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NOTES ON AERODYNAMIC FORCES - II.

Curvilinear Motion.

By Max M. Munk.

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Summary.

The laws of curvilinear motion are established and the transverse forces on elongated airship hulls moving along a curved path are investigated.

General Method.

This note deals with the steady motion of a rigid body on a curvilinear path through a perfect fluid otherwise at rest, so that the position of the body relative to the path remains constant. The body turns with constant angular velocity around a geometrical axis fixed with respect to space and with respect to the body. Hence the flow of the fluid is stationary relative to the body. The problem stands in close relation to that of the body moving straight forward, which indeed is a special case of the present problem. The methods employed with the investigation of the straight motion can partly be used for the more general problem too, but care must be taken that they are applied properly and only as far as they are still valid.

It is to be understood at the outset that the forces between the body and the fluid are now by no means the same whether the real motion is considered, or whether the body is supposed to be at rest and the entire fluid rotating about the center of rotation. This latter case would imply the presence of the centrifugal forces of the rotating fluid, giving rise to changes in the pressure distribution and of the forces between the body and the fluid. Nor is the pressure in each point now determined by the square of the velocity according to the law of Bernouilli, for the motion is not really stationary since the body is moving. In the case of non-stationary motion the pressure is the sum of the pressure due to the Bernoulli $(V^2 \rho/2)$ and the product of the change of the velocity potential per unit of time and the density (Lamb, p.19). The additional pressure can be transformed if, as in the present problem, the flow is quasi-stationary and changes only its position, each point of the configuration of the flow moving with a velocity U. Let ω be the potential, then the change $d\omega/dt$ of the potential per unit of time is $U\, d\omega/ds$ where ds is a linear element in the direction of U. Therefore the pressure additional to $V^2 \rho/2$ equals the product of the density, the velocity of the configuration in the point considered and the component of the flow in the same direction. The pressure is

$$- (V^2 \rho/2 - V U \cos \varphi)$$

where U is the velocity of the configuration of flow and φ is the angle between this velocity and the velocity of flow.

The apparent mass, moment and kinetic energy of the body can be used in the same way as with bodies moving straight. There is now also an apparent moment of inertia to which is corresponding a moment of momentum and a kinetic energy. As, however, the angular velocity is supposed to be constant, this does not give rise to any resultant force or moment between the fluid and the body.

A body moving straight forward in a perfect fluid experiences only a resultant moment. A drag, positive or negative, would not be compatible with the constancy of energy. From the point of view of the energy there is no reason why the body should not experience a lift, and indeed this can be the case in a two-dimensional flow. In a three-dimensional flow, however, a lift can occur only when there are free vortices in the fluid, and this is a flow different from the one under discussion. But the flow around the body rotating around a geometrical axis has constant energy and hence again only such resultant forces are possible, which do not supply or absorb energy. This cannot be a pure moment for the body possesses an angular velocity. It can only be a resultant force passing through the axis of motion. The direction of this force varies greatly for different shapes. The aerodynamic center of the body, or better, its central axis, may be that point, with reference to which the fluid possesses no moment of momentum if the body is moving rectilinearly at right angles to the axis. This center has a certain tangential velocity V, it moves around the center of rotation, along a circle, with the radius R. The resultant force between the body and the fluid possesses in gener-

al a tangential component parallel to V, a radial component at right angles to V and a resultant moment with respect to the aerodynamic center. For symmetrical bodies, the aerodynamic center of course coincides with the center of symmetry. The angle of attack may be measured between the tangential velocity V of the center and a proper axis of the body. The direction of the body which coincides with the tangential direction at the angle of attack zero may be called longitudinal, the direction at right angle to it transverse. Tangential and radial refer to the path, longitudinal and transverse refer to the body.

Let now the additional mass of the body, if moving straight and longitudinal, be $k_1 \rho$ and if moving straight and transverse, $k_2 \rho$. Then when moving with the velocity V, with the angle of attack α, at the same time rotating with any angular velocity zero or greater, the apparent additional momentum is $k_2 V\rho \sin \alpha$ in the transverse direction and $k_1 V\rho \cos \alpha$ in longitudinal direction. Hence the component in tangential direction is $V \rho \ (k_2 \sin^2 \alpha + k_1 \cos^2 \alpha)$ and the radial component is $V \rho/2 \ (k_2 - k_1) \sin 2\alpha$. The former component of the momentum gives rise to the radial force $\frac{V^2\rho}{R} \ (k_2 \sin^2 \alpha + k_1 \cos^2 \alpha)$, the latter is the cause of the tangential force $\frac{V^2}{R} \rho/2 \ (k_2 - k_1) \sin 2 \alpha$. This has to be supposed to act in the center of rotation, or if acting in the center of the moving body, it is accompanied by a moment around this center $V^2\rho/2 \ (k_2 - k_1) \sin 2 \alpha$. This is the same moment as if the body were moving rectilinearly with the velocity V and at the same angle of attack.

The radial force always tends to keep the path straight, it is a centrifugal force moving the body away from the center of rotation. The tangential forces can be a positive or negative drag, according to the angle of attack and to the shape of the body. A straight line, as two-dimensional problem for instance, may rotate under a small angle of attack and the distance between the leading edge and the center of rotation may be greater than the same distance to the rear edge. Then the resulting moment absorbs energy since it is opposite to the angular velocity of the line. Therefore the drag is negative, supplying the energy absorbed by the moment.

2. The Curvilinear Motion of a very Elongated Surface of Revolution.

The case of an airship hull moving along a circular path is the most important application of the results found in the last paragraph. An airplane is always surrounded by vortices and this case is not embraced by the discussion. The air forces acting on an airship hull are best understood by the consideration of a very elongated surface of revolution.

Such body has no longitudinal additional mass and its transverse additional mass is equal to its own mass if consisting of solidified fluid. This is discussed in the first note of this series. The resultant forces are therefore given by the formula at the end of the last paragraph by substituting zero for k_1 and the volume of the body for k_2. The remaining problem is the dis-

cussion of the distribution of the transverse air forces as their knowledge is important for the computation of the bending moment in the body.

This distribution is found by the consideration that each cross section has a transverse additional mass only and its additional momentum is the same as if it would be surrounded by the two-dimensional flow with the same transverse relative velocity. Now the motion of the body can be produced by superposing the motion along the circular path with the axis remaining parallel and the rotation of the body around the center with constant angular velocity. The aerodynamic center coincides with the center of volume.

At first the body may move in a tangential direction. Then the additional momentums are produced by the angular velocity V/R only, and the relative transverse velocity in each point of the axis in the distance x from the center is $V\,x/R$. Let S denote the area of the cross section in each point. Then the transverse momentum for a disk of the body with the length dx is $S\,V\,x/R\,dx$. The longitudinal component of velocity of each point is V. Hence a transvarse moment $S\,V^2\,\rho\,x/R\,dx$ is produced in each point, corresponding to the transverse force

$$\frac{V^2\rho}{R}\left(x\frac{ds}{dx}+S\right)dx$$

This distribution of transverse forces produces neither a resultant force nor a resultant moment. The term $\frac{V^2\rho}{R}\,S\,dx$ is exactly

equal to the centrifugal force of that portion of the hull, if having the same density as the fluid. Its direction is opposite to the centrifugal force. The forces represented by the term $\frac{y^2 \rho}{R} \frac{xds}{dx}$ are in direction of the centrifugal force.

If the body is inclined against the tangential direction, the fluid gives rise also to the unstable moment, produced by transverse forces distributed as with the straight motion. They are proportional to the dS/dx, as discussed in the first note of this series.

The radial force is distributed proportional to the cross section of the body.

3. Moderately Elongated Surfaces of Revolution.

The two additional masses of such bodies can be estimated by the comparison with similar ellipsoids, as discussed in the first note. If the body is not too short it can be supposed, that the bending moments due to the angular velocity are proportional to the transverse apparent mass, and they are to be diminished proportional to it, though in reality the distribution itself changes a little. The bending moments of the resultant moment are to be diminished proportional to $(k_1 - k_2)$ which also is only approximately true, but probably exact enough. There remains the radial force of the fluid. The part due to the angle of attack may be diminished proportional to the transverse apparent mass, but it remains proportional to the cross section of the body. Only the

second part of the radial force, due to the longitudinal additional mass and existing also at the angle of attack zero requires a somewhat fuller discussion.

This centrifugal force is due to the fact that the surface of revolution is not an extremely elongated one. Hence the previous methods which made use of this assumption must necessarily fail for the investigation of the distribution of the transverse forces giving rise to this centrifugal force. The entire force is not very great; with an ellipsoid with the ratio of elongation $L/D = 6$ it is only about $3\frac{1}{2}\%$ of the entire centrifugal force and with the elongation 9, it is only $2\frac{1}{2}\%$. However, the bending moments created by the corresponding transverse forces are comparatively greater, though they too are not exceedingly great. These bending moments are favorable for diminishing the resulting bending moments. The transverse forces change the sign along the axis so that the resultant centrifugal force is smaller than the sum of all positive forces and the bending moments are greater than expected at first. The reason is that these forces constitute the centrifugal force of the fluid accompanying the body on its longitudinal motion. Now the fluid near the front and the end has the same direction of velocity as the body. Along the sides, however, the fluid has a velocity opposite to the direction of motion. In consequence of this their centrifugal forces is negative, they tend to push the hull towards the center of rotation, whereas the transverse forces near the ends tend to push the hull away from the center of rotation.

As it is seen thus that the centrifugal force is an effect of a difference, the bending moments cannot safely be estimated beforehand without a closer examination in one case at least. This can be done by choosing a surface of revolution corresponding to one source and one sink. If the diameter is D, the intensity of the sources is approximately $V D^2 \pi/4$. The length may be denoted by L, then the velocity near the middle is about $\frac{VD^2}{2L^2}$ The consideration of a flow along a moderately elongated surface of revolution shows now that the velocity along the greatest part of the length does not greatly change, it is always negative and not very different from the velocity near the middle. The transverse component of the velocity can be neglected. The longitudinal velocity of rotation increases by V/R per unit of transverse length. The transverse gradient of the pressure is therefore

$$\frac{V^2 D^2 \rho}{2\ R\ L^2}$$

Almost the entire volume of the body is surrounded by this pressure gradient. Hence the sum of all negative centrifugal forces is about

$$\frac{\mathrm{Vol}\ V^2 D^2\ \rho}{2\ R\ L^2}$$

that is, the centrifugal force of the entire body (with the density of the fluid) times $\frac{1}{2}(D/L)^2$. For $L/D = 6$, for instance, this gives $1/72 = .14$ of the centrifugal force. The sum of all hydrodynamical centrifugal forces has the opposite sign and is about

twice as great. It follows then that the hydrodynamical centrifugal force consists of a negative portion distributed along the length and a positive portion about three times as great, acting on the two ends.

As a result of the preceding investigation, it appears that the transverse forces acting on a moderately elongated surface of revolution are of four different kinds: (a) due to angular velocity in connection with tangential velocity; (b) due to the angle of attack; (c) due to the centrifugal force of the longitudinal apparent mass and (d) due to the centrifugal force of the transverse apparent mass if under angle of attack. The magnitudes of these forces are discussed in the preceding paragraph, the forces (a) and (b) are of chief importance.