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NOTES ON AERODYNAMIC FORCES - I.

Rectilinear Motion.

By Max M. Munk.

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Summary.

The following note contains a number of interesting theoretical relations concerning air forces and a new and easy method of proving them.

1. Introduction.

The study of the motion of perfect fluids is of paramount importance for the understanding of the chief phenomena occurring in the air surrounding aircraft, and for the numerical determination of their effects. In connection with the computation of the air forces of airship hulls I recently employed successfully some simple methods for the investigation of the flow of a perfect fluid, new for the greatest part and at least never mentioned in connection with aerodynamical problems. These methods appeal particularly to the engineer who is untrained in the performing of laborious mathematical developments and computations, as they do away with these and allow one to obtain many interesting results

by the mere application of some general and well-known principles of mechanics.

In this note I apply these methods on problems of rectilinear motions. The fluid is always supposed to be perfect and the flow irrotational.

It is often mentioned that the forces between a fluid and a rigid body moving rectilinearly in it are the same whether the fluid flows around the resting body, or the body moves in the fluid otherwise at rest. This holds true whether the rectilinear motion is steady or accelerated. In consequence of this, the kinetic energy contained in the resting fluid added to the kinetic energy of the fluid flowing around the body gives the kinetic energy of the entire fluid moving with constant velocity. The resting body decreases the velocity of the fluid in front of it and behind it, thus diminishing the kinetic energy of the fluid by a certain amount and this negative energy is equal and opposite to the kinetic energy contained in the otherwise resting fluid surrounding the moving body. The energy of the moving fluid is further decreased for another reason: The fluid contained in the space occupied by the body is removed and hence the kinetic energy of this fluid moving with the velocity of motion is therefore missing. This energy however is contained in the system if the body has the same density as the fluid and if not only the kinetic energy of the fluid alone but also the complete kinetic energy of the system is considered. The kinetic energy of the fluid alone may be called additional kinetic energy.

The forces between the body and the fluid and hence the kinetic energy are proportional to the square of the velocity. It is for this reason that the effect of the surrounding fluid is the same as if an additional inertia has been imparted to the body. The sum of the additional inertia and the inertia of the body itself, provided its density be equal to that of the fluid, may be called the complete inertia. All methods of the ordinary mechanics of rigid bodies can be applied to the body with the increased inertia, and in particular use can be made of the momentum imparted to the body by the forces acting during the creation of a motion from rest.

There is however one important and interesting difference between the body immersed in a perfect fluid and an ordinary body. The latter has the same inertia in all directions. Hence the momentum has always the same direction as the velocity and is proportional to it. On the other hand, the immersed body has different inertias in different directions, and the direction of the momentum and the velocity does not agree in general. For if the body is unsymmetrical and moves in different directions the flow around it is quite different and there is no reason why the additional inertia or the corresponding momentum should be the same in all directions. This gives the explanation for many hydrodynamic phenomena and relations.

It can be seen in the textbooks (Lamb, p. 159), that there are always existing three directions of the body mutually at right

angles to each other, where the body experiences no moment. The direction of the momentum agrees then with the direction of the motion. It is easy to determine the momentum for any direction of the velocity. The velocity is to be divided into its three components in these three main directions, the momentum for each of them is determined, being proportional to the velocity component, and the three momenta thus obtained are the components of the entire momentum.

Before making use of the momentum and the kinetic energy of the system, I wish to make some remarks on the determination of the kinetic energy or the apparent mass in special cases. The entire kinetic energy is of course the integral over the product of half the masses of all the particles of the fluid and the square of their velocity. This space integral can be reduced first to an integral over the surface of the body by integrating along each tube bounded by stream lines. The kinetic energy appears then to be the integral over all products of the masses of fluid passing the surface per unit of time and their velocity potential. This refers to the case of the body moving in the otherwise resting fluid, and the passing fluid is determined by supposing the body to rest also but nevertheless the fluid to move as if the body were moving. This surface integral is sometimes quite convenient, if the potential is known. This is the usual method of computation. If, however, we know the fictitious sources and

vortices inside the body, which produce the outside flow, the computation can be considerably simplified. Instead of considering the flow of the moving body and hence the integral over the surface of the body, which is indeed the only one intersected by all stream lines, it is then more advisable to consider the flow of the moving fluid around the resting body and to consider the integral over a sphere of infinite radius, with the body near the center. Then only the second harmonic attributes to the integral and it appears:

The complete mass is the sum of all intensities of the fictitious sources each multiplied by the potential of the parallel flow of motion, and of all closed curves of fictitious vortices the intensity of each multiplied by the fluid passing any connecting diaphragm per unit of time, in virtue of the parallel flow.

This theorem can be impressed on the mind by means of the following conceptions. Each source and vortex experiences at each time a force as described in N.A.C.A. Report No. 114, Part III. That is, the force on the source is the product of the produced mass in unit of time and the velocity of the fluid in that point, acting in the direction of the velocity if the source is negative and in opposite direction if the source is positive. The vortices experience forces at right angles to the velocities in accordance with the theory of lift. Now all sources and vortices can be supposed to be first moved together into one point, producing no effect at all. Then they gradually move to their places, and the work required in order to move them is exactly

equal to the diminution of the complete kinetic energy.

No use is made of the fact that the original outside flow is rectilinear. The theorem indeed holds true under much more general conditions, and in particular also for a plurality of bodies. The theorem gives the complete kinetic energy of the last body added, considering the flow produced by the other bodies as original outside flow. The complete kinetic energy of several of the bodies is calculated in the same way with respect to the potential of the flow of the remaining bodies and the outside flow. With respect to their mutual increase of the kinetic energy, however, the factor $\frac{1}{2}$ is to be introduced, as otherwise each of the bodies would have been taken into account twice. This is proven in the same way as before.

The relations are in exact analogy to those in the theory of gravity, and indeed the forces between the sources are the same as between attracting masses. For the actual application, however, the value of these relations is sometimes diminished by the difficulties to find the sources, for instead of these the shape of the body is usually given.

2. Problems in Two Dimensions.

For the demonstration of the methods I begin with the most simple case, that is the straight line surrounded by a two-dimensional flow. If the line is parallel to the motion, the complete inertia, identical in this case with the additional inertia, is zero, and so is the kinetic energy and the momentum. Moving at

right angles to its direction, the complete mass equals that of the circle over the line, as shown in Lamb's "Hydrodynamics," Fourth edition, page 81. For any angle of attack the moment component in the direction of motion is $V \rho L^2 \frac{\pi}{4} \sin 2\alpha$ and $V \frac{\rho}{2} L^2 \frac{\pi}{4} \sin 2\alpha$ at right angles to it, as results from the consideration that the transverse flow alone has a momentum at right angles to the line.

Suppose the line to be resting and the fluid to be moving. The position at right angles to the motion is that of smallest energy, hence it is stable, for it cannot be conceived how any other position can be reached without external supply of energy. This consideration gives the direction of the moment acting on the line. Its magnitude can either be obtained by the consideration of the momentum or of the energy. If the line moves, the momentum $V \frac{\rho}{2} L^2 \frac{\pi}{4} \sin 2\alpha$ moves by the distance V per unit of time, hence requiring the moment $V^2 \frac{\rho}{2} L^2 \frac{\pi}{4} \sin 2\alpha$. Or, if the resting line revolves slowly by the angle $d\alpha$, the increase of the work $V^2 \frac{\rho}{2} L^2 \frac{\pi}{4} \sin 2\alpha$ is $V^2 \frac{\rho}{2} L^2 \frac{\pi}{4} \sin 2\alpha \, d\alpha$ giving the same moment as before.

I proceed now to two-dimensional sections which are almost straight lines, I mean symmetrical and thin ones. If they are first infinitely thin and obtain their shape by gradually growing, the pressure of the longitudinal flow along the surface is at first constant and to be taken as $V^2 \frac{\rho}{2}$ according to the preceding for the longitudinal flow, as the velocity is constant and equal to V along the surface. Hence the work done during the process of

creating the shape is equal to the area of the shape multiplied by $V^2 \frac{\rho}{2}$. That means the complete longitudinal mass is equal to the product of the area and the density of the fluid, and the additional mass is zero of higher magnitude than the area of the thin section. The pressure of the transverse flow is $V^2 \frac{\rho}{2} \times \frac{x^2}{x^2 - 1}$ for the length 2 of the line, reaching from $x = +1$ to $x = -1$. The thickness of the thin shape may be ξ in each part. Then the work absorbed during the process of increasing the thickness is

$$V^2 \frac{\rho}{2} \int_{-1}^{+1} \frac{\xi x^2 \, dx}{1 - x^2}$$

and the complete transverse inertia is increased by

$$\rho \int_{-1}^{+1} \frac{\xi x^2 \, dx}{1 - x^2}$$

The moment is proportional to the difference of the two complete masses, and it follows therefore that it is increased by

$$\rho \int_{-1}^{+1} \frac{\xi (2x^2 - 1)}{1 - x^2}$$

$$V^2 \frac{\rho}{2} \sin 2\alpha \; L^2 \left\{ \frac{\pi}{4} + \int_{-1}^{+1} \frac{\xi (2x^2 - 1)}{1 - x^2} \, dx \right\}$$

ξ is the thickness and x the coordinate in units of half the length. This shows that the moment is increased if the shape is made thicker near the ends but it is decreased if it is made thicker near the middle. The reason is easily understood.

The straight line may not be made thicker but it may be bent into the shape given by the coordinates ξ. At the angle of at-

tack α and for the length 2 as before, the transverse force is $V^2 \frac{\rho}{2} \sin 2\alpha \frac{x}{\sqrt{1 - x^2}} L^2 \frac{\pi}{4}$. Hence the work done is

$$L^2 \frac{\pi}{4} V^2 \frac{\rho}{2} \sin 2\alpha \int_{-1}^{+1} \frac{\xi \, x \, dx}{\sqrt{1 - x^2}}$$

For small α and ξ this increases the moment by the same amount as if the angle of attack is increased by $d\alpha = \int_{-1}^{+1} \frac{\xi \, x \, dx}{\sqrt{1 - x^2}}$ This result was formerly found by me by means of another method which is given in N. A. C. A. Report No. 142.

If the line becomes at the same time thicker and curved, the change of the moment is the sum of the two single changes.

The results are only valid for very thin sections. Conclusions for somewhat thicker sections can be drawn from the flow around ellipses. From the textbooks it can be seen that the additional inertias of an ellipse with the axes a and b are respectively $a^2 \cdot \rho \frac{\pi}{4}$ and $b^2 \rho \frac{\pi}{4}$ so that the difference depends only on the distance of the foci $\sqrt{a^2 - b^2}$. Hence all confocal ellipses have the same moment, if the angle of attack is the same. Now for elongated ellipses the focus is situated halfway between the end of the great axis and the center of the greatest curvature. It follows from this that for the calculation of the moment of an elongated section the end of the central curve is to choose half way between the end and the center of curvature of the end, as mentioned by me in N. A. C. A. Report No. 142.

The constancy of the moment of confocal ellipses reminds us of the constancy of their apparent additional moments of inertia when rotating in a fluid (Lamb, p.85). The kinetic energy of the external fluid is then given by

$$2\,T = \frac{1}{8}\,w^2\,\pi\,\rho\,\lambda^4$$

where w is the angular velocity and $2c$ the distance of the foci.

3. Problems in Three Dimensions.

I consider first a plane sheet. The inertia in all directions of the plane is zero. It follows therefore that the moment depends on the angle of attack but not on the angle of yaw.

If the plate has a small thickness, the same reasoning as before shows that the additional inertia is zero in all directions parallel to its plane. Hence the moment remains independent of the angle of yaw.

A circular disc has the apparent mass equal to that of the volume $\rho\,\frac{8}{3}\,r^3$ where r denotes the radius (Lamb, p.132). That is only 0.637 of the volume of the sphere with the same radius. Hence the moment is smaller as if each longitudinal element would be surrounded by the two-dimensional flow. This holds true for more elongated plan views too, but the difference is probably very small then.

Of considerable importance are bodies very elongated in one direction only. The axis may be straight. The same reasoning as before shows that the additional mass is zero for the longitudinal

motion. The complete masses in the other directions can be obtained by considering the transverse flow as two-dimensional.

Surfaces of revolution are particularly important. Their transverse complete masses are equal to twice the volume, their longitudinal masses to the volume only. The moment therefore is $V^2 \frac{\rho}{2}$ (Volume) $\sin 2\alpha$.

The influence of a moderate thickness can be studied by the comparison with ellipsoids of revolution. The following table gives the additional masses k_1 and k_2 for different ratios $\frac{\text{Length}}{\text{Diameter}}$ as computed by Lamb (R&M #623). The difference $(k_1 - k_2)$ indicates the fraction of the moment to that of a very elongated surface of revolution with the same volume. For the investigation of moderately elongated surfaces of revolution other than ellipsoids, the coefficient of the corresponding ellipsoid can be taken. If the shape of the surface is reasonable the ratio L/D of the corresponding ellipsoids is approximately

$$\sqrt{\frac{\pi}{6} \frac{L^3}{\text{Vol}}}$$

At last I proceed to the investigation of the distribution of the transverse forces of very elongated surfaces of revolution. I imagine the axis bent in a point by a small angle $d\alpha$. The work absorbed in the joint during the process of bending equals the change of the complete kinetic energy. One portion of the body may be resting, the other one turns around the joint. Its volume may be Vol'. The flow around each section is almost inde-

pendent of the position of the other sections and hence the change is the same as if a surface of revolution of the volume Vol' has increased its angle of attack by $d\alpha$. The bending moment in the joint appears therefore Vol' $V^2 \frac{\rho}{2} \sin 2\alpha$. From this follows that the transverse forces are distributed proportional to the change of section area per unit length of the axis.

TABLE.

Length / Diameter	k_2	k_1	$k_2 - k_1$
1.00	0.5	0.5	0
1.50	.621	.305	.316
2.00	.702	.209	.493
2.51	.763	.156	.607
2.99	.803	.122	.681
3.99	.860	.082	.778
4.99	.895	.059	.836
6.01	.918	.045	.873
6.97	.933	.036	.897
8.01	.945	.029	.916
9.02	.954	.024	.930
9.97	.960	.021	.939
10.00	1.000	0	1.000