# REPORT No. 676

## SURFACE HEAT-TRANSFER COEFFICIENTS OF FINNED CYLINDERS

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### SUMMARY

*An investigation to determine and correlate the experimental surface heat-transfer coefficients of finned cylinders with different air-stream cooling arrangements was conducted at the Langley Memorial Aeronautical Laboratory from 1932 to 1938. The investigation covered the determination of the effect of fin width, fin space, fin thickness, and cylinder diameter on the heat transfer. Wind-tunnel tests were made in the free air stream with and without baffles and also with various devices for creating a turbulent air stream. Tests were also made with blower cooling.*

*A variation of the initial turbulence of the tunnel air stream was found to have little effect on the heat transfer. Correlation of the surface heat-transfer coefficients was found possible by plotting a factor involving the heat-transfer coefficients, the fin space, and the conductivity of the cooling air against a factor involving the velocity, the density, and the viscosity of the cooling air and the fin space, the fin width, and the cylinder diameter.*


### INTRODUCTION

The large number of variables that affect the heat transfer from finned cylinders and the complexity of their relationships makes it difficult to determine optimum fin designs from experimental data alone. Investigations at this laboratory (reference 1) have shown that the heat transfer can be calculated from an equation involving the fin dimensions, the surface heat-transfer coefficient $q$, and the thermal conductivity of the metal. The value of $q$ depends on the fin design, the air-stream characteristics, and the method of cooling. The air-stream characteristics include the weight velocity, the conductivity, the viscosity, and the intensity and the scale of the turbulence of the cooling air. No satisfactory method of calculating the heat-transfer coefficient $q$ has been found and its value must therefore be experimentally determined.

In previous experiments, the value of $q$ was first established for cylinders having fins of various dimensions tested in a wind tunnel without cylinder baffles (reference 1). These tests were undertaken before baffles were generally used around the cylinders of radial air-cooled engines. Tests were later made to determine $q$ with several types of baffle. (See reference 2.) These tests were made prior to the introduction of the pressure-baffled engine and were made in the wind tunnel with the air stream passing around and through the baffles. At this time, an investigation was also made to determine the advantages of the completely jacketed finned cylinder in which the cooling air is forced through the jacket by means of a blower as compared with the pressure-baffled engine.

The surface heat-transfer coefficients and the pressure differences across the cylinders were determined with blower cooling for a number of cylinders having different fin designs (reference 3). Further tests in a wind tunnel (reference 4) were made on cylinders with fins more closely spaced than those tested in references 1, 2, and 3 in order to determine the coefficients with blower cooling and with and without baffles. In addition, tests were made to determine the pressure differences for a variation of air flow across a large number of jacketed cylinders and cylinders with baffles and also to determine the air flow between the fins (references 5, 6, and 7).

The investigation has been extended to obtain additional data on the effect of the fin dimensions on $q$. Cylinders of three different diameters have been tested to determine the effect of diameter on the pressure drop and the heat-transfer coefficient. Several copper cylinders with different fin widths and spacings and one aluminum-alloy cylinder were tested primarily to substantiate the effect of thermal conductivity in the heat-transfer equation previously mentioned. Data are also presented in this report showing the effect of turbulence on $q$ over a range of turbulence intensities.

The object of this report is to present the recent information that has been obtained on the heat-transfer coefficient $q$ and, in addition, to show that all available data on $q$ can be correlated for each air-flow arrangement in terms of functions defining a single curve and involving the fin dimensions, the cylinder diameter, and the air-stream characteristics. This information should permit the calculation of the best fin proportions for the conditions of air flow that are of general interest.

The present report includes tests on 58 cylinders with fins having widths from 0.37 inch to 3 inches, mean spaces from 0.010 inch to 0.50 inch, and mean thicknesses from 0.026 inch to 0.27 inch. The results of some of the tests have already been published. The variation in cylinder diameter was from 3.66 to 6.34 inches.

The tests were conducted at the Langley Memorial Aeronautical Laboratory from 1932 to 1938.

## APPARATUS

### TEST CYLINDERS

A description of the cylinders tested and the air-flow arrangements used is given in table I.

TABLE I.—*Cylinders tested*

| Cylinder | Fin space (in.) | Fin width (in.) | Fin thickness (in.) | Cylinder diameter at fin root (in.) | Material | Fin shape | Air-flow arrangement [1] |
|---|---|---|---|---|---|---|---|
| 1 | 0.330 | 1.47 | 0.270 | 4.66 | Steel | Tapered. | 1, 4. |
| 2 | .270 | 1.47 | .230 | | | | 1, 4. |
| 3 | .220 | 1.32 | .180 | | | | 1, 4. |
| 4 | .170 | 1.32 | .130 | | | | 1, 4. |
| 5 | .140 | 1.22 | .110 | | | | 1, 4. |
| 6 | .150 | 1.22 | .090 | | | | 1. |
| 7 | .185 | 1.22 | .065 | | | | 1. |
| 8 | .100 | 1.22 | .100 | | | | 1, 4. |
| 9 | .075 | .97 | .075 | | | | 1. |
| 10 | .210 | 1.22 | .040 | | | | 1. |
| 11 | .131 | 1.22 | .035 | | | | 1, 3. |
| 12 | .102 | 1.22 | .035 | | | | 1, 3. |
| 13 | .077 | 1.22 | .035 | | | | 1, 2, 3. |
| 14 | .048 | 1.22 | .035 | | | | 1, 2, 3. |
| 15 | .022 | 1.22 | .035 | | | | 1, 2, 3. |
| 16 | .210 | .97 | .040 | | | | 1. |
| 17 | .110 | .97 | .040 | | | | 1. |
| 18 | .101 | .97 | .035 | | | | 1, 2, 3. |
| 19 | .077 | .97 | .035 | | | | 1, 2, 3. |
| 20 | .048 | .97 | .035 | | | | 1, 2, 3. |
| 21 | .022 | .97 | .035 | | | | 1, 2, 3. |
| 22 | .210 | .67 | .040 | | | | 1. |
| 23 | .110 | .67 | .040 | | | | 1. |
| 24 | .101 | .67 | .035 | | | | 1, 2, 3. |
| 25 | .077 | .67 | .035 | | | | 1, 2, 3. |
| 26 | .048 | .67 | .035 | | | | 1, 2, 3. |
| 27 | .022 | .67 | .035 | | | | 1, 2, 3. |
| 28 | .210 | .37 | .040 | | | | 1. |
| 29 | .110 | .37 | .040 | | | | 1. |
| 30 | .060 | .37 | .040 | | | | 1, 4. |
| 31 | .101 | .37 | .035 | | | | 1, 2, 3. |
| 32 | .077 | .37 | .035 | | | | 1, 2, 3. |
| 33 | .048 | .37 | .035 | | | | 1, 2, 3. |
| 34 | .022 | .37 | .035 | | | | 1, 2, 3. |
| 35 | .080 | 3.00 | .035 | | Copper | Rectangular. | 3. |
| 36 | .055 | 3.00 | .035 | | | | 3. |
| 37 | .028 | 3.00 | .035 | | | | 3. |
| 38 | .035 | .37 | .035 | | | | 3. |
| 39 | .124 | .50 | .026 | 3.66 | | | 3. |
| 40 | .120 | .52 | .030 | 6.34 | | | 3. |
| 42 | .500 | 1.50 | .081 | 4.66 | Steel | | 3. |
| 43 | .375 | 1.50 | | | | | |
| 44 | .250 | 1.50 | | | | | |
| 45 | .125 | 1.50 | | | | | |
| 46 | .031 | 1.50 | | | | | |
| 47 | .500 | .75 | | | | | |
| 48 | .250 | .75 | | | | | |
| 49 | .125 | .75 | | | | | |
| 50 | .062 | .75 | | | | | |
| 51 | .031 | .75 | | | | | |
| 52 | .500 | .375 | | | | | |
| 53 | .250 | .375 | | | | | |
| 54 | .125 | .375 | | | | | |
| 55 | .062 | .375 | | | | | |
| 56 | .031 | .375 | | | | | |
| 57 | .010 | .67 | .035 | | | | |
| 58 | .200 | .85 | .050 | | | | |
| 59 | .200 | 1.66 | .050 | | Aluminum alloy | | |
| 60 | Smooth cylinder—no fins. | | | | Steel | | 1, 4. |

[1] Numbers indicate:
1, cylinder in free air stream, no baffles.
2, cylinder in free air stream, with baffles.
3, cylinder enclosed in jacket, blower supplying the air.
4, cylinder in free air stream with axis 45° to air stream.

The fin section in figure 1 shows the significance of $w$, fin width; $s$, fin space; $s_b$, fin space at the fin root; and $t$, fin thickness. The test cylinders were electrically heated and were tested with guard rings, as described in reference 1. The assembled over-all length of the test section and the guard rings was, in every case, approximately 10 inches. For convenience in referring to the finned cylinders, a nomenclature giving the fin space, the fin width, and the fin thickness has been devised. For example, the designation 0.101–0.67–0.035 represents a finned cylinder having an average fin space of 0.101 inch, a fin width

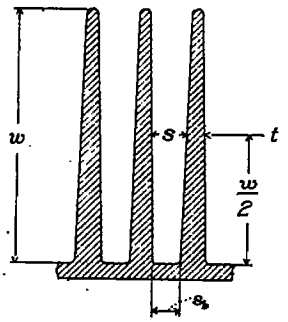
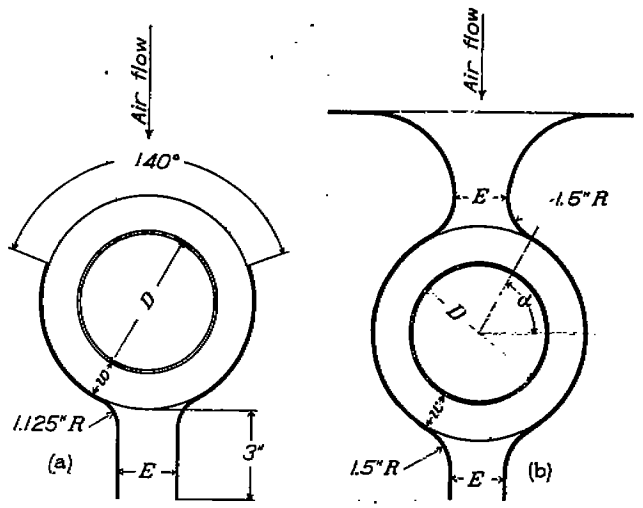



FIGURE 1.—Section showing significance of $w$, $s$, $s_b$, and $t$.

of 0.67 inch, and an average fin thickness of 0.035 inch. The designations corresponding to the cylinders in table I can be obtained by combining the values given in columns 2, 3, and 4.

### BAFFLES AND JACKETS

The type of baffle used for the tests of the cylinder with baffles in the free air stream is shown in figure 2 (a).




(a) Baffle. (b) Jacket.

FIGURE 2.—Type of baffle and jacket used in tunnel and blower-cooling tests.

The ratio of the area of the exit to the area between the fins was approximately 2.

The apparatus used in the blower-cooling tests is described in references 3 and 5. The jacket used in these tests is shown in figure 2 (b). The ratio of the entrance or the exit area of the jacket to the flow area between the fins was approximately 1.6 for all of the cylinders except the copper cylinders, for which the ratio varied from 1.0 to 1.7.

## TESTS

The tests covered in this report are listed in table I. In addition to the blower-cooling tests, this work included wind-tunnel tests with baffles, without baffles, and with the cylinder axis at 45° with respect to the air stream. The methods of testing have been described in references 1 to 5.

The tests to determine the effect of turbulence on the heat-transfer coefficient were made in the wind tunnel with the 0.077–0.37–0.035 steel cylinder with baffles starting 90° from the front of the cylinder (180° opening) and with various devices for creating different amounts of turbulence. Tests with small-scale turbulence were made for the following four conditions:

(1) With honeycomb.
(2) With screen.
(3) Without honeycomb or screen.
(4) With honeycomb and screen.

The honeycomb was constructed of 0.35-inch-diameter tubes of 3-inch length. The screen was of ½-inch mesh with wires of 0.092-inch diameter and was similar to a screen previously tested at the National Bureau of Standards for its turbulence-creating properties (reference 8). In the tests of reference 8, the screen was found to produce a very turbulent flow when placed 141 wire diameters ahead of the point of measurement. In the present tests, an indication of the intensity of turbulence was obtained for the foregoing conditions by the conventional sphere method. The turbulence factors were determined with a sphere 4 inches in diameter by the method described in reference 9. The four conditions of turbulence gave the following turbulence factors, which indicate the range covered:

| | |
|---|---|
| With honeycomb | 1.41 |
| Without honeycomb or screen | 1.89 |
| With screen | 2.96 |
| With honeycomb and screen | 3.21 |

A large value of the turbulence factor indicates a high degree of turbulence.

A larger scale turbulence was produced by placing a drum of 9-inch diameter various distances between 0.3 inch and 13 inches ahead of the test cylinder. These tests were made with the axis of the drum both perpendicular and parallel to the cylinder axis.

Tests previously reported (reference 10) showed high rates of heat flow when the fin-plane/air-stream angle was between 30° and 60°. Evidence shows that the front portions of cylinders placed within an N. A. C. A. cowling are cooled by a flow consisting mainly of large swirls incident upon the fins at various angles. It is very likely that, in the tests with an N. A. C. A. cowling, the flow phenomena between fins may be similar to that with different fin-plane/air-stream angles. In order to obtain more complete data on this subject, the range of the previous tests on fin-plane/air-stream angle has been extended to determine the effect of fin dimensions on the heat transfer of cylinders having a fin-plane/air-stream angle of 45°.

The heat lost from air-cooled engine cylinders by direct radiation is generally considered negligible as compared with the heat lost by convection. Some consideration has been given, however, to the merits of black-enamel surfaces for improving the heat transfer. Wind-tunnel tests were made to determine the effect of different thicknesses of black enamel and of oiled surfaces on the surface heat-transfer coefficient of a cylinder without fins. The use of a surface without fins avoided any effect of change of space between fins caused by the enamel or oil thickness. The enamel was a black baking japan applied by spraying and was baked for 1 hour at 400° F.

With blower cooling, measurements were made of the heat transfer, the weight of air used, and the pressure drop; with the other three air-flow arrangements, only heat-transfer measurements were obtained for a range of tunnel air speeds.

## SYMBOLS

$c_p$, specific heat of fluid at constant pressure, B. t. u. per pound per °F.

$d$, diameter of pipe, inches.

$d_e$, equivalent diameter of duct $[4ws/2(w+s)]$, inches.

$D$, cylinder diameter at fin root, inches.

$g$, acceleration of gravity, feet per second per second.

$k_m$, thermal conductivity of metal, B. t. u. per square inch through 1 inch per hour per °F.

$k_a$, thermal conductivity of air, B. t. u. per square inch through 1 inch per second per °F.

$l$, equivalent length for straight tube ($\varphi R_a$), feet.

$q$, surface heat-transfer coefficient, B. t. u. per square inch total surface area per hour per °F. temperature difference between surface and inlet cooling air.

$q_{av}$, surface heat-transfer coefficient, B. t. u. per square inch total surface area per hour per °F. temperature difference between surface and average cooling air.

$R_a$, average radius from center of cylinder to finned surface $\left(\frac{R_b}{12}+\frac{w}{2\times 12}\right)$, feet.

$R_b$, radius from center of cylinder to fin root ($D/2$), inches.

$s$, average space between adjacent fin surfaces, inches.

$s_b$, space between adjacent fin surfaces at fin root, inches.

$t$, thickness of fins, inches.

$U$, over-all heat-transfer coefficient, B. t. u. per square inch base area per °F. temperature difference between cylinder wall and cooling air per hour.

$V$, velocity of air in tunnel throat in air-flow arrangements 1, 2, and 4 or average velocity between fins in air-flow arrangement 3, feet per second. (See table I.)

$w$, fin width, inches.

$W$, quantity of air flowing, pounds per second.

$\rho_1 g$, specific weight of air in front of jacket in air-flow arrangement 3 or specific weight of air in tunnel throat in front of cylinder in air-flow arrangements 1, 2, and 4, pounds per cubic foot.

$\rho_2 g$, specific weight of air in rear of jacket in cooling method 3, pounds per cubic foot.

$\rho_{av} g$, average specific weight of air $\frac{\rho_1 g+\rho_2 g}{2}$, pounds per cubic foot.

$\rho_0 g$, specific weight of air at 29.92 inches Hg and 80° F. (0.0734 lb./cu. ft.), pounds per cubic foot.

$\varphi$, equivalent angle of curvature $(\alpha+\pi/2)$, radians.

$\alpha$, (See fig. 2(b)), radians.

$\mu$, absolute viscosity of air, pounds per second per foot.

$\Delta p_1$, pressure difference across cylinder, inches of water.

$\Delta p_2$, pressure difference caused by loss of velocity head from exit of skirt of baffle or jacket, inches of water.

$\Delta p_{total}$, total pressure difference across set-up $(\Delta p_1+\Delta p_2)$, inches of water.

## COMPUTATIONS

The surface heat-transfer coefficient $q$ was obtained by dividing the heat input per hour by the product of the total cooling-surface area and the difference between the average temperature of the cooling surface and the entering-air temperature. A complete description of the methods used in calculating the results has been presented in references 1, 3, 4, and 5. The viscosity and the conductivity of the air were based on an average of the cylinder and the mean air temperatures for air-flow arrangement 3 and on an average of the cylinder and the tunnel-throat temperatures for air-flow arrangements 1, 2, and 4. The temperature of the outlet air was calculated from the heat input to the cylinder, the quantity of air flowing over the cylinder, and the temperature of the inlet air. All tunnel air speeds and pressure differences have been corrected to a specific weight of air of 0.0734 pound per cubic foot.

In studies of the heat transfer from pipes, it is customary to base the average heat-transfer coefficient $q_{av}$ on the difference between the average surface and the average fluid temperatures. In the application of finned-surface heat-transfer data to engine cylinders and to the calculation of optimum fin constructions to meet various conditions of weight and pressure drop, it has been found advantageous to base $q$ on the difference between the entering inlet-air temperature and the average surface temperature. When the over-all heat-transfer coefficient $U$ is calculated from $q_{av}$, it is necessary to determine the temperature rise of the air. This temperature rise, in turn, depends upon the value of the $U$ that is being determined. For optimum fin calculations with limiting pressure differences, it is readily seen that the use of $q_{av}$ complicates the problem. If the effect of flow-path length on $q$, based on inlet-air temperature, is known, the calculations of the heat transfer can be made in a simple manner. In the present investigation, the effect of flow-path length on $q$ based on inlet-air temperature has been determined over a range of lengths that is large as compared with the range found on aircraft-engine cylinders.

An additional advantage in using $q$ or $U$ based on the inlet-air temperature is that the over-all heat-transfer coefficient $U$ is proportional to the rate of heat transfer, which is a function of the power developed. Thus an increase of $U$ is also an indication as to how much the power can be increased. The proportionality of the over-all heat-transfer coefficient to the rate of heat transfer does not hold for $U$ based on the average air temperature. The heat-transfer coefficients in the present report for the blower-cooling set-up have been calculated; both the difference between the average surface and the average air temperatures and the difference between the average surface and the inlet-air temperature were used. The heat-transfer coefficients based on the former temperature difference were calculated to determine whether factors used to correlate pipe data would be applicable to finned-cylinder data. For the other air-flow arrangements, only the inlet-air temperature difference was used.

## SURFACE HEAT-TRANSFER COEFFICIENTS

The over-all heat-transfer coefficient $U$ of finned cylinders can be calculated from the following equation, which was derived in reference 1:

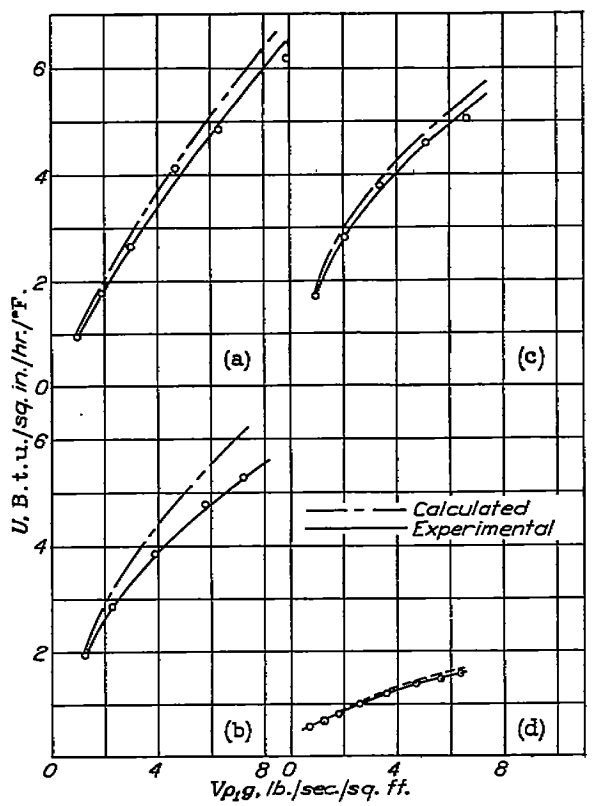
$$U=\frac{q}{s+t}\left[\frac{2}{a}\left(1+\frac{w}{2R_b}\right)\tanh aw'+s_b\right] \qquad (1)$$

where $a=\sqrt{2q/k_m t}$ and $w'=w+t/2$. Equation (1) has been experimentally verified for a large number of steel cylinders having a thermal conductivity $k_m$ of 2.17 B. t. u. per square inch through 1 inch per hour per °F.

In the present investigation, further verification has been made of this equation for pure copper, in which $k_m$ was taken as 18.04, and for aluminum Y alloy, in which $k_m$ was taken as 7.66. When aluminum or aluminum alloy is hereinafter mentioned, aluminum Y alloy is meant. The results of these tests are shown in figure 3 for three copper cylinders having different fin designs and for one aluminum-alloy cylinder. These cylinders were tested with blower cooling. The values of $U$ calculated from equation (1) conform reasonably well with the experimental values of $U$.

In equation (1), variables $k_m$ and $q$ depend upon experimental tests. Sufficiently accurate data are generally available for the values of $k_m$. The various possible factors that could affect the surface heat-transfer coefficient $q$ of finned cylinders may be enumerated as follows:

1. Characteristics of surface.
2. Intensity and scale of turbulence.
3. Fin and cylinder dimensions: space, width, thickness, and diameter.
4. The weight velocity, the conductivity, and the viscosity of the cooling air.




| | s (in.) | w (in.) | t (in.) | Set-up | Material |
|---|---|---|---|---|---|
| (a) | 0.028 | 3 | 0.035 | Blower cooling | Copper. |
| (b) | .055 | 3 | .035 | do | Do. |
| (c) | .08 | 3 | .035 | do | Do. |
| (d) | .20 | 1.66 | .050 | do | Aluminum. |

FIGURE 3.—Comparison of calculated and experimental values of $U$ for metals of high thermal conductivity.

The effect of each of these factors will now be considered.

### EFFECT OF SURFACE CHARACTERISTICS

The results of the tests on the smooth-enameled cylinder are shown in figure 4 for wind-tunnel tests at a constant air speed of 52 miles per hour. These tests show a slight improvement in $q$ with enamel thicknesses up to 0.002 inch; beyond this thickness, the heat transfer decreases and, with thicknesses greater than 0.005 inch, it becomes less than it is without enamel. The highest heat transfer is presumably obtained with just sufficient enamel to produce a smooth, black surface; any further increase in thickness results in a lowered heat transfer caused by decreased thermal conductivity.

In the tests to determine the effect of an oil film on the heat transfer, the effect of the air stream and the heated cylinder surface was continuously to decrease the oil-film thickness; and it was difficult to arrive at a satisfactory conclusion from these tests. The oil film had no measurable effect on the heat transfer obtained during these tests.

### EFFECT OF TURBULENCE

The results of the tests with different devices for creating turbulence are shown in figure 5. A single curve adequately represents the data for design purposes. A maximum deviation of the test points of

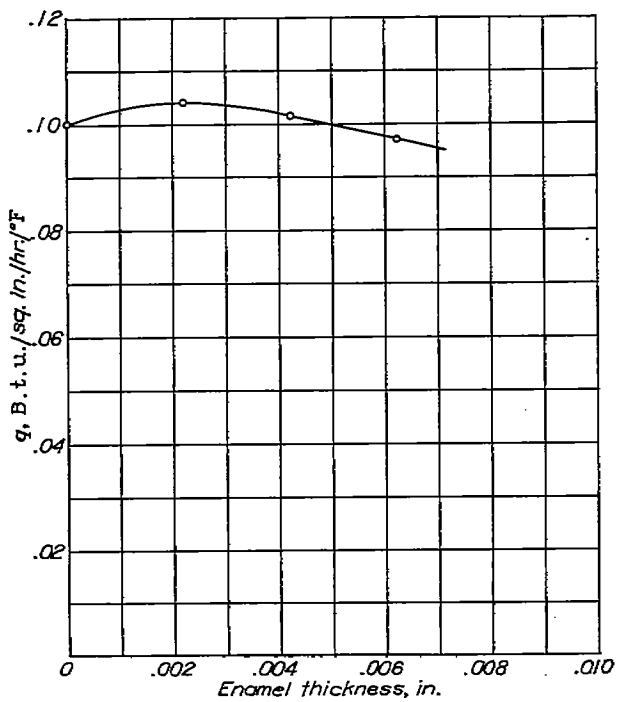



FIGURE 4.—Effect of enamel on surface heat-transfer coefficient $q$ of a smooth cylinder.

approximately 10 percent is caused by placing the 9-inch drum parallel and at distances between 0.3 inch and 13 inches from the test cylinder. The drum in this position caused a blocking effect in addition to the air swirl, and the condition is therefore outside the range of useful design. If the test points for this condition are disregarded, much less deviation is obtained and the conclusion may be drawn that the scale and the intensity of turbulence in the tunnel air stream for the range covered in these tests have an inappreciable effect on the heat transfer.

Although the turbulence impressed upon the air stream had little effect on the heat transfer, another flow condition produced between the fins when the fin-plane/air-stream angle lies between 30° and 60° has resulted in high rates of heat transfer (reference 10).

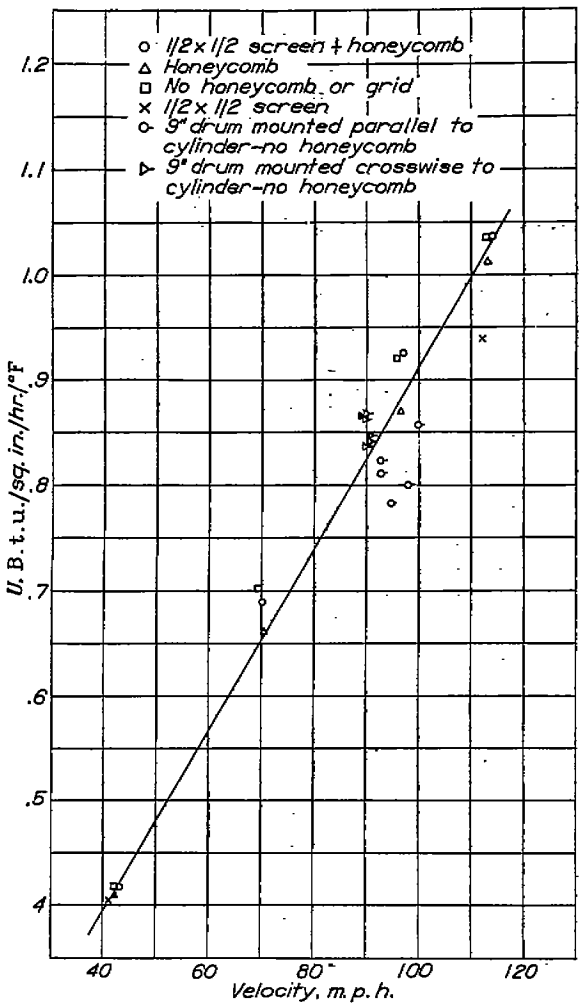



FIGURE 5.—Effect of turbulence devices on the over-all heat-transfer coefficient $U$ of the 0.077–0.37–0.035 cylinder with 90° baffles.

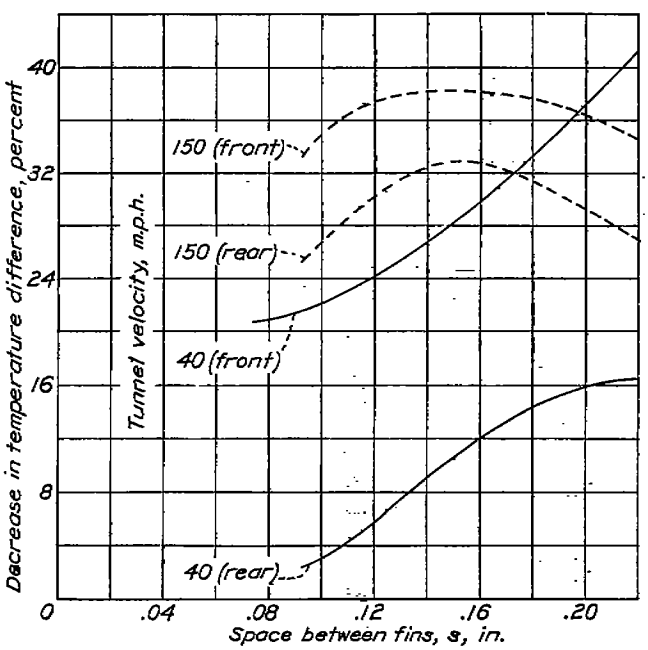



FIGURE 6.—Percentage decrease in temperature difference of cylinders with a fin-plane/air-stream angle of 45° as compared with the same cylinders at a fin-plane/air-stream angle of 0°.

Although the nature of the flow between the fins for this condition is not definitely known, lampblack-kerosene pictures of the flow over the fin surfaces indicate that, when the air stream strikes the fins at an oblique angle, vortices formed between the fins move around the cylinder with the general flow. Additional information on the result of this condition is given in figure 6. This figure shows the percentage improvement in cooling obtained with the 45° fin-plane/air-stream angle as compared with the parallel-flow condition for the front and the rear locations on the cylinder for a range of fin spacing. In general, the improvement becomes less with closely spaced fins.

## EFFECT OF CYLINDER DIAMETER

The data on $q$ for the 0.124–0.50–0.026 cylinder of 3.66-inch diameter and the 0.12–0.52–0.03 cylinder of

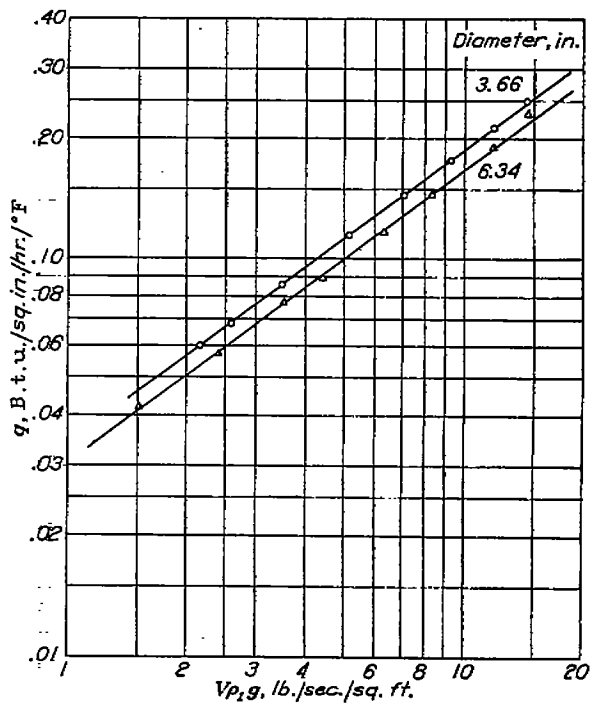



FIGURE 7.—Surface heat-transfer coefficients $q$ for two cylinder diameters.

6.34-inch diameter are plotted in figure 7 to show the effect of cylinder diameter. For the range of data obtained, the relation between $q$ and $D$ can be conveniently expressed by

$$q \propto \frac{1}{D^{0.25}}$$

Empirical functions determined from tests with coiled pipes show that $q$ varies as an inverse function of the radius of curvature of the coils (reference 11, p. 179). Little information is available to indicate more completely the relationship between the cylinder diameter and $q$.

### CORRELATION OF VARIABLES AFFECTING $q$

In the mechanism of heat transfer by convection in pipes, the velocity gradient across the stream does not follow any simple relation but apparently involves two zones: the film layer and the turbulent core. Three methods of attack have been used to predict the heat transfer, namely:

1. By mathematical analysis.
2. By analogy between results from heat transfer and friction.
3. By dimensional analysis.

With turbulent flow, the application of mathematical analysis has met with little success. Analogies between heat transfer and friction hold only for simple cases, such as for straight pipes. The use of dimensional analysis has been the most successful method of attack, the constants in the equations being experimentally determined. For the conditions of viscous flow, mathematical analysis has been used with greater success; here again the experimental results are checked only by applying empirical corrections to the theory.

The heat transfer of straight pipes and ducts has been correlated by means of various dimensionless factors. McAdams (reference 11, pp. 203–210) has recorrelated a greater part of the available data on straight pipes in the viscous region according to a theoretical equation that assumes a parabolic velocity distribution and an absence of free convection currents. This equation can be approximately represented, for values of $Wc_p/k_al$ greater than 10, by the equation

$$\frac{q_{av}d}{k_a}=1.65\left(\frac{Wc_p}{k_al}\right)^{1/3} \tag{2}$$

By the substitution of the weight velocity and the area for $W$, equation (2) can be rearranged as follows:

$$\frac{q_{av}d}{k_a}=1.5\left(\frac{V\rho_1 gdc_p}{k_a}\times\frac{d}{l}\right)^{1/3} \tag{3}$$

For air, the Prandtl number, $c_p\mu/k_a$, is approximately 1, and substitution in equation (3) gives the Nusselt number as a function of the Reynolds Number and the $d/l$ ratio of the pipe

$$\frac{q_{av}d}{k_a}=1.5\left(\frac{V\rho_1 gd}{\mu}\times\frac{d}{l}\right)^{1/3} \tag{4}$$

The designation of the symbols are as previously given but the dimensions are so chosen as to make both sides of equations (2), (3), and (4) dimensionless.

Colburn (reference 12) has pointed out that McAdams' correlations show that data on heating fall above the theoretical curve and that data on cooling fall about equally above and below the theoretical equation. Colburn attributed the discrepancy between the theory and the data to free convection effects and to velocity distributions different from that assumed in deriving the equation. Accordingly, Colburn amplified equation (2) to include these effects by introducing a factor $\Phi$, which involves the Grashof number and the ratio of the viscosity of the fluid, based on an arithmetic mean temperature of the fluid to the viscosity of the fluid based on an average fluid-film temperature. Equation (2) has been found, however, to apply with a fair degree of accuracy when the Grashof number and the ratio of the viscosities are approximately 1.

For the turbulent region, the Nusselt number has been found to be a function of the Reynolds Number and the Prandtl number; and, as the value of the Prandtl number is approximately 1 for air, it can be eliminated from the equations with little error.

In previous investigations, the heat-transfer data of rectangular ducts have been correlated in a manner similar to that of pipes by using an "equivalent" diameter equal to four times the area of the duct divided by the perimeter. Practically no data are available on the effect of curvature in ducts other than pipes. In tests on coiled pipes, the Nusselt number was found to be a function of the ratio of the radius of the pipe to the radius of curvature in addition to the factors already mentioned.

The correlation of heat-transfer data for finned cylinders is made more difficult than the correlation for pipes because of the complicated shape of finned cylinders and the fact that the boundary-layer velocity gradient between the fins from the front to the rear probably changes. The flow passages between the fins of finned cylinders can be considered as curved ducts, especially when baffles or a jacket is used. It is possible that the variables—heat-transfer coefficient, cylinder and fin dimensions, and air-stream characteristics—can be correlated by means of one or more of the dimensionless factors considered in the foregoing paragraphs. Owing to the difficulty of applying mathematics to this problem, recourse was had to the experimental results with blower cooling to determine empirical relationships for correlating the data.

Colburn (reference 12) found that, in the turbulent region, data on pipes could be correlated if the viscosity was based on the film temperature of the fluid, a mean of the average surface and the average fluid temperatures; whereas, in the viscous region, it made no difference whether the fluid or the film temperature was used. In the following correlation of heat-transfer data, the viscosity and the conductivity of the cooling air for both the turbulent and the viscous flow regions were based on the film temperature of the fluid.

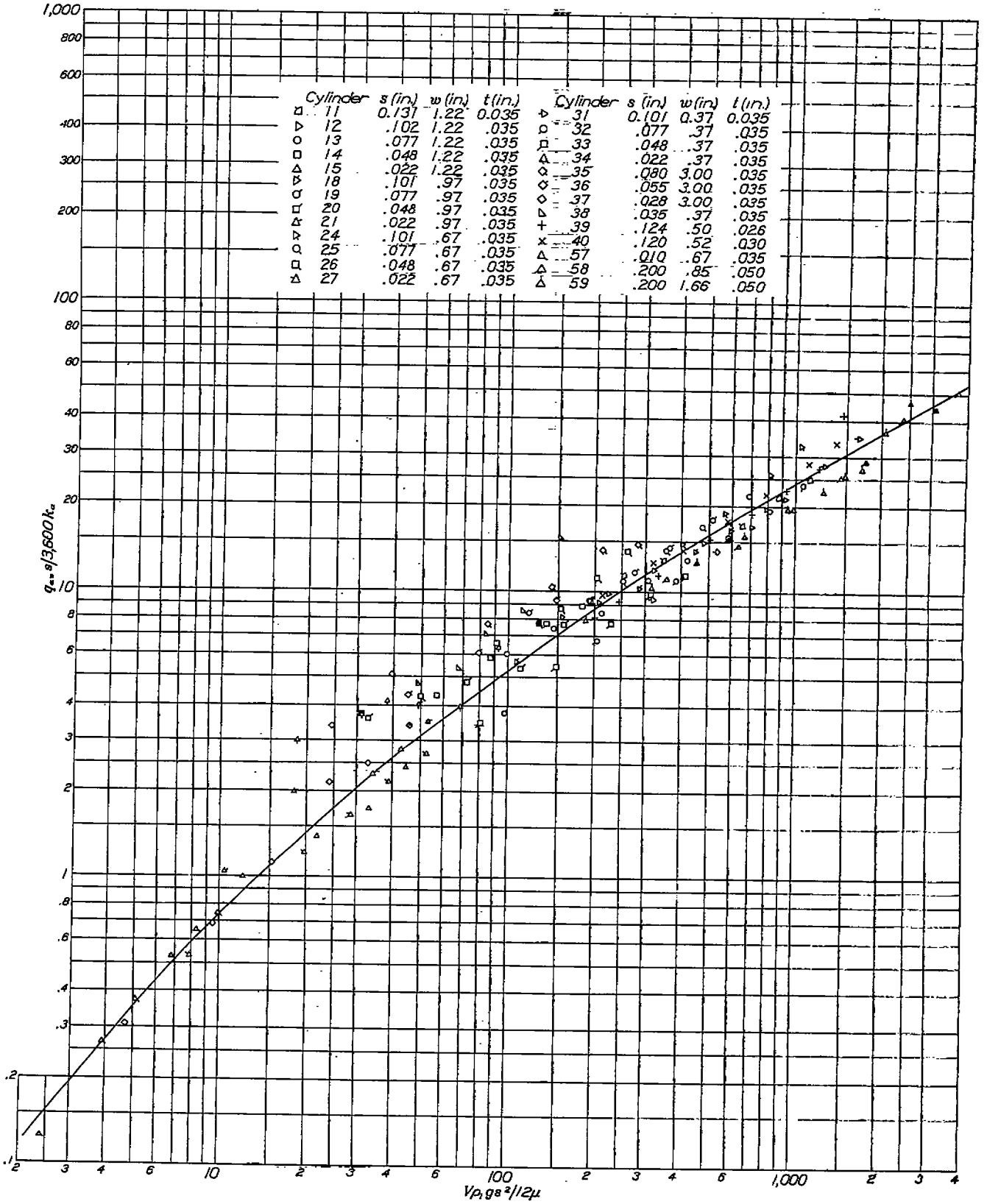
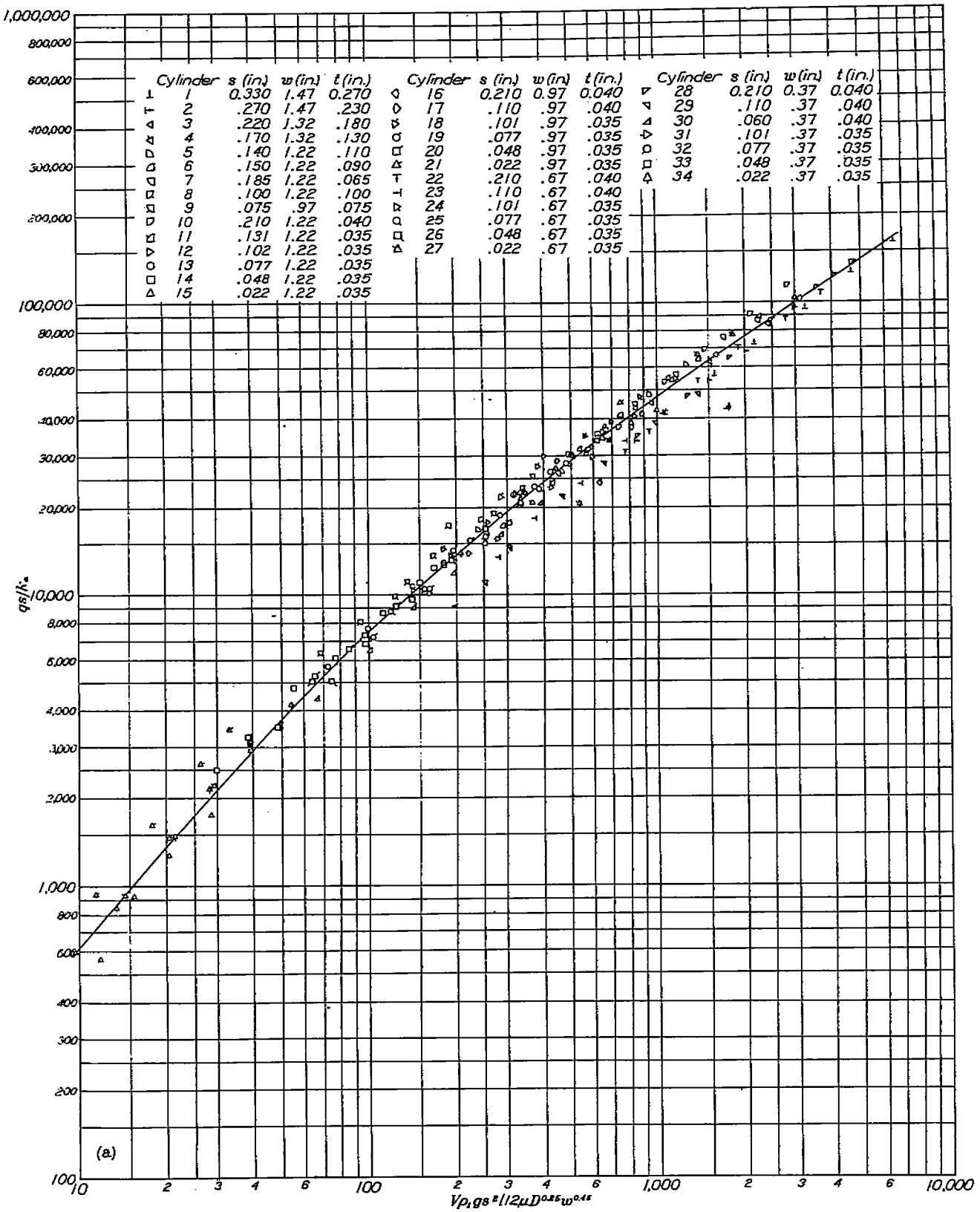
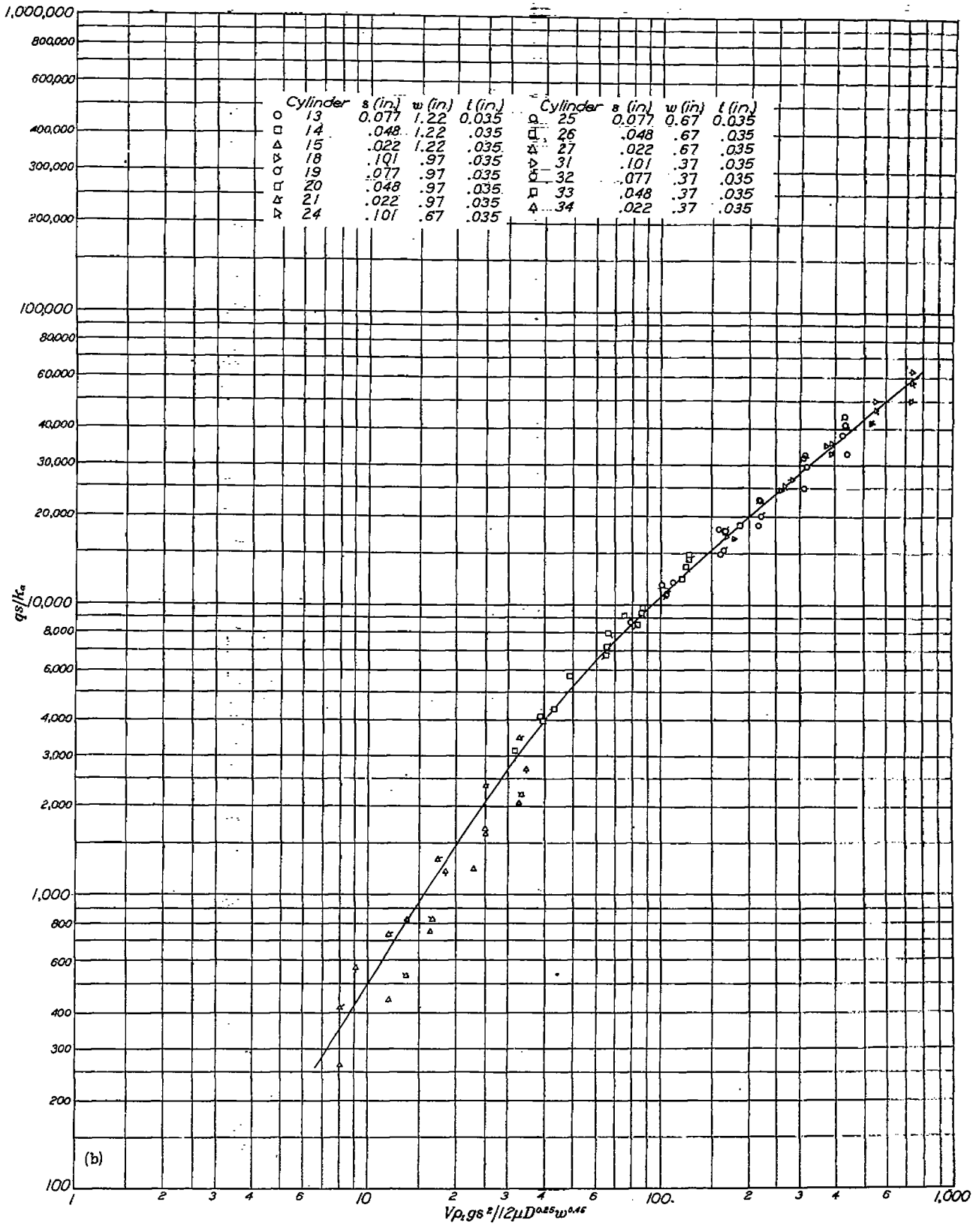



FIGURE 8.—Relation between factors involving $q_{av}$, fin dimensions, and air-stream characteristics for blower cooling.



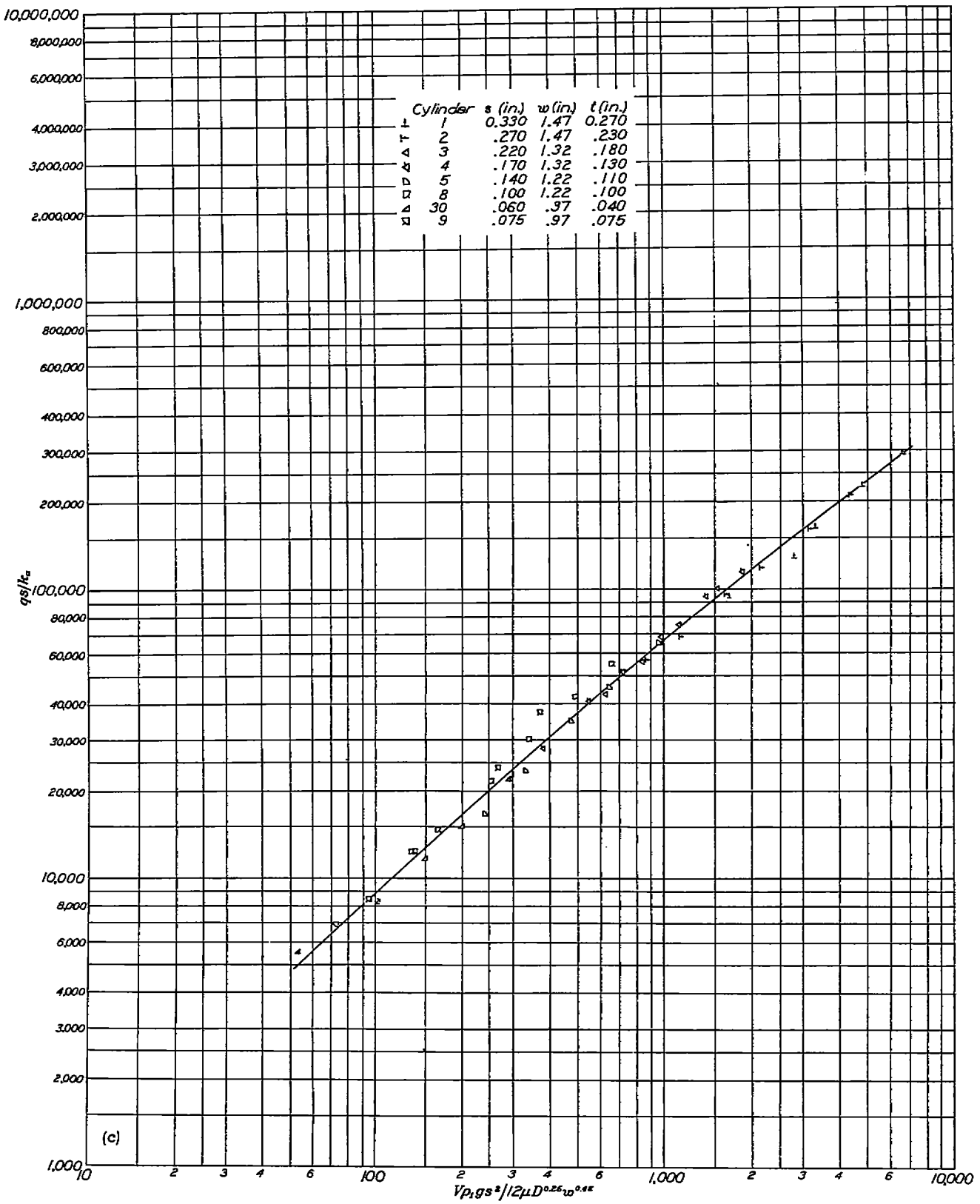

(a) Cylinder in free air stream, no baffles.

FIGURE 9.—Relation between factors involving $q$, fin dimensions, cylinder diameter, and air-stream characteristics.



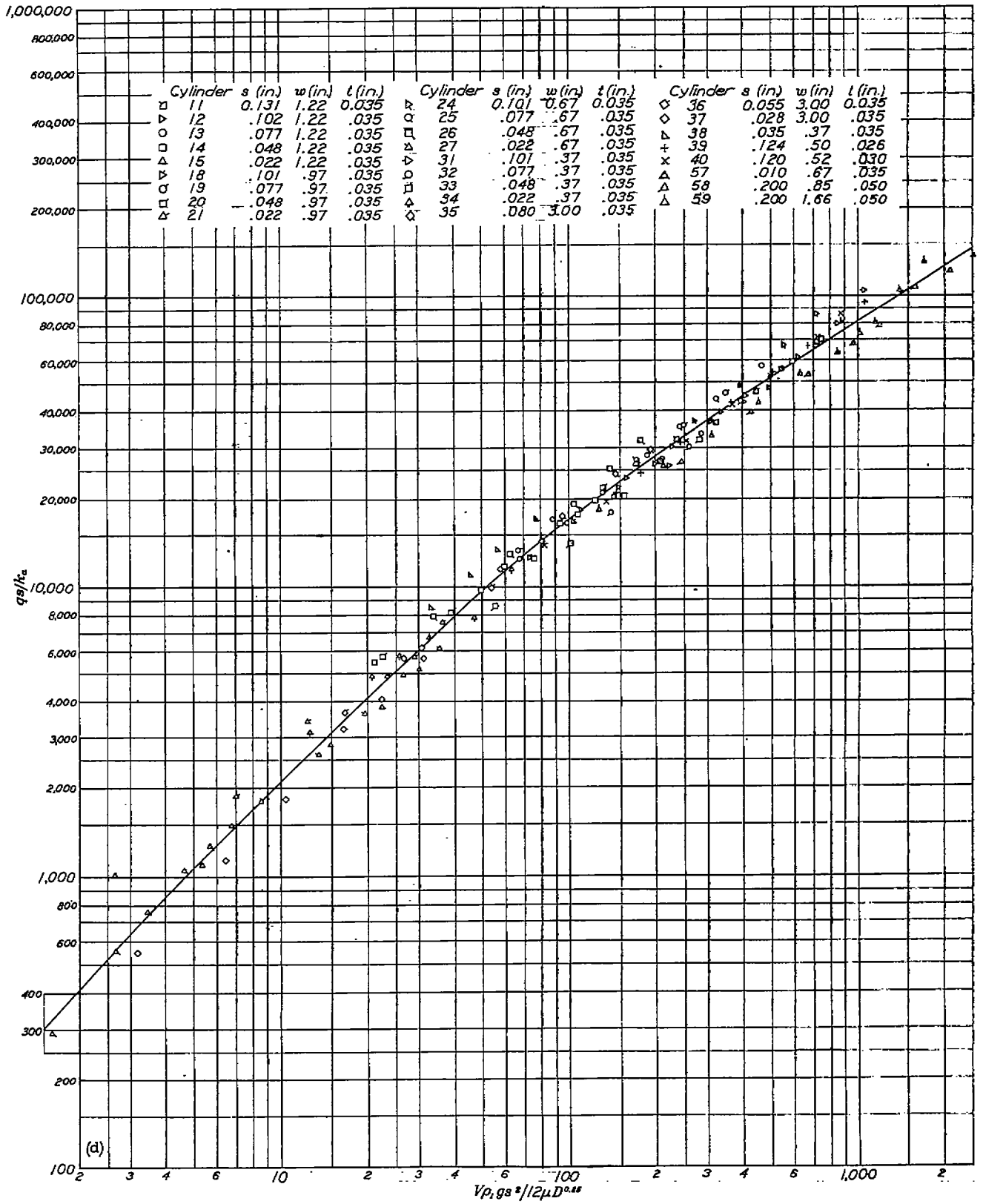

(b) Cylinder in free air stream, 140° baffles.

FIGURE 9—Continued.—Relation between factors involving $q$, fin dimensions, cylinder diameter, and air-stream characteristics.




(c) Cylinder in free air stream, cylinder axis 45° to air stream.

FIGURE 9—Continued.—Relation between factors involving $q$, fin dimensions, cylinder diameter, and air-stream characteristics.




(d) Cylinder enclosed in jacket, blower cooling.

FIGURE 9—Continued.—Relation between factors involving $g$, fin dimensions, cylinder diameter, and air-stream characteristics.

In the correlation of the data of the present tests, the heat-transfer coefficients were based on the difference between the average surface temperature and the average fluid temperature. The correct temperature difference to use in calculating heat-transfer coefficients depends upon the temperature distributions of the surface and the fluid along the duct length. In cases where the surface temperatures remain approximately constant throughout the length of the duct, the correct average temperature difference is the logarithmic mean average. In cases where the surface temperatures rise linearly with that of the fluid, as is practically always true for air-cooled finned cylinders, the difference between the average surface and the average fluid temperatures should be used.

In the present investigation, little success toward correlating the data was had by plotting Nusselt number against Reynolds Number as is done in the heat-transfer tests of pipes in the turbulent region or by plotting the results in which the factors in equation (4) were used. The equivalent diameter and the length of passage were used in these calculations. The calculations showed that the Grashof number and the ratio of viscosities mentioned previously were practically 1, so that the factor $\Phi$ was considered not to enter into the correlation.

After several trials, it was found that the best correlation of the data could be obtained by plotting as follows:

$$\frac{q_{av}s}{3600k_a}=f\left(\frac{V\rho_1 gs}{12\mu}s\right) \tag{5}$$

where $q_{av}s/3600\ k_a$ is a Nusselt number and $V\rho_1 gs/12\mu$ is a Reynolds Number. The replacement of $s$ by $d_e$, the equivalent diameter of the passage, resulted in a satisfactory correlation but with a greater dispersion of the test points than when $s$ was used. Figure 8 shows the results of the tests with blower cooling plotted according to equation (5). Efforts to correlate the data by inserting $w$, $D$, or $l$ in the right-hand side of equation (5) to make this factor dimensionless were unsatisfactory.

Figure 9 shows the variation of $q$ with fin and cylinder dimensions and air-stream characteristics for four different air-flow arrangements. The functions given for each arrangement are purely empirical and hold for the range of conditions tested. It was found that, for the cylinders with and without baffles and for the cylinders tested at 45° fin-plane/air-stream angle,

$$\frac{qs}{k_a}=f'\left(\frac{V\rho_1 gs^2}{12\mu D^{0.25}w^{0.45}}\right) \tag{6}$$

and, for the blower-cooling arrangement,

$$\frac{qs}{k_a}=f''\left(\frac{V\rho_1 gs^2}{12\mu D^{0.25}}\right) \tag{7}$$

The curves of figure 9 represent the data well enough for all practical purposes.

## PRESSURE DIFFERENCES WITH BLOWER COOLING

### EFFECT OF FIN SPACE AND WIDTH

The weight velocity required between fins to obtain the desired values of $q$ often requires much higher pressure differences than are available. For this reason, the pressure drop across finned cylinders is of considerable interest.

The pressure differences across cylinders enclosed in jackets similar to the one shown in figure 2 (b) have been determined for a large number of cylinders in addition to those tested in this investigation (reference 5). Faired curves of all these data are plotted in figure 10. The effect of the fin width on the pressure difference for the range of widths covered between 0.37 inch and 1.5 inches was negligible.

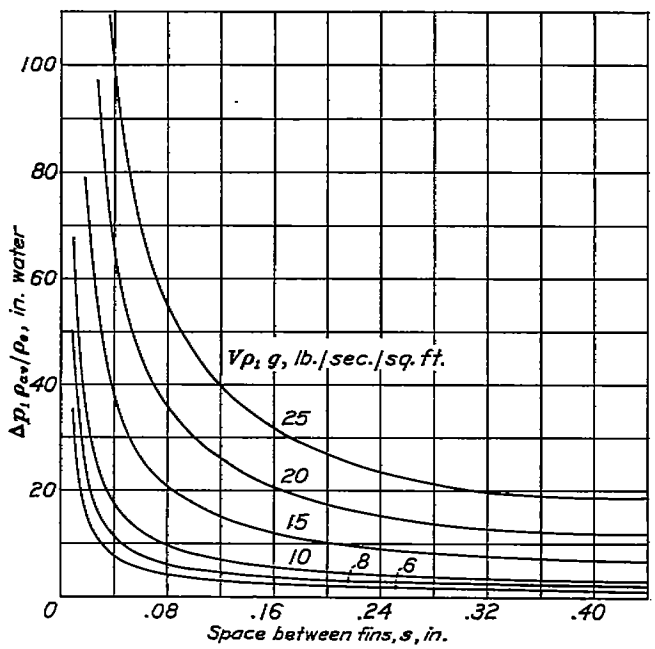



FIGURE 10.—Effect of fin space on pressure difference across finned cylinders at several weight velocities of the cooling air. Fin widths from 0.37 to 1.5 inches. Blower cooling.

Since the foregoing results were published, additional measurements of pressure drop have been made on a cylinder having a fin width of 3 inches and fin spaces of 0.028, 0.055, and 0.08 inch. These data show a higher pressure drop for the same fin spacing than was obtained with the narrower fin widths. Previous tests (reference 5) have shown that the pressure difference is directly proportional to the length $l$ of the flow path. The data obtained with the 3-inch fins are plotted in figure 11 in terms of $\Delta p_1/l$ together with curves obtained by cross-plotting figure 10 for the same values of fin space as were used in the cylinders with wide fins. The length $l$ of the flow path for the cylinders of figure 10 was taken as an average value corresponding to an average fin width of 0.935 inch. The data for the 3-inch fins were in good agreement with the curves cross-plotted from figure 10 for all but the smallest value of $s$. From these data it may be concluded that, for the useful range of

fin spacing, the pressure drop is proportional to the length of the flow path. The pressure difference is affected by the fin width in direct proportion to the effect of the width on the length of the flow path.

### EFFECT OF CYLINDER DIAMETER

The values of $\Delta p_1/l$ for the cylinders of diameters 3.66 and 6.34 inches are also plotted in figure 11 for comparison with the data from the 4.66-inch-diameter cylinder. With the exception of the data for the small-diameter cylinder at the low weight velocities, the pressure drop across the cylinders again varies as the length of the flow path and is affected by the cylinder diameter in direct proportion to the effect of the diameter on the length of the flow path.

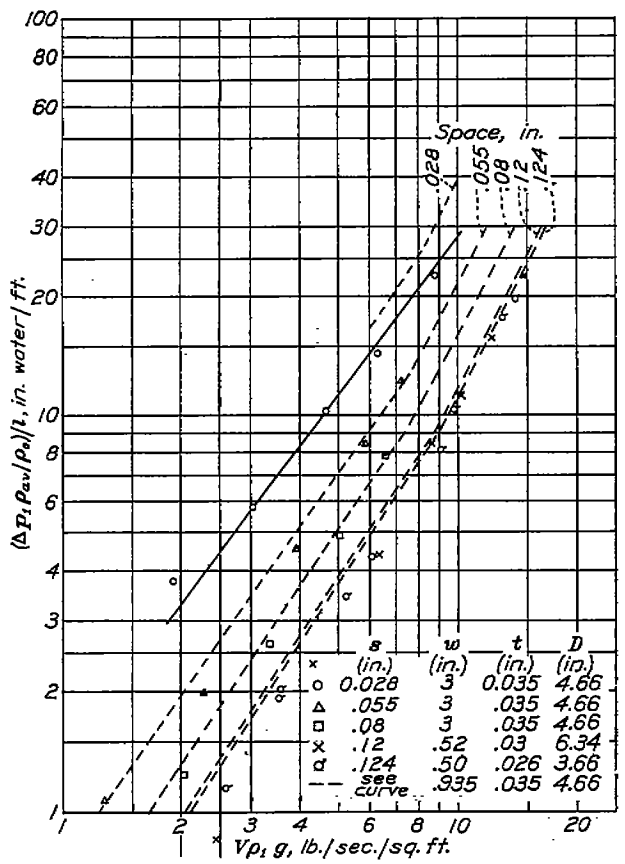



FIGURE 11.—Effect of weight velocity of cooling air on the pressure difference per foot length of path $l$ for cylinders of different fin widths and diameters.

## CONCLUSIONS

1. The surface heat-transfer coefficients of finned cylinders can be correlated for any one air-flow arrangement by plotting a factor involving the surface heat-transfer coefficients, the fin space, and the conductivity of the cooling air against a factor involving the velocity, the density, and the viscosity of the cooling air and the fin space, the fin width, and the cylinder diameter.

2. A variation of the initial turbulence in the tunnel air stream for the range covered in the present tests had very little effect on the surface heat-transfer coefficient.

3. The improvement in heat transfer obtained with a fin-plane/air-stream angle of 45° as compared with one of 0° is appreciably affected by the value of the space between fins.

4. The pressure difference across a finned cylinder is affected by the fin width, the cylinder diameter, and the front baffle opening in direct proportion to the effect that these three dimensions have on the length of path $l$ as defined in the report.

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *April 27, 1939.*

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