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Cylinders, thin-walled - Strength

No. 904

THE STRENGTH OF THIN-WALL CYLINDERS OF D CROSS SECTION IN COMBINED PURE BENDING AND TORSION

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By A. W. Sherwood

SUMMARY

The results of tests of 56 cylinders of D cross section conducted in the Aeronautical Laboratory of the University of Maryland are presented in this report. These cylinders were subjected to pure bending and torsional moments of varying proportions to give the strength under combined loading conditions. The average buckling stress of these cylinders has been related to that of circumscribing circular cylinders for conditions of pure torsion and pure bending and the equation of the interaction curve has been determined for conditions of combined loading.

SPECIMENS

The test specimens were formed by riveting together thin semicylindrical sheets of aluminum alloy to fabricated channels of the same thickness in such a way that the web of the channel formed a diameter of the cylinder. The specimens were carefully cut and fitted over the end bulkheads through which the loads were applied and were held in place and riveted with the aid of a simple jig. The rivets were spaced on $\frac{1}{2}$-inch centers and it was observed during tests that initial buckles of the specimens occurred as frequently at rivets as between them. The tests were conducted on cylinders fabricated of two-sheet thicknesses, two sizes of cross section, and with lengths varying from 3 to 20 inches. The specimens were carefully inspected prior to test to insure a close fit on the end bulkheads and the absence of wrinkles.

MATERIAL

The 24S-T sheet used in these tests was supplied in nominal thicknesses of 0.0125 inch and 0.020 inch by the Aluminum Company of America. The average thickness of the sheet used in each specimen was determined by means of a dial gage graduated in ten-thousandths mounted on a special jig. The average of readings at three or four points on each specimen is recorded in the data. It was found that the average deviation from mean values of 0.0115 inch and 0.0182 inch for the two thicknesses used was less than 0.0001 and so these average values were used in all calculations.

The modulus of elasticity was determined from measurements made on four specimens of each thickness milled accurately to a $1\frac{1}{2}$-inch width. The grips were fastened to the specimen with five machine screws passing through jig drilled holes and loaded through pin joints as shown in figure 6. The free distance between grips was $3\frac{1}{2}$ inches.

Loading was applied in a testing machine with strain measured by means of a Tuckerman strain gage mounted on one side of the sheet only. This procedure was felt to be justifiable since the specimens were carefully inspected for bends and wrinkles and the strain was not read until an initial stress of at least 3000 pounds per square inch was applied.

An average value of 10.5×10^6 was obtained for the thinner sheet and 10.9×10^6 for the thicker sheet.

APPARATUS AND METHOD

The testing machine that was used appears in figures 2a and 2b. This design is very similar to the one used by Donnell in his tests on circular cylinders (reference 1). The loading was accomplished with accurately calibrated lead weights, which were applied in increments of less than 2 percent of estimated load at failure. Friction was kept negligibly small through the use of ball universal joints and knife edges at all points subject to relative motion. The specimens were clamped to formed bulkheads consisting of a plywood core between two plates

of ¼-inch steel accurately machined to the half-circle contour. A flexible steel band encircled and clamped the specimen in place with a steel strip supporting the legs of the channel section against collapse. It will be noted that the torque arm is so positioned with respect to the universal joint that a torque load will produce a small bending moment as well. This moment was counteracted by the application of an appropriate bending load for the specimens tested in pure torsion.

In examining the method of application of the external loads, it will be noted that the bending moment is constant along the test specimen and that there is therefore no transverse shear which might otherwise combine with the shear stresses owing to torsion producing failure at lower loads. In addition, one end of the machine is supported on rollers precluding an axial load which would otherwise produce a stress which would combine with the bending stress. On the other hand, the method of clamping the specimens to the end bulkheads produced secondary bending stresses which, to some extent, affected the results. This error is considered negligible in the analysis to follow since the specimens tested were fitted very carefully over the end bulkheads with no perceptible wrinkles resulting from the tightening of the clamps. Several specimens were rejected prior to test because of failure in this regard.

In the specimens tested in pure torsion a diagonal wrinkle appeared in the web of the specimen at a very low load and continued practically unchanged until failure. Most of these failures appeared very near the clamps.

In the case of the failures of the specimens in pure bending, a ripple appeared in the legs of the channel and the final buckle developed on the compression side of the specimens almost anywhere between the clamps.

Judging by the relative movements of the torque and bending-moment arms of the machine up to failure, the failure was due primarily to torsional shear wherever the torsional-stress ratio R_T, as indicated in figure 7, was greater than 0.25.

DISCUSSION OF RESULTS

Since the author found only one other reference to tests on thin tubes of this type, it was considered

desirable to relate the strengths of these tubes to those of circular cylinders.

The unit shearing stress resulting from the application of a torsional load was calculated by the familiar formula $S_T = \frac{T}{2At}$ where T is the torque load at failure, A is the enclosed area of the median line of the section, and t is the average thickness of the sheet. The unit shearing stress S, determined in this way from test results, was then compared with the shearing stress of a circumscribed circular cylinder of the same material and thickness as given by the modification of Donnell's formula appearing in the ANC-5 (formula 1.51):

$$F_{st} = \frac{KE}{\left(\frac{D}{t}\right)^{5/4}\left(\frac{L}{D}\right)^{1/2}}$$

D diameter of cylinder

t thickness of cylinder

L length of cylinder

F_{st} unit buckling stress

K constant

E modulus of elasticity

As shown on figures 7 and 8, the test results agree satisfactorily with the values calculated by the above formula using a value of $K = 0.90$. In Donnell's report (reference 1) the corresponding value of K is given as 0.94 for circular cylinders to give comparable average buckling stresses. It will be noted by referring to figure 8 that the above equation with the modified K value is not equally satisfactory for all values of L/r, but it is felt that the difference is not great enough to warrant a change in Donnell's formula.

In computing the unit maximum compressive stress of the specimens resulting from the application of bending loads, the familiar flexure formula was used: $f_b = MC/I$.

This assumes that the stress varies linearly with the distance from the neutral axis, which is not in serious error since there is very little deflection or distortion of the cylinder prior to failure.

In developing a practical method for calculating the buckling unit stress of cylinders of this shape, reference was again made to the treatment of circular cylinders. A relation has been developed by Lundquist in NACA Technical Note No. 479 for the buckling unit stress (S_b) of a circular cylinder in compression as follows:

$$S_b = K_b E$$

K_b nondimensional coefficient depending on r/t ratio and imperfections of cylinder

E modulus of elasticity of material

In this technical note, the close parallelism between the buckling of a thin-wall circular cylinder in bending and in compression is shown. Reference is made to NACA Report No. 473 by the same author, which proposes the equation $S_c = K_c E$ for the strength of circular cylinders in compression. Values of K_c can be taken from curve C in figure 7 for minimum values of the buckling compressive stress corresponding to varying r/t ratios. As a result of these tests, it was suggested that values of K_b for buckling-bending stresses be taken 30 to 80 percent higher than corresponding values of K_c. In the analysis of these bending tests, it was concluded that the effect of the L/r ratio on the strength was completely overshadowed by the effect of initial imperfections in the range tested (L/r from 0.25 to 5.0).

Referring to figure 9, it will be noted that the value of K_b is a much smaller percentage of K_c than for circular cylinders. This is not surprising, since the web and the outstanding legs of the channel section wrinkled early in the tests and appeared to be taking very little of the load while over 50 percent of the moment of inertia of the section (I) is contributed by the channel. The D-section cylinder is therefore a much less efficient structural member than the circular cylinder for bending loads.

Figure 9 also shows a very marked decrease in strength with an increase in the L/r ratio. It will be noted, however, that the range is greater (L/r from 2 to 14) than for the cylinders tested by Lundquist.

For application to design problems, it was considered most satisfactory to present these results in the form of an interaction curve. In order to define this curve for a D-section cylinder of any dimensions, it is necessary to establish its strength in pure bending ($R_B = 1$) and pure torsion ($R_T = 1$) and the equation of the intermediate curve where the stress ratios R_B and R_T are defined as follows:

$$R_B = \frac{f_b}{S_b}$$

$$R_T = \frac{S_T}{F_{st}}$$

f_b unit bending stress at failure for cylinder subjected to combined loading, pounds per square inch

S_b unit bending stress at failure for cylinder subjected to bending alone, pounds per square inch

S_T unit shear stress at failure for cylinder subjected to combined loading, pounds per square inch

F_{st} unit shear stress at failure for cylinder subjected to shearing stress alone, pounds per square inch

As previously outlined, the value of F_{st} is found to agree closely with the calculated result obtained from Donnell's formula applied to a circumscribed circular cylinder of the same thickness and length, using $K = 0.90$. In the same way, S_b may be calculated as a percentage of the buckling stress of a corresponding circular cylinder in bending. This percentage varies with the L/r ratio of the cylinders as shown in figure 9.

The test results are shown in the form of an interaction curve in figure 7. The pattern of the points is seen to follow a circular arc fairly closely. It may be observed that the points with the greatest scatter from this curve correspond to specimens 1 to 12. This is believed to be due to two causes:

(1) The first specimens were not as expertly fabricated as later ones. While there were no perceptible wrinkles, technique in using the riveting hammers and bending tools was being acquired and small irregularities and imperfections may have resulted.

(2) The greater radius of curvature of these specimens makes them more sensitive to small imperfections and eccentricities.

CONCLUSIONS

1. The average buckling stress in pure torsion of the D-section tubes tested can be calculated by applying Donnell's formula to circumscribing circular cylinders of the same dimensions and material:

$$F_{st} = \frac{KE}{\left(\frac{D}{t}\right)^{5/4}\left(\frac{L}{D}\right)^{1/2}} \quad K = 0.90 \text{ for hinged ends}$$

2. The average buckling stress in pure bending can be computed with the formula:

$S_b = K_b E$ (E = modulus of elasticity of material). K_b is given as a varying percentage of K_c with the L/r ratio as shown on figure 9 of this report. K_c in turn is proportional to the buckling stress of a circumscribed circular cylinder of the same dimensions in direct compression as obtained from curve C of figure 7 in NACA Report No. 473.

3. The average buckling strength for D-section cylinders subjected to combined torsion and pure bending can be represented by an equation of the form $R_B^2+R_T^2 = 1$. R_B is the ratio of the average bending stress at failure to the calculated stress in pure bending and R_T is the ratio of the average shearing stress at failure to the calculated shearing stress. The minimum strength in combined loading can practically be represented by the equation $R_B^2 + R_T^2 = (0.88)^2$.

University of Maryland,
College Park, Md., April 1943.

REFERENCES

1. Donnell, L. H.: Stability of Thin-Walled Tubes under Torsion. Rep. No. 479, NACA, 1933.

2. Lundquist, Eugene E.: Strength Tests of Thin-Walled Duralumin Cylinders in Compression. Rep. No. 473, NACA, 1933.

3. Lundquist, Eugene E.: Strength Tests of Thin-Walled Duralumin Cylinders in Pure Bending. T.N. No. 479, NACA, 1933.

4. Lundquist, Eugene E., and Burke, Walter F.: Strength Tests of Thin-Walled Duralumin Cylinders of Elliptic Section. T.N. No. 527, NACA, 1935.

5. Lundquist, Eugene E., and Stowell, Elbridge Z.: Strength Tests of Thin-Walled Elliptic Duralumin Cylinders in Pure Bending and in Combined Pure Bending and Torsion. T.N. No. 851, NACA, 1942.

6. Cudhea, George G.: Stainless Steel Movable Control Surfaces. Jour. Aero. Sci., Dec. 1941.

TABULATION OF RESULTS

"D" SECTION REPORT

Spec. No.	Inner Radius In.	Thickness In.	Effective Length In.	Ultimate Torque "#s	Ult. Bend. Moment "#s	Torsional Stress #/in²	Bend. Stress f_b #/in²	Calcu. Torsional Stress #/in²	Calcu. Comp. Stress $S_c = K_c E$ #/in²	$\frac{K_B}{K_C}$ %	Torque Stress Ratio R_T	Bend. Stress Ratio R_B
1	2.53	.0115*	20.375	208	1415	900	5070				.383	.728
2	"	.0115	"	128	1845	555	6610				.242	.947
3	"	.0110	"	0	1880	0	6730		13650	51.2	0	.964
4	"	.0115*	"	58	1484	251	5320				.108	.76
5	"	.0115*	"	600	165	2600	591	2338			1.12	.086
6	"	.0109	"	428	422	1850	1510				.79	.219
7	"	.0110	"	328	1060	1420	3790				.608	.549
8	"	.0115*	"	520	725	2250	2600				.966	.372
9	"	.0112	"	128	1845	555	6610				.242	.947
10	"	.0112	"	0	1922	0	6900		13650	51.2	0	.987
11	"	.0115*	"	580	380	2510	1360	2338			1.07	.196
12	"	.0115*	"	328	1220	1420	4360				.61	.626
13	1.5	.0114	9	398	293	4900	3195				.942	.254
14	"	.0114	9	455	99	5600	1079	5180			1.08	.087

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Spec. No.	Inner Radius In.	Thickness In.	Effective Length In.	Ultimate Torque "#s	Ult. Bend. Moment "#s	Torsional Stress #/in^2	Bend. Stress f_b #/in^2	Calcu. Torsional Stress #/in^2	Calcu. Comp. Stress $S_c=K_cE$ #/in^2	$\frac{K_B}{K_C}$ %	Torque Stress Ratio R_T	Bend. Stress Ratio R_B
15	1.5	.0114	9	0	1130	0	12260		23100	55	0	.97
16	"	.0116	"	455	99	5600	1072				1.07	.087
17	"	.0115	"	328	548	4040	5950				.775	.47
18	"	.0113	"	375	488	4620	5300				.88	.42
19	"	.0113	"	352	714	4340	7750				.83	.62
20	"	.0114	"	306	784	3760	8500				.77	.68
21	"	.0113	"	228	913	2800	9900				.54	.78
22	"	.0113	"	341	592	4200	6420				.81	.51
23	"	.0116	"	114	1135	1400	12310				.266	.97
24	"	.0114	"	0	1070	0	11600		23100	55	0	.92
25	"	.0117	"	0	1235	0	13400		23100	55	0	1.06
26	"	.0116	"	0	1230	0	13350		23100	55	0	1.05
27	"	.0113	"	398	524	4900	5680				.95	.45
28	"	.0117	"	285	738	3500	8000				.67	.63

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Spec. No.	Inner Radius In.	Thickness In.	Effective Length In.	Ultimate Torque "#s	Ult. Bend. Moment "#s	Torsional Stress #/in^2	Bend. Stress f_b #/in^2	Calcu. Torsional Stress #/in^2	Calcu. Comp. Stress $S_c = K_c E$ #/in^2	$\frac{K_B}{K_C}$ %	Torque Stress Ratio R_T	Bend. Stress Ratio R_B
29	1.5	.0117	9	171	966	2100	10530				.41	.83
30	"	.0182	"	0	3400	0	23300		38200	55		1.11
31	"	.0181	"	1250	326	9700	2235	9430			1.03	.106
32	"	.0183	"	1090	1376	8450	9430				.90	.45
33	"	.0182	"	940	1835	7290	12580				.77	.596
34	"	.0180	"	845	1604	6550	11000				.70	.524
35	"	.0185	"	627	2476	4860	16950				.51	.81
36	"	.0181	"	467	3238	3620	22190				.38	1.05
37	"	.0184	"	318	3057	2462	20900				.261	1.00
38	"	.0181	"	156	3069	1210	21000				.128	1.00
39	"	.0181	"	1180	308	9150	2110	9430			.97	.102
40	"	.0184	"	0	3170	0	21700		38200	55	0	1.03
41	"	.0180	"	0	3200	0	21900		38200	55	0	1.05
42	"	.0113	3	674	0	8280	0	9000			.92	0

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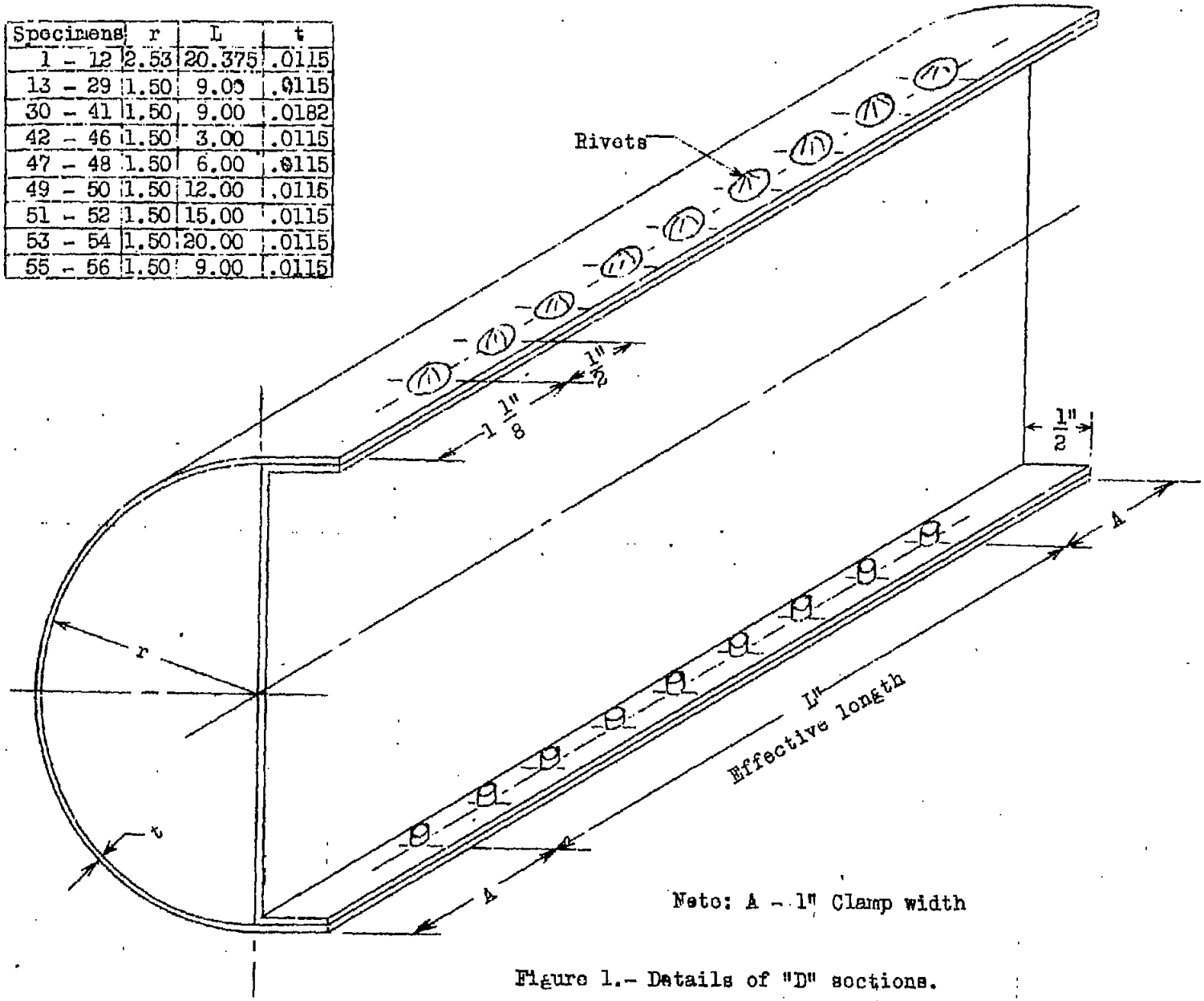
43	1.5	.0116	3	696	0	8560	0	9000			.95	0
44	"	.0116	3	627	0	7720	0	9000			.86	0
45	"	.0118	3	0	1384	0	15100		23100	64.5	0	1.01
46	"	.0117	3	0	1348	0	14690		23100	64.5	0	.98
47	"	.0116	6	0	1258	0	13700		23100	59.5	0	1.00
48	"	.0117	6	535	0	6570	0	6370			1.03	0
49	"	.0116	12	421	0	5180	0	4500			1.15	0
50	"	.0114	12	0	1082	0	11800		23100	51.5	0	1.00
51	"	.0115	15	0	1046	0	11400		23100	49	0	1.01
52	"	.0115	15	444	0	5450	0	4020			1.35	0
53	"	.0113	20	306	0	3765	0	3495			1.07	0
54	"	.0116	20	0	990	0	10800		23100	52	0	.997
55	"	.0115	9	445	0	5470	0	5180			1.06	0
56	"	.0115	9	466	0	5740	0	5180			1.11	0

* The thickness of these 5 specimens was not measured so average values were assumed.

Specimens	r	L	t
1 - 12	2.53	20.375	.0115
13 - 29	1.50	9.00	.0115
30 - 41	1.50	9.00	.0182
42 - 46	1.50	3.00	.0115
47 - 48	1.50	6.00	.0115
49 - 50	1.50	12.00	.0115
51 - 52	1.50	15.00	.0115
53 - 54	1.50	20.00	.0115
55 - 56	1.50	9.00	.0115



Note: A - 1" Clamp width

Figure 1.- Details of "D" sections.

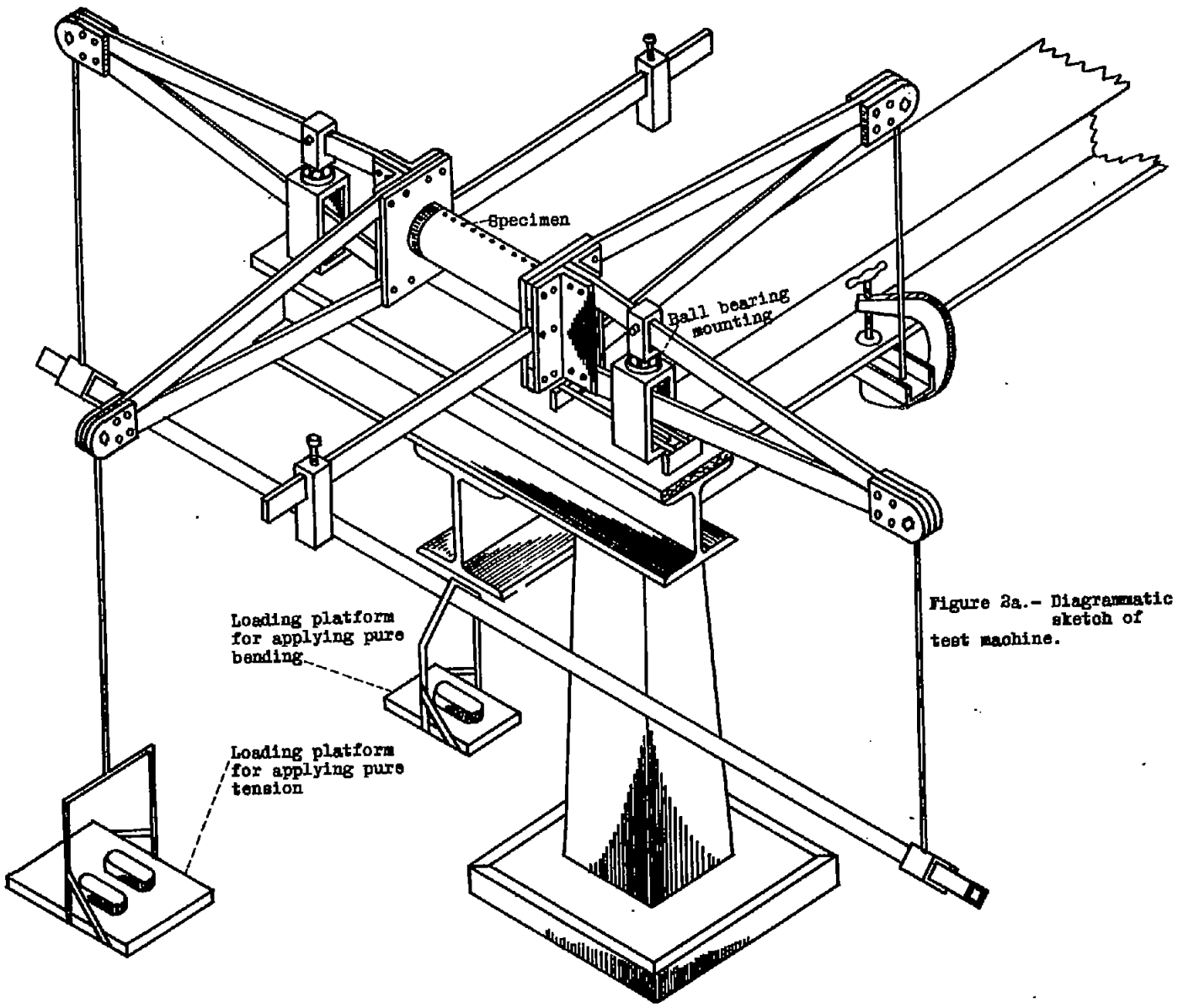


Figure 2a.- Diagrammatic sketch of test machine.

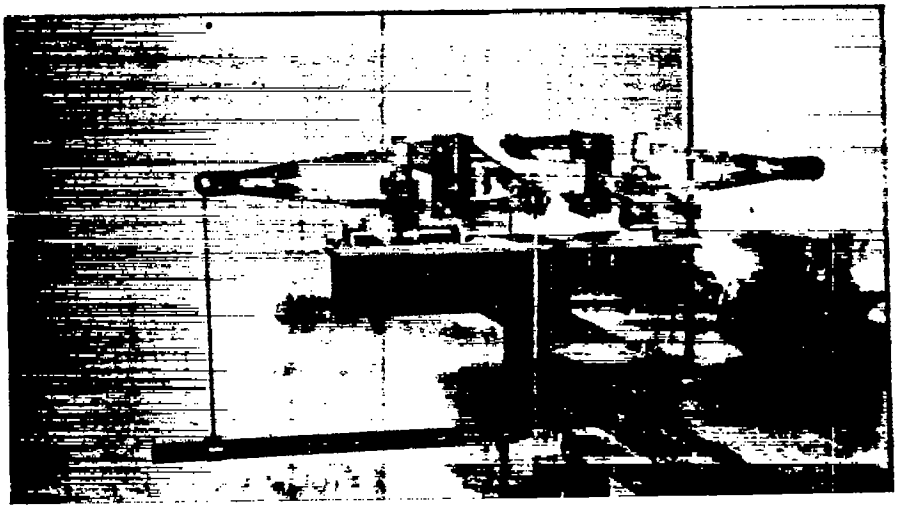


Figure 2b.- General arrangement of test machine.

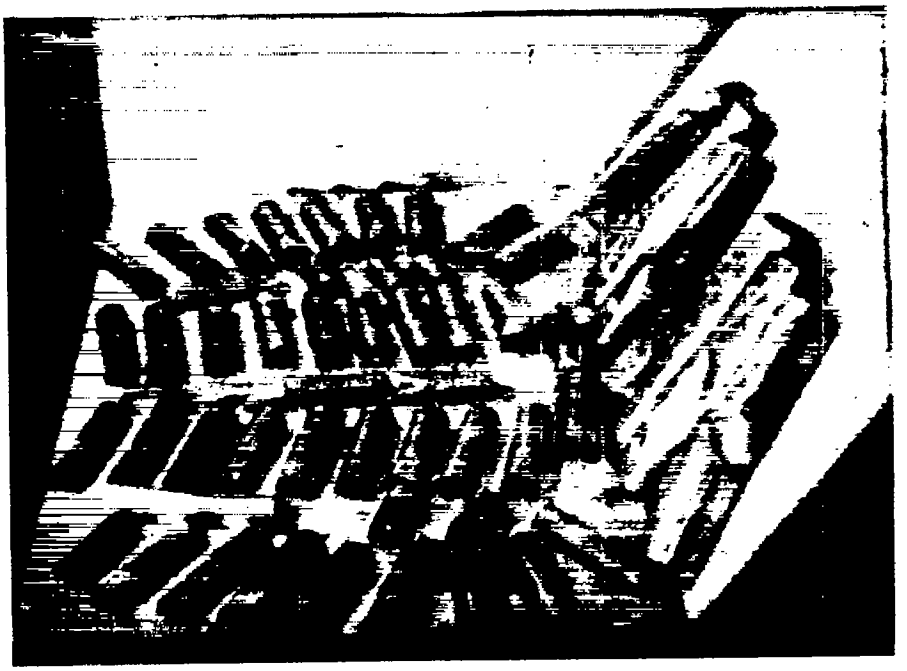
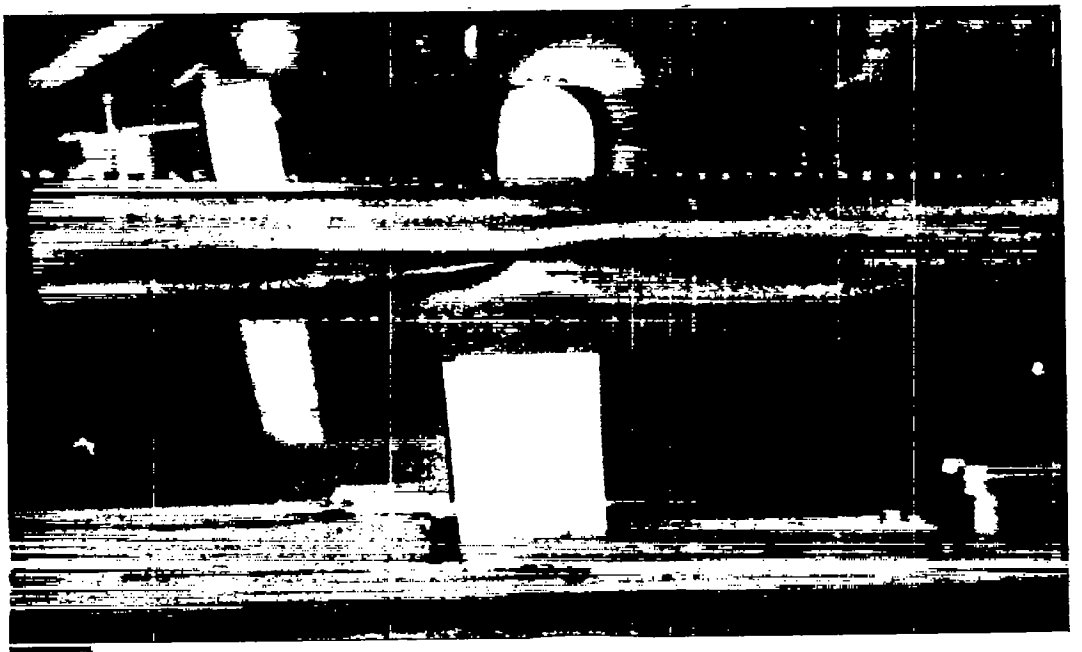
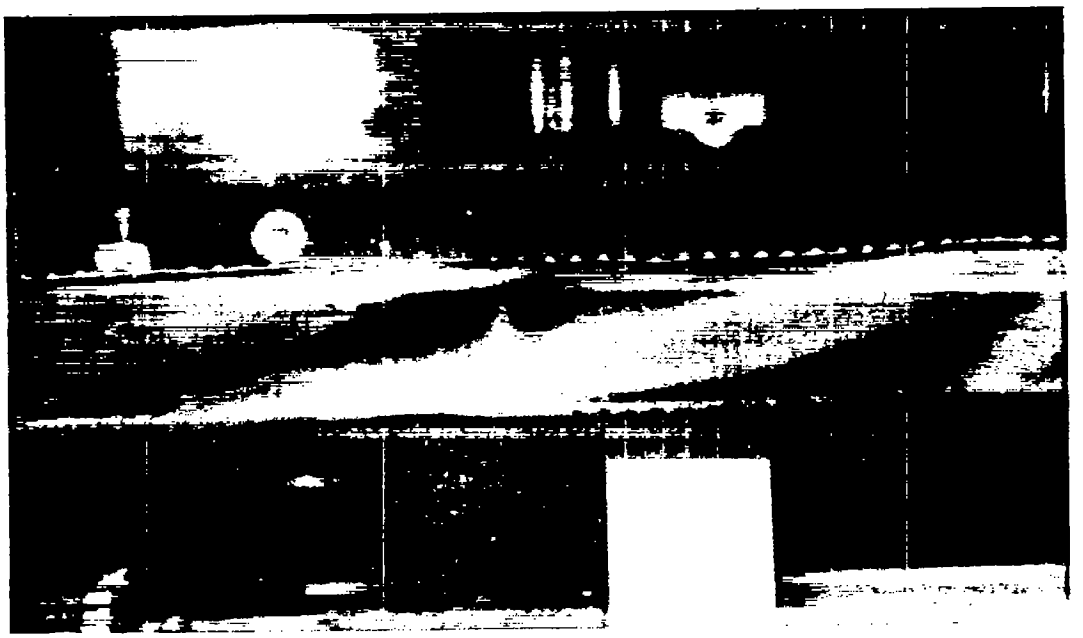


Figure 3.- Specimens subsequent to test.

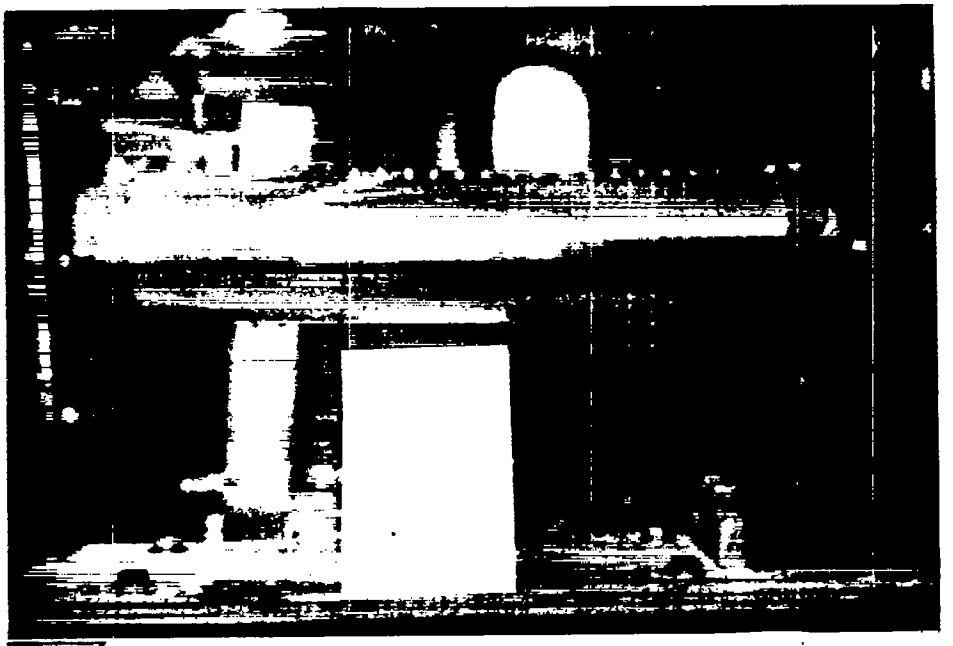


(4a)



(4b)

Figure 4(a,b).- Two views of failure in pure bending.



(5a)

(5b)

Figure 5(a,b).- Two views of failure in pure torsion.

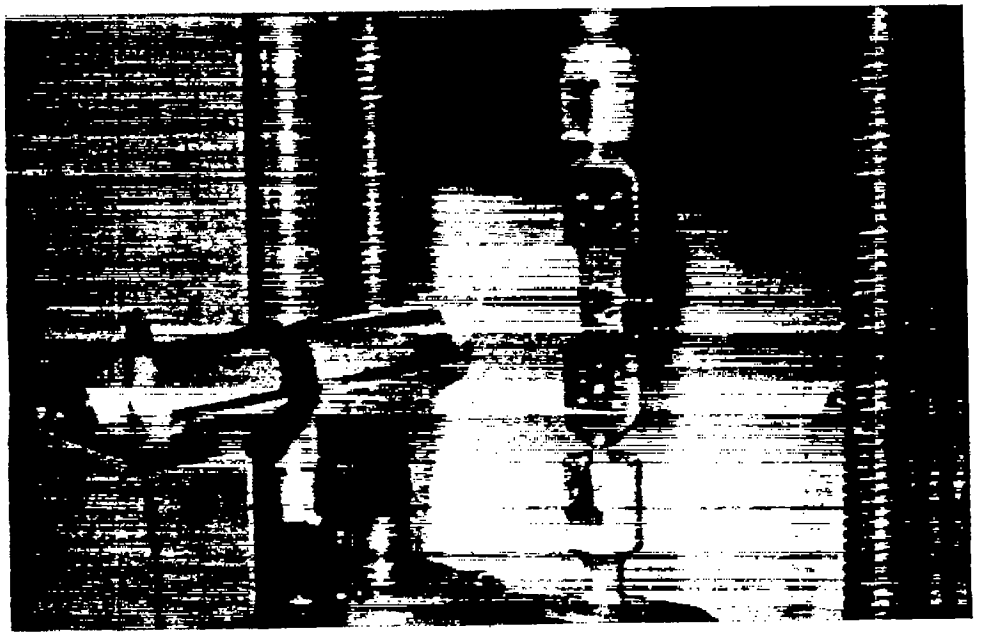


Figure 6.- Use of a Tuckerman optical strain gauge in determination of the modulus of elasticity of the sheet. The pin ended clamps were used to insure against eccentricity of the applied load.

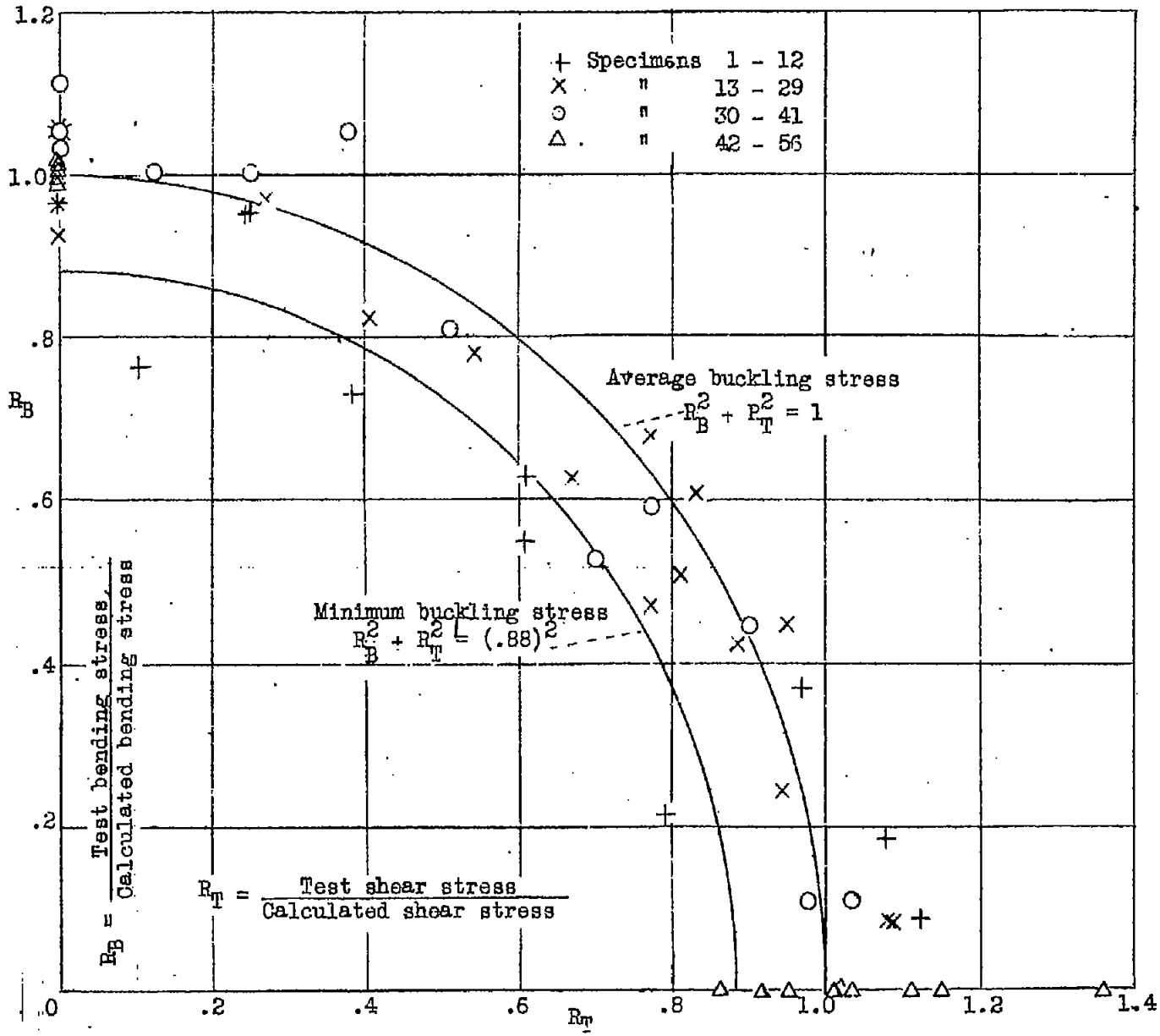


Figure 7.- Interaction curve for combined bending and torsion.

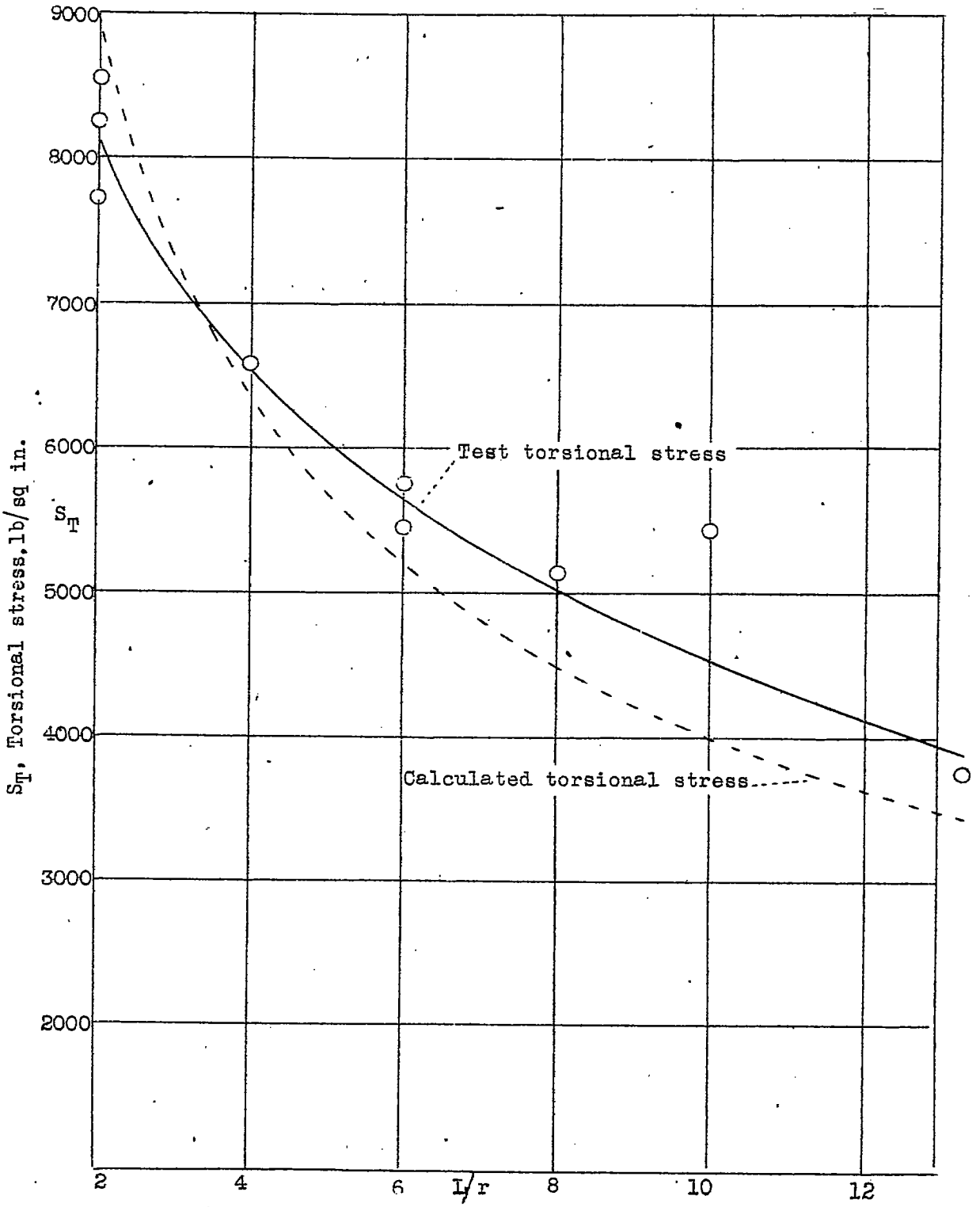


Figure 8.- Effect of L/r ratio on buckling torsional stress.

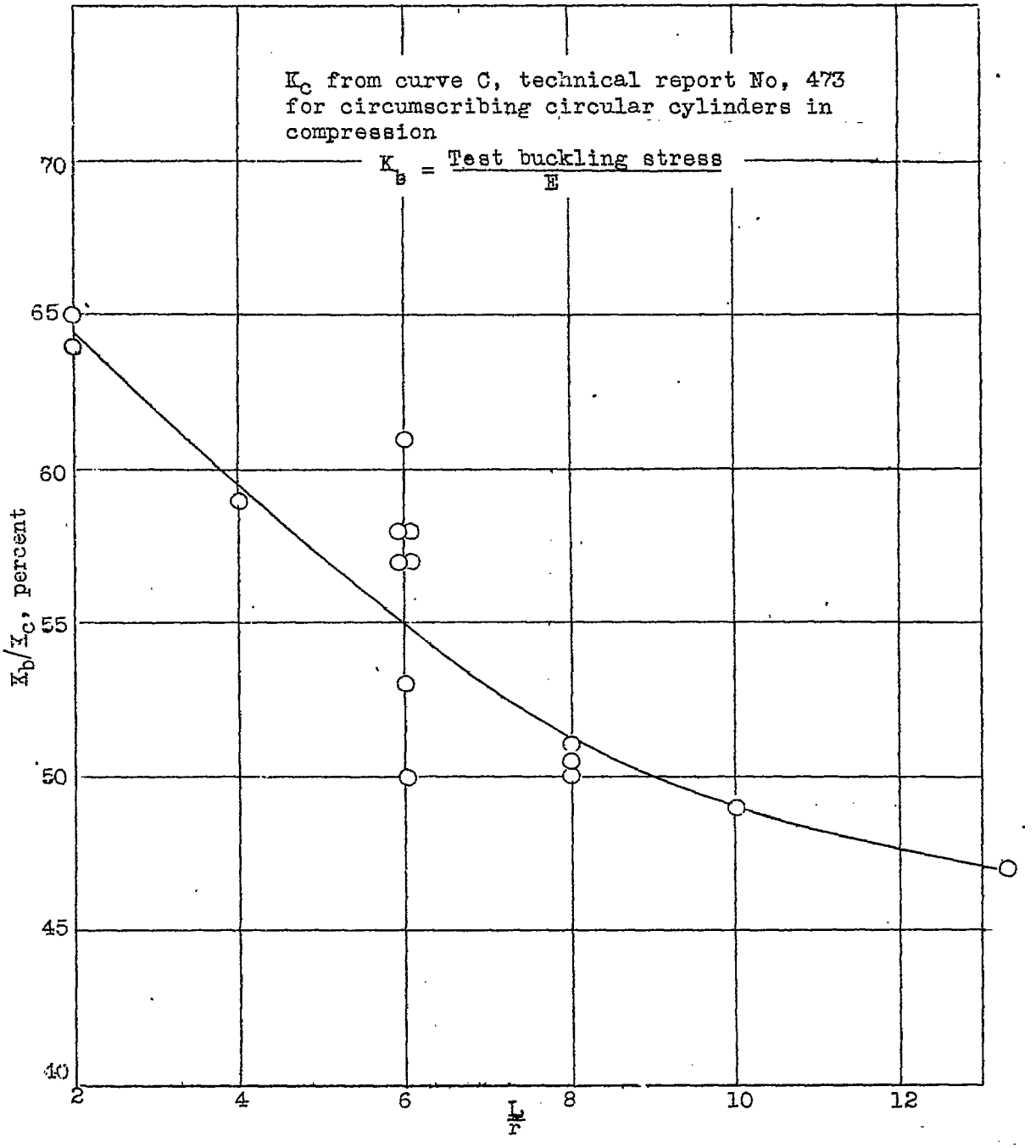


Figure 9.- Effect of $\frac{L}{r}$ ratio on strength in pure bending.