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DETERMINATION AND CLASSIFICATION OF THE AERODYNAMIC PROPERTIES OF WING SECTIONS

By Max M. Munk

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Summary

The following note, prepared for the National Advisory Committee for Aeronautics, contains several remarks on the possible improvement of the experimental determination of the aerodynamic properties of wing sections. It shows how errors of observation can subsequently be partially eliminated, and how the computation of the maxima or minima of aerodynamic characteristics can be much improved.

Special attention is given to the use of indices for classifying the aerodynamic properties of wing sections. The merits and disadvantages of such indices now in use are discussed, and a method for the determination of a new kind of indices is given.

Introduction

The practical value of a wing section depends on the suitability of its shape for structural design and on its aerodynamic properties. These latter have reference to the aerodynamic stability of the section, that is chiefly the location

and the travel of its center of pressure, and to the performance, or otherwise expressed, to the magnitude of the lift and the drag at various angles of attack.

This note deals chiefly with the aerodynamic properties of wing sections having reference to the performance only, and hence refers to an incomplete and very specialized portion of the whole question. Nevertheless, it aims at a broader point of view than do most papers written previously on this subject.

It is not always sufficiently realized that the aerodynamic properties of a wing are influenced by many considerations besides the wing section. I will at once dismiss the case where there are so many different cross-sections at different points that the wing cannot be said to have "one" wing section. Such wings are rare and of no practical interest at present. But even if the wing section does not vary much along the span, the performance also depends on other dimensions of the wing. The importance of the aspect ratio has become widely known. The shape of the plan form, more especially of the wing tips, may have an important influence on the maximum lift obtainable under certain conditions. There is, furthermore, the washout of the wings, also with a marked influence on the lift and drag. The most pronounced changes of the performance characteristics occur when several airfoils with a certain wing section are put together to form a biplane or triplane cellule. It then requires much experience to draw

reliable conclusions from the aerodynamic properties of the monoplane as to those of the multiplane with the same wing section, and there always remains some uncertainty about it.

Our knowledge about aerodynamic properties of wing sections is still largely based on experience and on experiments; the progress of the theory relating to this subject refers chiefly to the interpretation of the results from such tests and to their correct application to practical problems. The experimental methods involve an additional source of uncertainties about the actual properties of wing sections. The tests are mostly made with small models, and although much has been written, comparatively little is known about the "scale effect."

There sometimes arises the question whether, under these circumstances, the aerodynamic investigations of wing sections per se is of any practical value at all. The discussion has made it clear that no information regarding the aerodynamic properties of wing sections can be complete; what we know about wings is not strictly an exact science. Has then this incomplete information a parallel in other branches of technics, or otherwise expressed, is the uncertainty involved in the general information about the aerodynamic properties of wing sections detrimental to its practical use, or not? In my opinion it is not. On the contrary, the information, imperfect as it necessarily is, is still of the highest practical use, and most other physical data used in other branches of

technics are as imperfect and are still highly useful. Most other technical branches are older and the present generation has forgotten the imperfectness of the methods used. They are employed without scruples, because the methods have been proven satisfactory by their results. Aeronautics is still a very young branch of technics, and not even its fundamental conceptions have become generally known. What wonder, if there is a superabundance of talk about little difficulties, which will have been forgotten once the methods have been put into extensive use.

Limiting the investigation of a wing section to one condition only constitutes an enormous simplification, and this one condition is a substitute for a thousand or more conditions of practical importance, not investigated for want of time. In the same way, any table of aerodynamic characteristics of a wing section or any curve plotted as illustration of such table is the representative of a hundred or a thousand similar ones, omitted for want of time to prepare them and to study them.

When realizing this, many a reader may more patiently take in view every portion of such a table or of such a curve. A whole table or an entire curve is not easily grasped, but then one curve for a wing section stands for a thousand of its kind, and it required great pains to arrive at so simple a description of the aerodynamic properties of a wing section, requiring the contemplation of one single curve only. This

is really scant information. If a long and careful investigation has led to a new wing section of particularly promising properties, it is to be recommended that such a section be investigated under different conditions. It means over-doing research work to measure each wing section under more than one condition, but it is entirely legitimate, practical and sound to devote a large series of tests to the investigation of the aerodynamic properties of special wing sections which seem to be particularly promising and of practical importance.

The Determination of the Aerodynamic Properties.

Aside from the limitation to one test, under one set of standard conditions only, the investigation of a wing section has to face the imperfections of the wind tunnel methods. (a) The walls or boundaries of the air flow interfere with the creation of the flow pattern. (b) The air flow is never entirely parallel and of constant speed; it contains irregularities which may not even remain unchanged during long intervals of time. (c) The fastenings, connecting the model with the balance or balances, interfere likewise with the creation of the undisturbed flow pattern. These imperfections are not serious in themselves, but prevent the conditions from approaching the standard conditions, represented by a rectangular unwarped wing of aspect ratio 1 : 6 in an unobstructed and unlimited air flow of uniform velocity. The obstructions

are different in different wind tunnels or even with different tests in the same wind tunnel and therefore make direct comparisons of the results unsuitable. However, every airplane wing is surrounded by struts, by the fuselage and by other structural parts and a fastened wing much better imitates the actual conditions than would an airfoil being held in place without any fastening members. Little differences of the flow structure likewise impair the standard conditions, but neither is the motion of an airplane quite steady, nor is the atmosphere free from gusts and other motions. The wall interferences are not serious and are uniform for different tests.

Wind tunnel work is further difficult on account of particularities pertaining to the information desired. The aerodynamic properties are not always steady functions of the conditions, more specially of the angle of attack. They are sometimes even ambiguous; the drag, for instance, may have more than one value under the same set of conditions. The plotted curves show accordingly sudden breaks or even gaps, and in the latter cases several branches of the curves may overlap each other. When such properties of a wing section become evident from a test, they should be noted and taken down as such. The wing section is then found to have irregular aerodynamic characteristics under the condition tested, and very probably also under similar conditions. Under such

similar conditions, however, the details of the irregularities may differ widely, the break shifts to another angle, or the gap becomes smaller or larger. When such irregularities are observed, the investigator should remember that he makes one measurement to take the place of a hundred or of a thousand measurements, as discussed above. He is only interested in those properties common to all practical conditions, not in those alone for the one set of conditions selected as a characteristic one. It seems to me to be a waste of time to study systematically and with each new wing section any details of the irregularity which may never occur in the same quantitative way in actual flight. The investigator should keep his thoughts under control as is done in statistics, and he should learn to separate the facts merely observed in one case from those characteristics of the section in general. This requires skillfulness, but then the engineer is not a mere mathematician, but an artist as well, and "art" meant originally "skill."

In most cases, and over considerable portions of the range tested, the aerodynamic properties of wing sections are steady. The immediate object is then their determination for the conditions tested as accurately as can be done. The investigators should take unusual pains in making the latter determination as accurate as possible in order to make up partly at least for the imperfections of the method. Strange

to say, instead of doing that, the inaccuracies involved in the method lead the investigators often into the temptation to be careless in the exact measurement of the air forces. It can also often be observed that wind tunnel engineers invent their instruments and methods, instead of designing them as engineers do by making use of experiences in other lines of research.

Conspicuous examples are the balances used in wind tunnels. As a rule, they are different in each wind tunnel, and what is more surprising, the construction details are different too, and even different on the same balance. Now the construction of balances is as old as the oldest civilization. Modern balances have been developed so that they are more exact than most other instruments, and to capacities largely surpassing those needed in wind tunnels. A medium priced chemical balance measures up to 100 g say, to an exactness of 1/10000 of a gram, that is, up to 1/1000000 of its capacity. Other balances allow weighing loaded railroad cars. The same relative exactness for weighing air forces in a wind tunnel would require taking readings exact to 1/10 of a gram, as an average. This is often neglected. Some wind tunnel balances, measuring only a few kilograms, have an error of a full gram or more. It is true that the combination of several balances introduces certain errors, but still the balances should be sensitive enough so that any error arising from the weighing as such be entirely negligible.

It is known that experimental errors can be partly eliminated by first increasing the number of observations and by subsequently judiciously smoothing out and averaging up the results. The procedure consists in the end in eliminating by the use of the calculus of probability, errors that are distributed according to mere probability. It is not necessary for the present purpose to go into anything like full detail. It has chiefly to be emphasized that only those portions of readings that are steady, when there are no breaks or gaps in the curves, can be treated in the way now described. The procedure is largely simplified if one variable is changed by equal steps. Large modern tunnels give results which are worthy of increasing the original exactness by such statistieal methods. The variable density wind tunnel of the National Advisory Committee for Aeronautics, for instance, is free from scale effect, and the theory of wing sections has been brought to such perfection that the general character of the results can be anticipated. The exact details are the object of the tests. For these reasons, I would recommend to measure the air forces at equal intervals of the angle of attack and at smaller ones than has hitherto been done. The interval could be 2°, $1\frac{1}{2}^\circ$ or 1°, and the last interval seems by no means too small.

Suppose now the drag to be regular, and at certain angles, α_1, α_2, $\alpha_m \cdots \alpha_n$ with constant intervals $\alpha_m - \alpha_{m_1} = 1^\circ$,

let the drag measured be D_1, D_2, etc.

It is required to determine a value D_m', corresponding to an angle α_m, from all values D, in such a way that this value is improved and is probably more exact than the original D_m.

There are very many rules for the solution of this problem. Practically all of them are of the type

$$D_x' = a_0 D_x + a_1(D_{x-1} + D_{x+2}) + \ldots \qquad (1)$$

where the a's are positive and subject to the condition

$$a_0 + 2\,(a_1 + a_2 \ldots\ldots a_n) = 1$$

This latter condition is necessary to obtain $D_m' = D_m$ from (1) in case that all D's are equal.

Mr. Ugo Broggi (Reference 1) has recently proposed to use the convenient formula

$$D_x' = \frac{\epsilon - 1}{\epsilon + 1}\left[D_x + \frac{1}{\epsilon}\,(D_{x-1} + D_{x+1}) + \frac{1}{\epsilon^2}\,(D_{x-2} + D_{x+2}) + \ldots\right] \qquad (2)$$

which is a special case of equation (1).

In this equation, the number of terms used and the value of ϵ, $(\epsilon > 1)$ have to be determined from general considerations. I would think it sufficient to consider $n = 3$, that is, the value of the drag at six adjacent points. The choice of ϵ depends on the relative importance of the improvement to the original value. I would think that 7/8 of the observed value should be accepted for the drag and 1/8 left for correc-

tion, making $\epsilon = 15$.

The formula would then read

$$D'_x = \frac{7}{8}[(D_x + \frac{1}{15}(D_{x-1} + D_{x+1}) + \frac{1}{225}(D_{x-2} + D_{x+2}) + \\ + \frac{1}{3150}(D_{x-3} + D_{x+3})] \qquad (3)$$

The relation between the lift and the angle of attack is nearer to a linear one, and $\epsilon = 7$ may be more proper for the lift or moment. Then

$$L_x = \frac{3}{4}[L_x + \frac{1}{7}(L_{x-1} + L_{x+1}) + \frac{1}{49}(L_{x-2} + L_{x+2}) + \\ + \frac{1}{294}(L_{x-3} + L_{x+3})] \qquad (4)$$

The method can further be improved by first separating the profile drag coefficient from the observed drag coefficient by deducting the induced drag coefficient

$$C_{D_i} = \frac{C_L^2 \text{ (aspect ratio)}}{\pi}, \text{ i.e.,}$$

$$C_{D_i} = \frac{C_L^2}{6\pi} \text{ for aspect ratio} = 1/6$$

Such deduction leaves a more constant drag, making it easier to smooth out the errors. Moreover, the study of the profile drag coefficient is the aim of most investigators.

Formula (2) or (3) can and should likewise be employed for increasing the exactness of characteristics other than the drag, such as the lift and the resultant moment, provided that

the observed values change uniformly.

Classifying the Aerodynamic Properties of Wing Sections

I have tried to explain in the introduction that one standard test with a wing section is really the representative of many hundred tests and that likewise the results stand for many hundred sets of such results. It is customary to illustrate the results by plots, and it has become prevalent in most countries to plot coefficients of the air forces rather than the forces themselves, more particularly to plot the lift coefficient against the drag coefficient. The curve thus obtained is called a polar curve or lift curve. The magnitude of the angle of attack is usually inserted at several points of the curve by writing in its magnitude, not by plotting it. That saves inserting a second curve, which would make the diagram less clear and less impressive to the eye and mind. The parabola of the induced drag is usually drawn in. This parabola runs close to the polar curve and hence assists in impressing to the eye of the observer the latter's exact shape and location.

Although in publications the lift is usually not plotted against the angle of attack, it is recommended that the investigator make such a plot for his own benefit, and get better acquainted with the relation between the lift and the angle of attack.

The polar curve constitutes a kind of summary or average of a vast number of data condensed into one impressive curve; and its development required many years. And yet experience shows that this simplification has not yet been carried far enough. There is a demand for something more condensed, more impressive and more easy to memorize than any curve can be. This can only be a figure or a set of figures. Accordingly, and chiefly in this country, there are many different quantities proposed for characterizing polar curves. Not satisfied that a thousand curves are condensed into one curve, the public wants this one curve, and by it all the aerodynamic merits of a wing section, to be indicated by one or a few figures.

It is known that of two wing sections, one may be superior to the other for a certain type of airplanes and the other section may be superior to the first one for another type. Hence, it cannot be said generally that one of the two sections is superior to the other one. This excludes the possibility of arranging all wing sections according to their merit, and of finding index numbers for wing sections indicating by their magnitude the relative merits of the wing section.

Hence any index figures, however found, can only formally classify the aerodynamic properties of wing sections. This classification can be done in many ways and accordingly very many different kinds of classifying index figures have been proposed and have been used in the past. They again can be arranged in several classes. It can be said of all of them,

that they give quick, although somewhat superficial information about the aerodynamic properties of wing sections, and about small differences between such properties. In that respect they save time. They also save thought, but thinking should not be saved. There is always the danger that the classification indices are mistaken for an index of the merit of the wing section. This disadvantage can be largely diminished by using several kinds of classification indices at the same time. Let us take a glance at the several kinds of indices devised.

The Different Kinds of Classifying Indices

A great number of classifying indices used in the past are the maxima or the minima of some aerodynamic coefficient.

Before proceeding to them, it is pertinent to make some remarks on the determination of such a maximum or minimum quantity of a function which has been measured at certain intervals only. The intervals may be equal. Suppose the measured quantity y to be plotted against the variable (as for instance, the angle of attack) that was varied when measuring y. Suppose further, the points so obtained to be connected by a curve representing the value of y at each point x. Then there are two main reasons why the largest value of y measured is not the maximum of y in the interval investigated: (1) All measurements contain errors of observation; and (2) the maximum is probably situated between two points

x at which y was measured, and hence the maximum is larger (and the minimum is smaller) than the largest or smallest value of y measured.

To determine the maximum (or minimum) it will be sufficient to take into account the point x where the largest (smallest) value of y has been measured, together with the two adjacent points. For those three points, the observation errors should be diminished by means of a formula of the type of equation (1), say by the use of equation (3) or (4). In this way, three improved values y_1, y_2, and y_3 at the points x_1, x_2, and x_3, are obtained. It is supposed at present that $x_2 - x_1 = x_3 - x_2$ and that y_2, is larger (or smaller) than y_1 and y_3.

An improved value for the maximum of y can now be determined by replacing the y curve in the interval $x_1 - x_3$ by a parabola, having the same ordinates at x_1, x_2, and x_3 as the original y curve. The equation of this parabola would be

$$y = y_2 + \frac{y_3 - y_1}{x_3 - x_1} (x - x_2) + \frac{2(y_1 + y_3 - 2y_2)(x - x_2)^2}{(x_3 - x_1)^2} \qquad (5)$$

as can easily be verified by inserting in turn $x = x_1$, $x = x_2$, $x = x_3$, and by making use of the relations $x_3 - x_1 = 2(x_2 - x_1) = 2(x_3 - x_2)$. The position of the maximum will be found by differentiating equation (5) with respect to $(x - x_2)$

$$\frac{d\,y}{d\,(x - x_2)} = 0 = \frac{y_3 - y_1}{x_3 - x_1} + 4\,\frac{y_1 + y_3 - 2y_2}{(x_3 - x_1)^2}\,(x - x_2) \qquad (6)$$

or

$$x - x_2 = -\frac{1}{4}\,\frac{(y_3 - y_1)}{x_3 - x_1)}\,\frac{(x_3 - x_1)^2}{(y_1 + y_3 - 2y_2)} = -\frac{1}{4}\,\frac{(y_3 - y_1)\,(x_3 - x_1)}{(y_1 + y_3 - 2y_2)} \qquad (7)$$

Inserting (7) in (5) gives the maximum (or minimum)

$$y_{max} = y_2 - \frac{1}{4}\,\frac{(y_3 - y_1)}{y_1 + y_3 - 2y_2} + \frac{1}{8}\,\frac{(y_3 - y_1)^2}{y_1 + y_3 - 2y_2} =$$

$$= y_2 - \frac{1}{8}\,\frac{(y_3 - y_2)^2}{y_1 - 2y_2 + y_3} \qquad (8)$$

Formula (8) for the maximum or minimum contains the largest (or smallest) y observed, and in addition, the two adjacent values of y.

The maxima or minima used for classifying the aerodynamic properties of wing sections are (1) the maximum lift coefficient, (2) the maximum L/D ratio for the aspect ratio 1:6, (3) the minimum drag coefficient for the aspect ratio 1:6, (4) the ratio $\frac{\text{maximum lift coefficient}}{\text{minimum drag coefficient}}$ for the aspect ratio 1:6.

There are, furthermore, in use the maximum or minimum of more general combinations of the lift coefficient and of the

drag coefficient, in the form

$$\frac{C_L{}^p}{C_D{}^q}$$

more specially

$$\left(\frac{C_L{}^3}{C_D{}^2}\right)_{max}$$

and similar ones.

I consider as equally eligible for use the same expressions derived for infinite aspect ratio, that is, the same expressions in which the drag coefficients C_D derived from a standard test has been replaced by the coefficient of profile drag

$$C_D - \frac{C_L{}^2 \text{ aspect ratio}}{\pi} = C_{D\infty}$$

The aspect ratio 6 is nearer to the actual aspect ratio but classifying the sections for this aspect ratio 1 : 6 involves the danger that the designer does not give full attention to the aspect ratio, but uses the classifying indices directly rather than the coefficients referring to the actual aspect ratio of his wings. The drag coefficients for infinite aspect ratio are also much less variable than those for the aspect ratio 1 : 6, and the induced drag being eliminated, they give much clearer information about the effect of the wing section alone.

It must, however, be said in favor of the aspect ratio 1 : 6, that the results are directly obtained for this aspect ratio, that many designers are accustomed to figures with 1 : 6 and that there is a large amount of literature about wings of 1 : 6 ratio.

There is another point that should be considered before deciding on the use of certain coefficients for practice, and before selecting their maximum or minimum as a classification. The drag of the entire airplane is approximately the sum of the wing drag and of the parasite drag of the remaining parts of the airplane. Hence a coefficient like L/D, for instance, for the wings is not equal to the same coefficient for the airplane, since D is the drag of the wings and not the drag of the airplane. Now, it is comparatively easy to compute the coefficient for the airplane from the coefficient of the wings in the special case that the drag is contained as a direct factor in the coefficient, for instance, D/L, not L/D. For then

$$D/L\ (\text{airplane}) = D/L\ (\text{wings}) + \frac{D\ \text{parasite}}{L\ \text{wings}}.$$

This is the reason why the use of

$$\frac{C_D}{C_L^{3/2}}$$

is more convenient than the use of

$$\frac{C_L^3}{C_D^2}$$

The consideration of this point is important when making the decision about which classification indices should be used, if any.

The maximum lift coefficient is of particular practical importance and its use as a classifying index would easily suggest itself. There is, however, this difficulty, that its magnitude is much affected by secondary conditions, and by the imperfections of the wind tunnel methods. Many wing sections, furthermore, have no maximum of the lift coefficient within the flying range of the angle of attack. If, in view of its great practical importance, the maximum lift coefficient or expressions derived therefrom would be used as classifying index, the method of determining should be improved by varying the condition under which it is determined, and on the other hand, by specifying more closely what shall be understood under "maximum lift coefficient."

The point just discussed is of great importance for the discussion of the second class of classifying indices, now in use. Not the maxima or minima of the coefficients mentioned before, but their values at certain fractions of the maximum lift coefficient are used. That is sound in itself, but it requires the exact knowledge of the maximum lift coefficient, and this knowledge is generally lacking. The classifying indices of the latter kind are probably less valuable for comparison of different wing sections than the kind formerly dis-

cussed. An improvement would be obtained by choosing the values of certain coefficients at specified values of the lift coefficient. This, however, does not give a good comparison for judging the practical usefulness of the section. In all cases, the observation errors should be eliminated as far as possible.

The superiority of the maximum coefficients was founded on the easier way of determining their value; the superiority of the values of fractions of $C_{L_{max}}$ lies in their direct application to practical problems. It is desirable to combine those advantages. Such improved indices could be obtained by averaging the values of certain coefficients in the neighborhood of certain fractions of the maximum lift coefficients, so that the values nearer this specified $C_L = \frac{C_{L_{max}}}{n}$ would be given a larger "weight" in the summation.

In this way variations of the exact magnitude of the maximum lift coefficient would not result in serious changes of the classifying indices. Such summation can be done in very different ways. Formula (2) can be used for this end, giving ϵ a smaller value than for mere elimination of the observation errors, for instance, $\epsilon = 2$ or $\epsilon = 3$.

Conclusion

The progress of heavier-than-air craft design is closely interlinked with the improved determination of the aerodynamic properties of wing sections and with a most intelligent presentation of this information to the airplane designer. The use of classifying indices is helpful in the latter respect, but it is really helpful only if these indices are judiciously chosen and precisely determined.

References

1. Ugo Broggi: Ueber Mechanische Ausgleichung. Zeitschrift fur Angewandte Mathematik und Mechanik, Vol. V., No. 3, June, 1925, p. 267.