laboratories, however, is sure to yield plenty of new and reliable experimental data, and just on this account it becomes important to develop a theory which shall guide the experimenter and at the same time aid in the interpretation of his results.

It is hoped that the method of computing low pressure indicator cards outlined in this report will prove to be the first step in the development of such a theory. The writer particularly desires to emphasize the fact that this work is only a beginning, which can not at present be expected to yield results which are quantitatively reliable.

The method of calculation here outlined is based on the following assumptions:

(A) The mixture of air, burnt gas, and fuel in the cylinder may be treated as a perfect gas.

(B) The kinetic energy developed by the incoming charge as it passes through the throat of the inlet valve is converted into heat by eddies as fast as it emerges from the valve, so that we may treat the gas in the cylinder as if it had a definite temperature and pressure.

(C) The heat transmission between the gas and cylinder walls together with the heat absorption due to the vaporization of the fuel have a negligible effect on the pressure variations; that is, the processes under consideration may all be treated as adiabatic.

(D) The rate of air flow through the intake and exhaust valves may be calculated from the manifold pressures, the cylinder pressure, and the valve clear openings by the ordinary formulas for steady flow through an orifice with the aid of determinable coefficients of efflux.

These hypotheses are all to be regarded as rough approximations to the truth. The writer is of the opinion that the phenomena ignored through the introduction of these assumptions are of secondary importance in determining the pressure variations in the cylinder during the suction and exhaust strokes. In the last analysis, however, the justification of this opinion must come through a comparison of the results obtained from this theory with the results of experiment.

Assumptions B and D are apparently indispensable to the present method of attack on the problem. A and C, on the other hand, constitute convenient simplifications suitable to the present preliminary analysis, but not necessarily essential to the method. In connection with

hypothesis C it should be observed that even though the charge in the cylinder absorb a good deal more heat from the walls during the suction stroke than is required to complete the vaporization of the fuel, the increase in the heat content of the charge will not be accompanied by a correspondingly larger final pressure. This is because a slight increase in pressure due to a rise in temperature immediately reduces the rate of flow through the inlet valves.

In order to apply the method it is necessary that pressures in the intake and exhaust manifolds at points adjacent to the valves and the effective clear valve openings shall be known functions of the time. By the effective clear opening of a valve is meant the product of the minimum sectional area of the passage through the valve and the coefficient of efflux.

The use of the method is obviously restricted by the above requirements, since the manifold pressures are usually not known. In the case of a motor exhausting directly to atmosphere, however, the manifold pressure becomes the barometric pressure. Furthermore, certain types of multicylinder inlet manifolds have a nearly constant pressure, while a motor which draws air directly from the atmosphere and has its fuel injected directly into the cylinder will have atmospheric intake pressure, so that in these special cases the method is obviously applicable. In general the fluctuations in the manifold pressure may be expected to be small compared with those in the cylinder, and it is believed that the result of applying the method by replacing the actual varying pressure by its average value will be instructive, though quantitatively in error.

The coefficient of efflux may be expected to depend on the shape of the valve and port, the direction of flow, the ratio of the valve velocity to the velocity of the air through the valve, and the product of velocity of approach and the rate of valve closure. The coefficients for steady flow may be obtained from such experiments as those described in Report No. 24, Air Flow Through Poppet Valves, by Lewis and Nutting. The coefficients applicable in cases of intermittent flow might be obtained from a study of experimental light spring diagrams with the aid of the theory developed below. The writer is inclined from *a priori* considerations to the opinion that the coefficients for steady flow should not differ very greatly from those applicable to the intermittent flow obtained in practice provided that the pressures used in making the calculations are those actually existing at points close to the valves. (The pressures in the intake manifolds and induction pipes are different at different points as well as at different times.) This is by no means certain, however (Cf. "Pressure Drop Through Poppet Valves," by C. E. Lucke, Trans. A. S. M. E., vol. 27, 1905), and it is desirable that further experiments be made to determine the actual values of the coefficients of efflux in cases of rapidly varying flow. It should be pointed out in this connection that the errors involved in using the steady flow coefficients may be expected to increase with the ratio of the valve velocity to the velocity of the air through the valve and with product of the velocity of approach and the rate of valve closure.

## NOTATION.

Let $V, v$ = instantaneous volume of cylinder in cubic feet and cubic inches, respectively.

$S$ = specific volume in cubic feet per pound.

$P, p$ = absolute pressure in cylinder in pounds per square foot and pounds per square inch, respectively.

$P', p'$ = absolute pressure of air approaching inlet valve in pounds per square foot and pounds per square inch, respectively.

$P'', p''$ = absolute pressure in exhaust passage in pounds per square foot and pounds per square inch, respectively.

$\theta$ = absolute Fahrenheit temperature of cylinder contents.

$\theta'$ = absolute Fahrenheit temperature of intake charge.

$A_i, a_i$ = clear opening of inlet valves in square feet and square inches, respectively.

$C_i$ = coefficient of efflux of inlet valves.

$F_i, f_i \equiv C_i A_i, C_i a_i$ = effective clear opening of inlet valves in square feet and square inches, respectively.

$A_e, a_e$ = clear opening of exhaust valves in square feet and square inches, respectively.

$C_e$ = coefficient of efflux of exhaust valves.

$F_e$, $f_e \equiv C_e A_e$, $C_e a_e$ = effective clear opening of exhaust valves in square feet and square inches, respectively.

$M_i$ = rate flow through inlet valves in pounds per second.

$M_e$ = rate flow through exhaust valves in pounds per second.

$G$ = mass of air and gas in cylinder in pounds.

$t$ = time in seconds.

$\varphi$ = crank angle from head and dead center.

$R$ = gas constant.

$C_p$ = specific heat at constant pressure in mechanical units (foot-pounds per pound).

$C_v$ = specific heat at constant volume.

$\gamma = \frac{C_p}{C_v}$.

$P_o$, $V_o$, $\theta_o$ = assumed initial values of $P$, $V$, and $\theta$.

## GENERAL ANALYSIS.

*Case I.—Burnt gases escaping through exhaust valves.*

Treating the cylinder contents as a perfect gas (Assumption A), we write

$$PV = GR\theta \tag{1}$$

$$C_p = C_v + R. \tag{2}$$

The specific heat of a gas at constant volume is a measure of the increase of internal energy of unit mass for each degree rise in temperature. The product of the specific heat at constant volume (assumed constant), the mass, and the absolute temperature is therefore equal to the total internal energy. Consequently the rate at which energy accumulates in the cylinder is $\frac{d}{dt}(C_v G\theta)$. Substituting from (1) into this expression, we obtain

$$\frac{d}{dt}(C_v G\theta) = \frac{d}{dt}\left(\frac{C_v}{R} \cdot PV\right) = \frac{C_v}{R} \cdot \frac{d}{dt}(PV). \tag{3}$$

The rate at which external work is performed on the piston is $P\frac{dV}{dt}$. The rate at which the escaping exhaust gas carries away energy is $[C_v\theta + PS]M_e = C_p\theta M_e$. The rate at which the incoming charge brings energy in is $[C_v\theta' + P'S']M_i = C_p\theta' M_i$.

Neglecting the heat transmission to the cylinder walls (Assumption C), we equate the sum of the rate of accumulation of energy and the net rate of dissipation of energy to zero

$$\frac{C_v}{R}\frac{d}{dt}(PV) + P\frac{dV}{dt} + C_p\theta M_e - C_p\theta' M_i = 0, \tag{4}$$

or

$$\left(\frac{C_v + R}{R}\right)P\frac{dV}{dt} + \frac{C_v}{R}V\frac{dP}{dt} + C_p\theta M_e - C_p\theta' M_i = 0. \tag{5}$$

In virtue of (2), this becomes

$$\frac{C_p}{C_p - C_v} \cdot \frac{1}{V} \cdot \frac{dV}{dt} + \frac{C_v}{C_p - C_v} \cdot \frac{1}{P} \cdot \frac{dP}{dt} + \frac{C_p}{C_p - C_v}\frac{M_e}{G} - \frac{C_p\theta' M_i}{PV} = 0, \tag{6}$$

or

$$\frac{1}{\gamma - 1}\frac{d}{dt}\log_e P = -\frac{\gamma}{\gamma - 1} \cdot \frac{1}{V}\frac{dV}{dt} - \frac{\gamma}{\gamma - 1}\frac{M_e}{G} + \frac{C_p\theta' M_i}{PV}. \tag{7}$$

In case the inlet valve is closed, $M_i$ vanishes and $M_e$ becomes $-\frac{dG}{dt}$. Then

$$\frac{d}{dt}\log_e P = -\gamma\frac{d}{dt}\log_e V + \gamma\frac{d}{dt}\log_e G. \tag{8}$$

Hence

$$\frac{P}{P_o} = \left(\frac{V}{V} \cdot \frac{G}{G_o}\right)^\gamma = \left(\frac{P}{P_o}\frac{\theta_o}{\theta}\right)^\gamma,$$

or

$$\theta=\theta_0\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}. \tag{9}$$

This is the pressure temperature relation for adiabatic expansion which might have been anticipated from our assumptions without the aid of analysis.

Having determined the relation between the temperature and pressure in the cylinder, we can compute the value of $\frac{d}{dt}(\log_e P)$ for any given values of $P$, $V$, $\frac{dV}{dt}$ and $F_e$. Transforming equation (8), we obtain

$$\frac{d}{dt}\log_e P=\frac{-\gamma}{V}\left[\frac{dV}{dt}-\frac{R\theta}{P}\frac{dG}{dt}\right]. \tag{10}$$

But

$$\frac{dG}{dt}=-M_e=-F_e\Psi(P,\theta,P''). \tag{11}$$

Where the function $\Psi$, which denotes the theoretical rate of flow through a well rounded orifice of unit area, has the form

$$\Psi=\frac{K_1P}{\sqrt{\theta}} \tag{12}$$

when

$$\frac{P''}{P}<\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}},$$

and the form

$$\Psi=K_2\frac{P}{\sqrt{\theta}}\sqrt{\left(\frac{P''}{P}\right)^{2/\gamma}-\left(\frac{P''}{P}\right)^{\frac{\gamma+1}{\gamma}}} \tag{13}$$

when

$$\frac{P''}{P}>\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}.$$

The values of the constants $K_1$ and $K_2$ in the above equations are

$$K_1=\sqrt{\frac{2g\gamma}{R(\gamma+1)}}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}, \tag{14}$$

$$K_2=\sqrt{\frac{2g\gamma}{R(\gamma-1)}}. \tag{15}$$

If $\frac{P''}{P}$ is less than its critical value (10) becomes

$$\frac{d}{dt}\log_e P=\frac{-\gamma}{V}\left[\frac{dV}{dt}+\frac{R\theta F_e}{P}\Psi\right] \tag{16}$$

$$=\frac{-\gamma}{V}\left[\frac{dV}{dt}+K_1RF_e\sqrt{\theta_0}\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{2\gamma}}\right]. \tag{17}$$

If $\frac{P''}{P}$ is greater than its critical value, (10) takes the form

$$\frac{d}{dt}\log_e P=-\frac{\gamma}{V}\left\{\frac{dV}{dt}+K_2RF_e\sqrt{\theta_0\left(\frac{P''}{P_0}\right)^{\frac{\gamma-1}{\gamma}}}\sqrt{\left(\frac{P''}{P}\right)^{\frac{3-\gamma}{\gamma}}-\left(\frac{P''}{P}\right)^{\frac{2}{\gamma}}}\right\}. \tag{18}$$

Equations are to be evaluated by a graphical integration.

Case II.—*Fresh charge entering cylinder through inlet valves.*

In case the inlet valve is open and the exhaust valve is closed, equation (7) becomes

$$\frac{d}{dt}\log_e P=-\gamma\frac{1}{V}\frac{dV}{dt}+(\gamma-1)\frac{C_p\theta' M_1}{PV} \tag{19}$$

$$=-\frac{\gamma}{V}\left\{\frac{dV}{dt}-\frac{(\gamma-1)}{P}C_p\theta' F_1\Psi(P',\theta',P)\right\} \tag{20}$$

The ratio $\frac{P'}{P}$ will generally be less than its critical value and we may use the following approximate expression for $\Psi$ (Cf., Sanford A. Moss, American Machinist, Sept. 20 and 27, 1906):

$$\Psi=\frac{M_1}{F_1}=K_2\frac{P'}{\sqrt{\theta'}}\sqrt{\left(\frac{P}{P'}\right)^{2/\gamma}-\left(\frac{P}{P'}\right)^{\frac{\gamma+1}{\gamma}}}$$

$$=K_2\frac{P}{\sqrt{\theta'}}\sqrt{\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{P'-P}{P}\right)\left[1-\left(\frac{3}{2\gamma}-1\right)\left(\frac{P'-P}{P}\right)\right]} \tag{21}$$

Let

$$K_2'=K_2\sqrt{\frac{\gamma-1}{\gamma}}=\sqrt{\frac{2g}{R}}$$

Then

$$\Psi=K_2'\frac{P}{\sqrt{\theta'}}\sqrt{\frac{P'-P}{P}}\sqrt{1-\left(\frac{3}{2\gamma}-1\right)\left(\frac{P'-P}{P}\right)} \tag{22}$$

(20) becomes

$$\frac{d}{dt}\log_e P=-\frac{\gamma}{V}\left\{\frac{dV}{dt}-(\gamma-1)C_2F_1K_2'\sqrt{\frac{\theta'(P'-P)}{P}}\sqrt{1-\left(\frac{3}{2\gamma}-1\right)\left(\frac{P'-P}{P}\right)}\right. \tag{23}$$

Case III.—*Burnt gas escaping through inlet valve.*

In modern high speed internal combustion engines the pressure inside the cylinder will generally be greater than the pressure in the inlet passage when the inlet valve begins to open. Consequently we have to deal with outflow as well as inflow through the inlet passages. On the other hand the flow through the exhaust passages is always in the one direction. If the periods of valve opening overlap, the outward flow may occur simultaneously through both valves. This case can be dealt with by the addition to the right-hand member of (18) of a term to take care of the outward flow through the inlet valve.

Thus

$$\frac{d}{dt}\log_e P=-\frac{\gamma}{V}\left\{\frac{dV}{dt}+K_2R\left[F_e\sqrt{\theta_0\left(\frac{P''}{P_0}\right)^{\frac{\gamma-1}{\gamma}}}\sqrt{\left(\frac{P''}{P}\right)^{\frac{3-\gamma}{\gamma}}-\left(\frac{P''}{P}\right)^{\frac{2}{\gamma}}}\right.\right.$$
$$\left.\left.+F_i\sqrt{\theta_0\left(\frac{P'}{P_0}\right)^{\frac{\gamma-1}{\gamma}}}\sqrt{\left(\frac{P'}{P}\right)^{\frac{3-\gamma}{\gamma}}-\left(\frac{P'}{P}\right)^{\frac{2}{\gamma}}}\right]\right\}. \tag{24}$$

## GRAPHICAL INTEGRATION.

The above equations (17), (18), and (23) are of such a form that it is useless to attempt an analytical integration. They are of the general first order type.

$$\frac{dy}{dx}=f(x,y) \tag{25}$$

An accurate method for the graphical integration of this type of equation is given in Runge's "Graphical Methods." For our purpose, however, it will do to use a simpler means of getting an approximate solution.

The equation (25) defines not one curve, but a whole family of curves, one of which passes through every point in the region of definition of $f(x, y)$. $f(x, y)$ is the slope at the point $x, y$ of the particular curve which passes through $x, y$. We define the combination of the point $x, y$ with a short straight line through that point having the slope $f(x, y)$ as a lineal element of differential equation (25). $f(x, y)$ being continuous, the directions of these lineal elements vary continuously from point to point. Hence by plotting a large number of lineal elements we can estimate the direction of the integral curve passing through any chosen point (see plot 4). It happens that we are interested only in the particular integral curve passing through the point $\log_e P_0$, $t_0$. Consequently labor is saved if we proceed as follows: First plot the lineal element through $\log_e P_0$, $t_0$. Choose a slightly larger value of $t$, say $t_1$, and by a prolongation of the above lineal element to the line $x=t_1$, get an approximate value of the ordinate corresponding to this abscissa. Plot the lineal element through this new point and through two other

points having slightly greater and slightly smaller ordinates, respectively. With these lineal elements as a guide, it will be possible to sketch in by eye with fair accuracy the integral curve from its initial point on the line $x=t_0$ to its intersection with the line $x=t_1$, and to get an approximate value of the ordinate of its intersection with another suitably chosen vertical line, say $x=t_2$. By a simple repetition of this process we can extend the integral curve to any desired value of $t$ with a minimum of labor.

When the relation between the pressure in the cylinder and the time has been determined the mass of fresh charge drawn into the cylinder during the suction stroke can easily be calculated by means of a second simple mechanical integration. We denote this charge by the symbol $G_1$. It is equal to the gross charge flowing into the cylinder through the inlet valves minus the mass which flows out through these valves during the initial period of pressure equalization. Let $t'$ denote the time at which the inlet valve begins to open. Let $t''$ denote the time when the flow through the inlet valve reverses. Let $t'''$ denote the time of inlet valve closure. Then

$$G_1=-\int_{t'}^{t''}F_1(t)\Psi(P,\theta,P')dt+\int_{t''}^{t'''}F_1(t)\Psi(P',\theta,P)dt. \tag{26}$$

These integrals can be evaluated by means of a planimeter.

One of the equations (17) or (18) is applicable at all times when the flow through the valves is outward, even though part of the flow is through the inlet valve, although, if gas is escaping from the cylinder into the inlet manifold, we must replace in (18) by $(F_1+F_e)$. It is not to be expected that there will ever be any inward flow through the exhaust valves, or an inward flow through the inlet valves when the exhaust valves are open. Thus Equations (17), (18), and (23) cover all cases which will arise in practice.

## INTRODUCTION OF NUMERICAL VALUES.

Before making a definite application of the above equations, we introduce some rough assumptions regarding the properties of the gas in the cylinder. Normal exhaust gas from an engine using gasoline as fuel may be assumed to contain about 13.6 per cent by volume of water vapor, 12.4 per cent of $CO_2$, and 74 per cent of $N_2$. The percentages by weight are then 8.5 per cent ($H_2O$), 19.1 per cent ($CO_2$), and 72.4 per cent ($N_2$). The specific heats at 200° F. and at constant pressure may be taken to be 0.448 B. T. U. per pound, 0.202 B. T. U. per pound, and 0.244 B. T. U. per pound, respectively. The specific heat of the mixture in mechanical units computed from the above rough assumptions is 198 foot-pounds per pound. The gas constant of the mixture is 53.7 and the specific heat at constant volume works out to be

$$C_v=C_p-R=198-54=144$$

The ratio of the specific heats is

$$\gamma=\frac{198}{144}=1.375.$$

We take the above values of $R$ and $\gamma$ as the basis for our first rough calculation. At the high temperatures prevailing during the exhaust stroke a smaller value of $\gamma$ would probably be more accurate, while it is probable that a slightly larger value would be better for the suction stroke.

Introducing these values, we find that the critical pressure ratio is 0.5334. Further;

$$K_1=0.526;\ K_2=2.094;\ K_2'=1.094.$$

Equation (17) reduces to

$$\frac{d}{dt}\log_e P=-\frac{1.375}{V}\left[\frac{dV}{dt}+0.526\times 53.7\ F_e\ \sqrt{\theta_0}\left(\frac{P}{P_0}\right)^{0.1358}\right].$$

It is useful to replace $\log_e P$ by $\log_{10} p$, $V$ by $v$, and $F_e$ by $f_e$. Then

$$2.3026\frac{d}{dt}\log_{10}p=-\frac{1.375}{v}\left[\frac{dv}{dt}+0.526\times53.7\times12f_e\sqrt{\theta_0}\left(\frac{p}{p_0}\right)^{0.1358}\right],$$

$$\frac{d}{dt}\log_{10}p=-\frac{0.596}{v}\left[\frac{dv}{dt}+338.8f_e\sqrt{\theta_0}\left(\frac{p}{p_0}\right)^{0.1358}\right]. \tag{27}$$

Another useful change is to introduce the crank angle as the independent variable instead of the time. We denote the crank angle measured from the head end dead center by $\varphi$. We take the unit of measurement for this angle to be 10 degrees. Let $N$ denote the crankshaft speed in revolutions per minute. Then

$$\frac{d\varphi}{dt}=\frac{36N}{60}=0.6N$$

$$\frac{d}{d\varphi}\log_{10}p=-\frac{0.596}{v}\left[\frac{dv}{d\varphi}+\frac{338.8}{0.6N}f_e\sqrt{\theta_0}\left(\frac{p}{p_0}\right)^{0.1358}\right]. \tag{28}$$

Let $A$ = piston area in square inches.
$r$ = crank pin radius in inches.
$l$ = connecting rod length in inches.
$v_0$ = compression volume in cubic inches.
Then

$$v=v_0+A\{r(1-\cos\varphi)+l-\sqrt{l^2-r^2\sin^2\varphi}\}=v_0+A(l+r)-A[r\cos\varphi+\sqrt{l^2-r^2\sin^2\varphi}]; \tag{29}$$

$$\frac{dv}{d\varphi}=\frac{Ar\sin\varphi}{5.73}\left\{1+\frac{r\cos\varphi}{\sqrt{l^2-r^2\sin^2\varphi}}\right\}. \tag{30}$$

Equations (28), (29), and (30) enable us to determine the relation between pressure and crank angle from the time when the exhaust valve opens until the pressure ratio has been reduced to its critical value. From the latter point till the opening of the inlet valve we must use an equation of the form (18).

Introducing the value 1.373 for $\gamma$ and changing the independent variable from $t$ to $\varphi$ we derive the following equation from (18).

$$\frac{d}{d\varphi}\log_{10}p=-\frac{0.596}{v}\left[\frac{dv}{d\varphi}+2{,}250\frac{f_e}{N}\sqrt{\theta_0}\left(\frac{p''}{p_0}\right)^{0.1358}\sqrt{\left(\frac{p''}{p}\right)^{1.186}-\left(\frac{p''}{p}\right)^{1.456}}\right]; \tag{31}$$

$$\frac{p''}{p}>\left(\frac{2}{2.373}\right)^{3.68}=0.5334\,.$$

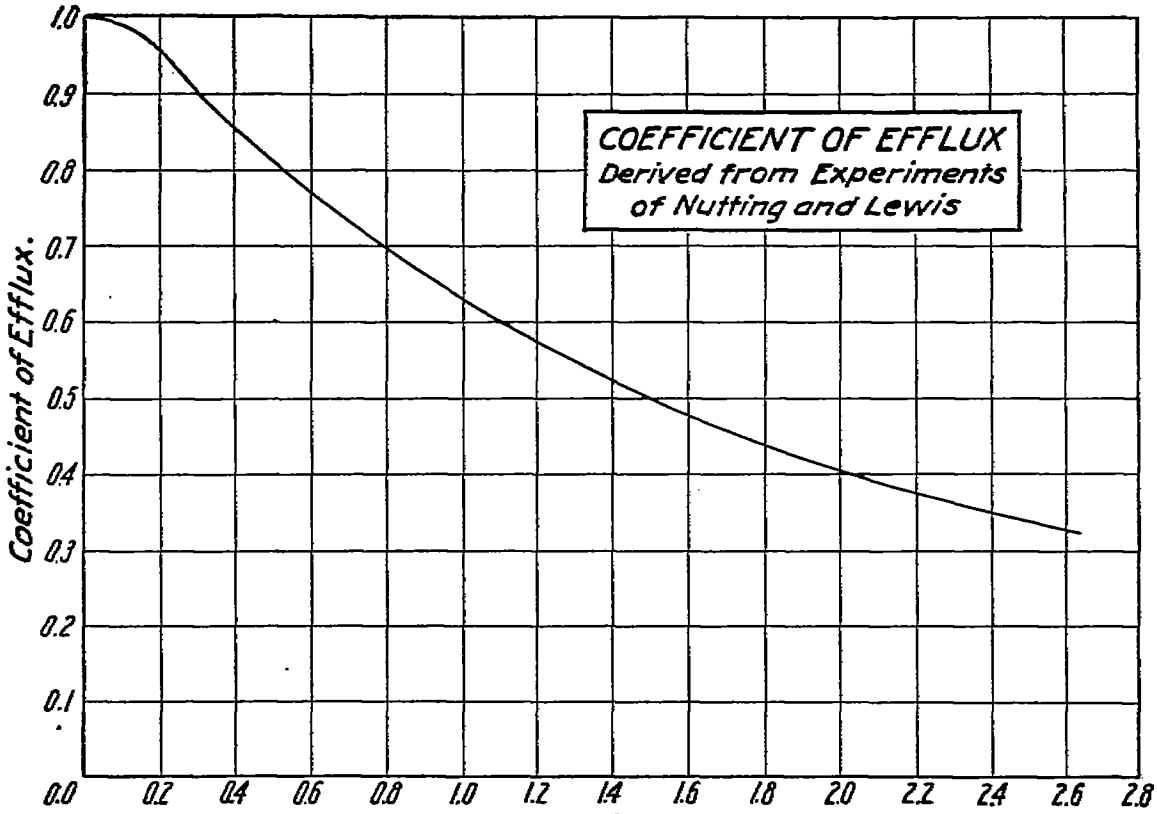
Making the same changes in (23) that we have made in equations (17) and (18), we derive the following equation applicable during the suction stroke.

$$\frac{d}{d\varphi}\log_{10}p=-\frac{0.596}{v}\left[\frac{dv}{d\varphi}-1{,}614\frac{f_1}{N}\sqrt{\frac{\theta'(p'-p)}{p}}\sqrt{1-.093\frac{(p'-p)}{p}}\right]. \tag{32}$$

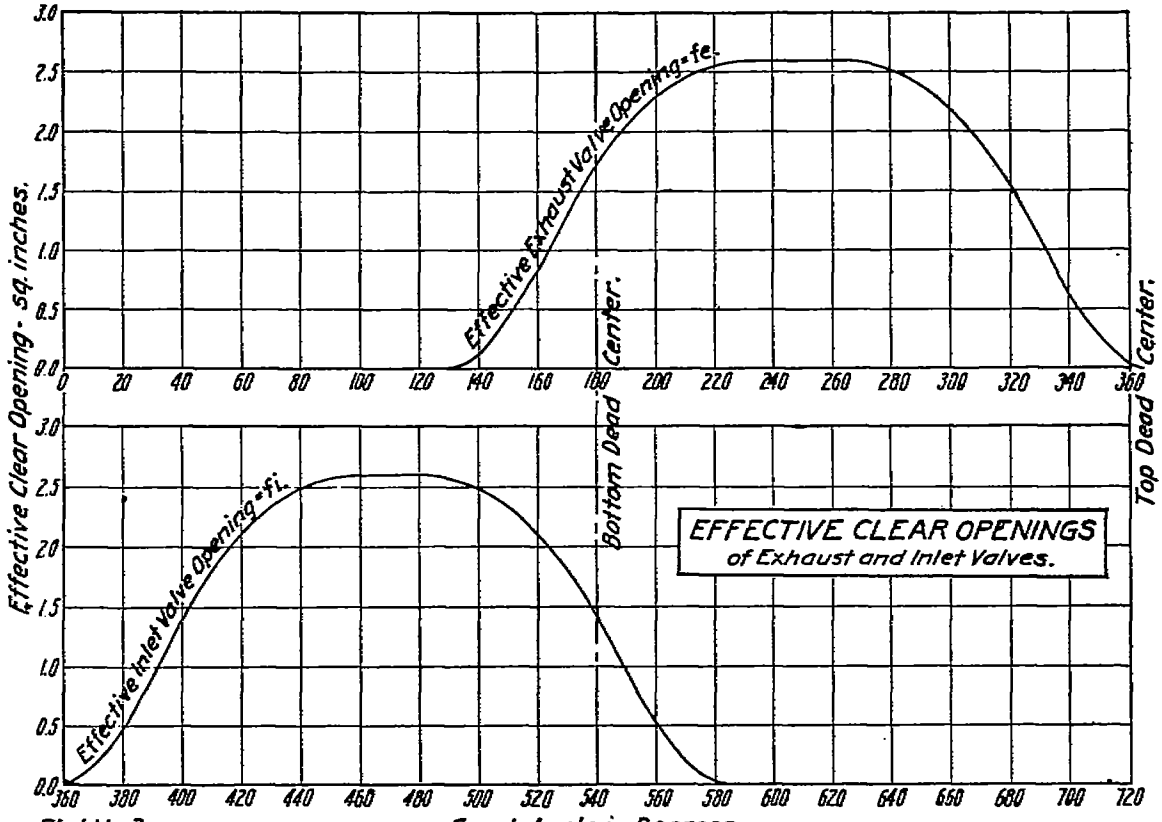
## EFFLUX COEFFICIENTS.

It will be assumed that efflux coefficients derived from steady flow experiments, such as those of Lewis and Nutting (Report No. 24), are applicable to intermittent flow. As a first approximation, the effects of valve design and seat angle, as well as the effect of reversing the direction of flow, will be neglected. In other words, it will be assumed that the efflux coefficient of any inlet or exhaust valve is a function only of the ratio of the clear opening of the valve to the net area of the valve passage (i. e. the gross area minus the valve stem area).

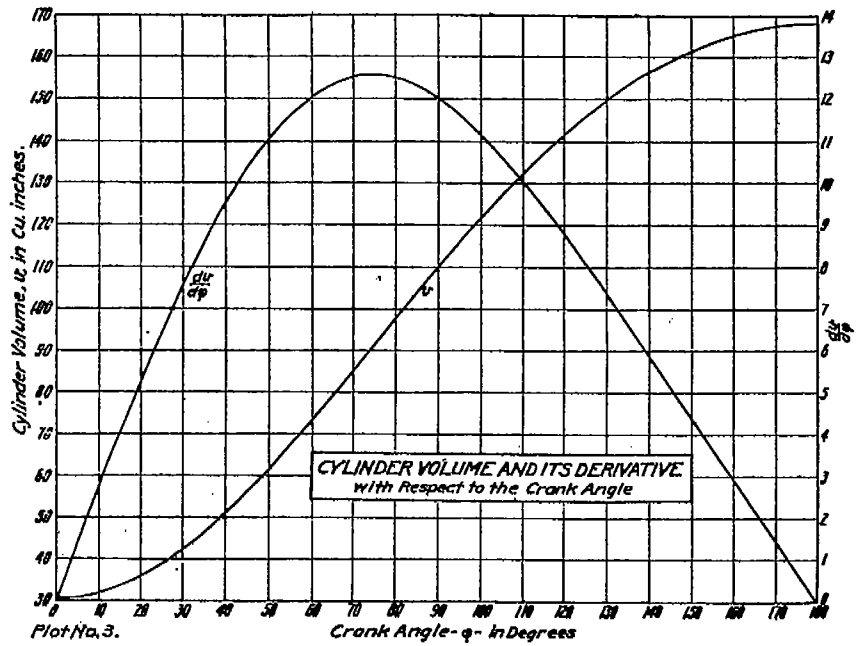
Plot 1 shows the relation between the above-mentioned ratio and the coefficient of efflux as computed from the experiments of Lewis and Nutting on 1¼-inch valves. Whereas the coefficients of efflux reported by Lewis and Nutting are based on the rough assumption that the clear opening of the valve is equal to the product of the valve lift and the port circumference, the values used in the curve on Plot 1 have been corrected by the substitution of clear valve areas computed in a more accurate manner by means of trigonometrical formulæ.



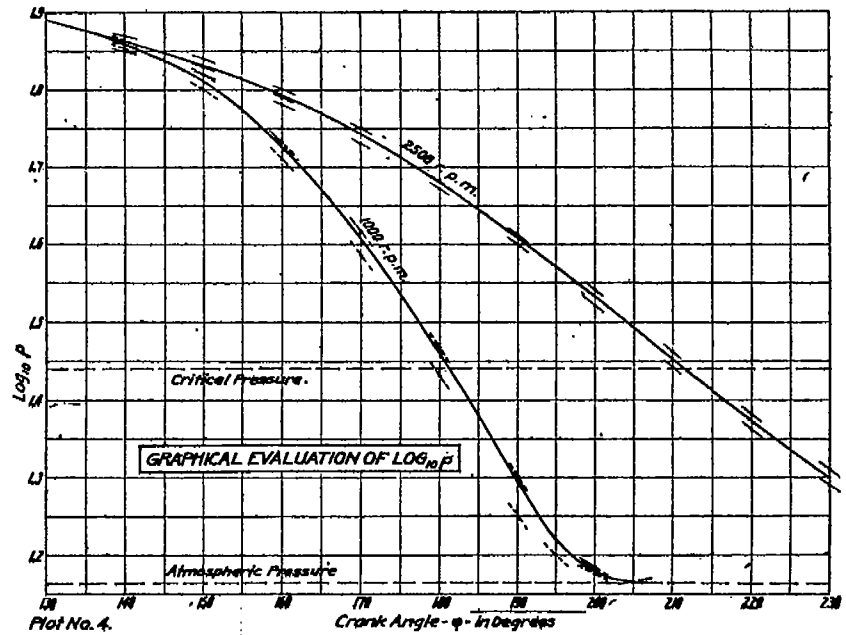

Plot No. 1. Ratio of Clear Opening to Net Port Area.



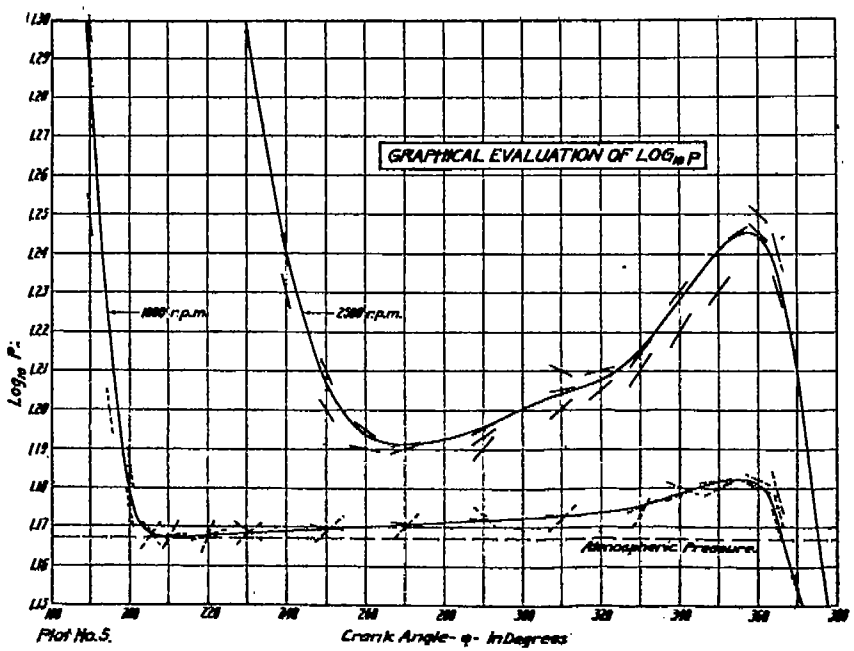

Plot No. 2. Crank Angle in Degrees



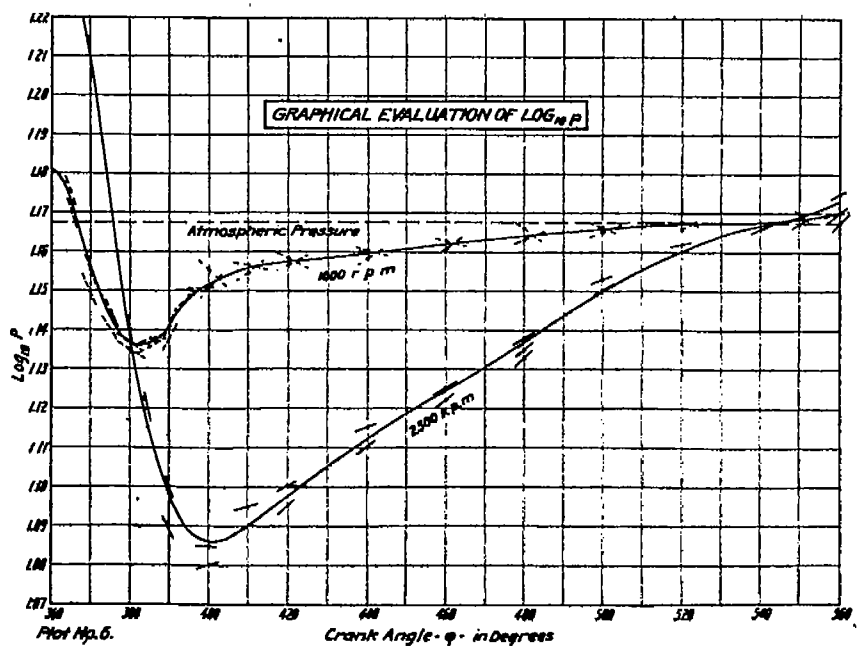

Plot No. 3.



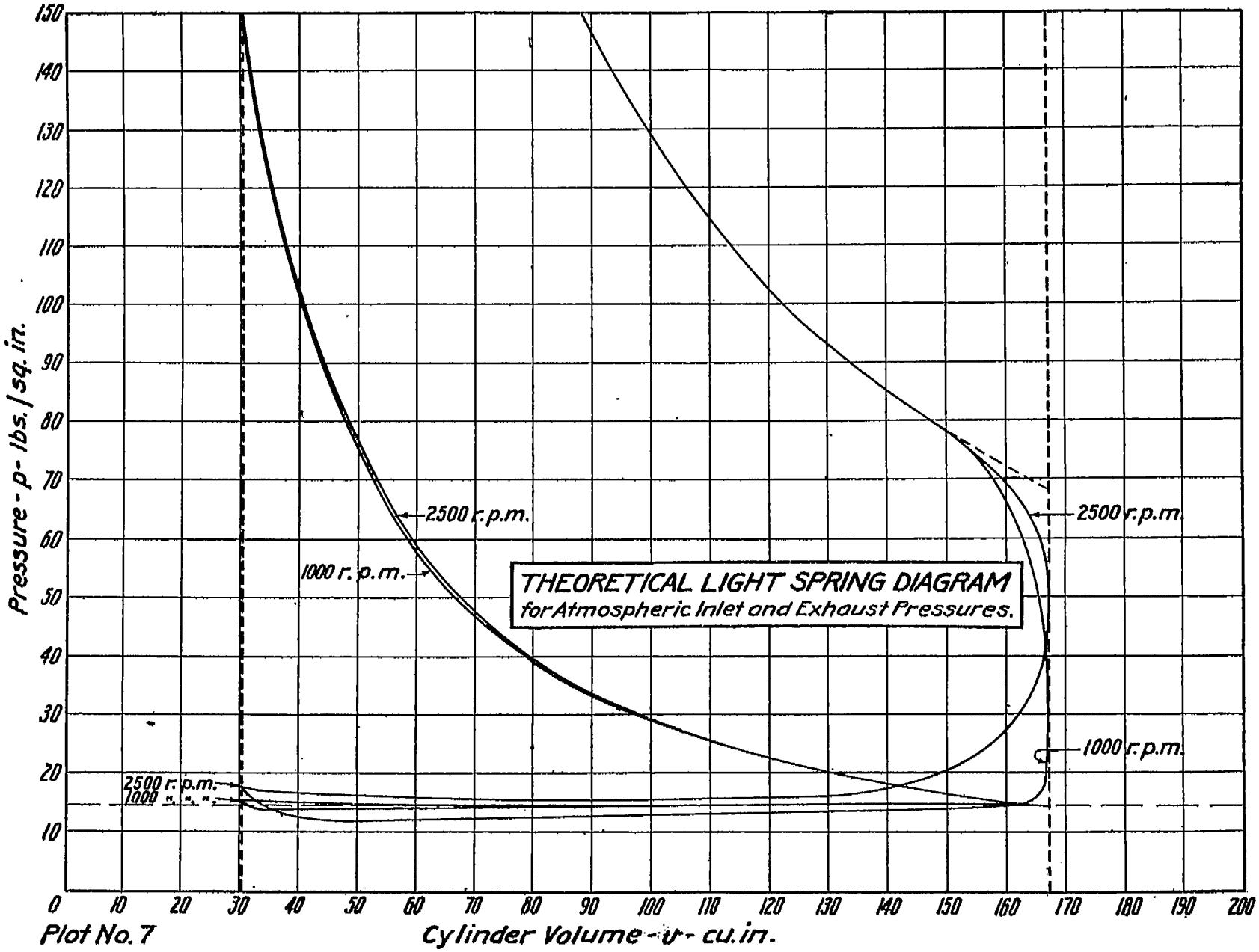

Plot No. 4.




Plot No. 5.




Plot No. 6.


THEORETICAL LIGHT SPRING DIAGRAM
for Atmospheric Inlet and Exhaust Pressures.
2500 r.p.m.
1000 r.p.m.
2500 r.p.m.
1000 r.p.m.
2500 r.p.m.
1000 " " "
Pressure - p - lbs./sq. in.
Cylinder Volume - v - cu. in.
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200

Plot No. 7

## APPLICATION TO TYPICAL MOTOR.

Plot 2 shows the effective clear opening (product of actual clear opening by efflux coefficient) for the exhaust and inlet valves of a typical engine plotted against the crank angle.

Plot 3 shows the cylinder volume $v$ and its derivative $\frac{dv}{d\varphi}$ as functions of $\varphi$.

We assume that pressure at the moment when the exhaust valve begins to open ($\varphi=130°$] is 78.0 pounds per square inch (abs.) and that the temperature is 3,000° on the absolute Fahrenheit scale.

Let the exhaust be direct to atmosphere. Then

$$p_0=78.0$$
$$\theta_0=3000°$$
$$v_0=149.7$$

Equations (28) and (31) become

$$\frac{d}{d\varphi}\log_{10} p=-\frac{0.596}{v}\left[\frac{dv}{d\varphi}+\frac{16,720}{N}f_e p^{0.1358}\right]; \qquad p>27.55 \quad (33)$$

$$\frac{d}{d\varphi}\log_{10} p=-\frac{0.596}{v}\left[\frac{dv}{d\varphi}+\frac{98,250}{N}f_e\sqrt{\left(\frac{14.7}{p}\right)^{1.185}-\left(\frac{14.7}{p}\right)^{1.456}}\right]; \quad p<27.55. \quad (34)$$

Let us further assume that the impact pressure in the passage leading to the inlet valve has the constant value 14.7 pounds per square inch, and that the impact temperature of the inflowing charge is 70° F. These assumptions set to one side the temperature and pressure changes which occur in the inlet manifold. Then

$$p'=14.7$$
$$\theta'=530°$$

Equation (32), for the pressure variation during the suction stroke, becomes

$$\frac{d}{d\varphi}\log_{10} p=-\frac{0.596}{v}\left\{\frac{dv}{d\varphi}-\frac{37,150}{N}f_i\sqrt{\left(\frac{14.7-p}{p}\right)\left[1-.093\left(\frac{14.7-p}{p}\right)\right]}\right. \quad (35)$$

The graphical integration of these equations for the determination of the relation between $\log_{10}p$ and $\varphi$ is shown on plots 4, 5, and 6.

Plot 7 shows the low-spring pressure-volume curves for 1,000 revolutions per minute and 2,500 revolutions per minute derived from plots 4, 5, and 6.

It will be observed that according to these diagrams the exhaust valve opens a little early at 1,000 revolutions per minute and a little late at 2,500 revolutions per minute. The inlet valve closing time is better adapted to 2,500 revolutions per minute than to the lower speed.

## CONCLUSION.

In conclusion it should be pointed out that we have here developed a method of computing theoretical low-spring diagrams which is capable of further development and refinement not justified by the experimental data now available. At present the absolute values deduced from the theory can not be relied on. It is to be hoped, however, that the results will prove to be of qualitative value and that future experiments will show us how to so modify the theory that it will give quantitatively correct values.

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