Structures Research Section
DEC 17 1940

203

TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 203

A SHORT METHOD OF CALCULATING TORSIONAL STRESSES IN AN AIRPLANE FUSELAGE.

By John E. Younger.
University of California.

FILE COPY
To be returned to
the files of the National
Advisory Committee
for Aeronautics
Washington, D. C.

September, 1924.

NASA Technical Library
3 1176 01425 5930

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL NOTE NO. 203.

A SHORT METHOD OF CALCULATING TORSIONAL STRESSES IN AN AIRPLANE FUSELAGE.

By John E. Younger.

Summary

This report deals with an investigation carried out in the Civil Engineering Laboratory of the University of California, to determine the accuracy of existing methods of computing stresses in an airplane fuselage when subjected to torsion, and to derive a simple approximate formula for the rapid calculation of these stresses. The formula is derived by using the customary least work equation and considering each bay separately. The errors due to assumptions in regard to members, sections, fittings, modulus of elasticity, etc., are made compensating as far as possible. The assumption is made that the wooden members, i.e., the longerons and struts may be neglected. It is further assumed that the wires in both directions in the side trusses are equal in size and length, that the wires in the top plane are the same size and length as those in the bottom plane, and that the fuselage is symmetrical with respect to the longitudinal axis.

In comparison with experimental results, it is shown that the derived formula gives more accurate results in this case than some existing standard formulas.

References

Advisory Committee for Aeronautics, Great Britain R. and M. No. 736, "Torsional Stresses in the Fuselage of an Aeroplane," by A. J. Sutton Pritchard.

"Aeroplane Structures," by Pippard and Pritchard.

R. and M. No. 368, "On the Torsion of a Fuselage," by H. M. Garner and Arthur Berry.

N.A.C.A. Report No. 76, "Analysis of Fuselage Stresses," by E. P. Warner and Roy G. Miller.

N.A.C.A. Report No. 82.

Introduction.

The exact theoretical method of calculating the torsional stresses in an airplane fuselage is extremely long and tedious, and uncertain conditions, such as the variation in the modulus of elasticity, or variation in the cross-section of the wire, slipping of a fitting, and the influence of turnbuckles and fittings, make the exact method of doubtful value. It seems, therefore, that a simple approximate method of computing these stresses, taking into account, as far as possible, the various uncertainties, which could be applied to the draughtman's layout, would be of considerable practical value.

Procedure and Apparatus.

To accomplish this end, tests were made on an airplane fuselage, and various methods of computing the stresses were tried

until one was found which was sufficiently simple, and gave sufficiently accurate results, as compared with the experimental stresses.

A Curtiss JN-4B fuselage, which was in excellent condition, was used for the test. The tail surfaces, landing gear, and fabric were removed, and the skeleton fuselage was wedged between the columns of a five hundred thousand pound hydraulic compression testing machine, as shown in Fig. 1 and Fig. 2. The wedging was placed around the bulkhead of the fuselage immediately behind the rear cockpit, leaving six bays free for the test. The torsion was applied by means of a lever at the bulkhead which supports the tail-skid. This bulkhead, being braced with steel rods instead of the usual light wire, may be assumed rigid. The lever (Fig. 2) was thirty inches long from the tail skid post to the weight pan support. Standard fifty pound weights were used for the load. The torque was not applied at the rudder post because the first bay was not strong enough to transmit sufficient torque to appreciably stress the wires in the sixth bay from the tail end. To prevent the first bay from affecting the results, the wires of that bay were made slack, and the rudder post was pulled loose from the longerons, leaving the longerons free at the ends.

To be certain that there was no vertical deflection in the fuselage due to the load, an Ames Dial, reading to .001 of an inch was used as shown in Fig. 2. The dial was screwed to a strut dropped from the top of a doorway, and the pin of the dial was attached by means of a stiff steel wire to the center of the rudder

post. A screw jack was placed under the tail skid post to maintain the dial reading at zero.

The stresses in the fuselage were determined by means of calibrated wires. The strain in the wires was measured by two Berry strain gages, equipped with Ames Dials. The gage length was eight inches. One full division on the dials indicated a strain of .0002 of an inch. Small cone-shaped holes were punched in the wires with specially hardened and ground punches, to fit the points of the gages. The gages were held in place by strips of rubber wound around the wire and the instrument at the two points as shown in Fig. 3 and Fig. 4. The wires were calibrated by standard weights as shown in Fig. 3.

The procedure was to take the strain in opposite wires simultaneously (Fig. 4). All wires of the top, bottom and sides which would not take a positive load were made perfectly slack. A reading was made of each gage for each fifty pound weight added, and for each fifty pound weight removed. These readings were then plotted, stress as a function of torque, and an average curve drawn for each wire. Frequent check tests were made, and although the load was applied and removed about eighty-five times the results obtained in the last runs checked within five percent of those obtained in the first runs (Fig. 8).

Every precaution was used to prevent the fittings from slipping on the longerons. However, this could not be entirely avoided. A check was made of the amount of slippage in the worst cases by

applying a Berry strain gage, one leg on the fitting and the other leg on the longeron. It was found that the movement of the fitting due to slipping was negligible, and that the greatest error was introduced by the fitting cutting into the longeron, and by the compression of the bulkhead strut sockets. The strain in every case was proportional to the applied torque, and the fittings would return to their original positions when the torque was removed. This elastic condition was also noted in measuring the stresses in the wires; the initial and final stresses in the wires invariably checked within ten or fifteen pounds of each other.

To dispose of irregularities due to the movement of the fittings, and other causes, curves were plotted (Fig. 9 and Fig. 10) with pounds stress in each wire as a function of the mean cross sectional area of the bay, this being one of the largest factors influencing the torsional stresses. There being no abrupt variation in the dimensions of the bays and wires, the curve should be smooth and should theoretically approach infinity as the area of the cross section approaches zero.

As each wire contained two loops, a turnbuckle, and two longeron fittings, a total length of eleven or twelve inches, there was a question as to the proper length of solid wire to consider in the calculations. To obtain data on this question, a strain gage was set with a gage length of the entire length of the wire, from the upper longeron fitting to lower longeron fitting. By comparing this data to the data obtained on the solid portion of

the wire, it was concluded that the strain in the combined length of the loops, turnbuckle and fittings was equivalent to 1.1 to 1.4 times the strain in an equal length of solid wire.

In order to determine the angle of twist, a thread was stretched (Fig. 5) parallel to the longitudinal axis of the fuselage at a horizontal distance of sixty-one inches in the plane of the top panel of the fuselage. Levers which carried small celluloid scales, set vertically at the outer ends, were then attached to the top strut of each bulkhead. The levers were adjusted so that the scales were held about one-sixteenth of an inch from the thread. The movement of the scale relative to the thread divided by the lever arm of sixty-one inches was taken as the angle in radians.

Comparison of Methods of Computing Torsional Stresses in an Airplane Fuselage.

In determining an approximate formula for calculating the stresses, the solution of the problem by several methods was carried out and the results compared with the experimental results. Prominent among these methods were:

(a) The graphical solution.

(b) The analytic solution by statics.

(c) Pippard and Pritchard's solution by the theorem of least work.*

Both (a) and (b) contain the inherent errors due to the assump-

* "Aeroplane Structures," by Pippard and Pritchard.

tions of rigidity found in methods of statics. The author has found in carrying out tests on airplane structures, that it is necessary to deal with the structure, in most cases, as an elastic body rather than a perfectly rigid one. The discrepancies may be more readily pointed out by referring to a specific solution. Take for instance, the solution by statics given by Zahm and Crook*: Referring to Fig. 7,

$$R' = \frac{\text{Torque}}{4l} \quad \ldots\ldots\ldots\ldots \quad (a)$$

$$P = \frac{R'd}{2l} \quad \ldots\ldots\ldots\ldots \quad (b)$$

$$Q = \frac{R'b}{2l} \quad \ldots\ldots\ldots\ldots \quad (c)$$

General case; applied torque in plane of truss base.

Denote stay stresses by R, T, L, B, longeron stresses by R', T', L', B'.

Denote stay direction cosines by R_x, R_y, R_z; T_x, T_y, T_z; L_x, L_y, L_z: B_x, B_y, B_z.

Denote longeron cosines by the same letters primed.

Then,

$$R_yR + R'_yR' + T_yT + T'_yT' + 2P = 0$$

$$T_zT + T'_zT' + L_zL + L'_zL' + 2Q = 0$$

$$L_yL + L'_yL' + B_yB + B'_yB' + 2P = 0$$

$$B_zB + B'_zB' + R_zR + R'_zR' + 2Q = 0$$

* National Advisory Committee for Aeronautics, Report 82.

Also,

$$R_xR + R'_xR' = 0$$

$$T_xT + T'_xT' = 0$$

$$L_xL + L'_xL' = 0$$

$$B_xB + B'_xB' = 0$$

The solution of the problem by this method as well as by any other method, depends upon the determination of P and Q.

Dividing equation (c) by equation (b):

$$\frac{Q}{P} = \frac{b}{d} \quad \text{or} \quad \frac{Q}{b} = \frac{P}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (d)$$

Now if the truss be assumed elastic, there will be a small rotation of the bulkhead in the direction of the applied torque. If ξ_p be the horizontal motion in the direction of P, and ξ_p be the vertical motion in the direction of Q, then

$$\xi_q : b = \xi_p : d$$

or

$$\frac{\xi_q}{b} = \frac{\xi_p}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (e)$$

Divided by (d)

$$Q : \xi_q = P : \xi_p \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (f)$$

From Young's modulus of elasticity:

$$Q = \Sigma \left(\frac{AE}{l}\right)_q \times \xi_q$$

in which

$\Sigma \left(\frac{AE}{l}\right)$ refers to the projections of the wires along the line of action of Q.

Likewise

$$P = \Sigma \left(\frac{AE}{l}\right)_p \times \xi_p$$

Substitute P and Q in (f):

$$\Sigma \left(\frac{AE}{l}\right)_q \times \xi_q \ : \ \xi_q = \Sigma \left(\frac{AE}{l}\right)_p \times \xi_p \ : \ \xi_p$$

or

$$\Sigma \left(\frac{AE}{l}\right)_q = \Sigma \left(\frac{AE}{l}\right)_p$$

Therefore, this method, with other methods of statics, will hold only when the summation of the ratios of the cross-sectional area times the modulus of elasticity to the length, of each member, these quantities being projected along the line of action of Q, is equal to the summation of the ratios of quantities projected along the line of action of P. This condition is rarely, if ever, obtained in the fuselage of an airplane. In the fuselage used in this test, the wires in the sides were nearly twice as large as those used in the top and bottom. We should expect results obtained by methods of statics, then, to be too small for the side wires, and too large for the top and bottom wires, which is the actual case as shown by the dotted curves in Figs. 9 and 10.

Another method of dealing with the torsion problem is to consider the four entire panels, top, bottom and the two sides as separate cantilever trusses, each truss taking a certain proportion of the force due to the torque, as expressed by the equation:

$$Qb + Pd = \psi$$

in which ψ is the torque (Fig. 7).

The problem, again, is to find P and Q. With these quantities known, the assumption is that the stresses may be determined for the entire panel, taken as a cantilever truss, by the graphical method. A very clever method of handling the problem from this standpoint, and considering the fuselage as an elastic body, is given by Messrs. Pippard and Pritchard in their book "Aeroplane Structures." The following assumptions are made:

1. The whole of the deformation is due to the horizontal and vertical panel wires.

2. The bulkhead bracing wires may be neglected.

3. The cross-section of the fuselage is everywhere rectangular.

4. The curvatures of the longerons are negligible.

The formulas derived are:

$$Q = \frac{\psi b \Sigma x}{d^2 \Sigma X + b^2 \Sigma x}$$

$$P = \frac{\psi d \Sigma X}{d^2 \Sigma X + b^2 \Sigma x}$$

in which

$$X = \frac{K^2 L}{A} \quad \text{and} \quad x = \frac{k^2 l}{a}$$

where the load in a side wire is KQ and the load in a top or bottom wire is kP.

Their description of the application is:

Stress diagram should be drawn for the horizontal and vertical bracing of the fuselage under consideration with a unit load applied in the appropriate direction. If the fuselage is symmetrical, two

stress diagrams only are needed.

The load in the wires under this unit of loading is the value of K or k.

The length of the wires L and l, are scaled direct from the frame diagram, and the areas A and a, are known from the sizes of wires employed. X and x can now be obtained, and hence the values of P and Q.

The loads in the fuselage are obtained by multiplying the unit loads obtained from the stress diagrams referred to above by the values of P and Q thus found, so that a second set of stress diagrams is unnecessary.

The results obtained by this method are plotted in Figs. 9 and 10.

Limitations of the Cantilever Truss Method.

The above theory is open to two objections: First, the work done by the wooden members has been neglected, which approximation introduces errors which are of considerable magnitude in certain members, as pointed out by Pippard himself in a subsequent report (R. and M. No. 736).

The theory, furthermore, is strictly applicable only when the curvature of the longerons is so slight as to be negligible. This condition seldom occurs in practice. It would be, therefore, advantageous if a theory could be devolved which would deal with each bay separately.

Points to Consider in Deriving a New Formula.

It may be noted that each bay, considered as a free body, contains four redundant members: two bulkhead wires and two side wires. Of these four members, experiment shows that the bulkhead wires may be neglected. The stress was measured in each of these wires, for the five bays, and in no case was the stress found to increase, but rather that there was a slight decrease due to the strain in the bulkhead struts. The variation in the lengths, modulus of elasticity, and area of the cross-section of the wires, generally found in a fuselage, makes the solution by statics alone, impossible.

Considering these conditions, it was therefore concluded that the purely analytical solution, considering each bay separately, would give more accurate results. However, analytical solutions are often long and tedious, so an effort was made to develop a formula containing as few terms as possible, and terms which could be taken directly from the working drawing with a small chance of error.

Derivation of the Formula.

The symbols used are as follows:

Let Q = vertical forces due to torque, pounds.

P = horizontal forces due to torque, pounds.

p = total stress in a wire, pounds.

s = total stress in a wooden member, pounds.

θ = angle between the wire and the bulkhead strut, degrees.

α = angle of twist of the fuselage, radians.

A = area of cross-section of side wires, sq.in.

a = area of cross-section of top and bottom wires, sq.in.

E = modulus of elasticity of steel wire, lb. per sq.in.

E_W = modulus of elasticity of the wooden members, lb./sq.in.

L = length of side wires, inches.

l = length of top and bottom wires, inches.

X = longitudinal projection of members, inches.

Y = transverse projection of members, inches.

Z = vertical projection of members, inches.

ψ = torque, pound inches.

b = width of bay, inches.

d = height of bay, inches.

c = length of bay, inches.

U = total work in one bay, inch pounds.

Let the subscripts,

T refer to top panel wire members.

B refer to bottom panel wire members.

L refer to left panel wire members.

R refer to right panel wire members.

t refer to the top panel wooden members.

b refer to the bottom panel wooden members.

l refer to the left side panel wooden members.

r refer to the right side panel wooden members.

Thus:

p_{LR} = Stresses in left and right panel wires.

p_{TB} = Stresses in top and bottom panel wires.

Assuming that each bay is a free body, the following relations may be noted:

$$\psi = Qb + Pd \quad \text{or} \quad P = \frac{\psi - Qb}{d} \quad \ldots\ldots\ldots\ldots \quad 1$$

$$p_L = \frac{Q}{\cos \theta_L} \quad \ldots\ldots\ldots\ldots \quad 2$$

$$p_R = \frac{Q}{\cos \theta_R} \quad \ldots\ldots\ldots\ldots \quad 3$$

$$p_T = \frac{P}{\cos \theta_T} = \frac{\psi - Qb}{d \cos \theta_T} \quad \ldots\ldots\ldots\ldots \quad 4$$

$$p_B = \frac{P}{\cos \theta_B} = \frac{\psi - Qb}{d \cos \theta_B} \quad \ldots\ldots\ldots\ldots \quad 5$$

The total work due to the strain is:

$$\Sigma U = \left[\Sigma \frac{p^2 L}{2AE}\right]_{LR} + \left[\Sigma \frac{p^2 l}{2aE}\right]_{TB} + \left[\Sigma \frac{s^2 L_w}{2A_w E_w}\right] \quad \ldots\ldots \quad I$$

In which the first term of the right-hand member of the equation refers to the side wires; and the second term refers to the top and

bottom wires; and the last term refers to the wooden members. It may be noted that the wooden members of a fuselage are located in such positions that the strain in the wires is not materially changed by the strains in the wooden members.

Neglecting the wooden members, we have:

$$U = \frac{p^2_L\ L_L}{2A_L\ E_L} + \frac{p^2_R\ L_R}{2A_R\ E_R} + \frac{p^2_T\ l_T}{2a_T\ E_T} + \frac{p^2_B\ l_B}{2a_B\ E_B} \quad \ldots\ldots\ldots\ldots \quad \text{II}$$

Substituting equations 1, 2, 3, 4, and 5, in II:

$$U = \frac{Q^2}{2}\left[\frac{L}{\cos^2\theta\ AE}\right]_L + \frac{Q^2}{2}\left[\frac{L}{\cos^2\theta\ AE}\right]_R +$$

$$\frac{(\psi - Qb)^2}{2d^2}\left[\frac{l}{\cos^2\theta\ aE}\right]_T + \frac{(\psi - Qb)^2}{2d^2}\left[\frac{l}{\cos^2\theta\ aE}\right]_B \quad \ldots \quad \text{II}$$

The work U, with respect to the force Q, must be a minimum. Differentiating and setting equal to zero:

$$\frac{dU}{dQ} = 0 = \frac{2Q}{2}\left[\frac{L}{\cos^2\theta\ AE}\right]_L + \frac{2Q}{2}\left[\frac{L}{\cos^2\theta\ AE}\right]_R$$

$$\frac{2b(\psi - Qb)}{2d^2}\left[\frac{l}{\cos^2\theta\ aE}\right]_T - \frac{2b(\psi - Qb)}{2d^2}\left[\frac{l}{\cos^2\theta\ aE}\right]_B \quad \ldots \quad \text{III}$$

Or:

$$Q = \frac{\dfrac{\psi b}{d^2}\left[\left[\dfrac{l}{\cos^2\theta\ aE}\right]_T + \left[\dfrac{l}{\cos^2\theta\ aE}\right]_B\right]}{\left[\left[\dfrac{L}{\cos^2\theta AE}\right]_L + \left[\dfrac{L}{\cos^2\theta AE}\right]_R\right] + \dfrac{b^2}{d^2}\left[\left[\dfrac{l}{\cos^2\theta aE}\right]_T + \left[\dfrac{l}{\cos^2\theta aE}\right]_B\right]} \quad \ldots$$

Since

$$\cos \theta_L = \frac{Z_L}{L_L}$$

$$\cos \theta_R = \frac{Z_R}{L_R}$$

$$\cos \theta_T = \frac{Y_T}{l_T}$$

$$\cos \theta_B = \frac{Y_B}{l_B}$$

and if

$$E_L = E_R = E_T = E_B = E$$

and if

$$A_L = A_R = A \quad \text{and} \quad a_T = a_B = a$$

the formula reduces to:

$$Q = \frac{\frac{\psi b}{d^2}\left[\frac{l_T^3}{Y_T^2\ a} + \frac{l_B^3}{Y_B^2\ a}\right]}{\left[\frac{L_L^3}{Z_L^2\ A} + \frac{L_R^3}{Z_R^2\ A}\right] + \frac{b^2}{d^2}\left[\frac{l_T^3}{Y_T^2\ a} + \frac{l_B^3}{Y_B^2\ a}\right]} \quad \ldots\ldots \text{III}$$

P may be found from equation (1)

$$P = \frac{\psi - Qb}{d} \quad \ldots\ldots\ldots\ldots \quad (1)$$

The formula for Q may be further simplified by the following assumptions:

1. That the longest projection of a member in an orthographic projection may be taken as the true length of the member. The formula for calculating the true length is:

$$L = \sqrt{X_B^2 + Y_B^2 + Z_B^2}, \quad \text{(for the bottom wire).}$$

This assumption is equivalent to omitting the smallest term under the radical. Take as an example, the case which will give the largest error in this problem, that of the bottom wire of bay No. 2:

$$L = \sqrt{21^2 + 12.25^2 + 3^2}$$

$$= \sqrt{441 + 150.1 + 9}$$

$$= \sqrt{591.1 + 9}$$

from which the true length is 24.5 inches and the length of the projection is 24.3 inches. The error is .82 per cent.

2. That b and d be taken at the middle of the bay, that is, the average b and d for the bay. The torque is equal at both ends of the bay but opposite in direction. Recalling the equation:

$$\psi = Qb + Pd \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (1)$$

It may be noted that the magnitudes of Q and P depend on the magnitudes of b and d. Now at one end of each bay, b and d are relatively small and at the other end relatively large. To take the average b and d seems to be the logical assumption.

3. $L_L = L_R = L$, taken as the longer of the two, and $l_T = l_R = l$, taken as the longer of the two. This assumption is experimentally justified. The strain in a wire member, including the turnbuckle, loops and fittings, was found to be slightly greater than for an equal length of plain wire. There is an alternative to

taking the average length, but another step in calculating is added thereby, and the final results do not justify the added work.

4. That $Z_L = Z_R = d$, and $Y_T = Y_B = b$. There is no error in this assumption if the fuselage is symmetrical with respect to the longitudinal axis.

Equation III now reduces to

$$Q = \frac{\frac{\psi b}{d^2}\left[\frac{l^3}{b^2\ a} + \frac{l^3}{b^2\ a}\right]}{\left[\frac{L^3}{d^2 A} + \frac{L^3}{d^2 A}\right] + \frac{b^2}{d^2}\left[\frac{l^3}{b^2 a} + \frac{l^3}{b^2 a}\right]}$$

Or,

$$Q = \frac{\psi l^3\ A}{b\ (L^3\ a + l^3\ A)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{IV}$$

and

$$P = \frac{\psi L^3\ a}{d\ (L^3\ a + l^3\ A)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{V}$$

since

$$p_{LR} = \frac{Q}{\cos\theta_{LR}} = \frac{Q}{\frac{d}{L}} = \frac{QL}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{a}$$

and

$$p_{TB} = \frac{P}{\cos\theta_{TB}} = \frac{P}{\frac{b}{l}} = \frac{Pl}{b} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{b}$$

Substituting (a) in IV and (b) in V:

$$p_{LR} = \frac{\psi L}{bd}\left[\frac{l^3\ A}{L^3\ a + l^3\ A}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{VI}$$

$$p_{TB} = \frac{\psi l}{bd}\left[\frac{L^3\ a}{L^3\ a + l^3\ A}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{VII}$$

It may be noted that these formulae are in terms of the constants, a and A, and ψ; and the variables L, l, b, and d, which may be taken directly from the isometric drawing.

If the section is square, and a = A, VI and VII reduce to the obvious formula:

$$Q = P = \frac{\psi}{2b} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad \text{VIII}$$

Formulae VI and VII have been used to compute the stresses in the wires of a Curtiss JN-4B fuselage when subjected to 9000 pound inches of torsion at the bulkhead supporting the tail skid. The wires were standard tipped aircraft wires, No. 12, for the top and bottom, and No. 10, for the sides. The area of the cross-section of No. 12 wire is .00515 square inch, and the area of the cross-section of No. 10 wire is .00817 square inch.

The work may be simplified by the following method of tabulation:

Tabulation of the Solution of the Torque Equations:

$$p_{LR} = \frac{\psi L}{bd}\left[\frac{l^3 A}{L^3 a + l^3 A}\right] \quad \text{and} \quad p_{TB}\ \frac{\psi l}{bd}\left[\frac{L^3 a}{L^3 a + l^3 A}\right]$$

Units = pounds and inches.

Bay	Wire	L,l	L and l	$L^3 a$ / $l^3 A$	b,d and bd	Term in brackets	Stress / ψ	Stress lb.
2	LR	26.8	19,248	99.1	15.0	.458	.0788	707
	TB	24.3	14,348	117.1	12.3	.542	.0604	544
				216.2	184.5			
3	LR	31.6	31,554	162.1	18.5	.423	.0557	502
	TB	30.0	27,000	220.2	17.7	.577	.0387	348
				382.3	327.5			
4	LR	35.3	43,986	226.0	22.2	.426	.0435	392
	TB	33.4	37,259	304.1	21.0	.574	.0305	274
				350.1	466.5			
5	LR	40.2	64,964	334.0	26.0	.445	.0370	333
	TB	37.3	51,895	424.0	23.2	.555	.0275	248
				758.0	603.5			
6	LR	43.7	83,453	529.5	29.5	.456	.0325	292
	TB	39.7	62,570	511.0	24.8	.544	.0247	222
				940.5	732.0			

See Figs. 6 and 7 for dimensions.

p_{LR} = Pounds stress in left and right diagonal members of each bay of a fuselage due to torsion.

p_{TB} = Pounds stress in top and bottom members.

L = Length of side members, inches.

l = Length of top and bottom members, inches.

b = Average width of bay in inches.

d = Average height of bay in inches.

A = Cross-sectional area of the side members, square inches.

a = Cross-sectional area of top and bottom members, sq. inches.

ψ = Pound inches torque.

Discussion of Results.

Reference to the curves of the calculated stresses as compared to the experimental stresses (Figs. 9 and 10), will show that the two curves parallel each other up to the first bay, and that the calculated values are slightly higher in each case. The calculated curve may be brought closer to the experimental curve by considering b as the true horizontal projection of the top and bottom wires, and d as the largest true vertical projection of the side members. The effect of the fittings may be considered more definitely, but each of these considerations tend to mar the simplicity of the application of the formulae. There is considerable consolation, also, in knowing that the calculated values will undoubtedly be on the safe side of the actual stresses.

The application of the formula to the last bay of the fuselage, which ends in the rudder post, is the same as for any other bay, b and d being taken at the middle of the bay; the top and bottom wires being along the longerons.

Angle of Twist.

The angle of twist for the five bays was found experimentally to be three degrees and twelve minutes for a torque of 750 pound feet. This angle may be calculated by equating the work done in torsion to the summation of the work done in all the members. The work done in torsion is:

$$U = \frac{\alpha\psi}{2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{IX}$$

The work done by all the members is given by equation I. The angle of twist is, however, affected very greatly by any movement of the fittings, which may be neglected in computing the stresses, so that it may be expected that the calculated angle will be too low a value.

Case of the Broken Wire.

Tests were also made to determine the distribution of stresses due to a broken wire. For instance, one of the side wires was removed and the torque of 9000 pound inches was applied as in the previous tests. The opposite side wire took only 95 pounds (bay 4) while the top wire took 445 pounds and the bottom wire 455 pounds. The bulkhead diagonal in this case took 140 pounds. The load in the side wire and the bulkhead diagonal was due to the stiffness of the longeron in taking the place of the removed wire. It is obvious that the longeron would break before even a very small bulkhead wire was stressed to the limit. It is quite possible that the presence of the bulkhead wires in this case weakens the structure as a whole, as the absence of these wires would minimize the bending in the longeron, and throw the entire stress due to torsion in the top and bottom wires. The calculated stress in the top and bottom wires, assuming that they take the entire torque, is 645 pounds each.

Conclusions.

The conclusions which may be drawn from these experiments and calculations are:

1. The bulkhead wires take no load except in the case of broken or loose top, bottom or side wire.

2. The turnbuckle, loops and fittings on a fuselage wire may be taken as equal to a length of solid wire 1.2 or 1.3 times their combined length.

3. Stresses in an airplane fuselage due to torsion cannot be computed by statics except in very exceptional cases.

4. The cantilever method of solving the torsion problem is subject to considerable error due to the irregularity in ratio between the heights and widths of bulkheads.

5. Formulae VI and VII may be used to compute the stresses due to torsion, in a fuselage of rectangular cross-section, with a reasonable degree of accuracy in ordinary cases. The method appears to be sufficiently accurate for preliminary calculations, and may prove sufficiently accurate for final results.

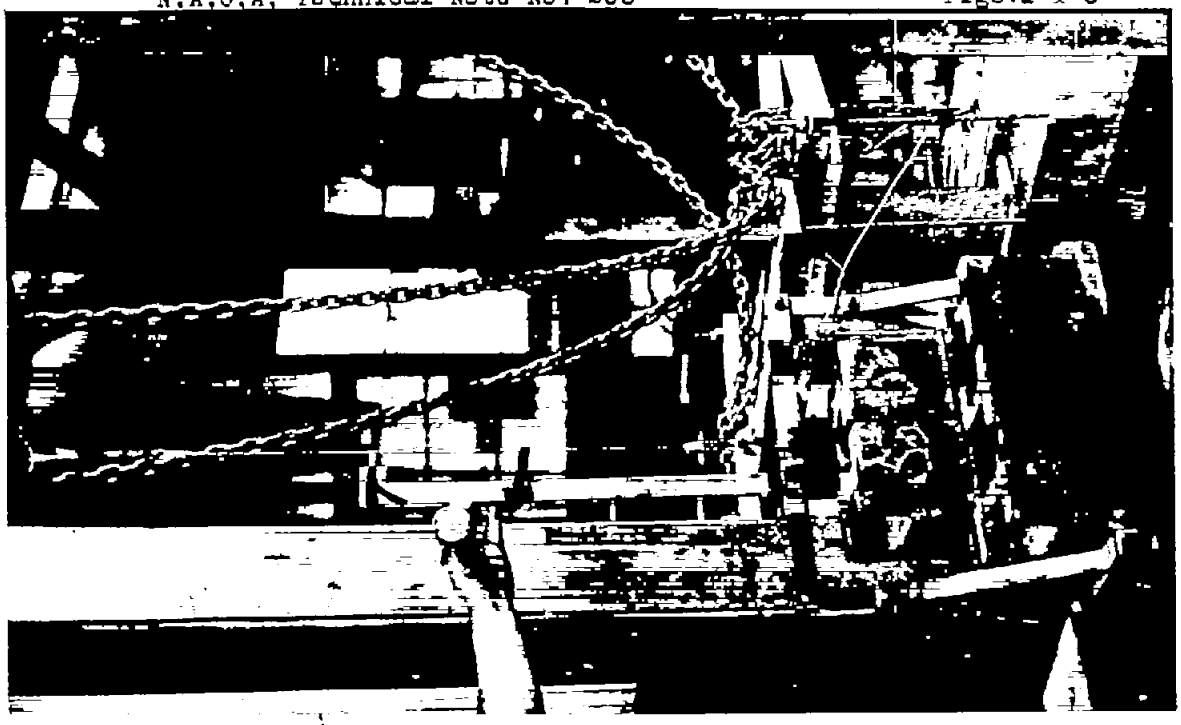

Fig. 3

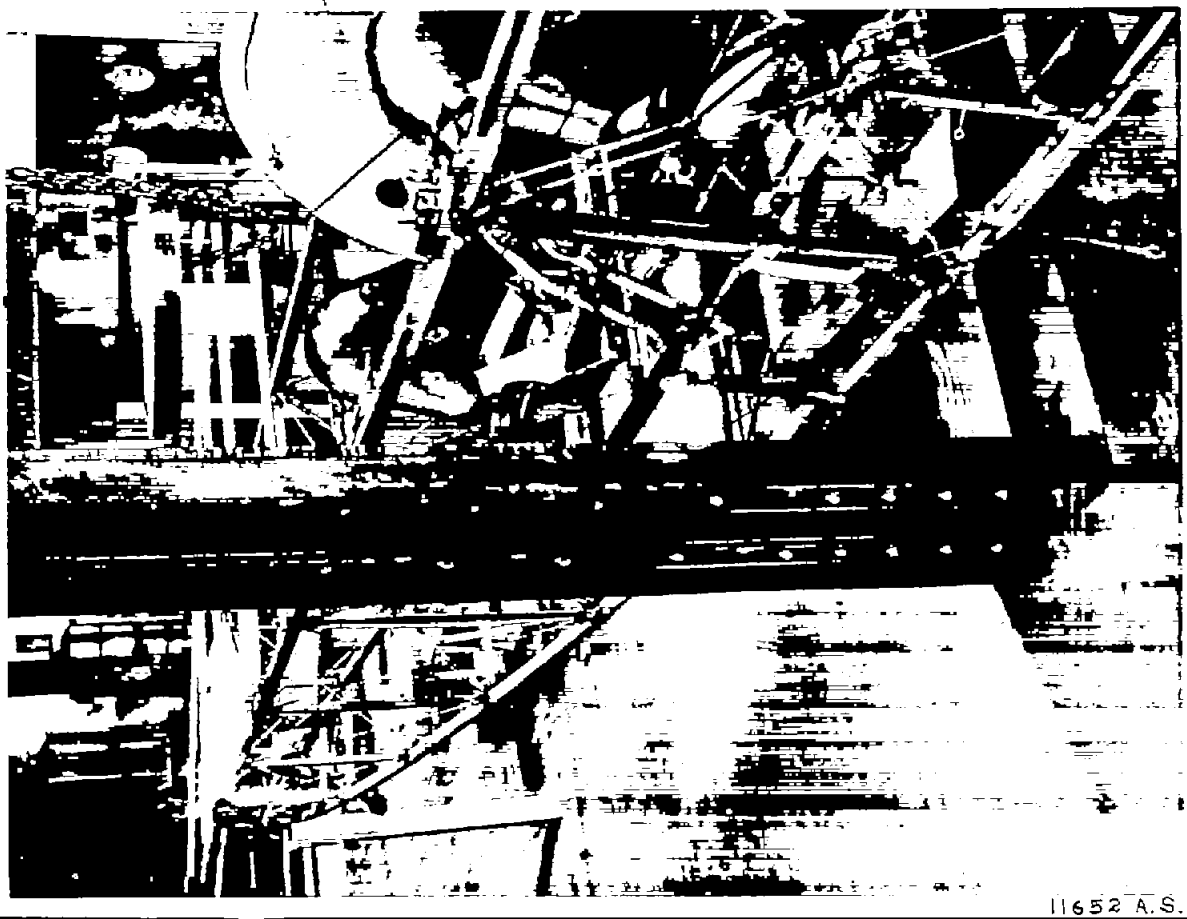

Fig. 1

11652 A.S.

Fig. 2

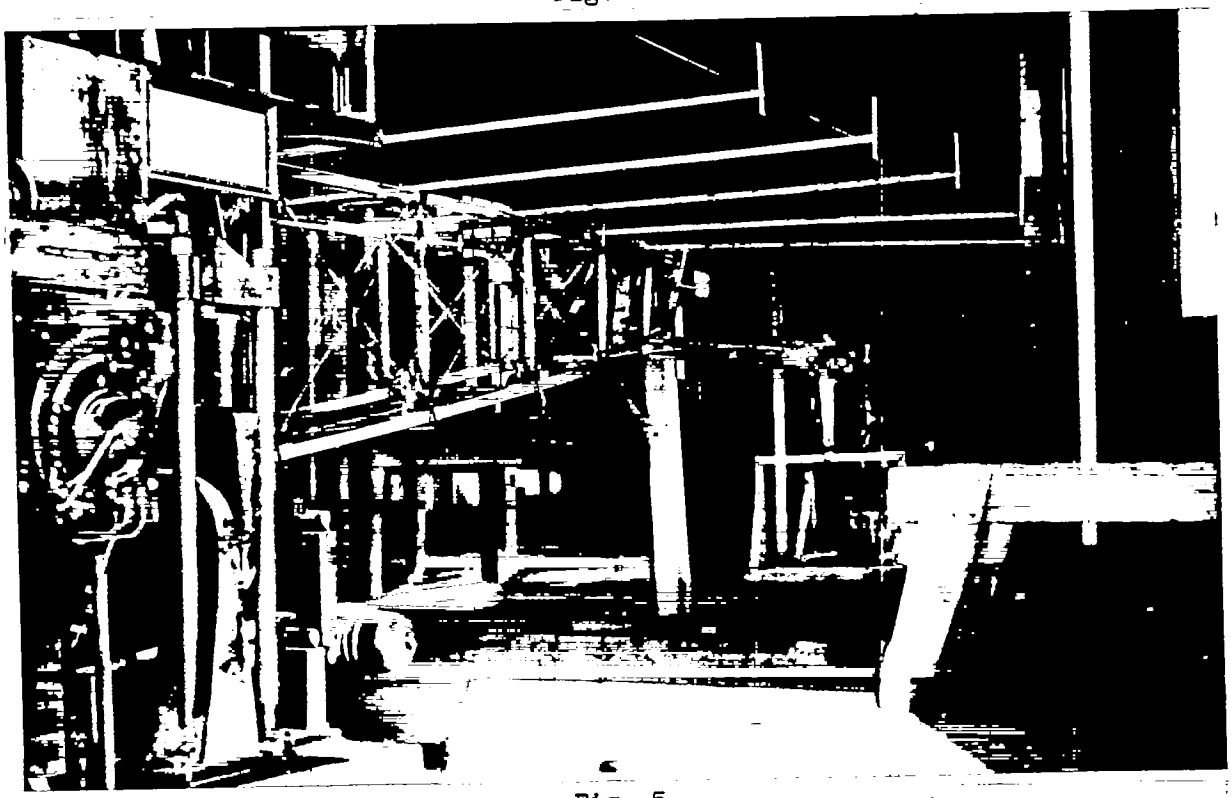

Fig. 5

11653 A.S.

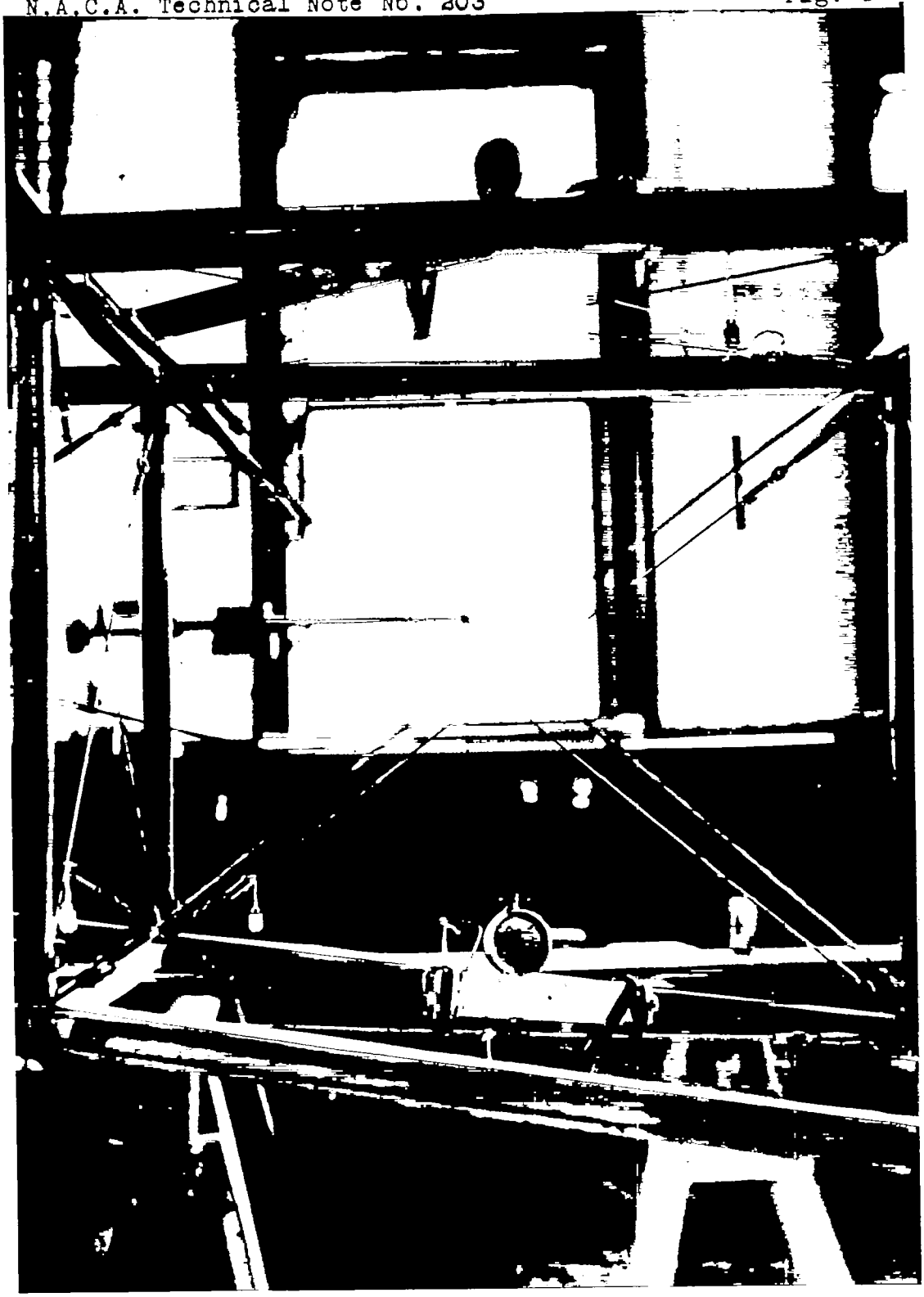

Fig. 4 11654 A.S.

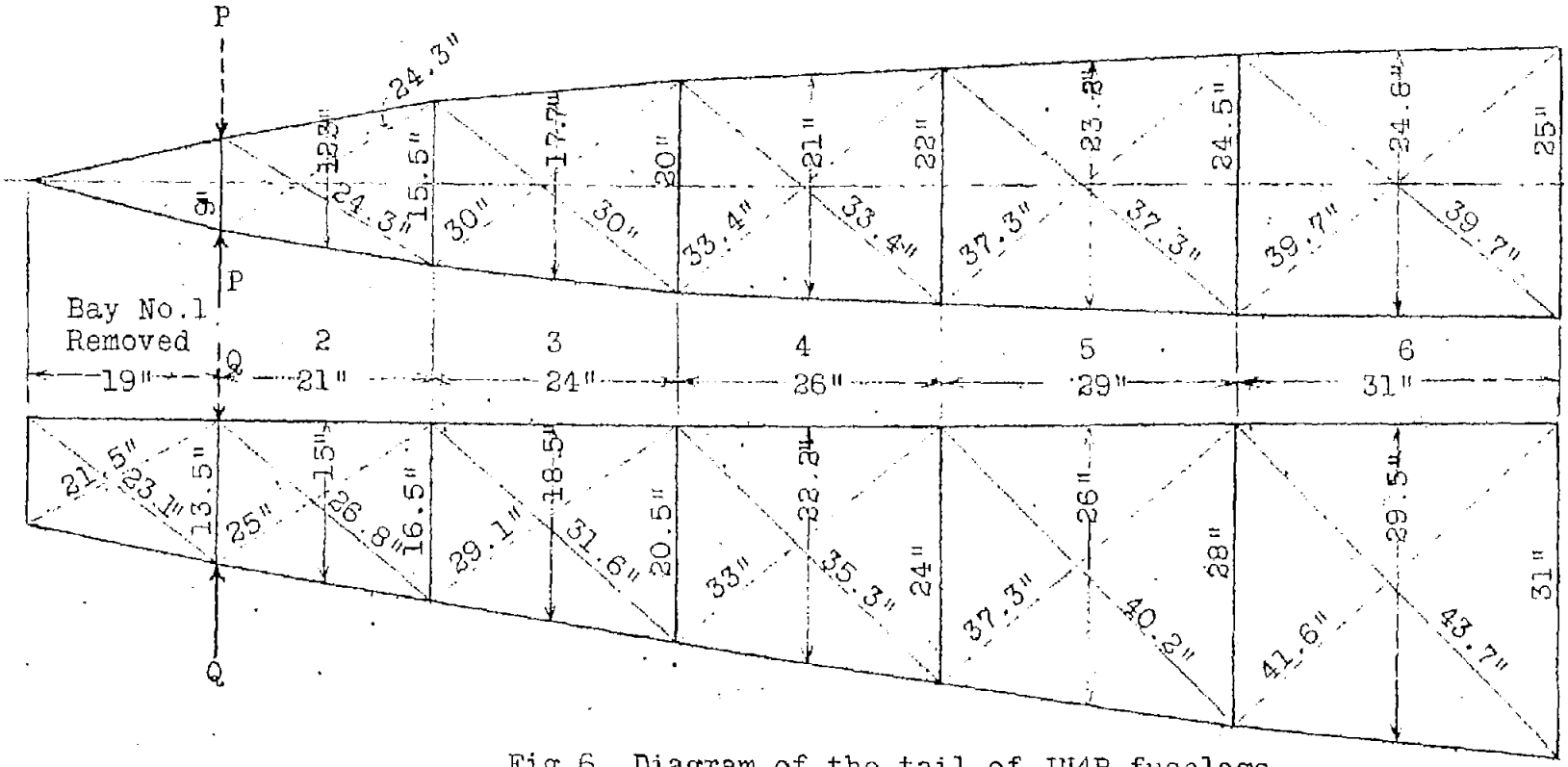

Fig.6 Diagram of the tail of JN4B fuselage.

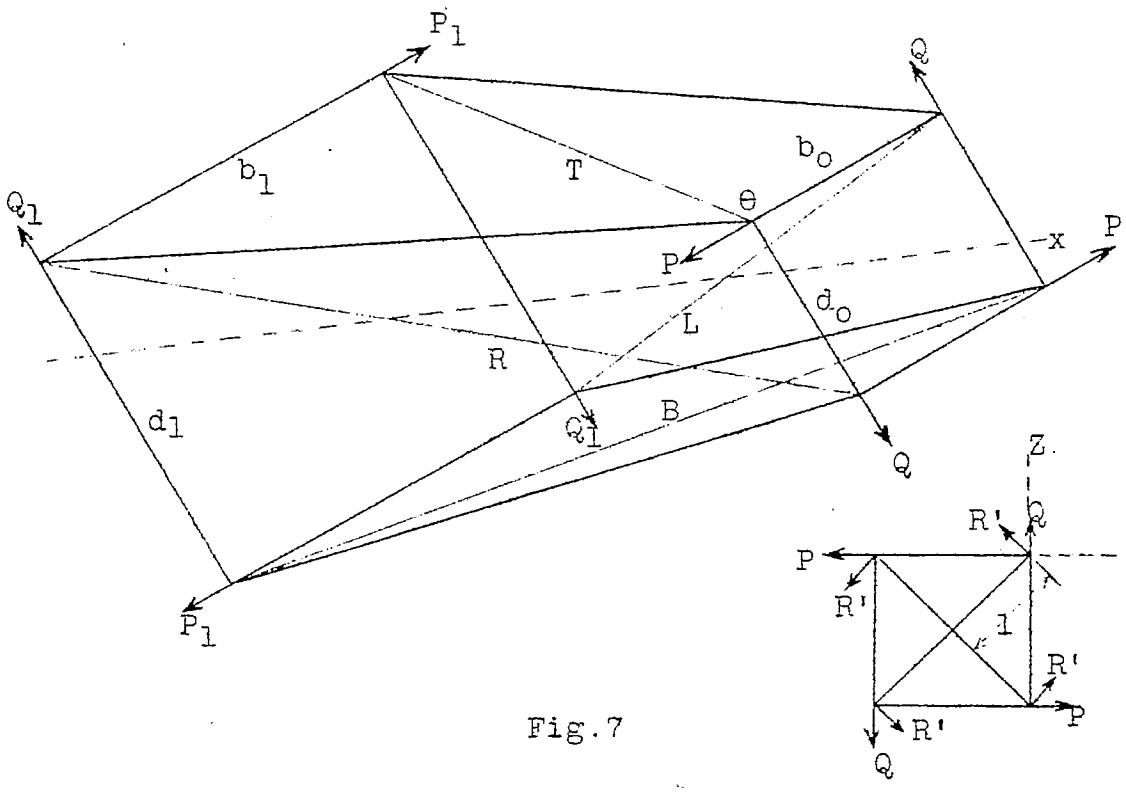

Fig.7

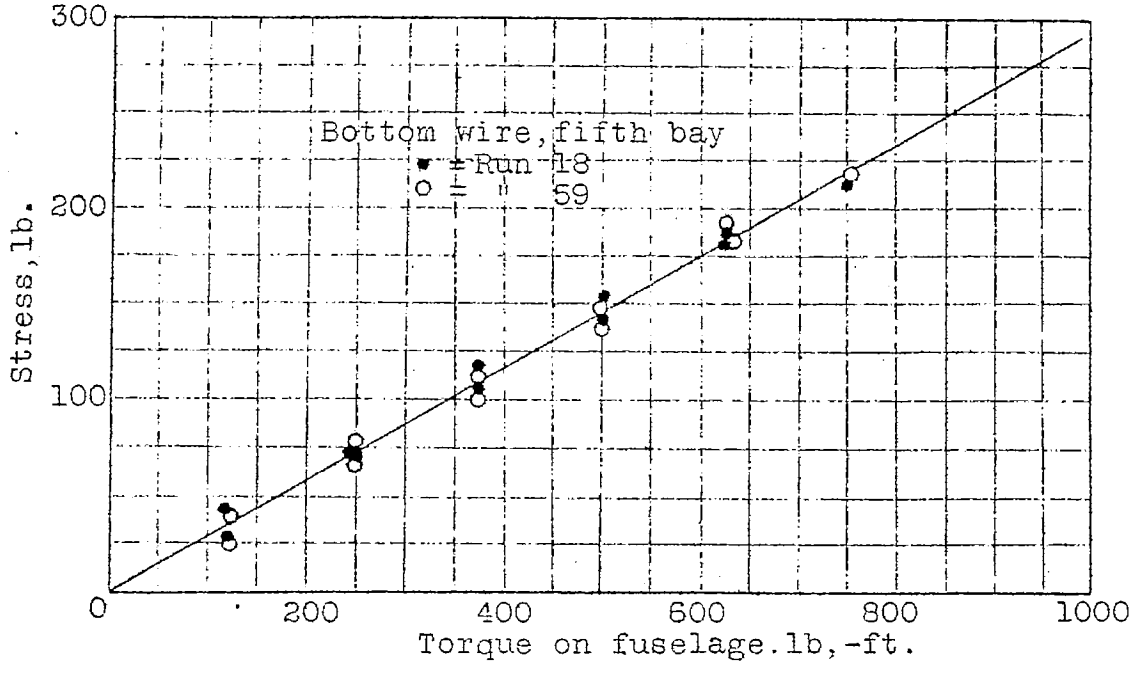

Fig.8 Typical stress-torque curve.

Comparison of calculated and experimental values of stresses.

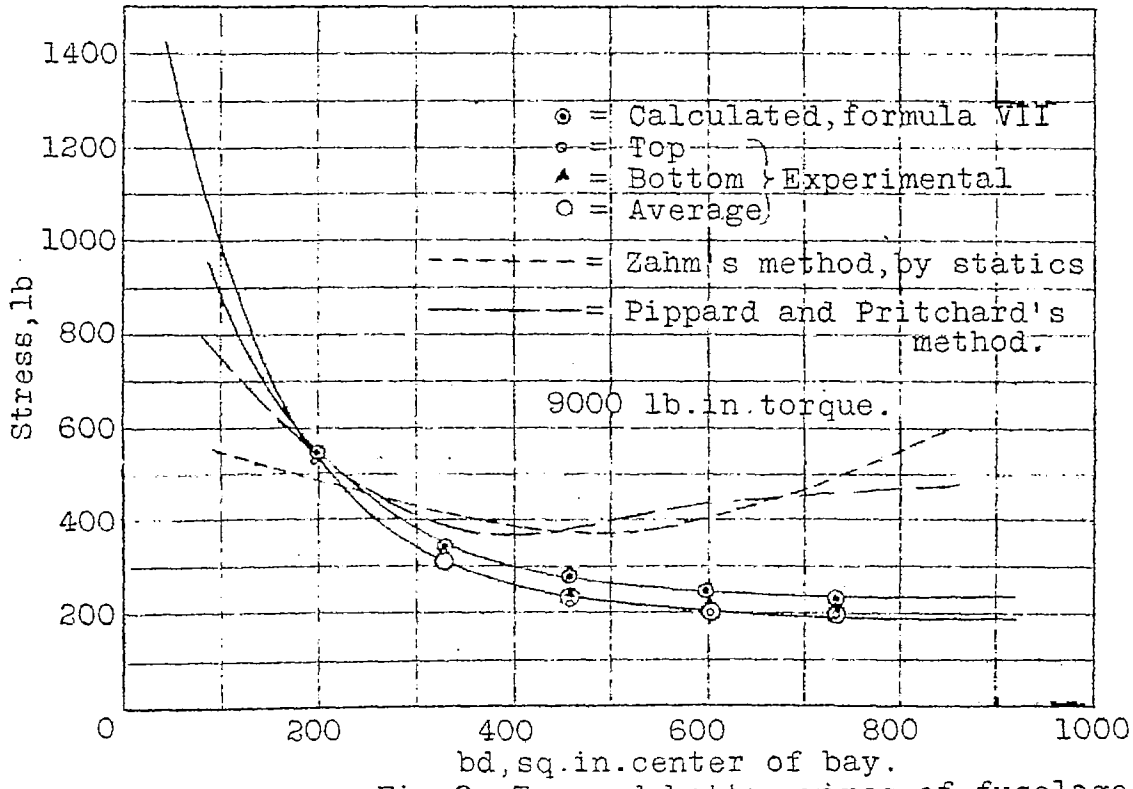

Fig.9 Top and bottom wires of fuselage.

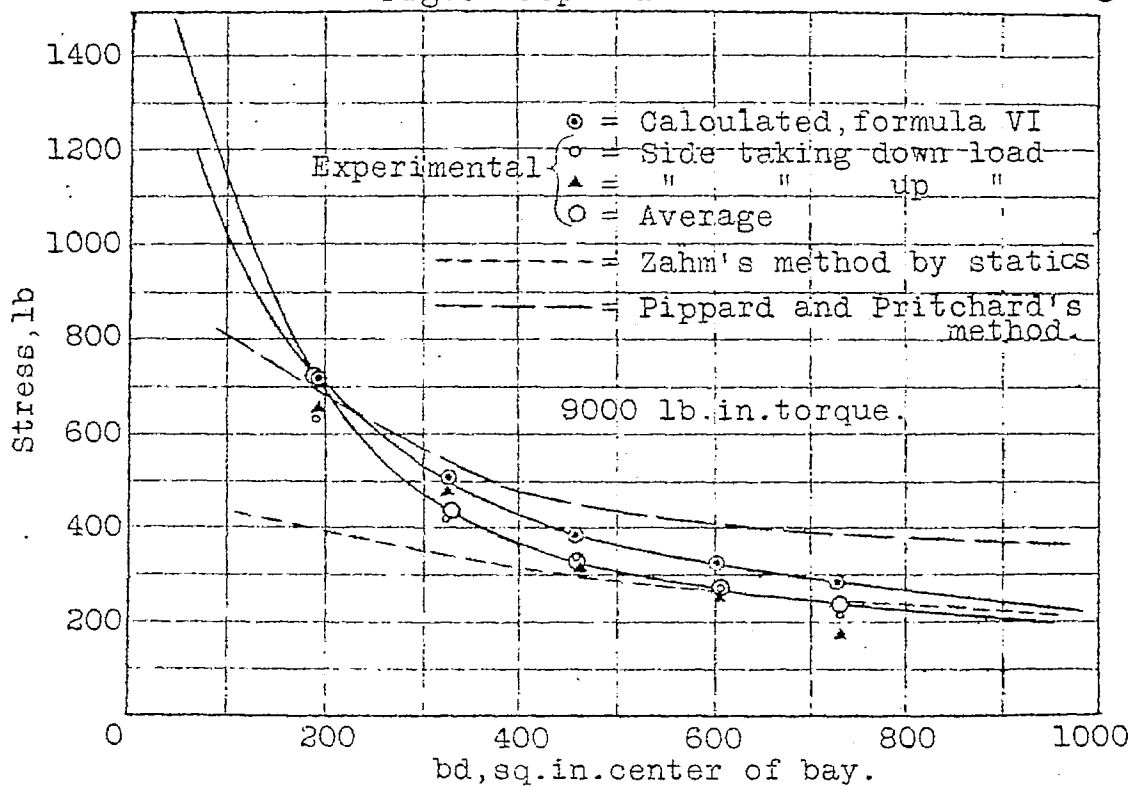

Fig.10 Side wires of fuselage.