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STRENGTH TESTS OF THIN-WALLED ELLIPTIC DURALUMIN CYLINDERS IN PURE BENDING AND IN COMBINED PURE BENDING AND TORSION

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STRENGTH TESTS OF THIN-WALLED ELLIPTIC DURALUMIN CYLINDERS IN PURE BENDING AND IN COMBINED PURE BENDING AND TORSION

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SUMMARY

An analysis is presented of the results of tests made by the Massachusetts Institute of Technology and by the National Advisory Committee for Aeronautics on an investigation of the strength of thin-walled circular and elliptic cylinders in pure bending and in combined torsion and bending. In each of the loading conditions, the bending moments were applied in the plane of the major axis of the ellipse.

INTRODUCTION

The strength of thin-walled circular and elliptic cylinders has been under investigation by the NACA for a number of years. Papers have been published giving the results of tests of thin-walled cylinders in torsion, compression, pure bending, and combined transverse shear and bending. The Massachusetts Institute of Technology completed tests made under contract with the NACA of cylinders under combined torsion and bending in the plane of the major axis. The data were recorded in a thesis written at M.I.T. in 1937 by W. E. Sangster and entitled, "Strength Tests of Duralumin Cylinders in Pure Bending and Bending Combined with Torsion." A report based on the data was made by Prof. Joseph S. Newell to the NACA.

The M.I.T. data have now been combined with the NACA data on thin-walled duralumin cylinders contained in references 1, 2, and 3 and the enlarged group of data has been analyzed.

MATERIAL

The material used in the M.I.T. tests was 17S-T aluminum alloy in sheet form with nominal thicknesses of 0.011 and 0.022 inch. The only important physical property of the material required for this paper is the modulus of elasticity E. The modulus of elasticity was determined by cutting a specimen from the undamaged portion of each cylinder after test and milling to the dimensions recommended by the American Society for Testing Materials for tensile tests on thin sheet. Extensometer tests with these specimens resulted in a value of 10.3×10^6 pounds per square inch for the modulus of elasticity.

The material used in the comparable NACA tests was 17S-T aluminum alloy in sheet form. The dimensions and properties of the sheet have been described in reference 1. In the tests of elliptic cylinders reported in reference 2, the nominal sheet thicknesses were identical with those of the sheets used in the M.I.T. tests, and the value of the modulus of elasticity E was 10.4×10^6 pounds per square inch. Thus, the material used in the two groups of tests was essentially the same.

SPECIMENS

The specimens used in the M.I.T. tests consisted of 48 thin-walled unstiffened cylinders, to which combined bending and torsional moments were applied, and of 32 similar cylinders, to which only pure bending moments were applied. Some of the cylinders were circular and others elliptical in cross section. The diameter of the circular cylinders was 15 inches; the major axis of all elliptical cylinders was 15 inches, and the minor axes of the elliptical cylinders were 12, 9, and 6 inches. The ratios of semiminor axis to semimajor axis b/a were thus 1.0, 0.8, 0.6, and 0.4. The lengths varied from 3.75 to 22.5 inches.

In the construction of the specimens, the sheet was cut approximately 1/16 inch shorter than the perimeter of the bulkheads. A butt strap 1 inch wide, having the same nominal thickness at the specimen, was used to complete the cylinder. A clearance of 1/16 inch between the butt strap and the bulkheads was allowed at each end. Care was taken to avoid injuring the specimens during their construction.

For $b/a = 1.0$, 0.8, and 0.6, the bulkheads used to transmit the loads to the cylinders were those previously used in NACA tests (references 1, 2, and 3). For those specimens with $b/a = 0.4$ a new set of similar bulkheads was made at M.I.T.

METHOD OF TESTING

In all cases the loads were applied to the bulkhead, which in turn transmitted the load to the cylindrical shell. The fact that the M.I.T. and the NACA test data were in agreement where overlapping occurred indicates that small differences in the method of applying the load to the bulkheads were unimportant. In both the M.I.T. and the NACA tests, loads were applied in increments of from 1 to 2 percent of the estimated load at failure.

COMPUTATION OF STRESSES

Bending.- The stress f_b at failure on the extreme fiber of the cylinder due to the application of a bending moment is assumed to be given by the formula

$$f_b = \frac{Ma}{I} \tag{1}$$

where

M bending moment at failure

a semimajor axis of ellipse

I moment of inertia of cross section about minor axis

If the wall thickness t is small compared with the semimajor and semiminor axes a and b, the moment of inertia I for the thin-walled elliptic cross section is given by the equation

$$I = \int y^2 t ds \tag{2}$$

where

y coordinate along major axis

ds elemental length of perimeter of elliptic shell

The result of the evaluation of this elliptic integral can be given in the form

$$I = k_b a\ t \tag{3}$$

Values of k_b for specimens considered in this report are:

b/a	k_b
1.0	π
.8	2.68
.6	2.24
.4	1.82

Values of k_b for other ratios of b/a may be read from the graph in figure 7 of reference 2.

Torsion.- The shearing stress S at failure in the plane of the skin due to the application of a twisting moment is assumed to be given by the formula

$$S = \frac{T}{2At} \tag{4}$$

where

T twisting moment at failure

A enclosed area of ellipse, πab

PURE BENDING TESTS

The original NACA data for pure bending are given in references 2 and 3. These data have been combined with the M.I.T. data and the combination has been analyzed to show the effect of the three ratios: L/a, b/a, and

a/t; these symbols have been defined previously except for L, which is the length of the cylinder.

In the analysis for the effects of L/a and b/a, the test data have been divided into groups according to the nominal sheet thickness of the material used in the construction of the specimens. Consequently, in the figures where the data have been so grouped, part of the scatter of test points is caused by small variations in the sheet thickness from the nominal thickness for that group. It is believed, however, that the improvement in clearness of the presentation justifies this grouping of the data.

Effect of the ratio L/a.- In order to show the effect of the ratio L/a of elliptic cylinders upon f_b under pure bending, figure 1 has been prepared. Inspection of this figure shows that, for the cylinders tested, L/a is an unimportant factor in the strength of thin-walled elliptic cylinders subjected to pure bending in the plane of the major axis. Consequently, in the following analysis of the bending strength of thin-walled circular and elliptic cylinders, the effect of L/a, and hence of length, is not considered.

Effect of the ratio b/a.- In order to show how f_b for an elliptic cylinder in pure bending is related to f_b for a circumscribed circular cylinder of the same wall thickness in pure bending, figure 2 has been prepared. The fact that the M.I.T. and NACA data are in agreement in this figure signifies that both sets of data are comparable in quality and represent similar care in the execution of the tests.

The experimental points for each nominal value of a/t in figure 2 show a wide scatter that is characteristic of the strength of unstiffened shells in compression or bending. The curves drawn in figure 2, which give the lower limit of the test data, are assumed to establish f_b for pure bending. These values are used later in this report in the analysis of the test data on combined bending and torsion.

The curves of figure 2 show that, for any value of a/t, the value of f_b for pure bending increases with decrease in the ratio b/a. This increase in strength with eccentricity of the elliptical shape evidently arises from the increased stability of the more highly curved portions of the cylinder wall at the ends of the

major axis where the bending stresses are highest. The increase in f_b for pure bending with decrease in b/a cannot be maintained indefinitely for, when $b/a = 0$, the cylinder is a deep narrow beam that is laterally unstable. The increase in f_b for pure bending as b/a decreases from 0.6 to 0.4 is very nearly zero, indicating the presence of a maximum value for f_b in this neighborhood.

Effect of the ratio a/t.- In figures 1 and 2 the values of f_b for pure bending are plotted without regard to the small effects of sheet thickness. This procedure was adopted in order that the data could be handled conveniently in groups, showing the effect of the ratio L/a in figure 1 and the effect of the ratio b/a in figure 2.

Although the ratio L/a has no important effect upon f_b for pure bending, the effect of the ratio b/a is large and cannot be neglected. It therefore becomes necessary to consider the effect of thickness upon the strength of the elliptic cylinders at a given value of b/a. The circular cylinder, for which $b/a = 1$, has been extensively studied and is therefore the most logical value of b/a for study of the effect of the ratio a/t. Thus, figure 3 has been prepared from the combined M.I.T. and NACA data. In this figure f_b/E for pure bending is plotted against the ratio a/t.

If sufficient data were available, other figures similar to figure 3 could be prepared for other values of b/a. In the absence of such data it is necessary to regard figure 3 as establishing a relationship between the ratio a/t and the value of f_b for pure bending of a circular cylinder. The increments by which these values of f_b should be increased in order to obtain values of f_b for pure bending of elliptic cylinders can then be established by referring to a figure such as figure 2, which correlates the bending strength of an elliptic cylinder with the bending strength of a circular cylinder of the same a/t ratio.

The curves A, B, and C in figure 3 are the curves given in figures 7 and 9 of reference 4, which presents the NACA study of circular cylinders in compression. Curves A and B, respectively, are the graphs of the equations

$$\frac{f_b}{E} = 0.605\,\frac{t}{a} \qquad \text{and} \qquad \frac{f_b}{E} = 0.363\,\frac{t}{a}$$

The plotted points in figures 7 and 9 of reference 4 representing the compressive strength of carefully constructed cylinders scattered between curves B and C.

In figure 3 the plotted points for cylinders in pure bending fall between curves B and C at large values of a/t and above curve B at small values of a/t. This trend toward higher values of the stress modulus ratio for pure bending as compared with compression must be ascribed to effects caused by changes in the thickness, because the radius a was held constant in the tests at 7.5 inches. Slippage of the thickest walled cylinders on the bulkhead during test has been suggested as a probable explanation of this trend. Inspection of the plotted data in figure 3 shows that the experimental points for the M.I.T. and the NACA data are in agreement. Consequently, if any slippage occurred in one set of tests, it must also have occurred in the other set in such a manner as to give substantially identical strengths. The experimental points in figure 3 actually represent the true bending strength of the cylinders tested. Whether or not a different method for bringing the loads into the skin from the bulkheads would give the same values of f_b/E is a matter for future research.

TORSION TESTS

The strength of thin-walled circular and elliptic cylinders in torsion is well established by the tests reported in references 1, 2, and 5. In reference 2 it was concluded that the shearing stress at failure for a thin-walled elliptic cylinder is the same as the shearing stress at failure for a circumscribed circular cylinder of the same length and wall thickness, when values of b/a lie between 0.6 and 1.0. It is reasonable to assume that this conclusion also holds for elliptic cylinders with $b/a = 0.4$ in the absence of torsion tests on such cylinders.

In reference 2 it was recommended that, for general design purposes, the strength of elliptic cylinders in torsion should be calculated on the basis of a circumscribed circular cylinder of the same length and wall thickness by use of Donnell's design formula of reference 5. This formula is, for the cylinders of this

report,

$$S = \frac{0.6}{1-\mu^2} E\left(\frac{t}{L}\right)^2 \left[4.6 + \sqrt{7.8 + 1.67\left(\sqrt{1-\mu^2}\,\frac{L^2}{2at}\right)^{3/2}}\right] \tag{5}$$

where μ is Poisson's ratio for the material.

COMBINED PURE BENDING AND TORSION TESTS

In order to correlate the data obtained from the M.I.T. tests on thin-walled elliptic cylinders under a combination of torsion and pure bending in the plane of the major axis with the data on pure bending alone and torsion alone, the following stress ratios have been computed:

R_b ratio of bending stress at failure in combined pure bending and torsion to bending stress at failure in pure bending alone

R_s ratio of shearing stress at failure in combined pure bending and torsion to shearing stress at failure in torsion alone

In the calculation of these stress ratios for the cylinders tested, the values of the bending stress at failure for pure bending alone are assumed to be given by the curves of figure 2 and the values of the shearing stress at failure for torsion alone are assumed to be given by equation (5).

The results of all the combined pure bending and torsion tests, as well as the results of the tests with pure bending alone and with torsion alone, are plotted in figure 4 with R_s as abscissa and R_b as ordinate. This method of plotting data on combined loading was used by Bridget, Jerome, and Vosseller in reference 6 and by Shanley and Ryder in reference 7.

Inspection of figure 4 shows that the data for combined torsion and bending scatter as widely as the data for torsion alone plotted along the axis of abscissa and for pure bending alone plotted along the axis of ordinates. The entire group of data therefore establishes a broad band that is characteristic of unstiffened shells in most loading conditions.

The data plotted in figure 4 lie almost entirely within the band established by two concentric circles about the origin as center with radii of 1.0 and 1.6. The circle with a radius 1.0 establishes the allowable stresses for circular and elliptic thin-walled cylinders in combined torsion and pure bending in the plane of the major axis. The equation of this circle is

$$R_s^2 + R_b^2 = 1 \tag{6}$$

This relation was also found by Shanley and Ryder to apply for combined torsion and bending of round steel tubing. (See fig. 2 of reference 7.) This same formula has also been recently confirmed by the National Bureau of Standards for combined torsion and bending of round tubing.

CONCLUSIONS

1. The ratio of length to semimajor axis of the ellipse is an unimportant factor in the strength of thin-walled elliptic cylinders subjected to pure bending in the plane of the major axis.

2. For any ratio of semimajor axis to wall thickness, the bending stress on the extreme fiber at failure for pure bending of thin-walled elliptic cylinders in the plane of the major axis increases with the eccentricity of the elliptical shape. This increase in strength with eccentricity evidently arises from the increased stability of the more highly curved portions of the cylinder wall at the ends of the major axis where the bending stresses are highest. As the ellipse becomes more and more eccentric the increase in strength cannot be maintained indefinitely, for in the limit the cylinder becomes a deep narrow beam that is laterally unstable. Data indicate that the strength in pure bending is a maximum when the ratio of minor to major axis is in the neighborhood of 0.4 to 0.6.

3. For the case of circular cylinders, the relation between the ratio of the bending stress at failure to the modulus of elasticity and the ratio of radius to thickness is established by a chart of f_b/E against a/t.

4. It was found that thin-walled elliptic cylinders subjected to combined torsion and pure bending in the plane of the major axis will fail when the stress ratio satisfy the equation

$$R_s^2 + R_b^2 = 1$$

where

R_b ratio of bending stress at failure in combined pure bending and torsion to bending stress at failure in pure bending alone

R_s ratio of shearing stress at failure in combined pure bending and torsion to shearing stress at failure in pure torsion alone

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., December 17, 1941.

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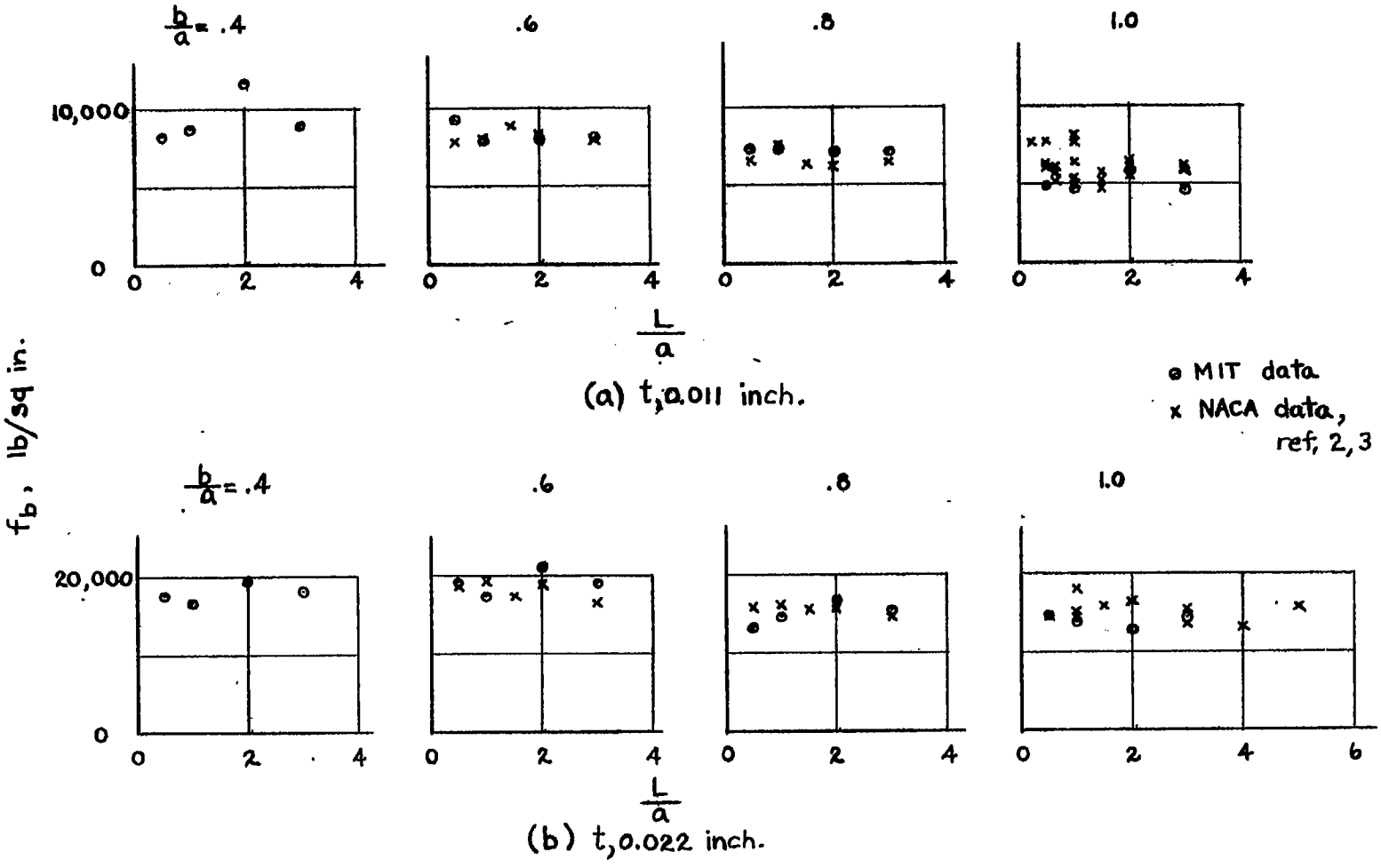

Figure 1. - Effect of the ratio L/a upon the bending stress f_b at failure of thin-walled elliptic cylinders under pure bending in the plane of the major axis.

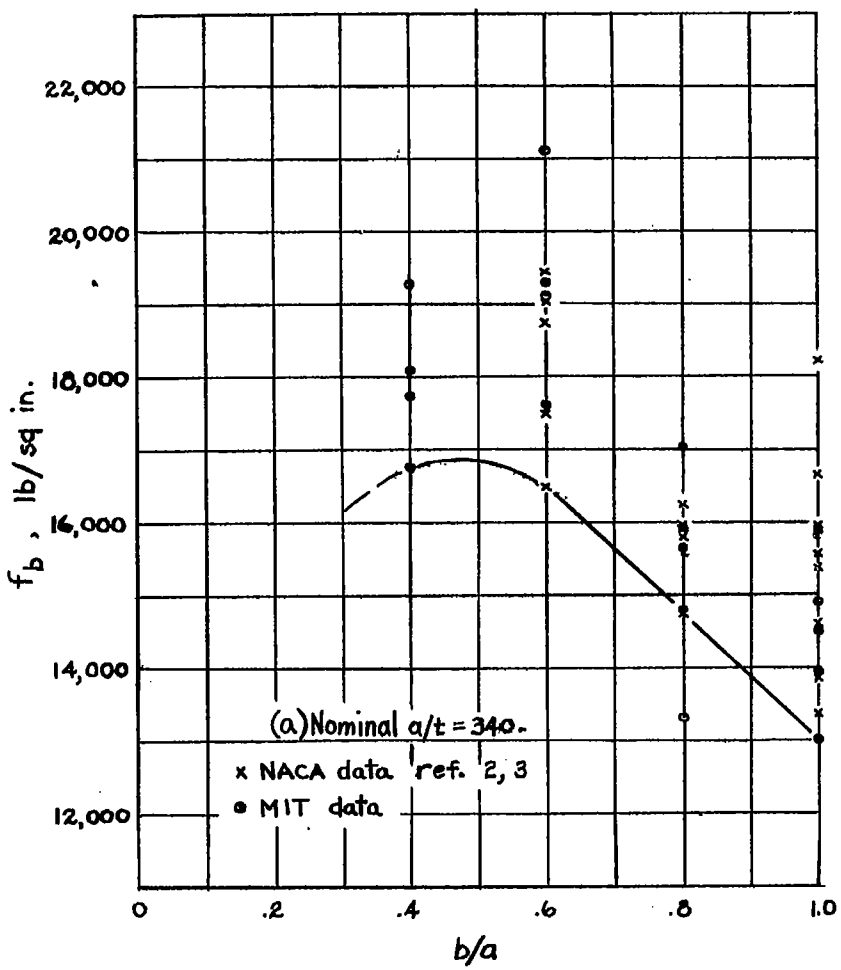

Figure 2.— Effect of the ratio b/a upon the bending stress at failure f_b of thin-walled elliptic cylinders under pure bending in the plane of the major axis.

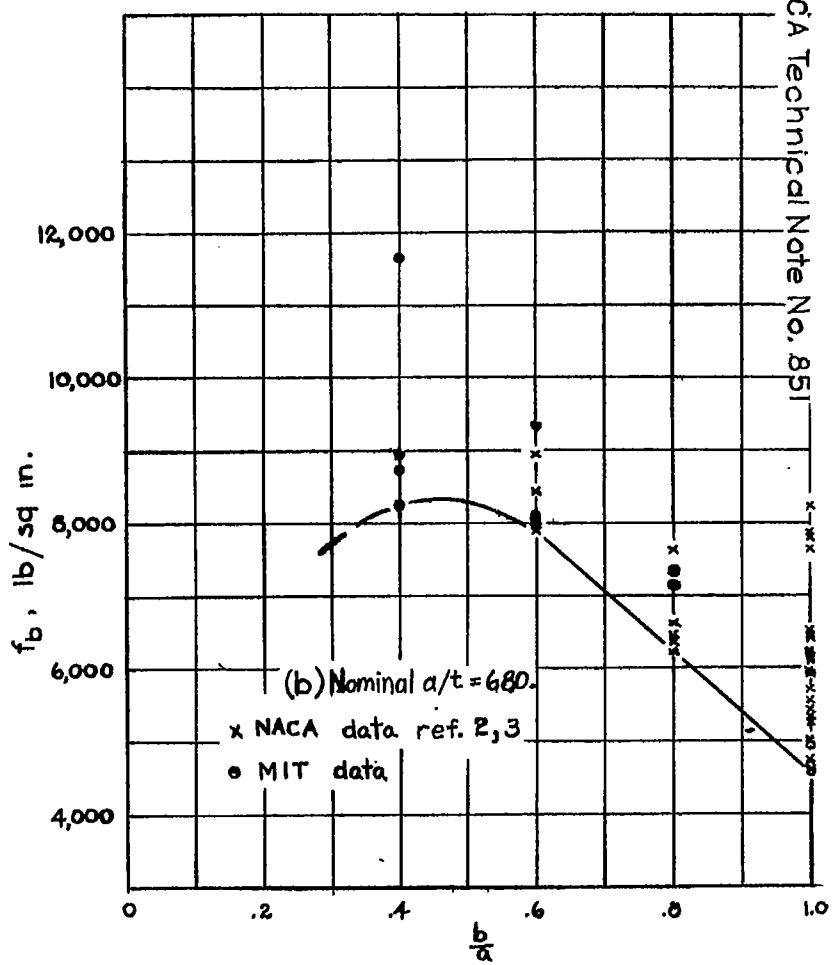

Figure 2.-Concluded.

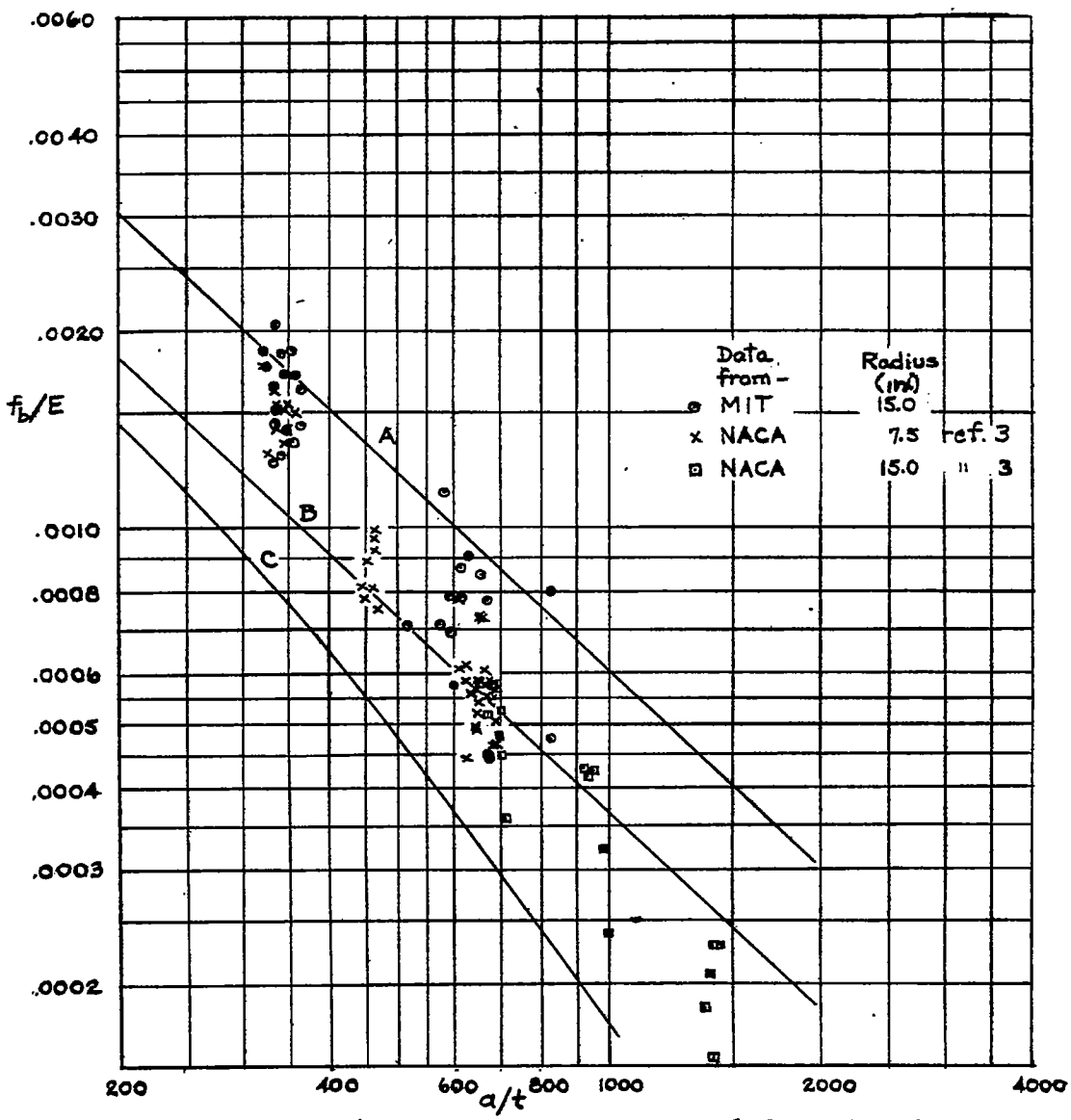

Figure 3. - Effect of the ratio a/t upon the value of f_b/E at failure for thin-walled circular cylinders under pure bending. (Curves A, B and C from figure 7, reference 3.)

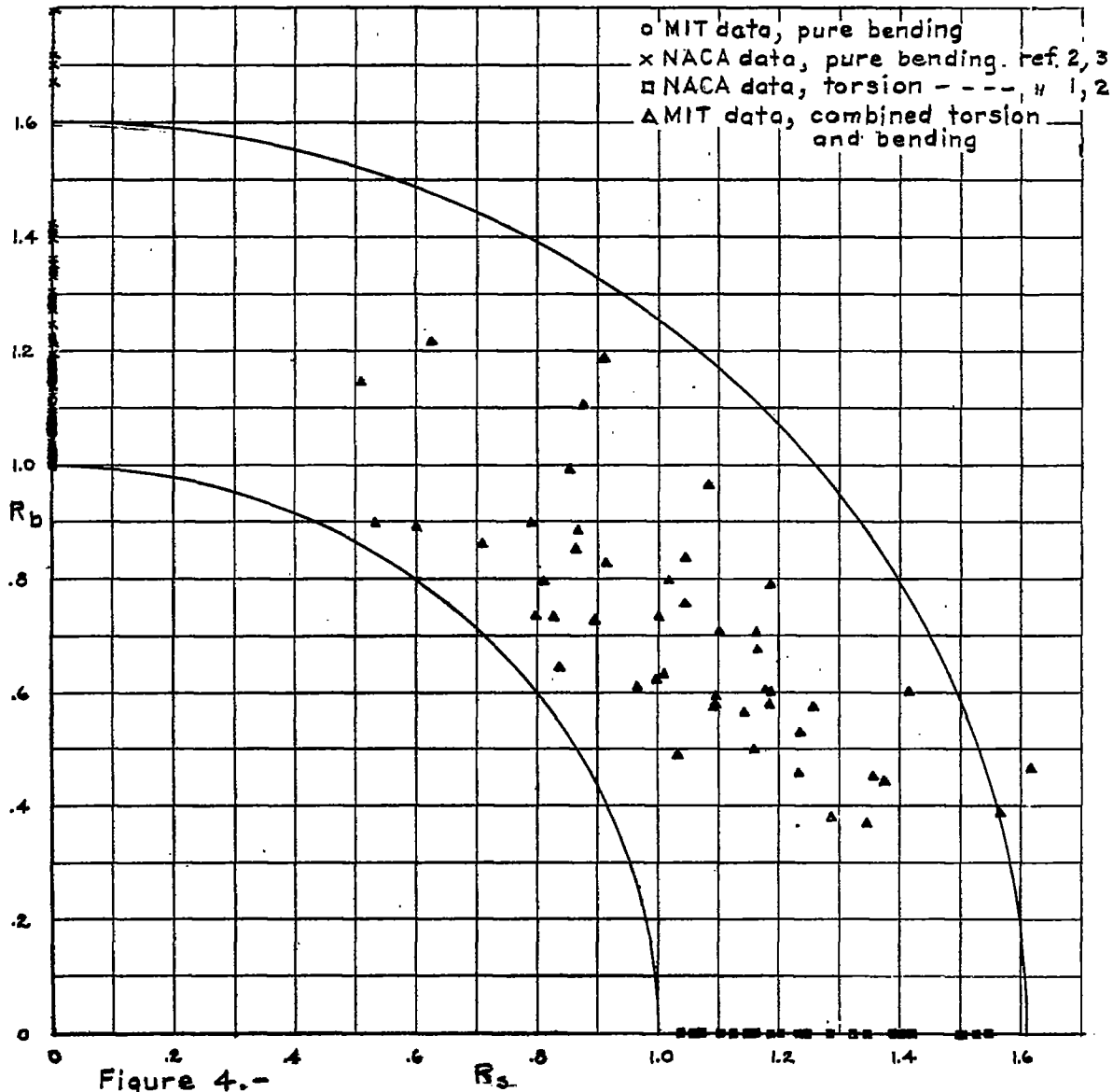

Figure 4.- Strength of thin-walled elliptic cylinders in combined torsion and pure bending in the plane of the major axis.