REPORT No. 153

CONTROLLABILITY AND MANEUVERABILITY OF AIRPLANES

By F. H. NORTON and W. G. BROWN
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SUMMARY.

This investigation was carried out by the National Advisory Committee for Aeronautics at Langley Field for the purpose of studying the behavior of the *JN4h* airplane in free flight under the action of its controls and from this to arrive at satisfactory definitions and coefficients for controllability and maneuverability. The method consisted in recording the angular velocity about the three axes, together with the air speed, control positions, and acceleration. An analysis of the records leads to the following results:

1. Both the maximum angular velocity and maximum angular acceleration are proportional to the displacement of the controls.
2. Both the maximum angular velocity and maximum angular acceleration for a given control movement increase with the air speed, rapidly immediately above the stalling speed, then nearly proportional to the speed.
3. The time required to reach each maximum angular velocity is constant for all air speeds and control displacements for a given airplane.
4. The minimum time required to reverse the direction of an airplane by a steeply banked turn is a rough indication of its general maneuverability.
5. Doubling the lateral moment of inertia of an airplane increases the time required to bank to 90°, with a maximum control angle, by only 10 per cent.
6. Controllability has been defined as applying to the moment produced about the center of gravity by the action of the controls and maneuverability as the resultant motion.
7. A simple method is described for measuring the controllability coefficients, $\frac{dL}{di_x}$, $\frac{dM}{di_y}$, and $\frac{dN}{di_z}$, and the maneuverability coefficients t_ϕ, t_θ, t_ψ and $\frac{(V\ \min)^2}{g}$.

These results are of practical value, as they give a quantitative means of measuring airplane maneuverability and controllability, which will allow designers to accurately compare the merits of different airplanes.

INTRODUCTION.

The combat airplane requires the four aerodynamic essentials of high rate of climb, high maximum speed, high diving speed, and maneuverability. It is not within the scope of this report to treat the relative importance of these quantities, but it may be stated that maneuverability in certain kinds of fighting is of extreme importance. The term maneuverability itself has been rather ill defined, but it is usually taken to mean that property of an airplane which allows the pilot to direct it into the desired position in the shortest possible time. Except for a few rather incomplete tests made by the British [1] to record the time required to bank an airplane up to a given angle, there has been no quantitative measurements made on maneuverability, this quantity being determined only by the personal opinion of the pilot or by the comparison of two machines when maneuvering together in the air. It is very difficult for a designer to improve the maneuverability of the airplane unless he can obtain actual quan-

[1] R. & M. Nos. 413 and 441.

titative measurements on the present machines to determine what factors are responsible for an improvement in this quality.

Controllability has usually been associated by pilots with the response of the airplane to correcting movements of the controls, especially at low speeds. The controllability of any airplane is satisfactory at high speeds, but it falls off more or less rapidly as the stalling speed is approached. Some work has been done in free flight to measure the angle at which the controls must be set to balance a known static movement applied to the airplane by means of weights, for determining the controllability under various conditions.[2,3] Unlike maneuverability, controllability may be measured directly in the wind tunnel by the moments exerted about the center of gravity with various positions of the controls. Altogether controllability is on a much more scientific basis than maneuverability.

Below are given a list of the principal references on controllability and maneuverability:

(1) The factors that determine the minimum speed of an airplane, N. A. C. A. Technical Note No. 54.
(2) Control in circling flight, N. A. C. A. Report No. 112.
(3) Practical stability and controllability of airplanes, N. A. C. A. Report No. 120.
(4) Full scale stability experiments, R. & M. Nos. 326 and 505.
(5) The longitudinal control of an airplane, R. & M. Nos. 470 and 629.
(6) The longitudinal control of "X" airplane, R. & M. No. 638.
(7) Control as a criterion of longitudinal stability, R. & M. No. 636.
(8) SE5A with modified control surfaces, T1504.
(9) The longitudinal motion of an airplane, R. & M. No. 121.
(10) The control of a laterally stable and a laterally unstable airplane, R. & M. No. 209.
(11) Lateral control of an aeroplane, R. & M. Nos. 413 and 441.
(12) Maximum control of elevators of different sizes, R. & M. No. 641.
(13) Full scale experiments with elevators of different sizes. R. & M. No. 409.
(14) Full scale experiments with different shapes of tail plane; R. & M. No. 532.
(15) Measurement of control moments on an airplane in flight, "Zeitschrift fur Flugtechnik und Motorluftshiffahrt," vols. Nos. 21 and 22.

The present report aims to devise some logical and universal definitions of maneuverability and controllability and means for easily making quantitative measurements of these qualities of the airplane. To do this the motion of the airplane was studied experimentally during various maneuvers and a set of coefficients were determined which would completely express the maneuverability and controllability of a given airplane. In a subsequent report the values of these coefficients will be determined for a considerable number of airplanes so that we may be able to determine more closely than at present what features of design lead to great maneuverability and controllability of the airplane.

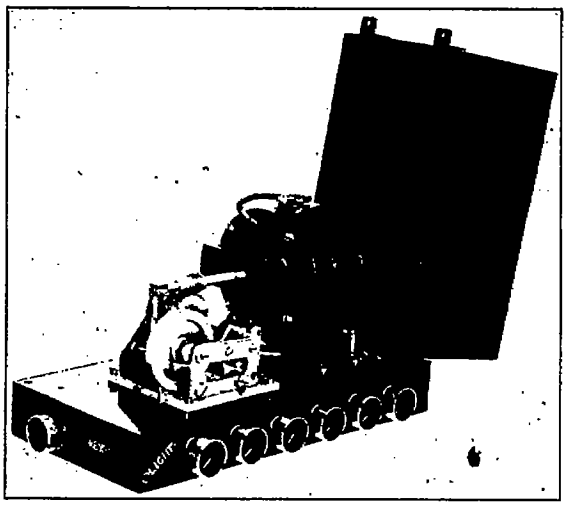


FIG. 1.—Electric chronometer for synchronizing records.

METHODS AND APPARATUS.

The instruments used in this investigation consisted of an angular velocity recorder, a recording air speed meter, a control position recorder, and an accelerometer. All of these instruments have been fully described in preceding reports.[4] The instruments were all synchronized by illuminating a timing light in each instrument simultaneously by means of a motor-driven chronometer. (Fig. 1.) The measurements of rolling angle were made by pressing a key at the instant the inner landing wires were aligned with the horizon, thus lighting the timing lights for an instant.

[2] Control in circling flight, N. A. C. A. Report No. 112.
[3] Measurement of control moments on an airplane in flight, "Zeitschrift fur Flugtechnik und Motorluftshiffahrt," vol. Nos. 21 and 22.
[4] N. A. C. A. Reports Nos. 99, 100, 112, and Technical Note No. 64.

In making the maneuverability tests the airplane was first flown steadily at the desired speed; all the instruments were started together with a common switch; then the given controls were moved to a definite angle as suddenly as possible and held at that angle until the machine had rotated through approximately 90°. This procedure was then repeated for various air speeds and motor speeds and with various angles of control movement. It may be contended that in these tests the personal element was not entirely eliminated as this might have come in, in the rapidity with which the controls are moved over. The control position records showed however, that the times to move the controls as quickly as possible to a given angle were very consistent; but if greater uniformity were desired, it would be quite easy to move the controls by means of a spring of known tension.

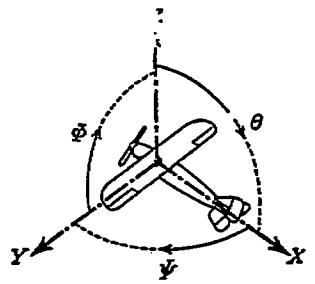


FIG. 2.—Giving the positive direction of axis and angles as shown by arrows.

The scope of the tests consisted in taking records when the machine was rotated about the X, Y, and Z axes with the ailerons, elevators, and rudder, respectively. Figure 2 shows the positive directions of axes and angles as shown by arrows. In each case the air speed was varied between 40 and 80 miles per hour with the engine turning at 1,400 revolutions per minute, and three separate control angles were used up to the maximum possible movement. In some cases the tests were repeated with the engine throttled.

Of the records taken, only those from the angular velocity recorder are of direct use, the others being merely to check the accuracy of the pilot's flying. Some examples of these records about the X axis are shown in Figure 3. As these records are angular velocities plotted against time, the height of the curve at any point from the zero line will give the angular velocity in radians per second, the slope of the line will give the angular acceleration at that time, and the area under the curve from any given time will give the angular displacement. In most of the records it was necessary only to measure the maximum angular velocity and the maximum angular acceleration.

FIG. 3.—Motion about the X axis.

The precision of the readings of angular velocity are of the order of 5 per cent and the precision of the angular acceleration is of approximately the same amount. The control positions were recorded with a precision of approximately 1°, while the timing is precise to two-fifths of a second. The values as plotted on the curves of angular velocity and acceleration are often as much as 20 per cent from the mean curve, which can be accounted for by unsteady air conditions or by a slight movement of the other sets of controls which were supposed to have been held stationary. The curves, however, plotted through these points should have a precision in all cases of better than 10 per cent, which is quite sufficient for the work in hand. It might be noted here that since these tests were completed the angular velocity recorder has been altered so that its errors have been decreased to the neighborhood of 1 per cent. Also in further tests of this kind the precision of the results can be considerably increased by locking the controls which are not being moved and by giving a more definite movement to the control under consideration.

RESULTS.

MOTION ABOUT THE X AXIS.

If L' be the moment due to the controls about the X axis it may be assumed that the angular motion of the airplane is determined by the following equation:

$$L = mK_A \frac{dp}{dt} + L_p p + L_q q + L_r r + L_u u + L_v v + L_w w,$$

where the derivatives have the usual meanings.

The motion about the X axis is a symmetrical case, while the motion about the other two axes are unsymmetrical. For this reason we will consider at first only the motion about the

X axis in order to illustrate the general behavior of the airplane when acted upon by the control surfaces.

If the ailerons are suddenly turned to some definite angle in steady flight and held there while the airplane rolls, the moment L is approximately expressed by:

$$L = K_A^2 m \frac{dp}{dt} + L_p\, p.$$

Where

L is the rolling moment due to the ailerons.
$K_A^2 m$, the lateral moment of inertia.
L_p is the damping coefficient due to roll.
p is the rolling velocity.

At the first instant the ailerons are moved, p will be very small, but $\frac{dp}{dt}$ will be large, so that

$$L = K_A^2 m \frac{dp}{dt}.$$

After a short interval, however, p becomes constant and

$$L = L_p\, p.$$

Perhaps the behavior of the machine can be made clearer by an actual example. Data were taken on a *JN4h* by the methods described previously in this report to show the action of

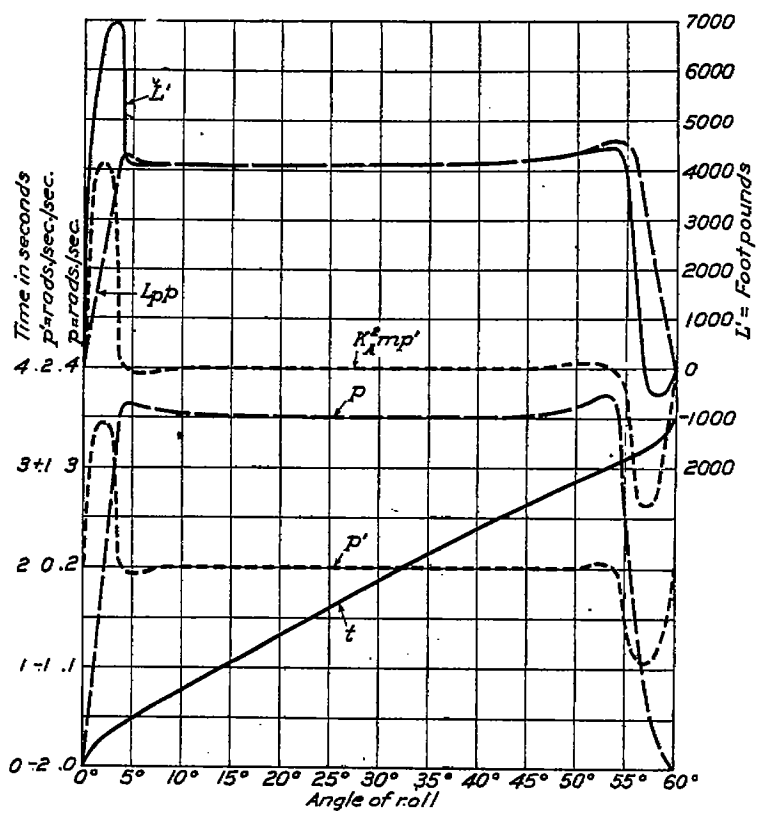


FIG. 4.—Rolling with ailerons up to a 60° bank on a JN4h at 80 M. P. H. Aileron angle=13°.

the airplane when rolling up to a 60° bank at an air speed of 80 miles per hour when the ailerons are suddenly turned to 13° and the other controls are held neutral. (Fig. 4.) The last end of the record is not as accurate as the other portions as the sideslip at this time had become large.

It will be seen that the angular velocity rises to a constant value of 0.35 radian per second in 4°, or 0.4 second, which is maintained up to 53°, when it decreases to zero. The angular acceleration reaches a maximum of 1.5 radians per (second)^2 at 2° and falls off to zero at 4°, and it remains zero until the rotation begins to be stopped at the end of the record. If the value of L_p is taken as -325 [5] for 80 miles per hour, the curve of L_p is similar in shape to that of p and the curve of $K_A^2 m \frac{dp}{dt}$ is similar to that of $\frac{dp}{dt}$. L is evidently the sum of these curves and is characterized by a high peak of 7,000 foot-pounds at 3°, which at once falls to a constant value of 4,100 pounds. It is interesting to note that the value of L when there is no rotation and with the ailerons at 13° is 9,000 foot-pounds.[2] From this data we may approximately plot the variations in L with the variations in p. (Fig. 5.) It is evident that L falls off as p increases, although the form of the curve is not determined here with any great accuracy.

If the angular velocity in roll is plotted against time, Figure 6, there is an approximately uniform increase up to a certain point and then a constant value. As will be shown later the

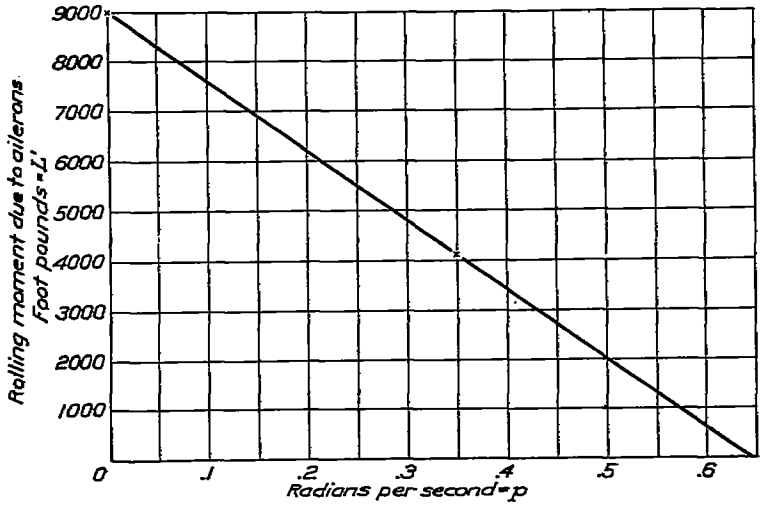


FIG. 5.—Effect on L' of changes in p, ailerons at 13°, 80 M. P. H.

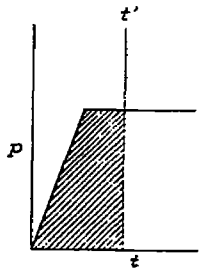


FIG. 6.

time required to reach the constant value is always the same under all conditions for a given airplane, so that the number of degrees turned in a unit time, that is, the area under the curve, will be proportional to the maximum angular velocity. Therefore we may conclude that the maximum angular velocity and the angle turned in a unit time are equivalent measures for a given airplane.

It might be thought at first that the quickest way to roll an airplane would be to put the ailerons hard over as suddenly as possible, but control-position records show that the pilot when making a barrel roll in which it is necessary to produce the greatest possible rolling moment, does not use the ailerons.[6] The manner in which the rolling moment is produced can be explained most clearly by taking a concrete example. Let us consider an airplane of the *JN4h* type which has the following characteristics:

$K_A = 6.3$ feet.
$m = 72$ slugs.
$L_p = -325$ at 80 miles per hour (ft. lbs. sec.).
$L_v = 0.4$ at 80 miles per hour and 20° yaw. (ft. lbs. sec.).
$a = 4.5$ g. (ft. sec.).

[2] Control in circling flight, N. A. C. A. Report No. 112.
[5] From a model test of a *JN3*, probably too large. Free flight determinations of this coefficient are now being made.
[6] A study of airplane maneuvers, with special reference to angular velocities, N. A. C. A. Report No. 155.

In performing a barrel roll the pilot first pulls back on the stick in order to produce a large normal acceleration, and a little later the rudder is kicked hard over, which produces a considerable angle of yaw at the same time that the acceleration reaches a maximum. Tests which have been made on this type of machine to determine the value of L_v at a given angle of yaw show that the value of L produced in this way is

$$L = 4.5\ mL_v v.$$
$$= 4.5 \times 72 \times 0.4 \times 118 \sin 20°.$$
$$= 5200 \text{ foot-pounds.}$$

This moment at first seems small, as it is only a third of the maximum static moment producible by the ailerons alone. There is this important difference, however; the aileron moment falls off to almost insignificant proportions as soon as rolling begins, whereas the moment due to yaw maintains its high value for a considerable length of time.

In order to bring together the data relating to the motion of the airplane about the X axis there has been plotted in Figure 7 the following quantities:

(*a*) The angular velocity.
(*b*) The maximum angular acceleration.
(*c*) The time required to roll up to 36°.
(*d*) The time required to roll up to 14°.
(*e*) The time required to reach a constant value of angular velocity.

All of these quantities are plotted on a common base of aileron angle in degrees. The results are further summarized in Figure 8 where a curve is plotted of the slope of the preceeding curves of angular velocity, against air speed. A similar curve is plotted for angular acceleration and a third curve for static moment.

An examination of these curves leads to the following conclusions:

1. The angular velocity rises quickly to a steady value which is maintained up to a large angle.

2. The maximum angular velocity is in all cases proportional to the aileron angle for any given air speed.

3. The time required to reach the maximum angular velocity is constant, at least up to an aileron angle of 18°, under all conditions, and for the $JN4h$ is 0.40 second.

4. The maximum angular acceleration is proportional to the aileron angle for any given air speed.

The values of both $\frac{dp}{di_x}$ and $\frac{dp^1}{di_x}$ are zero at stalling speeds, increasing rapidly at first and then more slowly; the increase from stalling speed being approximately proportional to the square root of the air speed. The time required to bank to a given angle decreases as the air speed increases and is a minimum with an aileron angle of approximately 20°.

MOTION ABOUT THE Y AXIS.

The consideration of the maneuverability about the Y axis is not as simple as for the symmetrical case. We have here not only to consider the angular movements of the machine itself, but also the form of the path taken by the airplane. For example, if we consider an airplane banked vertically and turning in a uniform horizontal circle, the change in direction of path must be equal to the angular velocity of the airplane. It is obvious that if the angular velocity of the airplane was maintained at a higher value than the angular velocity of the path, in a short time the airplane would be rotated around until it was moving through the air tail first, which would of course be an impossible condition. Longitudinal maneuverability depends then largely upon the minimum radius of turn for that particular machine.

In order to make a loop or vertically banked circle with the smallest radius of curvature, the airplane must have at all times an angle of attack corresponding to the maximum lift. The radius of turn under these conditions will then be equal to the square of the minimum flying speed divided by the acceleration of gravity, as will be shown later. Few airplanes have powerful enough longitudinal control to attain the angle of maximum lift in steady flight and this is evidenced by the elevator curves given in Figure 9. Actually, however, an airplane can

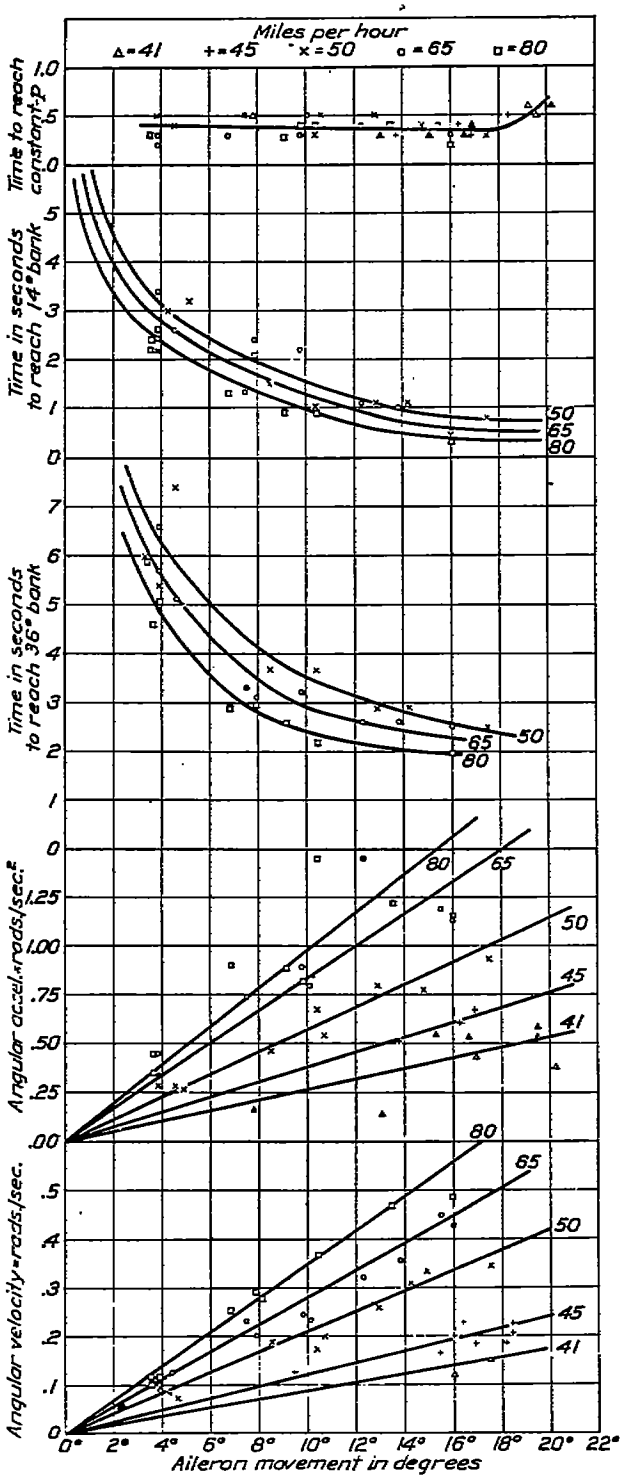


FIG. 7.—Motion about X axis.

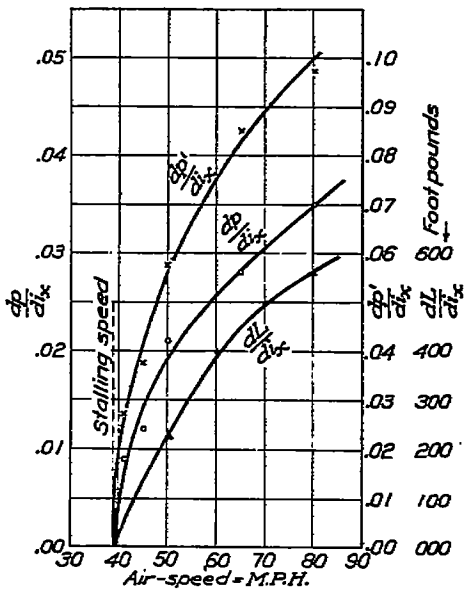


FIG. 8.—Motion about X axis.

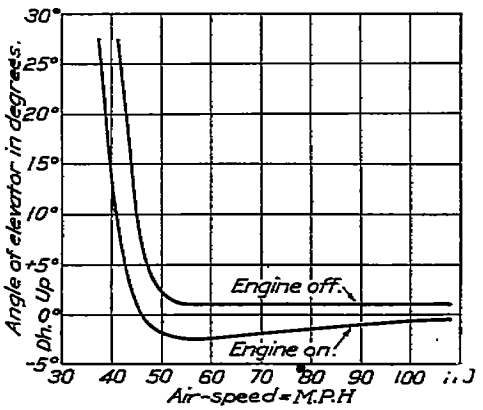


FIG. 9.—Elevator angle necessary to maintain a steady speed in the JN4h. From free flight tests.

be brought for a short time up to, or even beyond, the angle of maximum lift due to the impulse of the angular momentum. This is shown clearly by the curves in N. A. C. A. Report No. 105. Such a method of momentarily increasing the angle of attack is of little practical value in maneuvering, as the drag of the machine is enormously increased for a considerable length of time. For a satisfactory longitudinal control it is necessary, therefore, to have powerful enough elevators to overcome the natural stability of the airplane. This problem is a difficult one because of the great diving moment exerted by the wings and tail plane at high angles of attack. This is evidenced by Figure 9, referred to before. To decrease the air speed below 50 miles an hour requires a large elevator angle even on this airplane which has an especially powerful control. The only solution which presents itself is to increase the area of the elevator or even to eliminate the tail plane entirely as is done in the Salmson biplane.

When an airplane is flying in a correctly banked circle the angle of attack of the wings must be greater than for the same velocity in level flight in order to balance the component due

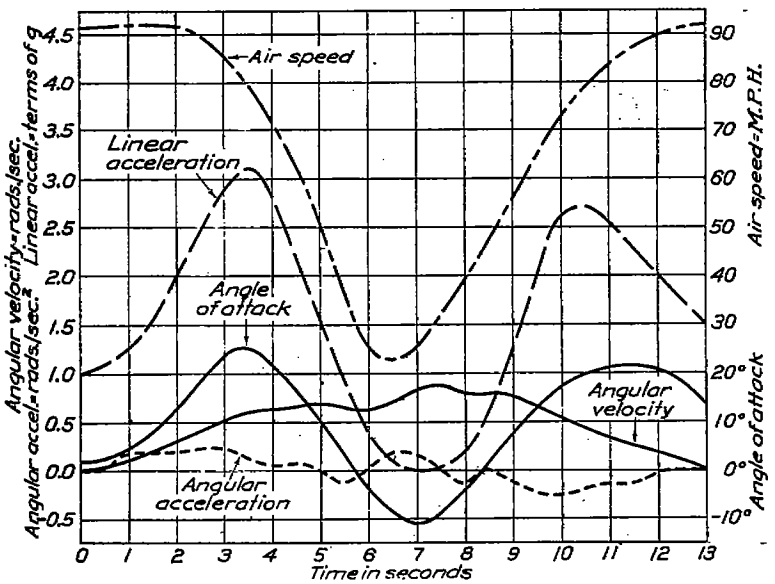


FIG. 10.—Performance of a JN4h airplane during a loop.

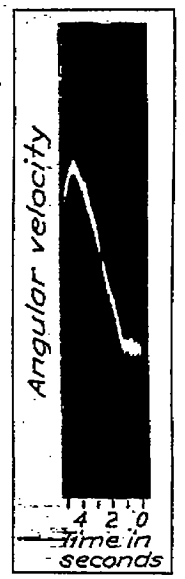


FIG. 11.—Motion about the Y axis.

to the centrifugal force. If F be the centrifugal force, W the weight of the airplane and V_1 and V_2 the velocities necessary for equilibrium in level flight and in the turn respectively for the same angle of attack, we can say:

$$F = W\frac{V_1^2}{V_2^2}$$

and the radius of turn r is

$$r = \frac{WV_1^2}{gF}$$

or

$$r = \frac{W}{g} \cdot V_1^2 \frac{V_2^2}{WV_1^2} = \frac{V_2^2}{g}$$

if V_2 is the minimum speed in level flight then r will be the minimum radius of turn.

In a vertically banked turn or a loop the minimum radius is independent of the air speed, so that the minimum time taken to complete the maneuver will be inversely proportional to the air speed. The air speed in rapid maneuvers is limited however by the strength of the airplane, for the loading on the wings equals $mg\frac{V^2}{V_2^2}$.

As an example let us assume that V_s is 40 miles per hour and the maximum allowable acceleration is 4 g., that is, $V=80$ miles per hour. The minimum radius of turn in a vertically banked circle will then be 107 feet, thus requiring 5.8 seconds to complete the circle.

To show more clearly the behavior of an airplane in a longitudinal maneuver there is plotted in Figure 10 all of the important characteristics during a loop as taken by recording instruments. Except for the angle of attack, which was separately measured on a similar loop, all the records were taken simultaneously. Examining the curve of angular velocity it is evident that there is a uniform acceleration up to 4 seconds, a nearly constant value for 5 seconds, and a uniform deacceleration for the last 4 seconds. It will also be noticed that at times the angle of attack reaches values considerably beyond that of maximum lift.

The angular velocity of an airplane was studied when the elevator was pulled up suddenly in the same manner as described for the ailerons. The resulting curves, however, differ considerably from those about the X axis as shown in Figure 11. In this case the angular velocity, q, rises rapidly to a maximum and then falls off more slowly. The reason that q is not maintained constant after arriving at its maximum value is due to the fact that the air speed falls off to

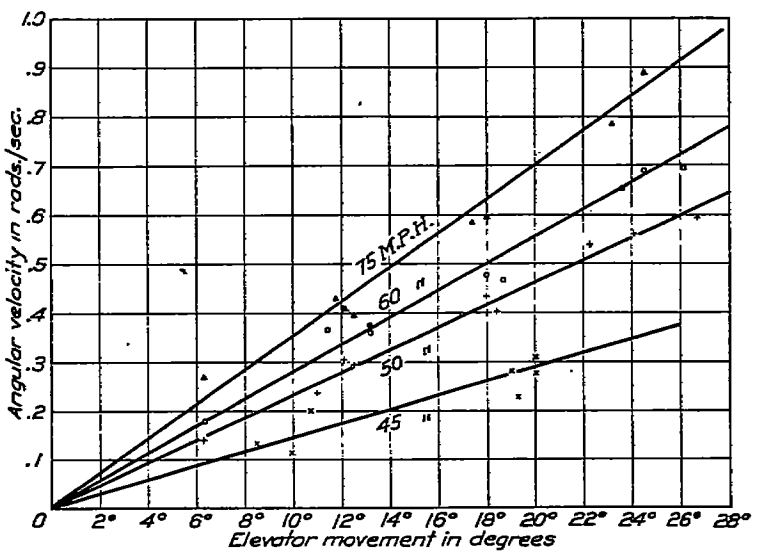


FIG. 12.—Motion about Y axis, engine on. R. P. M.=1,400.

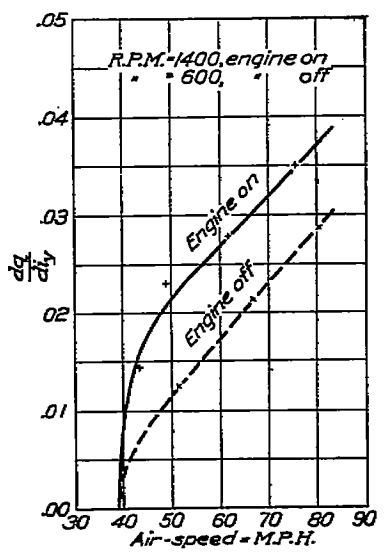


FIG. 13.—Motion about Y axis for JN4h.

low values after a few seconds and a large diving moment is set up. In the same way as for the ailerons, curves are plotted in Figures 12 and 13 to show the characteristics of the angular motion about the Y axis.

An examination of these curves leads to the following conclusions:

1. The maximum angular velocity for a given air speed is proportional to the change in elevator angle.

2. The maximum angular velocity is reached in approximately the same length of time for all conditions of flight, namely, 1 second.

3. The values of $\frac{dq}{di_y}$ when plotted against air speed rise rapidly from the stalling speed to 50 miles per hour, then more slowly in proportion to the air speed. The values with the engine throttled are about 0.017 unit lower than with the engine on, due to the decreased speed of the air over the tail in the latter case.

MOTION ABOUT THE Z AXIS.

The motion about the Z axis is, like the preceding case, unsymmetrical. In actual flight no maneuvers are made about this axis of any duration, so we need concern ourselves only with the small angular movements, and can neglect the cross wind force.

In Figure 14 is shown a typical curve of angular velocity against time produced by kicking the rudder over suddenly. It will be noticed that it is very similar to the preceding case—the curve rising to a sharp peak. The angular velocity falls off because of the large restoring moment brought into play when yawing, and because of the loss of air speed.

Curves are plotted in Figures 15 and 16 as for the preceding cases to show the angular velocity for various air speeds and rudder angles. The tests were carried out with engine on

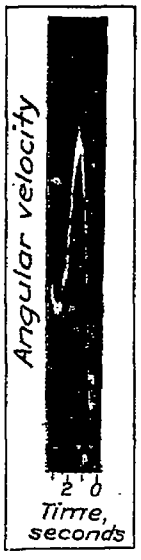


FIG. 14.—Motion about the Z axis.

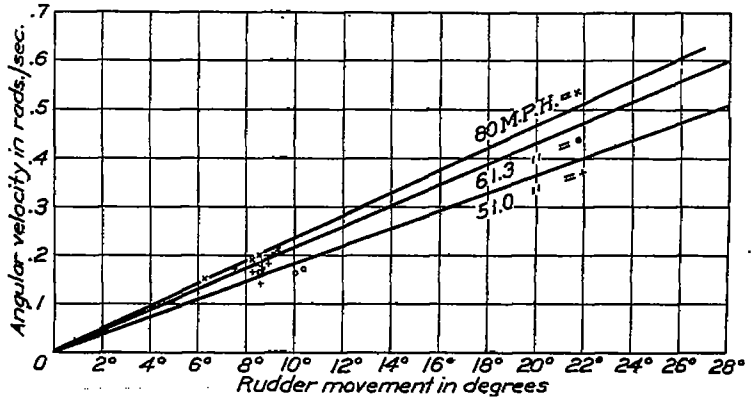


FIG. 15.—Motion about the Z axis, engine on.

and off, but the results from the latter are not very satisfactory. The following conclusions can be arrived at from an examination of the curves:

1. The maximum angular velocity for a given air speed is proportional to the change in rudder angle.

2. The maximum angular velocity is reached in approximately the same time for all conditions of flight, which for this machine is about 1.5 seconds.

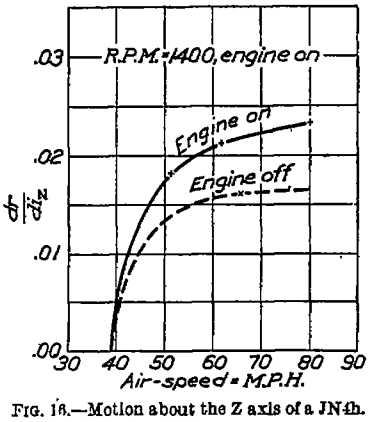


FIG. 16.—Motion about the Z axis of a JN4h.

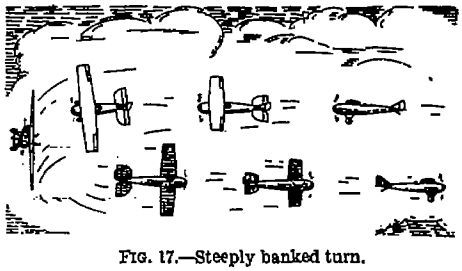


FIG. 17.—Steeply banked turn.

3. The values of $\frac{dr}{dv_z}$ when plotted against air speed rise rapidly from the stalling speed, but more slowly at higher speeds. With the motor throttled the values are considerably lower.

MOTION ABOUT ALL THREE AXES.

In general a maneuver requires the use of all controls and as these interact on each other it is difficult to analyze their separate effects. The general behavior of an airplane however has been carefully examined during all of the usual maneuvers in flying by means of recording

instruments.[7] It seems to be generally accepted that the time required to completely reverse the direction of the airplane is an important factor in a combat. As there are several methods of accomplishing this, they were all tried out on several machines and with several pilots.

The simplest way to reverse the direction quickly is to make a steeply banked turn (Fig. 17). The times required to accomplish this, averaged from a large number of runs, are given in the table below. The times were taken with a stop watch from the plane, and covered the space from horizontal flight in one direction to that in the other.

Banked turns.

Pilot.	Airplane.	Air speed at start.	Time for 180° in seconds.	Date.
Carroll	JN4h	60	13.1	Dec. 3, 1921.
Do	JN4h	70	13.4	Do.
Do	JN4h	80	15.1	Do.
Do	VE7	70	10.5	Do.
Do	VE7	80	8.5	Do.
Do	VE7	90	8.5	Do.
Do	JN4h	60	16.8	Dec. 7, 1921
Do	JN4h	70	15.5	Do.
Do	JN4h	80	14.6	Do.
Do	SE5	65-85	8.0	Dec. 6, 1921.
McAvoy	VE7	70	13.9	Dec. 7, 1921.
Do	VE7	80	12.1	Do.
Do	VE7	90	10.1	Do.

All of the times are the average of three to six readings.

The times show that the turn can be made more rapidly at higher speeds than at low, but the difference is less than was indicated previously in this report from theoretical considerations. This is undoubtedly due to the fact that the pilot unconsciously eases off at the higher speeds due to the large accelerations experienced. It is also evident that the time may vary by 10% between tests with the same machine and pilot on different days, and as much as 20% between different pilots. The comparison between the three machines used gives a ratio of times of about 15, 9, and 8 for the *JN4h*, *VE7*, and *SE5*, respectively. This ratio expresses very closely the average pilot's opinion of the maneuverability of the respective machines. It should be noted however that this is not a very scientific way of comparing machines as the times depend to a considerable extent on the pilot's willingness to punish his airplane. It serves however to give quickly an approximate value for the general maneuverability of an airplane.

While making the turns referred to above it was noticed that when all of them were divided into two groups, one for turns into the wind and the other for turns down wind, that there was a very consistent difference in time, the turns into the wind taking on the average 0.7 second less than those with the wind. Theoretically, of course, there should be no such difference and the explanation of it is at present unknown. It may be due to the structure of the air or simply to the ground speed unconsciously affecting the pilot or observer.

Another method of reversing the direction of flight quickly is an evolution termed a "wing over" as shown in Figure 18. In the table below are given the times required for this method of turning:

Pilot.	Airplane.	Air speed at start.	Time for 180° in seconds.	Date.
Carroll	JN4h	70	10.0	Dec. 3, 1921.
Do	VE7	80	9.1	Do.
McAvoy	VE7	80	13.7	Do.
Do	VE7	80	11.3	Do.
Do	VE7	95	8.0	Do.
Carroll	JN4h	70	10.1	Dec. 7, 1921
Do	SE5	80	7.5	Do.

[7] A study of airplane maneuvers, N. A. C. A. Report No. 155.

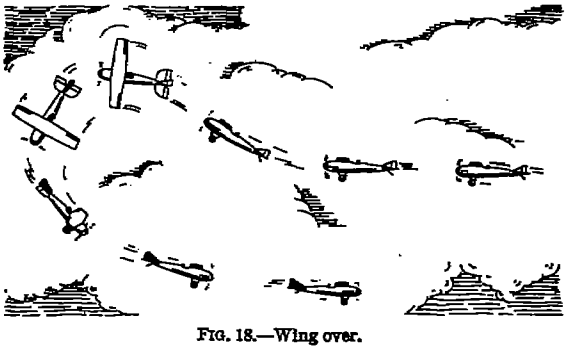
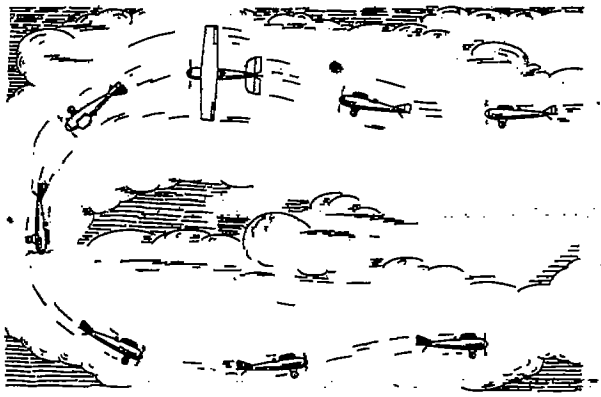


FIG. 18.—Wing over.

The times given here are not as consistent, as for the banked turn due to the greater complication of the maneuver, so that the relative values of the machines can not be judged accurately. On the whole this maneuver is one or two seconds quicker than the banked turn.

A third method of turning is the reverse turn (fig. 19), the times for which are given in the following table:



Pilot.	Airplane.	Air speed at start.	Time for 180° in seconds.	Date.
Carroll	JN4h	70	7.8	Dec. 3, 1921
Do.	VE7	80	9.2	Do.
McAvoy	VE7	80	7.7	Dec. 7, 1921
Carroll	JN4h	70	9.8	Do.

FIG. 19.—Reverse turn.

The times given here are not very consistent, as they depend on the pilot's willingness to punish the machine. There is no doubt, however, that this is the most rapid method of turning, but it has the disadvantage in combat work of necessitating a considerable loss in altitude and putting the plane for a time in an attitude in which it is not controllable.

THE EFFECT OF MOMENT OF INERTIA ON MANEUVERABILITY.

In designing airplanes for combat work where great maneuverability is desired it has always been recognized that a small moment of inertia was to be desired. Just what effect the moment of inertia has on maneuverability has been unknown, so that it seems desirable to study this problem quantitatively here. All the discussion and experiments refer to the motion about the X axis for simplicity but the other motions are analagous. We have seen that the rolling moment due to the ailerons is submitted to the condition:

$$L' = m\left(K_A^2 \frac{dp}{dt} - L_p p\right).$$

From the curve connecting L' and p shown in Fig. 5 we may also write:

$$L' = Kp + c$$

where K and c are the constants determining the straight line.

Then:

$$mK_A^2 \frac{dp}{dt} - p(mL_p + K) - c = o$$

and

$$p = \frac{c\left[e^{\frac{(K+mL_p)t}{mK_A^2}} - 1\right]}{K + mL_p}$$

if $t = o$ when $p = o$.

The angle rotated through in time t will be:

$$\phi=\int_0 p\,dt=\frac{c}{K+mL_p}\int_0^t\left[e^{\frac{(K+mL_p)t}{mK_A^2}}-1\right]dt=\frac{c}{K+mL_p}\left[\frac{K_A^2m}{K+mL_p}\left(e^{\frac{(K+mL_p)t}{K_A^2m}}-1\right)-t\right]$$

Taking from Figure 5 values of c and k corresponding to an aileron setting of 13° and an air speed of 80 miles per hour, curves of ϕ and t have been plotted in Figure 20 for values of K_A^2 corresponding to a normal *JN4h* and also one having twice the polar moment of inertia. The values of L_p from the model test of −325 as previously used in this report seemed to be too high to give results in good agreement with the actual performance, so a value of L_p just obtained on this airplane in free flight of −154 was used. This value is the only point so far obtained in flight so that it should be considered only tentatively, but it is probably closer than the wind tunnel result of twice the value.

It can be seen that if t is small, a change in the moment of inertia has a relatively large effect, but if t is large the effect is small. For if t=0.4 doubling the moment of inertia decreases it by 27 per cent; if t=1 it is decreased by 12 per cent, and if t=2 by 6 per cent. The above equations do not apply strictly to the actual conditions due to the fact that an appreciable time is required to move the controls.

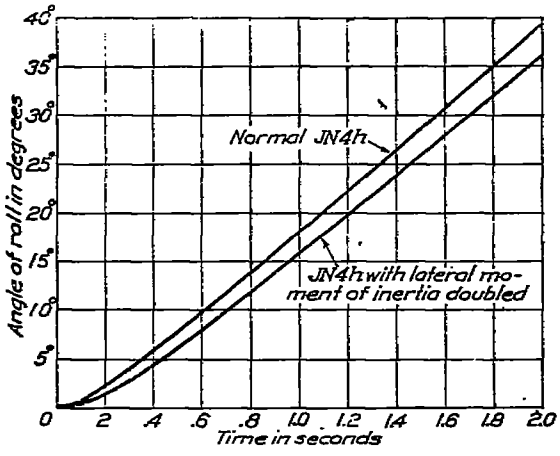


FIG. 20.—Effect at lateral moment of inertia on time required to bank a JN4h at 80 M. P. H. with ailerons at 13° (calculated).

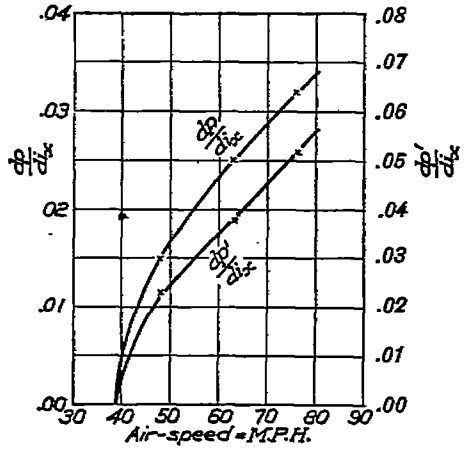


FIG. 21.—Motion about X axis with doubled K_A.

In order to determine experimentally the effect of increasing the moment of inertia of an airplane on its maneuverability, sufficient weight was applied to the wing tips to double the lateral moment of inertia of a *JN4h* airplane. As these weights amounted to approximately 150 pounds on each wing tip it was not considered advisable to make a landing with them in place; so that the same method was used for releasing the weights as described in N. A. C. A. Report No. 112. This method consisted in applying to each wing tip a streamline box filled with sand with a hinged bottom which could be opened from the cockpit of the machine. In making the tests the sand boxes were loaded up to their proper capacity and the tests were made in the usual way, then the boxes were emptied before landing.

The same series of runs was made with the sand boxes in place as had been made previously recording the angular velocity when the stick was moved suddenly over through various angles. The results obtained are plotted in Figure 21 which are strictly comparable with Figure 8 applying to the normal machine. An examination of the curves shows the following facts:

1. The maximum angular velocity is independent of the moment of inertia, as would be expected.

2. The maximum angular acceleration is reduced 50 per cent by doubling the moment of inertia, which agrees with theory.

3. The time required to reach the maximum angular velocity is constant and amounts to approximately 0.9 second which is slightly over twice that obtained for the normal airplane.

4. The increased time to bank to 14° is given in the following table:

"JN4h," 80 miles per hour.

Aileron angle.	Time to bank 14° with normal machine in seconds.	Increase in time by doubling moment by inertia.
		Per cent.
4°	2.3	1
8°	1.3	7
12°	0.7	20
16°	0.4	60

It may be noted from these results that the time necessary to bank to an angle of approximately 14° is increased by about 10 per cent when the time required is approximately 1 second; which corresponds to an aileron angle of 10°. This figure checks remarkably well with the value derived from the theory. If we use a relatively large aileron movement of 16° the time required to bank to 90° is only increased 10 per cent by doubling the moment of inertia. It would seem, then, from these figures that a small moment of inertia for a combat machine is not as necessary as has been previously supposed. Although the work done here has been only about the X axis there is no reason to believe that the figures would not apply equally well to motion about the other axes.

It might be well to bring out the fact here that it is practically of no value in a combat machine to have a small longitudinal moment of inertia, which is contrary to the ideas of most designers. If we took any airplane and went to the extreme of doubling the longitudinal moment of inertia we would increase the time to complete a loop, for example, by only 1 or 2 per cent. The reason why small rotary-motored airplanes are usually maneuverable longitudinally is not because of a small moment of inertia but because of a light wing loading and a short fuselage which decreases the value of M_q.

CENTER OF ROTATION.

If we consider the angular motion of an unrestrained body like an airplane in flight, there must be for each axis an instantaneous center of rotation. In a great deal of the work which has been done on airplanes, it has been assumed that all rotations occurred about the center of gravity. It is a well-known fact in mechanics, however, that an unrestrained body, when acted upon by an impulsive force (such as a suddenly applied tail load) need not rotate about its center of gravity. As the center of rotation is of considerable importance in determining the errors in the reading of an accelerometer or of an angle of attack vane, it may not be out of place to call attention to its location under various conditions.

If h = the distance in feet from the center of gravity to the center of rotation.

p = the distance in feet from the center of gravity to the application of the force.

k = radius of gyration.

$$hp = k^2$$

or

$$h = \frac{k^2}{p}$$

As an example let us consider an airplane where $K_A^2 = 40$ and $p = 12$. The center of rotation would then be 3.3 feet ahead of the center of gravity, a not unimportant distance. Of course all motion about the X axis will be symmetrical.

THE EFFECT OF ENGINE POWER ON MANEUVERABILITY AND CONTROLLABILITY.

It is an acknowledged fact that an airplane may be handled more rapidly when the engine is opened out than when it is throttled, and in the same way a high-powered machine is quicker than a low-powered one. This is due, first, to the fact that the velocity of the air over the tail

surfaces is dependent on the engine speed, and, second, that a large propeller thrust allows a higher and more uniform air speed throughout a maneuver.

The effect of engine power on the value of the coefficients can easily be determined in flight, but the direct relation of power to maneuverability has not been carefully studied and can well form the basis for further investigation. It is hoped that we shall soon be able to continue the work in this direction.

DEFINITIONS OF CONTROLLABILITY AND MANEUVERABILITY.

After careful consideration of the preceding data it was thought advisable to distinguish between the forces exerted by the controls and the resulting motion of the airplane. Controllability is applied to the first, and maneuverability to the second. This seems to make a clear and satisfactory distinction between the two words. The following definitions have been decided upon:

Controllability of an airplane is the ratio of M/i, where M is the additional moment about a principal axis applied to make the airplane perform the same motion under the same conditions except with a change i in the setting of the respective control surfaces.

The maximum control is the maximum moment that can be produced about a principal axis of the airplane by the movement of the respective control surfaces.

The efficiency of the control is the ratio of M to the force exerted on the stick or rudder bar to hold the respective control surface in a given position. (Moments in foot-pounds and forces in pounds.)

An airplane maneuver may be defined as any departure from uniform rectilinear motion.

The maneuverability of an airplane may be defined as that property which permits the pilot to direct it easily and quickly through any desired maneuver.

The definition of controllability is universal and applies to any type of flight, but it can not be readily measured except in uniform rectilinear flight. The controllability characteristics of an airplane are of course directly measured in the wind tunnel where moments are taken about the center of gravity of the machine with various settings of the control surfaces. In free flight however, the measurements are more difficult, as it is here necessary to apply known moments to the machine by the addition of weight and then to measure the angle at which the control surfaces must be set to fly in equilibrium. These measurements can be made easily and accurately so that controllability coefficients can be readily obtained upon any machine. The definition for the controllability coefficients is given below:

A coefficient of controllability is the slope of the curve of M plotted against i (moment in foot-pounds and angle in degrees).

The coefficients for the X, Y, and Z axes are respectively $\frac{dL}{di_x}$, $\frac{dM}{di_y}$, and $\frac{dN}{di_z}$.

Of course, the coefficients $\frac{dM}{di_y}$ and $\frac{dN}{di_z}$ will vary with the engine speed as well as air speed, due to the action of the slipstream.

The coefficients of maneuverability are not as easy to define and the measurements can not be made as directly. We must have a coefficient which will take into account the following quantities:

(1) The moment of inertia of the airplane.
(2) The moment of inertia of the control system.
(3) The damping due to rotation.
(4) The moment exerted by the controls.

It is obvious from the previous consideration that neither the maximum angular velocity nor the maximum angular acceleration will fulfill these conditions. Let us examine the typical curve of rolling moment plotted against time caused by a sudden change in the aileron setting. (Fig. 6.) It will be observed that the velocity increases rather steadily up to a maximum value which is then maintained steadily. As has been previously shown the time taken to reach this steady value is constant for any given airplane under all conditions of speed or control settings. If we then take a length of time which is longer than the time to reach the maximum angular velocity which we will call t, and then measure the area under the velocity curve up to this time, we will have the number of degrees turned in time t. If the process is repeated

with a curve of rotation about the Y or Z axis (fig. 22), we will obtain the same results except that the angular velocity does not maintain itself at a constant value but falls off rather rapidly after the maximum. If the time t, however, is not too long we should be able to get consistent results with this type of curve. This area under the angular velocity curve will then give us in one value the combination of all the factors which determine maneuverability. The use of this area, however, is an added advantage in that it can be determined directly on the airplane without the use of complicated instruments. To measure it, all that is necessary to do is to determine the angle through which the machine rotates in a given length of time t. In actual practice, however, we can obtain the same results more easily by taking the time required to rotate through a given angle.

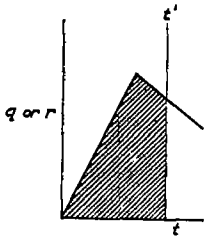


FIG. 22.

This method of measurement will then give us values of maneuver ability about each axis of the airplane. We have however to take into account another factor when considering the motion about the Y axis and this is the minimum radius of curvature of turning. For this reason it will be necessary to introduce a fourth coefficient of maneuverability which may be found from the minimum speed of the airplane. The four coefficients may then be defined as follows:

The coefficient of maneuverability is the time required in seconds for the airplane to rotate about a principal axis from rectilinear flight to a given angle (15°) after the pilot moves the respective control surfaces as suddenly as possible through a given angle (8°) from their equilibrium position with the other set of controls held stationary. For motion about the Y axis it is necessary to include a factor denoting the minimum radius of turn or the square of the minimum flying speed divided by g. The coefficients will be denoted by t_ϕ, t_β, t_ψ and $\frac{V\,min.^2}{g}$.

The angles given above for the rotation and the movement of the controls are the most satisfactory ones for the *JN4h*, but it may be necessary however to give them different values if the coefficients are to be used universally. This can be determined when tests have been made upon other types of airplane. The coefficients t_ϕ and t_ψ will vary with engine speed as well as with air speed due to the action of the slipstream.

CONCLUSIONS.

The study of controllability and maneuverablity has been particularly difficult, first because the subject is so intangible and second because there is so little previous work to follow. It is felt that the present investigation leaves much to be desired in the way of completeness, but it at least places the subject on a much more scientific footing than before, and will serve as a basis for further investigation.

In carrying out more research along this line it is recommended that the coefficients be determined for a number of machines so that we may have sufficient data to discover what factors are effective in producing controllability and maneuverability. It would also be advisable to make systematic changes in the control surfaces and study their effect on the coefficients. The important subject of the relation of stability to maneuverability has not been touched upon here, but will serve as the basis for both theoretical and experimental research in the near future. The effect of aspect ratio, number of planes, tail area, fuselage, length, etc., will also be studied as far as possible.