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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 121.

FURTHER INFORMATION ON THE LAWS OF FLUID RESISTANCE.

By C. Wieselsberger.

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FURTHER INFORMATION ON THE LAWS OF FLUID RESISTANCE.*

By. C. Wieselsberger.

1. In a former article (Physikalische Zeitschrift, 1921, Vol. 22, pp. 321-8 - For translation, see N.A.C.A. Technical Note No. 84), I described experiments connected with the dependence of the drag coefficient c on Reynolds number R.** These experiments were performed with cylinders of different diameters in a uniform stream and in a range of Reynolds numbers of $R = 4.2$ up to $R = 80000$. From these experiments there was found quite a complex relation of the drag coefficient to Reynolds number, which invalidates the quadratic law of drag for cylinders in a stream. Only in one region, between $R = 15000$ and $R = 180000$, is the drag coefficient nearly constant and therefore the pure quadratic law of drag fulfilled. The peculiar phenomena manifested in connection with the cylinder made it seem desirable to continue the experiments and include bodies of other shapes. A report of these experiments is made in the present article. First, the behavior of a cylinder of finite length, in as large a range as possible of Reynolds numbers, was determined,

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** The drag coefficient c is defined by $c = \frac{W}{\frac{\rho V^2}{2} F}$ in which W = drag, ρ = air density, V = velocity, and F = area of body projected on a plane perpendicular to the direction of the wind. Reynolds number is $R = \frac{Vd}{\nu}$, in which $\nu = \frac{\mu}{\rho}$, μ = coefficient of viscosity and d a linear dimension of the body experimented on.

as also the dependence of the drag coefficient on the ratio of th diameter of the cylinder to its length (for a given Reynolds number) and further the experiments were extended to the case of a sphere and of a disk perpendicular to the air stream. Lastly, several experiments, the results of which are likewise given in this article, were performed on the relation of the drag coefficient of rectangular plates, at right angles to the air flow, to the aspect ratio of the plates.

2. It was first determined in what manner the drag coefficient was affected in passing from the infinitely long cylinder to one of finite length, about both ends of which the air flowed. Six cylinders, of 4 to 300 mm. diameter were tested, the length of each cylinder being five times its diameter. The resistance or drag of the smaller cylinders (up to 13 mm diameter) was found according to the pendulum method by measuring the deflection under the influence of the air stream. The resistance of the larger cylinders was determined by means of an ordinary aerodynamic balance. Unfortunately, the experiments could not here be carried down to such small Reynolds numbers, as for an infinitely long cylinder. In the latter case, the steel wire cylinder passed through the air stream from above and served both as the experimental object and as its support, so that secondary resistances of the suspension device did not have to be considered. For small cylinders of finite length, on the contrary, the resistance of the suspension wire was very noticeable. Finally, the resistance of the suspension wire becomes considerably greater

than that of the object of the experiment, since there is a certain limit to the smallness of the suspension wire. The resistance of the cylinder then appears as the difference between two numbers of nearly the same magnitude, so that the accuracy of the experiment finally becomes insufficient. The smallest cylinder to give sufficiently accurate results was 4 mm in diameter and 30 mm long. The air velocity was varied between 1.55 and 35.5 m/sec. The experiments accordingly embraced a field of Reynolds numbers from $R = 400$ to $R = 700000$. The results are shown logarithmically in Fig. 1. The drag coefficient c is introduced as a function of the Reynolds number $R = Vd/\nu$. For the sake of comparison, there is also given the curve for the infinitely long cylinder already published in connection with my former article. The diameters of the cylinders tested are also given.

The course of the curve for the finite cylinder is similar in its main features to the one for the infinitely long cylinder. If we follow the curve from the smallest Reynolds numbers upward, we first find the depression, already known from the infinitely long cylinder, shifted somewhat to the left and, adjoining it, a long reach with a nearly constant drag coefficient. The critical Reynolds number, with the sudden diminution of the specific resistance, occurs at about the same point. The principal difference between the two curves lies in the lower values of the drag coefficients for the finite cylinder. This is evidently connected with the fact that, with a cylinder of finite length, the air can flow around both ends of the cylinder into the vortex region

immediately behind the cylinder, thereby considerably modifying the pressure distribution about the cylinder. This action, which has also been observed with differently shaped bodies, exerts an important influence on the drag. The details of this process, however, require further investigation. After passing the critical Reynolds number, the difference in the drag coefficients seems to diminish, although definite conclusions cannot be made, due to the limited upward range. In any event, however, these experiments demonstrate that the drag coefficient of a cylinder, placed at right angles to the air stream, not only depends in a high degree on the Reynolds number, but is also greatly affected by the ratio of the diameter to the length of the cylinder. It is therefore not possible to derive the drag of a finite cylinder directly from the drag of an infinitely long cylinder.

For the more accurate determination of the dependence of the drag coefficient on the ratio of the diameter to the length of the cylinder (this ratio being here denoted by λ), a special series of experiments was carried out with 8 cylinders. The measurements were made for a Reynolds number of 80000, since in this vicinity the drag coefficient is nearly constant. In Fig. 2, the drag coefficient c is plotted against λ and it is evident that the drag coefficient of the infinitely long cylinder is almost twice as great as that of a cylinder of the ratio $\lambda = 1$. The relation was found to be very similar in the case of rectangular plates of various aspect ratios placed at right angles to the air stream.

3. The next series of experiments was on the law of resistance of spheres. Here also, for the same reason as in the case of the finite cylinder, no such small Reynolds numbers could be found as in the case of the cylinder in a uniform flow. The diameter of the smallest sphere was 8 mm and that of the largest was 282.5 mm. The range of the Reynolds numbers for these diameters is from R = 790 to R = 770000. The results of these experiments are shown in Fig. 3, the diameters of the tested spheres being also given. The four largest were of hollow copper; the two smallest, of solid steel. All the spheres, except the largest one, were tested in the small air stream (diam. 1.2 m); the largest, in the 2.2 m. air stream. It is worth mentioning that, even for the two largest spheres, the law of similitude was very well fulfilled, although the experiments were tried in two different air streams. For the three smallest spheres, the measurements were made by the pendulum method. The critical Reynolds number, which is here shown with especial clearness, lies at R = 230000. The range of constant drag coefficient is shorter here than in the case of the cylinder. Even after passing the critical Reynolds number, the validity of the pure quadratic law is not assured. In creeping motion, i.e. with very small Reynolds numbers, the influence of the forces of inertia decrease more and more in comparison with the forces of friction. In theoretical hydrodynamics, an expression has been found for the resistance experienced by a sphere in a viscous fluid. This law was formulated by Stokes (H. Lamb, Hydrodynamics) and holds good

for Reynolds numbers which are small in comparison with unity. The drag coefficient, defined by us, is expressed according to the law of Stokes in the simple form $c = 24/R$, R being the Reynolds number corresponding to the diameter of the sphere. The range of validity of Stokes' formula was subsequently extended by Oseen, by considering to a certain extent the inertia members of the equation of motion. The resistance of the infinitely long cylinder was expressed by H. Lamb in a similar way, as mentioned in my former article. The drag coefficient according to the law of Oseen, which applies for Reynolds numbers below unity, is

$$c = \frac{24}{R}\left(1 + \frac{3}{16} R\right)$$

In Fig. 3, this law and also that of Stokes are represented by dash lines. The representation in the field of the small Reynolds numbers was further supplemented by experiments, which were performed by H. S. Allen in 1900 ("The Motion of a Sphere in a Viscous Fluid," Phil. Mag. Vol. 50, p. 323). Allen calculated the resistance from the ascending speed of very small air bubbles (0.05 to 0.3 mm in diameter) in a liquid and from the descending speed of amber and steel spheres in water. In the experiments with steel spheres falling in water, the Reynolds numbers were so large that they come within the range investigated by us. The six points of this series lie, however, somewhat below our values. There is moreover an easy transition from our curve to the first two series of Allen and it is seen that the experimentally found curve of the drag coefficients runs through the wedge-

shaped space formed by the curves of Stokes and Oseen. The resistance of the sphere is accordingly known for any Reynolds number below 770000.

4. For determining the resistance of disks at right angles to the direction of flow, a series of experiments was carried out, which covered a Reynolds range between $R = 3620$ and $R = 962000$ (with reference to the diameter of the disk). The ratio of the thickness to the diameter was about 1/100 for all the disks. The edges were sharp, not rounded. The results, which are also contained in Fig. 3 show that in the range investigated the coefficient of drag has a fairly constant value of $c = 1.1$. For a very slow motion, the drag of the disk was also calculated. Like the drag of a sphere, it is a special case of the drag of an ellipsoid (H. Lamb, Hydrodynamics). For the drag coefficient of a disk, we obtain

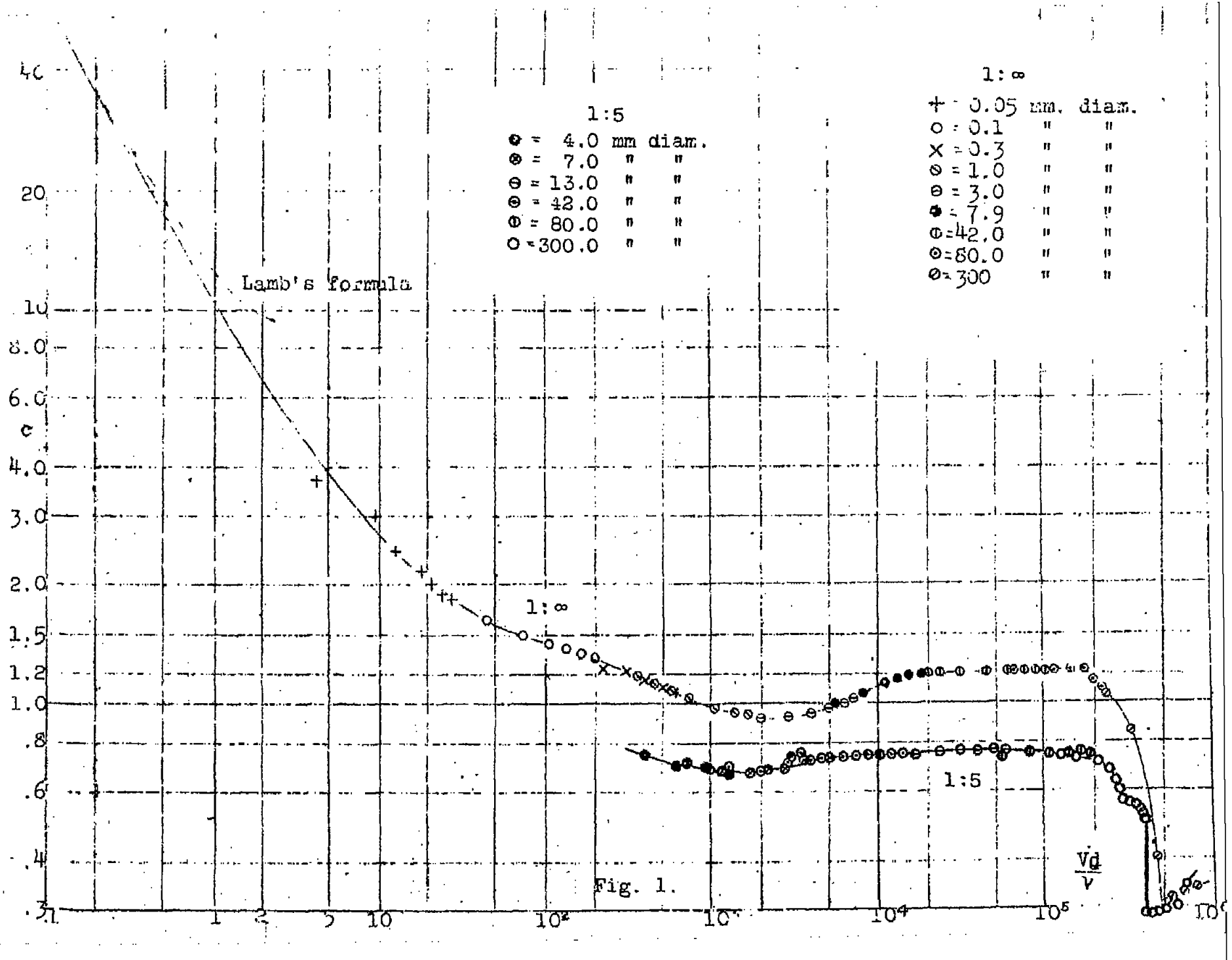
$$c = 17.4/R.$$

This law is represented by a dash line in Fig. 3. Between the experimental values and the theoretical, there is here a region in which the path of the curve has not yet been determined. The investigation of this region would necessitate an improved method of experimenting. With the apparatus here employed, it was not possible to measure accurately enough the drag for Reynolds numbers below 3600. Here there can be no critical Reynolds number, as in the cases of the cylinder and sphere, because the separation point, which is on the edge of the disk, cannot shift. We may therefore, in this case, calculate up to large Reynolds numbers

with a constant coefficient of drag. According to previous experience, this holds generally good for bodies in which the separation proceeds from a sharp edge, e.g. as for a prism.

5. For plates of any shape, in accordance with the preceding, the validity of the quadratic law of drag may be assumed. Here, however, the shape of the plate exerts an influence on the value of the drag coefficient only when the perimeter is greatly extended in one direction. This was clearly demonstrated by a series of experiments performed with rectangular plates of various aspect ratios (λ) varying between 1 and 0. The experiment with $\lambda = 0$ (infinitely long plate) was performed between smooth walls, in the same manner as the experiments with infinitely long cylinders (See former article in this publication, 1921, p. 323). The coefficient of drag was determined at four different speeds (10, 15, 20, and 25 m/sec) and their mean value taken. In Fig. 4, the drag coefficients are plotted against the aspect ratios of the plates. The drag coefficient varies between 1.1 (square plate) and 2 (infinitely long plate). Here also occurs the phenomenon, already observed in the case of the cylinder, that with infinitely long plates, hence with uniplanar flow, the coefficient of drag attains considerably higher values than is the case when the flow is in three dimensions, when owing to the sidewise flow the drag is diminished. This difference is of similar magnitude to that in the case of cylinders of different lengths, the drag coefficient of the infinitely long plate being almost twice as large as that of a square plate.

Translated by the National Advisory Committee for Aeronautics.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= 4.0 mm diam.
= 7.0 " "
= 13.0 " "
= 42.0 " "
= 80.0 " "
= 300.0 " "
1:∞
+ = 0.05 mm. diam.
o = 0.1 " "
x = 0.3 " "
= 1.0 " "
= 3.0 " "
= 7.9 " "
= 42.0 " "
= 80.0 " "
= 300 " "
Lamb's formula
1:∞
1:5
c
Vd/ν
40
20
10
8.0
6.0
4.0
3.0
2.0
1.5
1.2
1.0
.8
.6
.4
.3
.1
1
2
5
10
10²
10³
10⁴
10⁵

Fig. 1.

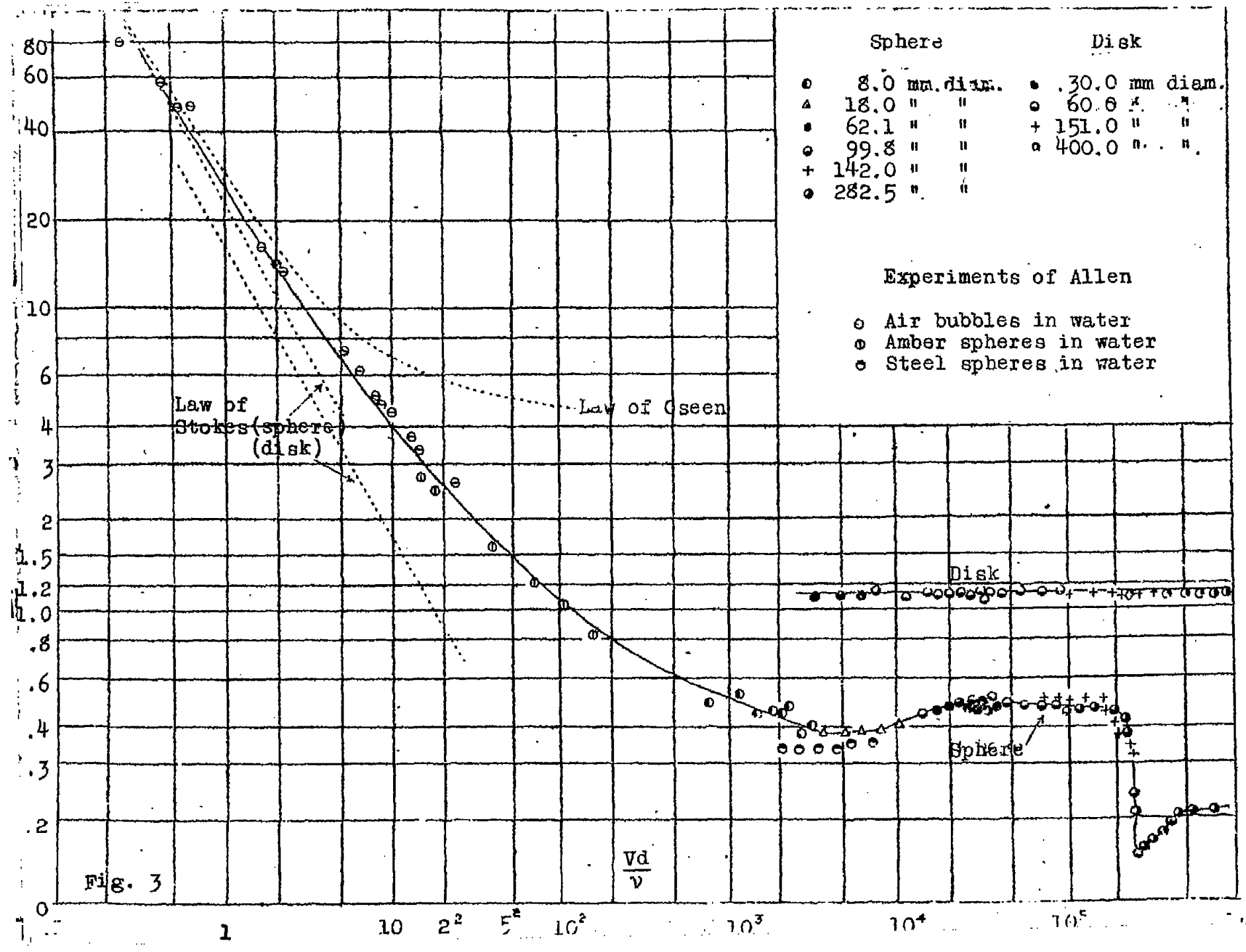
Sphere
8.0 mm. diam.
18.0 " "
62.1 " "
99.8 " "
142.0 " "
282.5 " "
Disk
30.0 mm diam.
60.8 " "
151.0 " "
400.0 " "
Experiments of Allen
Air bubbles in water
Amber spheres in water
Steel spheres in water
Law of Stokes (sphere) (disk)
Law of Oseen
Disk
Sphere
Vd/ν
80
60
40
20
10
6
4
3
2
1.5
1.2
1.0
.8
.6
.4
.3
.2
0
1
10
2²
5²
10²
10³
10⁴
10⁵

Fig. 3

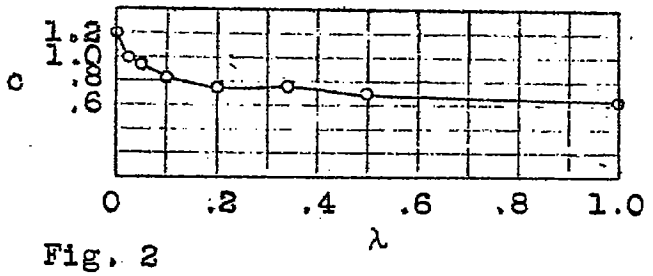

Fig. 2

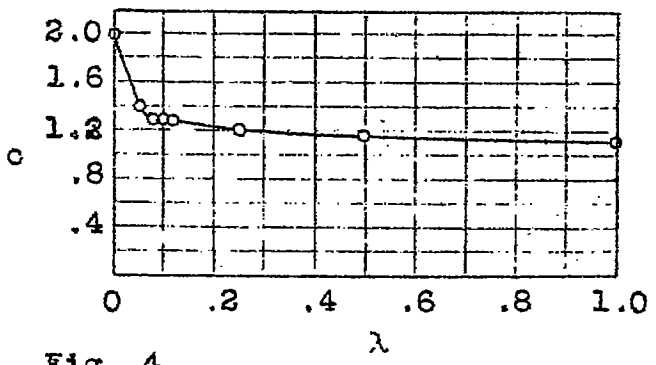

Fig. 4