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FLEXIBILITY OF BEARING SURFACES AND STRESS ON FABRICS.

By

A. Proll.

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FLEXIBILITY OF BEARING SURFACES AND STRESS ON FABRICS.*

By

A. Proll.

Experience has proved that the strength of airplane wings is in no slight degree dependent on the tension due to the covering of the bearing surfaces, which counteracts the deflection of the spars and ribs. This influence is particularly noticeable in the so-called "C loading," which occurs when an airplane makes an almost perpendicular nose-dive. The air forces on the wings are then principally reduced to the frontal resistance and to a couple in a plane perpendicular to the bearing surfaces, which causes strong torsion of the wings, so that one spar is deflected upwards and the other downwards. It may therefore be expected that there is great stress on the fabric in such cases. On account of the direct air forces, however, the load is extremely unsymmetrical even in case C, and the English measurements* previously published show that it is particularly heavy on the leading edge of the pressure and suction sides. Even under the simplified conditions of righting the airplane during steep gliding flight (Case A) both stresses may be added. In order to obtain a general survey, at least of the probable order of the values of the increase of tension, due to deflection, it is assumed in Fig. 1 that the deflection takes place in an upward direction (front spar in case A), and the top tension is

* See MUNK'S article on the "Distribution of Pressure on Bearing Surfaces According to English Measurements," "Zeitschrift für Flugtechnik und Motorluftschiffahrt," Vol. VII, 1916, p.133.

thereby extended, also in consequence of the great depression which ought to take place according to the measurements.

Curve radius of the elastic lines of the spars = ρh,

Rib spacing = l_1,

Spacing between the fabric covering and the neutral axis of of the spar = e,

Initial camber of the fabric (curve radius ρ_{1_0} depth of camber f_{1_0}).

(o = Negative in direction 1!)

The distance $A B = l_1$ is enlarged to $A'B' = l_1$ through the bending:

$$l_1' = \frac{\rho h + e}{\rho h} l_1 = 1 + \frac{e}{\rho h} \quad l_1 \quad \ldots\ldots\ldots\ldots \quad (1)$$

The air load p (kg/m²) simultaneously causes the originally negative camber of the fabric f_{1_0} to be transformed into the positive f_1, the value of which is estimated in the present instance by the diametrical measurements of the camber,* but which may generally be obtained by fundamental formulas.**

The original length of the fabric (Fig. 1a) is in accordance with a well-known approximate formula:

$$l_1 + \frac{8}{3} \frac{f_{1_0}^2}{l_1} \quad \ldots\ldots\ldots\ldots \quad (2)$$

with the requisite tension S_{1_0} (preliminary stretching).

The new length is (Fig. 1b)

$$l_1 \quad 1 + \frac{e}{\rho h} + \frac{8}{3} \frac{f_1^2}{l_1 (1 + \frac{e}{\rho h})} \quad \ldots\ldots\ldots \quad (3)^{***}$$

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* See Proll's "Fabric on Bearing Surfaces," Technische Berichte, Vol. III, p.234.

** See Proll's "Fabric on Bearing Surfaces," Vol. III, p.243.

*** The factor $1 + \frac{e}{\rho h}$ may be left out in the nominator of the second term.

and the accompanying tension S_1.

The fabric is therefore stretched to the following extent

$$\Delta l_1 = l_1 \frac{e}{\rho h} + \frac{8}{3} \frac{f_1^2 - f_{1_0}^2}{l_1}$$

and it is

$$\epsilon_1 = \frac{\Delta l_1}{l_1} = \frac{e}{\rho h} + \frac{8}{3} \frac{f_1^2 - f_{1_0}^2}{l_1^2} \quad \ldots\ldots\ldots\ldots (4)$$

The deflection of the spars will evidently cause a diminution of the depth of camber f as compared to its previously determined values. For the first calculation, however, the value known by former tests may be taken. From the fundamental equation for the doubly curved fabric surface (tension S_1 and S_2 in warp and woof), we get

$$p = \frac{S_1}{\rho_1} + \frac{S_2}{\rho_2} \quad \ldots\ldots\ldots\ldots (5)$$

the result, as shown elsewhere* assuming parabolically formed curves, is

$$S_1 = \frac{p\, l_1^2}{8\, f_1} \left(1 - \frac{S_2}{p\, \rho_2}\right) \quad \ldots\ldots\ldots\ldots (6)$$

and after inserting f_1 from (4)

$$S_1^2 \left(\epsilon_1 - \frac{e}{\rho h} + \frac{8\, f_{1_0}^2}{3\, l_1^2}\right) = S_1^2 \left(\epsilon_1 - \epsilon_1'\right) =$$

$$= \frac{p^2\, l_1^2}{24} \left(1 - \frac{S_2}{p\, \rho_2}\right)^2 \quad \ldots\ldots\ldots\ldots (7)$$

* See A. Proll's "Question of the Strength of Bearing Surface Tensions," Zeitschr. f. Flugtechnik und Motorluftschiffahrt, 1915, Nos. 3, 4, 5 and 6.

in which

$$\epsilon_1' = \frac{e}{\rho h} - \frac{8 f_{1_0}^{2}}{3 l_1^{2}} \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

was inserted.

Equation (7), in conjunction with the so-called standard characteristic (Normal-Charakteristik) "N.C." * renders it possible to form a calculation of the condition of the fabric by inserting, point for point, in the diagram of the "N.C.", the $S_1 \div (\epsilon_1 - \epsilon_1')$ curve (C curve, Fig. 2) derived from that equation. (Attention must be paid to the simultaneous alteration of S_2 [Note].). If there were no extension through the surface flexion, this curve would be THE GEOMETRICAL LOCUS FOR ALL THE POINTS UNDER THE GIVEN CONDITIONS. As the actual extension is greater, the real condition curve (C' curve) is higher by ϵ_1' at every point (Fig. 3).

* The best representation of the somewhat complicated conditions in the two-dimensional deformation of the covering fabric is given by the "Standard Characteristics," that is, two groups of curves showing, on occasion, extension and contraction in terms of the tension while taking into account the mutual influence of warp and woof. In the general case of any fabric whatever, the strength characteristics in the direction of warp or woof differ. Two standard characteristics are therefore necessary, one with S_1 as abscissae and ϵ_1 as ordinates (with constant S_2) as shown in Fig. 2; the other with S_2 as abscissae and the corresponding tensions ϵ_2 as ordinates and with curve groups of constant S_1. In the case of IMPREGNATED fabrics, however, the tests have shown that not only is the capacity for resistance considerably higher than in non-impregnated fabrics, but that the material, above all, becomes far more alike in BOTH directions, and the two standard characteristics are consequently but slightly different. Such impregnated fabrics being now almost exclusively used, it will suffice to apply only ONE normal standard in the first instance for the following considerations.

In Fig. 2, for instance, the full curves (S_2 = constant) represent the extensions $\left(\frac{\Delta l}{l}\right)_1 = \epsilon_1$, which occur in the WARP direc-

In order to determine the condition point, the extension in direction 2 (ϵ_2) must first be calculated.

According to Fig. 4, we find that the extension required in this case is

$$\epsilon_2 = \frac{8}{3\ l_2}(f_2^2 - f_{2_0}^2) \quad \ldots\ldots\ldots (9)$$

and

$$(f_2 - f_{2_0}) = (f_1 + f_{1_0}) \quad \ldots\ldots\ldots (10)^*$$

(f_{1_0} being a negative camber).

On the C curve, the point must be found for which the extension ϵ_2 occurs correctly, and in which the same tension S_2 is shown in both curve sheafs, at two points perpendicularly above one another.

(Continued from p.5)

tion with variable tension S_1, while the unalterable tension S_2 takes effect simultaneously. The dotted curves further show the extensions that occur simultaneously in the direction of the WOOF:

$$\left(\frac{\Delta l}{l}\right)_2 = \epsilon_2$$

Any condition of the fabric is then characterized by the TWO CONDITION POINTS P and P' respectively (Fig. 2). P is located on the extended curve S_2 = 100 kg/m., but the corresponding values are S_1 = 115 kg/m. and ϵ_1 = 0.76%. P' lies on the same ordinate line (S_1 again = 100; P' then gives S_1 = 115 kg/m. and S_2 = 100 kg/m. with the value ϵ_2 = 0.13%. Γ_1 and Γ_2 are the curves that correspond to the EQUAL tensions S_1 and S_2.

* In this case, the length l_2 is assumed to be always equal to the depth of the surface. This is only the case, however, when the rear surface connecting organ (wire) is not deflected. In case of its being deflected, however, the extension ϵ_2 is diminished, and with it also the tension S_2 (and S_1). These conditions must therefore always be taken into consideration, as they are of great importance, particularly in influencing tension S_2.

EXAMPLE. Assuming that $l_1 = 0.33$ m., $l_2 = 1.63$ m., $\rho_2 = 4$ m.,* p may be taken as150 kg/m.^{2}. THE DEFLECTION OF THE SPARS BEING LEFT OUT OF CONSIDERATION, according to equation (7), with ϵ_1' negative o

$$S_1^2 \epsilon_1 = 100 \quad 1 - \frac{S_2}{600}^2 \quad \ldots\ldots\ldots\ldots \quad (7a)$$

For the point by point drawing, we assume that $S_1 = 0$, 100, 200, and we get the following curves each time

$$S_1^2 \epsilon_1 = 100 \cdot \quad 1 - \frac{S_2}{600}^2$$

Their point of intersection with the corresponding S_2 lines are then the desired points of the C curve.

According to tests, it may now be assumed that $f_{1_0} = -5$ mm., $f_1 = +7$ mm., $f_{2_0} = 60$ mm., $f_2 = 72$ mm., also that $\epsilon_2 = 0.2\%$ (when the fabric does not contract towards the rear). Fig. 3 now shows how the condition point is found by means of tests. At point A ($S_2 = 100$), the corresponding extension (measured at point A') is too small (0.17%). When B $S_2 = 200$, it is far too large (0.66%) (at point B').

The correct condition point D ($S_2 = 108$) lies between them, with D', the characteristic data of which are: $\epsilon_2 = 0.2\%$, $\epsilon_1 = 0.24\%$, $S_1 = 167$ kg/m.**

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* In consequence of the camber of the fabric, there is a fairly considerable alteration in ρ_2, whereby

$$1 - \frac{S_2}{p\,\rho_2}$$

and also S_1 are diminished. It is therefore advisable to insert the approximately determined curve radius ρ_2 beforehand, NOT that of the fundamental camber. (The latter is 4.8 m. in the present instance.)

** It can be seen from the extremely steep course of the C curve that the tension S_1 remains ALMOST CONSTANT between 167 and 170, whereas S_2 and ϵ_1 are extremely variable.

It is now evident that $\epsilon_1' < \epsilon_1$ therefore S_1 is greater than before, and in this respect it differs from the fabric tension due to the deflection of the spars. The following must also be observed in this case: With a bending moment M of the spar in the space between two ribs, we get

$$\frac{1}{\rho h} = \frac{M}{EJ} \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

where E and J relate to spar material and cross-section. The actual bending moment M is, however, diminished in itself by the tension of the fabric as regards the moment M' without these; it is, namely.

$$M = M' - S_1 e \lambda \quad \ldots\ldots\ldots\ldots \quad (12)$$

in which λ designates that part of the depth of the ribs which comes into consideration for the loading of the spar in question. Equation (7) is thus replaced by:

$$S_1^2 \quad \epsilon_1 - \frac{e}{E\,J} \quad M' - S_1 e \lambda \quad - \frac{8}{3} \frac{f_{1o}^2}{l_1^2} =$$

$$= \frac{p^2 l_1^2}{24} \quad 1 - \frac{S_2}{p\,\rho_2}^{\,2} \quad \ldots\ldots\ldots \quad (13)$$

In this, we must again insert

$$\frac{e}{E\,J}(M' - S_1 e \lambda) - \frac{8}{3} \frac{f_{1o}^2}{l_1^2} \quad \ldots\ldots\ldots \quad (14)$$

---

* The determination of λ differs according to the loading condition, corresponding to the extremely variable value of the air forces per 1 meter square (p).

In this respect, it will suffice to assume that in the "A loading", λ is calculated from a point midway between the spars, towards the front or to the tips of the wings, for front and rear spars, and that a mean value is added for p (Fig. 5).

It must be observed that ρ_2 also changes in consequence of the deflection of the RIBS, also on account of the alteration of f_{2t} to f_2. As it would be to no purpose to establish an extremely complicated ratio between S_1, S_2 and ϵ_1, considering the simplifications that have been attained, it will be found preferable to calculate with successive approximations and to insert $M' = M$ in the first place.

This will now be proved by completing the example previously given. If the cross-section of the spar has a moment of inertia $J = 100$ cm.^{4}, and $E = 10^5$ kg.cm.^{2}, the greatest moment would be $M' = 9500$ kg. cm.* without taking into consideration the influence of the fabric. If it be calculated that $e = 5$ cm., $S_1 = 250$ kg.m. and $\lambda = 60$ cm., the fabric is responsible for a moment of $250 \times 0.06 \times 0.05 = 7.5$ kg.m. $= 750$ kg.cm., that is $\sim 8\%$ of M, and therefore $M = 8750$ kg.cm. According to equation (14) we now get:

$$\epsilon_1' = \frac{5 \cdot 8750}{100 \cdot 10^5} - \frac{8}{3} \left(\frac{0.5}{40}\right)^2 = 0.00435 - 0.0004 \qquad 0.004$$

By so much - that is, 0.4% (C' curve) the C curve is thus pushed upwards in a parallel direction. In the same way as before, the new condition point G (G') is found by assuming that

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* The greatest stress on the spars is thus

$$\sigma \max = \frac{M+}{\frac{J}{e}} = 475 \text{ kg/cm.}^2$$

without taking any eventual axial forces into consideration.

there is the same extension as before in direction 2: $\epsilon_2 = 0.002$ (0.2%). It is then found that:

$S_1 = 283$ kg.m. $\epsilon_1 = 0.43\%$ $S_2 = 153$ kg.m. $\epsilon_2 = 0.2\%$.

The warp threads are arranged in direction 1 (parallel to the spars) in this example.

The stress on the fabric is considerably increased through the deflection of the spars, especially in direction 1.

The above calculation may be corrected by taking the value $S_1 = 283$ kg.m. instead of the assumed value $S_1 = 250$ kg.m., though there would be but little difference in the result (S_1 would be somewhat less) and the value of λ on the other hand, cannot be so precisely estimated that the calculation would thereby be made more accurate.

The fact that the tension predominates so strongly in the direction parallel to the spars is partly due to the fact of the peculiarity of the characteristic standard curves, and particularly to the extensibility in the direction of the woof.

If, on the contrary, the fabric were STRETCHED AT AN ANGLE DIFFERING BY 90° (woof - spar, warp - ribs), the standard characteristic according to Fig. 6* is obtained, and an entirely similar rendering of the calculation again leads to a curve C' and the condition point G with $S_1 = 160$ kg.m. $\epsilon_1 = 0.002$ (presumably), $S_2 = 165$ kg.m. $\epsilon_2 = 0.0047$.

* All the indexes 1 and 2 must naturally be altered in this case, In the second standard characteristic for this instance, the downward (contraction) curves S_1 = constant are now curved lines; this corresponds to the dissimilar spacing between the extension curves S_2 = constant, in the first standard characteristic (Fig. 3). S_1 (tension in the direction parallel to the ribs) is now warp, while the woof direction is laid parallel with the spars and undergoes tension S_2.

The same temporary supposition that S_2 = 250 kg.m. was established in this case with regard to the tension caused by the flexion of the fabric. As it proves to be far too large in this case, a corrective calculation must be made. Assuming that S_2 = 170 kg.m., $S_2 e \lambda$ = 510 kg.cm. in that case, M = 9000 kg.cm., and ϵ_2' = 0.0041, which would scarcely produce any appreciable alteration.

For the fabric utilized in this case, AN INTERCHANGE OF WOOF AND WARP WOULD NOT ONLY LEAD TO GREATER UNIFORMITY OF TENSION, BUT ALSO TO A CONSIDERABLE DIMINUTION OF THE TENSION OF THE FABRIC. This is due to the fact that the fabric hasgreater extensibility in the DIRECTION OF THE WOOF, and it has been admitted that it is advisable to LAY THE DIRECTION IN WHICH THERE IS THE GREATER EXTENSIBILITY PARALLEL WITH THE SPARS.*

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* Without flexibility of the bearing surface (Fig. 6) lower curve;

S_2 = 130 kg.m., ϵ_2 = 0.36%, S_1 = 145 kg.m. ϵ_1 = 0.2%.

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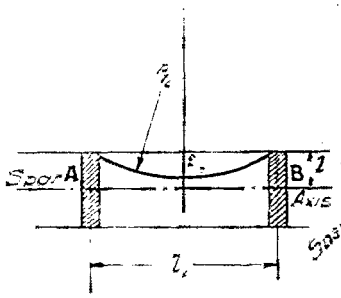


FIG. I a

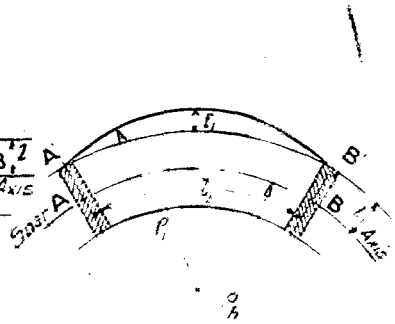
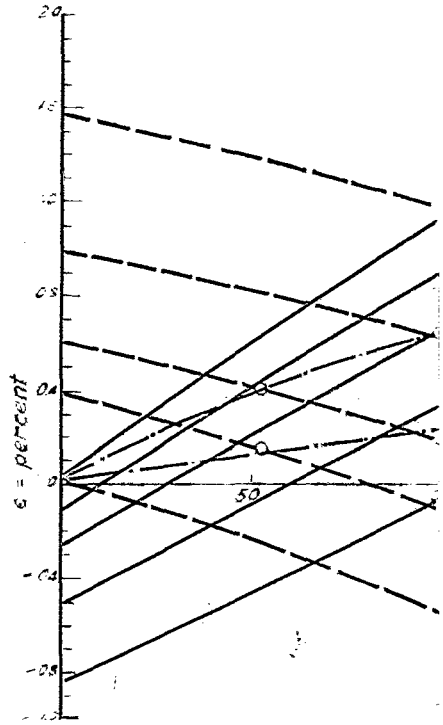


FIG. I b



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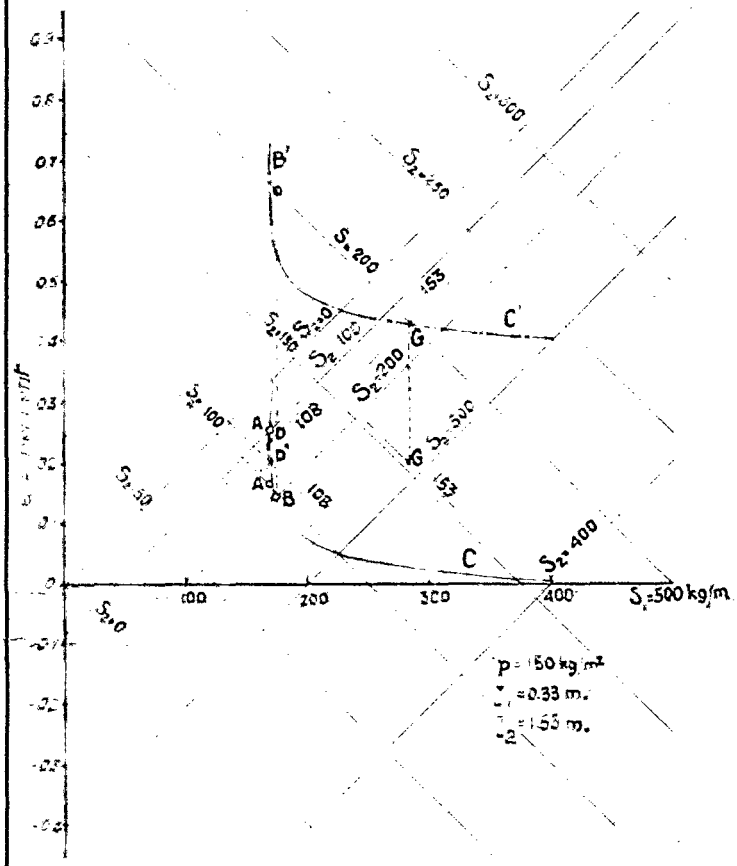

FIG 3

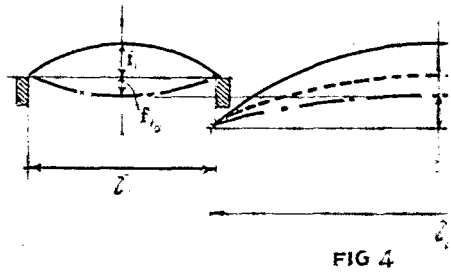

FIG 4

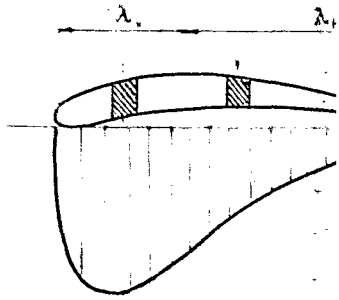

FIG. 5

FLEXIBILITY OF BEARING SURFACES AND STRESS ON FABRICS

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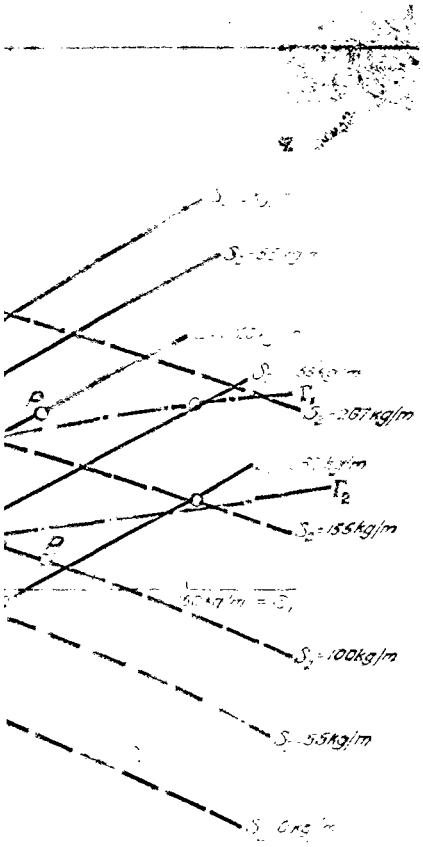
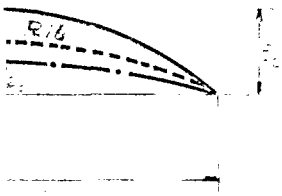


G 2



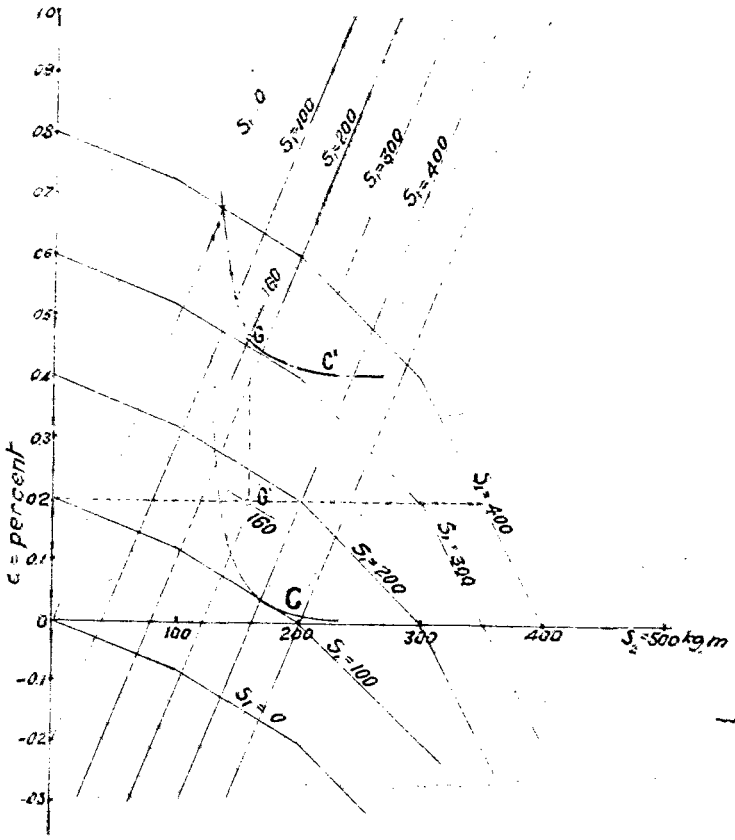


FIG.6