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TECHNICAL NOTES

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No. 122

THE DETERMINATION OF THE ANGLES OF ATTACK OF ZERO LIFT AND OF ZERO MOMENT, BASED ON MUNK'S INTEGRALS.

By Max M. Munk.

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Summary.

The integration is accomplished by the use of the mean camber of the section at particularly selected points. Simple graphical constructions of the zero directions are derived from the results.

References.

Max M. Munk.	General Theory of thin wing sections	N.A.C.A. Report No. 114.
" " "	Elements of Theoretical Aerodynamics	N.A.C.A. Report No. 171.

I have shown in the treatises referred to how the angle of attack of zero lift and the angle of attack of zero moment around the middle of the wing section, that is for the position of the center of pressure 50%, can be found by integrating the mean curve of the section multiplied by a certain function. If the radius of curvature at the leading edge is small, the mean curve extends practically up to the leading edge. This case may be considered first. Let the x-axis of a system of coordinates coincide with the chord of the section, as ordinarily taken.

The trailing edge may have the abscissa $x = +1$ and may coincide with the x-axis. The leading edge may have the abscissa $x = -1$ (Fig. 1). ξ may denote the ordinate of the mean curve at any point. This mean curve comprises all points having equal distance from the upper and lower curve of the section, but it is exact enough to replace it by the curve having ordinates ξ which are the mean of the ordinates of the upper and lower curve at the same abscissa x. $\xi = \frac{1}{2}(\xi_u + \xi_l)$. Then the angle of attack of zero lift is given by the integral

$$(1) \qquad \alpha_0 = -\frac{1}{\pi} \int_{-1}^{+1} \frac{\xi \, dx}{(1 - x)\sqrt{1 - x^2}}$$

This integral gives a finite value only if $\xi = 0$ at the point $x = 1$ that is, at the trailing edge: Hence, if the trailing edge is so thick that the rear end of the mean curve has an ordinate ξ of noticeable length, it is necessary to move the x-axis so as to make $\xi = 0$ at this point (Fig. 2). It is, however, sufficient to move the chord parallel to itself, that is, to diminish all ordinates ξ by the same amount.

The problem arises now to evaluate the integral (1) in the most convenient way and with the greatest accuracy of the result obtainable with the same amount of work. This requires the application of Gauss' method of numerical integration in a modified form.

Since ξ which is now regarded as a function of x, becomes necessary zero at the point $x = 1$, it can be written

$\xi = (1 - x)F(x)$ where $F(x)$ is finite over the whole chord and is defined by

$$(2) \qquad F(x) = \frac{\xi}{1 - x}$$

The integral (1) can now be written

$$(3) \qquad \alpha_o = -\frac{1}{\pi}\int_{-1}^{+1}\frac{F(x)\,dx}{\sqrt{1 - x^2}} = -\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} F(x)\,d\delta$$

where

$$(4) \qquad x = \sin\delta$$

The problem is now much simplified, being reduced to the evaluation of a simple integral of a function, not of a product of two functions, within the range from $\delta = -\frac{\pi}{2}$ to $\delta = +\frac{\pi}{2}$. This function $F(\delta)$ it is true, is probably and in most cases smaller near the leading edge than at the other parts of the intervals. But I will not take this into account. Then Gauss' method can now be applied directly.

This method consists in selecting the values of the function to be integrated at certain points $\bar{\bar{x}}$, multiplying them by certain factors A and adding the products obtained. The points are so chosen that they give a result more exact in general than the result obtained from the same number of points otherwise located. If, for instance, only one point and the value of the function at this point ξ_o shall be used for the integration, the best position of this point is in the middle of the interval,

in our case $\delta = 0$ i.e. $x = 0$. This is indeed the only point which gives the result absolutely correct not only if $F(x)$ is constant and hence can be written $F(x) = \xi_0$, but also if it is any linear function of δ, $F(x) = \xi_0 + \beta\,\delta$, containing the given point. The result is found by substituting for $F(x)$ any constant function having the same value as F at the point considered, that is $F = \xi_0$.

$$\alpha_0 = -\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \xi_0 \, d\delta = -\xi_0$$

This refers to the length of the chord 2. For any length of the chord c, the results appear $\alpha_0 = -\frac{2\,\xi_0}{c}$ in radians. It results therefore that for the first provisional determination of the direction of zero lift, using only one point of the mean curve of the section, the middle of the section, 50% of the chord, is the most characteristic point. The zero lift direction is, as first approximation, parallel to the line connecting the trailing edge and the mean curve at 50% of the chord (Fig. 3).

The wing sections used in practice are always smooth and regular, but still the use of one point only is often not exact enough. It is desirable to use at least two or three points. Even five points may sometimes be desired. For more than one point, Gauss computed a table, from which the following values are taken.

Gauss' Rule

Table 1.

$$\int_{-1}^{+1} F(\bar{x})\, d\bar{x} = A_1 \; F(\bar{x}_1) + A_2 \; F(\bar{x}_2) + \ldots + A_n \; F(\bar{x}_n)$$

n = 1	$\bar{x}_1 = 0$	$\frac{1}{2}A_1 = 1$
n = 2	$\bar{x}_1 = -\bar{x}_2 = .577\ 350\ 2691$	$\frac{1}{2}A_1 = \frac{1}{2}A_2 = \frac{1}{2}$
n = 3	$\bar{x}_1 = -\bar{x}_3 = .774\ 596\ 6669$	$\frac{1}{2}A_1 = \frac{1}{2}A_3 = 5/18$
	$\bar{x}_2 = 0$	$\frac{1}{2}A_2 = 4/9$
n = 5	$\bar{x}_1 = -\bar{x}_5 = .906\ 179\ 8459$	$\frac{1}{2}A_1 = \frac{1}{2}A_5 = .118\ 463\ 4425$
	$\bar{x}_2 = -\bar{x}_4 = .538\ 469\ 3101$	$\frac{1}{2}A_2 = \frac{1}{2}A_4 = .239\ 314\ 3352$
	$x_3 = 0$	$\frac{1}{2}A_3 = .284\ 444\ 4444$

When corresponding values are calculated for an equation of the type of (2), introducing proper limits for a wing of chord c, one obtains the following table. The formulas used for its computation, in agreement with equations (2) and (3) are

$$x_n = 50(1 - \sin[\bar{x}_n \tfrac{\pi}{2}])$$

$$f_n = \frac{90}{\pi} \quad \frac{A_n}{1-x}$$

Table 2 - Angle of Zero Lift in Degrees.

$$- \alpha_c = F_1 \left(\frac{\xi_1}{c}\right) + F \left(\frac{\xi_2}{c}\right) + + F_n \frac{\xi_n}{c}$$

n = 1

$x_1 = 50\%$ $\quad F_1 = 114.6$

n = 2

$x_1 = 89.185\%$; $x_2 = 10.815\%$

$f_1 = 264.9$; $f_2 = 32.12$

n = 2 less exact:

$x_1 = 90\%$; $x_2 = 10\%$

$f_1 = 286$; $f_2 = 31.9$

n = 3

$x_1 = 96.90\%$; $x_2 = 50\%$; $x_3 = 3.10\%$

$f_1 = 513.08$; $f_2 = 50.93$; $f_3 = 16.425$

n = 5

$x_1 = 99.458\%$; $f_1 = 1252.24$

$x_2 = 87.426\%$; $f_2 = 109.048$

$x_3 = 50\%$; $f_3 = 32.5959$

$x_4 = 12.574\%$; $f_4 = 15.6838$

$x_5 = .542\%$; $f_5 = 5.97817$

100% = Trailing Edge.

The computation with two points can be performed graphically in a very convenient way, which gives an improvement of the pro-

visional construction of the zero lift direction as mentioned before (Fig. 4). Connect the points of the mean section curve at 10.8% and 89.2% of the chord with the trailing edge. The line dividing the angle between these two connecting lines into equal parts is then the direction of zero lift.

I proceed now to the computation of the angle of zero moment, moments being taken around the origin, i.e. for the position of the center of pressure 50%. The integral for the computation of this angle is

$$(5) \qquad \alpha'_c = -\frac{2}{\pi} \int_{-1}^{+1} \frac{\pi \xi}{\sqrt{1 - x^2}} \, dx$$

This integral converges for any finite ξ, so it is not necessary for its evaluation that the trailing edge coincide with the x-axis. The conventional chord can always be used without any correction for the thickness of the trailing edge.

A closer examination of integral (5) shows that the factor of ξ is an odd function having opposite values for pairs of points at equal distance from the middle of the chord. It is therefore at once obvious that a symmetrical curve has the zero direction parallel to the x-axis, giving $\alpha'_c = 0$. Only an unsymmetric curve differently shaped at the front and at the rear part gives a finite angle. The degree of unsymmetry cannot be derived from one ordinate ξ only, a pair is at least required. One pair on the other hand is sufficient for most practical purposes.

Write $\xi = \xi_0 + x\,F(x)$ where ξ_0 is the value of ξ at $x = 0$. Then

$$\alpha'_o = -\frac{2}{\pi}\int_{-1}^{+1}\frac{F(x)\,x^2}{\sqrt{1-x^2}}dx = \frac{1}{\pi}\int_{-1}^{+1} F(x)\,d\left[\arc\sin x - x\sqrt{1-x^2}\right]$$

$$= \frac{2}{\pi}\int_{0}^{1}\left[F(x) - F(-x)\right] d\left[\arc\sin x - x\sqrt{1-x^2}\right]$$

The interval of $(\arc\sin x - x\sqrt{1-x^2})$ extends between 0 and $\frac{1}{2}\pi$. Hence the best ordinate to be chosen for the computation has the abscissa $(\arc\sin x - x\sqrt{1-x^2}) = \frac{1}{4}\pi$

Write $x = \sin\delta$: Then the condition is

$$2\delta - \sin 2\delta = \frac{\pi}{2};\ \left(\frac{\pi}{2} - 2\delta\right) + \cos\left(\frac{\pi}{2} - 2\delta\right) = 0$$

This equation has the solution $\delta = 66^\circ\ 10.4'$

$$x = \sin\delta = .91476 = 4.26\% \text{ and } 95.74\%.$$

The factor for these ordinates is easily found from the consideration that the straight line gives the exact direction, coinciding with its direction. The distance between the points is

$c\,[95.74\% - 4.26\%]$; the difference between the ordinates is $\xi_1 - \xi_2$ hence the angle between the x-axis and the straight line connecting the two points in degrees is

$$\frac{180}{\pi}\ \frac{\xi_1 - \xi_2}{c(.9574 - .0426)} = \frac{\xi_1 - \xi_2}{c}\ 62.634$$

Hence the factor is

$$F = \frac{180}{.91476 \cdot \pi} = 62.634$$

The angle of attack of zero moment is, therefore,

$$62.634 \quad \frac{(\xi_1 - \xi_2)}{c}, \text{ where } \xi_1 \text{ and } \xi_2 \text{ are}$$

the ordinates af the abscissae

$$x_1 = 95.74\%$$

$$x_2 = 4.26\%$$

This formula leads to a very simple graphical construction of the direction of zero moment, (Fig. 5). The straight line connecting the points of the mean curve at 4 1/4% and at 95 3/4% gives this direction.

I wish finally to call attention to a small improvement of the result if the leading edge of the section has a very great radius of curvature (Fig. 6). In this case the mean curve has to be elongated past the center of curvature of the leading edge by half the radius of curvature. 0 % of the chord is to be taken at this point rather than at the front point of the section. This small change excepted, all methods of computation remain unaltered.

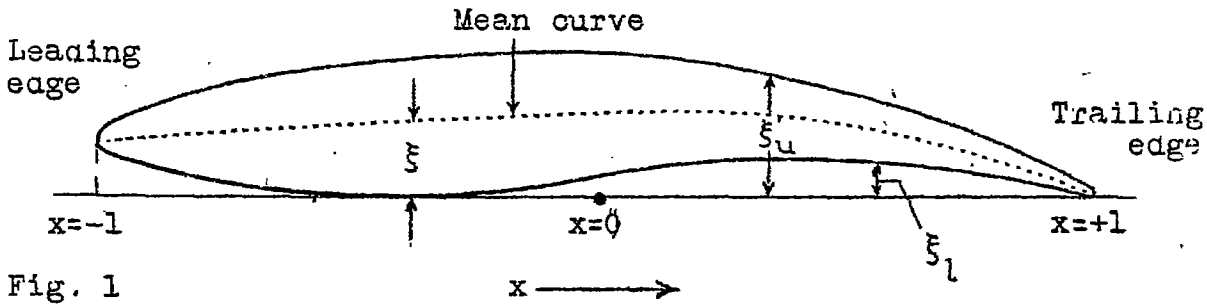

Fig. 1

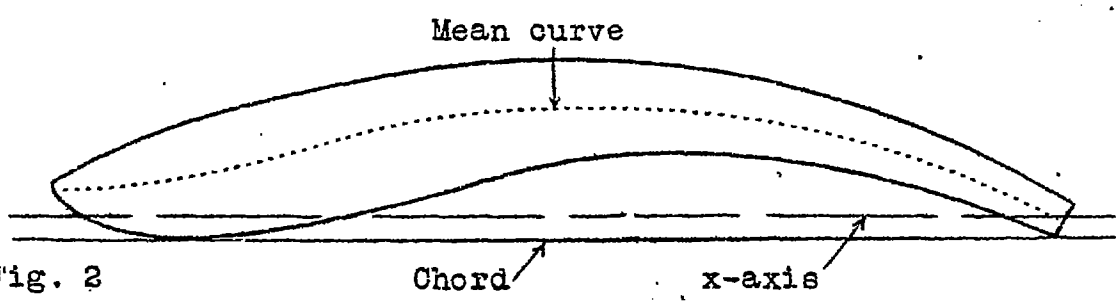

Fig. 2

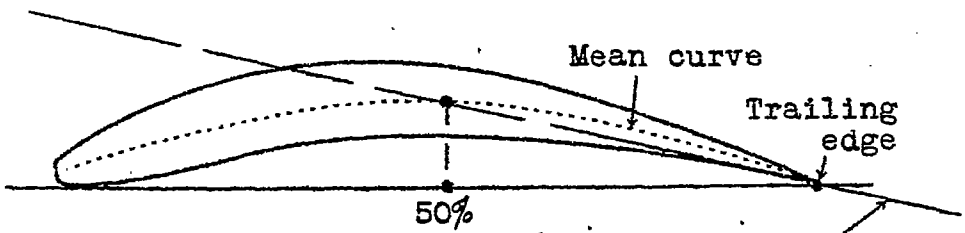

Fig. 3

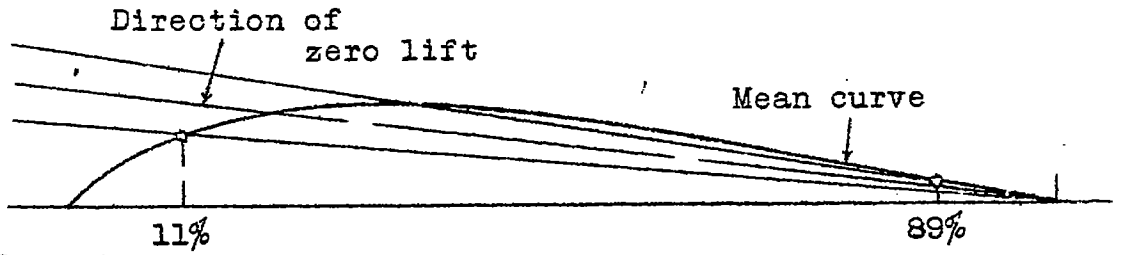

Fig. 4

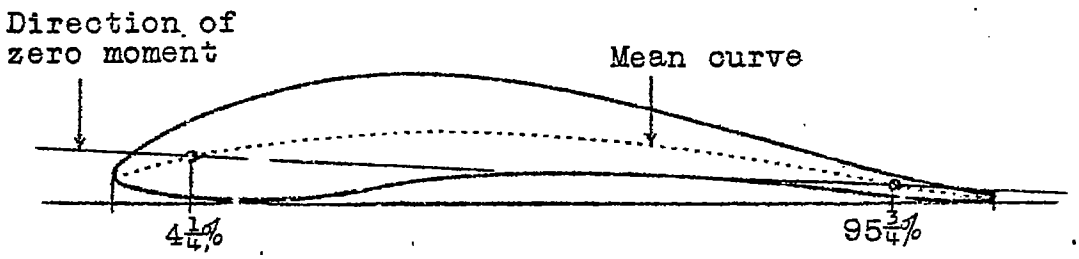

Fig. 5

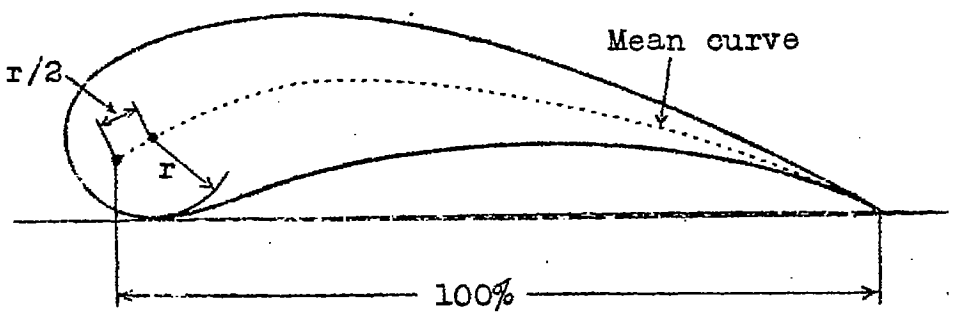

Fig. 6