REPORT No. 329

THE TORSIONAL STRENGTH OF WINGS

By C. P. BURGESS
Bureau of Aeronautics, Navy Department

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SUMMARY

This report is submitted to the National Advisory Committee for Aeronautics by the Bureau of Aeronautics, Navy Department. It describes a simple method for calculating the position of the elastic axis of a wing structure having any number of spars. It is shown that strong drag bracing near the top and bottom of a wing greatly increases the torsional strength. An analytical procedure for finding the contribution of the drag bracing to the torsional strength and stiffness is described, based upon the principle of least work, and involving only one unknown quantity.

The validity of the new method of analysis is tested by applying it to a two-fifths scale model of the large steel tubular 3-spar wing of the Huff-Daland XHB monoplane. The calculated stresses are checked by comparison with the strains observed by means of electric telemeter strain gauges secured to the spars during sand load tests in the static testing laboratory of the Army Air Service Engineering Division at Dayton, Ohio.

The torsional strength of a wing determines very largely the distribution of air forces upon it, and the tendency to flutter. Insufficient torsional strength produces wash-in or an increasing angle of attack toward the wing tips in the high incidence condition, further increasing the load on the front spar in the condition which is already the most severe. Conversely, torsional yielding in the low incidence and nose dive conditions produce washout of the wing shape and may exaggerate the critical condition for the rear spar.

The mathematical theory of the forces producing flutter is not yet sufficiently far advanced to determine by direct calculation the critical air speed at which flutter will commence. Comparison with successful practice must still be the principal criterion upon which to judge the adequacy of the torsional strength of a new design of wing. Obviously this comparison will be greatly facilitated by use of a coefficient of torsional rigidity including the principal factors in torsional strengths. A coefficient for comparing the torsional rigidity of different wings is derived in this report.

INTRODUCTION

The tendency of modern airplane design is largely toward monoplanes in which the wings are either full cantilevers, or more frequently, are cantilevered beyond a single pair of external struts. In either case, the cantilever portion of the wing must have sufficient torsional strength to prevent wing flutter at all flying speeds.

In the methods commonly prescribed and used for the structural analysis of 2-spar wings, it is assumed that the ribs act in a manner analogous to bridges resting upon the spars as abutments. It is further assumed that the torsional strength of the cantilever portion of the wing is derived solely from the spars acting independently without the assistance of drag bracing to resist torsion. Such a wing is not only inefficient in torsion, but it must have spars designed for two extreme positions of the center of pressure, so that when the front spar is carrying its maximum load, the rear spar is partially idle, and vice versa. The wing is therefore required to carry excess structural weight.

The advantages of strong drag bracing near the top and bottom surfaces of the wing are now generally conceded; but reasonably simple methods of calculating the contribution of the drag bracing to the torsional strength of the wing have not hitherto been presented. It is the

purpose of this paper to describe an improved method for calculating the torsional strength of wings having efficient drag bracing. Moreover, the method is not confined to 2-spar wings, but is equally applicable to multispar construction.

CONTRIBUTION OF THE DRAG BRACING TO TORSIONAL STRENGTH

Figure 1 represents a cross section through a wing having two spars, each consisting of two longitudinal members with diagonal bracing in the plane of each spar only. The arrows represent a torsional moment about the elastic axis or centroid. Without drag bracing, the torsion is resisted only by the strength of the spars in the vertical plane. The addition of drag bracing

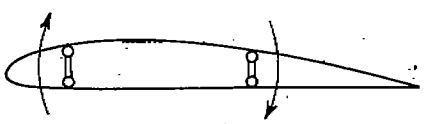

FIGURE 1.—Torsion in two spars without drag bracing

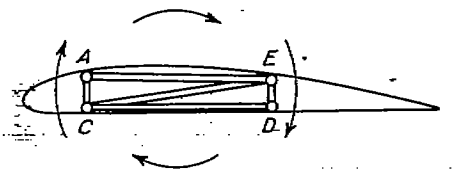

FIGURE 2.—Spars reinforced by drag bracing against torsion

in the top and bottom horizontal planes not only adds two more trusses to resist torque, but may be designed to eliminate entirely all forces in the longitudinal members due to torsion. For example, the longitudinal tubes A and D, in Figure 2, are in compression as members of the front and rear spars, but are in tension as members of the top and bottom drag trusses; and conversely for members C and E. By proper proportioning of the members, these tensile and compressive forces in the longitudinals can be made to cancel each other, with the result that the torsion is resisted entirely by the shear members in the spars and drag trusses. The nearness of the drag trusses to the centroid is largely compensated by their great depth in proportion to the spars.

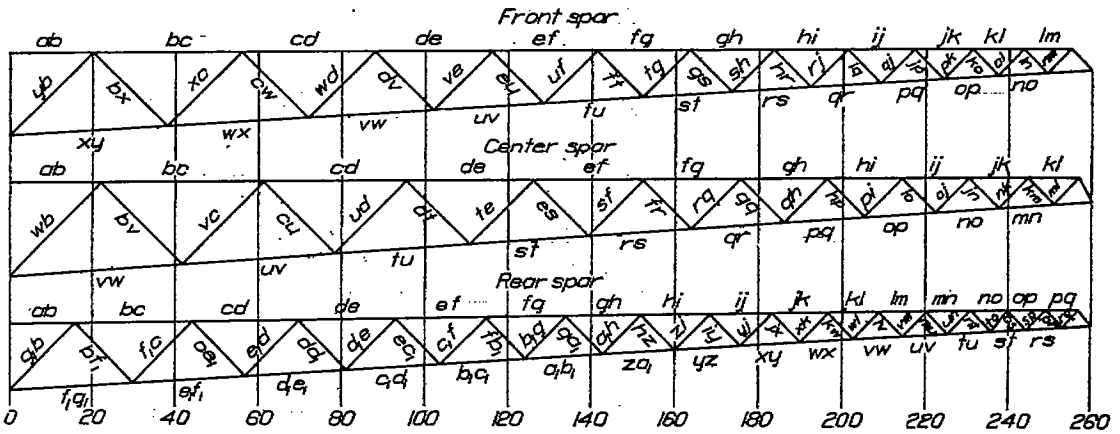

FIGURE 3.—Spar trussing in XHB wing

THE ELASTIC AXIS OR CENTROID.

The elastic axis or centroid of the wing is the line across which a transverse force may be applied without causing any rotation of the wing sections. In structural analysis of a wing having more than two spars, or drag bracing resisting torsion, it is convenient to divide the resultant force of the air pressure into forces acting through the elastic axis, and a pure torsional couple.

The distance of the centroid from the leading edge is calculated by taking moments of the factors which determine the rigidity of the wing spars. For example, in spars in which the shearing deflection is negligible, the rigidity, or resistance of the spars to deflection is directly proportional to the moment of inertia of their cross sections. In spars having very flexible shear bracing, an appreciable part of the deflection may be due to shear, and the rigidities of

the spars may be calculated from the internal work under a given load, on the principle that the rigidity is inversely proportional to the work. This follows directly from the well-known fact that with a given load, the work is proportional to the deflection, provided the stresses nowhere exceed the proportional limit.

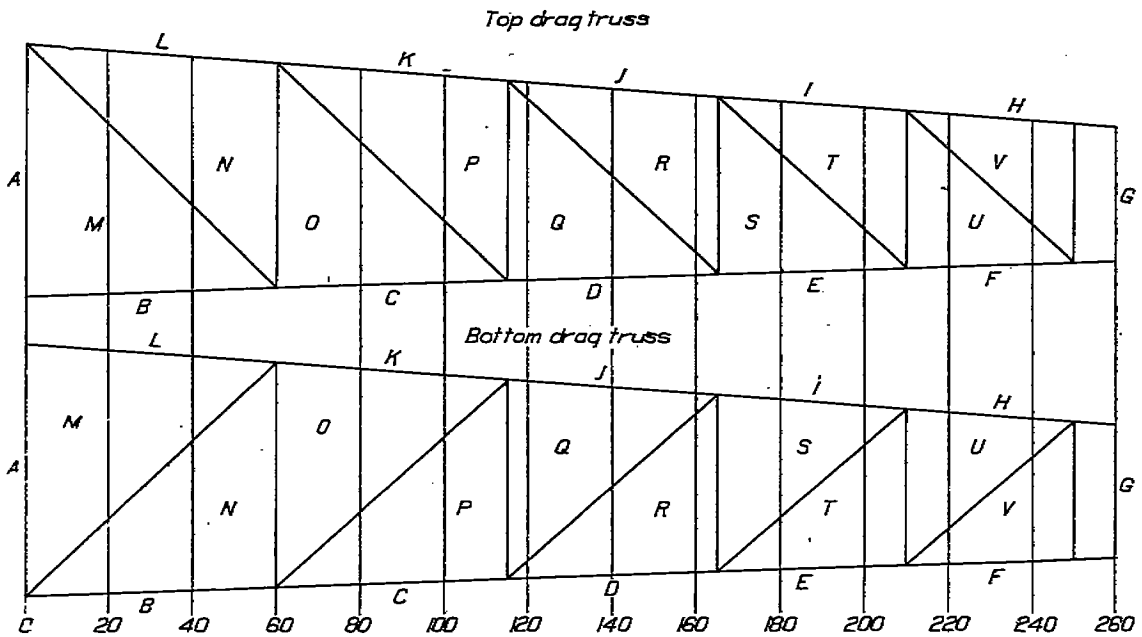

FIGURE 4.—Drag truss in XHB wing

The moment of inertia of the cross section, or the reciprocal of the internal work under a given load may be taken as rigidity factors of the spars. The sum of the moments of the rigidity factors about the leading edge of the wing, divided by the sum of these factors, gives the distance of the elastic axis from the leading edge. This principle is not confined to 2-spar wings, provided that as is usually the case, the ribs and rib bracing between the spars are sufficiently rigid to make the distortions of the wing sections negligible in comparison with the translational and rotational movement of the sections.

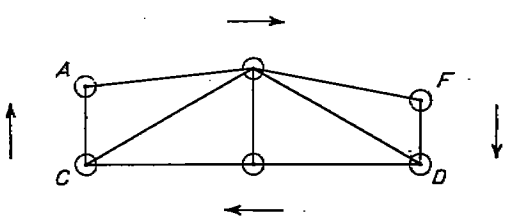

FIGURE 5.—Diagrammatic section through 3-spar wing

As an example, the position of the elastic centroid at Station 22 of the XHB wing, shown in Figures 3, 4, and 5, is calculated. This is a 3-spar, tapered wing in which the distances of the spars from the leading edge remain constant fractions of the tapering chord. It is therefore convenient to express the moment arms about the leading edge as fractions of the chord, rather than as definite lengths. The calculation is made in tabular form as follows:

TABLE I

Spar	I in.^{4}	% chord from leading edge	Moment
Front	44.4	15	666
Center	32.8	40	1,312
Rear	21.6	65	1,404
	98.8		3,382

The centroid is at 3,382/98.8=34.23% of the chord from the leading edge at Station 22.

DIVISION OF THE LOAD BETWEEN THE SPARS

When the resultant of the load normal to the wing chord passes through the elastic axis, there is no rotation of the wing sections, and if the ribs are rigid, all spars have equal deflections, and are loaded in direct proportions to their moments of inertia, I, provided the shearing deflection is negligible. It follows that the running load, w, on each spar due to a total running load, W, acting through the elastic centroid is given by:

$$w = WI/\Sigma I$$

The summation, ΣI, is taken over all spars at the station under investigation. In the XHB wing at Station 22, the division of load between the spars is as follows:

TABLE II

Spar	I	$I/\Sigma I$= fraction of normal load taken by spar
Front	44.4	0.450
Center	32.8	.332
Rear	21.6	.218
	98.8	1.000

If there were no drag bracing in the top and bottom planes of the wing, a torsional couple about the centroid would be opposed only by the rigidity of the spars, which would be loaded by the torsion in direct proportion to their moments of inertia and their distance from the centroid, i. e. in proportion to Is, where s is the distance of the spar from the centroid. The moments of the resistances of the spars to a torsional couple are equal to Is^2; and the loads w due to a torsional moment M are therefore given by:

$$w = MIs/\Sigma Is^2$$

The values of $Is/\Sigma Is^2$ for the present problem are given in the following table, where s is expressed as a percentage of the chord length.

TABLE III

Spar	I	s	Is	Is^2	$Is/\Sigma Is^2$
Front	44.4	−19.2	−852.5	16,368	−0.0225
Center	32.8	5.8	190.2	1,103	.0050
Rear	21.6	30.8	665.3	20,491	.0175
				37,962	

STRESSES IN THE MEMBERS AT STATION 22 DUE TO UNIT LOAD ON EACH SPAR

The unit load on each spar or drag truss is assumed to be 1,514 lb., distributed as shown in Table IV and Figure 6. Table V gives the abbreviations used for the various members in the subsequent analysis, and the stresses in the members at Station 22 due to the unit load being applied to each spar and drag truss in turn. The stresses have been determined by the ordinary analytical solution of determinate structures; it is not considered necessary to give the calculations here.

UNIT TORSIONAL MOMENT

The unit torsional moment is assumed to be the couple produced by 1,514 lb. distributed along the elastic axis in the same way as the unit spar load, acting downwards, and an equal force acting upwards at 1 per cent of the chord length forward of the elastic axis. The loads on the spars due to the unit torsional moment when there is no drag bracing are therefore 1,514 times the values of $Is/\Sigma Is^2$ calculated in Table III.

The depth of the wing at Station 22 is 18.6 per cent of the chord length. The loads on the drag trusses when they alone oppose the unit torsional couple are $\pm 1{,}514/18.6 = \pm 81.4$ lb.

LEAST WORK CALCULATION OF THE DISTRIBUTION OF TORSIONAL STRESSES

The diagonals of the top and bottom drag trusses are considered to be the redundant members in resisting torsional couples applied to the wing structure. In a pure couple, there can be no resultant drag force, and the forces in the two drag trusses must therefore be equal and opposite, so that there is only a single unknown force to be determined.

The least work analysis is applied to 1 inch length of the wing structure at Station 22. For mathematical exactitude, the least work calculations should be applied to the whole wing structure simultaneously, instead of only to the cross section at which the stresses are desired. It is believed, however, that the proposed method of procedure is not seriously in error provided there are no abrupt changes in the position of the elastic axis, and the sizes of the members in the spars and drag trusses taper out gradually. This limitation is also found in the ordinary

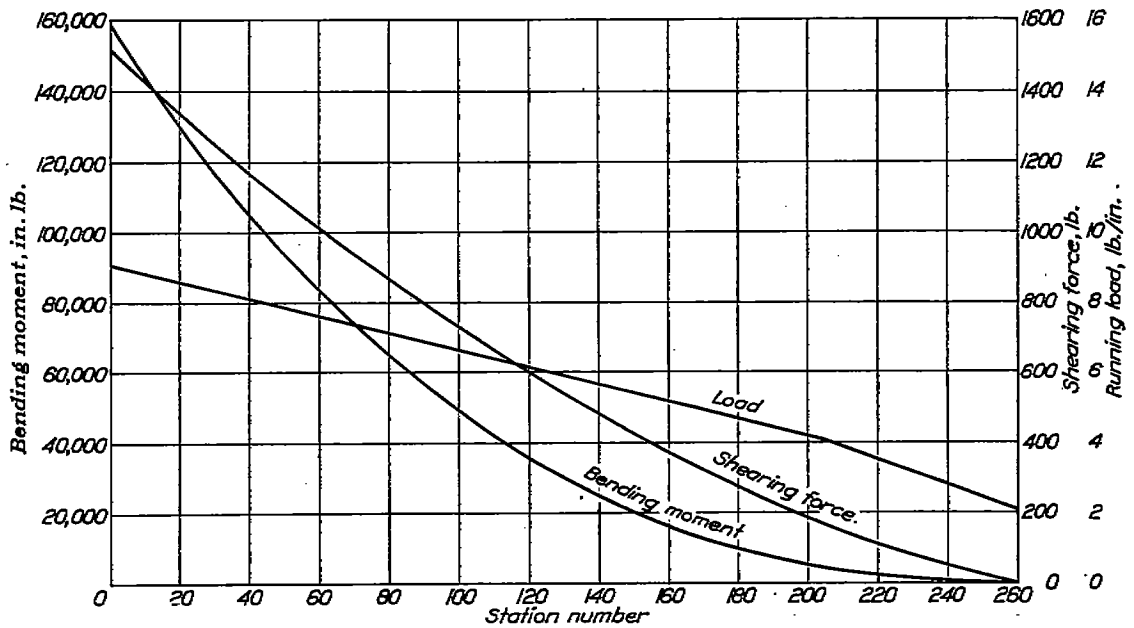

FIGURE 6.—Load, shearing force and bending moment with total distributed load of 1,514 pounds

beam bending theory which, as is well known, does not give the bending stresses correctly in the neighborhood of abrupt changes in the shape of the cross section of a beam.

In applying the principle of least work to the stress calculations the spars without the drag bracing are taken as the basic determinate structure, and the drag bracing as the redundant part. The stress, S, in any member of the structure, due to the unit torsional couple M, is regarded as consisting of two parts. One part, designated S_0, is the stress resulting from the couple M, opposed only by the spars of the determinate structure. The other part, XS_1, results from the forces in the redundant drag bracing. The stress, XS_1, is the product of X, the fraction of M resisted by the drag bracing; and S_1, the stress resulting from an imaginary condition of internal forces in which the spars and the drag bracing act against each other in torsion with an intensity equal to $-M$ in the spars, and $+M$ in the drag bracing. It follows that in any member, S_1 equals $-S_0$ plus the stress due to M applied to the drag bracing; and $S = S_0 + XS_1$.

In Table VI, the S_0 stresses (column 9) are the same as the stresses due to the unit torsion opposed only by the spars (column 7); and the S_1 stresses (column 10) are equal to $-S_0$ plus the stresses due to the unit torsion opposed only by the drag bracing (column 8).

The total internal work in the portion of the structure under consideration is designated by W, and is given by:

$$W=\Sigma\frac{S^2L}{2AE}=\Sigma\frac{(S_0+XS_1)^2L}{2AE}$$

W has its minimum value when $dW/dX=0$. By differentiating the above expression with respect to X, and equating to zero, the following equation for the solution of X is obtained:

$$X\Sigma S_1^2L/EA+\Sigma S_0S_1L/EA=0.$$

The summation is taken over all that part of the wing structure intercepted between two parallel planes 1 inch apart, perpendicular to the axis of the wing at Stations 22 and 23. The modulus E is the same for all members, and cancels out. Let $U=L/A$. Then

$$X=-\frac{\Sigma US_0S_1}{\Sigma US_1^2}$$

The least-work calculations are detailed in Table VI. The stresses due to the unit load of 1,514 lb. on each spar and drag truss are taken from Table V. The stresses, S_0, are the effect of the unit torsional moment opposed by the spars alone, without assistance from the drag bracing. These stresses are obtained by multiplying the stresses in each spar due to 1,514 lb. load by the values of $Is/\Sigma Is^2$ calculated in Table III. The S_1 stresses are equal and opposite to the S_0 stresses, plus the stresses in the drag trusses when they alone resist the unit torsion. The latter stresses, as shown on page 7, are the result of loads of 81.4 lb., acting forward in the upper drag truss, and in the reverse direction in the lower drag truss.

It is found by the computations in Table VI that $X=0.4919$. The stresses in all members at Station 22, due to the unit torsional moment of 1,514 lb. at 1 per cent of the wing chord aft of the elastic axis, are given in the column headed S in Table VI.

STRESSES AT STATION 22 DUE TO UNIT LOADS IN THREE FLIGHT CONDITIONS

The stresses at Station 22 are calculated for the unit load in the following three conditions of flight:

(a) High Incidence, normal load, 1,514 lb.; antidrag load, 210 lb.; center of pressure at 31 per cent of the chord length from the leading edge, or 3.23 per cent of the chord forward of the elastic axis.

(b) Low Incidence, normal load, 1,514 lb.; drag, 224 lb., center of pressure at 51 per cent of the chord from the leading edge, or 16.77 per cent aft of the elastic axis.

(c) Inverted Flight, normal load, −1,514 lb.; zero drag, center of pressure at 3.23 per cent of the chord forward of the elastic axis.

The drag force is assumed to act through the elastic axis, producing no torsion.

In calculating the stresses in each condition, the effects of the normal force, drag, and torsion are each found by proportionality from the stresses due to the unit loads as already computed. The normal load is distributed between the front, center, and rear spars in the ratios, 0.452, 0.332, and 0.218, respectively, as determined in Table II. The drag is divided equally between the two drag trusses; and the torsional stresses are equal to the final S stresses in Table VI multiplied by the distance of the center of pressure from the elastic axis in terms of per cent of the chord.

COMPARISON OF CALCULATED AND MEASURED STRESSES

The welded steel tubular wing truss, to which the foregoing stress calculations apply, was tested by sand loading at the static testing laboratory at McCook Field. Electric telemeter strain gauges, of the carbon pile resistance type developed by the Bureau of Standards, were clamped to the longitudinal members cut by Station 22.

The readings of the strain gauges, and the corresponding stresses are shown in Tables X, XI, and XII. The locations of the gauges, and the increments of stress per unit load are shown

in Tables XII, XIII, and XIV. The calculated stresses are also given in these tables for comparison with observation. The test at low incidence was the most illuminating because the largest amount of data was obtained, and the torsion of the wing was greatest, giving the best opportunity to check the theory. Inspection of Table XIII shows that the average stresses agreed fairly well with the theory. In fact, the stresses indicated by the gauges for different increments of load varied between themselves more than their averages differed from the theoretical stresses. It is therefore apparent that the theory is as good as the method of measuring stresses in this test, or else that the stresses were not proportional to the applied loads. It should be remembered that the strain gauges are not particularly accurate because of the tendency of the carbon piles to change their sensitivity to pressure and hence disturb their calibration. Moreover, the gauges were at considerable distances from the axes of the tubular members to which they were secured, and the steel was so hard that the points of the gauges could not be pressed into it. To overcome this difficulty, it was necessary to interpose pieces of aluminum, bored to fit the tubes, with flat exterior faces to receive the gauge points. A certain amount of lost motion was inevitable with this arrangement. In the case of compression members, the gauges were placed in pairs, one on each side of the tube. The two units of a pair often indicated quite different stresses, showing that there was buckling of the members. However, the outside radii of the tubes were only about one-third to one-half as great as the distances of the lines of action of the gauges from the neutral axes of the tubes, so that the buckling stresses were not nearly so great as indicated by the differences between the strains shown by the two gauges of a pair.

THE COEFFICIENT OF TORSIONAL RIGIDITY

The angular twist θ, of a wing in a given length of span, L, due to a given torsional bending moment, M, is inversely proportional to the absolute rigidity of the wing. When investigating the torsional rigidity of short lengths of span, it is convenient to replace θ by $L\mathrm{d}\theta/\mathrm{d}L$. It is to be expected that the torsional moment acting upon wings of similar sections, with homologous positions of the elastic axis, will vary as qFb, where q is the aerodynamic head, F the area of the wing, and b the chord length. It follows that for equal comparative torsional rigidities of different wings, $\frac{M}{L\mathrm{d}\theta/\mathrm{d}L}$ should be proportional to qFb. Hence a measure of comparative rigidity is the nondimensional coefficient, C_t, defined by:

$$\frac{M}{L\mathrm{d}\theta/\mathrm{d}L}=C_t\, q\, F\, b, \text{ or}$$

$$C_t=\frac{M\cdot \mathrm{d}L}{L\cdot q\cdot F\cdot b\cdot \mathrm{d}\theta}$$

The work done by a torsional moment M within the length L is given by

$$W=\frac{ML\cdot \mathrm{d}\theta}{2\cdot \mathrm{d}L}$$

And also:

$$W=\Sigma\frac{S^2L}{2EA}$$

where the summation is taken over all members of the structure within the length L.

Whence:

$$C_t=\frac{M^2}{2\,WqFb}=\frac{M^2}{2Fb}\Sigma\frac{EA}{S^2L}$$

Other things being equal, C_t is inversely proportional to W. The summations of the last two columns of Table VI show the comparative rigidities of the XHB wing at station 22, with

and without the drag bracing. The work in 1 inch length due to the unit torsion without drag bracing is given by:

$$W=\Sigma\frac{USo^2}{2E}=\frac{445,868}{58,000,000}=0.0077 \text{ in. lb.}$$

With the drag bracing in action,

$$W=\Sigma\frac{US^2}{2E}=\frac{73,284}{58,000,000}=0.00126 \text{ in. lb.}$$

That is to say, the torsional rigidity is six and one-tenth times greater with the drag bracing than without it.

CONCLUSIONS AND RECOMMENDATIONS

The structural analysis of a cantilever wing shows that the inclusion of strong drag bracing in the top and bottom of the wing enormously increases the torsional rigidity. It is recommended that the use of drag bracing to improve torsional rigidity be extended to other types of wing. Increasing the torsional stiffness improves the aerodynamic qualities of a wing, and raises the critical speed at which flutter commences. It also improves the structural efficiency by diminishing the shifting of the load between the front and rear spars due to movements of the center of pressure.

The customary procedure of assuming that the air load is divided between the front and rear spars of a 2-spar wing in inverse ratio to their distances from the line of the resultant air load is no longer valid when the drag bracing contributes to the torsional strength. For such wings, the position of the elastic axis should be computed, and if the resultant of the air load does not pass through the elastic axis, it should be resolved into normal and drag forces acting through and perpendicular to the elastic axis, plus a torsional moment about the axis. The stresses due to the bending of the elastic axis, and the twisting of the wing about it, should be computed separately and added together.

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TABLE IV

SHEAR AND BENDING IN WING WITH DISTRIBUTED LOAD OF 1,514 LB.

Station, in.	Load, lb.	Shear, lb.	Moment, in. lb.	Station, in.	Load, lb.	Shear, lb.	Moment, in. lb.
200		0	0	130		540.3	29,882
	23.0				60.1		
250		23	115	120		600.4	35,585
	26.2				62.6		
240		49.2	476	110		663.0	41,902
	29.5				65.0		
230		78.7	1,116	100		728.0	48,857
	33.0				67.6		
220		111.7	2,068	90		795.5	56,475
	36.8				70.0		
210		148.5	3,369	80		865.5	64,780
	40.8				72.4		
200		188.8	5,055	70		937.9	73,797
	42.8				74.9		
190		231.6	7,157	60		1,012.8	83,550
	45.3				77.4		
180		276.9	9,700	50		1,090.2	94,065
	47.7				79.9		
170		324.6	12,707	40		1,170.1	105,367
	50.2				82.3		
160		374.8	16,204	30		1,252.4	117,479
	52.7				84.8		
150		427.5	20,216	20		1,337.2	130,427
	55.2				87.3		
140		482.7	24,767	10		1,424.5	144,236
	57.6				89.7		
				0		1,514.2	158,929

TABLE V

Spar or drag truss	Member	Abbreviation	Stress, lb.
Front spar	Upper front longitudinal	U. F.	−6,010
	Lower front longitudinal	L. F.	6,932
	Front spar diagonal	F. D.	−1,305
Center spar	Upper center longitudinal	U. C.	−5,221
	Lower center longitudinal	L. C.	6,124
	Center spar diagonal	C. D.	−1,280
Rear spar	Upper rear longitudinal	U. R.	−8,424
	Lower rear longitudinal	L. R.	9,290
	Rear spar diagonal	R. D.	−1,225
Top drag truss	Upper front longitudinal	U. F.	2,702
	Upper rear longitudinal	U. R.	−1,591
	Upper diagonal	U. D.	−1,570
Bottom drag truss	Lower front longitudinal	L. F.	1,591
	Lower rear longitudinal	L. R.	−2,702
	Lower diagonal	L. D.	1,570

TABLE VI

CALCULATION OF TORSIONAL STRESSES BY THE METHOD OF LEAST WORK

Member	L	A	$U=L/A$	Stress due to 1,514 lb. load on each spar or drag truss		Stress due to unit torsion opposed only by—		S_0	S_1	$\frac{S_0S_1}{1,000}$	$\frac{S_1^2}{1,000}$	$\frac{US_0S_1}{1,000}$	$\frac{US_1^2}{1,000}$	$0.4919S_1$	$S=S_0+0.4919S_1$	US^2	US_0^2
				Spars	Drag trusses	Spars	Drag trusses										
	In.	*In.²*	*1/In.*	*Lb.*	*Lb.*	*Lb.*	*Lb.*										
U. F.	1.0	0.3312	3.02	−6,010	2,702	135.2	−145.3	135.2	−280.5	−37.92	78.68	−114.52	237.61	−138.0	−2.8	24	55,203
L. F.	1.0	.2041	4.90	6,932	1,591	−156.0	85.5	−156.0	241.5	−37.67	58.32	−184.58	285.77	118.8	−37.2	6,781	119,246
F. D.	1.414	.0786	17.99	−1,305		29.4		29.4	−29.4	−0.86	0.86	−15.47	15.47	−14.5	14.9	3,994	15,551
U. C.	1.0	.2041	4.90	−5,221		−26.1		−26.1	26.1	−0.68	0.68	−3.33	3.33	12.8	−13.3	867	3,333
L. C.	1.0	.1198	8.35	6,124		30.6		30.6	−30.6	−0.94	0.94	−7.85	7.85	−15.1	15.5	2,006	7,819
C. D.	1.414	.0786	17.99	−1,280		−6.4		−6.4	6.4	−0.04	0.04	−0.72	0.72	3.1	−3.3	196	737
U. R.	1.0	.2855	3.50	−8,424	−1,591	−147.4	85.5	−147.4	232.9	−34.33	54.24	−120.16	189.84	114.6	−32.8	3,765	76,044
L. R.	1.0	.1656	6.04	9,290	−2,702	162.6	−145.3	162.6	−307.9	−50.06	94.80	−302.86	572.59	−151.5	11.1	744	159,691
R. D.	1.414	.0786	17.99	−1,225		−21.4		−21.4	21.4	−0.46	0.46	−8.28	8.28	10.5	−10.9	2,137	8,239
U. D.	1.414	.0923	15.32		−1,570		84.4		84.4		7.12		109.08	41.5	41.5	26,385	
L. D.	1.414	.0923	15.32		1,570		84.4		84.4		7.12		109.08	41.5	41.5	26,385	
												−757.27	1,539.62			73,284	445,868

$X=-\Sigma\, US_0S_1/\Sigma\, US_1^2=757.27/1,539.62=0.4919.$

TABLE VII

STRESSES AT STATION 22 DUE TO UNIT LOAD AT HIGH INCIDENCE

Member	Normal forces, lb.	Drag force lb.	Torsional force lb.	Total force, lb.	Unit stress lb./in.²
U. F.	−2,705	−188	9	−2,884	−8,708
L. F.	3,119	−111	119	3,127	15,321
F. D.	−587		−48	−635	−8,079
U. C.	−1,733		43	−1,690	−8,280
L. C.	2,033		−50	1,983	16,553
C. D.	−425		11	−414	−5,267
U. R.	−1,836	111	105	−1,619	−5,671
L. R.	2,025	188	−36	2,177	13,146
R. D.	−267		35	−232	−2,952
U. D.		109	−133	−24	−260
L. D.		−109	−133	−242	−2,622

TABLE VIII

STRESSES AT STATION 22 DUE TO UNIT LOAD AT LOW INCIDENCE

Member	Normal forces, lb.	Drag force lb.	Torsional force lb.	Total force, lb.	Unit stress lb./in.²
U. F.	−2,705	200	−47	−2,552	−7,705
L. F.	3,119	118	−625	2,612	12,798
F. D.	−587		250	337	4,288
U. C.	−1,733		+223	−1,956	−9,584
L. C.	2,033		260	2,293	19,140
C. D.	−425		−55	−480	−6,107
U. R.	−1,836	−118	−551	−2,505	−8,774
L. R.	2,025	−200	186	2,011	12,144
R. D.	−267		−183	−450	−5,725
U. D.		−116	697	581	6,295
L. D.		116	697	813	8,808

TABLE IX

STRESSES AT STATION 22 DUE TO UNIT LOAD IN INVERTED FLIGHT

Member	Normal forces, lb.	Drag force, lb.	Torsional force, lb.	Total force, lb.	Unit stress, lb./in.²
U. F.	2,705		−9	2,696	8,140
L. F.	−3,119		−119	−3,000	−14,699
F. D.	587		48	635	8,079
U. C.	1,733		−43	1,690	8,260
L. C.	−2,033		50	−1,983	−16,553
C. D.	425		−11	414	5,267
U. R.	1,836		−105	1,731	6,063
L. R.	−2,025		36	−1,989	−12,011
R. D.	267		35	232	2,952
U. D.			133	133	1,441
L. D.			133	133	1,441

TABLE X

TEST OF THREE-SPAR WING, INVERTED FLIGHT

Load factor Total load, lb.	1.0 1,190		2.0 2,700		2.5 3,455		Load removed	
Gauge No.	Gauge reading	Stress lb./in.²	Gauge reading	Stress lb./in.²	Gauge reading	Stress, lb./in.²	Gauge reading	Stress, lb./in.²
4	+2.6	+4,840	+3.3	+12,300	+4.3	+16,000	+0.5	+930
5	−5.4	−10,050	−6.6	−24,600	−8.9	−33,100	−0.6	−1,115
6	−6.0	−11,170	−7.4	−27,550	−9.3	−34,600	−1.2	−2,230
7	+4.1	+7,630	+5.0	+18,600	+6.4	+23,800	+0.7	+1,300
8	−6.4	−11,900	−7.7	−28,650	−9.9	−36,800	+0.4	+745
9	−6.7	−12,480	−7.8	−29,000	−10.0	−37,200	−0.8	−1,490
10	+2.4	+4,460	+2.7	+10,050	+3.3	+12,300	−0.1	−186
11	−6.4	−11,900	−7.5	−27,900	−9.8	−36,500	−1.3	−2,420
12	−4.1	−7,630	−4.7	−17,500	−6.0	−22,340	−0.6	−1,115

TABLE XI

TEST OF 3-SPAR WING, LOW INCIDENCE

Load factor / Total load, lb.	2.0 / 2,700		2.5 / 3,455		3.0 / 4,210	
Gauge No.	Gauge reading	Stress lb./in.²	Gauge reading	Stress lb./in.²	Gauge reading	Stress lb./in.²
4	+6.8	+25,300	+9.1	+33,830		
5	−4.3	−16,000	−5.3	−19,700	−6.5	−24,200
6	−4.5	−16,750	−5.9	−22,000	−6.9	−25,700
7	+9.9	+36,800				
8	−4.8	−17,860	−5.3	−19,700	−6.5	−24,200
9	−6.5	−24,200	−7.7	−28,650	−9.2	−34,200
10	+5.4	+20,100	+6.3	+23,450	+7.5	+27,900
11	−2.9	−10,800	−3.6	−13,400	−4.2	−15,620
12	−4.5	−16,750	−5.6	−20,840	−6.6	−24,550

Load factor / Total load, lb.	2.5 / 3,455		2.0 / 2,700		Load removed	
Gauge No.	Gauge reading	Stress lb./in.²	Gauge reading	Stress lb./in.²	Gauge reading	Stress lb./in.²
4						
5	−5.8	−21,600	−4.6	−17,110	−0.8	−1,490
6	−6.0	−22,350	−4.9	−18,240	−1.3	−2,420
7						
8	−5.3	−19,730	−4.1	−15,270	+1.0	+1,860
9	−7.5	−27,900	−5.9	−22,000	+0.8	+1,490
10	+6.2	+22,100	+5.0	+18,600	−0.6	−1,116
11	−3.6	−13,400	−3.0	−11,160	−0.2	−372
12	−5.6	−20,840	−4.6	−17,110	−0.6	−1,116

TABLE XII

TEST OF 3-SPAR WING, HIGH INCIDENCE. LOAD FACTOR 2; TOTAL LOAD, 2,700 LB.

Gauge No.	Position of gauge: Spar tube	Position of gauge: Distance from root in.	Gauge reading	Stress, lb./in.²	Increment of stress per unit load, lb./in.²	Calculated increment, lb./in.²
11	Upper front	38	−3.9	−14,500	−8,100	−8,708
12	do	38	−4.6	−17,110	−9,560	−8,708
10	Lower front	25.4	+8.1	+30,150	+16,850	15,821
8	Upper center	48.5	−5.7	−21,200	−11,850	−8,280
9	do	48.5	−5.6	−20,840	−11,650	−8,280
7	Lower center	20.8	+8.3	+30,860	+17,250	16,553
5	Upper rear	26.9	−1.8	−6,700	−3,740	−5,671
6	do	26.9	−2.4	−8,940	−5,000	−5,671
4	Lower rear	17.4	+4.8	+17,860	+9,980	13,146

TABLE XIII

TEST OF 3-SPAR WING, LOW INCIDENCE

Load range			0 to 2	2 to 2.5	2.5 to 3	3 to 2.5	2.5 to 2	2 to 0	Mean increment lb./in.²	Calculated increment lb./in.²
Gauge No.	Position of gauge: Spar tube	Position of gauge: Inches from root	Increment of stress per unit load, lb./in.²			Increment of stress per unit load, lb./in.²				
11	Upper front	38	−6,030	−5,200	−4,440	−4,440	−4,480	−6,010	−6,600	−7,705
12	do	38	−9,350	−8,180	−7,420	−7,220	−7,460	−8,930		−7,705
10	Lower front	25.4	+11,200	+6,700	+8,900	+9,600	+9,000	+11,000	+9,400	12,798
8	Upper center	48.5	−9,980	−3,680	−9,000	−8,940	−8,920	−9,600	−10,100	−9,584
9	do	48.5	−13,500	−8,900	−11,100	−12,600	−11,800	−13,100		−9,584
7	Lower center	20.8	+20,600						+20,600	19,140
5	Upper rear	26.9	−8,940	−7,400	−9,000	−5,200	−8,960	−8,730	−8,280	−8,774
6	do	26.9	−9,350	−10,500	−7,400	−6,700	−8,220	−8,840		−8,774
4	Lower rear	17.4	+14,150	+17,060					15,600	12,144

TABLE XIV

TEST OF 3-SPAR WING, INVERTED FLIGHT

Load range			0 to 1.0	1 to 2	2 to 2.5	Calculated increment, lb./in.²
Gauge No.	Position of gauge		Increment of stress per unit load, lb./in.²			
	Spar tube	Inches from root				
4	Upper front	37	+6,150	+7,460	+7,400	8,140
5	Lower front	25.4	−12,800	−14,550	−17,000	−14,699
6	do	25.4	−14,200	−16,380	−14,100	−14,699
7	Upper center	49.4	+9,700	+10,970	+10,400	8,280
8	Lower center	19.5	−15,100	−16,750	−16,300	−16,553
9	do	19.5	−15,840	−16,520	−16,400	−16,553
10	Upper rear	27.5	+5,660	+5,590	+4,500	6,063
11	Lower rear	17.3	−15,100	−16,000	−17,200	−12,011
12	do	17.3	−9,700	−9,870	−9,680	−12,011