TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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A NEW METHOD OF TESTING MODELS IN WIND TUNNELS.

By

W. Margoulis, Aerodynamical Expert,
Paris Office, N.A.C.A.

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We know that the two essential conditions of the application of the law of proportionality of pressure to the product of density, the square of the linear dimensions and the square of the speed to the results of model tests are:

1) THE EQUALITY OF REYNOLDS' NUMBER.

$$N = \frac{vl}{\nu} = \frac{VL}{\nu_1}$$

v being the velocity of the airstream in the tunnel,

V being the speed of the machine in free flight,

l and L - respectively one of the principal linear dimensions of the model and of the full scale airplane.

ν and ν_1 - respectively the kinematic coefficients of viscosity of the fluid circulating in the tunnel and of the air in which the machine flies.

2) THE EQUALITY OF THE RATIOS OF THE SPEED TO THE VELOCITY OF SOUND (Law of Bairstow and Booth).

$$M = \frac{v}{w} \doteq \frac{V}{W}$$

w and W being respectively the velocities of sound in the tunnel and in the air.

The first of these conditions is due to viscosity and is important especially at low speeds (tests of model airplanes); the second condition is due to the consideration of compressibility and must be observed at high speeds (tests of model propellers).

Now, in existing laboratories utilizing a horsepower of 100 to 300, the models are generally made to a 1/10 scale and the speed is appreciably lower than the speeds currently attained by airplanes; the Reynolds' Number realized in the laboratories is thus from 15 to 25 times smaller than that reached by airplanes in free flight, while the ratio M varies between the third and three-fourths of the true ratio.

Thus, when a model airplane, for instance, is tested in such a laboratory, the streamline wires resist relatively twice as much, the struts of the rigging and undercarriage five times as much, while the wings carry 30% less on the model than on the airplane, so that RESULTS OBTAINED IN EXISTING LABORATORIES CANNOT BE PRACTICALLY UTILIZED.

We cannot appreciably increase Reynolds' Number by increasing either the diameter (d) of the tunnel, or the velocity v of the airstream, for the motive power required for working the fan producing the airstream is proportional to $d^{1.75}v^{2.75}$ and such increase would therefore lead to installations much too costly both as to establishment and upkeep.

Thus wind tunnels are now being planned having a diameter of 3 to 5 m., speeds of 60 to 75 m/sec., and horsepower of 1000

to 1500; but the Reynolds' Numbers attained in such tunnels will still be 8 times less than those of existing large airplanes.

We will show, however, that it is possible to have wind tunnels in which the Reynolds' Number will be greater than that now attained by airplanes, and in which the ratio of the velocity to the velocity of sound will also be greater than that realized in practice, and we will show that this can be done with an outlay for installation and upkeep much below that required by the laboratories now being planned.

In order to attain this result we have only to employ a gas other than air, at a pressure and temperature different from those of the surrounding atmosphere.

We will establish the expressions connecting the power:

1st. TO THE PHYSICAL CONSTANTS OF THE GAS.

2nd. TO THE VALUES OF THE TEMPERATURE AND PRESSURE.

3rd. EITHER TO REYNOLDS' NUMBER, OR TO THE SPEED OF THE AIRPLANE IN FREE FLIGHT, OR TO BOTH THESE VALUES SIMULTANEOUSLY.

1. - FUNDAMENTAL FORMULAS.

We will call

v and d respectively the speed and the diameter in the working section of a tunnel.

μ - the coefficient of viscosity at the absolute temperature T.

ρ - the density; ρ_0 - density at 1 kg/cm² and 273°.

ν - the kinematic coefficient of viscosity; $\nu = \mu/\rho$.

p, - the pressure.

The units employed are the kilogram (unit of force), the meter, and the second.

We assume with PRANDTL* that the power drop for any fluid whatever in a cylindrical conduit of length L, is:

$$\Delta = a\, L\, \mu^{0.25}\, \rho^{0.75} v^{1.75}\, d^{-1.25}$$

It can be shown** that the power drop in a closed circuit wind tunnel proceeds from two causes:

1st. The drop caused by friction, expressed in the form:

$$P_1 = A\rho \,.\, d^2 v^3 \,.\, \left(\frac{\mu}{\rho\, d\, v}\right)^{0.25} = A\rho\, d^2 v^3 \,.\, N_S^{-0.25}$$

(N_S being the Reynolds' Number of the tunnel.)

2nd. The second cause of power drop is the thickening of the airstream in the diffuser and the loss by impact in the bends:

$$P_2 = B\rho\, d^2 v^3$$

If we consider that these latter losses are very small as compared with those arising from the first-named cause, we can assume as an expression of the total power drop:

$$P_1 + P_2 = \rho\, d^2 v^3 \,.\, N_S^{-0.25} \left\{A + B\, N_1^{0.25} \,.\, \left(\frac{N_S}{N_1}\right)^{0.25}\right\} =$$

$$= \rho\, d^2 v^3\, N_S^{-0.25}\, (A + B\, N_1^{0.25}) = C\rho\, d^2 v^3 N_S^{-0.25}$$

N_1 being the mean value of N in the tunnel.

* "Abriss der Lehre von der Flüssigkeits und Gasbewegung" p.20.

** See in the last publication of the Eiffel Laboratory: "Résumé of principal Works executed during the War," p.184, my theory of the functioning of Wind Tunnels.

If we take as a basis the results obtained in the closed circuit tunnel of the Aeronautical Institute of Rome (50 m/sec. in a flue of 2 m. diameter with 113 h.p.; coefficient of utilization $\rho_s = 2.9$) for determining the value of $(A + B\,N_1^{0.25})$, we shall come to assume as an expression of the motive power required for working the fan of a tunnel of this type utilizing any fluid whatever:*

$$P_m = 0.47\,\mu^{0.25}\,\rho_o^{0.75} \cdot \frac{p^{0.75}}{T^{0.75}} \cdot v^{2.75} d^{1.75} \qquad (1)$$

We shall compute the Reynolds' Number for the tunnel by assuming that the span of the model is equal to 6/10 of the diameter of the flue:

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* The following Table gives the values of the coefficient of viscosity μ_o at 273° absolute, of the density ρ_o at 273° and 1 kg/cm² pressure, of the kinematic coefficient of viscosity ν_o at 273°, of the ratio γ of the specific heats at constant pressure and volume, and of the coefficient C of Sutherland's formula, giving the value of the coefficient of viscosity μ at the absolute temperature T of the gas.

$$\mu = \mu_o \frac{1 + C/273}{1 + C/T} \sqrt{\frac{T}{273}}$$

	$\mu_o . 10^6$ (kg.sec/m²)	ρ_o (kg.sec²/m⁴)	$\nu_o . 10^6$ (m²/sec)	γ	C
Air	1.69	0.127	13.3	1.4	114
CO_2	1.4	0.195	7.18	1.264	260
CH_3Cl (chloride of methyl)	0.99	0.251	3.94	1.2	454
X_e (xenon)	2.18	0.561	3.89	1.667	
Water	181	102	1.78	20830	

For water we have calculated the value of γ so as to be able to express the velocity of sound in water by a formula similar to that used for gas:

$$W = \sqrt{\gamma \frac{p_o}{\rho_o}}$$

$$N = \frac{0.6\ vd}{\nu} = 0.0164\ v\ d\ .\ \mu^{-1}\ \rho_o\ \ p\ T^{-1} \qquad (2)$$

The velocity of sound being equal to $\sqrt{\gamma \frac{p}{\rho}}$ where γ is the ratio of the specific heats, the condition of the constancy of the ratio of the speed of translation to the velocity of sound, leads to the expression:

$$v = 0.0183\ .\ \rho_o^{-0.5}\ \gamma^{0.5}\ \ T^{0.5}\ \ V \qquad (3)$$

v being the velocity of the fluid stream in the tunnel and

V the speed of the airplane in air at 273° and at a pressure of 1 kg/cm.²

Equations (1), (2), and (3) enable us to establish the following expressions for the power required for working the fan:

2. - 1st CASE - REYNOLDS' NUMBER IS GIVEN; TESTS OF MODEL AIRPLANES AND AIRSHIPS.

Eliminating v from equations (1) and (2) we have:

$$P_m = 37600\ \frac{\mu^3}{\rho_o^2}\ .\ \frac{T^2}{p^2}\ .\ \frac{N^{2.75}}{d} \qquad (4)$$

and

$$v = 61\ \frac{\mu}{\rho_o}\ .\ \frac{T}{p}\ .\ \frac{N}{d} \qquad (5)$$

Formula (4) shows that the power P_m is proportional to the term μ^3/ρ_o^2 characterizing the fluid, and to the term T^2/p^2 characterizing the conditions of temperature and pressure.

Continuation of footnote from p.5. For gases, compression and cooling diminish the value of ν, in consequence of the reduction of μ with the reduction of the temperature, and also on account of the increase of ρ; on the contrary, for water the reduction of the temperature increases the value of μ ($\mu = 19.10^6$ for $T = 423°$, 29.10^6 at 273 and 183.10^6 at 273°) while cooling and compression do not appreciably affect the density.

THE GAS WHICH IS PRACTICALLY MOST SUITABLE TO THE DIFFERENT CONDITIONS LAID DOWN BY THE ABOVE FORMULAS AND BY THOSE WHICH FOLLLW, IS CARBONIC ACID* (CO_2), AS A GAS HAVING A LOW COEFFICIENT OF VISCOSITY, HIGH DENSITY AND A LOW RATIO OF SPECIFIC HEATS.

For air, $\mu^3/\rho_o^2 = 301/10^{18}$; for CO_2, $\mu^3/\rho_o^2 = 72.4/10^{18}$; for $p = 15$ kg/cm² and $T = 253^o$, $\frac{T}{T_o} \cdot \frac{p_o}{p} = 0.0618$ and $\left(\frac{T}{T_o} \cdot \frac{p_o}{p}\right)^2 = 0.00382$.

WE THUS SEE THAT FOR A GIVEN REYNOLDS' NUMBER AND WITH EQUAL DIAMETER OF FLUES, THE USE OF CARBONIC ACID AT 273° AND 1 kg/cm² WILL ECONOMIZE 3/4 OF THE POWER REQUIRED WITH ATMOSPHERIC AIR AND THAT COMPRESSION TO 15 kg/cm² AND COOLING TO 253° WILL REDUCE THIS POWER IN THE RATIO OF 1088 TO 1, WHICH CERTAINLY CONSTITUTES A REMARKABLE RESULT.

THE SPEED WILL BE REDUCED IN THE RATIO OF 1.85 TO 1 IN THE FIRST CASE AND IN THE RATIO OF 30 TO 1 IN THE SECOND CASE.

3. - 2nd CASE - IT IS REQUIRED TO REALIZE THE SAME RATIO OF THE VELOCITY OF TRANSLATION TO THE VELOCITY OF SOUND AS FOR A FULL SCALE MACHINE FLYING AT A SPEED V.

Eliminating v from equations (1) and (3) we obtain:

$$P_m = 0.000008 \frac{\mu^{0.25} \gamma^{1.375}}{\rho_o^{0.625}} \cdot p^{0.75} T^{0.625} V^{2.75} d^{1.75} \quad (6)$$

and

$$v = 0.0183 \rho_o^{-0.5} \gamma^{0.5} T^{0.5} V^{0.5} \quad (3)$$

We see that though there is still an advantage in having

* Other gases, such as chloride of methyl (CH_3Cl) and Xenon would give better results but could not be practically employed. Thus μ^3/ρ_o^2 is equal to $571/10^{18}$ for water, to $15.6/10^{18}$ for CH_3Cl and to $33/10^{18}$ for Xe. It is evident that in the formulas we must always assume for water $p = 1$ kg/cm² and $T = 273^o$.

low values of μ, γ and T and a high value of ρ_o, there is, on the other hand, an advantage in REDUCING pressure.

Thus for T = 253°, $\left(\frac{p}{p_o}\right)^{0.75} \left(\frac{T}{T_o}\right)^{0.625} = 0.572$ for p = 0.5 kg/cm² and is equal to 0.164 for p = 0.1 kg/cm ; $\frac{\mu^{0.25}\,\gamma^{1.375}}{\rho_o^{\,0.625}}$ is equal to 0.208 for air and to 0.132 for carbonic acid; $\rho_o^{-0.5}\ \gamma^{0.5}$ is equal to 3.32 for air and to 2.54 for CO_2*.

WE SEE THAT THE USE OF CARBONIC ACID AT 273° AND 1 kg/cm² REDUCED THE POWER REQUIRED BY 40% AND THAT IF THIS GAS IS EXPANDED TO 0.5 kg/cm² AND COOLED TO 253°, THE POWER REQUIRED IS REDUCED BY 90%. THE VELOCITY IN THE FLUE IS IN THIS CASE EQUAL TO 0.74 OF THE SPEED OF THE FULL SCALE AIRPLANE.

4. - 3rd CASE - REYNOLDS' NUMBER N AND THE SPEED V ARE GIVEN.

If it is a question of establishing a new laboratory for well determined values of N and V, we shall employ the following formula obtained by eliminating d and v from equations (1(, (2), and (3):

$$P_m = 11.3\ \frac{\mu^2 \cdot \gamma^{0.5}}{\rho_o^{\,3/2}}\ \frac{T^{3/2}}{P}\ V \cdot N^{1.75} \qquad (7)$$

and the diameter will be given by the formula:

$$d = 3340\ \frac{\mu}{\rho_o^{\,0.5}\,\gamma^{\,0.5}} \cdot \frac{T^{0.5}}{p} \cdot \frac{N}{V} \qquad (8)$$

established by eliminating v from equations (2) and (3).

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* $\frac{\mu^{0.25}\,\gamma^{1.375}}{\rho_o^{\,0.625}}$ is equal to 1780 for water, to 0.0967 for CH_3Cl and to 0.0557 for Xe; $\rho_o^{-0.5}\gamma^{0.5}$ = 14.3 for water, 2.18 for CH_3Cl and to 1.72 for Xe.

Formula (7) shows that there is an advantage in reducing temperature and increasing pressure.

For computing POWER we note that the values of $\frac{\mu^2 \gamma^{0.5}}{\rho_0^{3/2}}$ are $\frac{74.5}{10^{12}}$ for air and $\frac{25.6}{10^{12}}$ for CO_2*.

For $T = 253^\circ$ and $p = 15$ kg/cm², $\left(\frac{T}{T_0}\right)^{3/2} \cdot \frac{p_0}{p} = 0.06$.

For computing DIAMETER we note that:

$\frac{\mu}{\rho_0^{0.5} \gamma_0^{0.5}} = \frac{4.01}{10^6}$ for air and $\frac{2.82}{10^6}$ for CO_2* and that for $T = 253^\circ$ and $p = 15$ kg/cm², $\left(\frac{T}{T_0}\right)^{0.5} \cdot \frac{p_0}{p} = 0.0642$.

In a laboratory constructed, when d is determined, we have:

$$P_m = 0.00339 \frac{\mu\gamma}{\rho_0} \cdot T \cdot d \cdot V^2 N^{0.75} \qquad (9)$$

established by dividing formula (7) by formula (8), thus eliminating p.

The value of p will be determined by formula (8), which may be written:

$$\bar{p} = 3340 \frac{\mu}{\rho_0^{0.5} \gamma^{0.5}} \cdot T^{0.5} \cdot \frac{N}{V d} \qquad (8')$$

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* $\frac{\mu^2 \gamma^{0.5}}{\rho_0^{3/2}} = \frac{4600}{10^{12}}$ for water, $\frac{9.34}{10^{12}}$ for CH_3Cl and $\frac{14.6}{10^{12}}$ for Xe.

$\frac{\mu}{\rho_0^{0.5} \gamma_0^{0.5}} = \frac{0.124}{10^6}$ for water, $\frac{1.8}{10^6}$ for CH_3Cl and $\frac{2.26}{10^6}$ for Xe.

$\frac{\mu \cdot \gamma}{\rho_0} = \nu_0 \gamma = \frac{37,100}{10^6}$ for water, $\frac{4.75}{10^6}$ for CH_3Cl and $\frac{6.48}{10^6}$ for Xe.

For determining the POWER we note that:

$\frac{\mu\gamma}{\rho_o} = \frac{18.6}{10^6}$ for air and $\frac{9.09}{10^6}$ for CO_2*; the values of $\frac{\mu}{\rho_o^{0.5}\gamma^{0.5}}$ are given above.

WE THUS SEE THAT IF WE WISH TO ESTABLISH A LABORATORY FOR WELL DETERMINED VALUES OF V AND N THE USE OF CARBONIC ACID AT 15 kg/cm² AND 253° REDUCES THE POWER REQUIRED IN THE RATIO OF 48.5 TO 1 AND REDUCES THE DIAMETER IN THE RATIO OF 22 TO 1.

IN A LABORATORY ALREADY BUILT THE USE OF CARBONIC ACID AT 253° REDUCES THE POWER REQUIRED IN THE RATIO OF 2.2 TO 1; THE PRESSURE SHOULD THEN BE EQUAL TO 68/100 OF THE PRESSURE REQUIRED WITH AIR*.

We would point out that the 3rd case rarely occurs in practice; generally we have only to meet the conditions of the first or second case. We wished to treat the 3rd case, however, in order to show the wide field our method opens for aerodynamical researches on the influence, separate or combined, of viscosity and compressibility.

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* See footnote, p.10.

* We know that in phenomena into which viscosity enters, and especially in the movement of fluids in ducts, the power drop per unit of length is represented by a formula of the following form:

$$\Delta = a\, v^{2-n} d^{-1-n}\, \mu^n\, \rho_o^{1-n}\, p^{1-n}\, T^{n-1}$$

It follows that the terms of formulas 4, 7, and 9, characterizing the fluid and the conditions of temperature and pressure, remain the same, whatever be the value of n (according to Prandtl, n = 0.25); only the numerical coefficient and the exponent of N vary. It is the same for all the terms of formulas 3, 5, and 8. In other words, the ratios of the power established in cases 1 and 3, and the ratios of the speeds and pressures established in cases 1, 2, and 3, constitute values independent of the assumed value of n.

REMARK I. - When, in the formulas giving the value of the power in the three cases considered, we compare the values of the terms characterizing the nature of the fluid, we find that water forms the least advantageous fluid for use in a laboratory, the more so as, being incompressible, its density cannot be varied.

We may remark, however, that heating water to 100° reduces its coefficient of viscosity in the ratio of 6 to 1; in the 1st case it then becomes more advantageous than air at atmospheric temperature and pressure. On this subject we may say that we may consider the use in wind tunnels not only of gas (that is, of fluids for which the temperature of saturation at 1 kg/cm² is below 273° absolute) but also of vapors, which must be heated so that their temperature is above the temperature of saturation at the pressure at which they are utilized. Thus we may consider using water vapor, although its characteristics ($\mu_0 = 0.89 \times 10^6$, $\rho_0 = 0.079$) are not favorable.

REMARK II. - In our theory of wind tunnels, we have called the coefficient of utilization of a tunnel (ρ_s), the ratio of the kinetic energy of the fluid stream in the working section to the power of the engine running the fan.

For the closed circuit tunnels which we are studying, this coefficient takes the very simple form:

$$\rho_s = \frac{0.393\ \rho\ d^2\ v^3}{7\ \rho\ d^2\ v^3\ N_s^{-0.25}} = 0.0561\ N_s^{0.25}$$

$N_s = vd/\nu = N/0.6.$

5. - PRELIMINARY PROJECT OF A WIND TUNNEL.

As a practical application let us consider a closed circuit tunnel, 2 m. in diameter, utilizing carbonic acid.

For tests of MODEL AIRPLANES we use carbonic acid compressed to 15 kg/cm² and cooled to 253° and we fix a speed of 30 m/sec. (corresponding, according to formula (3) to a speed in the air of 41 m/sec.).

The power required given by formula (1) will be 300 HP and the Reynolds' number realized will be 81.10^6 and equal to that realized by the largest existing airplanes (N is greater for large airplanes going slowly than for small planes flying at a high speed). This Reynolds' number will correspond to that of a racing plane flying at 650 km/hr.

The large laboratories now being planned will realize Reynolds' numbers 8 times smaller with powers about 4 times greater. If in one of these laboratories having a diameter of 3 m., we wished to attain $N = 81.10^6$, we should require a power of 300 x 1088 x 2/3 = 217,000 HP.*

For high speed tests, and especially for PROPELLER tests, the pressure must be below 1 kg/cm². Thus formula (1) shows that with carbonic acid at 0.5 kg/cm² and 253°, the speed realized with the same power of 300 h.p. would be:

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* This figure must be considered rather as a proof of the impossibility of realizing this Reynolds' number in an ordinary tunnel than as an exact value of the required power. As a matter of fact the speed in this case should reach 900 m/sec. and we have not the right to apply our formulas to such speeds, for which, moreover, the phenomena of compressibility would completely distort the results.

$$30 \times \left(\frac{15}{0.5}\right)^{0.75/2.75} = 30 \times 2.53 = 76 \text{ m/sec.}$$

equivalent to a speed in the air of 76/0.74 = 103 m/sec., that is 370 km/hr.

If the pressure is reduced to 0.1 kg/cm², the equivalent speed in the air would be 570 km/hr.. with an economy in power of 90%; to attain such a speed an ordinary tunnel of 3 m. would thus require $300 \times \left(\frac{3}{2}\right)^{1.75} \times 10 = 6000$ h.p.

Lastly, we would say a few words on the realization of this wind tunnel.

There should be a closed circuit flue with continuous wall formed of thick sheet metal and protected from over-heating. Two doors sliding perpendicularly to the axis of the flue will isolate the working section while the model is being handled. The measuring devices will be placed in an airtight cabin on the wall of the working section, so that the rods of the model supports can traverse the wall of the flue by joints which should not be airtight. The measurements will be registered automatically by apparatus installed either in the cabin and visible from outside, or actually installed outside the cabin. In the latter case the apparatus will consist of manometers connected with dynamometric capsules placed in the cabin. The propeller-fan will have adjustable blades so that it can be adapted to the density of the fluid used in the tunnel.

6. - GRAPHICAL REPRESENTATION OF THE FUNCTIONING OF A WIND TUNNEL OF OUR SYSTEM.

We will define the functioning of a tunnel by the correlative values of N and V which can be obtained in it.

For a tunnel of our system these values are given by equation (9), from which we deduce:

$$V^2 N^{0.75} = \frac{P_m}{0.00339 \; v_o \gamma \; . \; T \; . \; d} \qquad (9')$$

For the tunnel 2 m. in diameter, studied above, utilizing CO_2 at 253° we have:

$$V^2 N^{0.75} = 64500 \; . \; P_m \qquad (9'')$$

If in a system of rectangular axes (see Fig. 1) we lay off the values of V in abscissa and the values of N in ordinates, the functioning of the tunnel will be figured by a sheaf of iso-P_m lines.

The scales being logarithmic, this sheaf will be composed of parallel lines.

To each group of values of V and N will correspond a value of p, determined by formula (8'):

$$p = 3340 \; \frac{\mu}{\rho_o^{0.5} \; \gamma^{0.5}} \; . \; T^{0.5} \frac{N}{V \; d} \qquad (8')$$

or, for the tunnel under examination:

$$p = 74900 \; \frac{N/10^6}{V} \qquad (8'')$$

The values of p will thus be represented on Fig. 1 by a sheaf of iso-p lines plotted by formula (8").

If we take the maximum value of p to be 15 kg/cm and the minimum value 0.1 kg/cm², the field of realizable values of N and V will be limited:

1st. By the iso-P_m line corresponding to the maximum power of the engine of the tunnel; we have indicated the iso-P_m = 300 HP by a heavy line.

2nd. By the iso-p lines corresponding respectively to 0.1 and 15 kg/cm², also marked in heavy lines.

ANY GROUP OF VALUES OF V AND N FOUND WITHIN THESE LIMITS MAY BE REALIZED BY A TUNNEL OF OUR SYSTEM.

The value of N will be determined by multiplying the Reynolds' number of the full scale machine in flight by 0.6/1, 1 being the quotient of the dimension used in determining the Reynolds' number, and the diameter of the flue.

EXAMPLE. - Test of a 1/10 model of a strut 30 mm. in diameter, of an airplane whose speed is 50 m/sec. We have:

$$N_m = \frac{0.03 \times 50 \times 10^6}{13.3} \times \frac{0.6}{0.003/2} = 0.113 \times 10^6 \times 400 = 45.10^6$$

We see on Fig. 1 that the point $N = 45.10^6$, $V = 50$ m/sec. may be realized with 290 horsepower and a pressure of 7.2 kg/cm².

7. - COMPARISON OF AN ORDINARY TYPE TUNNEL WITH A TUNNEL OF OUR SYSTEM.

For an ordinary wind tunnel the Reynolds' number N is proportional to the speed V. The functioning of ordinary tunnels will thus be represented by a line parallel to the iso-p lines; the extremity of this line will correspond to the speed determined by the maximum power of the engine of the tunnel.

On Fig. 1 we have drawn the lines corresponding to the following tunnels: Eiffel (E_1 and E_2); St.Cyr (C_1 and C_2); Nat-

ional Physical Laboratory (N_1 ... N_5); Rome (R); Göttingen (G).

The graph shows in a very suggestive way the superiority of our system of tunnel; as a matter of fact, in our system there is much greater scope for variations of N and V (a surface instead of a line) and the power absorbed is much more advantageously utilized.

8. - COMPARISON OF THE VALUES OF REYNOLDS' NUMBER AND OF THE SPEED OF AIRPLANES, AIRSHIPS, AND PROPELLERS WITH THE VALUES REALIZED IN OUR TUNNEL.

Let l_a, l_d, l_H and l_h be the ratios of the dimension of the model used in determining Reynolds' number, to the diameter (d) of the tunnel.

We will assume:

l_a = span of the model airplane/d = 0.6

l_d = diameter of model airship/d = 0.15

l_H = diameter of model propeller tested alone/d = 0.5

l_h = diameter of model propeller tested on model airplane/d; this value varies according to type of machine.

As in all our computations we have assumed: $N = 0.6 \frac{vd}{\nu}$, the values of Reynolds' number must in each case be multiplied by 0.6/1; thus for airplanes $0.6/l_a = 1$; for airships $0.6/l_d = 4$; for propellers tested alone $0.6/l_H = 1.2$ and for propellers tested on a model airplane and on its scale $0.6/l_h$ = span of plane/diameter of propeller, so that $N_h = N_a$, which, moreover, was evident <u>a priori</u>.

The following Table gives the values of N_a, N_d, N_H and N_h for different existing machines, the value of ν being $\frac{13.3}{10^6}$ m^2/sec.

AIRPLANES	Span (m)	Speed (m/sec)	$N.10^6$	$N.10^6 \times 0.6/l_a$
Racing Plane (A_1)	6	90	40.6	40.6
Pursuit Plane (A_2)	9	70	47.4	47.4
Medium size transport plane (A_3)	14	55	58	58
Large transport plane (A_4)	25	43	81	81
Giant Airplane (A_5)	42	35	110	110

AIRSHIPS	Diameter			$N.10^6 \times 0.6/l_d$
Small flexible airship (D_1)	10	20	15	60
Large flexible airship (D_2)	15	25	28.2	113
Transport Zeppelin (D_3)	18.7	36.8	51.7	207
Military Zeppelin (D_4)	23.9	36.4	65.4	261

PROPELLERS	Diam. (m)	Speed (m/sec)	$N.10^6$	$N.10^6 . \frac{0.6}{l_H}$	$N.10^6 . \frac{0.6}{l_h}$
Racing Plane (H_1)	2.4	90	16.2	19.5	40.6
Pursuit Plane (H_2)	2.7	70	14.2	17.1	47.4
Medium Size Plane (H_3)	3	55	12.4	14.9	58
Large Plane (H_4)	3	43	9.7	11.6	81
Giant Plane (H_5)	5	35	13.2	15.8	110

IN EXAMINING THE POSITIONS OF THE POINTS REPRESENTING THE VALUES OF N, V FOR ANY MACHINE WHATEVER, IT SHOULD ALWAYS BE REMEMBERED THAT IT IS OF LITTLE IMPORTANCE IF THE MODEL OF A LOW SPEED MACHINE DOES NOT ATTAIN THE SAME SPEED IN THE TUNNEL AS IN FLIGHT AND ALSO THAT IT DOES NOT MUCH MATTER IF A HIGH SPEED MACHINE DOES NOT ATTAIN ITS REYNOLDS' NUMBER IN THE TUNNEL.

This being understood, we see that with 300 horsepower we realize for large airplanes the conditions of Reynolds' number and speed and that if the speed conditions are not realized for small rapid machines, that is of no importance so long as the Reynolds' number is attained.

For large airships only the half of N is reached, but, considering its very high value, this should not lead to error.

Finally, for propellers tested on the model and on the same scale as the model, we realize, as for large airplanes, the conditions of similitude imposed by the considerations of viscosity and compressibility.

For small high speed planes we shall adopt either mean values (for instance, for a pursuit plane: 60 m/sec., 4 kg/cm^2 and $N = 30.10^6$ instead of 47.10^6) of V and N, or extreme values of V and N according to whether we are studying the functioning of the propeller or of the airplane.

REMARK. - In the application just given of our system to a tunnel of 2 m. in diameter, we assumed, in order to deal with a general case, that the carbonic acid was cooled to -20° C.

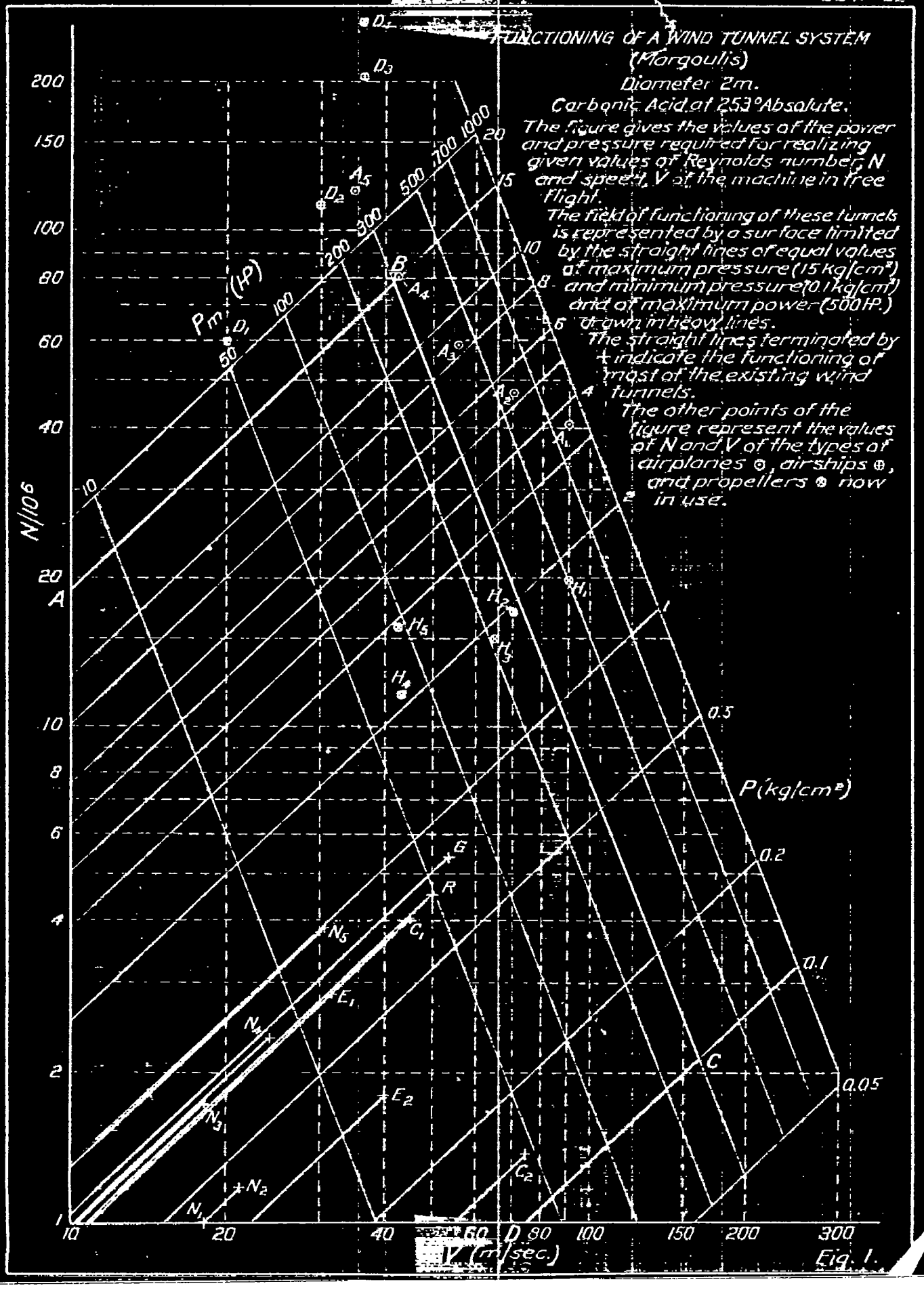
Practically, this cooling leads to complications in installation and functioning which the resulting small gain of power

does not justify (see formulas (4), (6), and (7).

We therefore consider it preferable to work at the temperature of the surrounding atmosphere. If we assume a mean temperature of +10° C., formulas (9) and (8') show that for the same values of V and N the power is increased by 11% and the pressure by 6%, with respect to those corresponding to a temperature of -20°.

In Fig.1 the graduations of the lines of equal values of P_m must thus be increased by 11% and the graduations of the lines of equal values of p by 6%.

IT FOLLOWS THAT THE LIMITS OF FUNCTIONING OF OUR WIND TUNNEL WILL BE THE SAME AS THOSE GIVEN ABOVE, ON CONDITION OF EMPLOYING 335 HP AT PRESSURES VARYING BETWEEN 0.106 AND 15.9 kg/cm².


FUNCTIONING OF A WIND TUNNEL SYSTEM
(Margoulis)
Diameter 2m.
Carbonic Acid at 253° Absolute.
The figure gives the values of the power and pressure required for realizing given values of Reynolds number, N and speed, V of the machine in free flight.
The field of functioning of these tunnels is represented by a surface limited by the straight lines of equal values of maximum pressure (15 kg/cm²) and minimum pressure (0.1 kg/cm²) and of maximum power (500 HP.) drawn in heavy lines.
The straight lines terminated by + indicate the functioning of most of the existing wind tunnels.
The other points of the figure represent the values of N and V of the types of airplanes ⊙, airships ⊕, and propellers ⊗ now in use.
N/10⁶
Pm (HP)
P (kg/cm²)
V (m/sec.)
A
D
Fig. 1.