REPORT No. 133

THE TAIL PLANE

IN FOUR PARTS

By MAX M. MUNK
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FORMULAS AND DEFINITIONS.

M = moment around c. g.
q = dynamical pressure = $V^2\rho/2$.
S = area of wings.
c = chord of wings.

$$c_m = \frac{M}{qSt} \text{ coefficient of pitching moment.} \tag{1}$$

I = moment of inertia of airplane around transverse axis = $k^2\, W/g$.
W = weight of the airplane.
k = radius of gyration.
g = acceleration of gravity.
c_L = coefficient of lift $\frac{L}{Sq}$

The period for fictitious statical oscillations is

$$T = 2\pi\sqrt{\frac{I}{d\,M/d\alpha}} = 2\pi\frac{k}{V}\sqrt{\frac{v_a}{v_m}} \tag{2}$$

During this period the airplane travels $2\pi k\sqrt{\frac{v_a}{v_m}}$ (3)

$$v_m = -\frac{d\,(M/q)}{d\alpha} \quad \text{(Definition)} \tag{4}$$

The metacentric height is

$$G_m = v_m \cdot q/\,W = \frac{v_m}{S\,(c_L)_o} \quad c_L \text{ constant} \tag{5}$$

The rate of the travel of the center of pressure is

$$-\frac{v_m}{Sc_L}c_L \text{ variable} \tag{5a}$$

The long period oscillations of an airplane, due to the variation of speed, have the period

$$T = \pi\sqrt{2}\cdot V/g \tag{6}$$

$$v_a = 2\,W/\rho g \quad \text{(Definition)} \tag{7}$$

$$a_w = S\frac{dc_L}{d\alpha} \quad \text{(Definition)} \tag{8}$$

S_t = area of the tail plane

c_{Lt} = lift coefficient of the tail plane

$$a_t = S_t \cdot \frac{dc_{Lt}}{d\alpha} \tag{9}$$

The coefficient of stability is

$$c_s = \frac{v_m \cdot v_a \cdot k^2}{(a_w\, k^2 - a_t\, l^2)^2} \tag{14}$$

The limit of aperiodic complete damping is

$$c_s \gtreqless 1/4$$

The limit of statical stability

$$c_a \geqq 0$$

The limit of real stability is

$$c_a \geqq -\frac{a_w \cdot k^2 \cdot a_t \cdot l^2}{(a_w \cdot k^2 - a_t \cdot l^2)^2} \tag{15}$$

or, otherwise expressed

$$v_m\, v_a \geqq -a_w\, a_t \cdot l^2 \tag{15a}$$

$\frac{d\alpha}{dc_L}$ rate of change of the angle of attack with reference to lift coefficient

$\frac{\delta\alpha}{\delta c_L}$ rate of change of the angle of attack due simply to section of aerofoil

$$\frac{d\alpha}{dc_L} = \frac{\delta\alpha}{\delta c_L} + \frac{S}{\pi b^2}$$

$$\frac{dc_m}{d\alpha} = \frac{dc_m}{dc_L} \cdot \frac{d\alpha}{dc_L} \tag{16}$$

The angle of downwash is

$$u/V = c_L \cdot 1.8\, \frac{S}{b^2\pi} \text{ for monoplanes, down to}$$

$$c_L \cdot 1.5\, \frac{S}{b^2\pi} \text{ for bi- and triplanes.} \tag{17}$$

The downwash of the propeller slip stream is

$$u/V = \frac{1}{2} \cdot \frac{T}{qD^2\pi/4} \sin\beta \tag{18}$$

The angle of the self-induced downwash is

$$u/V = \frac{c_L S_t}{\pi b^2} \tag{19}$$

The efficiency of the tail plane due to the self-induced downwash is

$$\eta_1 = \frac{1}{1 + \frac{S_t}{b_t^2\pi} \Big/ \frac{\delta\alpha}{\delta c_L}}$$

approximately

$$\eta_1 = \frac{1}{1 + 1.83\, S_t/b_t^2} \tag{20}$$

The efficiency due to the downwash of the airplane is

$$\eta_2 = 1 - 1.8\, \frac{S}{\pi b^2}\, \frac{1}{\frac{\delta\alpha}{\delta c_L} + \frac{1}{\pi}\frac{S}{b^2}} \tag{21}$$

(take 1.6 instead of 1.8 if biplane)

$$a_w = \frac{S}{d\alpha/dc_L} = \frac{S}{\frac{\delta\alpha}{\delta c_L} + \frac{S}{\pi b^2}}$$

$$a_t = S_t \cdot 5.7 \cdot \eta_1 \cdot \eta_2 \tag{22}$$

The lift coefficient of the tail plane at small angles is

$$\frac{0.1}{1 + S_t/\pi b^2_t}\, \frac{S_t\alpha + \beta k E}{S_t} \tag{24}$$

α and β in degrees, S_t entire tail plane, E elevator, k between 1.25 and 1.75.

REPORT No. 133.

THE TAIL PLANE.

By Max M. Munk.

INTRODUCTION.

The following paper which has been prepared for publication by the National Advisory Committee for Aeronautics deals with the calculation of the tail plane. I try to simplify the present theory of longitudinal stability for that particular purpose so as to obtain one definite coefficient characteristic of the effect of the tail plane. The more general theory hitherto developed gives no such compact result. It has indeed a wider aim, to calculate not only the tail plane, but the exact character of flight to be expected. The method is so complicated, however, and the results so uncertain that at present it seems to be unfit for application in design. For the present at least it has to be confined to the desks of some special investigators. It is not impossible on the other hand that the following method may be developed later so far that it covers also this more general task. In its present form the aim is not so ambitious.

The coefficient mentioned is obtained by substituting certain aerodynamic characteristics and some dimensions of the airplane in a comparatively simple mathematical expression. I take care to confine all aerodynamical information necessary for the caculation of the coefficient to the well-known curves representing the qualities of the wing section. I am able to do so by making use of the recent results of modern aerodynamics. All formulas and relations necessary for the calculation are contained in the paper. They give sometimes only an approximation to the real values. An example of calculation is added in order to illustrate the application of the method.

Moreover, it was necessary to discuss more general and simple questions concerning the subject before starting with the new contribution. Statical and dynamical stability, slope of the moment curve, travel of the center of pressure, metacentric height are conceptions which are old and comparatively simple. They have given rise, however, to controversies and there is not yet general agreement as to their meaning and usefulness. The discussion of some other questions related with the tail plane forms the last part of the paper.

The direct result of the investigation is the deduction of the coefficient of stability mentioned and the method for calculating and applying it. This coefficient indicates not only whether the effect of the tail plane is great enough, but also whether it is not too great. It appears that the designer has to avoid a certain critical length of the fuselage, which inevitably gives rise to periodical oscillations of the airplane. The discussion also shows the way and in what direction to carry out experimental work. For free flight tests more attention ought to be given to the relation between the position of the elevator and the corresponding equilibrium velocity. In model tests it appears that the moment of the air force on a wing about a fixed axis not too far from the average center of pressure ought to be preferred to the investigation of the travel of the center of pressure. It is easy to calculate the second quantity from the first, but not the first from the second. The other questions touched in this paper give suggestions for wind tunnel experimentation.

Before proceeding to attack the problem, I wish to touch one very general question which is intimately connected not only with this paper, but with the whole complex of technical and scientific questions. I obtain the essential coefficient by neglecting the variation of speed of the airplane during one oscillation after having investigated the expediency of this

procedure. I calculate the coefficient by substituting approximate theoretical formulas for real physical amd more complicated phenomena. What is the meaning of such procedure?

There are people who consider each omission and each approximation or simplification as a defect of the method. And they certainly are right sometimes, if the simplification is not justified. There are questions of abstract science where a more complete understanding and not a numerical calculation is the aim of the investigator, where the philosophical physicist reasons and does not make applications. In such cases any simplification may vitiate the purpose. The following paper is not written for philosophers and physicists, but for designers and engineers, no matter how much physical knowledge they may possess. And for such men the question is quite different. No engine is the result of a calculating machine, but of a brain. Certain quantities only can be and must be calculated, and even they approximately. It is impossible to take into account all details of all the physical processes. Their laws are generally known, it is true, but the actual calculation is either too difficult or impossible. It is, therefore, always a marked progress in technical science if an investigator succeeds in separating important and less important factors, and if he teaches how to calculate anything simply which could not be properly calculated at all before. The physical process forming the case of the simplified calculation, of course, can not be identical with the actual phenomenon, but if the method be a good one it forms, as it were, a skeleton of it. For instance, the most marked progress in the art of calculating the tension in beams was the discovery that it is allowable to neglect the shearing forces and to make the assumption that all plane sections remain plane during the deflection. This is not true, and it is well known that it is not true. But the method gives the right dimension of the beam and nothing else is wanted by the designer.

Neglecting the influence of the speed variation and not the variation of any other quantity is justified not alone by the result of the following discussion. It is already contained implicitly and hidden in the papers of previous writers. I begin honestly with this simplification and try to demonstrate its justification by physical reasons instead of only stating its possibilities without any explanation. By this systematic procedure I am enabled to investigate the remaining phenomena more closely and to obtain more compact and useful results. In the following treatise also the thrust of the propeller and its slip stream is not very exhaustively taken into account. It is perfectly correct to omit it for the gliding flight and, as the airplane must satisfy this condition of flight also, the omission is not likely to undermine grossly the result. Moreover, the increase of speed due to the slip stream is partially neutralized by the braking effect of the fuselage and the other parts which diminish the velocity of the passing air. The axis of the thrust never passes very far from the center of gravity. The resulting moment can be taken into account by assuming a correspondingly changed position of the center of gravity. The variation of the thrust depends on the propeller and the engine, and the calculation of its influence is far beyond the aim of this paper. The dimensions of the tail plane are hardly influenced by this consideration. In my opinion the omission of the thrust is by no means as essential as that of the variation of speed. It is only a detail which later can be elaborated.

It is hardly necessary to make any apologies for the use of general aerodynamical relations between the angle of attack, the lift, and the downwash. They are perfectly true under some simplifying assumptions, and they are always as exact as any other technical formulas and as the methods of calculation used in other parts of applied science. The greatest incorrectness of the result is not the consequence of this simplification, but is due to the uncertainty of the position of the center of gravity and to the uncertainty about the arodynamical quality of the wings. It must be emphasized that these difficulties are not peculiar to the special method adopted, for they are already contained in the original method of treating statical stability.

The reader who is not fond of mathematics may pass over the mathematical development. He finds all formulas he wants in the list at the beginning, and the example will show him how to apply the formulas. The next step, however, is not the application of the method for design, but its examination by applying it to actual airplanes. The method is subject to improvement, which can be made after such applications. I am wholly satisfied if I only succeed in giving a suggestion for an improvement of the method of calculating the tail plane.

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THE TAIL PLANE.

PART I.

THE ENTIRE AIRPLANE.

1. EQUILIBRIUM.

During a steady flight the tail plane makes the moment of the air forces around the center of gravity vanish, and hence the center of pressure is always at this point and does not "travel." To prevent its travel is the function of the elevator, the set of which is changed by the pilot on passing from one condition of steady flight to another. Without turning the elevator the new flight would not be steady, for then the center of pressure would have traveled and there would be a moment around the center of gravity. Through this center of pressure the line representing the effect of all the air forces would pass and this would be at some horizontal distance from the center of gravity. This line being almost vertical, the moment would equal the product of the horizontal distance from the center of gravity and the magnitude of the entire force. This magnitude is approximately equal to the weight of the entire airplane and therefore constant. This is well known, but it is necessary to mention it as an introduction. These statements, however, are very general and therefore very vague; the designer can not do very much with them. He needs to know how great the effect of the tail plane has to be, for the greater the desired effect, the greater the necessary dimensions to produce the effect. Now the moment mentioned is not due to the dimensions of tail and tail plane alone, but also, and to a very high degree, to the instantaneous velocity of flight, or, better expressed, to the dynamic pressure; and the velocity, or the dynamic pressure, is by no means constant, for the airplane has a different velocity, smaller, for instance, at a low number of revolutions of the engine than at a high number, and smaller if climbing than if diving. We think that we can not proceed more logically and usefully than by separating the two factors which produce the moment, and obtaining the true characteristic for the dimensions of tail and tail plane.

This is not difficult. Let M denote the moment around the center of gravity and q the dynamic pressure $\frac{1}{2}\rho V^2$. Then $\frac{M}{q}$ is the expression sought, as the reader can easily verify, by considering that the moment is proportional to the dynamic pressure q. Let there be a medium set of elevator (say zero) and a medium condition of steady flight; there is, then, no moment around the center of gravity and M/q is zero. The more the angle of attack is changed in one or the other direction, the greater is the positive or negative moment which must be neutralized by rotating the elevator, in order to obtain steady flight. If we divide this moment by the dynamic pressure of each particular steady flight, we obtain the expression M/q which is characteristic of the geometrical dimensions.

Imagine this expression M/q to be plotted against the angle of attack. Then the slope of the curve represents the effect necessary in order to change the angle of attack by a properly chosen unit of angle. We intend to use this curve actually and to plot it for a particular airplane. We must clearly understand, therefore, by what unit M/q can be measured and what kind of a physical quantity it is. We mentioned that this expression is characteristic of the geometrical dimensions simply. Now the "tail effect" is proportional to the area of the tail plane multiplied by the lever arm, that is to say, to a sort of volume. It is not in the least

surprising, therefore, that M/q is also represented by the cube of a length; it is a sort of volume and must be measured either by cubic feet or by any other measure of volume.

When comparing several different airplanes with respect to this "volume of moment," it is advisable to take its ratio to some standard volume given by the dimensions of the airplane. The wing area presents itself as a standard area, but the arm is not so easily found. We think it most convenient to take the chord of the wings or the mean chord, and the resulting expression,

$$c_m = \frac{M}{q\,S\,c} \tag{1}$$

may properly be called the coefficient of pitching moment and be denoted by c_m. In figure 1 this coefficient is plotted against the angle of attack. The values are taken from a model test and are not wholly reliable, the conditions not having been quite the same as in real flight. Neither was the situation of the center of gravity exactly known, and the shape of the curve depends on its position. The diagram, however, shows sufficiently what we wish to show. We have represented in a second curve and on a corresponding scale the travel of the center of pressure. The ordinates of this curve are $P/c\ (c_L)_0$ where P is the actual travel of the center of pressure, c is the chord and $(c_L)_0$ is the lift coefficient for steady flight. It should be noted that the curve has two branches asymptotic at infinity. These two curves of c_m and center of pressure travel coincide at the angle of equilibrium, where the moment as well as the distance between the center of gravity and the center of pressure is zero. Moreover, at this point the slope of the two curves agree also. But this agreement of curves and slopes is the case only because we have chosen the scale suitably. At all other angles of attack neither the two curves nor their slopes agree; they differ greatly.

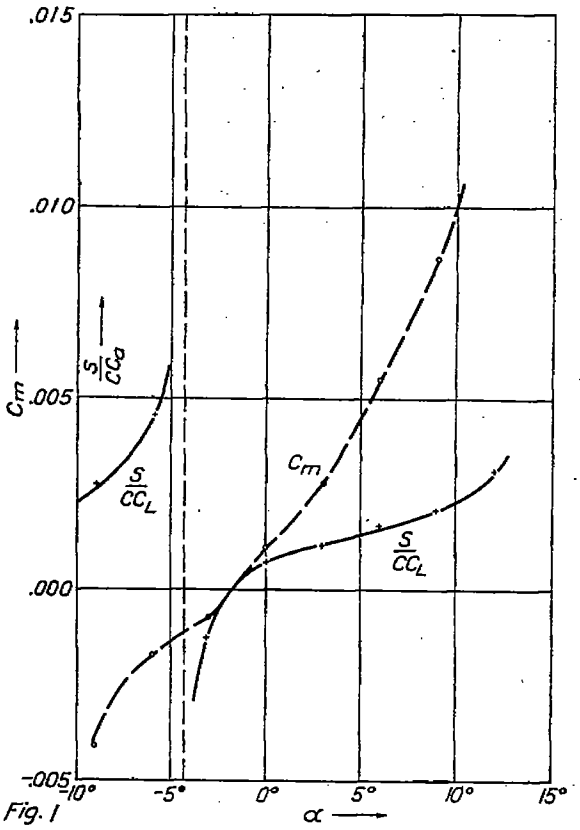


Fig. 1

If for any new project the designer knows the magnitude of the moments around the center of gravity and the velocity for the different angles of attack, he can draw the curve of c_m, and this curve will give him by its slope the first information concerning the condition of equilibrium that he wants for the design of the tail plane. It gives the information very much better than the curve of the travel of the center of pressure, which curve agrees with the former in one point only. It is true that the curve of the travel of the center of pressure is always the same, wherever the center of gravity may be situated, while the curve of the moment changes according to the position of the center of gravity.[1] If the moments are measured around a transverse axis which is at a distance x forward from the center of gravity, all that is necessary in order to obtain the values of c_m with reference to an axis through the center of gravity is to add to the measured values of c_m the quantity $\frac{xc_L}{c}$. This procedure certainly is not so troublesome that it does not pay to perform it, instead of calculating with a wrong curve.[2] There is

[1] Both can be influenced by the propeller thrust.

[2] Compare sec. 15, where it is shown that the change of the curve is performed by drawing one straight line.

so much the more reason to perform this labor, as this curve of c_m is important for other considerations which will be described in what follows.

As the result of this discussion we conclude:

(*a*) At a set zero of the elevator and for a medium angle of attack the moment around the center of gravity must be zero. For otherwise the elevator would not be fully utilized.

(*b*) The slope of the curve of moment must be moderate, for otherwise a very large tail plane and elevator would be necessary.

2. STATICAL STABILITY.

It is not sufficient that the airplane be in equilibrium, as the designer knows. The equilibrium must be stable or at least it should be only slightly unstable. Now the airplane is in a different equilibrium for every position of the elevator. Each equilibrium ought to be examined separately with respect to its stability. But we will proceed to show that this examination can be much simplified, and that it is sufficient to consider the curve of coefficient of the moment discussed in the preceding chapter.

We consider a small displacement, consisting of a change of the angle of attack, and assume the vertical velocity of the airplane and its velocity of flight to have changed so slightly that the changes can be neglected. The deflection from the original angle of attack then produces a moment proportional to the deflection and to the slope of the curve of moment, i. e., $dM=\frac{dM}{d\alpha}\cdot d\alpha$ and hence is proportional to $\frac{dc_m}{d\alpha}$. The airplane is said to be "statically stable," if the moment tends to bring it back to its original angle of attack. The degree of stability may be described by the time which would be taken for the airplane to swing back through its original angle of attack. The airplane is assumed in this case to be oscillating around the center of gravity and the time for one oscillation is calculated.

For steady flight and a definite tail setting there will be a definite angle of attack which may be called α_0; let the angle be α at any instant after the disturbance; let the moment corresponding to α be M, measured in the same sense as is α. Hence, the equation of motion, omitting all other influences, is

$$\frac{Wk^2}{g}\frac{d^2\alpha}{dt^2}=\frac{dM}{d\alpha}(\alpha-\alpha_0)$$

where W is the weight and k is the radius of gyration.

For reasons which will appear later, it is convenient to introduce two symbols:

$$v_a=\frac{2W}{\rho g}=\text{ twice the volume of air having a weight equal to that of the airplane}$$

$$v_m=-\frac{d(M/q)}{d\alpha}\text{ where } q=\frac{1}{2}\rho V^2$$

$$=-\frac{2}{\rho V^2}\frac{dM}{d\alpha}\text{ if } V\text{ is constant.}$$

Hence, on substitution,

$$\frac{d^2\alpha}{dt^2}=-\left(\frac{v_m}{v_a}\right)\frac{V^2}{k^2}(\alpha-\alpha_0)$$

There are two cases to consider:

(*a*) Static stability. $v_m>0$. This indicates oscillations with a period $2\pi\frac{k}{V}\sqrt{\frac{v_a}{v_m}}$. (2)

Hence the distance traversed by the airplane in this time is $2\pi k\sqrt{\frac{v_a}{v_m}}$ (3)

(*b*) Static instability. $v_m < 0$. The angle would continue to increase according to the expression $e^{\sqrt{-t\, v_m/v_a}\; V/k}$ i. e., it would increase to base (e) times its value in a time $\frac{k}{V}\sqrt{\frac{-v_a}{v_m}}$. The distance traversed by the airplane in this time would be $k\sqrt{\frac{-v_a}{v_m}}$.

Both these distances are independent of the velocity of flight. In case (a), as noted, there would be oscillations, and under such conditions the equilibrium condition of steady flight is said to be statically stable. Expressed mathematically, the condition for statical stability is, then,

$$\left[\frac{d(M/q)}{d\alpha}\right]_{\alpha=\alpha_0} < 0$$

We mentioned that the stability should be examined for every angle of attack and that every angle of attack has its own curve of moment, since for equilibrium the elevator has a different position. In figure 2 we have plotted three such curves for the same model as before, corresponding to the angles of the elevator $-5°$, $+5°$, and $+10°$. The reader sees that the curves are almost parallel and that therefore their slopes are almost equal for the same values of α, the angle of attack. That is always so and very natural. For changing the set of the elevator produces a moment approximately independent of the angle of attack. For this reason it is sufficient to consider one curve of moment only and to use its slope at any value of α, the angle of attack.

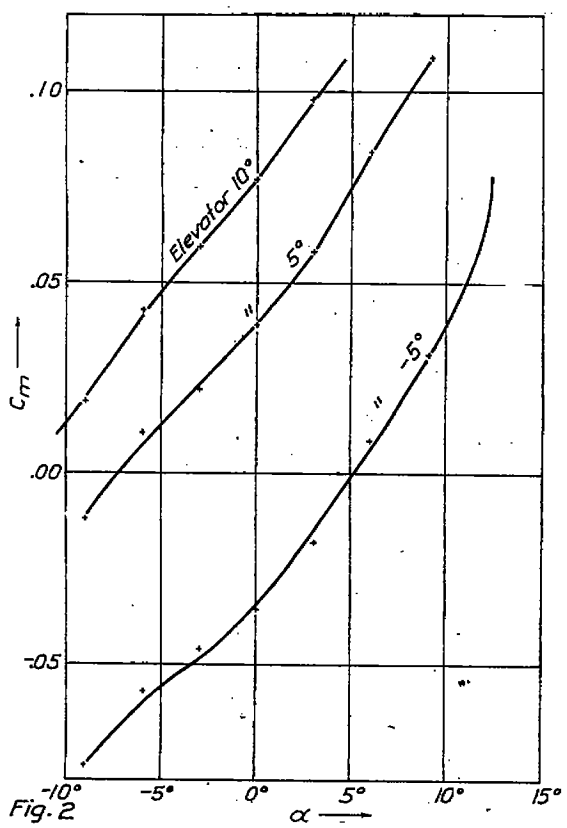


Fig. 2

The consideration of statical stability leads to almost the same rules as did that of equilibrium itself.

The slope of the curve of moment must be moderate, or zero, for otherwise we should obtain either great instability or short, quick oscillations.

If stability is desired, the inclination of the curve must be such that the moment brings the airplane back to its original angle of attack. The amount of the desired stability or of the allowable instability ought to be determined for the airplanes in actual use. No complicated method of testing is required, only the setting of the elevator must be determined for different conditions of flight. This is discussed more fully in section 16.

3. THE METACENTRIC HEIGHT.

In his most valuable paper on the fundamentals of aeronautical designing,[3] Commander J. C. Hunsaker, in accordance with the suggestion of Commander McEntee, suggests the application of the naval architects' metacentric height to calculate and measure the statical stability. Using the matacentric height as he defines it gives the same result as our method. He begins as we did with the curve of the pitching moment plotted against the angle of attack, and divides this moment by the weight of the airplane, thus obtaining the distance of the center of pressure

[3] Naval architecture in aeronautics. Aero. Journal, July, 1920.

from the center of gravity. The slope of this curve is the rate of travel of the center of pressure with reference to α, and we saw that the slope of the curves agree, in fig. 1, at the point of equilibrium. In this diagram we chose the scale of the travel of center of pressure according to Hunsaker's definition, dividing the ratio of the distance between the centers to the chord by the particular lift coefficient of equilibrium. The metacentric height is defined by

$$G_m = -\frac{1}{W}\frac{dM}{d\alpha}$$

The negative slope of the curve of the pitching moment divided by the dynamic pressure is written

$$v_m = -\frac{d(M/q)}{d\alpha} \tag{4}$$

Therefore, writing $W = (c_L)_o q\, S$, where $(c_L)_o$ is the lift coefficient for the value of α at equilibrium

$$G_m = v_m \cdot \frac{q}{W} = \frac{v_m}{S(c_L)_o} \tag{5}$$

The value of G_m is therefore proportional to the negative of the slope of c_m at the point of equilibrium.

The travel of the center of pressure equals M/L, where L is the lift and equals $c_L qS$; hence the rate of change with reference to the angle of incidence, provided c_L may be regarded as constant for a small range, is

$$-\frac{v_m}{c_L S} \tag{5a}$$

and only at the angle of equilibrium does it coincide with that of c_m.

These two expressions are apt to be confounded.

In the paper mentioned, Commander Hunsaker shows an interesting method of making the metacentric height visible. In the diagram for the center of pressure for different angles of attack he puts in for every center of pressure the vector of the lift alone, omitting the drag. The resulting diagram is the complete analogue of the diagram of the naval architect. The intersections of neighboring vectors of lift near the angle of equilibrium give the metacenter.

We think, however, that this diagram ought to be used only as an illustration for the geometric meaning of the metacentric height. It would require too much labor to draw a new diagram for every setting of the elevator; and this would be necessary, for his diagram has not the great advantage of ours, viz, of being valid for all angles of attack and all settings of the elevator.

4. WHY THE CALCULATION OF STABILITY CAN BE SIMPLIFIED.

There are many designers who would like to spend an hour or even two in order to obtain a more complete idea of the nature of longitudinal stability and to be able to calculate in a simple manner whether a certain tail plane will be sufficient or not. For such readers we will try to go a step further. We believe that then we have discussed the problem as completely as is ever necessary for the purposes of the designer.

Before discussing the next question, that is, the damping (which is a quality of stability as stability was a quality of equilibrium), we have to examine the justification of our assumptions in the last chapter. There we neglected the variation of vertical velocity and speed. This is allowable indeed for the calculation of the period only and not for the damping. For if the angle of attack has slightly increased, the lift increases too, and the airplane experiences an acceleration upwards. Therefore, it receives a vertical velocity component and the angle of attack no longer agrees with the angle between a fixed direction in the airplane and the horizontal. The difference equals the ratio, $-w/V$, of the vertical component to the original velocity, if we assume V to be great compared with the vertical component w. The effect is a gradual decrease of the amplitude of the original oscillation, for the effective angle of attack,

and therefore the stabilizing moment is increased when the change is opposed to the angular velocity and decreased when it has the same direction. This type of damping is the subject of the next chapter, together with the damping moment of the tail plane due to the angular velocity of the airplane. We merely state here, therefore, that the effect of the variation of altitude is chiefly a damping of the oscillation, and that, in making calculations of the damping, we are not allowed to neglect the variation of altitude.

The alteration of the horizontal velocity, however, the discussion of which we proceed to now, has no damping effect, but it is periodic. This oscillation and the first mentioned one, it is true, affect each other, but we will show that the interference can be neglected. We proceed by assuming in the beginning what we intend to prove and by examining the result of the assumption.

The smaller the influence of the variation of speed, the longer is the period of the oscillation caused by it; and, if it is very long, the influence of the original motion, due to the angular velocity, has so often changed its sign that the summed up effect on this long oscillation vanishes. During one short oscillation of the first kind, on the other hand, the velocity is almost constant. We assume, therefore, that as a first approximation the variation of the drag and of the angle of attack has no influence on the type of motion, the airplane altering its velocity, owing to alterations in gravitational energy. The oscillation consists in alternately changing the potential energy due to vertical height into kinetic energy of motion and conversely. The result is very interesting and simple. Let h denote the height and W the weight of the airplane, then the potential energy is $h.W$. The variation of this energy is $\Delta h.\ W$, if Δh denotes the variation of the height. The kinetic energy is $1/2(V+v)^2\ W/g$, if V is the original flight velocity and v its variation. If v is small when compared with V, this can be written $1/2\ V^2.\ W/g+Vv\ W/g$, and the second term is the variation of the kinetic energy. The lift is proportional to the square of the velocity, hence its ratio to the original lift W is $\frac{(V+v)^2}{V^2}$ and the variation is $\frac{2v}{V}$. Corresponding to this variation of lift, $\frac{2v}{V}\ W$, the vertical acceleration of the airplane is $2\frac{v}{V}\cdot g$. Now we have to eliminate the velocity v and to introduce instead of it an expression containing the height h. The sum of the variations of the two energies must, of course, be zero if the energy is conservative, i. e.,

$$\Delta h\cdot\ W+V\cdot v\frac{W}{g}=0$$

hence

$$v=\frac{-\Delta h\cdot g}{V}$$

and the vertical acceleration is therefore $-2\ \Delta h\ \frac{g^2}{V^2}$. The acceleration is opposite to and proportional to the displacement from the origin. This gives a simple oscillation with the period

$$T=\pi\sqrt{2}\ V/g \tag{6}$$

The distance traveled by the airplane during one period is

$$s=\pi\sqrt{2}\ V^2/g.$$

T and s depend only on the magnitude of the velocity of flight and are not influenced by the tail plane. If for instance $V=150$ foot seconds (about 100 mi./hr.), we obtain $T=20.8$ seconds. This indeed is so slow an oscillation that it can not have much influence on the character of the motion of the airplane during a short interval. During five seconds, say, the variation of speed may be neglected. Further, it is easily seen that this long oscillation is generally damped. The tail not only gives an additional drag when the airplane is turned, but during the continuous change of kinetic energy into energy of height and conversely the sum of the two decreases continuously. During the motion up and down a certain percentage of the energy is absorbed due to the variation of the drag, and the vertical amplitude grows smaller and smaller. This

holds true if the restoring moment is great, so that the angle of attack is almost constant. With actual airplanes the moment is very small, but the other factors tend to conserve the angle of attack. But even if the attitude of the airplane were constant with respect to the horizontal the damping would not be less. On the contrary, when descending, the angle of attack is increased and the drag and lift are both increased; when rising, both are smaller. In the valleys and on the summits of the waves the angle of attack is as before. Whence it follows that the amplitude of the vertical oscillation is generally decreased, while the period is not much altered, unless the propeller and its slipstream are the cause of a different behavior.

5. THE DAMPING.

We are now prepared to discuss the nature of longitudinal stability of an airplane. We realize that we may neglect the variation of speed, and hence we assume the center of gravity to move with constant horizontal velocity. The vertical velocity and the angular velocity are assumed to be small. The difficulty of such a mathematical investigation as the following one is only that of memorizing the different symbols. In order to facilitate this as far as possible, we shall introduce some new symbols and take care to choose them so that their physical dimensions are as simple as possible. In the preceding section 2, equation 3, we divided the weight W of the airplane by half the specific weight of the air, so obtaining the volume v_a

$$v_a = \frac{2\,W}{\rho g} \qquad \text{(definition)} \tag{7}$$

which is twice the volume of air having the same weight as the airplane. The moment of inertia was avoided by making use of the radius of gyration k, which of course is a length. Besides we need the distance l (length) between the tail plane and the center of gravity.

Now we proceed to the forces and moments acting on the airplane. We will confine ourselves to three, which are really known to the designer. These three are the lift of the wings, the lift of the tail plane, and the moment of the airplane around the center of gravity. The lift equals the product of the dynamic pressure and the "area of lift," which itself is the product of the lift coefficient and the area of wing. We are interested in the rate of change of the lift with respect to a change of the angle of attack, because in considering displacements the change in the lift must be introduced into the dynamical equations. This equals the product of the dynamic pressure and the rate of change of the area of lift with respect to the angle of attack, and is $a_w \cdot q$, where

$$a_w = S\,\frac{dc_L}{d\alpha} \tag{8}$$

a_w is an area and equals the slope of the curve of the area of lift when plotted against the angle of attack. In the same way we define a_t as the rate of change of the area of lift of the tail plane, i. e.,

$$a_t = S_t\,\frac{dc_{Lt}}{d\alpha} \tag{9}$$

The rate of change of the statical moment with respect to the angle of attack is described by the rate of change of M/q, which is discussed in the previous chapters. We write $v_m = -\frac{d(M/q)}{d\alpha}$. This has the dimensions of a volume. We describe thus all forces acting on the airplane and occurring in this discussion by two areas and one volume, which can be determined in each particular case. The moment contains the force on the tail plane resulting from its instantaneous angle of attack. We need a_t only for the calculation of the moment due to the angular velocity of the airplane. If this angular velocity is denoted by ω, the tail has a vertical velocity downward, $l \cdot \omega$; and its angle of attack is increased by $\frac{l\omega}{V}$. The corresponding change of the restoring moment of the tail plane is therefore

$$\frac{l\omega}{V} \cdot a_t \cdot l \cdot q$$

The change in the restoring or statical moment discussed in the previous chapter is

$$v_m \cdot q \cdot \alpha$$

where α is the change in the angle of attack, and the variation of lift is

$$a_w \cdot q \cdot \alpha$$

In the problem before us we shall assume a_w, a_t and v_m to be constants for the ranges discussed, and also that a_w and a_t are always positive; v_a is, of course, a positive constant.

In describing the motion of the airplane, the motion will be referred to moving axes, following Bryan and others. Let w be the velocity of the center of gravity along the Z-axis, and let ω be the angular velocity around the transverse axis. Then the acceleration along the Z-axis at any instant is $\frac{dw}{dt}+V\omega$, and the change in the angle of attack is $-\frac{w}{V}$. For small displacements the equations of motion for the three forces mentioned are

$$v_a\,\rho/2\left(\frac{dw}{dt}+V\omega\right)=-a_w\,q\,\frac{w}{V},\text{ i. e., }\frac{dw}{dt}=-\frac{a_w}{v_a}\,Vw-V\omega \tag{10}$$

and

$$v_a\,\rho/2\,k^2\,\frac{d\omega}{dt}=v_m\,q\,\frac{w}{V}-a_t l^2\,q\,\frac{\omega}{V},\text{ i. e., }\frac{d\omega}{dt}=\frac{v_m}{k^2 v_a}\,Vw-\frac{a_t l^2}{k^2 v_a}\,V\omega\,. \tag{11}$$

Hence by elimination

$$\frac{d^2w}{dt^2}+\frac{dw}{dt}\,V\left(\frac{a_w\,k^2+a_t l^2}{k^2 v_a}\right)+w\,V^2\left(\frac{v_m}{k^2 v_a}+\frac{a_w k^2\cdot a_t\,l^2}{k^4 v_a^2}\right)=0 \tag{12}$$

or the same differential equation for ω.

The solution of this equation is of the form $w=Ae^{\lambda t}$, where λ satisfies the algebraic equation

$$\left(\frac{\lambda}{V}\right)^2+\frac{\lambda}{V}\left(\frac{a_w\,k^2+a_t l^2}{k^2 v_a}\right)+\left(\frac{v_m}{k^2 v_a}+\frac{a_w\,k^2\cdot a_t\,l^2}{k^4 v_a^2}\right)=0 \tag{13}$$

Write this, for a moment,

$$\left(\frac{\lambda}{V}\right)^2+a\,\frac{\lambda}{V}+b=0$$

a may be assumed, for the case under consideration, an essentially positive constant, while b is a constant which may take either + or − values, owing to the presence in it of v_m.

The roots are

$$\frac{-a\pm\sqrt{a^2-4b}}{2}$$

Case 1. If $b<0$, one root is + giving rise to instability, one is − giving rise to a damped nonoscillatory motion, i. e., to stability.

Case 2. If $4b=0$, one root is zero; the other is negative, giving rise to a damped nonoscillatory motion.

Case 3. If $4b>0$, and $<a^2$, both roots are negative.

Case 4. If $4b=a^2$, one negative root.

Case 5. If $4b>a^2$, both roots are complex, with a negative real part, giving rise to a damped periodic motion.

Summary: Stability if $b\geqq 0$.

Damped aperiodic motion if $0\leqq 4b\leqq a^2$.

Damped periodic motion if $4b\geqq a^2$.

In the formula

$$a=\frac{a_w k^2+a_t l^2}{k^2 v_a}$$

$$b=\frac{v_m}{k^2 v_a}+\frac{a_w k^2\cdot a_t l^2}{k^4 v_a^2}$$

The condition for damped periodic motion is that

$$\frac{v_m}{k^2 v_a}+\frac{a_w k^2 \cdot a_t l^2}{k^4 v_a^2}>\frac{(a_w k^2+a_t l^2)^2}{4\ k^2 v_a^2}$$

i. e. that $v_m > \frac{(a_w k^2 - a_t l^2)^2}{4\ k^2 v_a^2}$

Let us define the absolute coefficient

$$c_s=\frac{v_m v_a k^2}{(a_w k^2-a_t l^2)^2} \tag{14}$$

Hence the condition for damped periodic motion is $c_s > \frac{1}{4}$. The limit of oscillations is then $c_s=\frac{1}{4}$.

Similarly, the condition for damped aperiodic motion is

$$0 \leqq \frac{v_m}{k^2 v_a}+\frac{a_w k^2 \cdot a_t l^2}{k^4 v_a^2} \leqq \frac{(a_w k^2+a_t l^2)^2}{4 k^4 v_a^2}$$

Near the limit of periodic motion, therefore $c_s \leqq \frac{1}{4}$

Near the limit for undamped motion

$$\frac{v_m}{k^2 v_a}+\frac{a_w k^2 \cdot a_t l^2}{k^4 v_a^2} \geqq 0$$

or

$$c_s \geqq -\frac{a_w k^2 \cdot a_t l^2}{(a_w k^2-a_t l^2)^2} \tag{15}$$

The limit of stability is when c_s equals this fraction. In case $a_w k^2$ nearly equals $a_t l^2$, it will be best to use the equivalent expression

$$v_m v_a \geqq -a_w a_t l^2 \tag{15a}$$

The condition for statical stability is $v_m > 0$, i. e. $c_s > 0$. So that it is apparent that cases may arise when there is static instability, i. e. $v_m < 0$, and yet there will be a damped aperiodic motion, and therefore the airplane is stable.

Summary:

$c_s > \frac{1}{4}$ damped oscillations.

$c_s = \frac{1}{4}$ limit between oscillations and aperiodic motion.

$-\frac{a_w k^2 . a_t l^2}{(a_w k^2 - a_t l^2)^2} \leqq c_s \leqq \frac{1}{4}$ aperiodic motion.

$$\left.\begin{array}{l} c_s = -\frac{(a_w k^2 . a_t l^2}{(a_w k^2 - a_t l^2)^2} \\ \text{or} \quad v_m v_a = -a_w a_t l^2 \end{array}\right\} \text{limit of true stability}$$

$c_s > 0$ statical stability.

$c_s = 0$ limit of statical stability.

$0 > c_s > -\frac{a_w k^2 a_t l^2}{(a_w k^2 - a_t l^2)^2}$ statically unstable, but damped aperiodic motion, i. e. stable.

Thus c_s is a quantity characteristic of the stability and it may be called the coefficient of stability.

6. THE AIRPLANE WITHOUT TAIL PLANE.

The exact determination of v_m, the slope of the volume of moment, is so difficult that it seems useless for the present to discuss the possible improvement of the other terms. We will rather proceed to discuss briefly the determination of those parts of v_m which are produced by the wings, although this is only indirectly connected with the subject of this paper. As for a_w, also produced by the wings, the same considerations are valid as for a_t and we have opportunity later to discuss them.

The moment produced by a wing depends on its situation with respect to the center of gravity. We have seen that for a condition of steady flight the entire moment must vanish. The tail plane has to neutralize the moment produced by the other parts of the airplane at that angle. Besides, the tail plane has to produce stability and damping. These three tasks are generally too much for it, the last two can be performed by it, but the first and second tasks are inconsistent with each other. For this would mean that the tail plane would not only be a stabilizing organ, but a lifting aerofoil too. In that case the system of wings extends up to the tail itself and an increased travel of the center of pressure is the consequence. This requires a greater tail plane, and fuselage and tail plane are heavier and move the center of gravity even more backward. This again requires a greater tail plane, etc. The case of nose heaviness is not much better. It is evident then, that the moment for all air forces, those of the tail plane excepted, must also be zero, or at least small, for a medium angle of attack. The tail plane must have such an inclination that at that angle its lift is zero. The angle of attack of the tail plane in steady flight has to be smaller, therefore, than that of the wings, a fact which is well known to every designer.

This is not all. Experience has shown that airplanes were never very satisfactory if the moment vanished as the result of large moments neutralizing each other. All good airplanes have wings which are so situated that they counterbalance themselves for a medium angle of attack by means of their position with regard to the center of gravity. Airplanes with large stagger are now rare, and are only chosen if there are important reasons for them. This is not surprising. The greater the single moments are, the more difficult is it to counterbalance them for a considerable range of angle of attack. A small deformation of the wing, a small error of the assembly or inexactness of fabrication is sufficient to produce a great disturbing moment. The center of gravity ought to be situated not far from the vertical through the center of pressure for the medium angle of attack of each wing. For most wing sections this is from about one-third to 0.4 of the chord from the leading edge.

The moment is the product of force and lever arm. If it is given with respect to any particular point, the moment with respect to any other point equals the sum of this moment and the moment of lift and drag with respect to the new axis if imagined to be applied at the first given point. This is well known.

When calculating from a model test, the slope of the moment or of its volume with respect to the angle of attack, it must be remembered that the center of pressure as well as the coefficient of moment belong to and are dependent on the coefficient of lift and not on the angle of attack. This must be noted especially if the aspect ratio of the wing is different from that of the model or if the model was a single aerofoil and the same section is used in a biplane. In all these cases a different angle of attack corresponds to the same coefficient of lift. It would be best to begin always with the slope of the coefficient of moment with respect to the change of the coefficient of lift, which is independent of the aspect ratio, and to divide it by the differential quotient $\frac{d\alpha}{dc_L}$, which depends on the aspect ratio and on the section.

$$\frac{dc_m}{d\alpha}=\frac{dc_m}{dc_L}\Big/\frac{d\alpha}{dc_L} \tag{16}$$

The influence of the wing section on the moment and its slope can be studied in the publications of the different wind tunnels. The slope of the travel of center of pressure is characteristic at that particular angle of attack only where the center of gravity coincides with the

center of pressure. Another method is to plot the coefficient of moment with respect to some particular point, not too far from the mean center of pressure, against the lift coefficient. This curve runs very regular, it is practically the same for models having different aspect ratios, and, as $\frac{d\alpha}{dc_L}$ is almost independent of the section, for small angles it is sufficiently characteristic of the slope of the coefficient of moment with respect to the angle of attack. In order to obtain the moment for any other position of the center of gravity, draw a straight line from the origin to that particular point of the curve where the moment around the new center of gravity is zero (Fig. 7). The distances between this line and the curve in the direction of the axis of moment give the new coefficient of moment, and the angle between the line and the curve gives the slope exactly enough.

We have discussed now the function of the tail plane, and have seen how both the equilibrium produced by different settings of the elevator and the statical stability are characterized by the same curve of the volume of moment around the center of gravity. Its slope, together with the two areas of damping of the airplane, that of the wings and that of the tail plane, provides us with the coefficient of stability. In the second part of this report we will proceed to discuss the details and see how they affect equilibrium, stability, and damping.

REPORT No. 133.

THE TAIL PLANE.

PART II.

THE TAIL PLANE.

7. EQUILIBRIUM.

We saw that for a medium angle of attack the airplane ought to be in equilibrium without the tail plane producing a lift. A small lift is indeed allowable, but it is better to make no use of it in the first design and to keep it back for an emergency in case the airplane turns out to be slightly nose heavy or tail heavy. The problem of the tail plane is simple, therefore, as far as equilibrium is concerned, and there remains only the question of its angle with a fixed direction in the airplane. This angle must be such that for the medium angle of attack of the airplane mentioned the effective angle of attack of the tail plane is zero. Without the downwash of the wings and the slip stream the direction of the tail plane which gives the lift zero would be parallel to the direction of flight. It must be turned by the angle of downwash and that of the slip stream. We will proceed to give the formulas for these two angles. We need not be too scrupulous in this respect, for a small subsequent change of the angle of the tail plane is always possible without great difficulties.

The angle of the downwash in degrees may be written [4]

$$\left.\begin{array}{l} 1.8\, c_L \cdot \dfrac{S}{\pi b^2} \cdot 57.3 \text{ for monoplanes, and} \\ 1.5\, c_L \cdot \dfrac{S}{\pi b^2} \cdot 57.3 \text{ for biplanes.} \end{array}\right\} \qquad (17)$$

For a more exact calculation we propose to take the factor $\frac{1.8}{k^2}$ where k is the factor of the effective span of the biplane. (See Technische Berichte II, p. 187.)

In these two formulas—

c_L denotes the coefficient of lift $\frac{L}{qS}$

S the area of wings.

b the maximum span of the wings.

L the lift.

V the velocity of flight.

The dynamical pressure $q = V^2\rho/2 = V^2/390$ for low heights, in lbs./sq. ft., if V is in mi./hr.

$\rho/2$ half the density of air.

The influence of the slip stream on the angle has not been discussed satisfactorily in any publication, as far as I know, and exact experimental work in this respect is defective too. We are dependent, therefore, on theoretical considerations, which we must greatly simplify in order to make its bulk agree with the value of the result. The question is complicated in the first place by the mutual influence of the propeller and the wings. We are forced to neglect this, the change of the final result by it is likely to be small because the effect is a double one and neutralizes itself partially. In the next place we will neglect the fact that a propeller with an inclined axis works unsymmetrically. For calculating the remaining main influence we

[4] See the papers of the author: Technische Berichte II, p. 187; Technische Berichte III, p. 10; Reports N. A. C. A., No. 121; where more exact data can be found.

may apply the ordinary momentum theory, the angle between axis and direction of flight being always small. This effect is partially neutralized by the wings, but this only holds true for the effect itself and not for its rate of change, which we have to consider afterwards. Let $V+v$ and u be the velocity components of the slip stream relative to the airplane at a point in the air some distance behind the propeller, u being downward. The mean components in the neighborhood of the propeller are $V+1/2\ v$ and $1/2\ u$ and the air at a great distance in front of the propeller has the components relative to it V and zero. The thrust may have the components T and $T \sin \beta$ where β denotes the angle between the axis of the shaft and the velocity of flight. Let D be the diameter of the propeller. Hence we have

$$T \sin \beta \frac{u}{2} = u^2 \frac{\rho}{2} V D^2 \frac{\pi}{4}$$

i. e.

$$\frac{u}{V} = \frac{T \sin \beta}{V^2 \rho D^2 \pi/4} = \frac{1}{2} \frac{T}{q D^2 \pi/4} \cdot \sin \beta \tag{18}$$

The angle in degrees is 57.3 times greater. The angle of the tail plane with the direction of flight is the sum of this angle and the angle of downwash, if the tail plane is wholly surrounded by the slip stream, otherwise a corresponding fraction can be taken.

Most designers wish the tail plane to lie within the slip stream. If the airplane flies at a great angle of attack and the propeller stops suddenly, the airplane should receive an impulse to decrease its inclination without the pilot acting. This takes place indeed if the tail plane is met by the slip stream, for at high angles the tail plane usually has a positive lift, which is increased by the slip stream, so that the positive lift decreases as soon as the slip stream ceases, thus setting up a turning moment. The same effect is obtained by an eccentric position of the center of gravity with respect to the axis of the thrust.

The problem of equilibrium also embraces the effect of the elevator necessary to produce equilibrium for all other angles of attack. It appears, however, that this consideration would give small values, the statical stability being almost zero. We shall discuss the question of the size of the elevator in the other sections.

8. STATICAL STABILITY.

The subject of this chapter is the rate of change of the moment if the setting of the elevator is fixed. We have to discuss in particular that part of this rate which is caused by the tail plane. It is chiefly due to the change of the lift of the tail plane; the travel of the center of pressure of the tail plane gives only a small rate of change of the moment.

If a single aerofoil or a biplane is inclined to the direction of the relative velocity of the air, the effective angle of attack does not agree with the apparent or geometrical angle of attack. The direction of air in its immediate neighborhood can be affected by some other body, so that the aerofoil is in a downwash and it is always affected by the aerofoil itself, so that it is in a self-induced downwash also. The angle of the self-induced downwash or shorter, the self-induced angle of attack has the magnitude [5]

$$\frac{c_L S_t}{b^2 \cdot \pi} \tag{19}$$

wherein

c_L denotes the coefficient of lift of the aerofoil $\frac{L}{Sq}$.
S_t denotes the area of the aerofoil.
b denotes the maximum span.
L denotes the lift.
q denotes the dynamical pressure $V^2\rho/2$.
V denotes the velocity of flight.
ρ denotes the density of air.

[5] See Technische Berichte I, p. 187.

The rate of change of the self-induced downwash with reference to the lift coefficient is, accordingly, $\frac{S_t}{b^2\pi}$. The rate of change of the entire angle of attack is the sum of this rate and the original rate, which may be written $\frac{\delta\alpha}{\delta c_L}$. This original rate depends on the section and indeed ought to be determined by model tests for the section used in the wings. It is not always very variable, however, and for small angles is seldom very different from $\frac{\delta\alpha}{\delta c_L}=10$ for the angle in degrees, 10/57.3 for the angle in radians. This holding true especially for small angles of attack, it will be sufficient in most cases to assume $\frac{\delta\alpha}{\delta c_L}=10/57.3=0.174$ for the tail plane. It follows that a particular lift requires an angle of attack about $\frac{(0.174+S/\pi b^2)}{0.174}$ times as great as without the existence of the self-induced downwash. The aerofoil behaves like one having a decreased area without self-induced downwash. The factor of decrease is approximately $\frac{1}{1+1.83\ S/b^2}$. The exact expression is

$$\eta_1=\frac{1}{1+\frac{S}{b^2\pi}\cdot\frac{1}{\delta\alpha/\delta c_L}} \tag{20}$$

The formula for the downwash of the wings is mentioned in the last chapter. Its rate of change with reference to the angle of attack depends on the section of the wings and on their effective aspect ratio b^2/S. Introducing the effect of the self-induced downwash of the wings on their effective area, it is expressed by the formula

$$1.8\frac{S}{\pi b^2}\frac{\frac{\delta c_L}{\delta\alpha}}{(1+1.83\ S/b^2)}$$

$\frac{\delta c_L}{\delta\alpha}$ is the rate of change of the lift coefficient with reference to the angle, due to the section only, and is not very different from 57.3/10 as mentioned before. The last factor is the rate of change of the lift coefficient with respect to the real angle of attack, including the effect of the aspect ratio of the wings S/b^2. For biplanes the first factor is smaller in the ratio 5:6. The effective angle of attack on the tail equals the angle of attack on the wing less the angle of downwash, i. e.,

$$d\alpha_t=d\alpha_w\left(1-1.8\frac{S}{\pi b^2}\frac{\frac{\delta c_L}{\delta\alpha}}{1+1.83\ S/b^2}\right)=\eta_2 d\alpha_w$$

where

$$\eta_2=1-1.8\frac{S}{\pi b^2}\frac{\frac{\delta c_L}{\delta\alpha}}{1+1.83\ S/b^2} \tag{21}$$

Hence

$$\frac{dc_L}{d\alpha_w}=\eta_2\frac{dc_L}{d\alpha_t}$$

so that the factor, which indicates the decrease of efficiency of an aerofoil due to the downwash of the wings is η_2 where again the factor 1.8 refers to monoplanes and is smaller by 17 per cent for biplanes.

We have thus obtained the two factors, one due to downwash, self-induced, and the other due to wings, which give the effective area for statical stability, and for dynamical stability and damping also. The final value of a_t, since $\frac{\delta c_L}{\delta\alpha}=5.7$, is

$$a_t=S_t\cdot 5.7\ \eta_1\cdot\eta_2 \tag{22}$$

The effect of the slip stream is neglected.

The investigation of the dynamical stability in the first part of this report has shown that the airplane is stable even if the statical stability is zero. For this reason the statical stability may be ever so small if only there is no statical instability, and this is what the designers aim to obtain. The calculation of course seldom gives the exact value zero for the stability but there remains always some definite value. In order to see whether the stability obtained from the calculation is small or not, the period of the statical oscillation imagined may be calculated from (3). If this period is a considerable fraction of one minute the stability is certainly small. The coefficient of stability gives a measure of the magnitude of the stability.

The preceding consideration gives exactly enough the size of the tail plane. We only need the dimensions of the airplane and the aerodynamical properties of the wing section. We will exemplify the method in the next part of this report; the reader will there convince himself that the whole calculation can be made in a fraction of an hour. We have to add still that the downwash is somewhat greater if the tail plane is very close to the wings; the formula given refers to the usual position. We realize thus that we have now determined, as it were, the statical moment of the tail plane surface $S_t \cdot l$. The next chapter deals with the moment of inertia $S_t \cdot l^2$.

9. DAMPING.

We have seen in the general discussion of dynamical stability that the coefficient of stability c_s indicates the degree of damping. $c_s=0$ indicates the limit of the statical stability though not at all that of the dynamical or real stability. A regular use of the elevator, however, requires a positive coefficient of stability. $c_s \geqq 1/4$ indicates the beginning of oscillations. Now the designer does not have it in his power to obtain the exact slope of moment which he wishes, nor is it possible to obtain a constant slope of moment for all angles of attack. He wishes, therefore, to have the range for this slope sufficiently great—at least as great as that of other successful airplanes. Our theory of the coefficient of stability shows how one may increase the range. The coefficient of stability has the magnitude

$$c_s=\frac{v_m\, v_a\, k^2}{(a_w\, k^2-a_t\, l^2)^2}$$

and has to lie within a particular range, say, between 0 and 1/4. The range of v_m, of the slope of the volume of moment is therefore proportional to the denominator $(a_w\, k^2-a_t\, l^2)$, where only the second term depends on the tail plane and its position. Whence, it follows: *Take care and do not let the damping moment of inertia of the tail plane* $(a_t\, l^2)$ *become equal to that of the wings* $a_w k^2$. This means for practical cases, as we shall see: Do not make the fuselage too long.

We proceed to consider the single terms which give the coefficient of stability. v_a was the volume of air which has twice the weight of the airplane. The specific weight of air is 0.080 lbs./cu. ft., therefore $v_a=\frac{W}{0.04}=25\ W$ where W denotes the weight of the airplane in pounds. This holds true, however, only for small heights; at about 20,000 feet height the specific gravity is only half this value, as is well known, and accordingly the coefficient of stability has twice the value as before. The airplane is damped much less than at small heights.

k is the radius of gyration of the airplane with respect to the horizontal transverse axis through the center of gravity. k^2 is the ratio of the moment of inertia to the mass of the airplane, and the moment of inertia is the sum of the mass of each single part of the airplane multiplied by the square of its distance from the axis mentioned. The calculation of the radius of gyration has no theoretical difficulty whatsoever; it is performed by using the list of weights. Each factory has the radii of gyration of their airplanes and can estimate the radius for a new project. There are, however, very few data published, and for the example in the next part the author is forced to take an estimation less reliable.

l is the distance between the center of gravity and the center of pressure of the tail plane. This center of pressure can be assumed to be constant at the first third of the chord.

Introducing the effect of self-induced downwash,

$$a_w = S \cdot \frac{1}{\frac{\delta\alpha}{\delta c_L} + \frac{S}{\pi b^2}} = S \frac{1}{\frac{d\alpha}{dc_L}} \tag{23}$$

as we have seen before. For the first estimation and especially for small angles, $\frac{\delta\alpha}{\delta c_L}$ can be considered to be constant and equal to 0.174. The denominator of (22) is independent then of the aerodynamical property of the wing section and depends only on the area and the shape of the wing.

The slip stream effect is not taken into consideration in this formula for the slope of the tail plane moment. This problem is so complicated that it can not be treated properly theoretically. A series of experimental tests would be useful. We will show, however, that the change due to the angle of downwash of the slip stream theoretically neutralizes about half the effect of the change of the velocity in the slip stream. The increase of the velocity in the slip stream is approximately [6]

$$w/V = \frac{1/2\ T}{D^2\pi/4\ q}$$

where

T denotes the thrust of the propeller.
D denotes the diameter of the propeller.
q denotes the dynamic pressure of flight.

The slip stream angle of downwash is given by equation (18), section 7, whence the rate of change with respect to the angle of the axis is seen to be

$$\frac{1}{2}\frac{T}{D^2\pi/4\ q}$$

Now the lift of the tail plane is proportional to the square of the velocity. Only half the effect is neutralized therefore. In reality it is often more than fully neutralized. The reason seems to be that the increase of speed is only inside the slip stream, the slip stream downwash also on top of it and below. See the very interesting results of R. & M. 629.

10. THE ASPECT RATIO.

The last chapters gave information only as to a_t, the effective area of the tail plane. Now, the same effect can be obtained with a greater area and a smaller aspect-ratio b^2_t/S_t and with a smaller area and a greater aspect ratio. We have considered only steady flight and small deflections and even then we were compelled to neglect some difficult factors. Besides, the tail plane must give its effect under extraordinary conditions of flight. The maximum lift of the tail plane and its slope at high angles of attack chiefly depend on its area, and so does the maximum elevator effect, provided the elevator is allowed to rotate sufficiently. It can be assumed that, for a certain type, a particular maximum effect of the tail plane is necessary which can not be calculated but is given by experience. Now, if the wings have a small aspect ratio b^2/S a great tail plane effect is required, and in this case it may be advisable to increase the effect of the tail plane by giving it a greater aspect ratio. If, however, due to a great aspect ratio of the wings and maybe also by the choice of a wing section of small instability, only a small tail plane effect is necessary and allowable, the choice of a small aspect ratio of the tail plane enables the designer to apply a great tail plane area without improperly increasing the stability. The aspect ratio of the tail plane ought to depend on the tail plane area desired, and this area, or, better, its product with the arm l, can be determined only from experience.

[6] Report No. 114, National Advisory Committee for Aeronautics.

11. THE SECTION.

The influence of the section on the stability of the airplane is only indirect. In the preceding discussion we have totally neglected it. It is known that the choice of an unsymmetrical section, however, has the effect of changing the angle of attack of the tail plane and in this respect is apt to change the equilibrium. The problem of the tail plane section is as difficult as that of the wing section. It is not so important, indeed, because the tail plane area is smaller than the wing area. That is one reason, we think, why almost no experimental work is extant on this subject, which is to be regretted, for this problem can only be solved by a great many tests.

Figure 3, which is taken from a publication of the author on tests with flat tail planes,[7] shows that for small angles the curves are straight and parallel and can be described by two co-

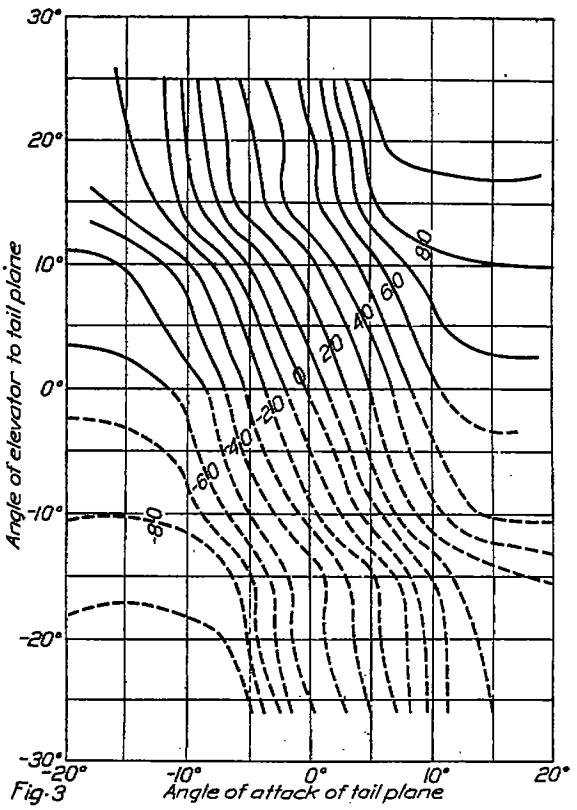


Fig. 3

efficients. We have repeatedly mentioned that at small angles the lift is proportional to the angle and the lift coefficient is about

$$c_L = \alpha \cdot \frac{0.1}{1+\frac{S}{\pi b^2}} \quad (\alpha \text{ in degrees})$$

A change of the setting of the elevator changes the effective angle of attack. If β is the angle of setting of the elevator, α should be replaced in this formula by $(\alpha + m\beta)$, where m is a certain factor, in order to take into consideration the setting of the elevator at small angles.

The angle of the elevator is measured in the same sense as that of the tail plane. This is the only natural method and the only one that leads to mathematical expressions which have the same meaning as usual. In all countries, accordingly, we find this method of measuring the setting of the elevator. We should not mention it as a matter of course if lately there had not appeared in this country one or two articles where the two angles are measured opposite to each other.

[7] Technische Berichte 1, p. 168.

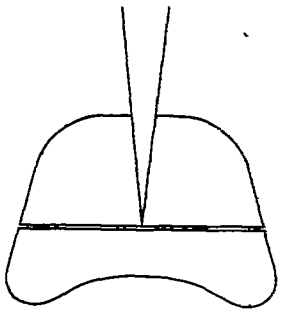
The factor m depends chiefly on the ratio of the elevator area, E, to the entire tail plane area, including the elevator, S_t, and only slightly upon the section and the plan form, and can be written $k\ E/S_t$. If the tail plane consists only of an elevator, $E/S_t=1$; and then obviously k is one too. Experiments of the author, mentioned before, show that k for flat sections equals 1.25 for E/S_t smaller than $\frac{1}{2}$. For curved sections k seems to be greater. We could find one test only, the arrangement of which allowed an exact interpretation. This test was made by the aeronautical staff of the wind tunnel of the Navy Yard, Washington, D. C., and is not yet published. This test gives $k=1.7$; the space between the stabilizer and the elevator was closed, however. The true factor seems to be between 1.25 and 1.75. This one test can not decide the question, nor does it contain anything on the moment of the elevator around the hinges. The lift coefficient of the tail plane is accordingly

$$\frac{0.1}{1+\frac{S_t}{\pi b_t^2}}\ \frac{\alpha\cdot S_t+k\beta E}{S_t} \tag{24}$$

α and β in degrees.

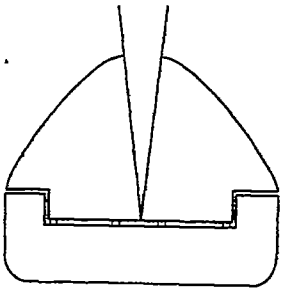
12. THE SHAPE.

The influence of the plan form of the tail plane defies calculation at present. Model tests show no great differences, nor does experience, for we find all kinds of shapes of the tail planes.



Curved elevator

Fig. 4



Elevator with projecting balance

Fig. 5

This does not include the influence of the aspect ratio, of course, which is discussed in the previous chapters.

Many elevators have a curved shape (fig. 4), for instance. There is one English publication (R. & M. No. 532) on full scale experiments with different shapes of tail planes. This author draws the conclusion from his experiments that a long rear edge is better than a long leading edge. A certain amount of damping of the elevator alone is necessary, and this damping is increased either by increasing the rear edge or by making it curved. A longer leading edge on the other hand decreases the hinge moment and the necessary deflection of the elevator for producing a particular lift. It seems to be generally known, but nowhere published, that if there is a projecting small part of the stabilizer in front of the counterbalancing part of the elevator it gives rise to oscillations and vibrations and ought to be avoided (fig. 5).

The scarcity of scientific experimental work with respect to the tail plane and its parts is compensated for by the experiments of the designers and factories, for no part of the airplane is so easily altered and exchanged as the tail plane. These results are not published directly, but they are found implicitly in all descriptions of successful airplanes. It seems useless to make an abstract of them, for it could never replace the original information.

REPORT No. 133.

THE TAIL PLANE.

PART III.

THE TAIL PLANE OF JN4H.

13. PREPARATION.

A method of calculating the tail plane can be justified only by the result, that is, if it gives the right tail plane, no matter whether the assumptions which are the foundation of the method are in reality facts or only fictitious. A quite exact agreement is not to be expected at all from our method, the calculation being too much simplified. But the result itself will be so much the better and the method the more natural, the more closely the fictitious phenomena are made to agree with the real ones. In order to give to our readers an opportunity to examine the exactness of the method, we choose as an example the JN4H. This airplane is already known to the regular readers of the National Advisory Committee for Aeronautics reports from reports Nos. 70 to 96. It happens that Mr. F. H. Norton has just finished a long series of tests with this airplane, and has determined the forces on the tail plane for many conditions of flight by measuring the pressure distribution over the tail plane. This very valuable and instructive paper furnishes us with some material for the comparison.

We have the following needed data on the airplane JN4H:

Upper main plane, area		203	sq. ft.
Lower main plane, area		150	sq. ft.
	$S =$	353	sq. ft.
Stabilizer, area		28.5	sq. ft.
Elevator, area	$E =$	21.8	sq. ft.
	$S_t =$	50.3	sq. ft.
Maximum span of wings	$b =$	43.6	ft.
Maximum span of tail plane	$b_t =$	10.26	ft.
Chord	$c =$	4.96	ft.
Gap		4.96	ft.
Stagger		1.46	ft.
"Length of fuselage"	$l =$	15.8	ft.

Center of gravity behind leading edge:

Upper wings		2.5	ft.
Lower wings		1.04	ft.
Weight	$W=$	2,281	lbs.
Horsepower		150	h. p.
Radius of gyration	$k =$	6	ft.

The angle between the two main planes is zero. A small difference would considerably change the stability in consequence of the stagger.

From these data we obtain the ratios

$$S/b^2 = 353/43.6^2 = 0.186 = 1/5.38 \quad \text{(wings)}$$
$$S_t/b_t^2 = 50.3/10.26^2 = 0.48 = 1/2.1 \quad \text{(tail plane)}$$
$$\frac{\text{stagger}}{\text{chord}} = 1.46/4.96 = 0.3$$

Mean center of gravity position; i. e., mean of ratio of distance from leading edge to chord

Upper wing 2.5/4.96 = 0.51
Lower wing 1.04/4.96 = 0.21
Load per unit area W/S = 6.5 lbs./sq. ft.

For the calculation of the mean center of gravity position it is not sufficient to take into account the fact that the upper wing area is $\frac{203}{150}=1.35$ greater than the lower wing; for at great angles of attack the upper wing, being in front, produces more lift than the lower does. The author found by experiments that for a stagger of 50 per cent of the chord this increase is about

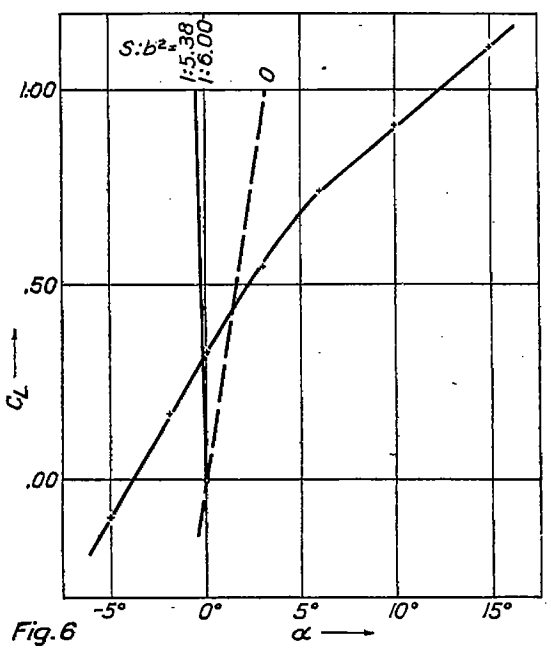


Fig. 6

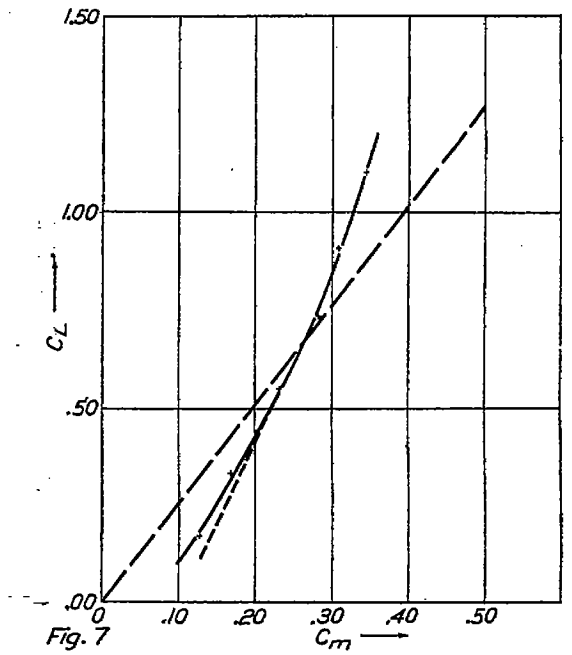


Fig. 7

20 per cent.[8] In our case we may assume, therefore, 20×30/50 = 12 per cent. Whence we obtain the mean center of gravity position

$$\frac{0.21+(0.51\times1.35\times1.12)}{1+(1.35\times1.12)}=0.39$$

But this is not all. The ratio of the lift of the upper wing to that of the lower wing is constant only for great angles of attack. For small angles the ratio is variable and depends considerably on the angle between the two wings. For parallel wings an additional instability results, and there is not yet known a method to calculate this, and we need a model test with the two main planes united to one model. It would be better still to have a test with the entire airplane, the tail plane excepted. Not having these tests, we must see how we can make the best of it and try to calculate the instability. We must content ourselves with a very imperfect method. We hope to have opportunity to discuss these questions more exactly in a later paper. They are indeed very loosely connected with the subject of this paper. From the experimental results mentioned it is seen that the moment curve of staggered biplanes is almost straight at its lower end. Besides, the inclination seems slightly greater than for no stagger. Accordingly, we will change the moment curve by adding a small increment of moment as can be seen in figure 7.

We need finally the aerodynamic properties of the wing section. The section is very similar to Eiffel 36 (N. A. C. A. Report No. 93, p. 70). The travel of the center of pressure

[8] Technische Berichte II, p. 187.

given there has to be transformed into the coefficient of the moment, and, for convenience, we will take this around the leading edge. Figures 6 and 7 give the angle of attack and the coefficient of moment around the center of gravity each plotted as abscissæ against the coefficient of lift. In order to obtain the values of angle of attack and c_m referred to leading edge for our particular airplane, we have to replace the two axes of ordinates by oblique straight lines through the origin and to measure the values of α and c_m by the horizontal distances of the curves to this line instead of to the original vertical axis. The new axis of the curve of the angle of attack runs through the point $c_L=1$, $\alpha=\frac{57.3}{\pi}\left(\frac{1}{6}-\frac{S}{6^2}\right)$.[9] The new axis for the moment diagram runs through $c_L=1$, $c_m=0.39$ (the position of the mean center of gravity). We want chiefly the slopes of the two curves. These slopes equal the original slopes increased by the slope of the new axis, that is to say, by constant values.

14. EQUILIBRIUM.

The diagram of moment shows the coefficient of moment to be zero at $c_L=0.70$. The diagram of the angle of attack gives for $c_L=.70$ the angle $\alpha=5.5°$. The downwash, according to our formula, is

$$57.3 \quad c_L \quad S/b^2 \quad 1/\pi\cdot 1.6$$
$$57.3 \quad 0.70 \quad 1/5.38\cdot 1/\pi\cdot 1.6=3.8°$$

The difference is $5.5°-3.8°=1.7°$
The actual angle as observed was 2°, showing satisfactory agreement.

15. STATICAL STABILITY.

The examination of statistical stability consists in calculating the volume of stability v_m

$$v_m=-S\cdot c\,\frac{dc_m/dc_L}{d\alpha/dc_L}+l\cdot a_t$$

From the two diagrams we take

c_L	.20	.40	.60	.80
$\frac{dc_m}{dc_L}$	.18	.18	.18	.19
$\frac{d\alpha}{dc_t}$	.21	.22	.26	.37

$\therefore$ First term of v_m -1500, -1430, -1210, -940

Factor of downwash $\eta_2=1-\frac{1.6\,S/b^2}{\pi d\alpha/dc_L}=1-\frac{1.6}{5.38\cdot\pi}\cdot\frac{1}{d\alpha/dc_L}$

.55	.57	.64	.74

Factor due to self-induced downwash

$$\eta_1=\frac{1}{1+1.83\ S/b^2}=\frac{1}{1+1.83/2.1}=.53$$

$a_t=$ S_t . 5.7 × factor of downwash × factor of self-induction.
$a_t=$ 50.3 × 5.7 × factor of downwash × factor of self-induction.

$\therefore a_t=$	835	866	960	113
$\therefore a_t l=$	1320	1370	1520	1790

Hence

$v_m=$	-180	-60	$+310$	$+850$

[9] Technische Berichte II, p. 187; N. A. C. A. Report No. 114.

It appears that the statical stability is considerable for great lift coefficients, that is, for small velocity. For high velocity the airplane is statically unstable. The length of the path of flight for an increase of amplitude to the "e" fold is for $v_m = -180$ cu. ft.

$$k \cdot \sqrt{-\frac{v_s}{v_m}} = \sqrt{\frac{36 \times 2281 \times 25}{180}} = 107 \text{ ft.}$$

The velocity is

$$\frac{L/S \cdot 850}{c_L} = \frac{6.5 \times 850}{.20} = 165 \text{ ft./sec.}$$

The time corresponding to the path 107 feet is $107/165 = 0.66$ second. This would be a short time only. The investigation of the dynamical stability will show, however, that the instability is damped out. Before proceeding to it we will compare the values obtained of v_m with those obtained from the actual flight. We think this test and its interpretation to be so important that it is worth while to discuss it in a special section.

16. CALCULATION OF STATICAL STABILITY FROM FREE FLIGHT TESTS.

The free flight of an airplane allows one to determine experimentally its statical stability by a very simple method. There are almost no preparations necessary for this test; every pilot can perform it, and the result is so important that the test ought to be made quite general with each new type. Up to now it seems to be made only occasionally.

The desired information is obtained from the observation of the position of the elevator for a series of velocities and of constant revolutions of the engine. Tests can be made at several numbers of revolutions. This observation of the elevator can easily be made, for instance, by observing the position of the control. The velocity and the density or the pressure of a Pitot tube ought to be determined too. It is hardly necessary to discuss the details of this simple test. The interpretation is founded on the fact that for an indifferent airplane the elevator would always have the same position for any steady flight. A change of the position of the elevator indicates an additional moment, and this moment can be calculated exactly enough from the proportions of the airplane if the change of the setting of the elevator is known. The moment is the same as if the airplane had no fixed stabilizer, but only the elevator increased by 25 to 50 per cent, according to the section and with the same aspect ratio as the entire tail plane and as if this fictitious elevator has rotated by the same angle. The subject is discussed in section 11. Let E be the area of the elevator and l the distance of the tail plane from the center of gravity. The volume of moment produced by the change of the elevator position is then approximately

$$\frac{5.7\, E l}{1 + 1.83\, S_t/b_t^2}$$

$V_m = \frac{5.7\, E\, l}{1 + 1.83\, S_t/b_t^2} \frac{d\beta}{d\alpha}$, where β denotes the position of the elevator. We take the required data from the report mentioned, by Norton. As an average from his approximate diagram, we draw a diagram where the set of the elevator is plotted against the velocity. The density of air is not given; we therefore take the normal density, the tests not being performed at a very great height and small differences of the density being of no importance. We recommend, however, for future tests that one always determines the dynamic pressure or the velocity or the density of air. From this diagram we take the first two lines of the table below and calculate the dynamic pressure and the coefficient of lift. The dynamic pressure is

$q = 1/390\ V^2$ lbs./sq.ft. if the velocity V is in mi./hr. The lift coefficient is $c_L = \frac{L/S}{q}$. For each lift coefficient we take the angle of attack from the diagram (fig. 6). Now we determine from a diagram the slope of the set of the elevator against the angle of attack.

The slope v_m of the volume of moment is calculated from the last formula. We obtain

c_a	20	40	60	80
α	-1.6	1.1	3.9	7.6°
$\frac{d\beta}{d\alpha}$	0.7	0.45	-0.08	-0.16
$v_m=$	$15.8\times$	21.8×1.25	$\frac{5.7}{1+\frac{1.83}{2.1}}$	$\frac{d\beta}{d\alpha}$
$\therefore v_m=$	-180	-60	$+310$	$+850$, whereas
$v_m=$	-920	-590	$+105$	$+210$ (from flight tests)

The agreement is not so very bad, inasmuch as the general type agrees. It is difficult to say which values are better, or, better said, worse. Our first calculation is not satisfactory because we had to change the curve of the moment coefficient somewhat arbitrarily. The values from the free flight test are qualified by the investigator himself as "approximately." But this lack of special data makes the method not less useful and our general discussion not less suited for general application. We think that first the method must be developed; afterwards it is not difficult to obtain the data for its application.[10]

17. THE COEFFICIENT OF STABILITY.

This coefficient is

$$c_s=\frac{v_m v_a k^2}{(a_w k^2-a_t l^2)^2}$$

We assume k, the radius of gyration, to be 6 ft., not knowing it more exactly. v_m and a_t are calculated already in the section on the statical stability. $v_a=25\ W=25\cdot 2280=57{,}000$ cu. ft. There remains only $a_w=S/\frac{d\alpha}{dc_L}$, which differential quotient is also contained in the section on statical stability. We obtain thus the following table:

	c_L	0.20	0.40	0.60	0.80
	a_w	1760	1600	1360	955
	$a_w k^2$	63,500	57,600	49,000	34,400
	a_t	83.5	86.8	95.8	113
	$a_t l^2$	20,900	21,700	23,600	28,250
	$(a_w k^2-a_t l^2)$	42,610	35,900	25,400	6,150
	$a_w k^2/a_t l^2$	0.31	0.38	0.48	0.82
Denominator					
	$1,000,000\times$	1770	1290	665	38
	v_m	-180	-160	310	850
Numerator					
	$1,000,000+$	-370	-329	635	1750
	c_s	-0.2	-0.23	0.97	46

That is to say, at a high velocity c_s is slightly negative, and in the range of ordinary flight c_s is smaller than 1/4 and each oscillation is damped out. At high angles of attack c_s grows very large in consequence of the denominator becoming almost zero. This only means oscillatory motion. It is chiefly due to the change of the slope of the well-known curve of the angle of attack against the lift coefficient. The damping moment of inertia of the wing a_w grows smaller and that of the tail plane greater. Their ratio is put into the table. The smallest ratio in the table is 0.31. The limit of stability for that ratio is $-0.31/(1-0.31)^2=-0.716$. This is considerably greater than c_s, and therefore the airplane turns out to be stable for all lift coefficients investigated.

[10] The question is also discussed by Rota, "Metodo per la determinazione sperimentale del coefficiente di stabilita longitudinale del velivole." This test gives the same result—statical stability for low velocity and instability for high velocity.

18. CONCLUSION.

Our theory of stability lies in between the imperfect calculation of statical stability only and the bulky theory of Bryan, Bairstow, and others. We believe that the simplified theory gives as good results as the latter. For the present, at least, our theory is more than sufficient considering the lack of knowledge of the aerodynamical properties. Our theory is the natural continuation of the statical procedure; the more complicated theory can be developed from ours, but we think, for scientists only, when all doubts about the coefficients have vanished.

It is remarkable that the coefficient most difficult to be obtained is that one which is also the foundation of the statical theory—v_m. Everything than can be said against the preceding calculation with respect to the uncertainty of the values proves nothing against the application of the coefficient of stability but holds true for any calculation of stability. The missing information on the radii of gyration can easily be obtained if there is general interest to procure it.

We think that the question of longitudinal stability and the necessary proportions of the tail plane are not settled by the previous discussion, but, rather, only begun. To investigate the stability of as many airplanes as possible in accordance with the discussion and to procure all information concerning the quantities occurring—that, we think, ought to be the next aim for solving the problem of the tail plane.

REPORT No. 133.

THE TAIL PLANE.

PART IV.

THE STRUCTURAL STRENGTH.

19. STATIC TESTS.

The tail plane is a small wing with a changeable section. The maximum forces per unit area on the tail plane are not very much smaller than those for the wings, although the forces on it during an ordinary steady flight are, or at least ought to be, smaller. The angle of attack is not very much smaller than that of the wings and the difference is neutralized by the greatest possible deflection of the elevator. At high angles of attack of the entire airplane the downwash decreases the effective angle of the tail plane and the greater aspect ratio of the tail plane has the same effect. But there are conditions imaginable when, in spite of it, the tail plane experiences a greater force per unit area than the wings. The only factor which definitely gives rise to a smaller maximum lift is the section of the tail plane, which has generally a smaller maximum lift coefficient. In accordance with this, the different specifications prescribe a smaller specific sand load for the tail plane than for the wings. For instance, the United States Army examines small airplanes with a sand load of 35 lbs./sq. ft. on the tail plane, but with eight times the unit load, i. e., about 6 lbs./sq. ft., which gives about 50 lbs./sq. ft. sand load for the wings.

There have been several investigations of the pressure distribution of different tail planes under different conditions. Norton measured a maximum average pressure of 2.3 lbs./sq. ft. But in the tests described he investigated only steady flight conditions, and the differences of the absolute maximum specific lifts are likely to be smaller. The tests show an odd and somewhat irregular distribution of pressure; it is almost surprising that the entire lift, the integral of these pressures, is so regular a function. The truth is, however, that the integral is not merely the sum, but it is subjected to particular physical laws, and it is not only the consequence but also the cause of the pressure distribution. We find similar phenomena in many parts of applied physics, for instance, the distribution of stresses or of brightness in several directions. Most of the results of the investigation of the pressure distribution refer to small lifts and hence are not applicable to our present problem. These distributions at the same time are the more irregular ones, as is to be expected.

The magnitude of the pressure at one particular point of the surface is not important in the consideration of the stresses in the structure. The cover will always be strong enough, and the sand load tests do not even try to test these sorts of stresses. There is always an uncertainty about the forces on the linen, for they depend on the pressure inside, and this pressure is generally not known. The pressure over small parts of the surface affects only the secondary members of the structure—the ribs. They are mostly strong enough, and the cover tends to distribute the differences of pressure and to neutralize them partially. Nor is the breaking of one rib the chief danger. The stresses of the primarily structural members of the beams depend almost solely on the magnitude of the entire force and on the position of its center of pressure. The moments of the air forces, while their sum is small, are not dangerous. Now there is no reason why the center of pressure at high lift should be situated very

far from its ordinary place; that is, the first third of the chord, the elevator included. The tests confirm this rule. The transversal center of pressure lies, roughly speaking, near the center of gravity of each side. The most natural method for performing the sand load test is, therefore, the application of the same rules of distributing the load as with the wings. There is no advantage in examining too carefully the method of distributing the sand, if only the center of gravity is in the right place, for the pressure produced is irregular and in no way in strict conformity with the distribution of the sand. The elevator, indeed, ought to be examined separately, too, chiefly in order to test the different contrivances for its rotation.

There is one particular condition of steady flight where the forces on the tail plane are considerable. This is flight vertically downwards, as occurs in air fights. For the calculation of the force on the tail plane we need the moment coefficient for zero lift [11] and also the drag coefficient; the knowledge of the center of pressure is less useful. The entire airplane is carried by the drag of the wings and that of the other parts, and we can calculate in each single case the fraction of the entire weight carried by the wings. The lever arm for that fraction is $c \cdot \frac{c_m}{c_D}$; that is, the chord multiplied by the ratio of the two coefficients mentioned. This arm, multiplied by the share of the wings in carrying the entire weight, gives the lever arm for the force equal to the entire weight, and the force on the tail plane has to produce the same moment around an axis lying in the wings. The resulting lever arm is sometimes equal to the chord, but can be as great as 2.5 chords. It depends on the type of the airplane and on the section. The prescribed safety can be smaller than that resulting from this calculation. The designer ought to give attention to this fact and choose a proper section, and the examination ought to be extended to this point.

20. THE OSCILLATIONS.

A phenomenon not yet much investigated, which tends to increase the structural stresses, is that of the oscillations and vibrations. The tail plane is more subject to them than the wings. There are two different kinds.

The motion of the passing air generally is not perfectly steady, even if the flight is. It changes rhythmically, and so does the force produced by the motion. The period depends on the shape of the body and on the velocity; it is l/V, where l denotes a characteristic length. This length is never very different from the transverse dimension of the body—in this case the thickness of the tail plane. There are not many experimental investigations on it.[12] The oscillation is only dangerous if its period agrees with the period of oscillations of the body the force acts on. Assuming the velocity to be 150 ft./sec. and the characteristic length 3 inches, we should obtain 150/0.25 oscillations—that is, 600 oscillations per second. That is very high, and, therefore, this kind of oscillation will generally not be dangerous. The period may, however, be greater at the burble point of the elevator. This kind of oscillation and the stresses arising from them can not be examined by moving the tail plane through water. The period is increased then and so is the damping of the water.

More important than these oscillations are those whose period is determined by the dimensions of the tail plane and whose amplitude is increased by the air forces. It is not possible to discuss all possible oscillations of this kind generally; each one must be examined with respect to the sign of the damping by air. As an illustration, consider a tail plane with two beams and let their proportions and the distribution of mass be such that the two beams have the same period of elastic oscillation. If they oscillate at the same time, but the beam in front is, say, 90° ahead in phase, the lift of the tail plane is greater when it moves upwards than when it moves downwards, and thus the air forces will increase the amplitude. The stresses can grow so intense that a fracture occurs. Whence it follows that the two beams, or their ends, must not have the same period. This oscillation and several other ones can not be discussed thoroughly without exact information for each single case. We, therefore, must confine our-

[11] Unfortunately the tables of the N. A. C. A. Reports Nos. 93 and 124 do not contain this very important coefficient.

[12] See paper by de Bothezat, N. A. C. A. Report No. 97.

selves to this general remark. It will be favorable with respect to such possibilities if the deflections are small, and it will be possible perhaps to decrease the prescribed load if the deflections at the same time are limited. The question of such oscillations ought to be examined experimentally by observing all possible kinds of oscillations and measuring their periods and, if possible, their damping. It is only then that a more detailed discussion can be made, and that we are less likely to forget the most important type of oscillation.

21. CONCLUSION.

There are many things concerning tail planes which we have not mentioned or not thoroughly discussed. The last part of this paper in particular is comparatively short, but the things discussed in it can not be decided without more experiments. We have tried to bring out only such matters as are not well known.

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