TECHNICAL NOTES.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 84.

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NEW DATA ON THE LAWS OF FLUID RESISTANCE.*

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While very noteworthy results have been obtained, especially in recent years, with the aid of the theory of the frictionless fluid, this is the case in a much smaller degree for the results of the theory based on a fluid with internal friction or viscosity. The fluids with which we actually have to do always possess some viscosity, which is the very reason for the resistance encountered by a body moving in a fluid. When this resistance or drag has been reduced to a minimum by streamlining the body, the effect of the viscosity becomes so small that the actual flow very nearly agrees with that calculated on the basis of the theory of the frictionless or non-viscous fluid.** This is not the case, however, with shapes which cause a great resistance, since the viscosity of the fluid here plays a decisive role. Thus far all attempts at the quantitative determination of the drag, on the basis of the theory of viscous fluids, with the exception of a few special cases, have met with but slight success. For this reason, whenever a more accurate knowledge of the drag is desirable, it

* From "Physikalische Zeitschrift," 1921, Vol. 22, pp. 321-328.

** Fuhrmann, Theorie und experimentelle Untersuchungen an Ballonmodellen, Dissert., Göttingen, 1911, also Jahrbuch der Motorluftschiffstudiengesellschaft, 1911-1912.

must be determined by experiment. In this article a few experimental results will be given on the drag of a cylinder exposed to a stream of air at right angles to its axis. It will be shown that the drag depends on the absolute dimensions of the body and the velocity and viscosity of the fluid in a much more complex manner than has heretofore been supposed.

It is customary to represent the drag D encountered by a body in moving through a fluid of the density ρ, by the formula

$$D = cS\frac{\rho v^2}{2}$$

in which V denotes the velocity with which the body moves through the fluid and S generally represents the projected area of the body on a plane perpendicular to the direction of motion. Instead of this area, we may take any other characteristic area of the body; for example, in the case of aerofoils, the greatest projected area. The dimensionless coefficient c is termed the coefficient of drag. For a long time the opinion held, mainly on the strength of Newton's conception of the resistance of the air, that for a given fluid this coefficient of drag is independent of the velocity and of the absolute size of the body and may accordingly be regarded as a constant whose value depends only on the geometrical shape of the body. It was thought possible, from the knowledge of the drag coefficient (obtained for a single velocity of a given body by means of the above drag formula), to determine the drag for any other size of the body and for any other velocity, geometrical similarity of shape being assumed.

In reality, as we shall see, the relations are not nearly so simple.

More accurate experiments on the mutual influence of the forces which produce the drag, have shown that the coefficient of drag remains constant only for geometrically similar flows. The latter do not however necessarily follow from geometric similarity of the bodies experimented upon. The decisive conditions for the production of geometrically similar flows were first determined by O. Reynolds. If any desired linear dimension of the body (which must however be identical in the cases compared) is designated by d and the kinetic viscosity by $\nu = \mu/\rho$ (in which μ is the coefficient of viscosity), the two flows are geometrically similar only when the quotient $\frac{Vd}{\nu} = R$ is the same in both cases. The coefficient R is dimensionless and is called Reynolds number from its discoverer.

Consequently, it cannot be expected that the coefficient of drag c (which characterizes the resistance of a body) will remain unchanged in the transition to another Reynolds number, for example, by changing the velocity or the size of the body. In fact, a dependence of the coefficient of drag on Reynolds parameter $\frac{Vd}{\nu}$ is observed for most bodies. The kind of change is determined by the geometrical shape of the body. The above expression is usually employed for the drag, even in the cases where c is not a constant. The least changes in the coefficient of drag occur for bodies with sharp edges, when the latter are perpendicular to the direction of flow. Thus, for example, according to

previous experiments on sharp-edged disks perpendicular to the flow, the coefficient of drag remains constant for a wide range of Reynolds numbers and has a value of about $c = 1.1$. On the contrary, bodies with convex upper surfaces may give very different results. With Reynolds numbers (which are small in comparison with unity) the drag increases directly as the velocity, as was first demonstrated by Stokes for the case of falling spheres. This flow is characterized by the fact that here the inertia completely disappears and the motion is only influenced by the forces of viscosity.

In order to trace the course of the coefficient of drag in at least one case, we recently carried out a series of experiments with cylinders. Each cylinder encountered the air stream at right angles to its axis. All measurements were made in "uniplanar flow", that is, where the particles were all moving parallel to a plane perpendicular to the axis of the cylinder and whereby moreover the same streamline form existed in all planes parallel to said plane. The coefficients of drag found were therefore for infinitely long cylinders. Since there was only a moderate range of velocity (between 1.2 and 36 m/sec) at disposal and since, on the other hand, the value of the kinetic viscosity was practically constant, so long as the experiments were performed in the same fluid, cylinders of different diameters had to be used for great variations in Reynolds number. It was found that Reynolds number could be varied in this way between very wide limits. Experiments were tried with

nine cylinders ranging in diameter from 0.05 to 300 mm. The experiments accordingly embraced a range of Reynolds numbers from 4.2 to 800,000,* with the adoption of the diameter of the cylinder as the characteristic length d. The drag of small cylinders (up to 8 mm. in diameter) was determined by suspending each one vertically in the air stream on a long wire, which was attached to the ceiling of the experiment chamber and carried a weight at the bottom, below the air stream. From the deflection of the thus-constituted pendulum, under the influence of the air stream, the drag was readily determined. In these experiments the cylinder extended through the air stream. Although deviations from the uniformity of the flow certainly occurred on the edges of the air stream, these could not materially affect the main flow, since in all cases the length of the cylinder was very much greater than its diameter (280 times, in the most unfavorable case) and the disturbances on the edge of the air stream extended over distances of only a few cylinder diameters. With cylinders of much larger diameter, however, this marginal disturbance could not be disregarded and some other method had to be employed. The thicker cylinders were accordingly placed between two flat rigid walls located inside of and parallel with the air stream. A special kind of packing (labyrinth packing) was placed between the ends of the cylinder and the two flat walls, so that the air

* The value of the kinetic viscosity for air at 760 mm pressure and 15°C is $\nu = 0.145$ cm²/sec.

could not pass between, and thus a uniform flow was produced.* A system of wires led from the cylinder to a balance which measured the drag.

Fig. 1 shows the results of all the experiments. The drag coefficient c is here plotted against Reynolds number $R = \frac{Vd}{\nu}$ on logarithmically divided coordinates. The logarithmic method of presentation was adopted in order to represent all fields uniformly side by side. It is first seen that the drag coefficient increases as the Reynolds number decreases. The experimental values of the latter extend down to about 4.2. Now a formula for the drag coefficient was given by Lamb (Phil. Mag., 1911, Vol. 21, p.120, "On the Uniform Motion of a Sphere through a Viscous Fluid") for motion with very small Reynolds numbers ("creeping motion"), on the basis of the theory of viscous fluids, similar to the one given by Stokes for the sphere. Lamb's formula for the drag coefficient of a cylinder reads, with our symbols,

$$c = \frac{8\pi}{R\,(2.002 - \ln R)}$$

in which R represents the Reynolds number with reference to the diameter of the cylinder. This formula is derived from an approximation theory and is only applicable for values of R which are small with reference to unity. The values corresponding to this

* A detailed description of this arrangement, which has hitherto been principally employed for testing aerofoils in a two-dimensional flow, is given in "Zeitschrift für Flugtechnik und Motorluftschiffahrt," 1919, p.95, and in "Ergebnisse der Aerodynamischen Versuchsanstalt," first report, 1921, pp. 54-56, published by R. Oldenbourg.

formula are represented by a dash line in Fig. 1. It is evident that the continuation of the curve passing through the experimental points connects well with the course of the calculated curve, so that the region in which the experiments can no longer be carried out, is bridged over. With $R \sim 2000$, there is a very noticeable downward deviation, confirmed however from another side..* From $R = 15{,}000$ to $R = 180{,}000$, the quadratic law of drag is approximately satisfied by the value of $c = 1.2$.

With $R \sim 200{,}000$, a very rapid fall of the drag coefficient (from 1.2 to 0.3) takes place. A very similar behavior had been previously observed in determining the resistance of spheres** and afterwards also for many other bodies with convex upper surfaces. The Reynolds number corresponding to this transitional region is usually designated as the "critical Reynolds number." The decrease of the drag coefficient is so great in the region, that even the absolute value of the drag for a cylinder of given diameter, contrary to all previous experience, decreases with increasing velocity. The quantitative relations are shown by Fig. 2, in which the drag in kg per meter length of a cylinder of 30 cm

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* E. F. Relf, "Discussion of the Results of Measurements of the Resistance of Wires, with some Additional Tests on the Resistance of Wires of Small Diameter," Technical Report of Advisory Committee for Aeronautics, 1913-1914, p.47.

** G. Eiffel, "Sur la Resistance des Spheres dans l'air en mouvement" Comptes rendus, 1912, No.155, p.1597; further, Capt. G. Constanzi, "Alcune esperienze di idrodinamica," Rendiconti delle esperienze e degli studi nello stab. di esp. e constr. aeron. del genio, Vol. II, No.4, Rome, 1912; L. Prandtl, Der Luftwiderstand von Kugeln Nachrichten der Koniglichen Gesellschaft zu Göttingen, Math. Phys. Klasse 1914; C. Wieselsberger, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1914, p.140.

diameter is plotted against the velocity of the air. It is seen that with the increase of the velocity from 15 to 30 m/sec. the absolute value of the drag falls from 4 to about 3.5 kg. The pure quadratic law of drag which is represented in Fig. 3 by the two dash lines (parabolas), is obeyed neither before nor after this critical point within a considerable region. In connection with Fig. 1, it should also be noted that the Reynolds law of similarity, in accordance with which it was necessary to have equal drag coefficients for equal Reynolds numbers, was very well satisfied, since the sections of the curve corresponding to the diameters of the different cylinders connected well with or covered one another.

Along with the magnitude of the drag coefficient, the forms of flow, corresponding to the different Reynolds numbers, are also of interest and are capable of shedding much light on the phenomena of flow. It has already been mentioned that, with very small Reynolds numbers, the nature of the flow is largely determined by the viscosity. On the basis of H. Lamb's article, already referred to, we have calculated the streamline form of the uniplanar cylinder flow for the Reynolds number $R = 1$, in which the coefficient of drag given by Lamb's formula is approximately correct. Fig. 3 shows the absolute, and Fig. 4, the relative streamlines of this flow. The absolute streamlines give, as may be here recalled, the direction of motion of the fluid particles for an observer at rest with reference to the fluid, while the relative streamlines, on the contrary, give the direction for an observer at rest with reference to the body. These two diagrams show that the flow is

not symmetrical with reference to a vertical plane passing through the axis of the cylinder and perpendicular to the direction of the undisturbed flow. The relative streamlines come less closely together behind the body than in front of it, which signifies that the flow behind the cylinder is considerably retarded in comparison with the undisturbed flow. This is clearly shown by the velocity curves in Fig. 4. Curve 3 shows that the velocity, at a distance of 7.5 cylinder diameters behind, has fallen off to less than half the value of the undisturbed velocity, while the retardation at the same distance in front of the body (curve 1) is only slight. On the surface of the cylinder the velocity of flow is zero (curve 2). The "wake" formed behind the cylinder is conditioned by the fact that Lamb's valuation for the flow about a cylinder does not entirely neglect the acceleration terms of the differential equation, as is the case in Stokes' flow about a sphere, but, following the example of Oseen's calculations for a sphere, takes them into account to a certain degree. If, in the case of the cylinder, we should consider only the effect of viscosity, as done in Stokes' calculation for the sphere, we would obtain a flow which is symmetrical with reference to a vertical plane passing through the axis of the cylinder and perpendicular to the direction of the flow (whereby in this case, however, the velocity in infinity would not have a finite value). With a decreasing Reynolds number the flow about a cylinder will therefore gradually approach a symmetrical form, while with an increasing Reynolds number up to about $R = 80$, the flow retains the character of Figs. 3 and 4. This was con-

firmed by a photograph of the flow with $R = 3.5$, hence already considerably outside the applicability of Lamb's formula.* A cylinder of 12.8 mm diameter was moved through a syrup solution and the moving particles of lycopodium, sprinkled on the surface of the liquid, were photographed, the camera being moved with the cylinder. The quantitative relations at fairly great distances from the cylinder can hereby make no claim to perfect agreement with the motion of an unlimited fluid, on account of its relatively small extent, the dimensions of the fluid being only 34 cm long, 34 cm wide and 8 cm deep. The character of Lamb's flow, especially the absence of vortices behind the body, is, however, clearly shown. A condition of transition to the flow with fully developed vortices behind the body is indicated by the wake's beginning to show an oscillatory motion, at about $R = 100$. With a further increase of the Reynolds number, very regular vortices were formed, which have been very thoroughly and successfully investigated by Von Karman.** The existence of these vortices can be easily demonstrated acoustically, since they set the air in vibration by their regular succession, thereby producing audible tones. In this manner we have demonstrated the presence of Karman vortices up to a Reynolds number of about 100,000. In excess of the critical Reynolds number, a considerable further change in the form of the flow takes place, in that the point on the surface of the cylinder where the forma-

* A cut in the original paper is omitted here.

** T. von Karman and H. Rubach, Ueber den Mechanismus des Flüssigkeits und Luftwiderstandes, Physikalische Zeitschrift, 1912, Vol. 13, p.49.

tion of vortices begins, the "separation point," is shifted more toward the rear. Both these forms of flow are shown diagrammatically in Fig. 5, where the vortex regions are indicated by cross-hatching. It is seen that, beyond the critical number, the width of the vortex region, which constitutes an approximate criterion for the magnitude of the drag, is considerably less. The point at which the smooth flow leaves the surface is designated by a. The pressure distribution on the cylinder in uniplanar flow, both below and above the critical point, is shown in Fig. 6, according to English experiments.* The angles recorded on the axis of abscissas are calculated from the foremost point ("rest-point") of the cylinder, while the ordinates indicate the ratio of the pressure measured at any point to the pressure at this point. The dash line indicates the pressure distribution resulting from the theory of the frictionless (or non-viscous) fluid, which would not give rise to any drag. This distribution is approximated considerably more closely by the distribution for the Reynolds number $R = 176,000$, than by the distribution below the critical point for $R = 64,000$. More thorough investigation now shows that the shifting of the separation point toward the rear is connected with the fact that the flow (influenced by the viscosity in the immediate vicinity of the surface, which originally consists of a smooth gliding of the fluid layers), above a certain Reynolds number, suddenly becomes permeated with small vortices. The surface layer is said to become

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* G. J. Taylor, Pressure Distribution Round a Cylinder, Technical Report of the British Advisory Committee for Aeronautics, 1915-1916, p. 30.

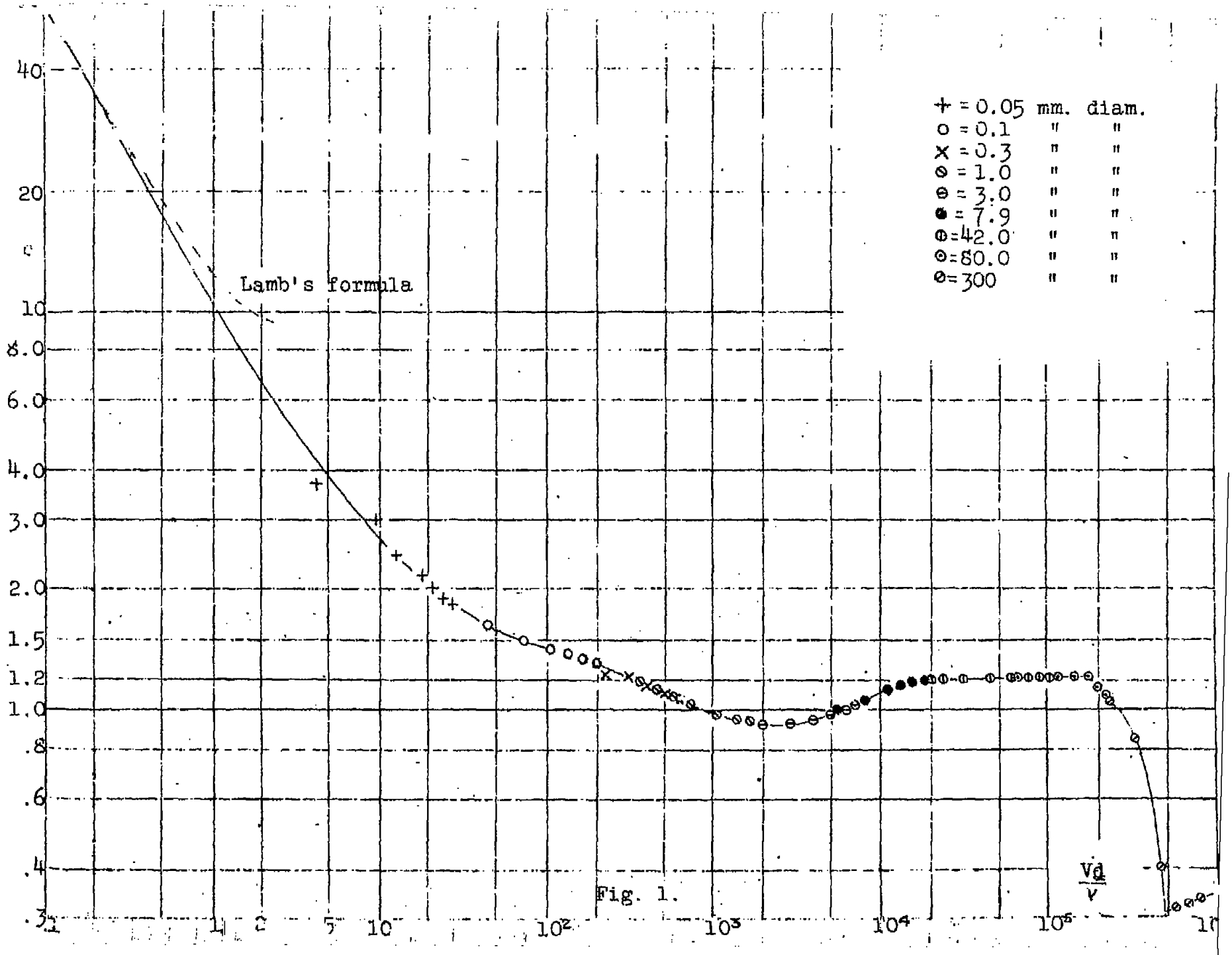
"turbulent." If this turbulence (which occurs automatically for a certain Reynolds number) is artificially created by special devices - for example, by placing a coarse sieve of suitable mesh in front of the experimental body or by means of obstacles (roughness) on its front surface - the location of the separation point can be shifted backward, even for smaller Reynolds numbers.

Even after the critical number is passed, very peculiar phenomena occasionally appear, as manifested in marked variations of the drag coefficient. Any roughness of the surface seems to play an especially important role here. Such a case is represented by Fig. 7. Here the coefficient of drag is again plotted against the Reynolds number $\frac{Vd}{\nu}$, in which d represents the thickness, perpendicular to the direction of the flow, of a cylinder tapered in the rear, as shown in the diagram. The continuous line was obtained with a perfectly smooth surface; the dash line, with a rough surface. It is seen that in the latter case, after passing the critical number, which is here about $R = 70,000$, a rapid increase of the drag coefficient again takes place, so that even for this region the quadratic law of resistance is by no means obeyed with a constant coefficient of drag. It will be an essential task for experimental aerodynamics to find the explanation of these peculiar phenomena.

In concluding, the writer wishes to express his heartiest thanks to Professor Prandtl for the active support he has given this work.

Aerodynamic Institute, Göttingen, April, 1921.

Translated by National Advisory Committee for Aeronautics.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Lamb's formula
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3.0
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10
10^2
10^3
10^4
10^5
$\frac{Vd}{\nu}$

Fig. 1.

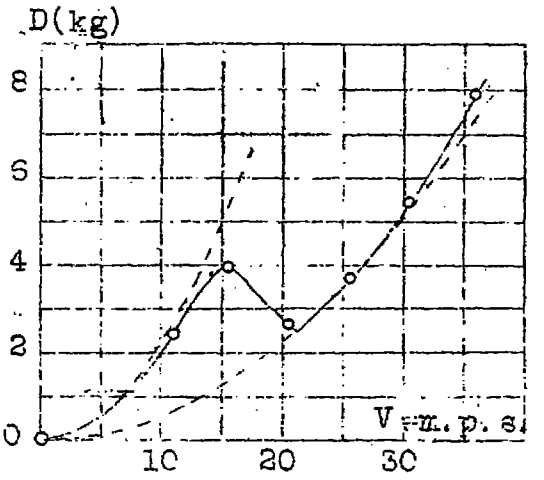

Fig. 2.

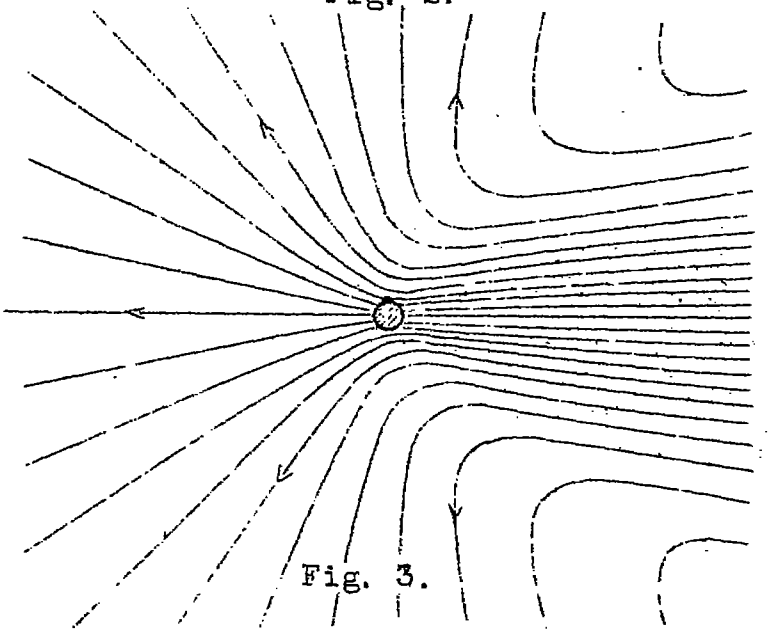

Fig. 3.

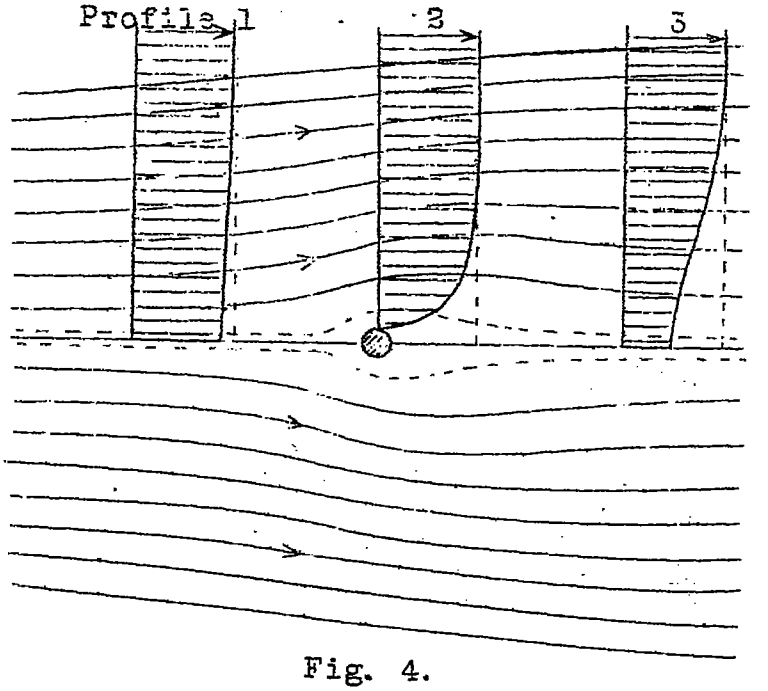

Fig. 4.

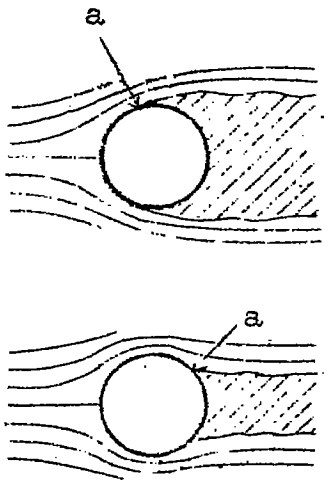

Fig. 5.

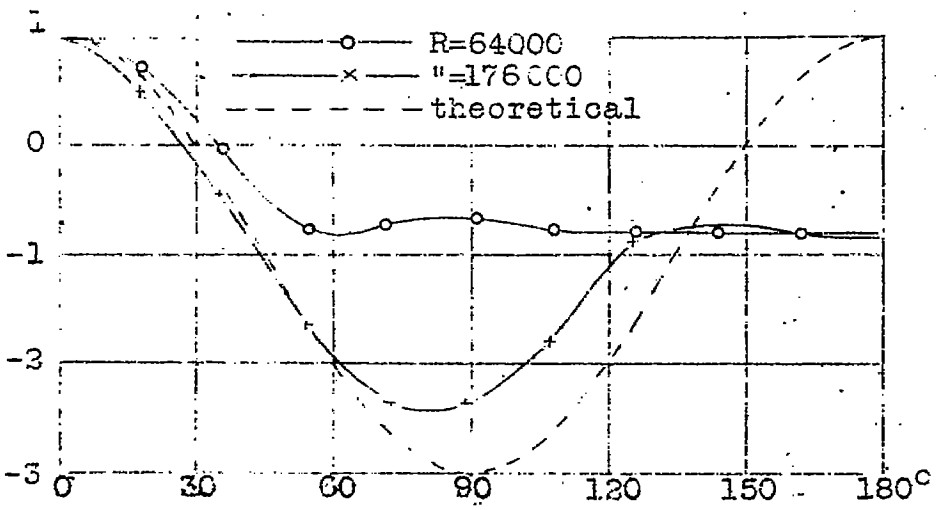

Fig. 6.

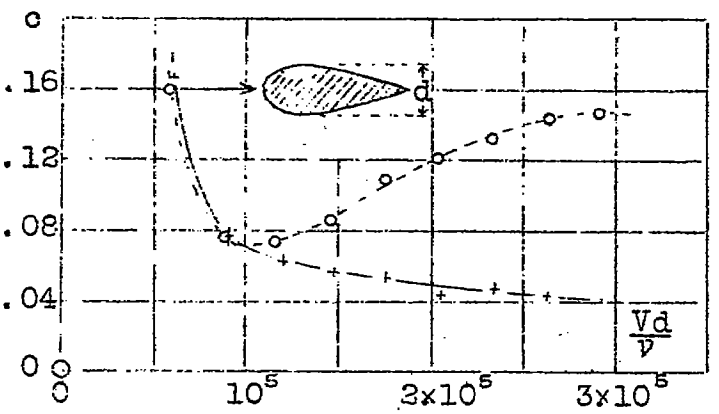

Fig. 7.