REPORT No. 458

RELATIVE LOADING ON BIPLANE WINGS

By Walter S. Diehl

SUMMARY

It is shown that the lift coefficients of the individual wings of a biplane are given by

$$C_{L_U}=C_L+\Delta C_{L_U}$$

and

$$C_{L_L}=C_L-\Delta C_{L_L}$$

Where C_{L_U}, C_{L_L}, and C_L are the lift coefficients for the upper wing, lower wing, and biplane, respectively.

For the upper wing it is shown that

$$\Delta C_{L_U}=K_1+K_2C_L$$

K_1 and K_2 being functions of gap/chord, stagger, aspect ratio, decalage, overhang and wing thickness. The combination of existing biplane theory and experimental data supply curves from which K_1 and K_2 can easily be determined for any biplane. This enables the designer to calculate with reasonable accuracy the relative loading for any condition of flight.

INTRODUCTION

The accuracy of a biplane stress analysis depends greatly on the accuracy with which the loads on each wing can be determined. The division of the load between biplane wings has usually been determined in the current stress analysis methods from a chart giving the "relative efficiency" as a function of gap/chord ratio and stagger. This "relative efficiency" or ratio of the lift coefficient of the upper wing to lift coefficient of the lower wing has been based on the average values at high lift coefficients and therefore does not necessarily hold true for all lift coefficients. Recent improvements in stress analysis methods have made it necessary to revise and to extend the loading curves to cover all conditions of flight. This paper is concerned with a study of existing biplane data in connection with such a revision.

A survey of theoretical biplane data, in which numerous comparisons were made between observed and calculated lift curves, showed that while the agreement between theory and experiment is reasonably close, the theoretical methods do not appear entirely satisfactory except at moderate lift coefficients. By combining the experimental and theoretical data, however, it is possible to derive a series of curves from which the lift curves of the individual wings of a biplane may be obtained.

BIPLANE THEORY

The first important contribution to biplane theory was due to Betz (reference 1). This theory was elaborated by Fuchs (reference 3), and is given in its final form by Fuchs and Hopf in chapter IV of their book Aerodynamik (reference 4). Denoting the upper and lower wing by the subscripts U and L, respectively, the lift equations are

$$\Delta C_{L_L}=-\frac{\mu}{2\pi}\frac{S_U}{b_Ub_L}C_{L_U}C_{L_L}-\frac{57.3}{4\pi}(\nu+\kappa)\frac{S_U}{b_Ub_L}C_{L_U}\left(\frac{dC_{L_L}}{d\alpha}\right) \quad (1)$$

$$\Delta C_{L_U}=+\frac{\mu}{2\pi}\frac{S_L}{b_Ub_L}C_{L_L}C_{L_U}+\frac{57.3}{4\pi}(\nu-\kappa)\frac{S_L}{b_Ub_L}C_{L_L}\left(\frac{dC_{L_U}}{d\alpha}\right) \quad (2)$$

Where S is area and b the span. μ, ν, and κ are functions of gap G, wing span and stagger β. If we let

$$\lambda_1=\frac{b_U+b_L}{2G} \text{ and } \lambda_2=\frac{b_U-b_L}{2G}$$

Then

$$\mu=\mu(\lambda_1)-\mu(\lambda_2) \quad (3)$$

$$\nu=\nu(\lambda_1)-\nu(\lambda_2) \quad (4)$$

and

$$\kappa=\kappa(\lambda_1)-\kappa(\lambda_2) \quad (5)$$

That is, the value of μ, ν or κ for a given biplane is the difference between the values for λ_1 and λ_2.

The variations of μ, ν, and κ with λ are given by the relations

$$\mu(\lambda)=\cos\beta\,[(1+\lambda^2\cos^2\beta)^{1/2}-1] \quad (6)$$

$$\nu(\lambda)=\sin\beta\,[(1+\lambda^2\cos^2\beta)^{1/2}-1]+\log_e\frac{(1+\sin\beta)\sqrt{1+\lambda^2}}{\sin\beta+\sqrt{1+\lambda^2\cos^2\beta}} \quad (7)$$

$$\kappa(\lambda)=\frac{1}{2}\log_e(1+\lambda^2) \quad (8)$$

Values of $\mu(\lambda)$, $\nu(\lambda)$ and $\kappa(\lambda)$ from the above equations are plotted in figures 1, 2, and 3. $\mu(\lambda)$ and $\nu(\lambda)$ vary with stagger but $\kappa(\lambda)$ is independent of stagger. Since stagger varies with angle of attack it will be found more convenient and more accurate to read values of $\mu(\lambda)$ and $\nu(\lambda)$ at some particular stagger and

then apply a correction for the stagger corresponding to each angle of attack. Figure 4 gives the variation of $\mu(\lambda)$ with stagger in terms of the value of $\mu(\lambda)$ for zero stagger. Figure 5 gives the variation of $\mu(\lambda)$ with stagger in terms of the value of $\nu(\lambda)$ for 30° stagger. The angle of stagger is to be measured between the lift direction and the line connecting the one third chord points (measured from the leading

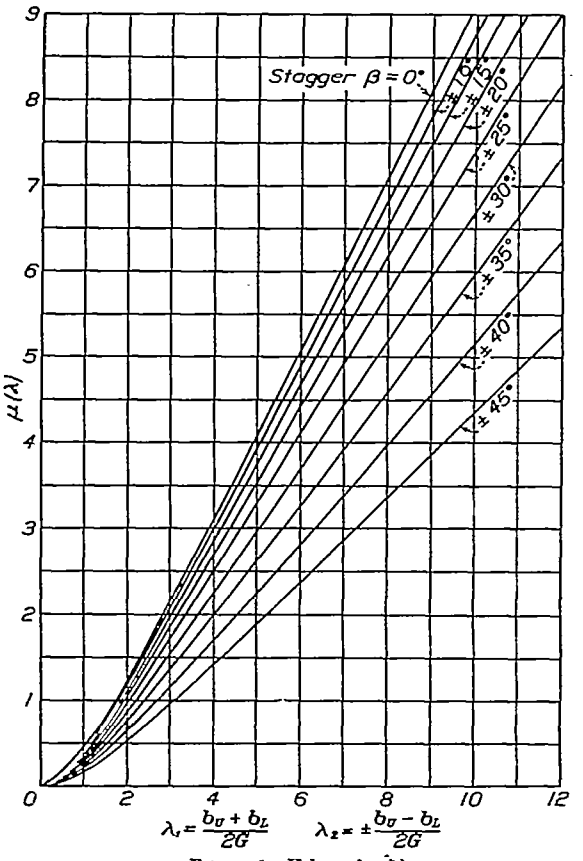

FIGURE 1.—Values of $\mu(\lambda)$.

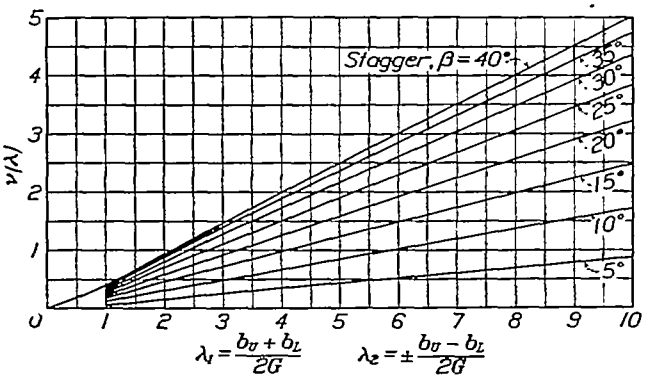

FIGURE 2.—$\nu(\lambda)$ for $-\beta° = -\nu(\lambda)$ for $+\beta°$.

edge) of the upper and lower wings. Stagger is positive when the third point in the upper wing is ahead of the lower wing.

Physically, $\mu(\lambda)$ is a factor which takes care of the velocity change due to the presence of each wing, while $\nu(\lambda)$ and $\kappa(\lambda)$ factors which allow for change in angle of attack due to the deflection of the air flow in the neighborhood of each wing.

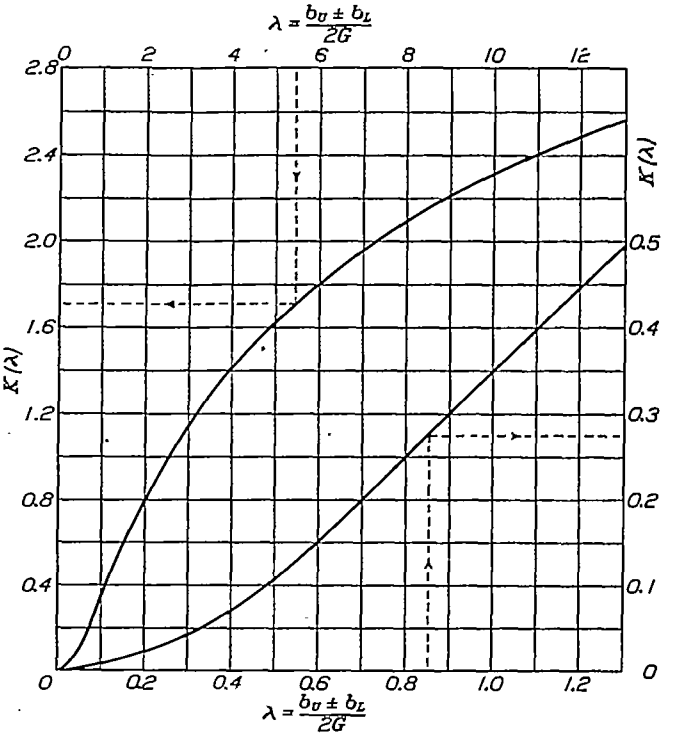

FIGURE 3.

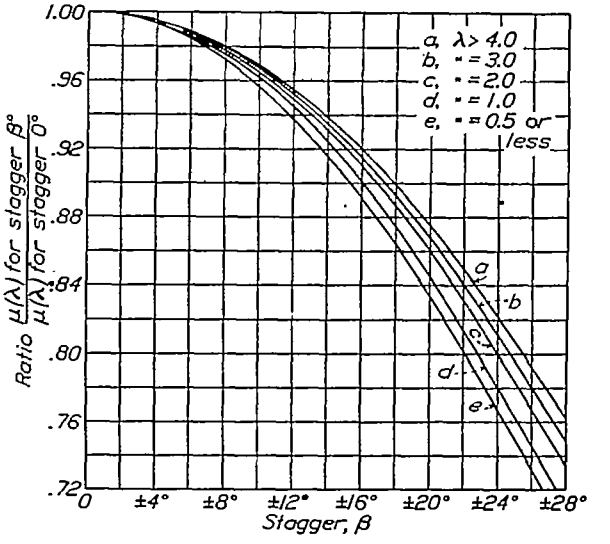

FIGURE 4.—$\lambda = \frac{b_U \pm b_L}{2G}$.

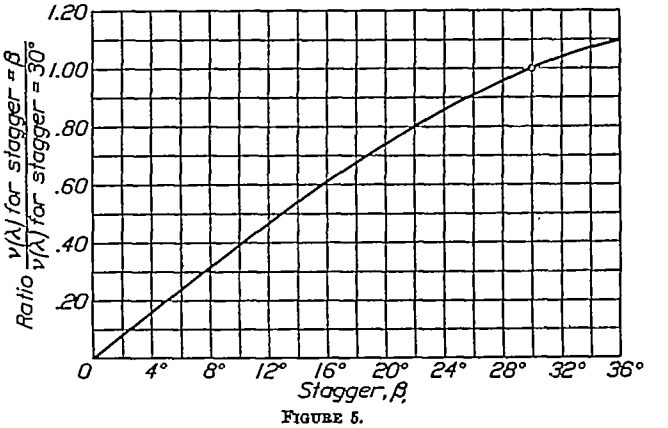

FIGURE 5.

Equation (1) gives the change in the lift coefficient of the lower wing due to the presence of the upper wing. Equation (2) gives the change in the lift coefficient of the upper wing to the presence of the lower wing. Similar equations for the change in drag coefficients are given in reference 4, as follows:

$$\Delta C_{D_L} = -\frac{\mu}{2\pi}\frac{S_U}{b_U b_L} C_{L_U} C_{D_L} - \frac{\nu+\kappa}{4\pi}\frac{S_U C_{L_U}}{b_U b_L}\left[57.3\frac{dC_{D_L}}{d\alpha} - C_{L_L}\right] \tag{9}$$

$$\Delta C_{D_U} = \frac{\mu}{2\pi}\frac{S_L}{b_U b_L} C_{L_L} C_{D_U} + \frac{\nu-\kappa}{4\pi}\frac{S_L C_{L_L}}{b_U b_L}\left[57.3\frac{dC_{D_U}}{d\alpha} - C_{L_U}\right] \tag{10}$$

Munk also finds the additional lift coefficient due to decalage of $\pm\delta$ as:

$$\Delta C_L = \pm 2\pi \frac{S}{2} B_o(1+2d)\delta \tag{12}$$

where $B_o(1+2d)$ is a factor obtained in his integration of the flow components. $B_o(1+2d)$ is given as a function of gap/chord ratio as follows:

G/c	2.02	1.46	1.11	.98	.79	.64	.56	.46	.39
$B_o(1+2d)$	1.03	1.06	1.10	1.12	1.19	1.25	1.30	1.38	1.48

These values are plotted in figure 7.

Millikan's treatment of the biplane theory (reference 7), is along lines very similar to that used in reference 4, but extending the theory. The resulting equations appear to give somewhat better agreement with test data than is obtained with the previous methods, but

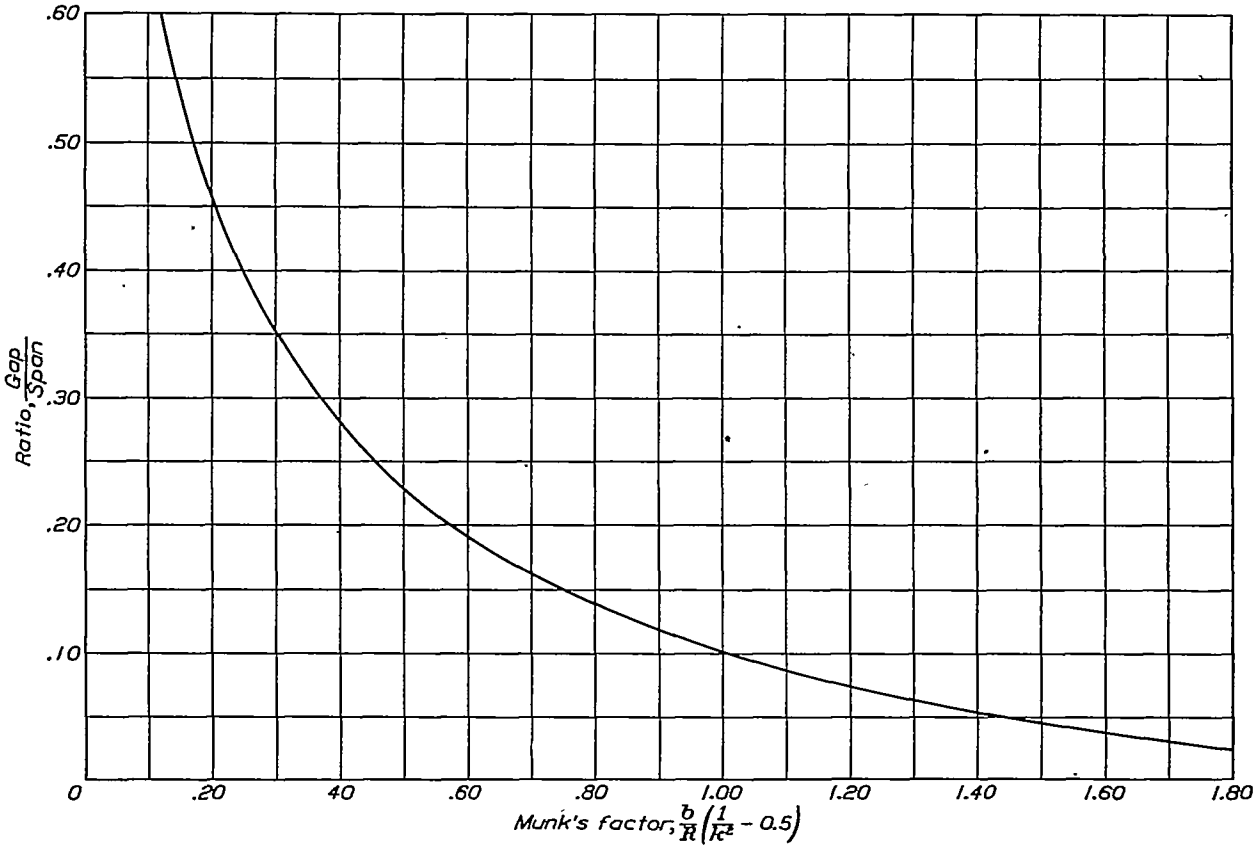

FIGURE 6.

Munk, in reference 5, derives comparatively simple formulas for the biplane. He finds that the additional lift coefficient of staggered wings is

$$\Delta C_L = \pm 2C_L \frac{S}{b^2}\left(\frac{1}{k^2}-0.5\right)\frac{b}{R}\frac{s}{c}\frac{c}{b} \tag{11}$$

where S is the total area, s the stagger, b the span, c the chord, k the equivalent monoplane span factor, and R a distance used in calculating the induced downwash. Munk gives $\frac{b}{R}\left(\frac{1}{k^2}-0.5\right)$ as a function of the ratio of gap to span G/b. His tabulated values have been plotted in figure 6.

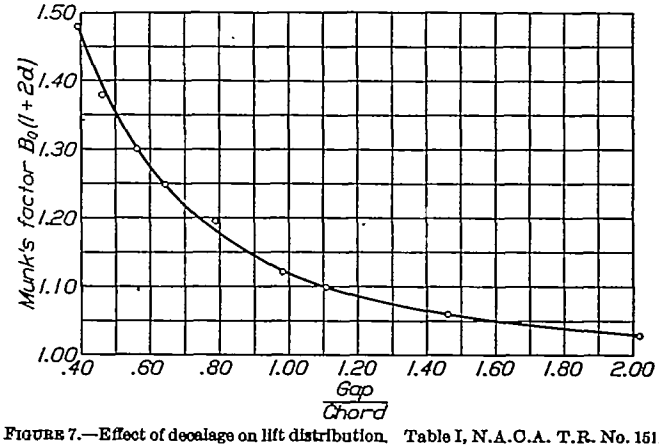

FIGURE 7.—Effect of decalage on lift distribution. Table I, N.A.C.A. T.R. No. 151.

it is very difficult for an engineer to follow the steps required in a typical calculation.

It is proposed to show how the foregoing theory may be used with test data in the derivation of working charts for routine use.

I. SIMPLE BIPLANES

It is desirable for the present to consider the simplest form of biplane in which the wings are of same chord and span, and to study the effect of stagger. The effect of unequal chords, decalage, and overhang can be considered later.

Equation (11) is equivalent to a statement that the lift coefficient of the upper wing (or lower wing) differs from that of the biplane by an amount depending directly on the biplane lift coefficient. That is,

$$C_{L_U} = C_L \pm \Delta C_L \tag{13}$$

or

$$C_{L_L} = C_L \mp \Delta C_L \tag{13a}$$

ΔC_L varying with stagger and gap/span ratio as indicated by equation (11) and figure 6.

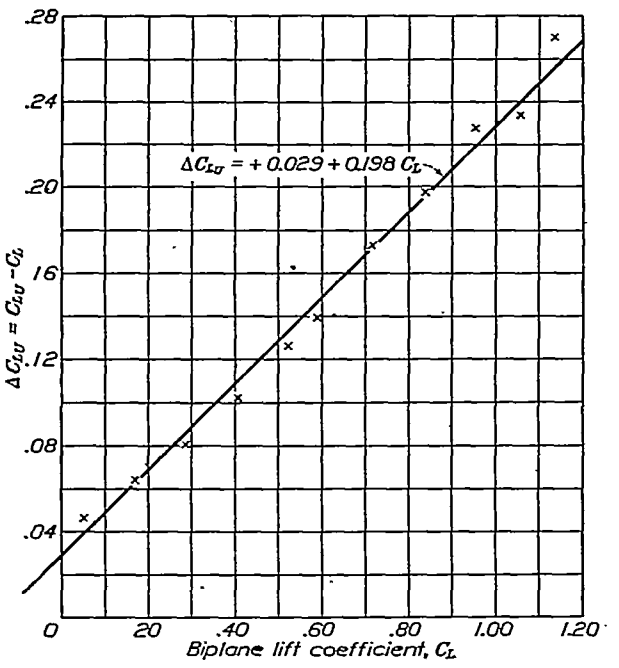

FIGURE 8.—U.S.A. TS-5 biplane $\frac{\text{gap}}{\text{chord}}=0.9$. Stagger 30°. Effective stagger= 0.64 c. Data from N.A.C.A. T.R. No. 256.

In order to verify the relation of equation (13), data from a number of biplane tests have been analyzed by the method illustrated in table I. The values of ΔC_{L_U} so obtained have been plotted against the biplane lift coefficient as in figure 8. In all cases the values of ΔC_{L_U} have shown a linear relation with C_L. The test data and calculations are too extensive for inclusion in this report but the equations of the lines are given in tables II to V inclusive. An inspection of these equations shows several outstanding facts, the most important of which is that the value of ΔC_{L_U} has the general form

$$\Delta C_{L_U} = K_1 + K_2 C_L \tag{14}$$

K_1 and K_2 being functions of gap/chord, stagger, decalage, overhang and wing thickness. The observed variation of K_2 with these factors is in surprisingly good agreement with the wing theory and in particular with the values given by Munk's equations, as will be shown later. The presence of the constant K_1 for biplanes *without* decalage is not indicated by existing theory but these data have been shown to Dr. Munk, who suggests that K_1 is due to the Venturi effect between the wings. In the case of the orthogonal biplane a simple integration of the flow between the wings on this basis gives a reduction in pressure of the order required by the average value of K_1.

Assuming that K_1 is due to the Venturi effect it should vary with the restriction, or the ratio of wing thickness to gap t/G, and with stagger. Table VI contains the results of an analysis on this basis of test data in which the stagger was varied with gap/chord constant. The fifth column of this table is the value of K_1 for zero stagger and the sixth column is the slope of K_1 when plotted against stagger. The values of K_1 for zero stagger are plotted in figure 9 and a probable curve is drawn through the points which are fairly consistent. Values of $\Delta K_1/\Delta s$ from column 6 of table VI are plotted on figure 10. As might be expected, the scattering of the points is greater here than in figure 9 since the difficulty of eliminating decalage is greater when stagger is present. It should be noted that the values of K_1 are quite small and correspond to an angular change of less than one half a degree, so that the usual error in measuring the alignment may become a relatively large item. The value of K_1 is greatly affected by decalage, as will be shown later. It would be highly desirable to determine the curve of figure 10 accurately by special wind-tunnel tests.

EFFECT OF GAP/CHORD ON THE COEFFICIENT K_2

Munk's relation, equation (11), indicates that K_2 varies with the ratios of gap/span and stagger/chord. Although the ratio of gap/span offers some advantages with no difficulties, the ratio of gap/chord is easier to visualize and the latter will, therefore, be used to study the effect of stagger. Table VII contains calculations for a set of typical curves showing the variation of K_2 with gap/chord. These curves are plotted as solid lines on figure 11. They are obtained by taking the values of $\frac{b}{R}\left(\frac{1}{k^2}-0.5\right)$ relative to the value for gap/chord = 1.00 and assuming values of K_2 for this condition. Observed values of K_2 for varying gap/chord with constant stagger, from tables II to V, are connected by broken lines in each series in the plotting on figure 11. The observed variation of K_2 with gap/chord is seen to be in excellent agreement with Munk's theoretical analysis. A set of correction curves may now be prepared from figure 6 and table VII for use in reducing observed values of K_2 to

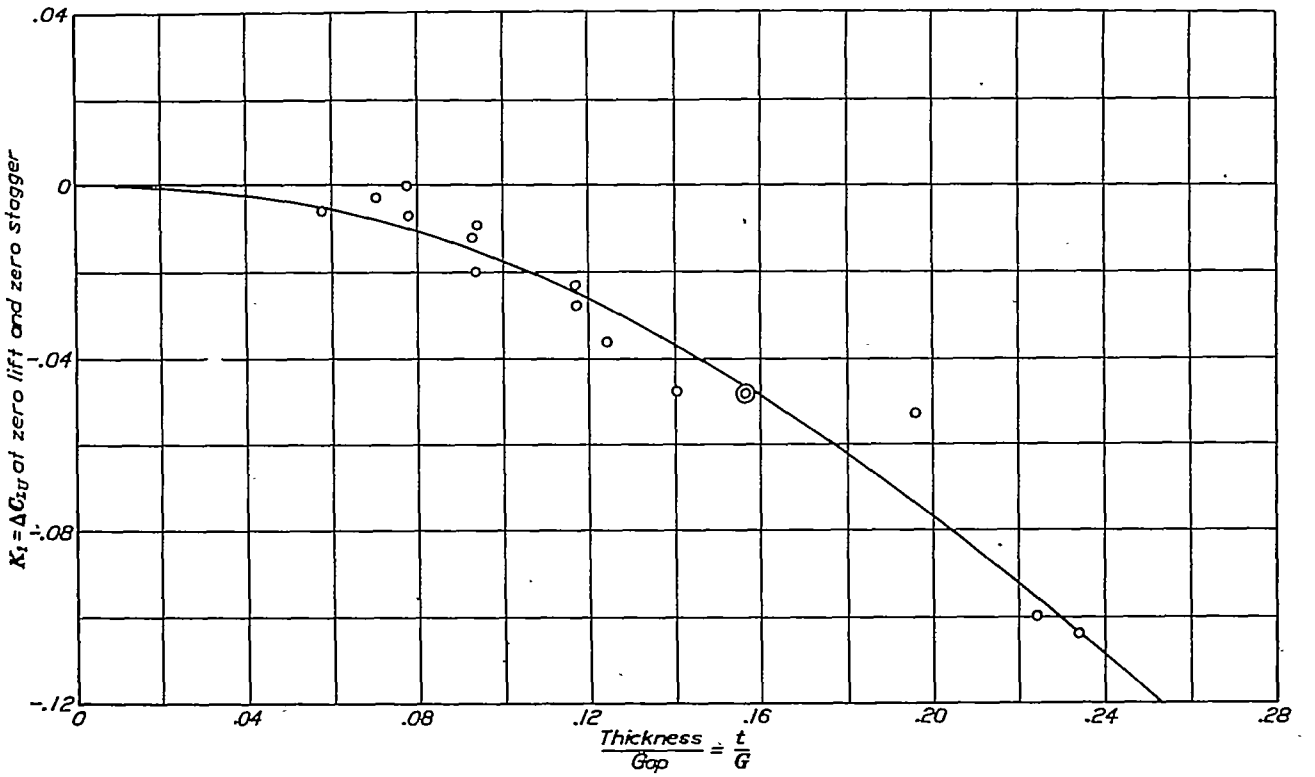

FIGURE 9.—Effect of wing thickness and gap on K_1.

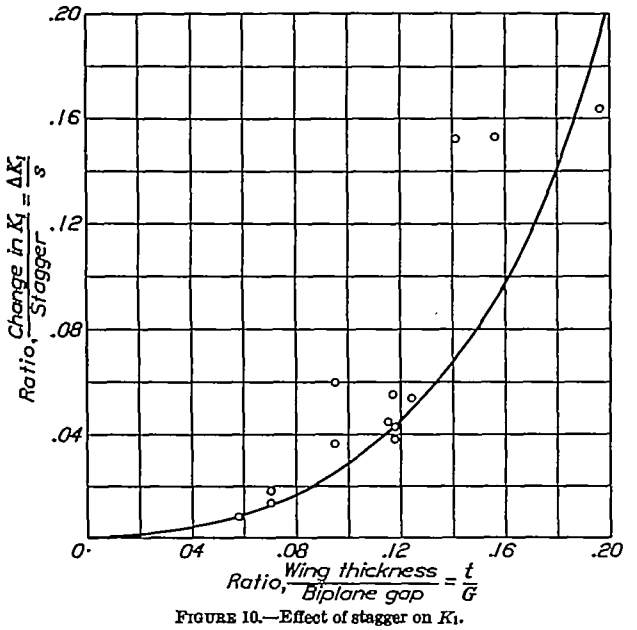

FIGURE 10.—Effect of stagger on K_1.

gap/chord = 1.00 and thereby separating the effect of gap/chord and stagger. The calculations are given in table VIII. For each assumed ratio of span to chord, the values of gap/span are calculated from the first column values of gap/chord. The factor $F = \frac{b}{R}\left(\frac{1}{k} - 0.5\right)$ is then read from figure 6. These values are then taken relative to the value for $b = 6c$ and gap/chord = 1.00, for which $F_o = 0.675$ from figure 6. The ratios are then multiplied by $\frac{36}{\left(\frac{b}{c}\right)^2}$ as required by equation (11). The resulting values which are plotted on figure 12, show the relative variation of K_2 with gap/chord.

EFFECT OF STAGGER ON THE COEFFICIENT K_2

Stagger may be given in terms of its ratio to either gap or chord, or in degrees. It should be measured from the line connecting the forward third points on the chords and in a fore and aft vertical plane. The true stagger varies with angle of attack but that given in the tabulation of data is usually measured from the zero angle of attack. In plotting up test data on widely different sections it was found that very much better agreement was obtained by using the stagger measured at the zero lift attitude. This may be called the "effective stagger." The effective stagger will therefore be used.

Observed values of K_2 from the tests with varying stagger listed in tables II to V have been collected in table IX and corrected to gap/chord = 1.00 by use of the curves of figure 12. The corrected values have been plotted on figure 13. With the exception of points at negative stagger and for low gap/chord ratios, the value of K_2 for gap/chord = 1.00 is given satisfactorily by the linear relation

$$K_2 = 0.050 + 0.17\frac{s}{c} \tag{15}$$

where s/c is the basic stagger measured at zero lift. The deviations of the points from this line are due partially to experimental errors and partially to the difficulty in determining the direction of the lines from which K_2 is read on the original plots of ΔC_{L_U}

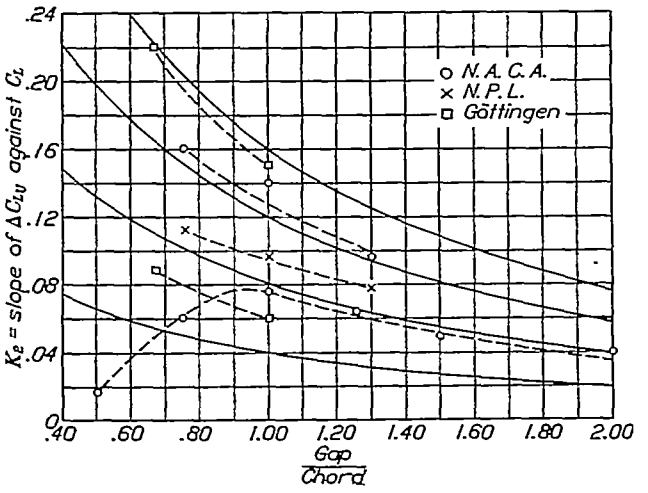

FIGURE 11.—Variation of K_2 with $\frac{\text{gap}}{\text{chord}}$ for constant stagger.

second case, the agreement is exact from 0.75 to 1.50, but the results for gap/chord ratio 0.5 deviate from the general curve. The theory can therefore be regarded as quite satisfactory in all practical applications. The deviation at the smallest gap implies that the theory must be examined more accurately in this case. In developing the theory it has been assumed that one wing may be treated as a lifting line as regards its influence on the other wing, and this assumption probably breaks down when the gap becomes as small as one half of the chord." Glauert also states in reference 6, "It will be noticed that the calculated values are in good agreement with the observed values for positive angles of stagger, but that there is a definite discrepancy in the case of negative stagger, for which no explanation has been found as yet."

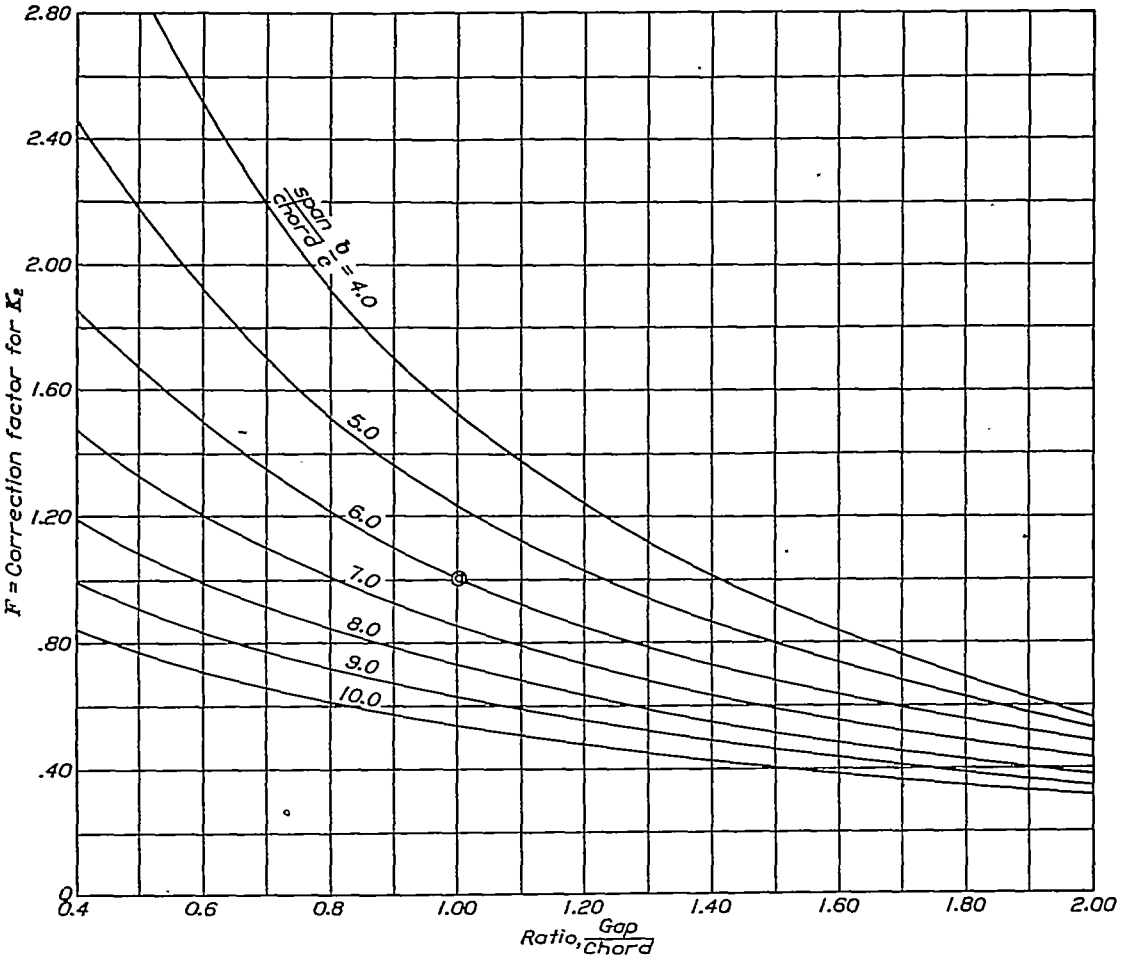

FIGURE 12.—Variation of K_2 with gap/chord.

against C_L, of which figure 8 has been given as an example. This uncertainty is, in general, of the order of 0.01 in the value of K_2. With this in mind the agreement is quite satisfactory.

In connection with the scattering of the points for low ratios of gap/chord, Glauert states in reference 6, "In the first case, exact agreement is obtained for gap/chord ratios ranging from 0.67 to 2.33. In the

It is fortunate that the interest in low gap/chord values and negative stagger is academic at present. There is some question, however, as to whether a biplane with small stagger at positive lifts acts like a biplane with negative stagger at negative lifts. No biplane tests covering the negative range are available to decide this point. Most of the available data, condensed in tables II to V, are not carried very far below

zero lift. Those that do extend to, say, $C_L = -0.30$ show no change in the value of K_2. Figure 13 indicates that there should be no change for small negative staggers, but this point cannot be determined without a revision of the theory and special tests.

II. BIPLANES WITH DECALAGE

Decalage has been defined as the acute angle between the wing chords of a biplane. This is equivalent to the

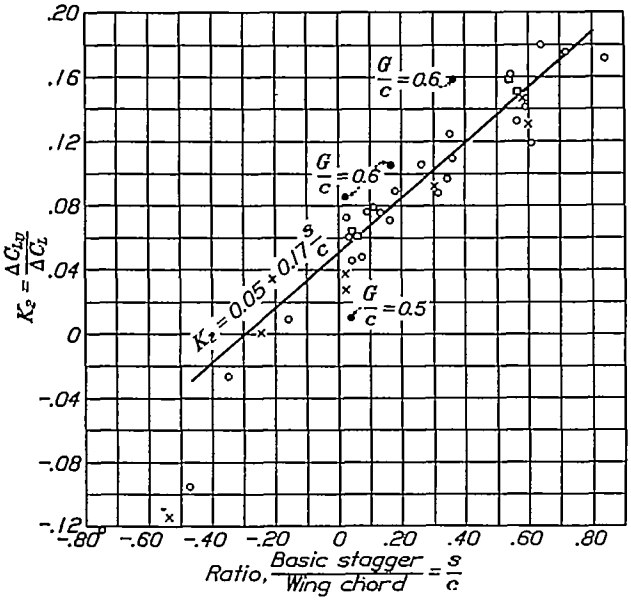

FIGURE 13.—Variation of K_2 with stagger for $\frac{\text{gap}}{\text{chord}} = 1.00$. Based on the effective stagger at zero lift.

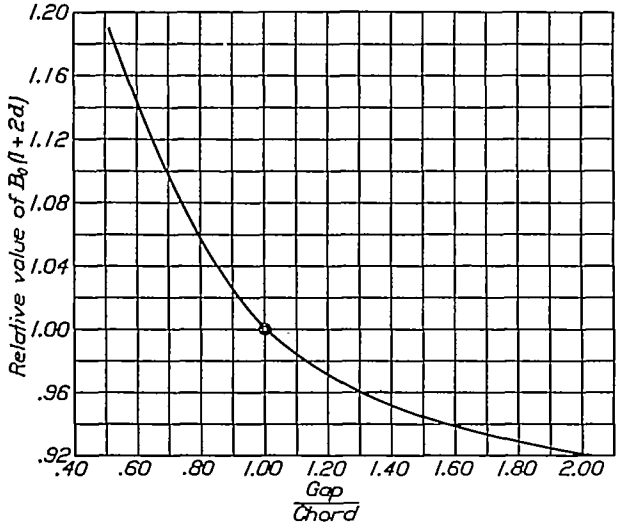

FIGURE 14.—Relative value of Munk's factor B_0 (1+2d). From table I, N.A.C.A. No. 151.

difference between the angles of incidence of the upper and lower wings. There is some confusion regarding the sign of the decalage, but the weight of authority and usage favors the definition of positive decalage for the lower wing at a positive angle with respect to the upper wing so that the chord lines of the upper and lower wings intersect forward of the leading edge.

The great influence of decalage on lift distribution and stability has not been fully appreciated by airplane designers. The definitions have been based on geometrical angles, which may be misleading. For the purpose of this study it is necessary to use aerodynamic decalage measured from the zero lift directions in the upper and lower wing, and not from the chord lines. The decalage will be considered positive when the zero lift direction lines intersect forward of the leading edge. The zero lift direction for each wing is further defined as the direction of the relative wind for zero lift on that wing.

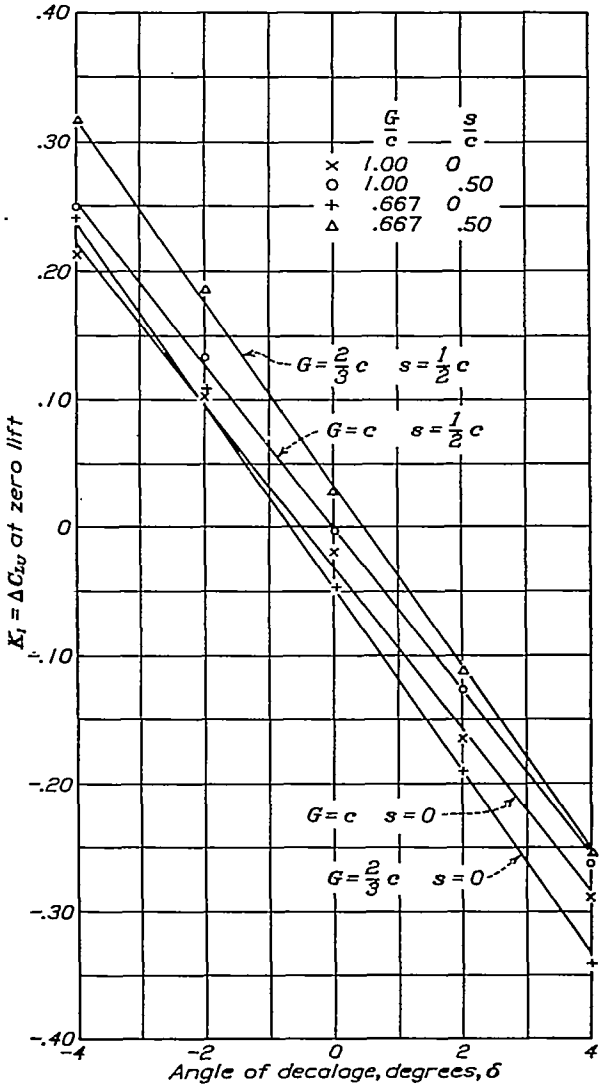

FIGURE 15.—Effect of decalage on K_1. From Munk's tests (reference 8).

According to Munk, equation (12), the effect of decalage is to change the lift coefficients of the individual wings by an equal and opposite increment which is a function of the gap/chord ratio and directly proportional to the decalage angle. That is, the chief effect of decalage is to change the value of K_1 in equation (14).

The factor B_o (1+2d) in equation (12) has been given in figure 7. This may be replotted to give values of B_o (1+2d) relative to the value for gap/chord = 1.00, as in figure 14. This form is convenient for comparing the theoretical and the observed variation in K_1.

Figure 15 is a plot of the values of K_1 against decalage from Munk's tests abstracted in table V. Data from Mock's tests (reference 14) are plotted on figure 16. In figure 15 the slope of the lines are as follows:

$\frac{\text{Gap}}{\text{Chord}}$	Stagger	$\frac{\Delta K_1}{\Delta \delta}$
1.00	0	−.0635
1.00	.50	−.0635
.67	0	−.071
.67	.50	−.071

Mock's tests (fig. 16) give $\frac{\Delta K_1}{\Delta \delta}=-.063$ for gap/chord=1.00. Munk's test data show that stagger does not affect the value of $\frac{\Delta K_1}{\Delta \delta}$.

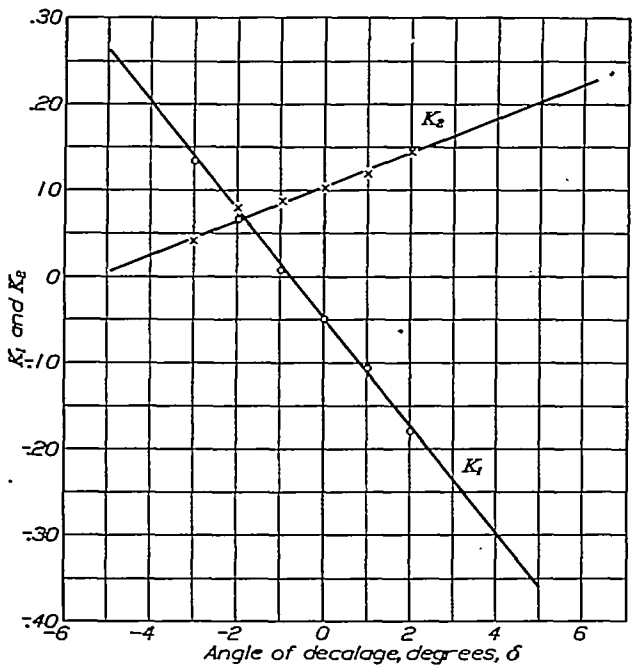

FIGURE 16.—Effect of decalage on K_1 and K_2. From Mock's tests (reference 14).

Figure 17 shows a curve similar to figure 14 derived by assuming $\frac{\Delta K_1}{\Delta \delta}=-.063$ at gap/chord=1.00. Munk's points obviously lie on a similar curve passing through $\frac{\Delta K_1}{\Delta \delta}=-.0635$ at gap/chord=1.00. The observed effect of decalage appears to follow very closely the theoretical effect predicted by Munk's equation.

The effect of decalage on K_2 is not covered by the theory but it is too great to be neglected. Values of K_2 for various decalage angles, as obtained from Mock's tests in reference 14, have been given on figure 16. A similar plot from Munk's data in table V is given on figure 18. The effect appears independent of gap/chord and stagger and is linear with decalage, the uniform slopes giving

$$\frac{\Delta K_2}{\Delta \delta}=0.0186 \tag{16}$$

Decalage therefore affects both K_1 and K_2 in equation (14), the effect on K_1 being given by figure 17 and the effect on K_2 by equation (16).

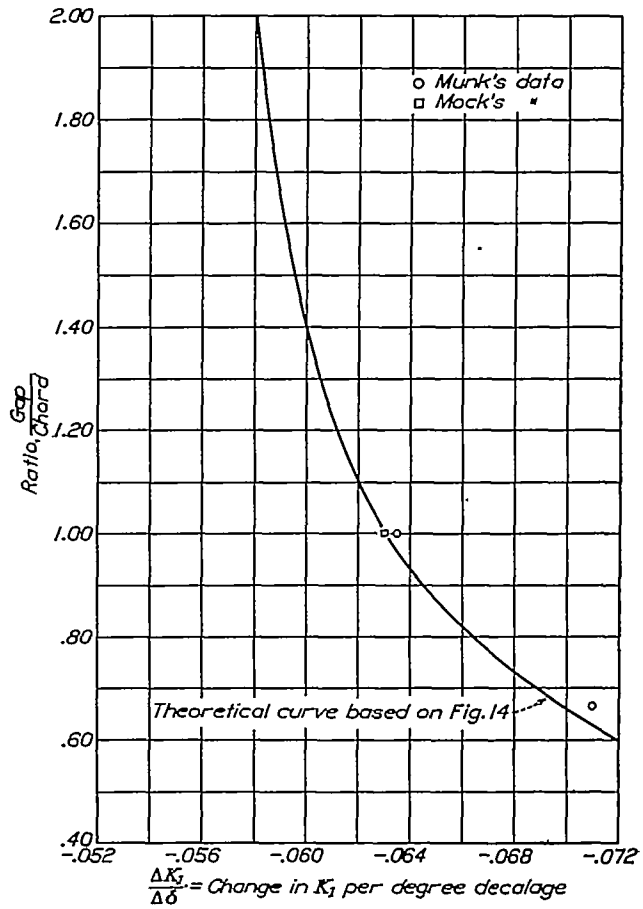

FIGURE 17.—Effect of decalage and gap/chord on K_1.

III. BIPLANES WITH OVERHANG

Overhang is defined as the ratio of the difference in wing spans to the span of the upper wing and is positive when the upper span is the greater. Overhang is usually given in percent of the upper wing span, or

$$\text{Overhang percent}=100\,\frac{b_U-b_L}{b_U}$$

where b_U and b_L are the spans of the upper and lower wings, respectively.

Limited tests on the effect of overhang are given in reference 13. These data are abstracted in table X and plotted on figure 19. The effect is surprisingly large. Calculations have been made by equations (1)

and (2) in order to check this point. These calculations are too long to be given in full, but the following results were obtained:

Overhang percent	K_1	K_2
−20	−0.025	+0.092
0	−0.017	+0.101
+20	−0.017	+0.100
+40	−0.014	+0.081
50	−0.012	+0.074
67	−0.007	+0.054

These are compared with the observed values of K_1 and K_2 on figure 20. The agreement is not entirely

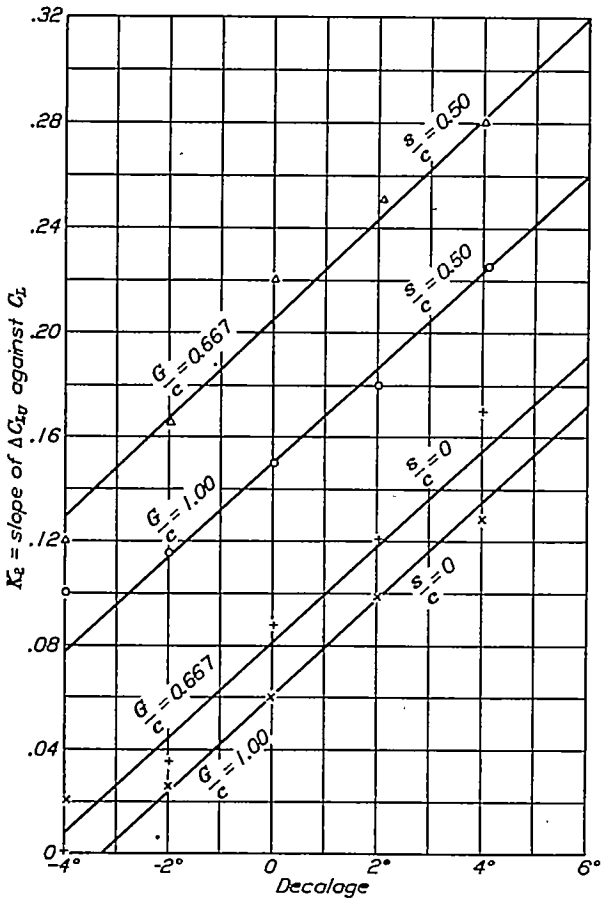

FIGURE 18.—Effect of decalage on K_2. From Munk's tests (reference 8).

satisfactory, although there is less difference than appears from a casual inspection of the curves. In the first place, the existence of the term K_1 is not predicted directly by the theory, equations (1) and (2) or (11). The values of K_1 given above have been obtained by extrapolating the lift curves through zero lift. Consequently, the fact that the values of K_1 so found are of the order obtained by wind-tunnel test is about all that can be expected. On the other hand, K_2 can be determined with better accuracy than K_1, so that the difference between theory and experiment is here of more importance. It appears highly desirable that special tests be made on biplanes with overhang to investigate these differences. In the meanwhile, the values of K_1 and K_2 for biplanes with overhang are probably best obtained from a contour plotting as in figure 21, which is based on the experimental values in table X. In using this plot the values

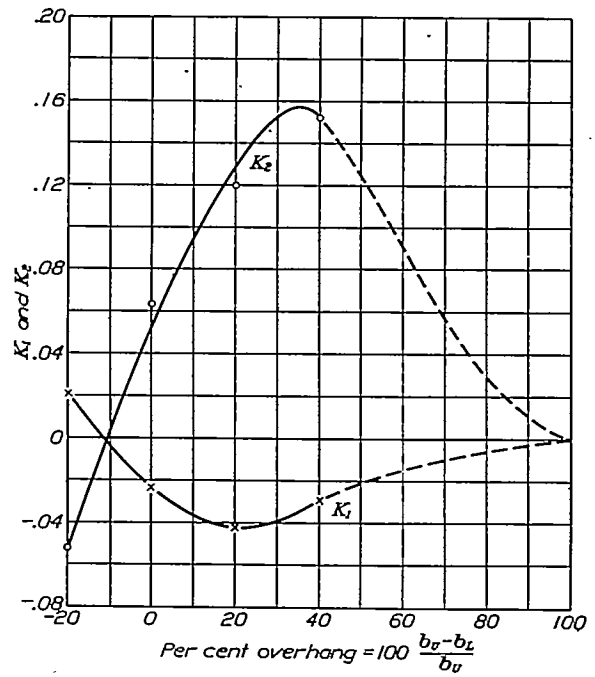

FIGURE 19.—Effect of overhang on K_1 and K_2. From data in reference 13. (See table X.)

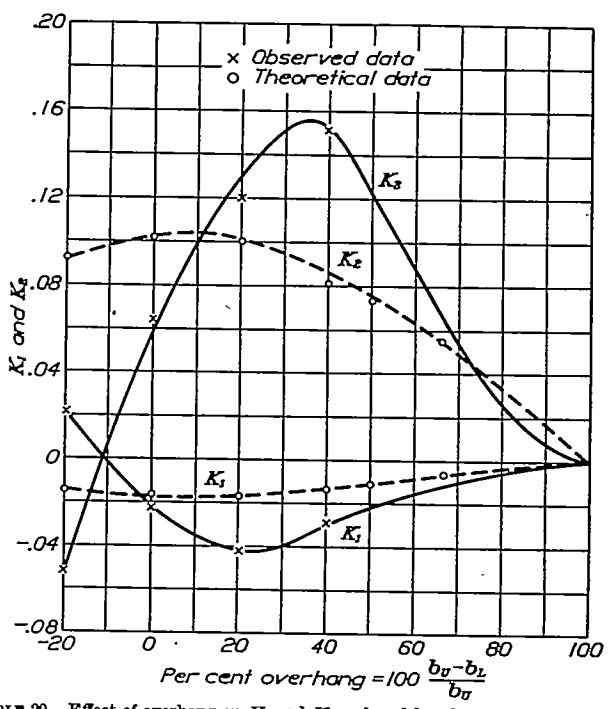

TABLE 20.—Effect of overhang on K_1 and K_2 as found by observation and by calculation from theoretical curves.

of K_1 and K_2 are determined first for a biplane without overhang but with the same stagger, gap and decalage as for the biplane in question. Spotting these points at zero overhang on figure 21, the corresponding point at the desired overhang will lie on curve similar to those given.

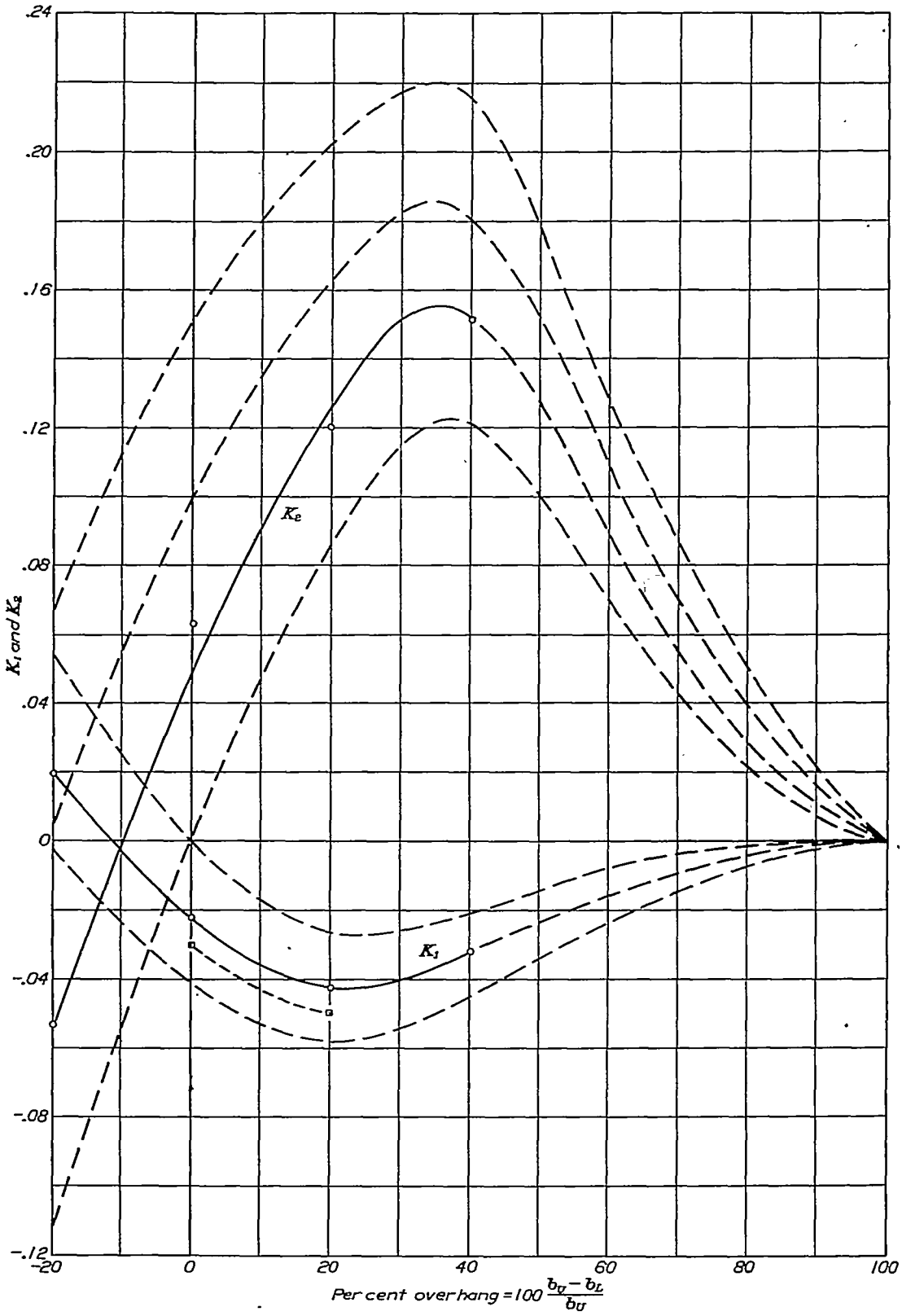
.24
.20
.16
.12
.08
.04
0
-.04
-.08
-.12
K_1 and K_2
K_2
K_1
-20
0
20
40
60
80
100
Per cent overhang = $100 \frac{b_U - b_L}{b_U}$

FIGURE 21.

PRACTICAL APPLICATION

The relative lift of the wings of any biplane may now be calculated from

$$C_{L_U} = C_L \pm \Delta C_{L_U} \tag{17}$$

$$C_{L_L} = C_L \mp \Delta C_{L_L} \tag{17a}$$

where C_L, C_{L_U}, and C_{L_L} are the lift coefficients for the biplane, the upper wing, and the lower wing, respectively.

It has been shown that

$$\Delta C_{L_U} = K_1 + K_2 C_L \tag{14}$$

where K_1 and K_2 are functions of gap/chord, stagger, decalage, overhang, and wing thickness. K_1 is numerically the lift coefficient on the upper wing when the biplane lift is zero, while K_2 determines the slope of the lift curve of the upper wing relative to that of the biplane. When the upper and lower wings are of equal area, the increments ΔC_{L_U} and ΔC_{L_L} are equal and of opposite sign. When the areas are unequal the increments are inversely proportional to the relative areas and of opposite sign. In any case:

$$\Delta C_{L_L} = -\Delta C_{L_U} \frac{S_U}{S_L} \tag{18}$$

where S_U and S_L are the areas of the upper and lower wings. It should be noted that ΔC_{L_L} is usually negative in equation (17a).

A convenient procedure for calculating ΔC_{L_U} is as follows:

1. Tabulate the average values of the ratios;

   $\frac{\text{maximum wing thickness}}{\text{chord}} = \frac{t}{c}$

   $\frac{\text{gap}}{\text{chord}} = \frac{G}{c}$

   $\frac{\text{stagger}}{\text{chord}} = \frac{s}{c}$

   Use the average gap and average stagger with an average chord defined by

   $c = \frac{1}{2}\left[\frac{S_U C_U + S_L C_L}{S_U + S_L}\right]$ where C_U and C_L are the chords of the upper and lower wings.

   With tapered wings the weighted average chord of each wing should be used.

   The effective stagger measured at zero lift from the third chord points must be used.

2. Calculate the ratio

   $\frac{\text{maximum wing thickness}}{\text{gap}} = \frac{t}{G} = \frac{t}{c} \div \frac{G}{c}$

3. Calculate the overhang if present by

   $\text{overhang} = 100\left[\frac{b_U - b_L}{b_U}\right]$

   where b_U and b_L are the actual spans of the upper and lower wings without reduction for fuselage or nacelle blanketing.

4. Calculate K_1 from $K_1 = K_{10} + K_{11} + K_{12} + K_{13}$ (19)

Where K_{10} is the value of K_1 for the equal wing orthogonal arrangement without decalage or overhang as read from figure 9, K_{11} is the change in K_1 due to stagger, K_{12} is the change in K_1 due to decalage and K_{13} is the change in K_1 due to overhang. The actual value of K_{13} is not determined directly since it is easier to pass from the value of K_1 with no overhang to the value of K_1 with overhang as will be explained later. The values of K_{10}, K_{11}, and K_{12} are determined as follows:

K_{10}: This is plotted as a function of t/G in figure 9;

K_{11}: This is obtained from figure 10 where $\Delta K_1/s$ is plotted against t/G and

$$K_{11} = \frac{\Delta K_1}{s} s \tag{20}$$

K_{11} is negative with negative stagger.

K_{12}: The effect of decalage is to change K_1 in a linear relation:

$$K_{12} = \left(\frac{\Delta K_1}{\delta}\right)\delta^\circ \tag{21}$$

$\left(\frac{\Delta K_1}{\delta}\right)$ varies with gap/chord as shown on figure 17. It has an average value of about -0.063, so that negative decalage, where the incidence of the lower wing is less than the incidence of the upper wing, gives a positive K_{12} which increases K_1.

K_{13}: The actual value of K_{13} need not be obtained, since it is more convenient to correct for overhang by the use of figure 21, and pass directly from the value of K_1 with no overhang to the value of K_1 with overhang. The value of K_1 with no overhang is the sum of $K_{10} + K_{11} + K_{12}$. This value may be spotted at zero overhang on the lower set of curves on figure 21. A curve similar to those given and passing through this point gives the value of K_1 at any other overhang as desired. It is unnecessary to draw the curve since the interpolation may be made visually with sufficient accuracy. For example, assume that without overhang $K_1 = K_{10} + K_{11} + K_{12} = -0.030$, then for $+20$ percent overhang $K_1 = -0.050$ as indicated on the lightly dotted curve.

5. Calculate K_2 from

$$K_2 = (K_{20})F + K_{21} + K_{22} \tag{22}$$

Where K_{20} is the value of K_2 for the desired stagger at gap/chord $= 1.00$, F_2 is a correction factor for gap/chord and aspect ratio, K_{21} is the change in K_2 due to decalage and K_{22} is the change in K_2 due to overhang. The values of these factors are determined as follows:

K_{20}: The effect of stagger, is either read from figure 13 or obtained from the equation

$$K_{20} = 0.050 + 0.17\frac{S}{c} \tag{15a}$$

F_2, the effect of gap/chord and aspect ratio

on K_2, is obtained from figure 12. In using this figure, the average aspect ratio of the two wings must be used, and not the effective aspect ratio of the combination.

K_{21}: The effect of decalage is obtained from equation (16) in the form

$$K_{21} = +0.0186\delta^0 \tag{16a}$$

where δ is the angle of decalage in degrees with its positive or negative sign. Positive decalage increases K_2, negative decalage decreases K_2.

K_{22}: The effect of overhang. This is obtained indirectly by the same procedure used for K_{13}. The value of K_2 without overhang is $K_2 = (K_{20})\ F + K_{21}$. This value is spotted at zero overhang on figure 21 and a line traced through it following the trend of the upper set of curves. This line gives the corrected value of K_2 at the desired overhang.

The relative unit lift or efficiency of the upper and lower wings of a biplane is defined by the ratio

$$e = \frac{C_{L_U}}{C_{L_L}} = \frac{C_L \pm \Delta C_{L_U}}{C_L \pm \Delta C_{L_L}} \tag{23}$$

which is now readily calculated. Obviously, e will vary over wide limits and in general it will become infinite at or near zero lift for the cellule. Any method that works directly with the ratio e must become unmanageable in the region of zero lift. The method here developed gives definite lift coefficients for any condition.

For the normal biplane, upper and lower wing of equal areas, with moderate stagger but without decalage, the values of K_1 and K_2 in equation (14) may be of the order of -0.020 and $+0.120$, respectively. That is,

$$\Delta C_{L_U} = -0.020 + 0.120\ C_L$$

and $\Delta C_{L_L} = -\Delta C_{L_U} = +0.020 - 0.120\ C_L$

so that equations (17) and (17a) would be

$$C_{L_U} = 1.12\ C_L - 0.020$$

and $C_{L_L} = 0.88\ C_L + 0.020$

When $C_L = 0$ for this biplane $C_{L_U} = -0.020$,

and $C_{L_L} = +0.020$ giving $e = -1.00$.

If $C_L = +0.01785$, $C_{L_U} = 0$ and $C_{L_L} = +0.0357$ giving $e = 0$. If $C_L = -0.0228$, $C_{L_U} = -0.0456$ and $C_{L_L} = 0$ giving $e = -\infty$. At negative values of C_L below -0.0228, e will again be positive. Since the vertical location of the aerodynamic mean chord depends on the value of e, it is obvious that the vertical location of the mean chord is a function of the lift coefficient. It therefore follows that biplane arrangements having positive or very small negative values of K_1 tend to give a high location for the mean chord at low lift coefficients, which tends to improve the static longitudinal stability. This is one method of explaining the improvement in longitudinal stability due to negative decalage.

The steps involved in the calculation of C_{L_U} and C_{L_L} may perhaps be clarified by a numerical example. Assume a biplane with the following characteristics:

Upper wing: span $b_U = 40$ feet, chord $= 6$ feet, area $S_U = 230$ square feet.

Lower wing: span $b_L = 36$ feet, chord $= 5$ feet, area $S_L = 170$ square feet.

Mean gap: $G = 70$ inches.

Chord (weighted average) $c = 67$ inches.

Stagger measured on leading edge at zero lift $= 34$ inches.

Stagger measured on the ⅛ chord points at zero lift $s = 30$ inches.

No decalage $\delta = 0°$.

Wing section Clark Y.

Then

$$\frac{t}{c} = 0.117 \text{ for Clark Y}$$

$$\frac{\text{gap}}{\text{chord}} = \frac{G}{c} = \frac{70}{67} = 1.045$$

$$\frac{\text{wing thickness}}{\text{gap}} = \frac{t}{G} = \frac{0.117}{1.045} = 0.112$$

$$\frac{\text{stagger}}{\text{chord}} = \frac{s}{c} = \frac{30}{68} = 0.44$$

$$\text{overhang} = 100\ \frac{b_U - b_L}{b_U} = +10 \text{ percent}$$

K_1 is now found as follows:

From figure 9, $K_{10} = -0.023$.

From figure 10, $\frac{\Delta K_1}{S} = 0.038$, hence,

$$K_{11} = 0.038 \times 0.44 = +0.017$$

$$K_{12} = 0 \text{ since } \delta = 0°$$

$$\therefore\ K_{10} + K_{11} + K_{12} = -0.023 + 0.017 = -0.006$$

From figure 21, a value of $K_1 = -0.006$ for zero overhang gives $K_1 = -0.022$ for 10 percent overhang. Hence $K_1 = -0.022$.

K_2 is now found as follows:

From figure 13, or equation (15a)

$$K_{20} = 0.050 + 0.17 \times 0.44 = 0.125$$

Since the average aspect ratio of the two wings is

$$\frac{1}{2}\left[\frac{(40)^2}{230} + \frac{(36)^2}{170}\right] = \frac{1}{2}[6.95 + 7.62] = 7.3,$$

the value of F from figure 12 is $F = 0.82$, so that $K_{20}F = 0.125 \times 0.103$. For zero decalage $K_{21} = 0$. Hence $(K_{20}\ F) + K_{21} = 0.103$. From figure 21 a value of $K_2 = 0.103$ for zero overhang gives $K_2 = 0.138$ for 10 percent overhang.

The lift increment for the upper wing is

$$\Delta C_{L_U} = -0.022 + 0.138 C_L$$

and for the lower wing it is $\Delta C_{L_L} = -\Delta C_{L_U} \frac{S_U}{S_L}$

$$= -[-0.022 + 0.138\ C_L]\ \frac{230}{170}$$

$$= +0.030 - 0.187\ C_L$$

Hence

$$C_{L_U}=C_L-0.022+0.138\ C_L$$
$$=1.138\ C_L-0.022$$

and

$$C_{L_L}=C_L+0.030-0.187\ C_L$$
$$=0.813\ C_L+0.030$$

The relative lift is

$$e=\frac{C_{L_U}}{C_{L_L}}=\frac{1.138\ C_L-0.022}{0.813\ C_L+0.030}$$

CONCLUSIONS

The method here outlined for calculating the lift coefficients of the individual wings of a biplane has been based on a combination of theoretical and experimental data. In some respects there is excellent general agreement between theory and experiment, as follows:

1*a*. The effect of gap/chord stagger and aspect ratio on K_2 as shown by figure 11, table IX, and figure 13. (See equation 11.)

2*a*. The effect of decalage on K_1 as shown by figure 17. (See equation 12.)

The experimental data are consistent and fairly complete in other items such as:

1*b*. The effect of wing thickness and gap/chord ratio on K_1 with zero stagger as shown by figure 9.

2*b*. The effect of decalage on K_2 as shown by figures 16 and 18.

The remaining factors that need further investigation are:

1*c*. The effect of stagger on K_1. Special tests to obtain greater accuracy in figure 10 are highly desirable.

2*c*. The effect of overhang on K_1 and K_2. Special tests to obtain greater accuracy in figure 21 are required.

3*c*. The extension of test data to maximum negative lifts. Available test data indicate no appreciable change in K_2 at zero lift. Special tests should be made to investigate this effect.

Several conclusions may be drawn from a study of the method developed in this report, in the light of the foregoing summary.

1. The calculation of the individual wing lift coefficients is the only practical method of determining the ratio e at low lift coefficients.
2. The method here presented is not difficult to use.
3. In general, the existing biplane theory is verified by experiment, but further investigation is desirable to cover the interaction at zero lift and the effects of overhang.
4. Special biplane tests to cover the items listed under 1*c*, 2*c*, and 3*c* above should be made in order to eliminate the present uncertainty in these items.

Bureau of Aeronautics,
Navy Department,
Washington, D.C., *February 15, 1933.*

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TABLE I

LIFT COEFFICIENTS FOR U.S.A. TS-5 BIPLANE [1]

$\frac{\text{Gap}}{\text{chord}}=0.9$ Stagger$=+30°=+0.69$ c

Angle of attack	Lower wing C_{L_L}	Upper wing C_{L_U}	Biplane C_L	$\Delta C_{L_U}=C_{L_U}-C_L$
−9	+0.003	+0.097	+0.050	+0.047
−7	.102	.229	.165	.064
−5	.203	.363	.283	.080
−3	.300	.504	.402	.102
−1	.395	.646	.520	.126
0	.447	.724	.585	.139
2	.544	.890	.717	.173
4	.635	1.030	.833	.197
6	.725	1.180	.952	.228
8	.823	1.290	1.057	.233
10	.921	1.340	1.130	.210

[1] Data from N.A.C.A. Technical Report No. 256.

TABLE II

DIFFERENCE BETWEEN LIFT COEFFICIENT OF UPPER WING AND BIPLANE [1] NO DECALAGE

Gap/chord	Stagger/chord Nominal	Stagger/chord Effective	$\Delta C_{L_U}=$	Section
0.9	−0.51	−0.47	$-0.007-0.104\ C_L$	R.A.F.-15.
1.2	−.69	−.74	$+.011-.104\ C_L$	Do.
.6	0	+.02	$-.028+.127\ C_L$	Do.
.8	0	+.02	$-.010+.088\ C_L$	Do.
1.0	0	+.03	$-.004+.060\ C_L$	Do.
1.2	0	+.04	$-.006+.038\ C_L$	Do.
.6	.16	.17	$-.034+.157\ C_L$	Do.
.9	.24	.26	$-.008+.116\ C_L$	Do.
1.2	.31	.35	$-.005+.104\ C_L$	Do.
.6	.35	.36	$-.014+.236\ C_L$	Do.
.9	.52	.54	$-.005+.178\ C_L$	Do.
1.2	.69	.72	$-.001+.147\ C_L$	Do.
.9	−.51	−.35	$-.036-.030\ C_L$	U.S.A. TS-5.
.9	0	+.16	$-.053+.077\ C_L$	Do.
.9	+.51	+.64	$-.051+.198\ C_L$	Do.

[1] From N.A.C.A. Technical Report No. 256.

TABLE III

DIFFERENCE BETWEEN NORMAL FORCE COEFFICIENT OF UPPER WING AND BIPLANE

From N.A.C.A. Pressure Distribution Tests

Clark Y Section—Circular Tips [1]

Gap/chord	Stagger/chord Nominal	Stagger/chord Effective	$\Delta C_{N_{F_U}}$
0.50	0	+0.04	$-0.104+0.017\ C_{N_F}$
.75	0	.07	$-.048+.060\ C_{N_F}$
.75	.25	.31	$0\ +.111\ C_{N_F}$
.75	.50	.56	$+.047+.160\ C_{N_F}$
1.00	−.25	−.16	$-.024+.009\ C_{N_F}$
1.00	0	+.09	$-.027+.076\ C_{N_F}$
1.00	+.25	.34	$-.007+.096\ C_{N_F}$
1.00	.50	.59	$+.001+.141\ C_{N_F}$
1.00	.75	.84	$+.006+.172\ C_{N_F}$
1.25	0	+.11	$-.011+.064\ C_{N_F}$
1.25	.25	.36	$+.010+.088\ C_{N_F}$
1.25	.50	.61	$+.023+.096\ C_{N_F}$
1.50	0	.13	$0\ +.050\ C_{N_F}$
2.00	0	.18	$+.033+.040\ C_{N_F}$

[1] N.A.C.A. Technical Report No. 417.

TABLE IV

DIFFERENCE BETWEEN LIFT COEFFICIENT OF UPPER WING AND BIPLANE [1]

R.A.F.-15 Section—No Decalage

Gap/chord	Stagger/chord Nominal	Stagger/chord Effective	$\Delta C_{L_U}=$	Reference
1.00	+0.57	0.60	$+0.015+0.130\ C_L$	R. & M. 589.
1.00	0	+.02	$+.002+.027\ C_L$	R. & M. 589.
1.00	−.57	−.54	$-.005-.115\ C_L$	R. & M. 589.
1.00	0	+.02	$-.008+.036\ C_L$	R. & M. 774.
1.00	.56	.58	$+.006+.146\ C_L$	R. & M. 857.
1.00	.27	.30	$-.005+.091\ C_L$	R. & M. 857.
1.00	−.28	−.25	$-.004+0$	R. & M. 857.

[1] From British A.R.C. Reports and Memoranda as indicated.

TABLE V

DIFFERENCE BETWEEN LIFT COEFFICIENT OF UPPER WING AND BIPLANE [1] GÖTTINGEN 133 SECTION

Gap/chord	Stagger/chord Nominal	Stagger/chord Effective	Decalage degrees	ΔC_{L_U}
1.00	0	0.06	−4	$+0.213+0.020\ C_L$
			−2	$+.103+.025\ C_L$
			0	$-.020+.060\ C_L$
			+2	$-.165+.120\ C_L$
			+4	$-.290+.130\ C_L$
1.00	.50	.56	−4	$+.250+.100\ C_L$
			−2	$+.132+.115\ C_L$
			0	$-.002+.150\ C_L$
			+2	$-.125+.180\ C_L$
			+4	$-.262+.225\ C_L$
.667	0	.04	−4	$+.240+0$
			−2	$+.105+.035\ C_L$
			0	$-.048+.088\ C_L$
			+2	$-.192+.120\ C_L$
			+4	$-.342+.170\ C_L$
.667	.50	.54	−4	$+.315+.120\ C_L$
			−2	$+.185+.165\ C_L$
			0	$+.028+.220\ C_L$
			+2	$-.113+.250\ C_L$
			+4	$-.243+.280\ C_L$

[1] From Technische Berichte II-2.

TABLE VI

VARIATION OF $K_1=\Delta C_{L_U}$ AT ZERO LIFT AS A FUNCTION OF CHORD, GAP, STAGGER AND WING THICKNESS

Gap/chord $\frac{G}{c}$	Wing section	Wing thickness $\frac{t}{c}$	Thickness/gap $\frac{t}{G}$	K_1	Change in K_1 with stagger $\frac{\Delta K_1}{\Delta S}$	Reference
0.50	Clark Y	0.117	0.234	−0.104	--------	N.A.C.A. T.R. 417.
.60	RAF-15	.070	.117	−.028	0.045	N.A.C.A. T.R. 256.
.67	G-133	.094	.140	−.048	.152	T.B. II-2.
.75	Clark Y	.117	.156	−.053	.153	N.A.C.A. T.R. 417.
.90	USA TS-5	.174	.196	−.053	.164	N.A.C.A. TR. 256.
1.00	G-133	.094	.094	−.020	.036	T.B. II-2.
1.00	Clark Y	.117	.117	−.027	.055	N.A.C.A. T.R. 417.
1.00	RAF-15	.070	.070	−.008	.014	R. & M. 589.
1.00	do	.070	.070	+.002	.018	R. & M. 774 and 857.
1.20	do	.070	.058	−.006	.008	N.A.C.A. T.R. 256.
1.25	Clark Y	.117	.094	−.011	.060	N.A.C.A. T.R. 417.
1.50	do	.117	.078	0	--------	Do.

TABLE VII

EFFECT OF $\frac{\text{GAP}}{\text{CHORD}}$ RATIO ON K_2

Gap/chord	Gap/span for $b=6c$	$\frac{b}{R}\left(\frac{1}{k^2}-0.5\right)$ Munk's factor	Relative values of Munk's factor: As calculated	Relative values of Munk's factor: Faired	Typical curves			
0.4	0.067	1.25	1.850	1.850	0.074	0.148	0.222	0.296
.6	.100	1.00	1.482	1.490	.059	.118	.178	.239
.8	.133	.825	1.223	1.210	.048	.097	.145	.194
1.0	.167	.675	1.000	1.000	.040	.080	.120	.160
1.2	.200	.570	.845	.845	.034	.068	.101	.135
1.4	.233	.490	.726	.725	.029	.058	.087	.116
1.6	.267	.425	.630	.630	.025	.050	.076	.101
1.8	.300	.370	.548	.550	.022	.044	.066	.088
2.0	.333	.320	.474	.475	.019	.038	.057	.076

NOTE.—The typical curves are based on assumed values of K_2 for gap/chord=1.00

TABLE VIII

CALCULATIONS FOR CORRECTION CURVES GIVING EFFECT OF GAP/CHORD ON K_2

Gap/chord $\frac{G}{c}$	$b=4c$: Gap/span $\frac{G}{b}$	$b=4c$: F	$b=4c$: $\frac{F}{F_s}\times\frac{36}{16}$	$b=5c$: Gap/span $\frac{G}{b}$	$b=5c$: F	$b=5c$: $\frac{F}{F_s}\times\frac{36}{25}$	$b=8c$: Gap/span $\frac{G}{b}$	$b=8c$: F	$b=8c$: $\frac{F}{F_s}\times\frac{36}{64}$	$b=10c$: Gap/span $\frac{G}{b}$	$b=10c$: F	$b=10c$: $\frac{F}{F_s}\times\frac{36}{100}$
0.4	0.100	1.00	3.33	0.080	1.145	2.45	0.050	1.435	1.196	0.040	1.555	0.829
.6	.150	.750	2.50	.120	.892	1.91	.075	1.185	.988	.060	1.325	.706
.8	.200	.574	1.910	.160	.706	1.51	.100	1.00	.833	.080	1.145	.610
1.00	.250	.455	1.52	.200	.574	1.227	.125	.865	.721	.100	1.00	.533
1.2	.300	.368	1.226	.240	.473	1.012	.150	.750	.625	.120	.892	.476
1.4	.350	.298	.993	.280	.400	.855	.175	.655	.546	.140	.793	.422
1.6	.400	.245	.817	.320	.338	.723	.200	.574	.479	.160	.706	.376
1.8	.450	.202	.673	.360	.287	.613	.225	.505	.421	.180	.638	.340
2.0	.500	.167	.557	.400	.245	.524	.250	.455	.379	.200	.574	.306

$F_s=0.675=F$ for $\frac{b}{\ }=6$ and $\frac{G}{c}=1.0$.

TABLE IX

VARIATION OF K_2 WITH STAGGER

Stagger/chord: Nominal	Stagger/chord: Basic	Gap/chord	Munk's factor F from figure 12	$K_2=\frac{\Delta C_{L_U}}{\Delta C_L}$: Observed	$K_2=\frac{\Delta C_{L_U}}{\Delta C_L}$: Corrected to $\frac{G}{c}=1$	Reference
0	0.04	0.50	1.66	0.017	0.010	N.A.C.A. T.R. 417.
0	.07	.75	1.27	.060	.047	Do.
.25	.31	.75	1.27	.111	.087	Do.
.50	+.56	.75	1.27	.160	.126	Do.
−.25	−.16	1.00	1.00	.009	.009	Do.
0	+.09	1.00	1.00	.076	.076	Do.
.25	.34	1.00	1.00	.096	.096	Do.
.50	.59	1.00	1.00	.141	.141	Do.
.75	.84	1.00	1.00	.172	.172	Do.
0	.11	1.25	.81	.064	.079	Do.
.25	.36	1.25	.81	.088	.109	Do.
.50	.61	1.25	.81	.096	.119	Do.
0	.13	1.50	.67	.050	.075	Do.
0	.18	2.00	.47	.042	.089	Do.
0	.06	1.00	1.00	.060	.060	T.B. II-2.
.50	.56	1.00	1.00	.150	.150	Do.
0	.04	.67	1.39	.068	.063	Do.
.50	.54	.67	1.39	.220	.158	Do.
−.51	−.47	0.9	1.10	−.104	−.095	N.A.C.A. T.R. 256.
−.69	−.66	1.2	.84	−.112	−.133	Do.
0	+.02	0.6	1.49	+.127	+.085	Do.
0	.02	0.8	1.21	.088	.072	Do.
0	.03	1.0	1.00	.060	.060	Do.
0	.04	1.2	.84	.038	.045	Do.
+.16	.17	0.6	1.49	.157	.105	Do.
.24	.26	0.9	1.10	.116	.105	Do.
.31	.35	1.2	.84	.104	.124	Do.
.35	.36	0.6	1.49	.236	.158	Do.
.52	.54	0.9	1.10	.178	.161	Do.
.69	+.72	1.2	.84	+.147	+.175	Do.
−.51	−.35	0.9	1.10	−.030	−.027	Do.
0	+.16	0.9	1.10	+.077	+.070	Do.
+.51	+.64	0.9	1.10	.198	.180	Do.
.57	.60	1.00	1.00	.130	.130	R. & M. 589.
0	+.02	1.00	1.00	.027	.027	Do.
−.57	−.54	1.00	1.00	−.115	−.115	Do.
0	+.02	1.00	1.00	.036	.036	R. & M. 774.
+.56	.58	1.00	1.00	.146	.146	R. & M. 857.
+.27	+.30	1.00	1.00	.091	.091	Do.
−.28	−.25	1.00	1.00	0	0	Do.

TABLE X

EFFECT OF OVERHANG ON LIFT DISTRIBUTION

Overhang percent	Gap/chord	Stagger/chord	Decalage	$\Delta C_{L_U}=$
−20	1.00	0	0	$+0.020-0.053\ C_{NF}$
0	1.00	0	0	$-.023+.063\ C_{NF}$
+20	1.00	0	0	$-.043+.120\ C_{NF}$
+40	1.00	0	0	$-.030+.152\ C_{NF}$

Overhang percent $=100\frac{b_U-b_L}{b_U}$