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The Distribution of Thrust Over a Propeller Blade.

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Summary.

The best distribution of the thrust over the length of the propeller blade is investigated, taking into account chiefly the slip stream loss and the friction between the blades and the air.

The energy losses of the propeller depend noticeably on the distribution of the thrust over the length of the blades, and the losses can be diminished by a favorable distribution. The two induced losses of smaller importance, the loss due to the finite number of blades and the loss due to the rotation of the slip stream, call for a gradual decrease of the thrust per unit of propeller disc area towards the inner and outer end of the blades. Near the center the thrust is naturally less dense, and hence the loss from rotation is generally kept reasonably small without special effort of the designer and no further improvement is here possible. The breadth of the blades, however, is not always tapered towards the tips as much as would be desirable in order to keep small the loss due to the finite number of blades. It is

true, the actual thrust distribution is almost as favorable in spite of it and the density of thrust always decreases properly, because it is physically impossible for a wing to produce a finite density of lift quite close to its end. Still the wing works then under less favorable conditions and with smaller efficiency, and the weight of the propeller and the centrifugal force is unnecessarily great too. Now, the following investigation will show that a gradual decrease of the density of thrust towards the ends is also desirable for other reasons. It is sufficient therefore to keep in mind that the wing tips have to be round, and to consider in the following investigation only the two chief energy losses, the energy absorbed by the air friction and the slip stream loss.

A small variation of the distribution of the thrust hardly changes noticeably the entire loss, especially if the distribution is already close to the best distribution. Hence the problem is less the exact determination of the best distribution of thrust than the derivation of a simple expression which gives quickly an idea as to how the thrust has to be arranged.

The conditions are quite different from those for ordinary wings. There, the inductive losses form a much greater part of the entire losses, and the other part, that is the loss of friction, does not depend on the distribution of the lift at all. Hence, with ordinary wings the distribution of the lift is determined by the consideration of the induced drag exclusively. With

the propeller, however, the friction is the dominant part, and both, not only the slip stream loss but the loss of friction too, depend on the distribution of the thrust, for the velocity of the blade elements relative to the air is variable and greater at greater distance from the axis. And whether the ratio lift/drag be constant or not, the ratio of the useful work done by the lift to the energy absorbed by the drag is quite different from it and certainly not constant in general. The useful work is done in the direction of the constant velocity of flight, but the friction is multiplied by the relative velocity of the blade element and the loss is the smaller the smaller the relative velocity is. Therefore the consideration of the friction alone calls for a great density of the thrust near the center. The slip stream loss alone however calls for constant density of the thrust. Hence the smallest loss occurs at a compromise, that is, that the thrust is neither concentrated near the center nor distributed uniformly, but that the density of thrust decreases uniformly towards the outside.

A short calculation will be sufficient to give numerical information on the desirable decrease. Let C_p denote the density of the thrust per unit of the disc area divided by the dynamical pressure of the velocity of flight, that is

$$C_p = \frac{dT}{V^2 \frac{\rho}{2} df}$$

where dT denotes the infinitesimal thrust acting on the small area df of the propeller disc. C_p may be assumed to be small,

say up to .50. Then the slip stream loss originated by this thrust is approximately 1/4 C_P multiplied by the useful work done by the same element of thrust. The density of drag measured in the same way is $C_P \; C_D/C_L$ and the work absorbed by this drag is $C_D/C_L \; v/V$ times the useful work, where v denotes the velocity of the blade element relative to the air and V the velocity of flight. For simplicity's sake, I will replace the velocity v by the tangential velocity of the blade element, which is somewhat smaller but not so much that it will greatly injure the final result.

The problem can now be stated in the following way. Let r denote the radius of the blade element, that is, its distance from the axis. The entire thrust is easily found to be

$$(1) \qquad T = 2 \pi q \int_0^{D/2} C_P \, r \, dr$$

The slip stream loss for the path of length l of the airplane can be taken as

$$(2) \qquad 2 \pi q \frac{l}{4} \int_0^{D/2} C_P^2 \, r \, dr$$

The energy absorbed by the friction during the same time is

$$(3) \qquad \frac{(2\pi)^2 \, q \, n}{V} \int_0^{D/2} \frac{C_D}{C_L} C_P \, r^2 \, dr$$

The coefficient of thrust density C_P variable along the length of the blade, is to be determined in such a way that for a given

thrust (1) the sum of the two losses (2) and (3) becomes a minimum.

The solution is extremely easy. It can be seen that by transferring an element of thrust from one place to another the entire loss must not be changed, nor the entire thrust, and hence the ratio of the local change of loss to the change of thrust must be equal to a constant, say λ. The equation which determines the desired function is therefore

$$\delta \left(\lambda (1) + (2) + (3) \right) = 0$$

where δ means that the variation originated by any variation of C_P as to form. That is the same condition as if

$$2 \pi q \int_0^{D/2} \left[\frac{1}{4} C_P^2 \, r + \frac{2 \pi n}{V} \frac{C_D}{C_L} C_P \, r^2 + \lambda C_P \, r \right] dr$$

is to be made a minimum. The variation is proportional to the integral over

$$\frac{1}{2} C_P \, r + \frac{2 \pi n}{V} \frac{C_D}{C_L} r^2 + \lambda r = 0$$

which then has to be zero at every point r. Hence the solution of the problem is

$$(4) \qquad C_P = 2 \lambda - 4 \frac{\pi n}{V} \frac{C_D}{C_L} r$$

It shows, as was to be expected, that the thrust density has to be constant only if the drag coefficient of friction $C_D = 0$. But if the friction is taken into account, it is not constant but has

to decrease towards the outside. The simplifying assumptions make it appear as a linear function of the radius, and that is just what we wanted, and exactly enough. It remains only to determine the value of the Constant from the condition that the entire thrust has a particular value in order to obtain the final expression for the density of thrust. By substituting equation (4) in equation (1) it appears that

$$T = 2\pi q \int_0^{D/2} \left(2\lambda - \frac{4\pi n}{V} \frac{C_D}{C_L} r\right) r\, dr$$

$$\text{i.e.,} \quad T = 2\pi q \lambda \frac{D^2}{4} - \pi q \frac{\pi n}{3V} \frac{C_D}{C_L} D^3,$$

$$\text{Hence} \qquad 2\lambda = \frac{T}{D^2 \frac{\pi}{4} q} \quad \frac{4}{3} \frac{C_D}{C_L} \frac{\pi n D}{V}$$

$$\text{and finally,} \quad C_P = \frac{T}{D^2 \frac{\pi}{4} q} + \frac{4}{3} \frac{C_D}{C_L} \frac{\pi n D}{V} - 2 \frac{C_D}{C_L} \frac{\pi n \cdot 2r}{V}$$

At a radius 2/3 of the greatest radius, the thrust has the mean density, the same that it had to have without friction. At the blade tips, the density has to be

$$(5) \qquad \frac{T}{D^2 \frac{\pi}{4} q} - \frac{2}{3} \frac{C_D}{C_L} \frac{\pi n D}{V}$$

This expression can become negative. But then one essential assumption for the proceeding is no longer valid: C_L/C_D is no

longer constant but changes its sign. For this reason a negative thrust appears uneconomical in accordance with what everybody expects. Equation (5) becoming negative rather indicates that the diameter has reached its most economical value. This appears, then, to be

$$D^3 = \frac{T\ V}{n\ q}\ \frac{C_L}{C_D}\ \frac{6}{\pi^2}$$

Let us see now by means of an example, what the derived formulas give for usual conditions. They will give different results for the same propeller under different conditions of flight a reason more why to confine the calculation to a moderate exactness conveniently to obtain.

Illustration:

Diameter 9 ft., that is, disc area 63.5 sq. ft.

n 25 revolutions per second, that is, n D = 706 ft/sec.

Thrust 400 lbs.

V 100 mi/hr.

C_L/C_D 22

q 25 lbs/sq. ft dynamical pressure of flight.

$$D^3 = \frac{400 \cdot 100 \cdot 1.47 \cdot 22.6}{25 \cdot 25\ \pi^2} = 1260\ ft^3$$

$$D = 10.6\ ft.$$

That is pretty near to the actual diameter. However, the conditions of great velocity are favorable for a small economical diameter. Suppose on the other hand the velocity of the same pro-

peller to be 60 mi/hr only and the thrust to be 575 lbs. The dynamical pressure q may be 10 lbs/sq.ft.

Now,
$$D^3 = \frac{575 \cdot 60 \cdot 1.47 \cdot 22.6}{10 \cdot 25\ \pi^2} = 2700\ ft^3$$

$$D = 14\ ft.$$

Even in this case, the diameter results only 14 ft., which indicates that the improvement theoretically possible cannot be exceedingly great.

In the first case the mean value of C_p appears .252 and and C_p at the blade tip has to be .106. The mean value agrees with the desired value at 2/3 of the radius as always. That is a rather great variability, which corresponds to the fact that the diameter is only slightly smaller than the most economical diameter. In the second example, the mean value of C_p is .90. This is comparatively high and in consequence of it the developed formulas give too large a diameter, for the factor .25 for the induced loss is too high for large values of C_p. Paying no attention to this, the coefficient at the blade tips appears .658. The density is much more constant now, according to the greater predominance of the slip stream loss.

The results obtained are only approximate. The formula for the diameter is not to be considered as a literal prescription. The weight of the propeller is not considered nor the resulting tip velocity, which cannot be increased without limit. Besides,

the diameter is more often determined by the general lay-out of the airplane. The formula is only to show whether an increase of the diameter means an improvement at all, and to give an indication how much.