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By O. Walchner

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THE EFFECT OF COMPRESSIBILITY ON THE PRESSURE READING OF A PRANDTL PITOT TUBE AT SUBSONIC FLOW VELOCITY*

By O. Walchner

SUMMARY

The effect of compressibility of a flow on the pressure reading of a normal Prandtl pitot tube (fig. 2) was investigated while the air speed approaches the velocity of sound. Errors arising from yawed flow were also determined up to 20° angle of attack. In axial flow, the Prandtl pitot tube begins at $w/a \approx 0.8$ to give an incorrect static pressure reading, while it records the tank pressure correctly, as anticipated, up to sonic velocity. Figures 5 to 7 illustrate the recorded pressure errors for different angles of attack. If errors up to within ±1 percent are permissible in the speed prediction, the pressure difference $p_1 - p_2$ recorded by the Prandtl pitot tube can be evaluated through the equation

$$p_1 - p_2 = \frac{\rho}{2} w^2 \left[1 + \frac{1}{4}\left(\frac{w}{a}\right)^2\right]$$

whereby the range of validity extends to $w/a \leqq 0.95$ and yawed flow up to 10°. The equation is plotted for standard atmosphere in figure 9.

Owing to the compressibility of the air, the Prandtl pitot tube manifests compression shocks when the air speed approaches velocity of sound. This affects the pressure reading of the instrument. Because of the increasing importance of high speed in aviation, this compressibility effect is investigated in detail. (The results of similar investigation by Panetti (reference 1) are not directly comparable with the present findings because his instrument was not of normal dimensions.)

*"Über den Einfluss der Kompressibilität auf die Druckanzeige eines Prandtl-Rohres bei Strömungen mit Unterschallgeschwindigkeit." Jahrbuch 1938 der deutschen Luftfahrtforschung.

INTRODUCTION

Every stationary compressible or incompressible parallel flow may be visualized as being the creation of loss-free discharge from a tank in which the medium is at rest under a higher pressure. The speed can be predicted according to Bernoulli if the static pressure of the flow and the tank pressure are known.

The introduction of a body in such a flow produces in the stagnation point the same state as in the tank so long as there is no energy loss on the streamlines toward the stagnation point. In incompressible fluid, the flow toward the stagnation point is free from loss, and likewise in compressible flow so long as the speed remains below the velocity of sound. But in supersonic flow a compression shock forms before the obstacle ~~which~~ and divides the zones with supersonic speed upstream from the subsonic speed downstream toward the stagnation point. Energy is lost in this compression shock, hence the pressure in the stagnation point becomes lower than the tank pressure.

Thus a Prandtl pitot tube with its forward orifice in the stagnation point indicates the <u>tank pressure</u> of the flow if the medium is incompressible, or, when in compressible fluid the speed remains subsonic.

The field of disturbance which is impressed upon the undisturbed flow in the circulation about the Prandtl pitot head disappears again downstream. Now, the normal Prandtl pitot is so designed that the static orifice is at the place where in incompressible fluid the undisturbed state is re-established. Then the speed can be predicted on the basis of the pressure readings of the Prandtl tube.

In compressible fluid the curvature of the streamlines around the head of the tube is amplified because of the change of density with the pressure. So long as the local velocity of sound is not exceeded at any point (the velocity of sound changes with the temperature along a stream filament) the field of disturbance is merely distorted while qualitatively remaining similar to that in incompressible flow. But its character changes as soon as the flow velocity becomes high enough to create a zone with supersonic speed during the circulation about the Prandtl tube, which downstream must again change to subsonic flow with a compression shock. This causes a more

or less severe disturbance in the flow at the static orifice of the Prandtl pitot, with the result that the pressure of undisturbed flow can no longer be measured at that point.

Strictly speaking, the speed prediction with the Prandtl pitot therefore is contingent upon the knowledge of the errors of the pressure readings from calibration of the instrument and their correction. The experimental determination of this correction is related in the following.

Notation

w, velocity in undisturbed flow;

a, velocity of sound in undisturbed flow;

ρ, density in undisturbed flow;

p, pressure in undisturbed flow;

p_0, tank pressure;

p_1, pressure at forward orifice of Prandtl pitot;

p_2, pressure at lateral orifice of Prandtl pitot;

κ, ratio of specific heat at constant pressure and constant volume, respectively.

DESCRIPTION OF TEST EQUIPMENT

The high-speed tunnel (Prandtl type) employed is shown in figure 1. A container B is pumped empty so that the atmospheric air (pressure p_0, density ρ_0, and speed $w_0 = 0$) flows, after opening of cock H, as a free jet through the test chamber M into the container. On leaving the entrance cone E the air stream attains the speed w, the pressure expands to p and the density to ρ; a indicates the velocity of sound in the free jet. The wind velocity w is regulated with the nozzle V whose narrowest section is adjustable. So long as the pressure in B stays low enough to produce velocity of sound in the narrowest section of V, the jet remains stationary and the flow volume and jet velocity, respectively, change only

when the narrowest section of V is changed, with the provision that the section of the entrance cone is greater than the narrowest section of nozzle V.

The speed w and the Mach number $\frac{w}{a}$, respectively, in the free jet follow, according to Bernoulli, from the pressure ratio $\frac{p_o}{p}$. The pressure p is tapped through a wall orifice shortly before the exit from the entrance cone, and the <u>tank pressure</u> p_o, which, of course, must not be confused with the pressure in container B, corresponds to the atmospheric air pressure. Then Bernoulli's equation for stationary compressible flow without substantial local height differences reads:

$$\frac{w^2}{2} + \int \frac{d\,p}{\rho} = \text{const} \tag{1}$$

Since adiabatic change of state may be assumed ($\kappa = 1.405$ for air) in the flow through the entrance cone, hence

$$\frac{1}{\rho} = \frac{1}{\rho_o}\left(\frac{p_o}{p}\right)^{1/\kappa} \tag{2}$$

the integral in equation (1) becomes

$$\int \frac{d\,p}{\rho} = \frac{\kappa}{\kappa - 1}\,\frac{p_o^{\,1/\kappa}}{\rho_o}\,p^{\frac{\kappa - 1}{\kappa}} = \frac{\kappa}{\kappa - 1}\,\frac{p}{\rho} \tag{3}$$

For the state of rest p is replaced by p_o, ρ by ρ_o, and w by $w_o = 0$, whence the constant of the Bernoulli equation

$$\text{const} = \frac{\kappa}{\kappa - 1}\,\frac{p_o}{\rho_o} = \frac{\kappa}{\kappa - 1}\,\frac{p}{\rho}\left(\frac{p_o}{p}\right)^{\frac{\kappa - 1}{\kappa}} \tag{4}$$

Entering equations (3) and (4) in equation (1) gives the flow velocity in the free jet at

$$w = \sqrt{\frac{2\,\kappa}{\kappa - 1}\,\frac{p}{\rho}\left[\left(\frac{p_o}{p}\right)^{\frac{\kappa - 1}{\kappa}} - 1\right]} \tag{5}$$

This equation is known as the discharge formula for the case where a flow with velocity w, pressure p, and density ρ is produced through adiabatic discharge from a tank in which the medium rests under pressure p_0.

Then the introduction of the velocity of sound in the free jet

$$a = \sqrt{\frac{d\,p}{d\,\rho}} = \sqrt{\kappa \frac{p}{\rho}} \tag{6}$$

gives the Mach number

$$\frac{w}{a} = \sqrt{\frac{2}{\kappa - 1}\left[\left(\frac{p_0}{p}\right)^{\frac{\kappa - 1}{\kappa}} - 1\right]} \tag{7}$$

Thus, w, ρ, a, p, and κ denote the state quantities of undisturbed flow for the Prandtl pitot tube, the dimensions of which are given in figure 2. The pressure at the forward orifice is hereafter indicated with p_1 and the pressure at static orifice on the side with p_2. For determining the effect of yawed flow, the tube could be turned in the sense of the arrow in figure 1.

The measurements included the incorrect readings caused by compressibility effects and yawed flow, i.e., the pressure differences $p_0 - p_1$ and $p - p_2$ over a speed range from around 0.55 times sonic velocity up to near the velocity of sound. The pressure field in the vicinity of the forward part of the Prandtl tube was treated by the Schlieren method and photographed.

RESULTS

The first appearance of a shock wave was recorded by the Schlieren method at a Mach number of $w/a \approx 0.7$. The intensity of the shock increases very little at first by an increasing Mach number. Figure 3 is a Schlieren record at $w/a = 0.85$. The dark area denotes a compression of the medium in flow direction and the light areas an expansion. Note the compression toward the stagnation point followed by expansion at circulation about the head. Behind this expansion a compression follows again. The surprising fact now is the location of the shock wave in the middle of the

expansion zone (light area). It probably is no genuine compression shock, which would have to lie at the transitional point from supersonic to subsonic flow, but rather a condensation shock which in its effect is very much like the genuine shock. Such expansion shocks in the expansion zone are, for instance, frequently observed in Laval nozzles when inducting atmospheric air. Illumination of the dark field readily reveals the onset of condensation (nebulosity) to be coincident with the shock.

If, during flow around an obstacle in the wind tunnel of the described type, a condensation shock occurs, this process is not transferable to the case of a body moving at the same Mach number in calm atmospheric air. For the condensation shock depends on the relative humidity in the undisturbed flow and it is greater in the wind tunnel because of the lower pressure in the free jet than in the atmosphere. Further investigation shows, however, that shocks of the low intensity of figure 3 scarcely falsify the static pressure record with the Prandtl tube, so that the defective model similitude as regards humidity causes no appreciable error.

Another Schlieren record taken at $\frac{w}{a} = 0.95$ is shown in figure 4. On the schlieren emanating from the head of the Prandtl tube a Mach angle of about 52° can be observed, i.e., a local velocity of about 1.26 times the local velocity of sound. The transition from this supersonic zone in subsonic flow is consummated in a strong compression shock.

Figures 5 to 7 illustrate the pressure records of the Prandtl tube in relation to the corresponding values of the undisturbed flow - as an ideal instrument should record - for different p_0/p, along with the correlated $\frac{w}{a}$ from equation (7). The measurements at the different angles of attack are indicated with different marks.

As anticipated, the pressure p_1 in axial flow recorded by the Prandtl tube agrees with the tank pressure p_0 of the undisturbed flow throughout the entire subsonic zone (fig. 5).* With increasing yawed flow, p_1 drops below p_0, as in incompressible fluid. But this deparature is at first very slight and independent of the

*See footnote on page 7.

Mach number. Up to 15° angles of attack, the error due to yaw amounts to $\leqq 1$ percent. It increases, however, rapidly with increasing angle of attack, and the measurements themselves manifest then a relationship with the Mach number.

In contrast to p_1, the pressure p_2 recorded with the static orifice of the Prandtl pitot already discloses perceptible deviations with respect to the pressure of undisturbed flow p in the subsonic flow. In axial flow they begin to show at $w/a \approx 0.8$, although the shock waves ascertained by the Schlieren method are still so faint at around $w/a < 0.8$ as to cause no perceptible disturbance in the adiabatic flow.

From $\frac{w}{a} \approx 0.8$ on, p_2 begins a gradual rise relative to p. At first, the opposite might have been expected, since energy is lost in the shock. But the records were well reproducible and they give a definite, even though small, increase of $\frac{p_0}{p}$. The change in streamline curvature with increasing Mach number might play some part herein. No increase in p_2 relative to p under the effect of compression shock alone is expected until the local velocity prior to the shock has exceeded the velocity of sound to such an extent that a velocity substantially lower than that of undisturbed flow is produced after the shock.

The records indicate a rapid decrease of $\frac{p_2}{p}$ shortly before the air speed approaches velocity of sound. This ties in with the fact that the local supersonic zone expands with increasing Mach number, the compression shock being ultimately placed behind the static opening which then records the pressure in the supersonic zone. The Schlieren record disclosed a marked decrease of $\frac{p_2}{p}$ at

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*At supersonic velocity, p_1 becomes smaller than p_0 as a result of pressure loss through the compression shock formed before the instrument. Then the increment of pressure in the stagnation point with respect to the pressure of undisturbed flow is, according to Prandtl:

$$p_1 - p = \frac{\rho}{2} w^2 \left\{ \frac{\kappa + 1}{\kappa} \left[\frac{(\kappa + 1)^2}{4\kappa - 2(\kappa - 1)\left(\frac{a}{w}\right)^2} \right]^{\frac{1}{\kappa - 1}} - \frac{2}{\kappa}\left(\frac{a}{w}\right)^2 \right\}$$

the exact instant where the shock passes over the static test point.

The effect of yawed flow on the static pressure record is the same as for p_1, but of greater percent.

Figure 7 finally shows the pressure difference $p_1 - p_2$ in ratio to $p_0 - p$ for different angles of attack.

PREDICTION OF SPEED

The measurement of the flow velocity in the compressible medium by Prandtl pitot tube requires, strictly speaking, first, the replacement of the recorded pressures based on the described calibration, by the corresponding values for undisturbed flow, i.e., the solution of equations (5) and (7), respectively. But this procedure is not convenient for practical use.

In fact, since exact accuracy is a secondary consideration in many cases and a small error is frequently admissible, some approximate solutions are attempted. Expressing equation (7) in the form

$$\frac{w}{a} = \sqrt{\frac{2}{\kappa - 1}\left[\left(1 + \frac{p_0 - p}{p}\right)^{\frac{\kappa - 1}{\kappa}} - 1\right]} \tag{7a}$$

and substituting the pressure p with equation (6) by

$$p = \rho\,\frac{a^2}{\kappa} = \frac{\rho}{2}\,w^2\,\frac{2}{\kappa(w/a)^2} \tag{6a}$$

equation (7a) gives after solution with respect to $p_0 - p$

$$p_0 - p = \frac{\rho}{2}\,w^2\left\{\left[1 + \frac{\kappa - 1}{2}\left(\frac{w}{a}\right)^2\right]^{\frac{\kappa}{\kappa - 1}} - 1\right\}\frac{2}{\kappa(w/a)^2} \tag{8}$$

Thus equation (8) gives the pressure gradient necessary to create by adiabatic process from state of rest a flow with the characteristic quantities p, ρ, w, and a.

With binomial development of the bracketed term, the first approximation gives for this pressure gradient, as for incompressible fluid,

$$(p_o - p)' = \frac{\rho}{2} w^2 \tag{8a}$$

Second approximation gives

$$(p_o - p)'' = \frac{\rho}{2} w^2 \left[1 + \frac{1}{4}\left(\frac{w}{a}\right)^2\right] \tag{8b}$$

and in third approximation leaves

$$(p_o - p)''' = \frac{\rho}{2} w^2 \left[1 + \frac{1}{4}\left(\frac{w}{a}\right)^2 + \frac{2 - \kappa}{24}\left(\frac{w}{a}\right)^4\right] \tag{8c}$$

Figure 8 gives these approximate values in relation to the exact values of equation (8), along with the value of $p_1 - p_2$ in relation to $p_o - p$ as recorded with the Prandtl pitot tube in axial flow. It is seen that the pressure difference $p_1 - p_2$ recorded by the Prandtl pitot departs qualitatively from the corresponding value for undisturbed flow $p_o - p$ just as the second approximation (equation (8b)) does from the exact value (equation (8)). For $\frac{w}{a} \leqq 0.95$, the difference between $(p_o - p)''$ and $p_1 - p_2$ likewise becomes small quantitatively and amounts to less than ±2 percent.

Hence $(p_o - p)''$ in equation (8b) can be replaced by $p_1 - p_2$, which thus affords an empirical equation

$$p_1 - p_2 = \frac{\rho}{2} w^2 \left[1 + \frac{1}{4}\left(\frac{w}{a}\right)^2\right] \tag{9}$$

from which the speed w in the range of $\frac{w}{a} \leqq 0.95$ can be determined to within less than ±1 percent, when ρ and a are given. The omissions in the approximate calculation at large Mach numbers are approximately equalized by the effects of the compression shocks. Even in yawed flow up to 10° equation (9) leaves within $w/a \leqq 0.95$ a speed

error of less than 1 percent. Equation (9) is shown plotted for standard atmosphere in figure 9. The curves with parameter H give the flying speed w with respect to the pressure difference $p_1 - p_2$ to be recorded with the Prandtl pitot tube for different altitudes up to H = 11 kilometers. Lines with constant Mach number up to $\frac{w}{a} = 0.95$ have been included.

With a potential error in speed of $\leqq \pm 1$ percent, the evaluation of $p_1 - p_2$ gives according to equation (9) or (9a) a result which is accurate enough for many practical cases.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

REFERENCE

1. Panetti, M.: Problemi della tecnica sperimentale alle alte velocita. Convegno di scienze fisiche, matematiche e naturali. Roma, Reale Accademia D'Italia, 1936, XIV.

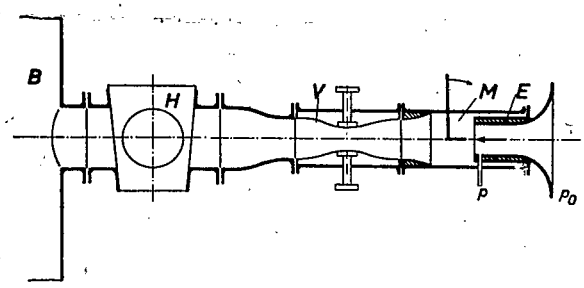

Fig. 1.- Prandtl type high-speed wind tunnel.

pressure - true indicated
impact - p_0 p_1
static - p p_2

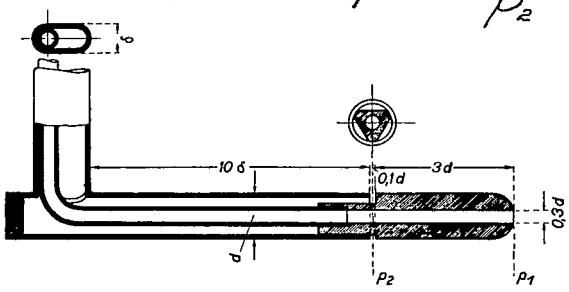

Fig. 2.- Dimensions of normal Prandtl pitot tube.

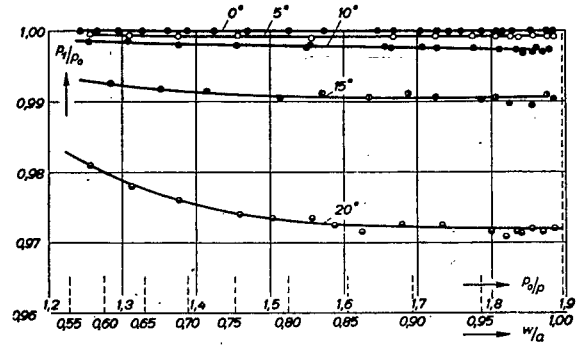

Figure 5

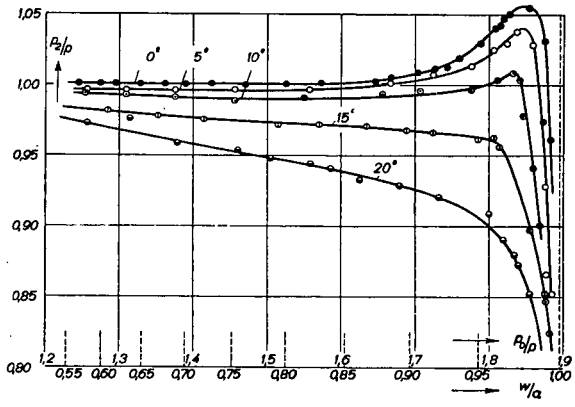

Figure 6

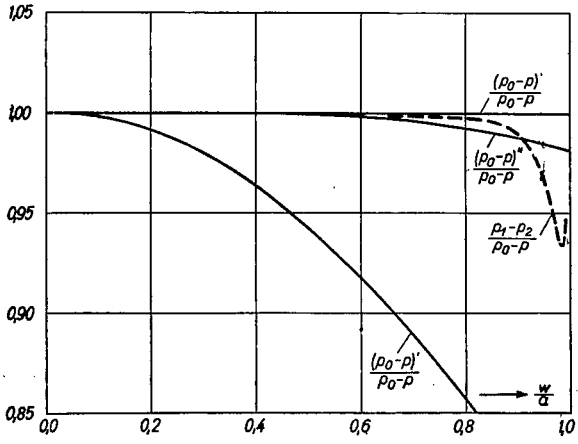

Fig. 8.- Approximate solutions of pressure gradient necessary to adiabatic formation of flow form rest position (eq. 8a,8b) compared to pressure record p_1-p_2 with Prandtl pitot tube.

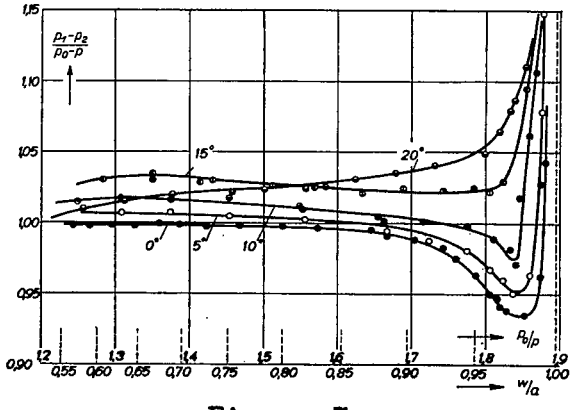

Figure 7

Figure 5-7.- Pressure errors recorded by Prandtl tube as a result of compressibility effect and yawed flow.

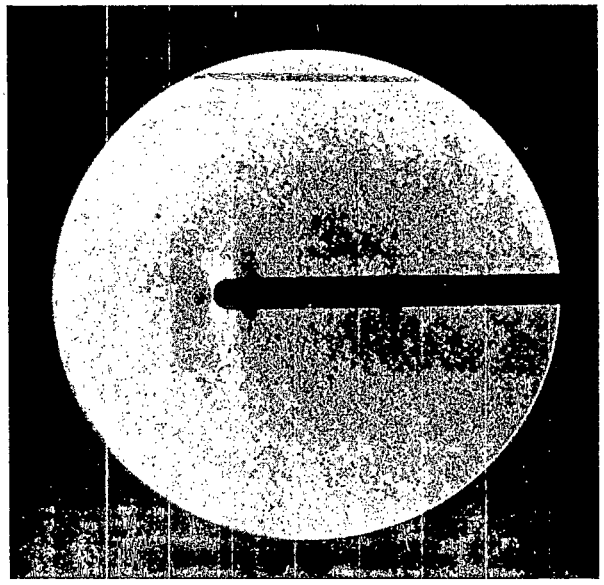

Figure 3.- Schlieren record at w/a = 0.85.

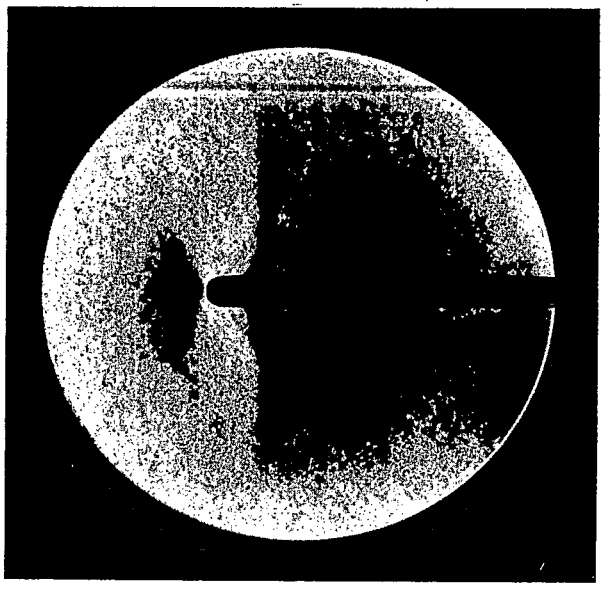

Figure 4.- Schlieren record at w/a = 0.95

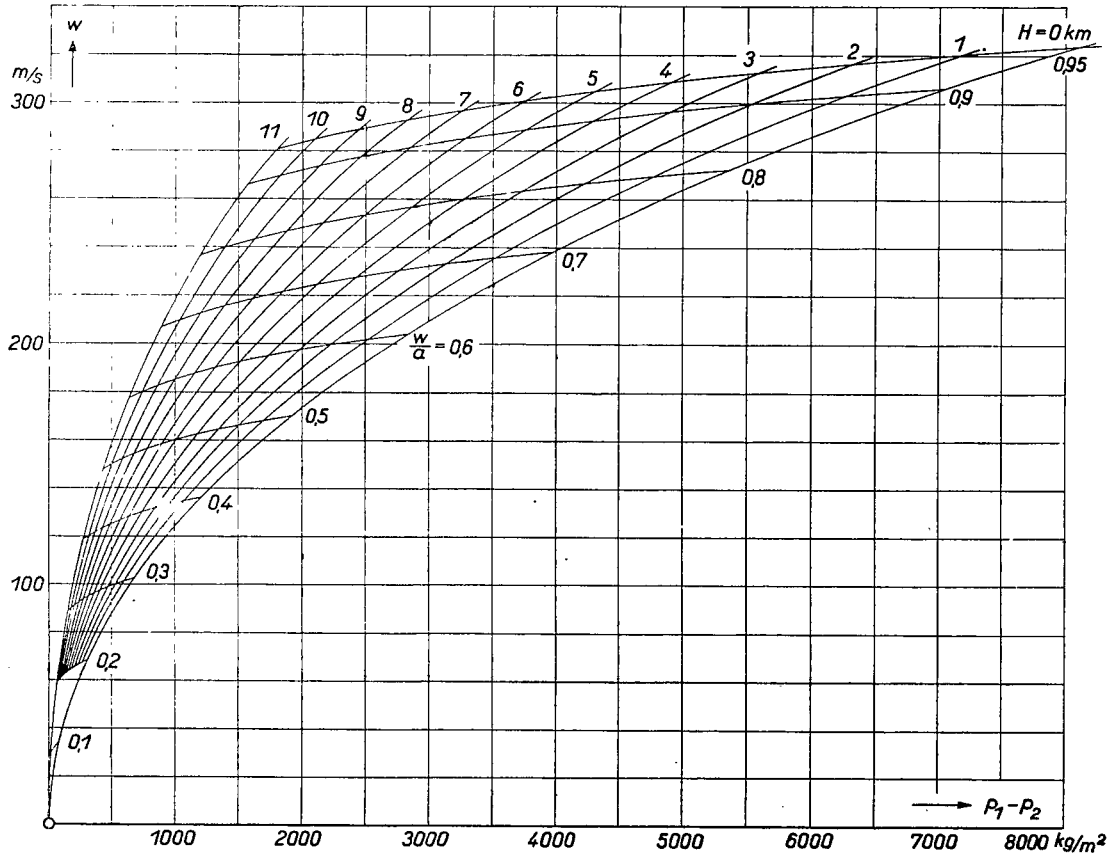

Figure 9.- Correlation of eq. (9) for flight at standard altitude.

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