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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 782

STATUS OF WING FLUTTER

By H. G. Küssner

Luftfahrtforschung
Vol. XII, No. 6, October 3, 1935
Verlag von R. Oldenbourg, München und Berlin

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By H. G. Küssner

SUMMARY

This report presents a survey of previous theoretical and experimental investigations on wing flutter covering thirteen cases of flutter observed on airplanes. The direct cause of flutter is, in the majority of cases, attributable to (mass-) unbalanced ailerons.

Under the conservative assumption that the flutter with the phase angle most favorable for excitation occurs only in two degrees of freedom, the lowest critical speed can be estimated from the data obtained on the oscillation bench. Corrective measures for increasing the critical speed and for definite avoidance of wing flutter, are discussed.

I. INTRODUCTION

The forced oscillations on airplane wings are oscillations created solely by the air stream and have as a rule nothing to do with the vibrations set up by the inertia forces of the engine. They are therefore best designated by the term "flutter" since they revert to the same underlying causes as the fluttering of a flag.

Flutter starts at the so-called "critical speed," which depends chiefly on the oscillation frequency and on the wing chord. The lower the frequency and the smaller the chord, the lower the critical speed will be. The oscillation frequency of a wing, in turn, depends on the stiffness and on the mass of the wing.

Flutter in an air stream is possible only when a plate - in whole or in part - is free to rotate about at

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*"Augenblicklicher Entwicklungsstand der Frage des Flügelflatterns." Luftfahrtforschung, October 3, 1935, pp. 193-209.

least two axes or, which is the same, has at least two degrees of freedom of oscillation.

A wind vane of sheet metal made to rotate about one axis only does not flutter. However, when the flagpole is not rigid but free to swing laterally, thereby pivoting the vane about an axis below it and parallel to the wind, flutter is possible. If the vane is of cloth rather than metal, it can turn about infinitely many axes and is therefore particularly susceptible to flutter. So also is a wind vane made of two pieces of sheet metal hinged together, because then the flagpole and the hinge line between the two pieces form the two axes of rotation.

A similar condition exists when mounting a rudder R with tab H to a practically rigid fin F (fig. 1). The two axes of rotation are A_1 and A_2. With such an arrangement flutter has actually been observed (reference 1).

Far more importance, from the practical point of view, attaches to the case of an airplane wing fitted with an aileron. When oscillating, the wing turns about some nodal axis which may, for instance, coincide with the wing center line or the axis of the strut connections. Besides, the aileron itself can turn about its hinge.

The first records of wing flutter go back to the early days of flying, when the lateral control obtained by twisting the wing tips, was abandoned in favor of the aileron-control method

During the World War several cases occurred where flutter caused the ailerons to break and tear off. Likewise, almost all cases observed later on disclosed upon investigation, that the ailerons were the cause of the accident. Even a rigid plate can flutter, as stated above, when free to rotate about two axes. If the wing tip bends and twists simultaneously, it can flutter even without ailerons, although this case is much less frequent than the one described first.

In the following, the results of past investigations on wing flutter are given without resorting to mathematical deductions, while one section contains a discussion of the theoretical relations.

II. DEVELOPMENT OF METHODS OF ANALYSIS

The exploration of the causes of wing flutter is marked by the diversity of methods employed with a view to obtaining technically useful solutions of this extremely complicated problem.

1. Theorem of Linear Differential Equations for Steady Aerodynamic Forces

In the first flutter investigations, the air loads on the oscillating wing were assumed to be steady and dependent on the dynamic angle of attack, the dynamic angle of attack being defined as the angle between the wing chord and the momentary direction of motion of the oscillating wing. Some authors also took into account the lift due to dynamic profile camber. A wing oscillating about some axis, while its wing chord describes a curved surface line in flight, is identical with a wing in steady flight whose profile curvature changes at measured intervals.

This substitution is, in fact, strictly correct. Even these elementary assumptions afford a physical explanation of the phenomenon of flutter through a system of linear differential equations, the number of which depends on the number of degrees of freedom. Flutter is possible whenever undamped oscillations of constant amplitude, i.e., harmonic oscillations, are possible. Routh's discriminant thereby served as a criterion from which the critical speed may be computed.

The first calculations of this kind were made by Blasius in June 1918, at the request of the Inspection Section of the German Air Corps (reference 2), incident to the investigation of the flutter on the lower wing of the Albatros D3 biplane which, having only one spar, was of low torsional stiffness. There were no ailerons on the lower wing. The accidental circumstance which prompted the investigation of that particular case at all, was due to the fact that at that time the significance of the aileron as promoter of flutter, was not sufficiently appreciated. Similar investigations were subsequently made by v. Baumhauer and Koning, Bairstow, Frazer and Duncan, Blenk and Liebers, Hesselbach, and were extended to include oscillating ailerons (references 3 to 14).

Indeed, the calculation of the simple elastic-mass oscillations of an airplane wing on the oscillating bench, stipulated a number of simplifying assumptions. Other simplifying assumptions consisted in disregarding the energy-consuming, unsteady system of vortices and the premises of material damping proportional to the rate of deformation. The accuracy of such calculations therefore is, as a rule, quite small. Numerical agreement between calculation and experiment has been obtained only in cases where the assumptions could be made to fit the particular case.

Agreement was more readily obtainable on cantilever than on braced wings. At first it was believed that cantilever monoplanes were particularly susceptible to flutter, but subsequent experience proved otherwise.

One important result was the following rule: The mass axis of the wing shall lie ahead of the elastic axis if feasible; the aileron c.g. shall lie in its hinge axis in order to avoid flutter.

2. Calculation of Vortex Separation

Whereas in the early stages of development, wing flutter was treated as a mechanical problem, the aerodynamical side now received more attention and it was attempted to trace the source of the unsteady lift of the oscillating wing and the correlated separation of vortices, at least for the case of two-dimensional flow.

The problem of the oscillating wing was first attacked by Birnbaum (references 15 and 16). He introduced the important concept of the reduced frequency ω, which is π times the ratio of wing chord to wave lengths. If n is the oscillation frequency (in minutes), t, the wing chord (in meters), and v, the flying speed (in kilometers per hour), the reduced frequency is:

$$\omega = 0.06\,\pi\,\frac{nt}{v} \tag{1}$$

The air loads on the oscillating wing are functions of this nondimensional parameter. Following the example of Prandtl, Birnbaum replaced the wing by a system of bound vortices and postulated that the sum of bound and free vortices must remain constant with time; he obtained an equation which he could solve for small values of the reduced frequency $\omega \leq 0.12$. Beyond this point his development was not convergent.

This solution, however, was unsuitable for the elucidation of the problem under consideration, because wing flutter always occurs at materially higher values of ω. Küssner found the general solution of Birnbaum's equation and extended it to include the case of the co-oscillating aileron (reference 17). Since the creation of harmonic oscillations is always considered as the oscillation criterion, it seemed natural to write the equations from the very first for harmonic oscillations, which offers the added advantage of utilizing the labor-saving method of complex presentation. The oscillation criterion then is the disappearance of the complex denominator determinant, which yields two equations for calculating the oscillation frequency and the critical speed. This obviates the use of the linear differential equation and Routh's discriminant. One particular advantage accruing from the use of the harmonic oscillation is that the material damping can be introduced in a simple and physically correct manner as phase difference of the elastic force. This possibility does not exist with the linear differential equation, where it is even necessary to make a physically incorrect assumption of the damping in order to obtain a linear equation.

On this basis it was then possible to calculate several examples of an oscillating flat plate in order to elucidate systematically the influence of mass distribution, elastic forces, and material damping. It was found that with two degrees of freedom - bending and torsion - the critical speed depends chiefly on:

1. The torsional oscillation frequency of the wing.

2. The backward position of the c.g. of the wing.

3. The material damping.

The result of material damping is that flutter is possible only up to certain maximum ω. In oscillations at higher ω, the energy obtainable from the air stream would become inadequate for compensating the damping losses. This rule holds not only for the two degrees of freedom under discussion - bending and torsion - but is of general validity, as will be shown later.

Theoretically the effect of material damping is so much greater, as the ratio of bending stiffness to torsional stiffness is higher, which is approximately equivalent to the ratio of wing chord to length of overhang.

As the total damping is not accurately determinable, no fairly close agreement can be expected between calculation and observation except for cantilever wings of high aspect ratio.

Similar investigations have been made in England. Glauert calculated the unsteady air loads on an oscillating wing for the two degrees of freedom - bending and torsion (reference 18). He proceeded from H. Wagner's concept of the area of discontinuity; but as his numerical calculations extend only to $\omega = 0.5$, they are insufficient for the mathematical treatment of flutter, with its much higher ω values.

Duncan and Collar extended the calculation to a wing oscillating with increasing amplitude (reference 19). Lately, Theodorsen has calculated the air forces on an oscillating airfoil (reference 19a).

3. Model Experiments

Model experiments are another means of investigating wing flutter, but if such model tests are to afford practical conclusions the models must be constructed dynamically similar. Dynamic similarity is the more difficult to attain as the model scale, i.e., the model, is smaller. Since the model scale depends moreover on the jet diameter of the available wind tunnel, the dynamic similarity was disregarded at first and the simply constructed model wings were mounted in the air stream to a wall representing the plane of symmetry of the wing (references 9, 22, and 23). Such models were sufficient for exploring the effect of c.g. position, damping, and mass unbalance of the aileron. But the values of the reduced frequency obtained in these tests are considerably less than the experimental values cited below.

The British have investigated a great number of actual cases of flutter besides model testing since 1925, and have shown great skill in their choice of assumptions which afforded agreement between calculation and observation (references 7 to 13). Model experiments were frequently used as basis for computing the still unknown damping forces, the linear differential equations forming the starting point, while Routh's discriminant was expressed as determinant, whereby some fields of the determinant remained empty.

The method of calculation given in reference 17 was checked at the D.V.L. by wind-tunnel experiments on model cantilever wings, which were, of course, fairly heavy and as a result, oscillated at a lower reduced frequency $\omega \leq 0.3$ (reference 22). The observed critical speed on five model wings was from 14 to 24 percent higher than the theoretical, which may be attributed to the flow being other than two-dimensional and to the energy absorption of the disregarded trailing vortices.

Subsequently two dynamically similar models of the He 60 type were constructed at 1:5.6 scale with a span of 2.4 meters. These model tests were intended to trace the cause of the accident described elsewhere and to test the efficacy of certain structural changes with a view to preventing flutter. The problem was solved, although a number of unexpected difficulties were encountered in this first attempt at constructing dynamically similar models. The highest reduced frequency obtained in the tests was 0.76, a figure which is fairly close to the probable $\omega = 0.93$ at the time of the accident. Since complete dynamic similarity is not attainable and the model usually has more damping than the full-scale wing, the expected ω value for the model will in any case be less than for the full-size wing.

4. Statistical Investigation

Admittedly, the methods of investigation described so far suffice to explain observed cases of flutter and to prove the underlying causes of such flutter, wherein the actually observed critical speed always constituted a check on the correctness of the assumptions. But these methods did not lend themselves to computing the critical speed on a new type of airplane within a fair degree of accuracy, particularly when applied to braced wings. Any further analytical treatment of flutter was precluded, since it was impossible to compute the purely elastic oscillations of a wing on the stand with a reasonable amount of paper work, unless the construction was fairly simple, such as monospar, cantilever wings. As a result it was attempted to establish a simple dimension rule, suitable for practical use by the designer, to prevent wing flutter due to torsion within the normal speed range.

Since the wing mass and its backward c.g. position are little amenable to influence, aside from the fact that

the material damping should also be considered as predetermined, the only valid means for raising the critical speed is the torsional stiffness. The German design specifications carried a provision for torsional stiffness as far back as 1918 for army airplanes. The angle of twist in the terminal dive was not to exceed 5°; this was reduced to 3.5° in the 1926 design specifications. However, this was primarily with a view to static torsional stability of the wing rather than to wing flutter. With the increasing use of airfoils with fixed c.p., this requirement became useless.

The 1930 specifications contained a rule of thumb for torsional stiffness, based on a few theoretical examples and similarity considerations (reference 17).*

$$D_{(y)} = M_d \frac{dy}{d\varphi} \geq k \rho_o v_k^2 F_{(y)}^2 \quad (\mathrm{kgm}^2) \tag{2}$$

In this formula, k was at first put at $k = 0.12$ to 0.24, but subsequent calculations brought about a change to $k = 0.5$ (1934 design specifications). With this assumption it is already very probable that the true critical speed lies above that given in formula (2). It was therefore permissible to introduce the terminal diving speed v_c in formula (2).

It is worthy of note that this formula, originally merely intended for the degrees of freedom - wing torsion and bending - proved practical also for a number of airfoil-aileron combinations, because the observed maximum values of the reduced frequency for this type of oscillation are of the same order of magnitude as the frequencies stipulated for wing bending and torsion.

Roxbee Cox checked formula (2) against ten actual flutter cases (references 20 and 21). He applied torque M_d at the wing tip A-A, measured the angle of twist φ, and computed therefrom the constant

$$k' = \frac{M_d}{\rho_o v_k^2 F_{(y)}^2} \frac{y_o}{\varphi} \tag{3}$$

Its numerical values are given in table I.

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*For symbols, see section IV, 1.

TABLE I. Torsional Stiffness and Flutter

Type	$F(y)$ m^2	t_m m	V_k km/h	k'	k
Gloster "Gamecock"	6.68	1.60	258	0.135	0.405
Gloster "Gamecock" in pull-out	6.68	1.60	403	.074	.224
Gloster "Gorcock", wood	6.58	1.60	290	.106	.318
Gloster "Gorcock", metal	6.58	1.60	217	.257	.771
Short "Satellite"	7.87	1.68	145	.033	.066
Gloster "Grebe"	6.54	1.60	258	.116	.348
Desoutter Mark II	6.97	1.55	225	.295	.590
Martinsyde F 4	6.08	1.68	323	.098	.294
DeHavilland "Puss Moth"	8.80	1.83	314	.129	.258
Simmond "Spartan"	5.02	1.37	274	.072	.216

$m^2 \times 10.7639$ = sq.ft. km/h × .62137 = mi./hr.

The characterization of the torsional stiffness solely through angle of twist at the tips is a rather summary procedure. Consequently, the k' values scatter considerably. If the increase in angle of twist at the tip of a monoplane wing is twice as great as the mean value over the whole wing and three times as great for a biplane wing, then the k values given in the last column of table I are comparable to the mean value $k = 0.35$ (formula (3)). Only two values lie above the maximum value of 0.5 stipulated in the 1934 design specifications. It seems reasonable to assume that these two cases at least involve flutter with wing flexure and aileron motion. Unfortunately the British report fails to give the modes of oscillation and the flutter frequencies. Index values for the bending stiffnesses were established in a similar manner. However, it serves no useful purpose to analyze these makeshift dimension rules, because section IV contains a method which affords a better estimate of the critical speed.

The stiffness formulas are makeshift substitutes for the calculation of the purely elastic oscillation frequencies of a cellule - a calculation which is often quite difficult. This difficulty may be overcome by subjecting the finished airplane to a static oscillation test. To this end the airplane is elastically mounted, an unbalanced rotor is attached below the fuselage and driven at varying speeds by an electric motor, the mode of oscillation and the frequency being recorded in resonance conditions.

Even when the data of such oscillation tests are available, it is still extremely difficult and tedious to analyze the critical flutter speed for the three degrees of freedom - bending, torsion, and aileron motion - because the calculation still contains important simplifying assumptions, especially that of two-dimensional flow, as a result of which the possibility of error should not be underestimated. It is true, however, that this error is usually on the safe side, as shown by the comparison between calculations and model tests mentioned above, because any damping, neglected in the calculation, will raise the critical speed. In such a calculation, made with the utmost care, for the braced He 9a monoplane a reduced frequency of $\omega = 1.13$ was established, the possible error being estimated at -20 percent. The chief drawback of the operation lies in the physically correct terms for the complex determinant rather than in the evaluation of the determinant.

Presumably no substantially greater mathematical accuracy can be obtained even after the calculation has been improved and refined, because flutter does not always start at the same speed even in the wind-tunnel test. The turbulence of the air stream, the angle of attack of the wing, and accidental small differences in the hinge friction - all have some influence. Past experience has been that flutter often starts in gusty weather, from which it may be concluded that gust shocks have overcome the initially excessive friction forces.

Once flutter has started - in this or some other manner - it frequently continues until the pilot has reduced the speed to two thirds or less of its original value. Possible causes for this are: rupture of the aileron control cables, the consistently smaller proportion of hinge friction to the total damping as the amplitude increases, and lastly, the effect of change in angle of attack.

Another fact should be mentioned in this connection. At very low amplitudes the laws of potential flow do not hold because then the viscosity of the air is no longer negligible. Consequently, the air forces are smaller for very small amplitudes than they should be according to the potential theory, and therefore do not induce flutter. This effect was observed by Birnbaum (reference 15, p. 292). It apparently is a boundary-layer effect. The wing flops around, so to speak, in its boundary layer, without encountering any resistance.

In flight free from oscillations and at uniform speed through still air, a wing could exceed its critical speed by any amount without starting to flutter. It would take a shock of a certain minimum size, e.g., a gust shock, to start flutter which, on the other hand would, of course, then be extremely violent. Notable in this connection is the fact that flutter has often been observed during or directly following a pull-out from a steep dive, particularly in vicious cases. In a normal, mild pull-out from high speeds, only small changes of angle of attack are possible. It is improbable that the quotient $d\,c_a/d\alpha$, on which the air forces depend, changes very materially within such a small range of angle of attack. One may suspect, therefore, that the disturbance of the boundary layer during transition from gliding to pull-out or pull-out to level-off was the trigger effect in these cases.

Summing up these facts deduced from experience and considering in particular the great amount of time required for the calculation, which is not justified by the small degree of accuracy, one comes to the conclusion that the analytical method, while adequate for explaining the fundamental relations, is scarcely suitable for the prediction of the critical speed of a new type of airplane.

Once a physical process is no longer amenable to analytical treatment because it contains variables which cannot be observed and numerically defined, then it must be explained statistically, based on a large number of observations. This statistical method, indicated during the formulation of the stiffness formula (2), can now be applied in a more comprehensive manner to the problem of wing flutter, because within the last few years a number of cases of flutter have been investigated in detail, even though this number is as yet not very large from the point of view of statistical research

The most important parameter introduced in the analytical treatment is the reduced frequency. An attempt was therefore made to determine the reduced frequency in the observed cases of flutter.

One may differentiate between "mild" and "vicious" cases. In mild cases flutter occurs with small amplitudes which are well below the ultimate strength of the wing. The flutter usually stops at a speed slightly below that at which it started, so that the flutter may be stopped very quickly by pulling the stick back. These mild cases, while few in number, can be demonstrated with comparatively little danger and are therefore suitable for flutter investigations in free flight. A test of this kind made on the He 46c, is described elsewhere in the report. The recorded air speed, frequency, and mode of oscillation in flight affords the true value of the reduced frequency and the ratio of the amplitudes for each degree of freedom.

This determination is more difficult in the vicious cases. In these cases flutter is, in a way, actually delayed by the very causes cited above and does not start until the theoretical critical speed has been exceeded; then, however, it begins with such violence as to cause failure of the wings or ailerons. If the airplane is still able to land, it is repaired after the flight and subjected to an oscillation test. The dangerous mode of oscillation is that at which the lowest frequency is accompanied by torsional oscillations of the wing or aileron for the reason that, aside from wing flexure, it requires one of these two degrees of freedom to give increasing amplitudes. However, this does not imply that flutter must occur at the frequency observed in the oscillation test, because the air forces existing during flutter may modify the mode of oscillation and the frequency. In particular, a difference in phase angle is always to be expected between bending and torsion, because it is only under these conditions that the energy for increasing the amplitudes can be taken out of the air stream. Even so, the oscillation test affords a certain basis, which is the more reliable as the resonance condition appearing in the oscillation test is more definitely expressed; i.e., as the damping is smaller. (See section IV, 4.)

If the airplane is destroyed by the accident, another airplane of the same type will be subjected to the oscillation test. The flight speed at the time of the accident can rarely be given very accurately for obvious reasons.

In serious accidents one may have to rely on statements of eyewitnesses on the ground.

The possibility of errors introduced when determining the reduced frequency is therefore great for the vicious cases. It is also necessary to decide whether the calculation of the reduced frequency is to be effected at the speed at which flutter started or at which it stopped. But since the start of flutter is decisive for flight operation, the speed at incipient flutter is customarily preferred. In this manner the observed values discussed in the next section have been obtained. Disregarding the possible errors, they range between $\omega = 0.58$ and $\omega = 1.14$, from which it appears that the reduced frequency in new types of airplanes will not exceed $\omega_h = 1.14$.

Testing an airplane on the oscillation bench and observing the dangerous mode of oscillation with the frequency n, the lowest possible value of the critical speed can be roughly estimated on the basis of the assumed maximum value ω_h of the reduced frequency. If t_m is the mean chord of the outer part of the oscillating wing, the lowest possible value of the critical speed is

$$v_k = 0.06\,\pi\,\frac{n\,t_m}{\omega_h}\ \text{km/h} \tag{4}$$

Obviously such a statistical appraisal is worthless unless the particular type of airplane is not substantially different from all the airplanes which showed flutter in the indicated range of reduced frequencies by having incorporated special features which minimize flutter hazard. When these investigations on flutter were started, the probability of finding such a type of airplane was very small, but in time there will be an ever-increasing number of types on which such preventative measures may be effected with at least the partial success of lower reduced frequency. This being so, the rough statistical estimate may be replaced by an improved method (section IV) which permits the inclusion of proved preventative measures.

For mass-balanced ailerons or wings without ailerons, the lowest possible critical speed is higher, and the reduced frequency consequently lower, than the maximum value given above. Practical data are very scarce on this subject, because in all cases of flutter described hereinafter, the ailerons contributed to the growth of oscillations;

at least, it was impossible to state whether in one case or the other, flutter would also have occurred if the ailerons had been rigidly connected to the wings. The one case of flutter without aileron which had been definitely established, prompted the first investigation of flutter without aileron (section II, 1).

Naturally, as the speed of airplanes increase, the region of flutter without aileron will also be reached more frequently, and any appraisal of the critical speed based on the static oscillation must allow for this possible mode of oscillation also.

III. RESULTS

1. Analysis of Observed Cases

a) Braced DP 9 (references 4 and 24).- The strut is short, so that a long overhang exists. This model developed two cases of vicious flutter in the spring and autumn of 1925, starting during pull-up from a steep glide at about 180 km/h. In one case it led to complete fracture of the wing; in the other, to fracture of the ribs in the overhang and of the aileron control cables. In gusty weather it started a slight flutter at 135 km/h.

After the wing was mounted on a rigid test frame, it showed a flexural oscillation frequency of 548/min., and a torsional oscillation frequency of 494/min. The frequency of the free oscillation may be rated at 520/min. The wing chord was 1.5 m; the aileron chord 0.32 m; and the aileron c.g. was 126 mm behind the hinge line. The reduced frequency is

$$\omega = 0.1885 \frac{520 \times 1.5}{180} = 0.82$$

b) Braced He 8a monoplane.- This airplane crashed in the fall of 1928, due to fracture of the wings during an exhibition flight. From the reports of eyewitnesses, it seems quite safe to conclude that flutter was the cause. The flight speed is estimated at 350 km/h. An airplane of the same type was tested on the oscillation stand. The dangerous mode of oscillation lies probably at 540/min., and has a nodal line running from the rear strut fitting toward the point where the curved tip joins the straight leading edge (fig. 3).

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m × 39.37 = in. mm × .03937 = in. km/h × .62137 = mi./hr.

The wing chord was 3.0 m, the aileron chord 0.3 m, the aileron c.g. was 50 mm behind the hinge line. The reduced frequency is

$$\omega = 0.1885 \frac{540 \times 3.0}{350} = 0.87$$

c) Braced L 78 biplane.- This model, of which quite a number had been built, had often been dived at 350 km/h, when in May 1930, it developed a case of mild flutter while flying at a speed of about 210 km/h. It started with an oscillation of the strut between the lower and upper ailerons at great amplitude, then the wings fluttered so severely that the pilot was unable to hold the stick. As soon as the pilot cut his speed, the oscillations died out. The dangerous mode lies at 860/min. The lower wing oscillates in bending, the nodal line being near the strut fittings.

The aileron connecting strut shows severe lateral deflections which cause the upper-wing ailerons to oscillate in torsion (fig. 4).

The mean chord of the overhang of the lower wing is 1.36 m; the aileron chord from hinge line to trailing edge is 115 mm; the aileron c.g. is 23 mm behind the hinge line. The reduced frequency is

$$\omega = 0.1885 \frac{860 \times 1.36}{210} = 1.05$$

d) Unbraced He 60 biplane.- This model is a rather less conventional design. The lower wing is braced against the floats while the two struts on each side reach only to the front spar. No wire bracing is used between the wings. While in other versions of this type the spars had been made of wood, this particular type (He 60) utilized steel, providing the same strength for the same spar height. The ratio of Young's modulus to ultimate strength for steel being substantially lower than for wood, it assured low natural frequencies of the wing. In addition, the aileron system had an unbalance of 75 cm kg. Apprehensions were therefore voiced from the very beginning that flutter might occur at speeds lower than the prescribed diving speed of 365 km/h.

In the attempt to reach the prescribed diving speed, the airplane crashed in December 1931, as a result of a

torn upper wing. According to the testimony of eyewitnesses, it was a case of dangerous flutter. The speed was estimated at 350 km/h.

The dangerous mode lies at 780/min. The nodal line of the upper wing was in the overhang close to and almost parallel with the leading edge in the inner bay; at approximate wing center it runs parallel to the wing axis (fig. 5). The wing chord is 2.2 m; the aileron chord 0.4 m. The reduced frequency is

$$\omega = 0.1885 \frac{780 \times 2.2}{350} = 0.93$$

e) Braced He 46c biplane.- The He 46c is a braced biplane with a small lower wing developed from a high-wing monoplane. After a long period of service, it finally revealed a mild case of flutter at 260 km/h which, however, disappeared immediately as soon as the speed was reduced. It was therefore decided to obtain some oscillation photographs in flight with this airplane, taking, of course, proper precautions. The records showed the flexural oscillation of the lower wing, coupled with turning of the unbalanced aileron system as the cause of the increase; the aileron c.g. was 52 mm behind the hinge line. The aileron chord of the lower wing is 335 mm, and that of the upper wing, 500 mm.; the wing chord is 1.4 m on the lower, and 2.0 m on the upper wing.

The oscillation test disclosed between 520 and 755/min., a series of antisymmetrical oscillation modes of the whole cellule about the longitudinal, vertical, and transverse, axes, accompanied in part by severe aileron motions (aileron control by means of torque tubes). The remarkable feature is that these modes do not induce flutter. This may be attributable to a slight mass-coupling with the aileron oscillation as a result of the small amplitude of the aileron hinge line and the shifting of the location of the nodal line closer to the trailing edge. Possibly the damping of the antisymmetrical oscillations of the whole cellule is greater. The first symmetrical natural bending frequencies of the wings lie between 815 and 895/min. The nodal line of the lower wing lies at 50 percent or more of wing chord forward of the leading edge (fig. 6). The resonance conditions are not pronounced.

But the flutter frequencies recorded with the optigraph, lie in this range. Flutter started with a frequency

of 830-860/min., and dropped to 810/min. as speed and amplitude increased. The increase of flutter amplitude was probably favored by the method of mounting of the aileron connecting strut, which sloped about 25° toward the plane of the struts. Thus bending of the upper wing caused motion of the lower aileron.

The flight records ranged between altitudes of from 4,000 to 600 m, so as to establish the effect of air density. Flutter started at 260 to 275 km/h. The reduced frequency of the lower wing was found at

$$\omega = 0.87 \left(\frac{\rho}{\rho_0}\right)^{0.2} \tag{5}$$

The air-density effect ρ is therefore relatively small.

f) Cantilever biplane KL1A "Schwalbe".- This type, built since 1927, had been in service quite awhile when, in the spring of 1932, several of them developed flutter below the level top speed which could not be called mild because it resulted in fracture of the ailerons. An airplane of this type was therefore subjected to an oscillation test.

The dangerous mode lies at 675/min. It is the symmetrical fundamental bending mode of both wings (fig. 7). The nodal line lies far forward of the leading edge of the wing. The ailerons are in phase opposition; their chord is 240 mm, their c.g. is 103 mm behind the hinge line. This results in a strong mass coupling between wing bending and aileron motion. The mean chord of the extremely oscillating wing tips is 1.3 m. The experiments were temporarily interrupted to permit the airplane to take part in an air circus. During this air circus in July 1932, it was stunted at speeds up to 200 km/h without developing flutter; but as soon as the pilot started to land, it suddenly began to flutter very severely at 145 km/h, which ended in the breaking of the ailerons and damage to the plywood covering. The flutter continued up to 100 km/h speed. This case shows very clearly the unpredictability of flutter.

The reduced frequency at start of flutter is

$$\omega = 0.1885 \frac{675 \times 1.3}{145} = 1.14$$

while at the end it reaches the high value of $\omega_e = 1.65$.

g) Cantilever Do 10 monoplane.- This is an all-metal high-wing design. The three-spar wing is braced by short struts. The cantilever length is 72 percent of the semi-span. The leading edge of the wing is approximately a semiellipse, the trailing edge is straight. During a flight on September 9, 1932, it developed such a severe case of vicious flutter at about 450 km/h, that both wing tips broke off to the length of one chord. While leveling off from a dive at 2,500 m to 1,500 m, the pilot noted oscillations on the ailerons as far as the wing tips which, within about 3 seconds, resulted in broken wing tips and ailerons. The pilot was able to land safely and the airplane was subsequently repaired and tested on the oscillating stand.

The wing reveals a series of oscillation modes in the 500 to 1,250/min. frequency zone, whereby the nodal line gradually shifts from the front toward the rear spar. Although the aileron, with a chord of 355 mm, has its c.g. 41 mm behind the hinge line, these modes do not induce flutter because the aileron control is very rigid (push rods), so that the aileron motion does not build up to large amplitudes at these frequencies. The dangerous mode lies at 1,400 to 1,500/min. At 1,400/min. the nodal line is exactly coincident with the principal line of failure of the wing tips, which slopes 30° outward and backward from the leading edge in the direction of flight. At 1,500/min. the outer nodal line, in form of a quarter circle about the wing tip, is in part coincident with the line of the secondary failure. The inner nodal line runs from the point of intersection of the trailing edge and plane of struts at an angle of 30° outward, and passes directly through a region in which the internal bracing was broken (fig. 8).

Without the lines of failure as clues, it would indeed be difficult in this case to ascertain the dangerous mode from the static oscillation test alone. The next section contains various factors which should help to facilitate this decision.

Another source of error lies in the estimate of the mean chord of that part of the wing which oscillates most severely, because of its pronounced taper in plan. Appraising the mean chord of the severed wing tip at 1.6 m and the flutter frequency at 1,450/min., the reduced frequency is

$$\omega = 0.1885 \frac{1450 \times 1.6}{450} = 0.97$$

h) Braced L 102 monoplane.- This airplane is the braced high-wing type. The wing structure is of duralumin; has two spars, and is covered with fabric. The ailerons are split. The aileron chord from hinge line to trailing edge is 300 mm; the aileron c.g. is 35 mm behind the hinge line. With this c.g. position, vicious flutter started at 290 km/h, and persisted to 120 km/h. The rivets of the torsion structure were sheared off from the outboard aileron hinges of both wings. This explains perhaps the rather extended range of speeds during which flutter persists. With perfect mass balance, a speed of 340 km/h had previously been obtained without flutter.

The airplane was then tested on the oscillating bench. Asymmetrical and symmetrical fundamental bending modes occurred at 500/min. and 580/min. The natural frequency of the ailerons lies at 730/min. The dangerous mode is the torsional oscillation of the wing at 835/min. The nodal line runs over the entire span between front and rear spar. The aileron amplitudes are high (fig. 9).

For a 1.56 m wing chord, the reduced frequency at incipient flutter amounts to

$$\omega = 0.1885 \frac{835 \times 1.56}{290} = 0.85$$

as against the abnormally high $\omega_e = 2.05$ at its termination; the failure of the torsion structure itself may perhaps have lowered the flutter frequency.

i) AC 12 E cantilever monoplane.- This is a cantilever high-wing design of wood with tapered wings. While competing in the 1932 International Challenge Contest, it developed a mild case of flutter at 220 km/h, but only in rough, gusty weather. In fair weather it reached a speed of 270 km/h without flutter.

The mean chord of the oscillating wing tip is 1.4 m; the aileron chord is 300 mm; the aileron c.g. is 112 mm behind the hinge line.

Tested on the oscillation bench, this airplane revealed bending oscillations with very indefinite resonance conditions at 585 to 830/min. frequencies. The dangerous mode

apparently lies between 800 and 830/min., because then the aileron motion caused by mass coupling and elasticity of the control cables, has a phase difference of about 90° against the bending oscillation; that is, is in resonance with the bending oscillation (fig. 10). The reduced frequency is

$$\omega = 0.1885 \frac{815 \times 1.4}{220} = 0.98.$$

j) Do 12 cantilever monoplane "Libelle".- This is a high-wing all-metal amphibian. The two-spar wing is tapered in plan form.

The airplane showed vicious flutter on September 27, 1933, at 180 km/h. The oscillations started when the pilot opened the throttle after leveling off from a glide. The oscillations were so severe that one aileron jumped out of its hinges and both wings were badly damaged. The wing flutter was preceded by tail buffeting, initiated apparently when opening the throttle, and which in turn started the wing flutter. The pilot made a safe landing, however, after which the airplane was repaired and tested on the oscillation bench.

The dangerous oscillation mode of the wing lies at 580/min., which at the same time is the principal resonance mode of the horizontal tail surfaces. It is an antisymmetrical bending oscillation; the nodal line starts at the inner aileron and runs outwardly at an angle of 15° in the direction of flight (fig. 11). The aileron oscillates in torsion. The mean chord of the outer oscillating part of the wing was estimated at 1.3 m; the aileron chord is 400 mm. The aileron is not mass-balanced; its c.g. position was estimated at 100 mm behind the hinge line. The aileron control cables are not very rigid. The reduced frequency is

$$\omega = 0.1885 \frac{580 \times 1.3}{180} = 0.79.$$

k) M 28 monoplane.- This is a cantilever low-wing design, of duralumin with wings tapering in plan only.

After extensive testing, the airplane developed a mild case of flutter at 220 km/h, which was started by the (mass-) unbalanced ailerons. It stopped when the speed was reduced to 180 km/h. The pilot had the impression

that the flutter started each time after a bump, even when very slight. The engine r.p.m. was 1,750 at the beginning. Later the airplane was dived to 250 km/h at 1,950 r.p.m. without developing flutter.

The airplane was then subjected to an oscillation test. The wings have a symmetrical fundamental flexural oscillation at 480/min. and antisymmetrical bending oscillations at 850 and 770/min. The resonance points are very clearly expressed. The dangerous mode seems to lie at 770/min., because this is the frequency at which out-of-phase oscillation of the ailerons is first noticed, which likens it much to the dangerous mode of the Do 12. The nodal line runs outward from a point near the inboard end of the aileron at an angle of 10° in the direction of flight (fig. 12).

The mean chord of the outer oscillating part of the wing is taken at 1.05 m, the aileron chord at 350 mm; the c.g. is 140 mm behind the hinge line. The reduced frequency is

$$\omega = 0.1885 \frac{770 \times 1.05}{220} = 0.69.$$

It is planned to make flutter measurements in flight on this airplane in order to determine the flutter frequencies exactly.

l) Biplane S 24 "Kiebitz".- This is a biplane of wood construction, braced in one plane. In the spring of 1932, the airplane went into a long, unexpected dive with a burning engine and started to flutter, finally breaking the cellule. The calculated terminal velocity is 280 km/h.

Another airplane of the same type was subjected to an oscillation test. At 490 and 615/min. the whole cellule started to oscillate; at 825 and 1000/min., the overhang went into flexural oscillations. The dangerous mode lies at 1215, because this was the frequency at which the ailerons first revealed phase opposition. The nodal line runs from the intersection of the strut plane and trailing edge to the first third of the edge strip (fig. 13).

The mean wing chord is 1.18 m, the aileron chord 240 mm, the c.g. of the aileron system is 21 mm behind the hinge line. The reduced frequency is

$$\omega = 0.1885 \frac{1215 \times 1.18}{280} = 0.97.$$

n) Biplane Ar 66 C.- This is a braced biplane with a smaller lower wing. In a dive at 340 km/h, the airplane started to flutter, which led to the beginning of a fracture of the lower front spar as well as of the plywood covering on the lower side of the wing. Since the stick did not oscillate, the mode was symmetrical. The chord of the lower wing is 1.65 m; the c.g. of the aileron lies 45 mm behind the hinge line.

The airplane was subjected to an oscillation test by the manufacturer. The bending oscillation of the lower wing at 790/min. was considered as the dangerous mode. At this mode, the inboard part of the wing pivots roughly about the front spar, while the overhang bends.

The reduced frequency is

$$\omega = 0.1885 \frac{790 \times 1.65}{340} = 0.72$$

After finishing this report, flutter was again observed after the ailerons had been completely mass-balanced and were perfectly quiet in the oscillation test. Following some minor changes in the shape of the aileron, it developed vicious flutter at 420 km/h, leading to complete destruction of the cellule. This gives

$$\omega = 0.1885 \frac{790 \times 1.65}{420} = 0.58$$

This might have been a case of flutter in combined bending and torsion, although not without some probable aerodynamic coupling effect of the aileron motion.

TABLE II. Observed Cases

No.	Type	t_m m	t_n m	s_R m	n_o $\frac{1}{min}$	n_q $\frac{1}{min}$	Δn $\frac{1}{min}$	v_k $\frac{km}{h}$	ω
1	DP 9	1.5	0.32	0.126	520	-	-	180	0.82
2	He 8	3.0	.42	.055	540	290	-	350	.87
3	L 78	1.36	.20	.023	860	-	30	210	1.05
4	He 60	2.2	.41	.150	780	910	40	350	.93
5	He 46 c	1.4	.27	.052	845	550	40	268	.87
6	KL 1 A	1.3	.24	.103	675	675	45	145	1.14
7	Do 10	1.6	.30	.041	1450	-	60	450	.97
8	L 102	1.56	.30	.035	835	730	35	290	.85
9	AC 12 E	1.4	.30	.112	815	700	120	220	.98
10	Do 12	1.3	.40	.100	580	460	50	180	.79
11	M 28	1.05	.35	.140	770	1410	60	220	.69
12	S 24	1.18	.24	.021	1215	-	70	280	.97
13	Ar 66 C	1.65	-	.051	790	-	-	340	.72

TABLE IIa. Observed Cases

No.	Type	Mode of oscil-lation	Rigidity of controls	Aspect	Wing struc-ture
1	DF 9	symmet-rical	small	vicious	wood
2	He 8	"	"	"	"
3	L 78	antisym-metrical	great	mild	"
4	He 60	-	-	vicious	metal
5	He 46 c	symmet-rical	small	mild	wood
6	KL 1 A	"	"	vicious	"
7	Do 10	antisym-metrical	great	"	metal
8	L 102	-	small	"	"
9	AC 12 E	symmet-rical	"	mild	wood
10	Do 12	antisym-metrical	"	vicious	metal
11	M 28	"	great	mild	"
12	S 24	-	small	vicious	wood
13	Ar 66 C	symmet-rical	"	"	"

2. Conclusions

The numerical data of these 13 test cases are appended in tables II and IIa. The mean value for incipient flutter is, according to table II:

$$\omega_m = 0.90 \pm 0.12.$$

The majority of cases are vicious; only 30 percent are mild (table IIa). The structural material does not seem to have any effect.

When the stiffness of the ailerons and their controls is great, flutter occurs in an antisymmetrical mode because only then can the ailerons oscillate freely and transmit energy.

If the stiffness of the aileron control is small, flutter may also occur in a symmetrical mode, but then only at a frequency high enough above the natural aileron frequency to permit motion in phase opposition. Oscillations of the whole cellule of a biplane about the vertical and the transverse axis may not necessarily lead to flutter, even when combined with additive wing torsion. The dangerous mode, however, is frequently the first natural mode of the wing independent of the cellule.

The reason for this behavior lies in the damping of the oscillations through the precessional moment of the rotating propeller. Heretofore, all airplane oscillation tests have been, almost without exception, static tests, i.e., with the engine standing still. Under these conditions, a number of cellule oscillation modes may develop, during which the fuselage oscillates slightly in torsion. In flight with full r.p.m., the precessional moment of the propeller damps such oscillations very effectively, so that flutter is very rare. The KL 1 A revealed such cellule oscillations on the stand at the dangerous frequency. The sudden entry of flutter when starting to land, was probably attributable to the diminished damping of the propeller as a result of the smaller r.p.m. One two-engine airplane tested on the oscillation bench manifested a marked difference in oscillation modes, depending on whether the engines were running or not. On the other hand, purely symmetrical wing oscillations, during which the thrust axis is merely translatory, do not prevent flutter as proved by the Ar 66 C airplane.

Table II also gives the natural frequencies of the aileron oscillations n_q for various airplanes, together with the width of the resonance curve Δn defining the damping of the flutter oscillation. This width is measured at 71 percent of the maximum amplitude.

IV. DETERMINATION OF LIMITING CONDITIONS FOR FLUTTER

Section II shows that further analytical treatment of concrete cases of flutter is impractical because of the many secondary circumstances which are not amenable to numerical treatment. Even so, it is useful from the point of view of flutter prevention, to establish analytically the limiting conditions under which flutter is possible.

Flutter is obviously possible only when the oscillating wing is able to take energy out of the air stream in order to equalize the ever-present damping losses. For a general survey, the calculation may be restricted to the limiting conditions of energy absorption in two-dimensional flow.

The reduction of lift due to tip vortices of the oscillating wing tip, will have the effect of increasing the damping and narrowing the range in which flutter is possible. This additional damping effect can be estimated, although it is neglected in the following derivation.

1. Notation

$A\,l$	m	stroke (bending) amplitude of (aerodynamic) neutral axis (quarter-chord point).
B		torsional amplitude of wing.
C		deflection amplitude of aileron.
b	m	effective span of oscillating wing tip.
b_q	m	aileron span.
d_F		damping factor of wing.
d_R		damping factor of aileron.
$D_{(y)}$	kg m²	torsional stiffness of wing at abscissa y.
e_F	m	distance from nodal line of wing to center line of wing.
E	m kg	energy of oscillating airplane.
E_F	m kg	energy of wing.

E_R	m kg	energy of aileron.
F	m^2	wing area.
g		damping phase angle of wing oscillation.
h		damping phase angle of aileron oscillation.
k	1) 2) m	stiffness constant. radius of gyration of wing section, referred to nodal line.
k_R	m	radius of gyration of aileron, referred to aileron nodal line.
l	m	semichord of wing.
$l_1 \ldots l_9$		energy coefficients.
L_m	m kg/s	mean aerodynamic energy.
L_{dm}	m kg/s	mean damping power.
M_d	m kg	torsional moment.
m	kg s^2/m	mass.
m_F	kg s^2/m	mass of oscillating wing tip.
m_R	kg s^2/m	aileron mass.
n	1/min.	oscillation frequency.
n_o	1/min.	oscillation frequency at resonance.
Δn	1/min.	width of resonance curve in oscillation test.
p	m	absolute oscillation amplitude.
s		position of wing c.g. behind neutral point (quarter-chord point).
s_R		position of aileron c.g. behind hinge line.
t	m	wing chord.
t_R	m	aileron chord.

v	m/s; km/h	flight speed.
y	m	coordinate along span.
ϑ		index of amplitude ratio for wing.
ϑ_R		index of amplitude ratio for aileron.
$\mu = \frac{G_F}{F t_m}$	kg/m³	specific weight of wing.
ν	1/s	frequency of wing oscillation in radians.
ρ	kg s²/m⁴	air density.
ρ_o		air density at sea level.
$\tau = \frac{t_R}{t}$		aileron chord ratio.
φ		1) angle of twist. 2) phase angle between bending and torsional oscillation.
ψ		phase angle between bending and torsional oscillation of aileron.
χ		phase angle between torsional oscillation of wing and torsional oscillation of aileron.
$\Phi_1 \ldots \Phi_{12}$		functions of aileron chord ratio.
ω		reduced frequency.
ω_h		maximum value of reduced frequency.

2. Aerodynamic Energy of Oscillation

With $A\,l$ denoting the stroke (bending) amplitude, B and C, the amplitudes of angular motion of wing and aileron; and index ' signifying the real part, index " the imaginary part of these amplitudes; a bar – their absolute magnitude, the time average of the aerodynamic energy of oscillation is,* according to (16) and (17):

*The theoretical development of the ensuing formulas is to be published in a separate report.

$$
\begin{aligned}
L_m = \frac{1}{2}\pi\rho v v^2 l^3 b\,[&\bar{A}^2 l_1 + \bar{B}^2 l_2 + \bar{C}^2 l_3 \\
&+ (A'B' + A''B'')\, l_4 + (A'B'' - A''B')\, l_5 \\
&+ (A'C' + A''C'')\, l_6 + (A'C'' - A''C')\, l_7 \\
&+ (B'C' + B''C'')\, l_8 + (B'C'' - B''C')\, l_9
\end{aligned}
\tag{6}
$$

where the energy coefficients:

$$
\left.
\begin{aligned}
l_1 &= 1 + T' \\
l_2 &= 1 \\
l_3 &= \frac{1}{2\pi^2}\left[\Phi_1\Phi_8\frac{T''}{\omega} + \frac{\Phi_2\Phi_8}{2}(1 + T') + \Phi_{11}\right] \\
l_4 &= \frac{T''}{\omega} + 2 + T' \\
l_5 &= \frac{1 + T'}{\omega} - T'' \\
l_6 &= \frac{1}{\pi}\left[\Phi_1\frac{T''}{\omega} + \frac{\Phi_2 + \Phi_8}{2}(1 + T') + \Phi_3\right] \\
l_7 &= \frac{1}{\pi}\left[\Phi_1\frac{1 + T'}{\omega} - \frac{\Phi_2 + \Phi_8}{2}T''\right] \\
l_8 &= \frac{1}{2\pi}\left[\Phi_6 + \Phi_9 + \Phi_8\left(\frac{T''}{\omega} + 1 + T'\right)\right] \\
l_9 &= \frac{1}{\pi}\left[\Phi_5 + \frac{\Phi_8}{2}\left(T'' - \frac{1 + T'}{\omega}\right)\right]
\end{aligned}
\right\}
\tag{7}
$$

The functions $T'(\omega)$, $T''(\omega)$ based on Hankel's cylinder functions, are given in table III. The functions $\Phi_n(\tau)$ derived from trigonometrical functions, are given in table IV. From these the energy coefficients l_n have been computed for aileron chord ratios of $\tau = 0.15$, 0.20, 0.25 with a 20-inch slide rule (table V). The energy is negative when taken out of the air stream.

TABLE III. Functions $T'(\omega)$, $T''(\omega)$

ω	0	0.2	0.4	0.6	0.8	1.0	
T'	1	0.455	0.250	0.158	0.108	0.079	
T''	0	-.377	-.330	-.276	-.233	-.201	
ω		1.2	1.4	1.6	1.8	2.0	∞
T'		0.060	0.047	0.038	0.031	0.026	0
T''		-.175	-.156	-.140	-.126	-.115	0

TABLE IV. Function $\Phi_n(\tau)$

τ	0.10	0.15	0.20	0.25	0.30
Φ_1	1.244	1.510	1.727	1.913	2.076
Φ_2	.334	.610	.935	1.299	1.698
Φ_3	.164	.296	.447	.614	.793
Φ_4	.0264	.0719	.1459	.2518	.3924
Φ_5	1.080	1.214	1.280	1.299	1.283
Φ_6	.615	1.077	1.577	2.094	2.612
Φ_7	.0506	.1347	.2672	.4507	.6860
Φ_8	.0079	.0192	.0400	.0707	.1129
Φ_9	.0459	.1246	.2519	.4330	.6718
Φ_{11}	.0546	.1803	.4180	.7978	1.345
Φ_{12}	.0056	.0281	.0876	.2128	.4371

TABLE V. Energy Coefficients

ω		0.4	0.6	0.8	1.0	1.2	1.6	2.0
	l_1	1.250	1.158	1.108	1.079	1.060	1.038	1.026
	l_2	1	1	1	1	1	1	1
	l_4	1.425	1.698	1.817	1.878	1.914	1.951	1.968
	l_5	3.455	2.205	1.618	1.279	1.059	.788	.628
0.15	l_3	0.00829	0.00880	0.00904	0.00916	0.00923	0.00931	0.00935
	l_6	-.1773	-.0106	.0652	.1059	.1300	.1561	.1696
	l_7	1.534	.954	.688	.538	.442	.326	.258
	l_8	.1926	.1934	.1938	.1939	.1941	.1942	.1943
	l_9	.3757	.3796	.3815	.3825	.3832	.3840	.3846
0.20	l_3	0.0195	0.0207	0.0212	0.0215	0.0217	0.0219	0.0219
	l_6	-.1175	.0695	.1542	.1993	.2265	.2553	.2697
	l_7	1.769	1.104	.798	.624	.513	.379	.300
	l_8	.2940	.2956	.2964	.2967	.2970	.2973	.2976
	l_9	.3854	.3934	.3970	.3993	.4006	.4024	.4034
0.25	l_3	0.0377	0.0400	0.0410	0.0416	0.0419	0.0422	0.0424
	l_6	-.0343	.1682	.2596	.3078	.3374	.3686	.3841
	l_7	1.975	1.235	.895	.701	.576	.426	.335
	l_8	.407	.410	.412	.412	.413	.413	.414
	l_9	.3746	.3886	.3952	.3991	.4016	.4045	.406

3. Damping

As the damping energy was not measured directly in the oscillation tests, the energy absorbed by material damping and external friction must be estimated from the phase difference between elastic force and deformation.

The previous assumption of a constant phase difference gives an elliptic hysteresis loop independent of frequency, which agrees closely with observed facts. The product of this phase difference and π equals the usual logarithmic decrement. A detailed exposition of these relations will be found in B. v. Schlippe's article (reference 25).

The damping energy is obtained from the relation

$$2\pi\ g = \frac{\text{energy of damping for a complete cycle}}{\text{mean energy of oscillating system}}.$$

The phase angles g and h of the wing and of the aileron oscillations may be determined from the width of the resonance curves of the oscillation test. If the width of the resonance curve Δn is measured at 71 percent of the resonance amplitude (fig. 14), the phase angle is

$$g = \frac{\Delta n}{n_0} \tag{8}$$

At higher amplitudes the phase angle increases, the increase being, as a rule, greater for wooden wings than for metal wings.

An exception to this is the damping due to friction in the aileron hinge bearings. With constant frictional moment the damping is proportional to the (angular) amplitude C, while the energy of the oscillating aileron rises proportionally to C^2.

By the same argument, the phase angle h of the aileron is inversely proportional to the amplitude. Thus, at very low amplitudes, the phase angle of the damping can be so great as to make flutter impossible. Flutter cannot set in unless some outside cause imparts a momentum of a certain minimum rise to the aileron. At high amplitudes only the residual damping due to material damping of the control cables remains. With careful installation of the ailerons and cables this effect is less pronounced.

The phase angles of wing bending and wing torsion damping may be considered as being about equal. Table VI gives the values of phase angle g obtained from oscillation tests with constant excitation and with impulse excitation (deflection tests). They are multiples of the values measured on smooth test bars, because in built-up members develop additional losses at the joints. On the average $g_m = 0.061$. The phase angle of the aileron damping is according to (6) $h = 0.20$, and according to a resonance curve, $h = 0.08$.

The mean energy content of the oscillating airplane at the dangerous mode may be determined by forming the integral

$$E = \frac{\nu^2}{2} \int p^2 \, dm \tag{9}$$

over the whole airplane, whereby p denotes the absolute amplitude of the mass element dm measured in the oscillation test. This calculation is so tedious, however, that in most cases an estimate is preferable.

Visualizing the outer end of the wing as a rigid plate oscillating in torsion at amplitude B about its nodal line while the aileron oscillates in torsion about the hinge line at amplitude C, the energy content of wing and aileron is:

$$\left.\begin{aligned} E_F &= \frac{\nu^2}{2} \left[m\, k_F^2 + \pi\, \rho\, l^2\, b \left(\frac{l^2}{8} + e_F^2\right)\right] B^2 \\ E_R &= \frac{\nu^2}{2} \left[m_R\, k_R^2 + \rho\, l^4 \left(b_q\right) \frac{\Phi_{12}}{4\,\pi}\right] C^2 \end{aligned}\right\} \tag{10}$$

If the wing oscillates in pure bending, then $k = e = \infty$, $B\,k = B\,e = A$.

Very often the oscillation amplitude and the wing chord are very variable near the tip. As the energy transfer involves the square of the amplitude, it is advisable in this case to form the mean values:

$$\left.\begin{aligned} (\bar{A}\, t)_m^2 &= \frac{\int \bar{A}^2\, t^3 \cdot dy}{\int t\, dy} \\ t_m &= \frac{\int \bar{A}^2\, t^3\, dy}{\int \bar{A}^2\, t^2\, dy} \end{aligned}\right\} \tag{11}$$

Exactly corresponding averages must be formed for the amplitudes B and C. The mean damping energy is

$$L_{dm} = \nu(g\ E_F + h\ E_R) = \frac{1}{2}\ \pi\ \rho\ \nu^2\ v\ l^3\ b\ \omega(d_F\ B^2 + d_R\ C^2) \qquad (12)$$

For the sake of brevity, the following nondimensional damping factors have been introduced into equation (12):

$$\left.\begin{aligned} d_F &= g\left[\frac{m}{\pi\ \rho}\frac{k^2}{l^4\ b} + \frac{1}{8} + \frac{e_F{}^2}{l^2}\right] \\ d_R &= h\left[\frac{m_R}{\pi\ \rho}\frac{k_R{}^2}{l^4\ b} + \frac{\Phi_{12}}{4\ \pi^2}\frac{b_q}{b}\right] \end{aligned}\right\} \qquad (13)$$

Table VI gives damping factors for several airplanes. The factor $d_F{}'$ applies to pure bending oscillation. The torsional oscillation about the neutral point is estimated at $0.4\ d_F{}'$ because the mass distribution along the chord is not very unlike in the various types. The aerodynamically effective wing span b may be estimated from the dangerous oscillation mode. Only a portion of the outer oscillating part of the wing between wing tip and nodal line or plane of struts can be considered as aerodynamically effective, because the trailing vortices lower the efficiency of the wing tips as compared with two-dimensional flow.

It should be emphasized that this estimate of the damping is quite rough and therefore merely affords the approximate magnitude of the damping energy. For this reason, the direct determination of the energy of damping from oscillation tests is very much desired.

4. Ratio of Amplitudes

In order to be able to compare the amplitude ratios of the forced oscillations with those of the oscillation test, the elastic oscillation of a flat plate covered with a mass was investigated. The plate is assumed to be pivoted about the axis P and elastically restrained against torsion, so that it oscillates with a natural torsional frequency n_o (fig. 15).

TABLE VI. Wing and Aileron Damping

No.	Type	g	$\mu = \frac{G_F}{F t_m}$ kg/m³	$d_F' \sim (2\mu+1)g$	$\tau = \frac{t_R}{t_m}$	d_R
1	DP 9	0.050	4.8	0.53	0.21	-
2	He 8	.030	2.4	.17	.14	-
3	L 78	.034	6.9	.50	.15	-
4	He 60	.051	3.6	.42	.19	0.0029
5	He 46	.047	7.2	.72	.19	.0039
6	KL 1 A	.067	5.8	.84	.19	.0039
7	Do 10	.041	7.2	.63	.19	.0017
8	L 102	.042	5.0	.46	.19	--
9	AC 12 E	.147	5.4	1.67	.21	.0046
10	Do 12	.086	5.1	.96	.31	--
11	M 28	.078	5.4	.92	.33	.0134
12	S 24	.058	4.0	.52	.20	.0062
13	Ar 66 C					

Assume that axis P executes forced vertical oscillations at amplitude $A\,l$ and frequency n. The amplitude ratio of this forced oscillation is:

$$\left.\begin{aligned} \frac{A}{B} &= \vartheta\left(\frac{n_0^2}{n^2}\, e^{ig} - 1\right) \\ \vartheta &= \frac{m\,k^2 + \frac{3}{8}\pi\rho\, l^4\, b}{m\,s\,l + \frac{1}{2}\pi\rho\, l^4\, b} \end{aligned}\right\} \qquad (14)$$

The comparative factor ϑ depends only on the mass distribution and on the air density. The second terms represent the effect of the co-oscillating air masses.

The three following limiting cases must be considered:

1. $n_0 = 0$, free pivoting about the axis P.
   $\frac{A}{B} = \vartheta$, phase angle $\varphi = \pi$.

2. $n_0 = n$, forced frequency = natural torsion frequency.
   $\frac{A}{B} = \vartheta\, g$, phase angle $\varphi = \frac{\pi}{2}$.

3. n_0 very high, great torsional stiffness.
   $\frac{A}{B} \sim \vartheta \frac{n_0^2}{n^2} \sqrt{1 + g^2}$, phase angle $\varphi \sim g$.

Similar relations hold for a plate with attached aileron, which is pivoted with elastic restraint about the hinge axis. With $A_R\, l$ (fig. 16) as the forced amplitude of the aileron hinge, the amplitude ratio of the forced oscillation is:

$$\left.\begin{aligned} \frac{A_R}{C} &= \vartheta_R \left(\frac{n_0^2}{n^2}\, e^{ih} - 1\right) \\ \vartheta_R &= \frac{m_B\, k_R^2 + \rho\, l^4\, b_q\, \frac{\Phi_{12}}{4\pi}}{m_H\, s_R\, l + \rho\, l^4\, b_q \left(\frac{A}{A_R}\, \frac{\Phi_4}{2} + \frac{B}{A_R}\, \frac{\Phi_7}{4}\right)} \end{aligned}\right\} \qquad (15)$$

The values for the amplitude ratio in the three limiting cases correspond to those given above. The cases between the first and second limiting cases are of particular significance for the forced oscillation, because here the phase angle lies in the second quadrant; that is, it approaches the optimum phase angle which almost always lies in the third quadrant. This is borne out by experience. Flutter usually occurs either antisymmetrically - that is, with freely oscillating ailerons, so to say - or symmetrically, at a frequency which lies above the natural oscillation frequency of the ailerons.

The values of the amplitude ratio obtained from the oscillation test are, on the whole, smaller than according to limiting case 1, because the elasticity of the control system shifts the oscillation mode toward limiting case 2.

It should be noted that finite values of ϑ_R are still obtainable even if the aileron c.g. lies in the hinge line, because the co-oscillating air masses exert a mass-coupling moment. This explains the fact that an aileron, whose static c.g. lies only very little behind the hinge line, can still excite the oscillations to flutter (cf. section III,1,g,h,c).

For the customary method of estimating the critical velocities, as well as for the new method using oscillation tests and recommended hereinafter, it is important to know whether flutter occurs in approximately the same mode as the oscillation of equal frequency on the stand.

A very simple way is to visualize the wing tip as a mass-elastic system. Posing the torsion of the wing tip about the nodal line at

$$\beta = B\, e^{i\nu t},$$

whereby amplitude B is real, the inertia force moment is

$$\theta \frac{d^2 \beta}{dt^2} = - \theta\, B\, \nu^2\, e^{i\nu t},$$

the elastic moment

$$f_o\, e^{ig}\, \beta = f_o\, B\, e^{i\nu t + ig},$$

and the excitational moment

$$= M\, e^{i\nu t}.$$

In steady oscillation, the sum of these three moments must always disappear; that is,

$$- m\, B\, \nu^2 + f_o\, B\, e^{ig} + M = 0$$

Separating this equation into real and imaginary parts, affords

$$- \theta\, B\, \nu^2 + f_o\, B \cos g + M' = 0$$

$$i\, f_o\, B \sin g + i\, M'' = 0$$

Now, the aerodynamic exciattion could occur in such a manner that the "blind component" $M' = 0$. Then the

oscillation frequency with aerodynamic excitation would not differ at all from the natural frequency and from the frequency $M' = 0$ of the oscillation test. In actual flutter, it usually is $M' < 0$. The phase angle most favorable for the excitation lies generally in the third quadrant, as will be shown below. The oscillation frequency is, as a result, lower. On the other hand, the frequency increases with the flying height because the air mass co-oscillating with the wing, decreases with decreasing density.

The estimate will be fairly correct when assuming the phase angle of the exciting moment at $\varphi \sim 225^\circ$; that is, posing

$$M' = M'' = - f_0 \, A \sin g$$

The average for 12 wings is $\sin g \sim 0.06$. The change of frequency in this case is -3 percent; that is, still within the resonance width Δn. Model experiments and flight measurements in several instances proved that these assumptions agree with the facts in approximate magnitude, particularly for coupled wing and aileron oscillations.

Greater blind components and consequently greater discrepancies in frequency are to be expected when the damping is great and the phase angle of excitation is close to 180° or 360°. This is the case, e.g., on model wings without ailerons which are not dynamically similar; have high frictional damping and oscillate with a low reduced frequency. The limiting case is the "oscillation" with 0 frequency - the aperiodic twisting off of the wing due to static torsional instability. In this case,

$$M' = - f_0 \, B \cos g.$$

However, the wing flutter observed up to date, has been so far from this limiting case that the assumption of identical modes for actual flutter and for oscillation test may be considered as a close approach, provided, of course, the oscillation test is made with different methods of excitation since, for example, symmetrical modes appear only indistinctly or not at all, with antisymmetrical excitation. Furthermore, it should not be expected that oscillation modes due to aerodynamic coupling, are reproducible on the oscillation bench.

5. Criterion for Flutter

Wing flutter with constant amplitude occurs when the energy of damping equals the input of aerodynamic energy; that is, when

$$L_m + L_{dm} = 0 \tag{16}$$

If, for simplification, we set $A' = \bar{A}$, $A'' = 0$, we obtain from equations (6), (12), (16) the condition under which oscillations with constant amplitude occur:

$$\begin{aligned}
&\bar{A}^2(l_1 + \omega\, d_F') + \bar{B}^2(l_2 + \omega\, 0.4\, d_F') + \bar{C}^2(l_3 + \omega\, d_R) \\
&\quad + \bar{A}\, B'\, l_4 + \bar{A}\, B''\, l_5 + \bar{A}\, C'\, l_6 + \bar{A}\, C''\, l_7 \\
&\quad + (B'\, C' + B''\, C'')\, l_8 + (B'\, C'' - B''\, C')\, l_9 = 0
\end{aligned} \tag{17}$$

As the oscillation test is primarily a means for recording the oscillation amplitudes, it is advisable to consider the amplitudes $\bar{B}$ and $\bar{C}$ as independent variables and to set

$$B' = \bar{B} \cos \varphi \qquad C' = \bar{C} \cos \psi$$

$$B'' = \bar{B} \sin \varphi \qquad C'' = \bar{C} \sin \psi$$

The phase angles φ and ψ should be so determined that the energy assumes an extreme value. Partial differentiation of (10) according to φ and ψ gives:

$$\left.\begin{aligned}
&\bar{A}(-l_4 \sin \varphi_0 + l_5 \cos \varphi_0) + \bar{C} \\
&\quad [l_8 \sin(\psi_0 - \varphi_0) - l_9 \cos(\psi_0 - \varphi_0)] = 0 \\
&\bar{A}(-l_6 \sin \psi_0 + l_7 \cos \psi_0) - \bar{B} \\
&\quad [l_8 \sin(\psi_0 - \varphi_0) - l_9 \cos(\psi_0 - \varphi_0)] = 0
\end{aligned}\right\} \tag{18}$$

From (17) and (18) the limiting values of the amplitudes $\bar{B}$ and $\bar{C}$ may be computed as functions of the reduced frequency. Admittedly, the calculation is quite complicated for three degrees of freedom.

Restricted to two degrees of freedom, the results are more simple and elucidating. This limitation is particu-

larly permissible when the aileron is the main cause of flutter and the rest of the wing simply oscillates with it. Then the ensuing mode of wing oscillation may be said to have only one degree of freedom. The wing then oscillates about some fixed nodal line as in the oscillation test. This has also been observed in model tests. The four following modes of oscillation with two degrees of freedom, are investigated.

1. $\bar{C} = 0$, wing bending + wing torsion.

2. $\bar{B} = 0$, wing bending + aileron motion.

3. $\bar{A} = 0$, wing torsion about quarter-chord point + aileron motion.

4. $\bar{A} = - B'$, $B'' = 0$, wing torsion about three-quarter chord point + aileron motion.

Writing the extreme conditions (18) in (17) results in:

$$\left.\begin{aligned}
&1.\ \bar{A}^2(l_1 + \omega\, d_F') + \bar{B}^2(l_2 + \omega\, 0.4\, d_F') - \bar{A}\bar{B}\sqrt{l_4^2 + l_5^2} = 0 \\
&\qquad \sin\varphi_0 = -\frac{l_4}{\sqrt{l_4^2 + l_5^2}},\quad \cos\varphi_0 = -\frac{l_5}{\sqrt{l_4^2 + l_5^2}} \\
&2.\ \bar{A}^2(l_1 + \omega\, d_F') + \bar{C}^2(l_3 + \omega\, d_R) - \bar{A}\bar{C}\sqrt{l_6^2 + l_7^2} = 0 \\
&\qquad \sin\psi_0 = -\frac{l_6}{\sqrt{l_6^2 + l_7^2}},\quad \cos\psi_0 = -\frac{l_7}{\sqrt{l_6^2 + l_7^2}} \\
&3.\ \bar{B}^2(l_2 + \omega\, 0.4\, d_F') + \bar{C}^2(l_3 + \omega\, d_R) - \bar{B}\bar{C}\sqrt{l_8^2 + l_9^2} = 0 \\
&\qquad \sin\chi_0 = -\frac{l_8}{\sqrt{l_8^2 + l_9^2}},\quad \cos\chi_0 = -\frac{l_9}{\sqrt{l_8^2 + l_9^2}} \\
&4.\ \bar{B}^2(l_1 + l_2 - l_4 + \omega\, 1.4\, d_F') + \bar{C}^2(l_3 + \omega\, d_R) \\
&\qquad - \bar{B}\bar{C}\sqrt{(l_8 - l_6)^2 + (l_9 - l_7)^2} = 0 \\
&\qquad \sin\chi_0 = -\frac{l_8 - l_6}{\sqrt{(l_8 - l_6)^2 + (l_9 - l_7)^2}} \\
&\qquad \cos\chi_0 = -\frac{l_9 - l_7}{\sqrt{(l_8 - l_6)^2 + (l_9 - l_7)^2}}
\end{aligned}\right\} \quad (19)$$

Excepting l_ε the energy coefficients are consistently positive; therefore, the optimum phase angles usually lie in the third quadrant. For mode 2, the optimum phase angle passes from the second into the third quadrant at $\omega \sim 0.5$; for the fourth mode, it passes from the fourth into the third quadrant at $\omega = 1.5$.

6. Energy Graphs

In order to circumscribe the influence of damping on flutter, the calculation is based on the following extreme values:

$d_F = 0$	0.25	1.25
$d_R = 0$	.0025	.025

The values $d = 0$ are only of theoretical interest. The corresponding curves show the range within which energy may be taken from the air stream.

The four oscillation modes finally afford energy graphs, with four curves each, for the different damping values. The abscissa is the logarithm of the amplitude ratio; the ordinate is the reciprocal value of the reduced frequency

$$\frac{1}{\omega} = \frac{v}{\nu l}$$

In the following, the energy graphs (figs. 17-23) are compared with experience.

a) Oscillation: wing bending and wing torsion.- Figure 17 shows that the best condition for flutter exists at the amplitude ratio $A/B = 0.9$. This corresponds approximately to a pure elastic oscillation about the three-quarter chord point. The reduced frequency $\omega_h = 0.85$ is not exceeded even with very low damping $d_F' = 0.25$, and may therefore be considered as the practical limit. With greater damping, as it occurs particularly on models, the upper limit drops to $\omega = 0.52$, so that in especially unfavorable cases, flutter in bending and torsion is likely to occur. In the region $\omega = 0.52$ to 0.85, this type of flutter has been analytically investigated on the Junkers A 20 (reference 17). The tapered wing has a chord of 2.36 m at the root and a mean chord of 1.70 m at the severely oscillating tip. Referred to the wing chord of 1.70 m,

the calculation gave in second approach, the reduced frequency $\omega = 0.45$. The torsional stiffness of the thick cantilever Junkers wing is so great that the critical speed would still lie above the operating range even if the reduced frequency were twice as high.

b) Oscillation mode: Wing bending and aileron motion.- This mode is especially predominant in the He 46, KL 1A, AC 12E, Do 12, M 28.

Figures 18 and 19 are calculated for aileron chords of $\tau = 0.15$ and 0.25, and show first that the flutter zone becomes smaller as the aileron chord becomes greater. According to (15) the backward c.g. position of the aileron affects the amplitude ratio. The reduced frequency is so much higher as the amplitude ratio is smaller and the aileron c.g. moves backward, although increases only to $\omega \sim 2$ with very small damping and $\omega \sim 1$ with very great damping. Practically only the right-hand parts of the curves come in question, because the amplitude ratio of the flutter oscillation cannot drop below a certain value determined by the mass distribution of an unbalanced aileron. The value $A/C = 0.2$ to 0.3 may be regarded as the practical limit.

If the amplitude ratio is low the aileron damping exerts a profound influence on the reduced frequency. At $A/C = 0.25$, e.g., ω increases from 1.13 to 1.4 when the aileron damping drops from an initially high value of 0.025 to the low value of 0.0025. Flutter therefore continues until the speed has dropped to 80 percent of its initial value.

A comparison of the ϑ values as computed for several airplanes with the energy graphs, discloses ω values which are of the same order of magnitude as the observed ones. As the interpolation for different aileron chords and dampings by means of figures 18 and 19 is not very exact, a closer agreement can be obtained when establishing a special energy graph for each airplane, using the damping values and amplitude ratios obtained in the oscillation test and taking the ω values from these graphs. The essential factor is the correct reproduction of the damping effect and the backward c.g. position of the aileron on the flutter phenomenon.

c) Oscillation mode: Wing torsion about a quarter-chord point and aileron motion.- This mode occurs approximately in models DP 9, He 8, He 60, L 102, S 24. The fact that 80 percent of these types of airplanes were completely destroyed, proves that this mode is by far the most dangerous. The energy graphs (figs. 20 and 21) show that extremely high values of the reduced frequency are obtainable with small damping. If the aileron damping drops from an initially high value to a small final value after flutter starts, a large excess of energy is available, which inevitably must lead to complete failure.

One interesting feature is that such flutter is possible only within a limited range of the amplitude ratio. The practical upper limit is for the aileron chords:

$$\tau = 0.15 \qquad B/C \leq 0.37$$

$$\tau = .25 \qquad B/C \leq .45$$

This would stipulate a certain minimum distance of backward c.g. position of the aileron.

An aileron with the rather conventional characteristics: $\tau = 0.15$; $k_R = 0.12\ l$; $s_R = 0.06\ l$; $m_R = 0.25\ \pi\ \rho\ l^2\ b_q$ has, for example, according to (15), the amplitude ratio

$$\frac{B}{C} = \frac{1}{1.2}\ \frac{A_R}{C} = 0.192$$

if freely oscillating. According to figure 20, this value lies exactly at the point of minimum critical velocity for small wing damping, and gives $\omega = 1.25$. In the neighborhood of amplitude ratio $B/C = 0.2$, reduced frequency values up to $\omega = 0.9$ are possible even with maximum damping. With small damping, very high ω values are possible, as actually observed on the L 102 at the end of flutter.

d) Oscillation mode: Wing torsion about three-quarter chord point + aileron motion.- In this mode no reduced frequency values in excess of $\omega \sim 1.0$ are possible, even with small damping (figs. 22 and 23). This mode is therefore less dangerous.

The aileron chosen above as example, requires twice the unbalance ($s_R = 0.12\ l$) to give an amplitude ratio

$$\frac{B}{C} = \frac{1}{0.2}\frac{A_R}{C} = 0.89$$

which, in figure 22, meets the right-hand side of the curve for small damping at $\omega = 0.62$. With the aileron c.g. farther forward, this value could be obtained only when the amplitude C is increased at the same time by resonance of the control system.

7. Application to a Practical Example

These energy graphs are supposed to give a general view of the new method of estimating the critical speed. In actual cases, it usually involves oscillations at which the nodal lines do not exactly correspond with the third nor with the fourth type of the illustrated examples. The aileron chords also vary within a great range. However, with the aid of the figures given in tables III and IV, supplemented by the damping energy from the oscillation test, a particular energy graph can be obtained for each individual case.

For example, if the oscillation test shows that the nodal line in the outer part of the wing lies at three-quarter chord point, that the (angular) amplitude C of the aileron motion equals twice the wing torsion amplitude B, and that the wing damping factor is $d_F' = 0.25$, then figure 23 gives for $B/C = 0.5$

$$\frac{1}{\omega} = 1.06 \text{ to } 1.10$$

$$\omega = 0.94 \text{ to } 0.91$$

depending on the value of the aileron damping d_R. In this case the effect of d_R on the reduced frequency is quite small. In fact, the reduced frequency will be lower than the values found from the graph, because the assumption of optimum phase angle is not exactly fulfilled. Even so, it is possible to estimate the lowest critical speed at the observed mode of oscillation, which may prove very valuable under certain circumstances.

Preventative measures against flutter, particularly mass balancing of the ailerons, show their effect in the low aileron amplitude C in the oscillation test. The amplitude ratio B/C can then increase quite easily beyond 1, so that no flutter at all is possible for a nodal line

at the quarter-chord point (figs. 20 and 21) or at any rate that a substantially higher value of $1/\omega$ will be found on the right-hand branch of the curves in figures 18, 19, 22, and 23.

V. PREVENTION OF FLUTTER

The previously described conditions for oscillation modes of amplitude ratio and damping - while admittedly necessary - do not, however, constitute adequate conditions for flutter. The thus-estimated critical speeds are therefore on the safe side. In the energy calculation, the premises were optimum phase angles which, as is known, are closely approached in many cases in practice, especially with aileron oscillations. On the other hand, it is conceivable that cases may occur wherein the phase angle cannot even approach the optimum value and in which no flutter is at all possible, even if the oscillation modes as recorded on the oscillation bench, were indicative of flutter.

The very simple form of the energy method compared with the exact method, is simply the result of omitting the elastic forces as well as the mass forces and their distribution from the calculation. But these forces are far from negligible as far as the magnitude of phase angle is concerned. To illustrate: It can be proved that with two degrees of freedom - wing bending and wing torsion - flutter is possible only when the product

$$m\ s > \frac{\pi\ \rho\ t^3}{16} \tag{20}$$

whereby m is the wing mass per unit length of the span and s the distance of the c.g. of the wing element behind the quarter-chord point. Applied to a cantilever wing without aileron, this simply means shifting the c.g. of the individual wing sections near the quarter-chord points, in order to prevent flutter at any air density ρ. Such wings, although with ailerons, are found on the M 20.

The majority of flutter cases described in section III, probably could have been prevented by careful mass balancing of the ailerons. This method, originally pointed out by von Baumhauer and Koning (reference 3) in 1923, has frequently been discussed since then in the literature. It is a fact, however, that practically all older models had un-

balanced ailerons and that flutter was less frequent and le less dangerous at the then comparatively low flying speeds.

On the other hand, mass-balancing the ailerons after an airplane is built, means an expense of weight, requiring up to 0.5 percent of the airplane weight aside from increased drag. Quite obviously, subsequent modification of all existing types was therefore out of the question, particularly as the need for such measures did not seem very apparent as far as the older types were concerned.

For the new types the DLA at first recommended mass balancing; later this was incorporated into the airplane-design specifications. An aileron originally designed for mass balance is not much heavier than one designed without balance. The purpose of mass balancing is to reduce the aileron amplitude to harmless magnitude in all modes that may cause flutter. For modes such as shown in figures 22 and 23, even a "partial mass balance" may be all that is required.

In other cases, however, it is necessary to effect a complete and careful balance because it requires a ten-times-greater aileron amplitude in order to get out of the range of the minimum of critical speed, according to the diagrams. This fact has not always been sufficiently recognized.

The success of these preventative measures should be checked on the oscillation bench, because even a completely balanced aileron may oscillate. Possible causes are:

1. Mass coupling due to the co-oscillating air mass, particularly when the gap between wing and aileron is small and the aileron is not aerodynamically balanced.

2. Kinematic coupling with complicated and indistinct static structure of the cellule.

3. Lack of torsional stiffness of ailerons.

4. Natural oscillations of the system: left aileron, controls, right aileron.

The latter oscillation is particularly dangerous when coincident with a symmetrical natural mode of the wings. In the vicinity of the resonance point, the phase

angle of the aileron motion relative to the wing motion changes profoundly with the frequency, so that the phase angle most favorable for flutter can easily occur. Remedies are: changes in the natural frequency of the aileron control or artificial damping of the aileron deflection.

Some of the above cases prove, at any rate, that even ailerons nearly or completely mass-balanced may occasionally develop flutter, in which case, however, the reduced frequency seems to be below the average, i.e. $\omega < 0.9$.

If the aileron were to be considered as nonexisting so far as flutter is concerned, they would have to have not only zero amplitude on the oscillation bench but also complete aerodynamic balance - at least, within the range of small angles of attack and aileron deflection. It can be proved theoretically that otherwise the circulation may cause a purely aerodynamic coupling, which lowers the critical speed relative to that of the wing flutter in bending and torsion.

If all these conditions for preventing aileron oscillation were fulfilled, the displacement of the c.g. axis of the wing near to the quarter-chord point, would practically suffice for flutter prevention. Obviously, this is predicated on the assumption that the two-dimensional theory of wing flutter is substantially correct, which cannot be summarily taken for granted with complicated wing shapes. The wing stiffness of such an airplane could be arbitrarily low, provided no other lower stiffness limits existed.

Such flutter prevention, however, requires a large number of design changes of such a radical nature that in many cases it would be tantamount to a new departure in design methods. In view of this fact, it seemed more expedient to increase the wing stiffness as long as consistent with minimum weight. This was the reason why this preventative measure was resorted to at first. Greater stiffness leads to higher wing frequencies and consequently higher critical speed. As a result, the flutter, while not altogether prevented is, however, moved up into a speed range above the highest speed which can be reached. A contributing factor was the consideration that the wing itself must have a certain minimum stiffness in order to prevent static torsional instability and reversal of aileron effect at high flying speed.

With further increase of speed, however, a point is reached where the simple expedient of increased stiffness is no longer compatible with the weight and where it will be necessary to combine all known measures for the prevention of flutter.

Translation by the National Advisory Committee for Aeronautics.

REFERENCES

1. Duncan, W. J., and Collar, A. R.: A Theory of Binary Servo-Rudder Flutter, with Applications to a Particular Aircraft. R. & M. No. 1527, British A.R.C., 1933.

2. Blasius, H.: Über Schwingungserscheinungen an einholmigen Unterflügeln. Z.F.M., vol. 16, no. 3, 1925, p. 39.

3. von Baumhauer, A. G., and Koning, C.: Onstabile Trillingen vaan een draagvlak-klap system. Verslagen en verhandelingen vaan den rijksstudiedienst voor de Luchtvaart, Report A 48, Amsterdam, vol. II, 1923.

   von Baumhauer, A. G., and Koning, C.: On the Stability of Oscillations of an Airplane Wing. T.M. No. 223, N.A.C.A., 1923.

4. Blenk, H., and Liebers, F.: Gekoppelte Torsionsschwingungen von Tragflügeln. Z.F.M., vol. 16 no. 23, 1925, p. 479.

5. Blenk, H., and Liebers, F.: Flügelschwingungen freitragender Eindecker. D.V.L. Yearbook, 1928, p. 63.

6. Blenk, H., and Liebers, F.: Gekoppelte Biegungs-Torsions- und Querruderschwingungen von freitragenden und halbfreitragenden Flügeln. D.V.L. Yearbook, 1929, p. 257.

7. Bairstow, L.: The Theory of Wing Flutter. R. & M. No. 1041, Appendix II, British A.R.C., 1927.

8. Frazer, R. A.: An Investigation on Wing Flutter. R. & M. No. 1042; British A.R.C., 1926.

9. Frazer, R. A., and Duncan, W. J.: The Flutter of Aeroplane Wings. R. & M. No. 1155, British A.R.C., 1929.

10. Frazer, R. A., and Duncan, W. J.: A Brief Survey of Wing Flutter with an Abstract of Design Recommendations. R. & M. No. 1177, British A.R.C., 1928.

11. Frazer, R. A., and Duncan, W. J.: Wing Flutter as Influenced by the Mobility of the Fuselage. R. & M. No. 1207, British A.R.C., 1929.

12. Frazer, R. A., and Duncan, W. J.: Conditions for the Prevention of Flexural-Torsional Flutter of an Elastic Wing. R. & M. No. 1217, British A.R.C., 1930.

13. Duncan, W. J.: The Wing Flutter of Biplanes. R. & M. No. 1227, British A.R.C., 1930.

14. Hesselbach, B.: Über die gekoppelten Schwingungen von Tragflügel und Verwindungsklappe. Z.F.M., vol. 18, no. 20, 1927, p. 465.

15. Birnbaum, W.: Das ebene Problem des schlagenden Flügels. Z.f.a.M.M., vol. IV, no. 4, 1924, p. 277.

16. Birnbaum, W.: Der Schlagflügelpropeller und die kleinen Schwingungen elastisch befestigter Tragflügel. Z.F.M., vol. 15, Nos. 11-12, 1924, p. 128.

17. Küssner, H. G.: Schwingungen von Flugzeugflügeln. D.V.L. Yearbook, 1929, p. 313.

18. Glauert, H.: The Force and Moment on an Oscillating Aerofoil. R. & M. No. 1242, British A.R.C., 1929.

19. Duncan, W. J., and Collar, A. R.: Calculation of the Resistance Derivatives of Flutter Theory. Part I. R. & M. No. 1500, British A.R.C., 1933.

19a. Theodorsen, Theodore: General Theory of Aerodynamic Instability and the Mechanism of Flutter. T.R. No. 496, N.A.C.A., 1935.

20. Cox, H. Roxbee: Statistical Method of Investigating Relations between Elastic Stiffnesses of Aeroplane Wings and Wing-Aileron Flutter. R. & M. No. 1505, British A.R.C., 1933.

21. Cox, H. Roxbee: Problems Involving the Stiffness of Aeroplane Wings. Jour. R.A.S., vol. 38, no. 278, 1934.

22. Essers, J.: Untersuchung von Flügelschwingungen im Windkanal. D.V.L. Yearbook, 1929, p. 345.

23. Younger, J. E.: Wing Flutter Investigation on Brady's Wind Tunnel Model. A.C.I.C. No. 608, Matériél Division, Army Air Corps, 1928.

24. Raab, A.: Flugelschwingungen an freitragenden Eindeckern. Z.F.M., vol. 17, no. 7, 1926, p. 146.

25. v. Schlippe, B.: Die innere Dampfung, Berechnungsansatze. Ing.-Archiv, vol. VI, 1935, p. 127.

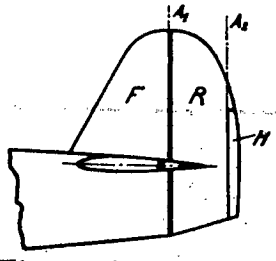


Figure 1.- Fin and rudder with servo-rudder

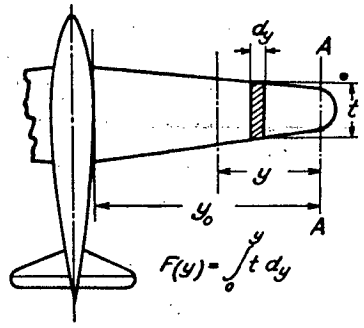


Figure 2.- Wing section

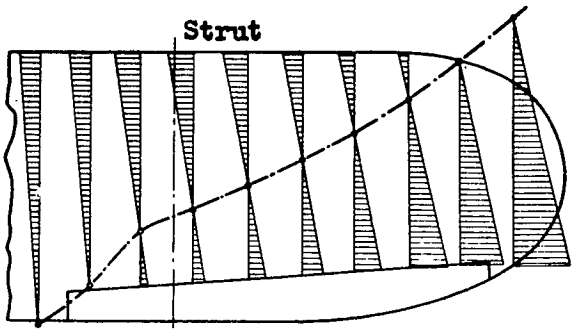


Figure 3.- Critical form of oscillation of the He 8 at 550/min. scale of wing contour 1:80

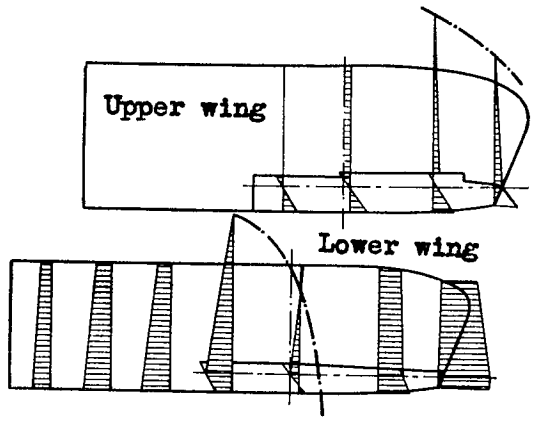


Figure 4.- Dangerous oscillation of L 78 at 860/min. scale 1:80

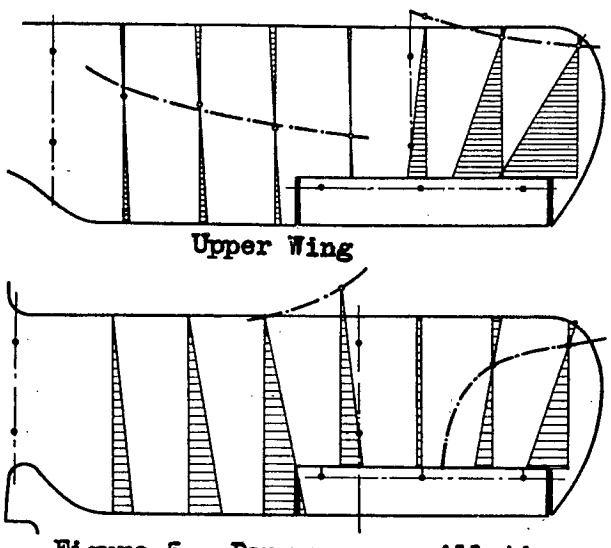


Figure 5.- Dangerous oscillation of He 60 780/min. scale 1:80

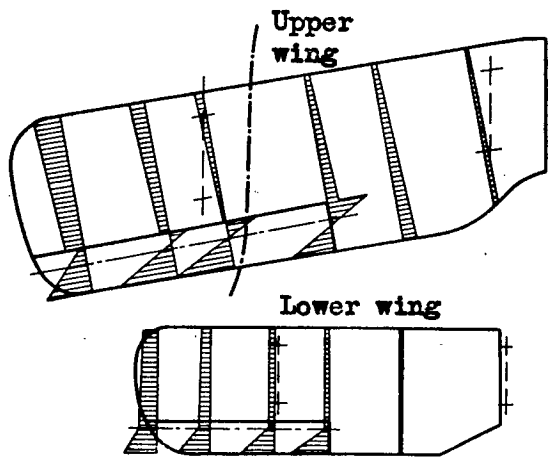


Figure 6.- Dangerous oscillation of He 46c at 815/min. scale 1:80

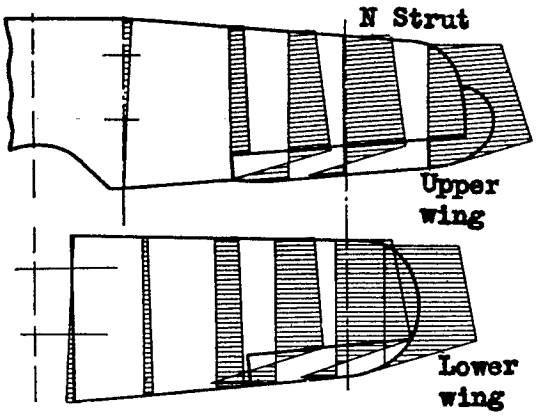


Figure 7.- Dangerous oscillation of KL 1 A at 675/min. scale 3:200

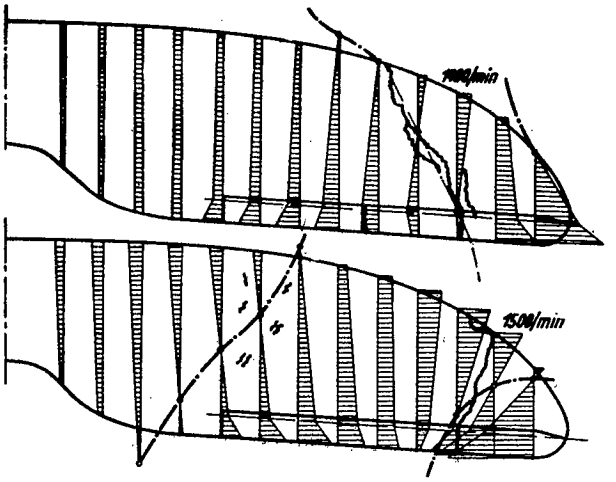


Figure 8.- Dangerous oscillation of Do 10; scale 1:100
Lines of break on left and right wing
Break or stretch of cables

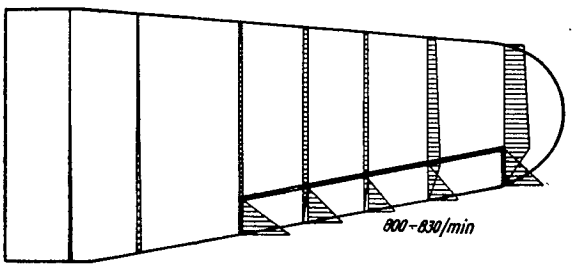


Figure 10.- Dangerous oscillation of AC 12 E; scale 3:200

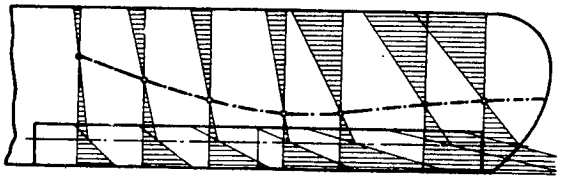


Figure 9.- Dangerous oscillation of L 102 at 835/min. scale 1:80

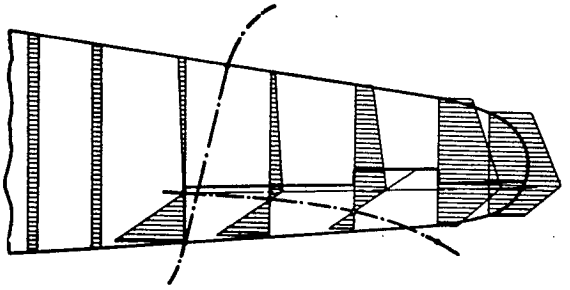


Figure 11.- Dangerous oscillation of Do 12 at 580/min asymmetrical excitation scale 3:100

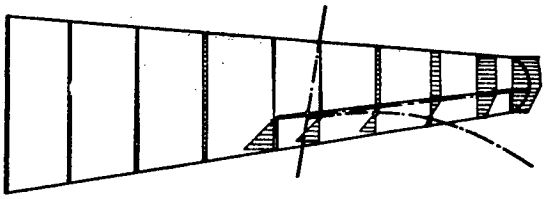


Figure 12.- Dangerous oscillation of M 28 at 770/min. asymmetrical excitation scale 1:100

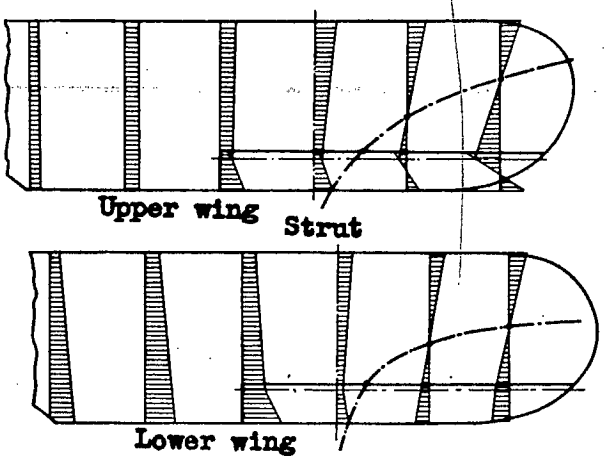


Figure 13.- Dangerous oscillation of S 24 at 1215/min. scale 1:150

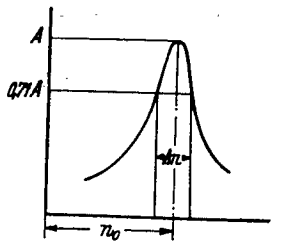


Figure 14.- Width of resonance curve

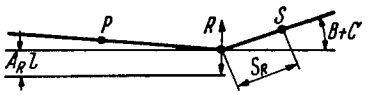


Figure 16.- Aileron vibration

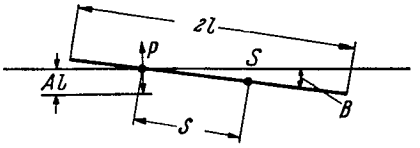


Figure 15.- Plate vibration

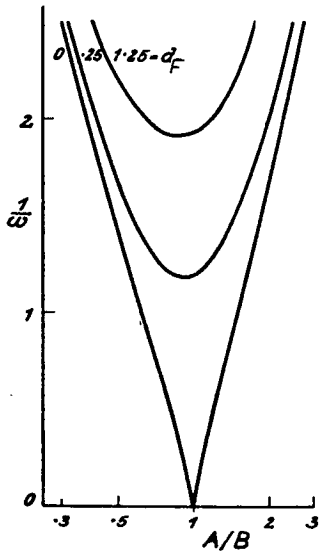


Figure 17.- Wing bending and wing torsion

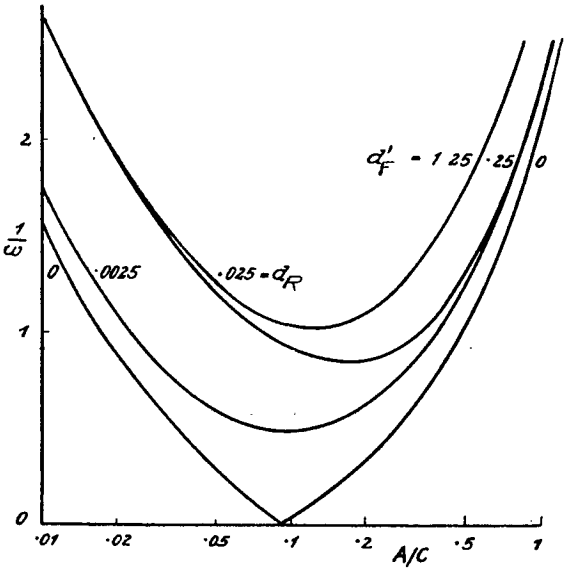


Figure 18.- Wing bending and aileron motion, $\tau = 0.15$

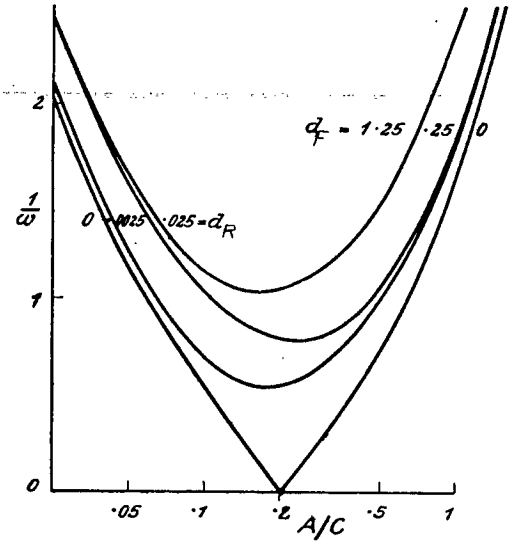


Figure 19.- Wing torsion and aileron motion, $\tau = 0.25$

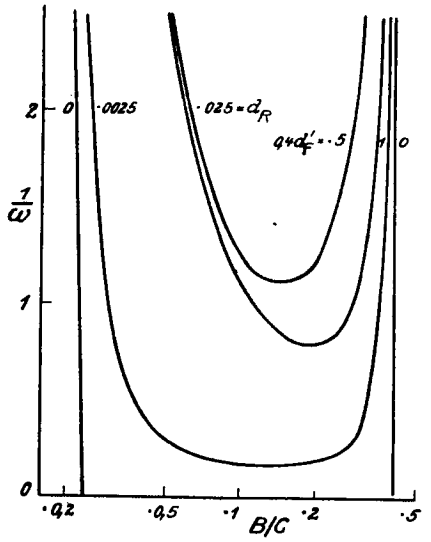


Figure 20.- Wing torsion about 1/4 chord point and aileron motion, $\tau = 0.15$

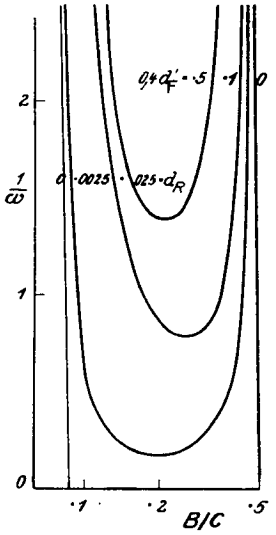


Figure 21.- Wing torsion about 1/4 chord point and aileron motion, $\tau = 0.25$

Figure 22.- Wing torsion about 3/4 chord point and aileron motion, $\tau = 0.15$

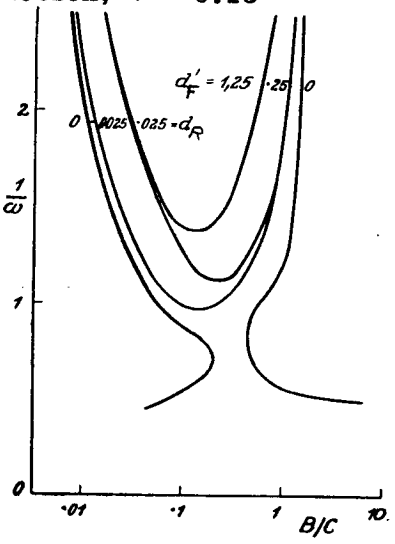
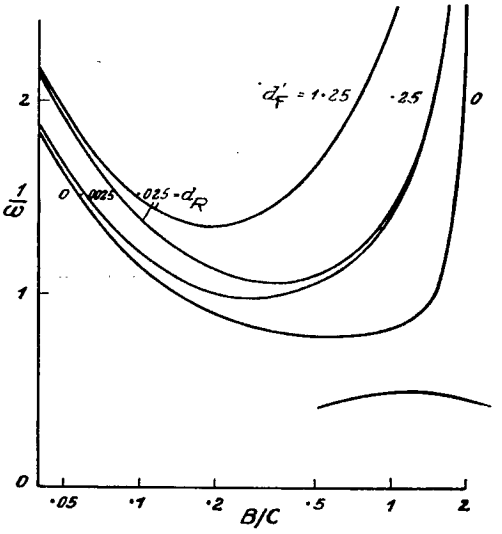


Figure 23.- Wing torsion about 3/4 chord point and aileron motion, $\tau = 0.25$



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