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A GENERAL REPRESENTATION FOR AXIAL-FLOW FANS AND TURBINES

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SUMMARY

A general representation of fan and turbine arrangements on a single classification chart is presented that is made possible by a particular definition of the stage of an axial-flow fan or turbine. Several unconventional fan and turbine arrangements are indicated and the applications of these arrangements are discussed.

INTRODUCTION

A study of the idealized two-dimensional blade arrangements of axial-flow fans and turbines, or turbomachines, indicates that, on the basis of a certain definition of the turbomachine stage, various fan and turbine arrangements can be represented on a single chart. This report presents a definition of the turbomachine stage and the resulting classification chart, which was developed from an analysis of the general velocity diagram for the fan or turbine stage. Some unconventional fan and turbine arrangements are indicated in the chart; these arrangements and some possible applications are discussed. Turbomachines having variable axial-flow area are shown to be amenable to the same general representation.

SYMBOLS

The following symbols are used in the text and on the figures:

A axial-flow area

a parameter characterizing change in axial-flow area

C_H total-pressure coefficient, $\frac{\Delta H}{\frac{1}{2}\rho u^2}$

C_p pressure coefficient, $\frac{\Delta p}{\frac{1}{2}\rho u^2}$

H total pressure

p static pressure

u design peripheral speed of rotor

v axial velocity

x ratio of rotor-entrance relative tangential-velocity component to rotor peripheral speed

y ratio of rotor-exit relative tangential-velocity component to rotor peripheral speed

ρ mass density

Subscripts:

r fan or turbine stage

f rotor

sd downstream stator

su upstream stator

0 entrance to upstream stator

1 entrance to rotor

2 exit from rotor or entrance to downstream stator

3 exit from downstream stator

GENERAL REPRESENTATION SCHEME

A single stage of an axial-flow fan or turbine may be defined as consisting of an upstream stator, a rotor, and a downstream stator, as shown in figure 1. The flow, assumed to be two-dimensional, incompressible, and frictionless, is considered to enter the upstream stator and to leave the downstream stator with a velocity v in a purely axial direction. The stage velocity diagram is shown in figure 1. The velocity-diagram vectors are shown relative to the corresponding blading. Positive directions of the velocity vectors are indicated.

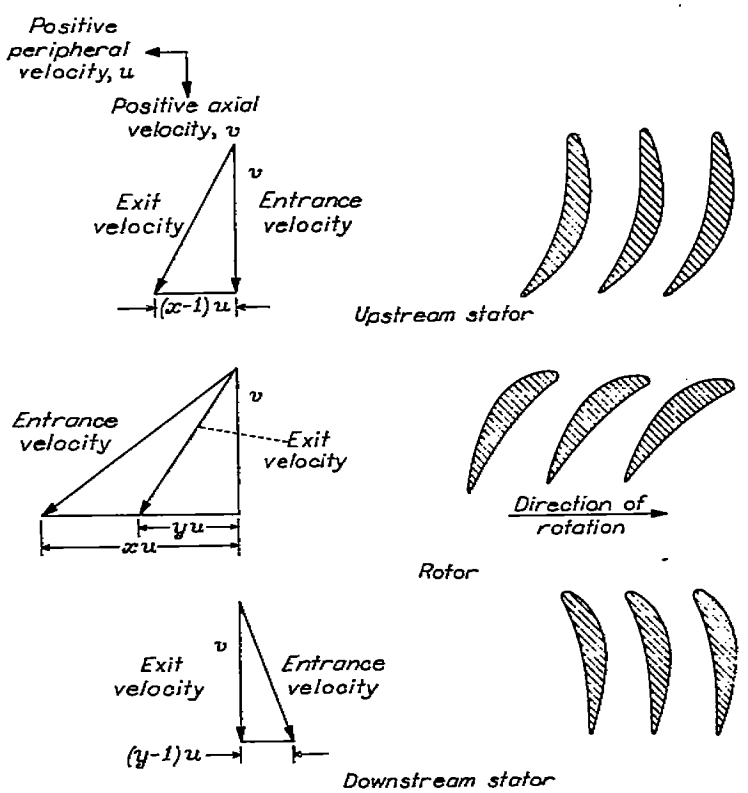

Figure 1.—Velocity diagram of single stage of axial-flow fan with constant axial-flow area.

The velocity-diagram parameters used in the stage analysis are x and y. The peripheral speed of the rotor is denoted by u. The quantity x represents the ratio of the rotor-entrance relative tangential-velocity component to the rotor peripheral speed. The quantity y represents the ratio of the rotor-exit relative tangential-velocity component to the rotor peripheral speed. In figure 1, the upstream stator imparts a tangential-velocity component $(x-1)u$ to the fluid. Relative to the rotor, the incoming tangential-velocity component is xu and the outgoing tangential-velocity component is yu. The downstream stator removes the tangential-velocity component $(y-1)u$ from the fluid.

The nondimensional pressure coefficients across the various stage elements are, by Bernoulli's equation,

Upstream stator $$C_{p,su}\equiv\frac{\Delta p_{su}}{\frac{1}{2}\rho u^2}=-(x-1)^2 \tag{1}$$

Rotor $$C_{p,r}\equiv\frac{\Delta p_r}{\frac{1}{2}\rho u^2}=x^2-y^2 \tag{2}$$

Downstream stator $$C_{p,sd}\equiv\frac{\Delta p_{sd}}{\frac{1}{2}\rho u^2}=(y-1)^2 \tag{3}$$

The stage, or over-all, pressure coefficient $C_{p,f}$ is the sum of the component pressure coefficients:

$$C_{p,f}=2(x-y) \tag{4}$$

The rotor pressure coefficient $C_{p,r}$ and the over-all pressure coefficient $C_{p,f}$ thus are functions of the velocity-diagram parameters x and y. Either x or y may be eliminated from equations (2) and (4) to give $C_{p,f}$ in terms of $C_{p,r}$ and one of the parameters; that is:

$$\left(\frac{1}{2}C_{p,f}-x\right)^2=-C_{p,r}+x^2 \tag{5}$$

$$\left(\frac{1}{2}C_{p,f}+y\right)^2=C_{p,r}+y^2 \tag{6}$$

The families of parabolas representing equations (5) and (6) are shown in the classification chart (fig. 2). The quantity $x-y$ is, by the momentum principle, the tangential force or torque per unit radius on each rotor blade per unit mass flow per unit circumferential speed. Positive torque corresponds to fan action, in which energy is transferred to the fluid by means of the rotor. Negative torque corresponds to turbine action, in which energy is transferred from the fluid by means of the rotor. Positive values of $C_{p,f}$ therefore correspond, by equation (4), to fan action and negative values to turbine action. Figure 2 thus represents a general designation of the characteristics of axial-flow fans and turbines.

DISCUSSION

FANS AND TURBINES WITH CONSTANT AXIAL-FLOW AREA

Conventional fans and turbines.—Operating conditions for various fan or turbine arrangements can be obtained from

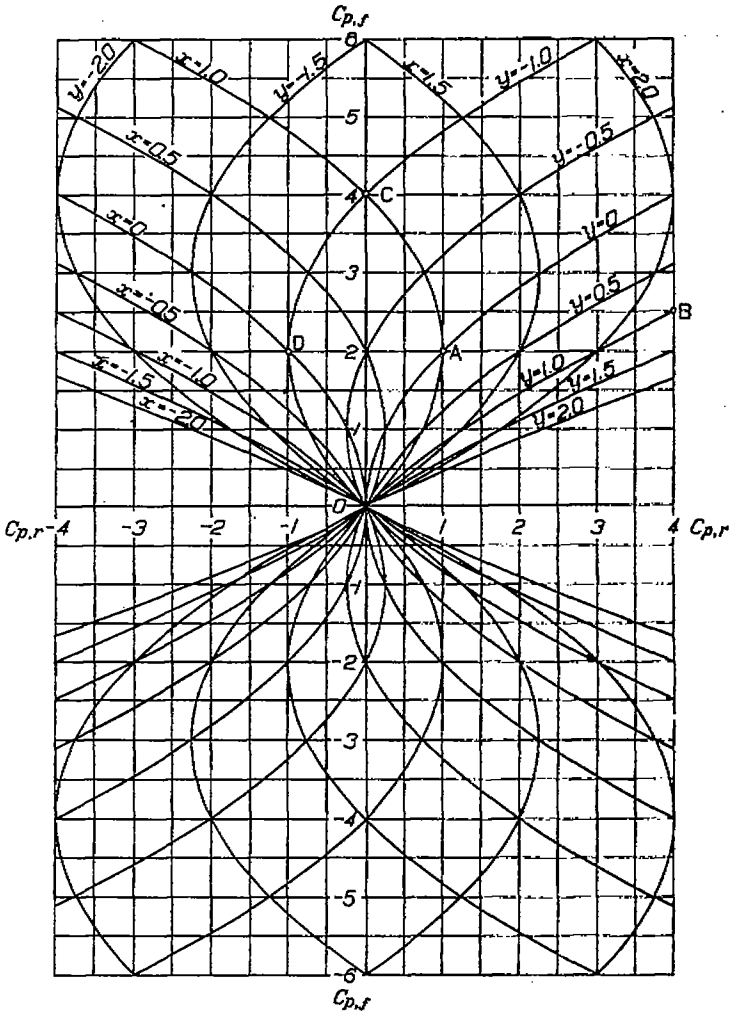

FIGURE 2.—Classification chart relating pressure coefficients of axial-flow fans and turbines and velocity-diagram parameters.

the chart in figure 2. Fan arrangements consisting of a rotor followed by a stator will be designated rotor-stator fans. Fan arrangements consisting of a stator followed by a rotor will be designated stator-rotor fans. All single-stage rotor-stator fans, for which evidently $x=1$, are represented on the first-quadrant parabolic arc OA. All single-stage stator-rotor fans, for which evidently $y=1$, are represented on the first-quadrant parabolic arc OB. For an over-all pressure coefficient $C_{p,f}=1$ in the single-stage rotor-stator fan, the rotor contributes three-fourths and the stator one-fourth of the over-all pressure-rise coefficient. For an over-all pressure coefficient $C_{p,f}=1$ in the single-stage stator-rotor fan, the rotor contributes five-fourths and the stator minus one-fourth of the over-all pressure-rise coefficient. In general, the first quadrant of figure 2 contains the design operating points of conventional fan arrangements, that is, arrangements in which the rotor pressure coefficients and the over-all pressure coefficients are both positive.

Similarly, the third quadrant of figure 2 contains the design operating points for conventional turbine arrangements in which the rotor pressure coefficients and the over-all pressure coefficients are both negative.

Turbine-type fans.—It is apparent from the chart that positive over-all pressure coefficients may also be obtained for fan arrangements in which the rotor pressure coefficients are negative. Such arrangements, which are represented in the second quadrant, will be designated turbine-type fans. These fans utilize the rotor to transfer kinetic energy of rotation to the fluid, the transfer thus accounting for the pressure drop across the rotor. The downstream stator converts the kinetic energy of rotation to pressure energy. In this way the problem of high blade loading in the presence of an over-all adverse pressure gradient need be solved only for the stator blades. The known aerodynamic methods of increasing the efficiency of diffusion—in particular, the various methods of boundary-layer control—may now be attempted without incurring the difficulties presented by application of these methods to blades rotating at high speed. Such arrangements appear to offer definite advantages in developing fans with high pressure rise per stage.

Representative turbine-type fan arrangements are presented in figures 3 and 4. The fan shown in figure 3 is designed for operation at point C of figure 2. The rotor velocity triangle is similar to that of an impulse-turbine rotor. The fan shown in figure 4 is designed for operation at point D of figure 2. The rotor velocity triangle is similar to that of a reaction-turbine rotor. Although the rotor-blade pressure distributions in the examples given are similar to the favorable pressure distributions that occur in turbine blading, the fan rotors transfer energy to the fluid in the form of kinetic energy of rotation.

Fan-type turbines.—The fourth quadrant of figure 2 contains the design operating points for turbines having positive rotor pressure coefficients. These turbines, which will be designated fan-type turbines, bear the same relation to conventional turbines that turbine-type fans bear to conventional fans.

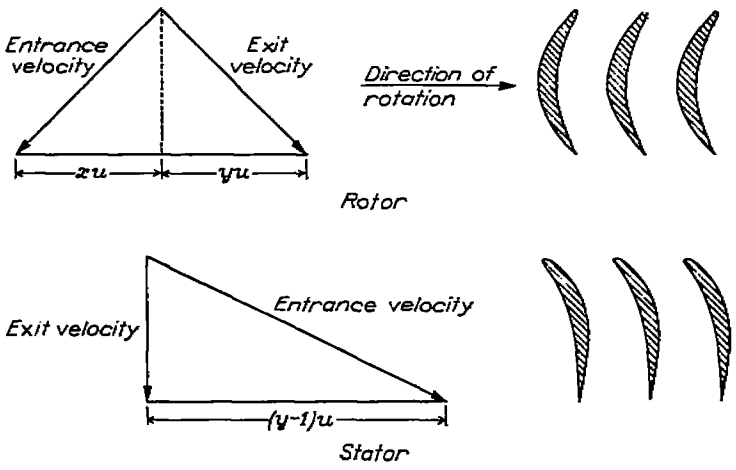

FIGURE 3.—Fan design for over-all pressure coefficient $C_{p,f}=4$ with rotor pressure coefficient $C_{p,r}=0$. Velocity vectors shown are relative to corresponding blading. Design values: $x=1$; $y=-1$.

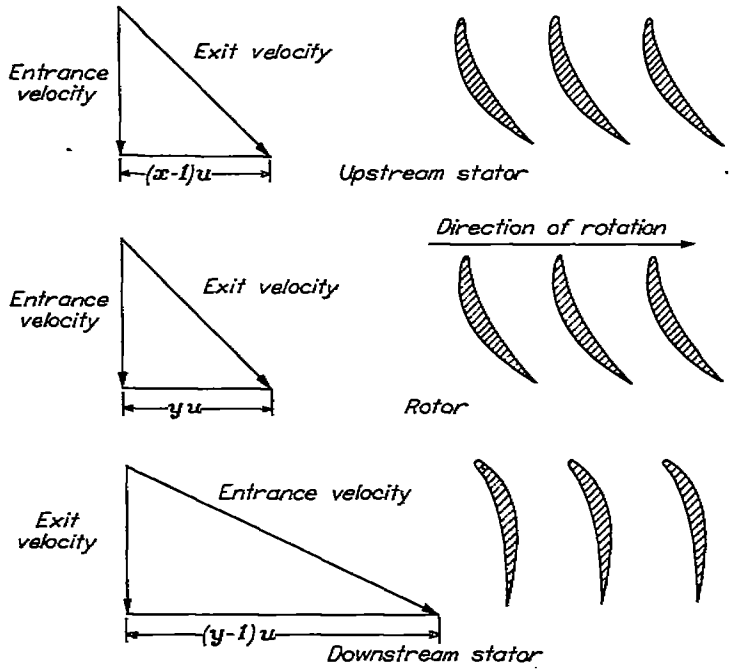

FIGURE 4.—Fan design for over-all pressure coefficient $C_{p,f}=2$ with rotor pressure coefficient $C_{p,r}=-1$. Velocity vectors shown are relative to corresponding blading. Design values: $x=0$; $y=-1$.

Generalization of stage definition.—It may be noted that the results embodied in figure 2 hold unchanged if the stage definition is generalized so that the entrance velocity to, and the exit velocity from, the stage are not necessarily axial but are equal in magnitude. With this extension, the results are particularly applicable to multistage blading.

Consider, for example, multistage fan or turbine blading in which the tangential velocities relative to a rotor are equal and opposite to the tangential velocities relative to its downstream stator. The locus in figure 2 of all such blading is obtained (a) by setting the axial and tangential components of the stage entrance and exit velocities equal to v and $(x-1)u$, respectively (see fig. 1), and (b) by setting $xu=-(y-1)u$ or

$$x+y=1 \tag{7}$$

Step (a) does away with the upstream stator and consequently fixes the change in tangential velocity through the rotor equal to the change in tangential velocity through the downstream stator; step (b), in effect, equates the entrance velocity relative to the rotor to the entrance velocity relative to the downstream stator. Equations (7), (6), and (5) yield

$$C_{p,f}=2C_{p,r} \tag{8}$$

This equation would be represented in figure 2 by a straight line through the origin and, of course, expresses the fact that the rotor and the stator contribute equally to the resultant pressure change across the stage.

FANS AND TURBINES WITH VARIABLE AXIAL-FLOW AREA

Velocity-diagram analysis.—The analysis may be extended to include a variable axial-flow area through the fan or the turbine. The velocity diagram to be considered is shown in figure 5. Radial-velocity components are assumed

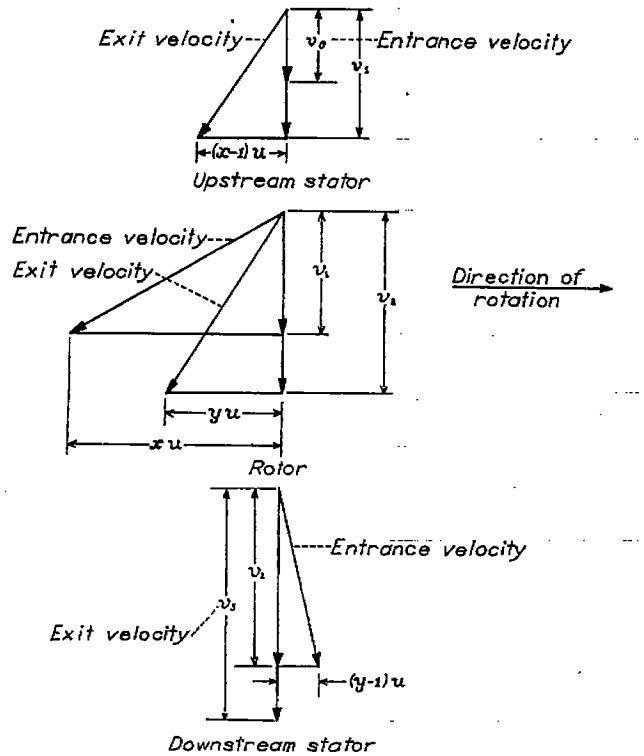

FIGURE 5.—Velocity diagram of single stage of axial-flow fan with variable axial-flow area.

to be nonexistent in the ultimate upstream and downstream conditions. The parameters characterizing the change in axial-flow area are

$$\left.\begin{aligned} a_0 &\equiv \frac{A_0}{A_1}=\frac{v_1}{v_0} \\ a_1 &\equiv \frac{A_1}{A_2}=\frac{v_2}{v_1} \\ a_2 &\equiv \frac{A_2}{A_3}=\frac{v_3}{v_2} \end{aligned}\right\} \tag{9}$$

where

A_0 axial-flow area at entrance to upstream stator
A_1 axial-flow area at entrance to rotor
A_2 axial-flow area at entrance to downstream stator
A_3 axial-flow area at exit from downstream stator

The pressure coefficients of the stage elements are, by Bernoulli's equation,

$$\text{Upstream stator} \quad C_{p,su}=-(x-1)^2+\left(\frac{v_0}{u}\right)^2(1-a_0^2) \tag{10}$$

$$\text{Rotor} \quad C_{p,r}=x^2-y^2+\left(\frac{v_1}{u}\right)^2(1-a_1^2) \tag{11}$$

$$\text{Downstream stator} \quad C_{p,sd}=(y-1)^2+\left(\frac{v_2}{u}\right)^2(1-a_2^2) \tag{12}$$

The over-all pressure coefficient is the sum of the components:

$$C_{p,f}=2(x-y)+\left(\frac{v_0}{u}\right)^2(1-a_0^2)+\left(\frac{v_1}{u}\right)^2(1-a_1^2)+\left(\frac{v_2}{u}\right)^2(1-a_2^2) \tag{13}$$

The coefficient $C_{p,f}$ is not indicative of the total energy transferred to the fluid because of the change in axial-flow area. The total energy change is given by the difference in total pressures, based on absolute velocities, upstream and downstream of the rotor. For the frictionless flow assumed, no change in total pressure occurs across either of the stators. If H_1 and H_2 are the total pressures upstream and downstream of the rotor, respectively,

$$\left.\begin{aligned} H_1&=p_{r,1}+\frac{1}{2}\rho\left[(x-1)^2u^2+v_1^2\right] \\ H_2&=p_{r,2}+\frac{1}{2}\rho\left[(y-1)^2u^2+v_2^2\right] \end{aligned}\right\} \tag{14}$$

The total-pressure coefficient for the stage is

$$C_{H,f}\equiv\frac{H_2-H_1}{\frac{1}{2}\rho u^2}=2(x-y) \tag{15}$$

A comparison of equations (11), (13), and (15) with equations (2) and (4) shows that the relation between the coefficients

$$\left.\begin{aligned} C_{H,f}&=C_{p,f}-\sum_{n=0}^{n=2}\left(\frac{v_n}{u}\right)^2(1-a_n^2)=2(x-y) \\ C_{p,r}'&\equiv C_{p,r}-\left(\frac{v_1}{u}\right)^2(1-a_1^2)=(x^2-y^2) \end{aligned}\right\} \tag{16}$$

may be determined by the values given in figure 2, where $C_{p,r}'$ may be regarded as an equivalent rotor pressure coefficient.

Applications.—In the conventional axial-flow fan the rotor transfers energy to the fluid in the form of pressure energy. In the turbine-type fan the rotor transfers energy to the fluid in the form of kinetic energy of rotation. The Schicht, or constant-pressure, fan described in reference 1 transfers energy to the fluid in the form of increased energy of axial motion. The constant-pressure fan utilizes a reduction in axial-flow area to increase the axial-flow velocity and thereby converts to axial-flow kinetic energy the static-pressure rise that would otherwise occur. Such fans are indicated in the first quadrant of the chart in figure 2. The coefficients $C_{H,f}$ and $C_{p,r}'$ of equations (16) must be used because of the change in axial-flow area through the fan. Which of the three methods of transferring energy to a fluid is most efficient depends upon the purpose of the fan design under consideration. Experimental data on this problem are limited.

The possible utility of applying variable axial-flow area to the design of constant-pressure turbines is also of interest. Such turbines are indicated in the third quadrant of figure 2. In the constant-pressure turbine the axial-flow area would be increased in the downstream direction, and the axial-flow velocity would thus be decreased. The static-pressure drop of the working fluid that would occur if the axial-flow area were constant may be nullified by a change in axial-flow area. The turbine thus delivers shaft work at the expense of a reduced kinetic energy in an axial direction rather than at the expense of reduced static-pressure energy.

Constant-pressure turbines with increasing axial-flow area might prove of use in the design of short diffusers. The diffuser lengths required to transform the kinetic energy of a flow to pressure energy are often excessive. In combination with a compressor mounted immediately downstream on the same shaft, the constant-pressure turbine would perform the same function as a diffuser, in a shorter length.

CONCLUDING REMARK

The general representation for axial-flow fans and turbines presented in this paper indicates several unconventional arrangements of fans and turbines that appear to offer definite advantages for high-pressure-rise fans and for diffuser application.

AIRCRAFT ENGINE RESEARCH LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
CLEVELAND, OHIO, *May 4, 1945*.

REFERENCE

1. Sörensen, E.: Constant-Pressure Blowers. NACA TM No. 927 1940.