REPORT No. 825

ANALYSIS OF WIND-TUNNEL STABILITY AND CONTROL TESTS IN TERMS OF FLYING QUALITIES OF FULL-SCALE AIRPLANES

By Gerald G. Kayten

SUMMARY

The analysis of results of wind-tunnel stability and control tests of powered airplane models in terms of the flying qualities of full-scale airplanes is advocated. In order to indicate the topics upon which comments are considered desirable in the report of a wind-tunnel stability and control investigation and to demonstrate the nature of the suggested analysis, the present NACA flying-qualities requirements are discussed in relation to wind-tunnel tests. General procedures for the estimation of flying qualities from wind-tunnel tests are outlined.

INTRODUCTION

At the laboratories of the National Advisory Committee for Aeronautics and at laboratories maintained by various universities and independent agencies, wind-tunnel tests are often made to investigate the stability and control characteristics of particular airplane designs. Upon these tests are based, to a large degree, the decisions regarding changes and improvements necessary in order to make the final product a satisfactory airplane. Unfortunately, however, the test results are in most cases not presented in such a manner as to permit their immediate use in making these decisions; instead, the wind-tunnel data are usually presented in coefficient form in a voluminous series of curves and tables.

As judged in flight testing or service, the flying qualities of an airplane are characterized not by dimensionless coefficients but by forces, velocities, accelerations, angles, and other measurable quantities which actually define the stability and control characteristics of an airplane in flight.

Various means may be employed for determining from wind-tunnel test data the particular dimensional values describing the airplane's flying qualities, but this type of analysis has not generally been considered the province of wind-tunnel personnel. It is believed, however, that the value of wind-tunnel tests would be increased considerably if an analysis of this nature were included in every stability and control investigation. Although the analysis would augment rather than replace the measured data as usually presented, the greater portion of the wind-tunnel report would consist of a discussion of the actual flying qualities of the airplane. The inclusion of such a discussion would eliminate the confusion often caused by mere presentation of the test results, facilitate the practical application of tunnel data, and provide assurance that no flight difficulties will pass undetected because of failure to put the accumulated information to its proper use. Moreover, test programs designed for this purpose could be planned more efficiently with regard to the amount of required testing. Many programs in the past have been laid out arbitrarily without a complete understanding of the manner in which the resulting data should be applied. This lack of understanding has at times resulted in insufficient data concerning trim conditions and considerable unnecessary data for untrimmed conditions.

The purpose of this paper is to outline a suggested form of presentation of the results of a stability and control investigation in terms of flying qualities as defined in reference 1 and to systematize and review briefly the analytical work required for this type of presentation. No effort is made to specify definite test procedures.

Reference 2 contains a review of testing technique for use with powered wind-tunnel models and a fairly complete discussion of most of the standard tests necessary for the collection of data used in the suggested analysis. It should prove useful in the preparation of any stability and control test program. Although flight measurements and observations are made with respect to the airplane body axes, the use of the "stability" axes as recommended in reference 2 will probably be satisfactory for the normal range of test conditions, and these axes are used throughout the present paper.

It is assumed that all necessary tunnel corrections will be made before any analysis is attempted and that the measured data will be sufficiently accurate for use in predicting flight characteristics with reasonable precision.

The NACA requirements for satisfactory flying qualities, as explained in detail in reference 1, are used as a basis for the procedure suggested herein and constitute the list of subjects on which it is believed comments should be made in the presentation of wind-tunnel data relating to stability and control. The complete series of tests is not considered essential for every airplane; the list of requirements is included in its entirety for the purpose of pointing out the desired form of analysis for any phase of stability and control investigated.

It is realized, of course, that the requirements for satisfactory flying qualities may undergo constant revision with time. By methods similar to those indicated in the present paper, however, wind-tunnel tests may be used for the investigation of any revisions of the present requirements or for the investigation of a completely different set of criterions.

For purposes of clarity and convenience, each of the present NACA flight requirements is given in the text, accompanied by recommendations regarding its relation to tunnel testing.

Unless otherwise specified or implied, the requirements should be investigated for all conditions of flight, special attention being given the conditions that appear to be the most critical.

COEFFICIENTS AND SYMBOLS

W airplane weight, pounds

S area of wing (unless accompanied by subscript a, e, or r denoting aileron, elevator, or rudder), square feet

b wing span, feet

c mean aerodynamic chord (M. A. C.), feet

$\bar{c}_e$ root-mean-square elevator chord, feet

$\bar{c}_a$ root-mean-square aileron chord, feet

$\bar{c}_r$ root-mean-square rudder chord, feet

l tail length (distance from center of gravity to elevator hinge line), feet

i_t angle of stabilizer setting with respect to thrust line, degrees, positive when leading edge is up

δ_a aileron deflection, degrees, positive when trailing edge is down

δ_e elevator deflection, degrees, positive when trailing edge is down

δ_r rudder deflection, degrees, positive when trailing edge is to left

α airplane angle of attack (thrust line), degrees

α_t angle of attack at tail, degrees

β angle of sideslip, degrees, positive when right wing is forward

ψ angle of yaw, degrees, positive when left wing is forward ($\psi=-\beta$)

V airspeed, feet per second

ρ mass density of air, slugs per cubic foot

q dynamic pressure, pounds per square foot $\left(\frac{1}{2}\rho V^2\right)$

m airplane mass, slugs (W/g)

μ relative-density factor $\left(\frac{m}{\rho S l}\right)$

C_L lift coefficient $\left(\frac{\text{Lift}}{qS}\right)$

C_m pitching-moment coefficient about center of gravity $\left(\frac{\text{Pitching moment}}{qSc}\right)$

C_l rolling-moment coefficient about center of gravity $\left(\frac{\text{Rolling moment}}{qSb}\right)$

C_n yawing-moment coefficient about center of gravity $\left(\frac{\text{Yawing moment}}{qSb}\right)$

C_Y lateral-force coefficient $\left(\frac{\text{Lateral force}}{qS}\right)$

p rolling velocity, radians per second

C_{h_e} elevator hinge-moment coefficient $\left(\frac{\text{Elevator hinge moment}}{q\bar{c}_e^2 b_e}\right)$

C_{h_a} aileron hinge-moment coefficient $\left(\frac{\text{Aileron hinge moment}}{q\bar{c}_a^2 b_a}\right)$

C_{h_r} rudder hinge-moment coefficient $\left(\frac{\text{Rudder hinge moment}}{q\bar{c}_r^2 b_r}\right)$

C_{l_p} rolling-moment coefficient due to rolling $\left(\frac{\partial C_l}{\partial\left(\frac{pb}{2V}\right)}\right)$

C_{n_p} yawing-moment coefficient due to rolling $\left(\frac{\partial C_n}{\partial\left(\frac{pb}{2V}\right)}\right)$

F stick force, pounds

x linear travel at top of control stick, feet

$\frac{dC_h}{d\alpha}$ change in hinge-moment coefficient per degree change in angle of attack

$L.F.$ maximum allowable load factor

$\frac{dC_m}{di_t}$ change in pitching-moment coefficient per degree change in stabilizer setting

$\frac{dC_{h_e}}{di_t}$ change in elevator hinge-moment coefficient per degree change in stabilizer setting

g acceleration of gravity, feet per second per second

Δa_n change in normal acceleration, g units

t time, seconds

DETERMINATION OF FLYING QUALITIES

The requirements for satisfactory flying qualities of airplanes have been given in reference 1 under three main headings, namely:

I. Requirements for Longitudinal Stability and Control
II. Requirements for Lateral Stability and Control
III. Stalling Characteristics

The present paper follows the outline of reference 1 and each of the flying-quality requirements is quoted directly.

I. Requirements for Longitudinal Stability and Control.

Requirement (I–A).—Characteristics of uncontrolled longitudinal motion.

"When elevator control is deflected and released quickly, the subsequent variation of normal acceleration and elevator angle should have completely disappeared after one cycle."

The technique of applying wind-tunnel data to flight has not yet developed to a state sufficiently advanced for the quantitative investigation of the short-period oscillation with controls free, which is the important factor in the consideration of this requirement.

Control-free stability has been shown in reference 3 and by recent unpublished investigations not to depend as critically upon the stability characteristics of the airplane as upon the design of the control system, particularly upon the weight moment, the aerodynamic balance, and the friction. These factors must therefore be considered in any special study of this requirement. Work is now in progress at the Langley Laboratory for the purpose of establishing procedures to make such studies practicable.

Requirement (I–B).—Characteristics of elevator control in steady flight.

I–B–1. "The variation of elevator angle with speed should indicate positive static longitudinal stability for the following conditions of flight:

a. With engine or engines idling, flaps up or down, at all speeds above the stall.

b. With engine or engines delivering power for level flight with flaps down (as used in landing approach), landing gear down, at all speeds above the stall.

c. With engine or engines delivering full power with flaps up at all speeds above 120 percent of the minimum speed."

For each of the specified flap and power conditions, data are assumed to be available in the form of pitching-moment and hinge-moment coefficients against lift coefficients, as shown in figure 1. These curves may be obtained either from constant-power test runs or from cross-plotted constant-thrust data. In the absence of definite idling data for the engine-propeller combination, the zero-thrust condition is considered a satisfactory representation of the idling condition.

The slope of the curve of C_m against C_L is a valid measure of static stability in power-on flight only where $C_m=0$. The slope at the trim point might be considered for each of a number of elevator settings, but a curve of δ_e against C_L or airspeed represents a more direct comparison with flight and is useful in investigating several other requirements. The curve shown in figure 2(a), which was obtained by cross-plotting for $C_m=0$ on figure 1, possesses a negative slope indicative of the required stable variation. The curve should be plotted to the stall in every case; although full-power stability below 120 percent of the minimum speed is not an absolute requirement, it is highly desirable.

A sufficient number of elevator deflections within the trim range should be tested to provide for a dependable curve of δ_e against C_L or airspeed. Complete test runs with elevator deflections that will not provide trim in the flight range, however, are not necessary and need not be made. For the investigation of elevator effectiveness at high deflections for landing conditions and maneuvers, tests need be made only at high lift coefficients.

In addition to the regular elevator runs, one set of measurements should be taken for the investigation of the take-off requirement, as explained in item (I–E).

The terms "minimum speed" and "maximum speed" as used in this and subsequent requirements should be interpreted as the minimum and maximum speeds specified by the designer or estimated for the airplane in free air. (In each test, the maximum and minimum values referred to are for the particular flap position and power being investigated.) Conditions specified in terms of percentages of minimum or maximum speed should be represented by using the lift coefficients corresponding to the appropriate full-scale speeds at the appropriate altitudes, no attempt being made to use small-scale tunnel data for the determination of maximum and minimum speeds. If, however, the lift coefficient for a condition near minimum speed falls on a nonlinear portion of the model lift curve, representation of this low-speed condition should not be attempted and a higher-speed condition should be substituted.

The center-of-gravity position considered as critical in the investigation of static stability should be that specified by the designer as the most rearward position.

Of considerable value in a study of this sort, particularly since the final airplane loading conditions are generally not established at the time of the wind-tunnel tests, is the determination of the most rearward center-of-gravity position for neutral stability. The distance, in chord lengths, from this neutral point to the center of gravity under consideration is the so-called static margin. With power off, the static margin may be considered numerically equal, but opposite in sign, to the slope of the pitching-moment curve at $C_m=0$, and the neutral point may be located by adding this value to the center of gravity about which the moments are plotted. This procedure is not correct for constant-power operation because of the effect upon stability of the different tail loads associated with trim at different center-of-gravity locations. The power-on neutral point can be determined by plotting $\frac{dC_m}{dC_L}$ against two or three center-of-gravity positions and extending the curve to $\frac{dC_m}{dC_L}=0$. As demonstrated in reference 4, this curve will be a straight line, provided that the tail-lift curve is linear.

In order to use wind-tunnel data in this manner, it is not necessary to recompute the pitching moments for each horizontal center-of-gravity location. A graphical method is illustrated with the curves of figure 1, which are for a $0.25c$ center of gravity. It is desired, for example, to find the neutral point at $C_L=0.8$. From the point at which $C_L=0$ and $C_m=0$, straight lines are drawn radiating outward and intersecting curves for various elevator angles at $C_L=0.8$. For $\delta_e=-6°$, the pitching-moment slope is -0.095. The slope of the radiating line for this point is 0.103. The slope about a center-of-gravity location of $0.147c$ $(0.250-0.103=0.147)$, then, is equal to $-0.095-0.103=-0.198$. For $\delta_e=-3°$, the measured slope is -0.104 and the slope of the radiating line is 0.032, giving a pitching-moment slope of -0.136 about a $0.218c$ center of gravity. Repetition of the process for $\delta_e=0°$ and $\delta_e=2°$ supplies the data for the curve of $\frac{dC_m}{dC_L}$ against center of gravity shown in figure 3(a). Extended to zero slope, this curve shows the neutral point to occur at $0.372c$. The procedure should be repeated for several lift coefficients, permitting the construction of a curve of neutral points plotted against lift coefficients.

If the pitching-moment curves are so shaped that the radiating lines may be drawn tangent to the curves, the slope of each radiating line is numerically equal (but opposite in sign) to the static margin for the lift coefficient at the point of tangency, and the neutral point is determined with no further consideration of different center-of-gravity locations. Figures 3(b) and 3(c) illustrate the determination of neutral points by this method. (The dashed lines represent pitching-moment curves and the solid lines A and B are the radiating tangent lines.)

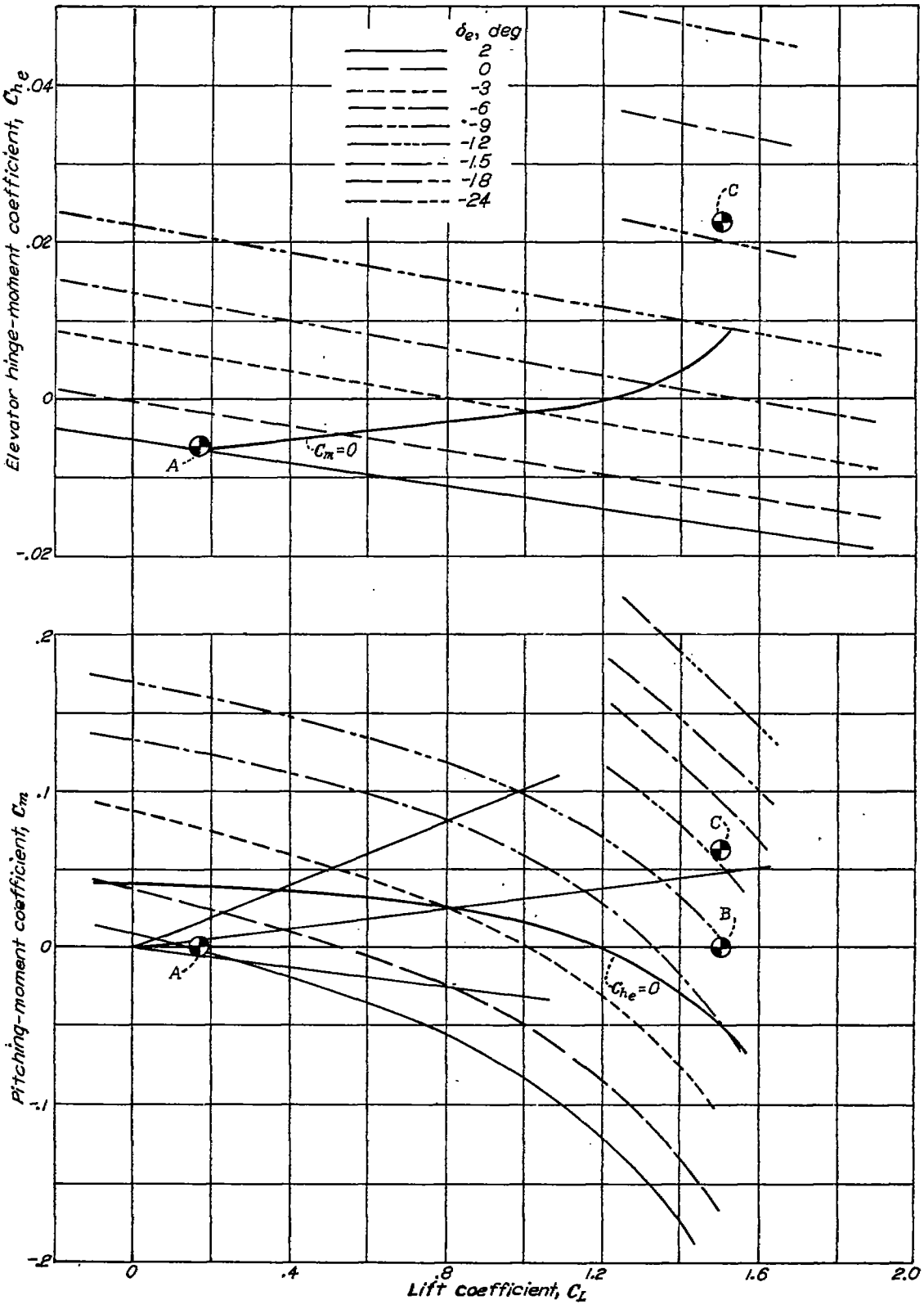
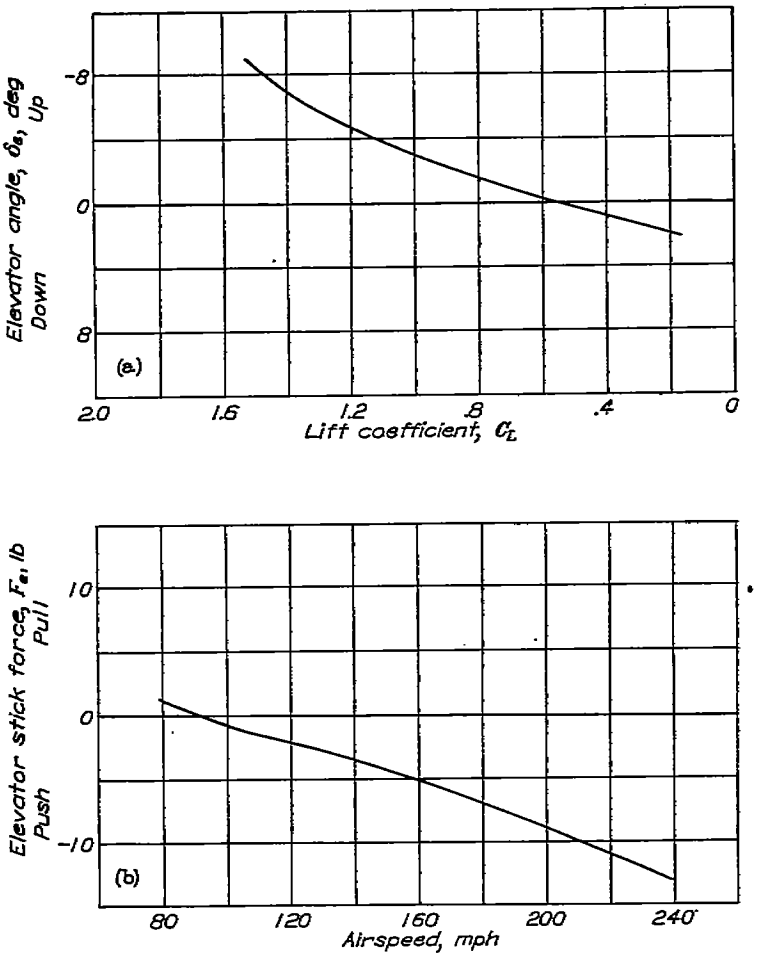


FIGURE 1.—Pitching-moment and hinge-moment curves for a typical fighter airplane. $\delta_f=0°$; idling power; $T_c=0$.



(a) Variation of elevator trim setting with lift coefficient for a typical fighter airplane. δ_t=0°; idling power.

(b) Variation of elevator stick force with trim airspeed for a typical fighter airplane at sea level. δ_t=0°; idling power.

FIGURE 2.—Elevator angles and stick forces for trim.

Both methods involving the radiating lines are based upon the "rotation" method of moment transfers. This method is subject to some error at very high lift coefficients inasmuch as it neglects the pitching moment due to drag. As shown in reference 5, however, the method is sufficiently accurate for use in the foregoing analysis.

It is possible to express the graphical method illustrated in figures 1 and 3(a) in terms of a mathematical formula involving the slopes of the pitching-moment curves for any two elevator settings at the specified lift coefficient, the two pitching-moment coefficients, and the lift coefficient. Thus, for a given lift coefficient the static margin is found by the following expression:

$$\text{Static margin}=\frac{\frac{C_{m_A}}{C_L}\left(\frac{dC_m}{dC_L}\right)_B-\frac{C_{m_B}}{C_L}\left(\frac{dC_m}{dC_L}\right)_A}{\left(\frac{dC_m}{dC_L}\right)_A-\frac{C_{m_A}}{C_L}-\left(\frac{dC_m}{dC_L}\right)_B+\frac{C_{m_B}}{C_L}} \qquad (1)$$

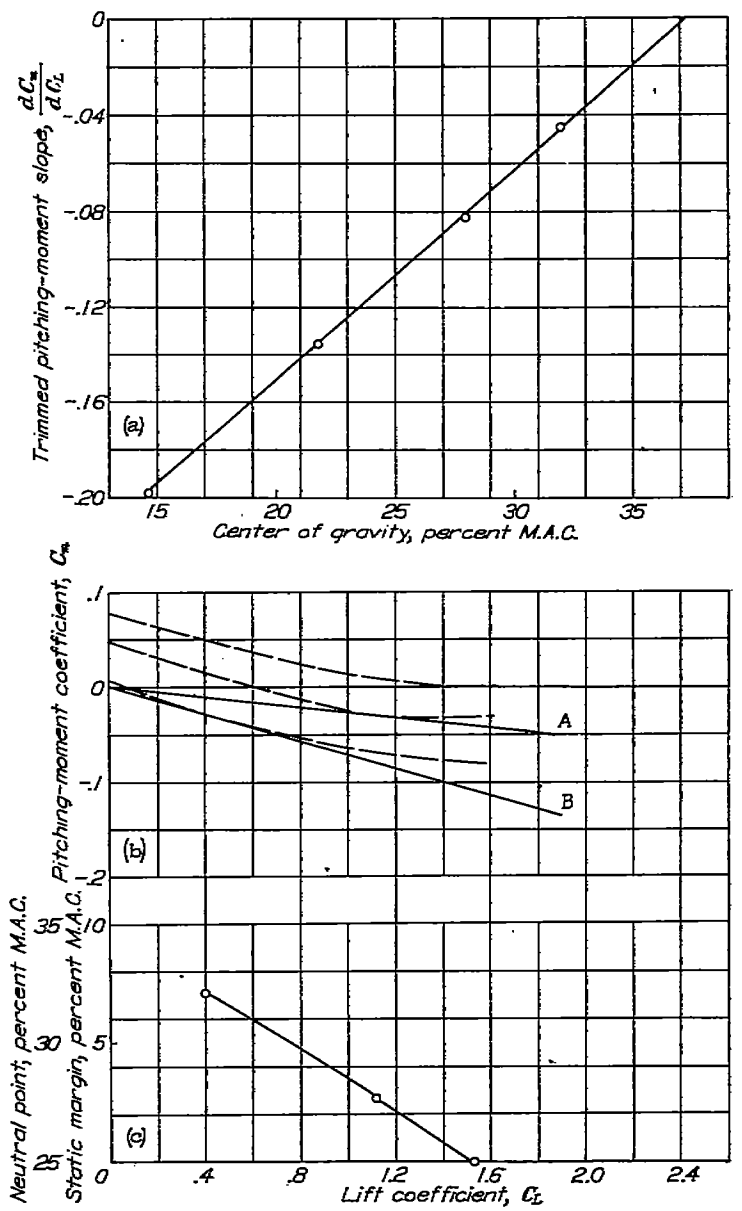
where C_{m_A} and C_{m_B} are the measured pitching-moment coefficients for the two curves at the given lift coefficient, and $\left(\frac{dC_m}{dC_L}\right)_A$ and $\left(\frac{dC_m}{dC_L}\right)_B$ are the pitching-moment slopes at each point.

If equation (1) is applied to the curves of figure 1 for C_L=0.8 and elevator curves of −6° and −3°,

$$\text{Static margin}=\frac{\frac{0.083}{0.8}(-0.104)-\frac{0.025}{0.8}(-0.095)}{-0.095-\frac{0.083}{0.83}+0.104+\frac{0.025}{0.8}}$$

$$=0.123$$

The neutral point, then, is at 0.250+0.123=0.373 (or 37.3 percent M. A. C.). This point agrees with the value found graphically in figure 3(a).



(a) Pitching-moment slope against center-of-gravity location.

(b) Determination of neutral point from pitching-moment curves. Rated power; center of gravity at 25 percent M. A. C.; flaps neutral.

(c) Variation of neutral point with lift coefficient. Rated power; flaps neutral.

FIGURE 3.—Determination of neutral point.

When consideration of vertical center-of-gravity movement is necessary, as it may be for an airplane on which large vertical center-of-gravity movement is possible, the procedure can be repeated for two or three vertical locations and a locus of neutral points (which theoretically should be a straight line) can be set up for each desired lift coefficient. Reference 4 contains an explanation of this method, as well as a more complete discussion of the determination of neutral points from wind-tunnel data.

I–B–2. "The variation of elevator control force with speed should be such that pull forces are required at all speeds below the trim speed and push forces are required at all speeds above the trim speed for the conditions requiring static stability in item 1."

Although this requirement is shown to be met if, for each of the specified conditions, the elevator-free ($C_{h_e}=0$) pitching-moment curve cross-plotted as shown on figure 1 crosses the zero ordinate only when it possesses a stable slope, subsequent requirements make it desirable to plot an actual curve of stick force against trim airspeed. This curve (fig. 2(b)) is obtained by converting to forces the hinge-moment coefficients for trim at different lift coefficients (fig. 1) and should be drawn for different trim-tab, flap, and power conditions.

The requirement should be studied for the trim-tab settings giving trim at appropriate ranges of speed in each condition. In the absence of reliable test data for various tab settings, reasonable estimates may be made from the tab-neutral data, by proper adjustment of the hinge-moment curves to simulate the effects of small tab changes. The shift in the hinge-moment curve may not be constant throughout the speed range but will depend on the dynamic-pressure ratio at the tail.

I–B–3. "The magnitude of the elevator control force should everywhere be sufficient to return the control to its trim position."

Figure 2(b) shows a curve of stick force plotted against trim airspeed, as used in item I–B–2. In the absence of quantitative information concerning friction in the elevator control system, a value of 0.05 pound per mile per hour, as suggested in reference 1, may be used as a minimum value for the slope near trim of the curve of elevator stick force against airspeed (fig. 2(b)). For very large airplanes, however, this amount may not be sufficient.

I–B–4. "It should be possible to maintain steady flight at the minimum and maximum speeds required of the airplane."

Examination of curves of elevator deflection against lift coefficient (or airspeed) of the type shown in figure 2(a) will supply the desired information. It is assumed that the tests are made at a Reynolds number sufficiently high that data near the stall will be fairly reliable. When these data are not available, the minimum-speed trim cannot be investigated unless the curves can be extrapolated beyond the model stall.

Requirement (I–C).—Characteristics of the elevator control in accelerated flight.

I–C–1. "By use of the elevator control alone, it should be possible to develop either the allowable load factor or the maximum lift coefficient at every speed."

Although it may be desirable to check this requirement at several lift coefficients, an investigation at only one critical lift coefficient is necessary if the previous static-stability requirements have been met. The initial lift coefficient to be studied will be $C_L=\frac{C_{L_{max}}}{L.F.}$. The model arrangement considered in the accelerated flight studies should be that corresponding to the airplane in its maneuvering condition.

Reference 6 gives a method for estimating the elevator deflections required for different normal accelerations in a pull-up maneuver. With a slight variation, the method can be applied to steady-turn maneuvers. It is possible to rearrange and simplify the formula of reference 6, expressing it in a form more suitable for use with wind-tunnel data for a particular airplane. The required change in elevator deflection can then be determined graphically from the equation

$$\Delta\delta_e\frac{dC_m}{d\delta_e}=-\Delta C_m-\Delta\alpha_t\frac{dC_m}{di_t} \tag{2}$$

where $\Delta\delta_e$ is the change in elevator deflection from trim setting at the initial lift coefficient and ΔC_m is equivalent to the static pitching-moment coefficient at the final lift coefficient with the elevator set for trim at the initial lift coefficient.

Some difference may exist between computations made for steady turns and for pull-ups because of the difference in the change in angle of attack at the tail due to the curved path $\Delta\alpha_t$. In a steady turn tightening from C_L to $C_{L_{max}}$,

$$\Delta\alpha_t=\frac{57.3C_L}{2\mu}\left(\frac{C_{L_{max}}}{C_L}-\frac{C_L}{C_{L_{max}}}\right)$$

In general, unless the amount of static stability and the relative density are quite small, the difference between the two maneuvers will be slight enough to permit coverage of the various accelerated-maneuver requirements by the consideration of the steady turn alone, and the examples given in the following paragraph are confined to this maneuver. If it is desired to investigate the pull-up from C_L to $C_{L_{max}}$, the identical procedure may be used but the change in angle of attack at the tail will be

$$\Delta\alpha_t=\frac{57.3C_L}{2\mu}\left(\frac{C_{L_{max}}}{C_L}-1\right)$$

In both maneuvers, the normal acceleration

$$a_n=\frac{C_{L_{max}}}{C_L}$$

and the change in normal acceleration, if a start from level flight is assumed, will be

$$\Delta a_n=\frac{C_{L_{max}}}{C_L}-1$$

In the following steady-turn examples, sea-level operation is assumed. It should be remembered, however, that the altitude must be taken into account in any actual investigation.

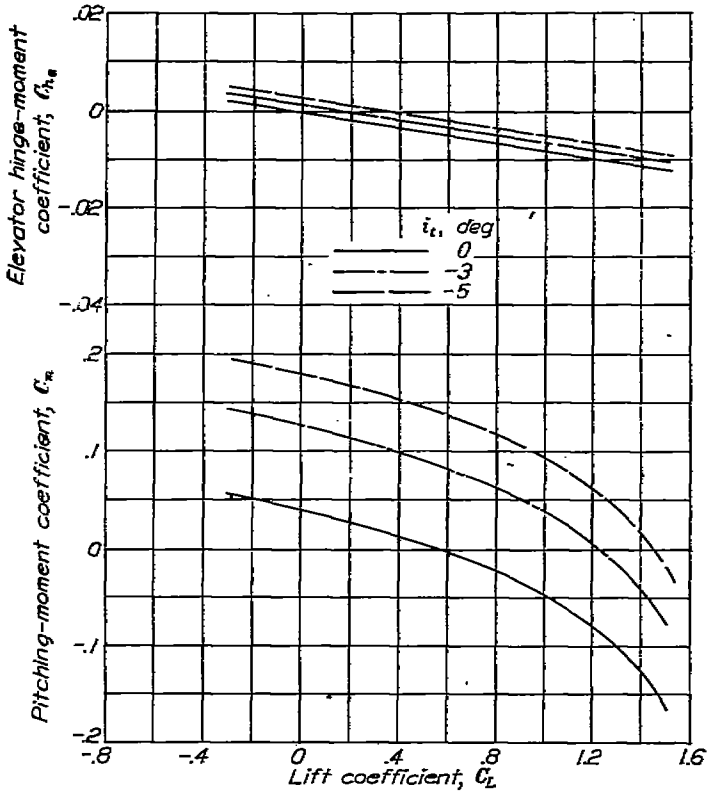


FIGURE 4.—Pitching-moment and hinge-moment curves for a typical fighter airplane. $\delta_f=0°$; $\delta_s=0°$; $T_c=0$.

The determination of the elevator characteristics in accelerated maneuvers involves the use of pitching-moment and hinge-moment curves for both stabilizer and elevator variations, as shown in figures 4 and 1. These curves should be plotted for a constant thrust coefficient, the value of which is determined by C_L and the rated power for the maneuvering condition. If the initial lift coefficient investigated is low (as may be expected in the case of pursuit airplanes if normal values of load factor and $C_{L_{max}}$ are used), the thrust coefficient may be sufficiently close to zero to permit the use of idling or zero-thrust data.

As indicated by the equation, the elevator deflection must supply enough pitching moment to balance the two factors ΔC_m and $\Delta\alpha_t \frac{dC_m}{di_t}$. The amount of deflection necessary to balance ΔC_m is merely the difference between the elevator settings for trim in steady flight at C_L and at $C_{L_{max}}$. The additional deflection is required to overcome the damping of the horizontal tail. (For a conventional airplane, the damping effects of the wing, fuselage, and other airplane components are considered negligible in comparison with the damping of the horizontal tail.)

The use of equation (2) can best be demonstrated by a sample solution for a steady-turn maneuver for a 6000-pound airplane with a tail length of 16.5 feet and a wing area of 250 square feet. The airplane has a maximum up-elevator travel of 25° for a 10-inch travel at the top of the control stick, with an essentially linear variation of δ_e with x. Pitching-moment and hinge-moment coefficients for the maneuvering condition are shown in figures 1 and 4. If a maximum lift coefficient of 1.5 and an allowable load factor of 9 are assumed, the lift coefficient investigated is $C_L=\frac{1.5}{9}=0.17$. Figure 4 gives -0.028 as the value for $\frac{dC_m}{di_t}$. The relative-density factor

$$\mu=\frac{6000}{32.2\times0.002378\times250\times16.5}$$
$$=18.99 \text{ at sea level}$$

The term

$$-\Delta\alpha_t\frac{dC_m}{di_t}=\frac{57.3}{2\mu}C_L\left(\frac{C_{L_{max}}}{C_L}-\frac{C_L}{C_{L_{max}}}\right)0.028$$
$$=\frac{57.3\times0.17}{37.98}\left(\frac{1.50}{0.17}-\frac{0.17}{1.50}\right)0.028$$
$$=0.063$$

The elevator setting for trim at $C_L=0.17$ is 1.8°, as shown by point A in figure 1. The setting for trim at $C_L=1.5$ (point B) would be $-8.6°$. Adding a pitching-moment increment of 0.063 at this point results in point C, which shows the elevator angle required for the maneuver to be $-12.5°$. A total upward deflection of 14.3° from the original trim position is required; this deflection is within the limit of available travel.

I–C–2. "The variation of elevator angle with normal acceleration in steady turning flight at any given speed should be a smooth curve which everywhere has a stable slope."

If all previous criterions are assumed satisfied, this requirement will be met if the pitching-moment curves of figure 1 and the variation of C_m with δ_e and with i_t as determined by cross-plotting from figures 1 and 4 are all smooth curves.

If so desired, the method used in item I–C–1 can be applied by keeping C_L constant at any value and by using in place of $C_{L_{max}}$ a lift coefficient that gives any particular value of Δa_n. Various normal accelerations can be used and plotted against the required elevator angles as in figure 5.

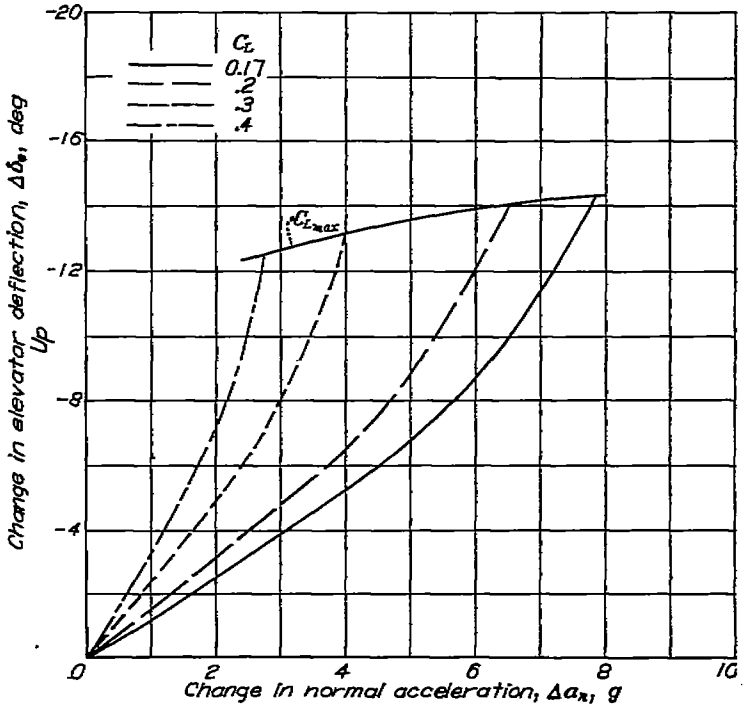


FIGURE 5.—Elevator deflection in steady-turn maneuvers.

I–C–3. "For airplanes intended to have high maneuverability, the slope of the elevator-angle curve should be such that not less than 4 inches of rearward stick movement is required to change angle of attack from a C_L of 0.2 to $C_{L_{max}}$ in the maneuvering condition of flight."

The elevator deflection for the steady turn starting from $C_L=0.2$ in straight flight and tightening to $C_{L_{max}}$ may be taken directly from figure 5. The angle can be converted to stick movement by consideration of the mechanical linkage, data for which were previously given. Figure 5 shows the deflection to be 14.1°, corresponding to a stick movement of 5.6 inches.

I–C–4. "As measured in steady turning flight, the change in normal acceleration should be proportional to the elevator control force applied."

If curves of C_h cross-plotted against δ_e for $C_m=0$ (as represented in fig. 1) are smooth, this requirement will be met, provided that requirement I–C–2 has been satisfactorily fulfilled.

I–C–5. "The gradient of elevator control force in pounds per unit normal acceleration, as measured in steady turning flight, should be within the following limits:

a. For transports, heavy bombers, etc., the gradient should be less than 50 pounds per g.

b. For fighter types, the gradient should be less than 6 pounds per g.

c. For any airplane, it should require a steady pull force of not less than 30 pounds to obtain the allowable load factor."

Although this item may be investigated for several initial lift coefficients, theory indicates that the force per unit normal acceleration is independent of the trim speed, neglecting compressibility effects. Unless the slope of the pitching-moment and hinge-moment curves varies appreciably with lift coefficient, the force in pounds need only be computed for the elevator deflection required for the development of allowable load factor (item I–C–1). This force should be less than $6\Delta a_n$ for pursuit types, less than $50\Delta a_n$ for heavy bombers, and more than 30 pounds in any case.

The stick force for the maneuver may be computed by the equation

$$F_e=\frac{d\delta_e}{dx}\left(\Delta C_{h_e}+\Delta\alpha_t\frac{dC_{h_e}}{di_t}\right)\frac{1}{2}\rho V^2\bar{c}_e^2 b_e \qquad (3)$$

where

ΔC_{h_e} difference between C_{h_e} for the initial elevator setting at C_L and C_{h_e} for the final elevator setting at $C_{L_{max}}$ (item I–C–1)

The operation of the suggested formula can be demonstrated by an example applied to the airplane investigated in item I–C–1. The root-mean-square chord of the elevator is 1.4 feet and the elevator span is 13 feet. The difference between C_{h_e} at point A and C_{h_e} at point C (fig. 1) measures 0.0286; figure 4 gives −0.0006 as the value for $\frac{dC_{h_e}}{di_t}$. The computations then proceed as follows:

$$\frac{d\delta_e}{dx}=\frac{25/57.3}{10/12}$$

$$=0.524$$

$$\Delta\alpha_t\frac{dC_{h_e}}{di_t}=\frac{57.3}{37.98}\,0.17\left(\frac{1.50}{0.17}-\frac{0.17}{1.50}\right)(-0.0006)$$

$$=-0.0013$$

$$\frac{1}{2}\rho V^2=\frac{W/S}{C_L}=\frac{6000}{250\times0.17}=141.2$$

$$F=0.524\times(0.0286-0.0013)\times141.2\times(1.4)^2\times13$$

$$=51.5 \text{ pounds}$$

The force gradient, then, is $\frac{51.5}{9-1}=6.4$ pounds per g, which is reasonably close to the specified limit for pursuit airplanes, and the force is greater than 30 pounds.

It is possible, through the use of the principle of axis rotation employed in item I–B–1, to investigate the elevator deflections and stick forces in accelerated maneuvers for any center-of-gravity position. If, for example, the steady-turn maneuver discussed in the previous paragraphs were investigated for a center of gravity located at 21.8 percent of the mean aerodynamic chord rather than at the 25-percent position used in figure 1, the radiating line representing a forward center-of-gravity movement of 0.032c (fig. 1) would be used. At $C_L=0.17$, this line shows the elevator setting for trim at the new center-of-gravity location to be 1.5°. At $C_L=1.5$, the distance $C_m=0.063$ (the damping term $\Delta\alpha_t\frac{dC_m}{di_t}$, which for practical purposes may be considered independent of center-of-gravity position) is measured from the radiating line rather than from the $C_m=0$ axis and indicates a final elevator setting of −16.7°. The hinge-moment coefficients for $\delta_e=1.5°$ at $C_L=0.17$ and for $\delta_e=-16.7°$ at $C_L=1.5$ are now used instead of points A and C to determine the value of ΔC_{h_e} to be used in place of 0.0286 in the stick-force equation.

Requirement (I–D).—Characteristics of the elevator control in landing.

I–D–1. "(Applicable to airplanes with conventional landing gear only.) The elevator control should be sufficiently powerful to hold the airplane off the ground until three-point contact is made."

I–D–2. "(Applicable to airplanes with nose-wheel type landing gear only.) The elevator control should be sufficiently powerful to hold the airplane from actual contact with the ground until the minimum speed required of the airplane is attained."

The elevator effectiveness in the presence of the ground may actually be investigated by use, for example, of the plate method described in reference 7 or of the combination half-ground-board and image model as employed in reference 8. These procedures, however, should not be necessary for consideration of the landing requirement. Through the use of

the methods and data of reference 8, the change in angle of attack at the tail $\Delta\alpha_t$ can be estimated. The elevator deflection for the landing can then be found in the same manner as point C on figure 1 (item I–C–1) by use of $\Delta\alpha_t \frac{dC_m}{di_t}$ as the pitching-moment increment. For the case of the landing requirement, however, the data used will be for the flap-down idling condition, and the lift coefficient will be either the airplane maximum lift coefficient or the lift coefficient corresponding to the angle of attack determined by the three-point ground angle of the airplane at rest. Extrapolation beyond the model stall may be necessary for the representation of the full-scale landing condition.

I–D–3. "It should be possible to execute the landing with an elevator control force which does not exceed 50 pounds for wheel-type controls, or 35 pounds where a stick-type control is used."

The stick force for landing can be computed from equation (3) where

ΔC_{h_e} elevator hinge-moment coefficient (with stabilizer or trim tab set for speed of approach) for elevator deflection and lift coefficient (item I–D–1 or I–D–2) required for landing

$\Delta\alpha_t$ change in angle of attack of tail computed for item I–D–1 or I–D–2

The remaining terms have already been defined in item I–C–5.

Requirement (I–E).—Characteristics of elevator control in take-off.

"During the take-off run, it should be possible to maintain the attitude of the airplane by means of the elevators at any value between the level attitude and that corresponding to maximum lift after one-half take-off speed has been reached."

For an airplane with a conventional landing gear, force and moment measurements at $\alpha=0°$ should be made for the take-off condition with the elevator set at the maximum available positive deflection and the center of gravity at its most rearward position. The power condition represented should be that corresponding to take-off power at one-half take-off speed. The measurements should, if possible, be made in the presence of a ground board or some similar device. In the absence of actual ground representation, an approximation may be made neglecting the slipstream displacement and the resulting change in elevator effectiveness. By the same method as that employed in item I–D–1, $\Delta\alpha_t$ due to the presence of the ground can be estimated. The effective pitching-moment coefficient $C_m'=C_m+\Delta\alpha_t \frac{dC_m}{di_t}$, where $\frac{dC_m}{di_t}$ is obtained from curves similar to figure 4 but for the correct power condition. This coefficient can be converted to an effective pitching moment $M'=C_m'q_1Sc$, where q_1 corresponds to the sea-level dynamic pressure at a speed equal to one-half the take-off speed. The lift at this speed is $L_1=q_1C_LS$; it is believed that the ground effect on lift is sufficiently small that it may be ignored in this problem.

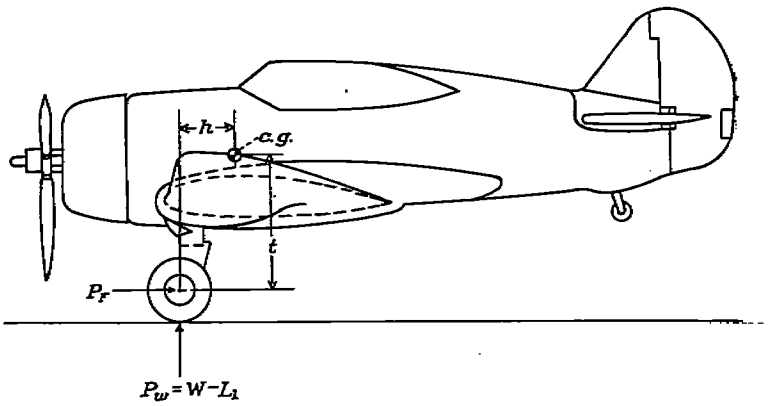


FIGURE 6.—Landing-gear center-of-gravity relationship.

Figure 6 presents a sketch showing the information necessary for the study of the criterion. The normal force at the wheel P_w equals $W-L_1$. The friction force $P_F=fP_w$, where f is the coefficient of rolling friction, the value of which can be obtained from reference 9. The requirement is met if the term $M'+hP_w-tP_F$ does not have a positive value.

If the sample airplane is used as an example, it may be assumed that: The values of C_m and C_L are measured as -0.32 and 0.40, respectively; the dynamic pressure at one-half take-off speed is 6.0; reference 8 shows $\alpha_t=5.6°$; curves similar to figure 4 for the flaps-neutral, take-off power conditions give $\frac{dC_m}{di_t}=-0.040$; $h=1.1$ feet and $t=4.9$ feet; and reference 9 gives $f=0.03$. Then,

$$C_m'=-0.32+5.6(-0.04)$$
$$=-0.54$$
$$M'=-0.54\times6.0\times250\times6.45$$
$$=-5225 \text{ foot-pounds}$$
$$L_1=6.0\times0.40\times250$$
$$=600 \text{ pounds}$$
$$P_w=6000-600$$
$$=5400 \text{ pounds}$$
$$P_F=0.03\times5400$$
$$=162 \text{ pounds}$$
$$M'+hP_w-tP_F=-5225+(1.1\times5400)-(4.9\times162)$$
$$=-5225+5940-794$$
$$=-79 \text{ foot-pounds}$$

The total pitching moment has a negative value, and the requirement is met.

If the geometry of the airplane is such that the weight moment in the three-point attitude appears sufficiently increased to overbalance the diving moment (which in the three-point attitude is increased considerably because of the airplane's stability and the more potent ground effect), the procedure may be repeated for this attitude.

For an airplane with a tricycle landing gear, a similar analysis may be made. In this case, however, the critical angle of attack will be that determined by the three-wheel ground angle, and the most forward center-of-gravity position will be critical for a given airplane weight. Maximum up-elevator deflection is required here rather than maximum down-elevator deflection, and the requirement will be met if the resultant pitching moment is greater than zero in a positive direction.

Requirement (I–F).—Limits of trim change due to power and flaps.

I–F–1. "With the airplane trimmed for zero stick force at any given speed and using any combination of engine power and flap setting, it should be possible to maintain the given speed without exerting push or pull forces greater than those listed below when the power and flap setting are varied in any manner whatsoever.

a. Stick-type controls—35 pounds push or pull.

b. Wheel-type controls—50 pounds push or pull."

I–F–2. "If the airplane cannot be trimmed at low speeds with full use of the trimming device, the conditions specified in item 1 should be met with the airplane trimmed full tailheavy."

By comparison of the stick-force curves (fig. 2(b)) for the various flap and power conditions the combination of flap-setting and power changes that will cause the greatest changes in trim can be determined. At any speed the difference in forces required for the two extreme conditions at the same trim setting should not exceed the specified limit.

For a complete investigation curves for different trim-tab settings covering the range of settings required for trim in the different flight conditions should be used. When reliable curves for different tab settings are not available, the investigation should be restricted to the determination of power and flap trim changes with trim tabs neutral. This investigation may not reveal the most critical conditions, however, because of the possible variation in trim-tab effectiveness in different flight conditions.

The critical center-of-gravity position for the trim changes will in most cases be the most forward position rather than the most rearward position investigated for static stability.

Requirement (I–G).—Characteristics of the longitudinal trimming device.

I–G–1. "The trimming device should be capable of reducing the elevator control force to zero in steady flight in the following conditions:

a. Cruising conditions—at any speed between high speed and 120 percent of the minimum speed.

b. Landing condition—any speed between 120 percent and 140 percent of the minimum speed."

This item can be investigated for each condition by two elevator-free curves similar to the curve shown in figure 1. The curve for maximum tailheavy trim-tab or stabilizer setting and the curve for the maximum noseheavy setting should intersect the $C_m=0$ line at two points which cover the specified range of speeds.

If the trimming device is a small tab, the study may be feasible only in large-scale investigations. It may be possible, however, to estimate tab effectiveness theoretically from previously accumulated data—for example, the data presented in reference 10 and in the various papers listed therein as references.

I–G–2. "Unless changed manually, the trimming device should retain a given setting indefinitely."

Although the load on the trimming device is, of course, determined by aerodynamic forces, this requirement (I–G–2) is chiefly a problem involving the construction of the full-scale mechanism; its investigation in connection with wind-tunnel models will ordinarily be of no value.

II. Requirements for Lateral Stability and Control

Requirement (II–A).—Characteristics of uncontrolled lateral and directional motion.

II–A–1. "The control-free lateral oscillation should always damp to one-half amplitude within two cycles."

II–A–2. "When the ailerons are deflected and released quickly, they should return to their trim position. Any oscillations of the ailerons themselves shall have disappeared after one cycle."

II–A–3. "When the rudder is deflected and released quickly, it should return to its trim position. Any oscillation of the rudder itself shall have disappeared after one cycle."

In general, wind-tunnel study of this item, which is concerned with the control-free lateral oscillation and the oscillations of the lateral control surfaces, is impracticable for the reasons advanced in the discussion of uncontrolled longitudinal motion. As stated in reference 1, however, the requirement for damping of the control-free oscillation is not considered critical as a design consideration; experience to date has indicated that the uncontrolled lateral motion will usually be satisfactory when other requirements of fin area and dihedral are met.

Requirement (II–B).—Aileron-control characteristics (rudder locked).

II–B–1. "At any given speed, the maximum rolling velocity obtained by abrupt use of ailerons should vary smoothly with the aileron deflection and should be approximately proportional to the aileron deflection."

Aileron data will be assumed to be available in the form shown in figure 7. The aileron tests should be made for all flap conditions; power-off runs will ordinarily be sufficient, but the use of the power most commonly associated with each flap condition would be desirable if convenient.

The information necessary for the study of this requirement can be obtained directly by inspection of the curves of rolling moment against aileron deflection (fig. 7), which should be smooth and approximately straight lines. For standard-type ailerons, the aileron-control characteristics should be most carefully considered in the high angle-of-attack range. For unusual lateral-control devices, such as spoilers, the low speeds may not be critical, and equal attention should be paid to the moment curves at lower angles of attack.

II–B–2. "The variation or rolling acceleration with time following an abrupt control deflection should always be in the correct direction and should reach a maximum value not later than 0.2 second after the controls have reached their given deflection."

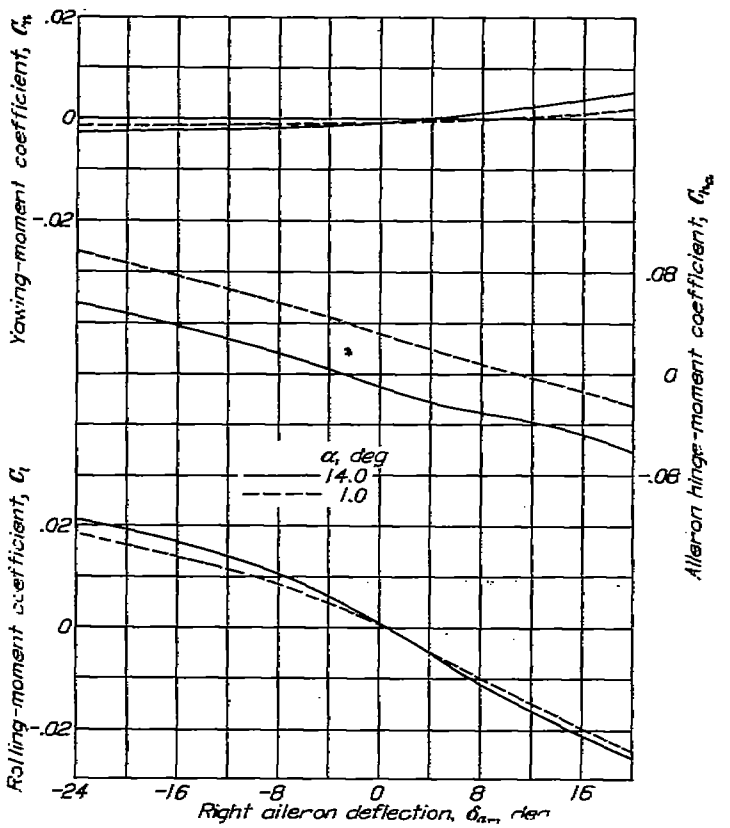


FIGURE 7.—Typical aileron curves. Power off; $\delta_f=0°$.

This requirement, which is intended for spoilers or other unusual lateral controls that may exhibit lag in the development of the rolling moment or initial rolling tendencies in the wrong direction, need not be given consideration in tests of airplanes equipped with the conventional type of aileron.

If necessary, the time lag and the initial adverse rolling moment associated with the action of a lateral-control device can be measured in special wind-tunnel tests. Investigations of this nature are reported in reference 11.

II–B–3. "The maximum rolling velocity obtained by use of ailerons alone should be such that the helix angle generated by the wing tip, $pb/2V$, is equal to or greater than 0.07 . . . ."

The helix angle $pb/2V$ for the rolling velocity produced by a given aileron deflection, the effects of sideslip being neglected, is equal to C_l/C_{l_p} where C_l is the rolling-moment coefficient resulting from aileron deflection. It may be necessary to correct the value of C_{l_p} determined from reference 12 for the section-lift-curve slope of the wing in question $\left(\text{i. e., multiply by } \frac{\text{Section-lift-curve slope per degree}}{0.099}\right)$. The rolling-moment coefficient used should represent the maximum total rolling moment produced by both ailerons at any one stick position. This position may not represent the maximum deflection if the aileron effectiveness drops off noticeably at high deflections.

Inasmuch as 0.07 has been set as an absolute minimum value, any fairing, extrapolation, or estimation necessary in the analysis should be conservative rather than optimistic. Values of $pb/2V$ computed from wind-tunnel data are likely to be somewhat higher than full-scale flight values. Adverse yaw at low speeds and wing twist and compressibility at high speeds are largely responsible for this discrepancy. Although actual consideration of the losses in each case would be desirable if practical, an arbitrary correction factor appears to offer the simplest means of arriving at valid flight values through the use of tunnel data. Several recent attempts at correlation made at the Langley Laboratory have indicated that $0.8C_l/C_{l_p}$ may be used as a reasonable approximation of the airplane $pb/2V$.

For example, if the sample airplane has a wing with an aspect ratio of 6 and a taper ratio of 0.5, reference 12 gives a value of 0.46 for C_{l_p}. Figure 7 shows the total rolling-moment coefficient for maximum aileron deflection (24° up, 20° down) to be $0.021+0.026=0.047$. The estimated airplane $pb/2V$, then, is $0.8\left(\frac{0.047}{0.46}\right)=0.082$, a satisfactory value. A curve of $pb/2V$ plotted against total aileron deflection is shown in figure 8. The setting of each aileron for any total deflection is obtained from a linkage curve (fig. 9).

For combat-type airplanes, rolling velocity in degrees per second at various speeds is often desirable as an expression of the actual rolling maneuverability independent of span.

II–B–4. "The variation of aileron control force with aileron deflection should be a smooth curve. The force should everywhere be great enough to return the control to trim position."

The aileron hinge-moment data of figure 7 can be used to develop a curve of stick force against aileron deflection. The 80-percent maximum-speed condition and one low-speed condition should be investigated. The requirement that the control force should everywhere be great enough to

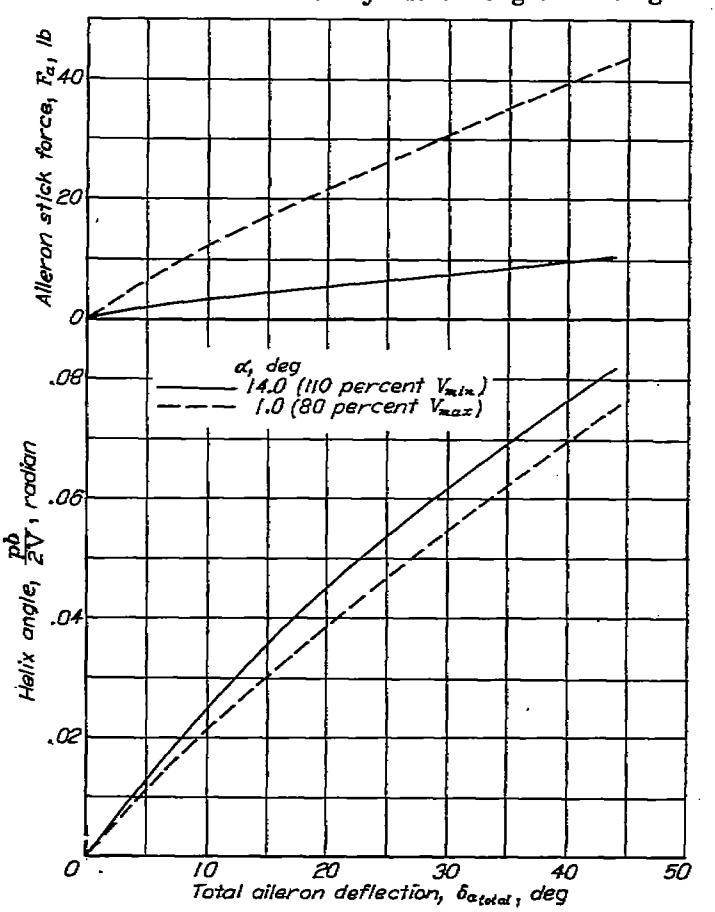


FIGURE 8.—Variation of estimated helix angle and stick force with aileron deflection.

return the control to trim position, aside from ruling out the possibility of a reversal of forces, is not specific; but in this case, as in that of item I–B–3, an arbitrary value might be set as a minimum slope of the curve of stick force against aileron deflection for a given speed.

From data for the appropriate lift coefficient, the total aileron force for an airplane rolling steadily can be computed as

$$F_a=\frac{\frac{1}{2}\rho V^2 b_a \bar{c}_a^2}{57.3}\left[\frac{-d\delta_{a_u}}{dx}\left(C_{h_{a_u}}+\frac{dC_{h_a}}{d\alpha}\Delta\alpha\right)-\frac{d\delta_{a_d}}{dx}\left(C_{h_{a_d}}-\frac{dC_{h_a}}{d\alpha}\Delta\alpha\right)\right] \tag{4}$$

where $C_{h_{a_u}}$ and $C_{h_{a_d}}$ are the aileron hinge-moment coefficients at the given up and down deflections, and $\frac{d\delta_{a_u}}{dx}$ and $\frac{d\delta_{a_d}}{dx}$ are the slopes in degrees per foot of the curves of aileron deflection against stick travel for each aileron at the appropriate deflection. A linkage curve of this nature is shown as figure 9. If the linkage curve is given in terms of angular stick deflection θ the term $\frac{d\delta_a}{dx}$ should be replaced by $\frac{d\delta_a}{d\theta}$ divided by the stick length in feet.

The term $\Delta\alpha$ used in equation (4) is the change in effective angle of attack of the downgoing wing. This term has a positive value and can be found for a given $pb/2V$ as

$$\Delta\alpha=\frac{pb}{2V}\frac{114.6y}{b}$$

The length y is the spanwise distance from the fuselage center line to a point on the wing. The location of this point varies with aileron shape and, to a certain extent, with aspect ratio and taper. As indicated by the results of unpublished investigations, however, it may be assumed for most conventional ailerons to be at 10 percent of the aileron span outboard of the inboard aileron tip. The value of $\frac{dC_{h_a}}{d\alpha}$ is obtained from curves similar to figure 7 for different angles of attack. Unless $\frac{dC_{h_a}}{d\alpha}$ varies considerably with aileron deflection, an average value may be used throughout the δ_a range.

Figure 8 presents curves obtained by application of this method to the sample airplane (b=38.75, y=13.4, b_a=9.88, $\bar{c}_a$=1.09).

II–B–5. "At every speed below 80 percent of maximum level-flight speed, it should be possible to obtain the specified value of $pb/2V$ without exceeding the following control-force limits:

a. Wheel-type controls: ±80 pounds applied at rim of wheel.

b. Stick-type controls: ±30 pounds applied at grip of stick."

The stick force for the required $pb/2V$, as determined from figure 8, should not exceed the given limits. In the present example, figure 8 shows the force at $\frac{pb}{2V}$=0.07 to be 39.9 pounds for the high-speed (80 percent maximum speed) condition.

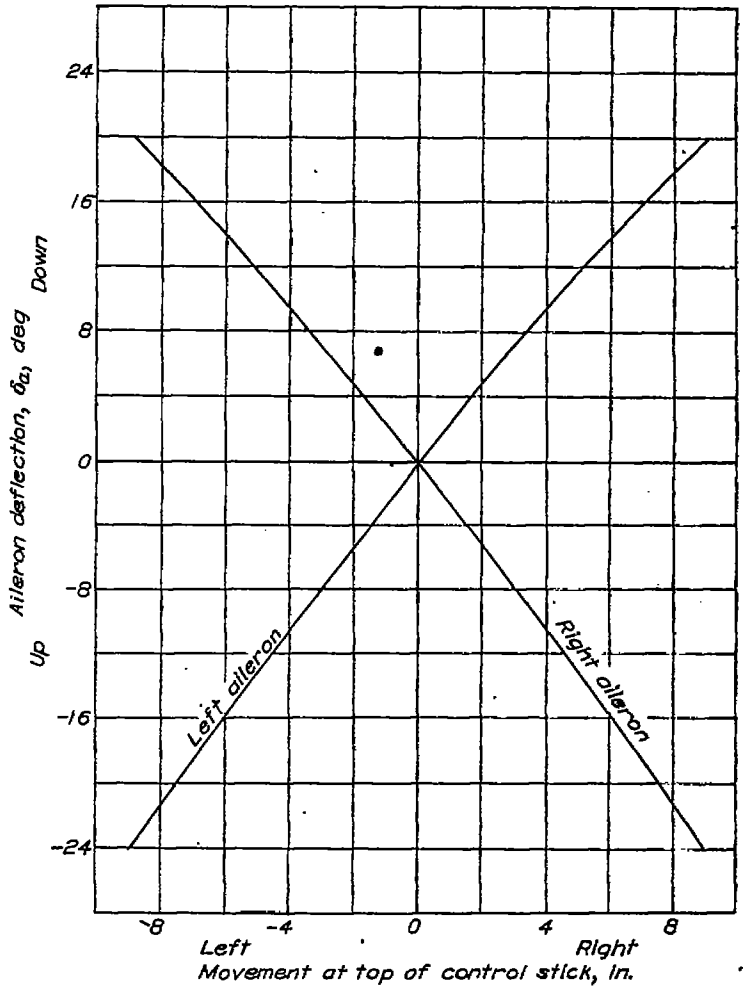


FIGURE 9.—Aileron linkage for typical fighter airplane.

Requirement (II–C).—Yaw due to ailerons.

"With the rudder locked at 110 percent of the minimum speed, the sideslip developed as a result of full aileron deflection should not exceed 20°."

Because the amount of sideslip developed in a full aileron roll depends largely upon dynamic factors, it has not yet been found practicable to estimate the sideslip simply on the basis of static tunnel test data. The complete solution involves the use of standard dynamic-stability equations similar to those discussed in reference 6. With the aid of certain simplifying assumptions, however, it is possible to set up a general expression for the sideslip as a function of time

$$\beta=57.3[k_1+k_2\cos pt+k_3\sin pt -e^{At}(k_4\cos Bt+k_5\sin Bt)] \tag{5}$$

The angle of sideslip can then be computed by substitution in the formula for several values of time. The maximum sideslip should not exceed 20°. An explanation of the constants and a more complete discussion of the problem and its solution are given in the appendix.

Tunnel data necessary for the investigation of the requirement include aileron curves for the low-speed condition (figs. 7 and 8) and yaw curves similar to those of figure 10 for high and low power conditions, flaps both extended and retracted, at the lift coefficient corresponding as closely as possible to 110 percent of the minimum speed in each case.

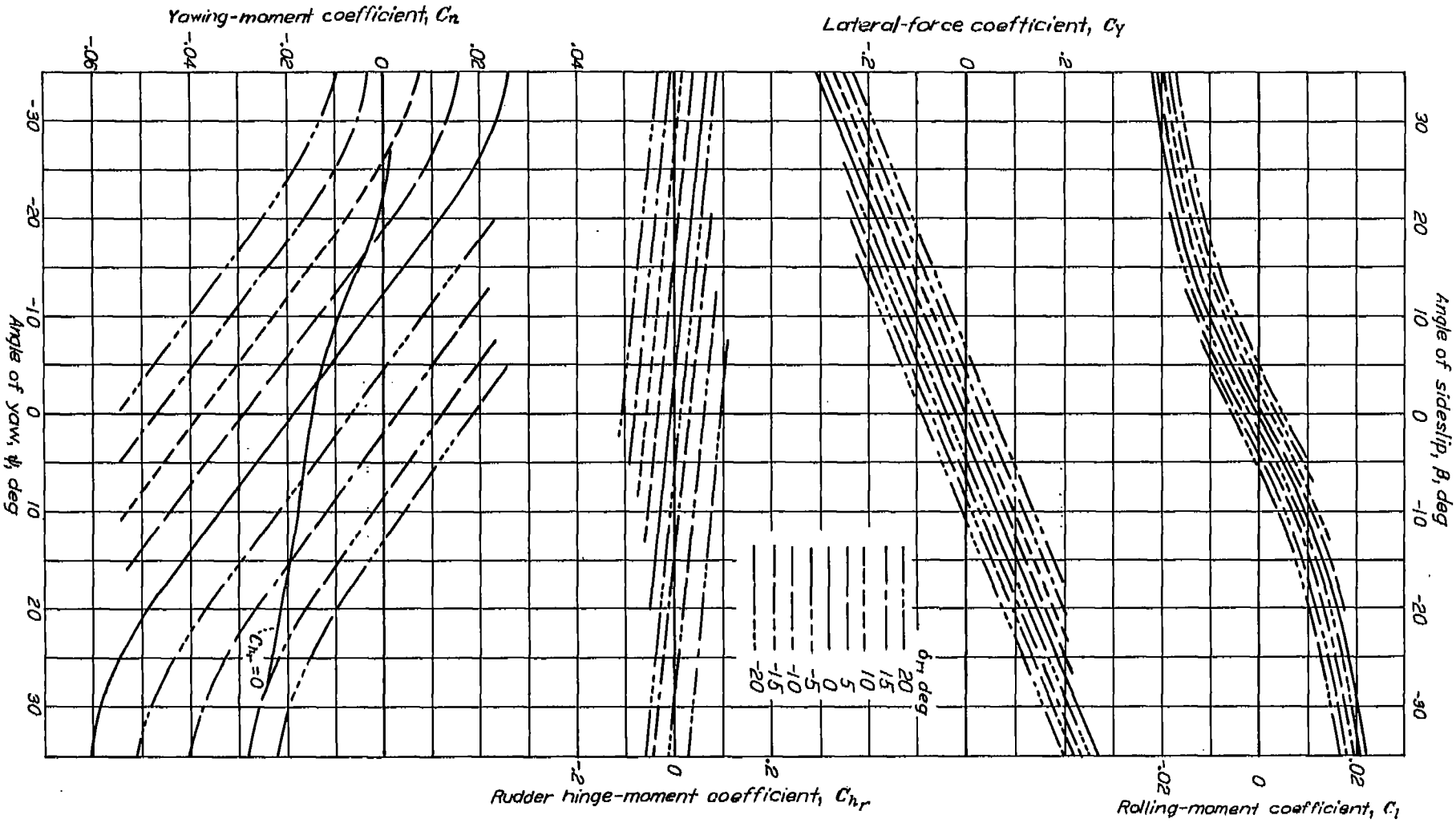


FIGURE 10.—Yaw curves for a typical fighter airplane. Rated power; $\delta_f=0°$; $\alpha=14°$.

From the aileron curves, the aileron yawing-moment coefficient C_{n_a} and the helix angle $pb/2V$ for the maximum aileron deflection can be found. These values and the directional-stability data obtained from the yaw curves are used in the determination of the constants in equation (5).

Requirement (II–D).—Limits of rolling moment due to sideslip (dihedral effect).

II–D–1. "The rolling moment due to sideslip as measured by the variation of aileron deflection with angle of sideslip should vary smoothly and progressively with angle of sideslip and should everywhere be of a sign such that the aileron is always required to depress the leading wing as the sideslip is increased."

Rolling-moment coefficients can be obtained directly in yaw tests of the model with neutral ailerons. Curves of C_l against β (fig. 10), drawn for idling, cruising, and high powers, should be smooth and should possess negative slopes throughout, indicating positive dihedral effect. The range of sideslip angles included in the tests should extend at least 10° beyond the angle at which maximum rudder deflection will give trim in yaw.

II–D–2. "The variation of aileron stick force with angle of sideslip should everywhere tend to return the aileron control to its neutral or trim position when released."

This requirement may be investigated by a study of the stick-free effective dihedral, in a manner similar to that used in the study of stick-free stability in item I–B–2.

For each of several combinations of left and right aileron deflections determined by the known aileron linkage, flap-up and flap-down yaw tests should be made. If practicable (that is, if the rudder is equipped with remote control), the rudder should be set as closely as possible for trim in yaw at each point.

Curves of hinge-moment and rolling-moment coefficient against angle of sideslip (as in fig. 10) can be drawn for various total aileron deflections. The hinge-moment coefficient should be the total aileron hinge-moment coefficient $C_{h_{a_u}}\frac{d\delta_{a_u}}{dx}+C_{h_{a_d}}\frac{d\delta_{a_d}}{dx}$, where the slope $\frac{d\delta_a}{dx}$ is that measured at the appropriate up or down aileron setting.

The zero hinge-moment points can be spotted on the C_l curves in the same manner as the zero elevator hinge-moment points were spotted on the C_m curves in figure 1. The resulting curve will be a measure of the stick-free effective dihedral, which should be positive in order to satisfy the requirement.

II–D–3. "The rolling moment due to sideslip should never be so great that a reversal of rolling velocity occurs as a result of yaw due to ailerons (rudder locked)."

Although this requirement actually demands a theoretical study similar to that of the yaw due to aileron deflection, a simple check of tunnel data may be made which should give adequate indication of the ability of the airplane to meet the requirement. The check should reveal that the rolling-moment coefficient at the angle of sideslip developed with full aileron deflection and neutral rudder (determined in item (II–C)) is always considerably less than the rolling-moment coefficient contributed by the full aileron deflection at zero yaw.

Requirement (II–E).—Rudder-control characteristics.

II–E–1. "The rudder control should everywhere be sufficiently powerful to overcome the adverse aileron yawing moment."

The total adverse yawing-moment coefficient $\left(C_n \text{ due to aileron plus } C_{n_p}\frac{pb}{2V}\right)$ developed with maximum aileron deflection in the 110-percent minimum-speed condition, as previously determined for item (II–C), should always be less than the yawing-moment coefficient at zero yaw contributed by maximum opposite rudder deflection.

II–E–2. "The rudder control should be sufficiently powerful to maintain directional control during take-off and landing."

The problem of rudder control near the ground will ordinarily be most critical on the take-off at the high-power low-speed condition. In this attitude (with the flaps set to the prescribed angle for take-off) the rudder deflection necessary for $C_n=0$ and $C_Y=0$ is found. This required deflection should not approach too closely the maximum available travel.

In figure 10, for example, the airplane shows simultaneous trim in yaw and lateral force with 15° right rudder deflection at 8.5° left sideslip. This amount of sideslip is normal in the take-off of a highly powered single-engine airplane.

Although ground effect as related to this requirement may merit further study, the information available at present appears to indicate that the requirement may be investigated with sufficient accuracy with no ground representation.

II–E–3. "On airplanes with two or more engines, the rudder control should be sufficiently powerful to provide equilibrium of yawing moments at zero sideslip at all speeds above 110 percent of the minimum take-off speed with any one engine inoperative (propeller in low pitch) and the other engine or engines developing full rated power."

For the specified power conditions, flaps in take-off position, curves of C_n and C_{h_r} similar to those of figure 10 should be drawn for different rudder deflections at the 110-percent minimum-speed attitude. Yawing moment should equal zero at a rudder deflection well within the limits of travel.

The "inoperative" engine should be the one the failure of which would cause the maximum asymmetry of thrust. It should be run at windmilling rather than at idling power. Reference 13 may be used as an aid in setting up test conditions to simulate the action of a dead engine being turned by a propeller.

II–E–4. "The rudder control in conjunction with the other controls of the airplane should provide the required spin-recovery characteristics."

Examination of this quality by means of usual wind-tunnel programs is not feasible.

II–E–5. "Right rudder force should always be required to hold right rudder deflections, and left rudder force should always be required to hold left rudder deflections."

This requirement may be considered satisfactorily met if a curve of C_n against β for $C_{h_r}=0$ (as shown spotted on fig. 10) shows a reversal in the sign of C_n only where it possesses a positive slope. Although a stable slope of this curve would be desirable throughout the sideslip range, it is not considered

absolutely essential provided the curve does not cross the zero ordinate at any point at which its slope indicates instability.

In the event that large trim-tab deflections appear necessary for reduction of pedal forces in straight flight, the possibility of a force reversal with the required tab setting should be considered.

II–E–6. "The rudder forces required to meet the above rudder-control requirements should not exceed 180 pounds (trim tabs neutral)."

The highest pedal force computed from the rudder hinge-moment coefficients associated with the deflections necessary to meet the foregoing rudder-control requirements should be less than 180 pounds.

Requirement (II–F).—Yawing moment due to sideslip (directional stability).

II–F–1. "The yawing moments due to sideslip (rudder fixed) should be sufficient to restrict the yaw due to ailerons to the limits specified in requirement (II–C–1)."

This item is included in consideration of requirement (II–C).

II–F–2. "The yawing moment due to sideslip should be such that the rudder always moves in the correct direction; i. e., right rudder should be required for left sideslip and left rudder should be required for right sideslip. For angles of sideslip between $\pm 15°$, the angle of sideslip should be substantially proportional to the rudder deflection."

A curve of δ_r against β can be plotted for $C_n = 0$ (fig. 11), supplying the necessary information for both portions of this requirement. The words "left" and "right" should be interpreted as being relative to the normal trim point, inasmuch as rudder-neutral trim may not occur at zero sideslip in some power conditions.

In order that the curve of δ_r against β may be more truly representative of the variation in actual flight, a correction to the rudder deflection should be made for the yawing moment associated with the aileron setting required for balance at each angle of sideslip.

II–F–3. "The yawing moment due to sideslip (rudder free) should be such that the airplane will always tend to return to zero sideslip regardless of the angle of sideslip to which it has been forced."

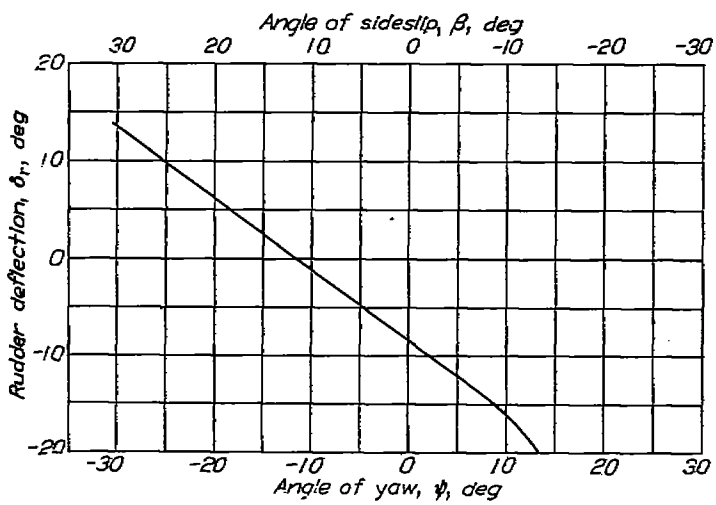


FIGURE 11.—Variation of trim rudder deflection with angle of sideslip.

The rudder-free curves drawn for item II–E–5 (fig. 10) supply the necessary information for this requirement. As already mentioned, a stable slope is not demanded at every angle of sideslip; but the signs of the yawing-moment coefficient must not reverse in an unstable direction.

In tests of wind-tunnel models in yaw, data should be considered for sideslip angles extending at least as far as 10° or 15° beyond the angle at which the maximum rudder deflection will provide trim because an airplane may be forced accidentally to angles of this magnitude. The characteristics of the curves at greater angles of sideslip are of no concern in flight, inasmuch as the angles represent attitudes impossible for the airplane to attain.

A considerable amount of testing and computing time may be saved by eliminating test points at high positive sideslip angles with large amounts of right rudder deflections, and at high negative angles with large left rudder deflections.

II–F–4. "The yawing moment due to sideslip (rudder free with airplane trimmed for straight flight on symmetric power) should be such that straight flight can be maintained by sideslipping at every speed above 140 percent of the minimum speed with rudder free with extreme asymmetry of power possible by the loss of one engine."

A yaw test in the 140-percent minimum-speed attitude with the rudder free to float seems to be the most direct approach to the study of this requirement. With the rudder trim tab set for straight flight on symmetric power, the model should be operated with all motors simulating full rated power except the one, failure of which would result in the greatest asymmetry of power. This outboard engine should simulate the windmilling condition.

Typical curves of yawing-moment and rolling-moment coefficients measured in the rudder-free yaw run are shown in figure 12. From aileron curves similar to those of figure 7, the total aileron deflection required to balance the rolling moment at each angle of sideslip and the yawing-moment coefficient C_{n_a} associated with the total deflection can be found. A curve of $-C_{n_a}$ is then superposed on the C_n curve of figure 12. In order for the requirement to be met, the C_n curve must cross the $C_n=0$ axis and the curves of C_n and $-C_{n_a}$ must intersect.

Requirement (II–G).—Cross-wind force characteristics.

"The variation of cross-wind force with sideslip angle, as measured in steady sideslips, should everywhere be such that right bank accompanies right sideslip and left bank accompanies left sideslip."

Inspection of the yaw curves should show that the slope of the curve of C_Y against β is negative, as is indicated in figure 10.

Requirement (II–H).—Pitching moment due to sideslip.

"As measured in steady sideslip, the pitching moment due to sideslip should be such that not more than 1° elevator movement is required to maintain longitudinal trim at 110 percent of the minimum speed when the rudder is moved 5° right or left from its [original] position for straight flight."

At the lift coefficient corresponding to 110 percent of the minimum speed, the trim rudder deflection for each power condition can be found as in item II–E–2. For a rudder deflection first 5° to the left and then 5° to the right of the trim setting, and at the angle of sideslip for $C_n=0$ in each

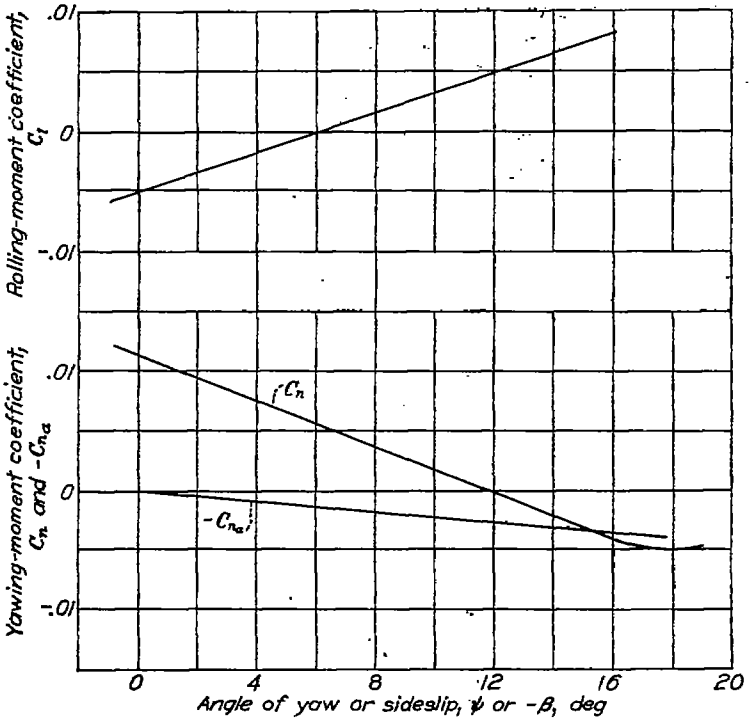


FIGURE 12.—Yawing-moment-coefficient and rolling-moment-coefficient curves required for item II-F-4.

case, the pitching-moment coefficient should not be sufficiently different from the pitching-moment coefficient at the trim point to account for a difference of more than 1° elevator deflection. The angles of sideslip involved are relatively small; it appears, therefore, that the use of elevator effectiveness data for unyawed flight (fig. 1, for example) is justified.

Requirement (II-I).—Power of rudder and aileron trimming devices.

II-I-1. "Aileron and rudder trimming devices should be provided if the rudder or aileron forces required for straight flight at any speed between 120 percent of the minimum speed and the maximum speed exceed 10 percent of the maximum values specified in requirements (II-B-5) and (II-E-6), respectively, and unless these forces at cruising speed are substantially zero."

The rudder hinge-moment coefficients for straight flight at the specified speeds are obtained from yaw curves (fig. 10), the method of item II-E-2 being used for the determination of straight-flight conditions. The rolling-moment coefficient for each straight-flight attitude appears on the same curves. The aileron settings required for trim in roll and the hinge-moment coefficients for these deflections are determined from aileron curves (fig. 7).

If the difference between the maximum and minimum forces computed from the aileron hinge-moment data exceeds 8 pounds at the rim of a wheel or 3 pounds at the grip of a stick, the provision of an aileron trimming mechanism should be recommended. A similar recommendation should be made for a rudder trimming device if the difference between the maximum and minimum rudder forces exceeds 18 pounds. If the forces at the cruising condition are not substantially zero but the variations fall within the desired limits, the trim can be changed by rigging and tabs need not be recommended.

II-I-2. "Multiengine airplanes should possess rudder and aileron trimming devices sufficiently powerful, in addition, to trim for straight flight at speeds in excess of 140 percent of the minimum speed with maximum asymmetry of engine power."

It is presumed that if aileron tabs are supplied for the model, data will be obtained in some form permitting construction of curves similar to those shown in figure 13. For this requirement, as for requirement II-D-2, the hinge-moment and rolling-moment coefficients are for the combined settings of both ailerons; these aileron data need not be obtained with the specified asymmetric power, unless the configuration is such that the ailerons and tabs may be influenced significantly by engine operation.

The rolling-moment coefficient due to asymmetric power C_{l_1} can be measured in the straight-flight attitude representing 140 percent of the minimum speed with the model motors representing the condition of maximum asymmetry of power. For this measurement, the rudder should be set to provide approximately $C_n=0$. A tab setting within the

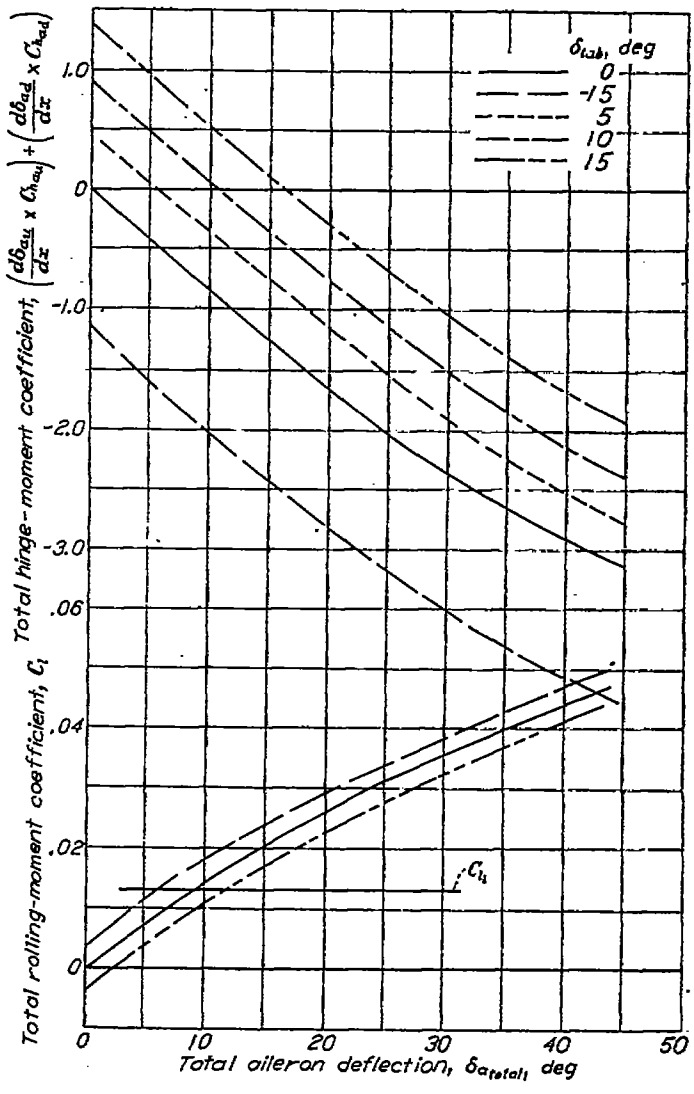


FIGURE 13.—Aileron tab characteristics.

limits of travel should provide zero hinge-moment at an aileron deflection required to balance this amount of rolling. On figure 13, for example, a tab angle of approximately 10° meets the requirement.

A similar procedure can be used for the rudder. It might be desirable, if feasible, to study the requirement in a more direct manner by making straight-flight measurements of rolling-moment and yawing-moment coefficients at various tab settings with rudder and properly linked ailerons free to float. The requirement would then be met if C_n and C_l were found to equal zero at points within the limits of tab travel.

Because the study of this item depends on the action of small tabs, the investigation of this requirement will probably be advisable only on models of fairly large scale.

II–I–3. "Unless changed manually, the trimming device should retain a given setting indefinitely."

This item does not appear suited for normal wind-tunnel investigation.

III. Stalling characteristics

III–1. "The approach of the complete stall should make itself unmistakably evident through any or all of the following conditions:

a. The instability due to stalling should develop in a gradual but unmistakable manner."

Tuft studies appear to be mandatory in connection with this phase of the investigation and with the stalling problem in general. Information concerning the flow phenomena and hence, to some extent, the behavior of the airplane at the stall can be acquired from observations of the action of tufts on the model as the angle of attack is increased until the complete stall is reached. The development of instability is usually gradual when stalling appears first at the wing roots and spreads gradually forward and outward in such a manner that the flow over the ailerons is the last to become disturbed.

b. "The elevator pull force and rearward travel of the control column should markedly increase."

The curve of elevator stick force against airspeed (fig. 2 (b)) and the curve of elevator deflection against lift coefficient (fig. 2 (a)) should each show a marked increase in slope as the stalling speed is approached.

c. "Buffeting and shaking of the airplane and controls produced either by a gradual breakdown of flow or through the action of some mechanical warning device should provide unmistakable warning before instability develops."

This item is best studied in the wind tunnel through the tuft observations. Tuft behavior, for example, that indicates an initial flow breakdown near the wing center section and, consequently, turbulence in the flow over the tail, is usually an indication that tail buffeting will be present. If a mechanical stall-warning device is intended for use on the airplane, the angle of attack at which the instrument gives its warning should be appreciably lower than the angle at which the tunnel results indicate that instability will develop.

III–2. "After the complete stall has developed, it should be possible to recover promptly by normal use of controls."

Mechanical difficulties and the present lack of correlative data tend to diminish the value of any quantitative tunnel measurements beyond the stall. However, the tendencies suggested by measurements of pitching moment through the stall may be of some use in consideration of this requirement; that is, the direction and sharpness of pitching-moment breaks at and beyond the stall, and the apparent degree of control effectiveness beyond the stall, should bear a direct relationship to the possibility of recovery from stalled flight. Further investigation may be required as to the nature and consistency of this relationship.

III–3. "The three-point landing attitude of the airplane should be such that rolling or yawing moments due to stalling, not easily checked by controls, should not occur in landing, either three-point or with tail-first attitude 2° greater than that for three-point contact."

The angle of attack corresponding to the three-point ground angle of the airplane can be determined from drawings of the complete airplane. The angle of attack at which tuft studies indicate bad stalling in the flap-down, idling-power condition should be more than 2° greater than the ground angle.

CONCLUDING REMARKS

An attempt has been made in the present paper to present methods of analysis of wind-tunnel tests in terms of flying qualities of airplanes. The suggested methods have been presented in an effort to demonstrate the practicability of this type of analysis and also to stimulate interest and discussion among design and test personnel. It is hoped that with the cooperation of interested groups, the present methods will be extended, improved, refined, and—most important—proved by application and correlation.

In the present rather general treatment of the subject, it has naturally been impossible to cover unusual cases that may require special treatment. If, for example, an airplane is known to be provided with some mechanical device that influences the control forces in certain maneuvers although the measured items in the wind tunnel show no effect, this device should be considered in the study of the relevant requirements. In short, every effort should be made to regard the subject of the investigation as an actual flying airplane and not as a scaled-up reproduction of a model.

In conclusion, it is believed that wind-tunnel tests of powered models can, if properly analyzed, be used to examine the flying qualities of airplanes and to determine the extent to which any particular airplane will satisfy requirements for satisfactory stability, control, and handling characteristics in flight. It is recommended that this type of testing, analysis, and presentation of data be generally employed in wind tunnels engaged in testing airplane models for stability and control.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *April 1, 1943.*

APPENDIX

SIDESLIP IN AILERON MANEUVERS

SYMBOLS AND DEFINITIONS

The following terms, in addition to those previously defined, are used in the computation of the sideslip angle:

C_{n_r} yawing-moment coefficient due to yawing $\left(\frac{\partial C_n}{\partial\left(\frac{rb}{2V}\right)}\right)$

r yawing velocity, radians per second

k_Z radius of gyration about Z-axis, feet (mk_Z^2=Yawing moment of inertia)

ϕ angle of bank, radians

N_β and N_r lateral-stability derivatives in terms of unit moment of inertia of airplane

$$N_\beta=\frac{g}{C_L}\frac{b}{k_Z^2}\left(-57.3\frac{dC_n}{d\psi}\right)$$

$$N_r=\frac{g}{C_L}\frac{b}{k_Z^2}C_{n_r}\frac{b}{2V}$$

$$A=\frac{N_r}{2}$$

$$B=\sqrt{N_\beta-\frac{N_r^2}{4}}$$

The value of C_{n_p} for the appropriate angle of attack is found by the use of reference 12. Reference 14 may be used for the estimation of C_{n_r}.

SIDESLIP FORMULA

As stated in item (II–C), the angle of sideslip developed by a conventional airplane in a rudder-fixed aileron roll may be expressed as

$$\beta=57.3\ [k_1+k_2\cos pt+k_3\sin pt-e^{At}\ (k_4\cos Bt+k_5\sin Bt)] \tag{5}$$

where

$$k_1=\frac{C_{n_p}\frac{pb}{2V}+C_{n_a}}{57.3\frac{dC_n}{d\psi}}$$

$$k_2=\frac{gp\ [(N_\beta-p^2)-N_r^2]}{V[(N_\beta-p^2)^2+p^2N_r^2]}$$

$$k_3=\frac{-gN_rN_\beta}{V[(N_\beta-p^2)^2+p^2N_r^2]}$$

$$k_4=k_1+k_2$$

$$k_5=\frac{pk_3-Ak_4}{B}$$

After the constants have been evaluated, the sideslip angle may be determined for any value of time by substitution in equation (5). This substitution should be made for half-second or smaller intervals covering a range of time sufficient to allow the airplane to reach an angle of bank of 90° (or less in the case of an extremely large airplane).

DISCUSSION

The angles of bank reached in a full-aileron roll are far too large to permit the use of the usual assumption that $\phi=\sin\phi$. The customary solution must therefore be further complicated by the introduction of the sine of the angle in the equations of motion.

The expression presented herein offers a somewhat simplified solution but suffers a corresponding loss in accuracy. The greatest possible source of error lies in the fact that the derivation assumes the airplane to be rolling at a steady rate. The error introduced by neglect of the small initial period of acceleration is not believed to be serious. If, however, the assumption is applied to an airplane with pronounced spiral instability, it may be considerably in error after several seconds because the rolling velocity of the airplane would be increasing rather than remaining constant. Although the error in the assumed rolling velocity would lead to erroneous values of computed sideslip angles (indicating a peak rather than an ever-increasing sideslip), it is believed that, for conventional airplanes with reasonably effective ailerons, the brevity of the maneuver will permit the use of this method with reasonable accuracy.

The probable inaccuracy introduced by nonlinearity of the yawing-moment curves can be minimized by selection of an average value of $\frac{dC_n}{d\psi}$ over a range, for example, of 10° to −10° of sideslip. Ordinarily, an airplane that meets requirement II–F–2 will exhibit yaw curves sufficiently linear to permit the use of this method with reasonable accuracy. If the curves are considerably nonlinear over the range of 15° to −15° of sideslip, the use of constant slopes may result in appreciable error, either by this method or by more rigorous conventional solution of the complete equations. In that event, a more exact solution could, if desired, be obtained through the use of a step-by-step procedure such as that described in reference 15.

The effect of the lateral-force derivative, which is relatively small, is neglected. The equations upon which the solution is based are

$$\frac{d\beta}{dt}=\frac{g\sin\phi}{V}-r$$

$$\frac{dr}{dt}=\beta N_\beta+rN_r+N_0$$

where

$$N_0=\frac{g}{C_L}\frac{b}{k_Z^2}\left(C_{n_p}\frac{pb}{2V}+C_{n_a}\right)$$

$$\phi=pt$$

and β is expressed in radians.

The equations may be combined and expressed as

$$\frac{d^2\beta}{dt^2}-N_r\frac{d\beta}{dt}+\beta N_\beta=\frac{pg}{V}\cos pt-\frac{gN_r}{V}\sin pt-N_0$$

The solution for β then becomes

$$\beta=\frac{pg}{V(\lambda_2^2+p^2)(\lambda_1^2+p^2)}[(\lambda_1\lambda_2-p^2)\cos pt-p(\lambda_2+\lambda_1)\sin pt]$$
$$+\frac{gN_r}{V(\lambda_2^2+p^2)(\lambda_1^2+p^2)}[(p^2-\lambda_1\lambda_2)\sin pt-p(\lambda_1+\lambda_2)\cos pt]$$
$$-\frac{N_0}{\lambda_1\lambda_2}+C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t} \tag{6}$$

where λ_1 and λ_2 are roots of the equation

$$\frac{d^2\beta}{dt^2}-N_r\frac{d\beta}{dt}+\beta N_\beta=0$$

or

$$\lambda_1,\ \lambda_2=\frac{N_r\pm\sqrt{N_r^2-4N_\beta}}{2}$$

and C_1 and C_2 are constants determined from the knowledge that β and $\frac{d\beta}{dt}$ are both equal to zero when $t=0$. The equation (6) for sideslip may be used for any distinct values of λ_1 and λ_2 other than zero. For a conventional airplane with even slightly better than neutral directional stability, however, the term $N_r^2-4N_\beta$ will be negative, and the roots will take the form $A\pm Bi$. Substitution of these roots for λ_1 and λ_2 results in the expression

$$\beta_{radians}=k_1+k_2\cos pt+k_3\sin pt$$
$$-e^{At}(k_4\cos Bt+k_5\sin Bt)$$

It should be noted, then, that if the term $N_r^2-4N_\beta$ is positive, the intermediate expression involving λ_1 and λ_2 must be used instead of the final expression given in item (II–C) in terms of A and B.

REFERENCES

1. Gilruth, R. R.: Requirements for Satisfactory Flying Qualities of Airplanes. NACA Rep. No. 755, 1943.
2. Recant, Isidore G., and Swanson, Robert S.: Determination of the Stability and Control Characteristics of Airplanes from Tests of Powered Models. NACA ARR, July 1942.
3. Greenberg, Harry, and Sternfield, Leonard: A Theoretical Investigation of the Lateral Oscillations of an Airplane with Free Rudder with Special Reference to the Effect of Friction. NACA Rep. No. 762, 1943.
4. Schuldenfrei, Marvin: Some Notes on the Determination of the Stick-Fixed Neutral Point from Wind-Tunnel Data. NACA RB No. 3I20, 1943.
5. Jenney, William: The Effect of Horizontal Variations in Center of Gravity Position on the Static Longitudinal Stability of Airplanes. Jour. Aero. Sci., vol. 3, no. 2, Nov. 1935, pp. 43–45.
6. Donlan, C. J., and Recant, I. G.: Methods of Analyzing Wind-Tunnel Data for Dynamic Flight Conditions. NACA TN No. 828, 1941.
7. Recant, I. G.: Plate Method of Ground Representation for Wind-Tunnel Determination of Elevator Effectiveness in Landing. NACA TN No. 823, 1941.
8. Katzoff, S., and Sweberg, Harold H.: Ground Effect on Downwash Angles and Wake Location. NACA Rep. No. 738, 1943.
9. Wetmore, J. W.: The Rolling Friction of Several Airplane Wheels and Tires and the Effect of Rolling Friction on Take-Off. NACA Rep. No. 583, 1937.
10. Ames, Milton B., Jr., and Sears, Richard I.: Determination of Control-Surface Characteristics from NACA Plain-Flap and Tab Data. NACA Rep. No. 721, 1941.
11. Wenzinger, Carl J., and Rogallo, Francis M.: Wind-Tunnel Investigation of Spoiler, Deflector, and Slot Lateral-Control Devices on Wings with Full-Span Split and Slotted Flaps. NACA Rep. No. 706, 1941.
12. Pearson, Henry A., and Jones, Robert T.: Theoretical Stability and Control Characteristics of Wings with Various Amounts of Taper and Twist. NACA Rep. No. 635, 1938.
13. Hartman, Edwin P., and Biermann, David: The Negative Thrust and Torque of Several Full-Scale Propellers and Their Application to Various Flight Problems. NACA Rep. No. 641, 1938.
14. Campbell, John P., and Mathews, Ward O.: Experimental Determination of the Yawing Moment Due to Yawing Contributed by the Wing, Fuselage, and Vertical Tail of a Midwing Airplane Model. NACA ARR No. 3F28, June 1943.
15. Weick, Fred E., and Jones, Robert T.: The Effect of Lateral Controls in Producing Motion of an Airplane as Computed from Wind-Tunnel Data. NACA Rep. No. 570, 1936.

843107—50——35