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TECHNICAL NOTE

No. 1016

PRESSURE DISTRIBUTIONS FOR REPRESENTATIVE AIRFOILS AND RELATED PROFILES

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SUMMARY

Selected pressure-distribution diagrams for a number of conventional and special airfoils are presented. The conventional airfoils include the NACA 22-, 44-, and 230-series airfoils and the Clark Y family. The special airfoils consist of circular-arc sections of several thicknesses and curvatures and a number of synthetic airfoils, including several with zero moment coefficient. The pressure distributions are in most cases given for several lift coefficients including that at the ideal angle of attack.

INTRODUCTION

In the last decade the importance of obtaining a smooth and continuous pressure distribution on an airfoil has been increasingly recognized. With improvements in the experimental technique of measuring pressure distributions, the agreement between experimental and theoretical pressure distributions has steadily improved. A low-drag airfoil has a distribution which is in almost perfect agreement with theoretically calculated values, since little allowance has to be made for the boundary layer. Furthermore, it is possible to determine by inspection in what manner an airfoil may be improved, particularly if peaks or fluctuations are evident in the pressure curve.

The early development of the pressure-distribution theory is contained in the works of Munk and Glauert. The theory of arbitrary, thick airfoils was published in 1931 (reference 1). Further details of the theory of arbitrary airfoils were presented in reference 2, particularly in regard to the calculation of moment coefficients.

The present paper contains an analysis of three groups of airfoils with their associated pressure distributions; that is, conventional airfoils, circular-arc airfoil sections, and synthetic airfoils, including several with zero moment coefficient. The purpose of this paper is to present data on a wide selection of airfoils that are suitable for general purposes or as a basis for variations.

METHOD

Conventional airfoils.- The airfoil pressure distributions for the conventional airfoils are computed by the method of references 1 and 2. The symbols and formulas of this method are given in reference 3 and are repeated herein for convenience.

x, y coordinates of airfoil; origin at center of airfoil and chord line of approximately 4 units along x-axis

θ $2 \sin^2\theta = p + \sqrt{p^2 + y^2}$ where $p = 1 - \left(\frac{x}{2}\right)^2 - \left(\frac{y}{2}\right)^2$

ψ $2 \sinh^2\psi = - p + \sqrt{p^2 + y^2}$. Since y is generally small for airfoils, $\sinh \psi = \frac{y}{2 \sin \theta}$ may be preferable. Near the leading (or trailing) edge ψ is given approximately by $\psi = \sqrt{\frac{\sigma}{2}}$ where σ is the radius of curvature at the leading (or trailing) edge

ϵ $$\epsilon(\varphi') = -\frac{1}{2\pi}\int_0^{2\pi} \psi(\varphi) \cot \frac{\varphi - \varphi'}{2} \, d\varphi$$

ϵ' obtained graphically from ϵ, θ curve $\left(\frac{d\epsilon}{d\theta}\right)$

ψ' obtained graphically from ψ, θ curve $\left(\frac{d\psi}{d\theta}\right)$

$\varphi = \theta + \epsilon$

$$\psi_o = \frac{1}{2\pi}\int_0^{2\pi} \psi(\varphi) \, d\varphi = \text{Constant}$$

α angle of attack with respect to x-axis

β angle of zero lift, given by value of ϵ for $\theta = \pi$

$k = \dfrac{e^{\psi_0}(1 + \epsilon')}{\sqrt{\left(\sinh^2\psi + \sin^2\theta\right)\left(1 + \psi'^2\right)}}$ (value of k is independent of angle of attack)

$\frac{v}{V}$ ratio of local velocity at airfoil surface to uniform stream velocity $\left(k[\sin(\alpha + \varphi) + \sin(\alpha + \beta)]\right)$

$\frac{p}{q}$ ratio of local superstream pressure to dynamic pressure (the term "superstream pressure" is used to designate difference of local pressure and static pressure in undisturbed uniform stream) $\left(\frac{p}{q} = 1 - \left(\frac{v}{V}\right)^2 \text{ and } q = \frac{1}{2}\rho V^2\right)$

c segment of x-axis intercepted by airfoil boundary

C_L lift coefficient $\left(C_L = \dfrac{L}{\frac{1}{2}\rho c V^2} = \dfrac{8\pi e^{\psi_0}}{c} \sin(\alpha + \beta)\right)$

F point designated the focus of the airfoil. The complex constants c_1 and c_2 may be defined as

$$c_1 = me^{i\delta}$$

$$= A_1 + iB_1$$

$$= \frac{e^{\psi_0}}{\pi}\int_0^{2\pi} \psi(\varphi)(\cos\varphi + i\sin\varphi)\,d\varphi$$

$$c_2 = A_2 + iB_2$$

$$= \frac{e^{2\psi_0}}{\pi}\int_0^{2\pi} \psi(\varphi)(\cos 2\varphi + i\sin 2\varphi)\,d\varphi$$

Then

$$b^2c^{2i\gamma} = 1 + \frac{c_1^2}{2} + c_2$$

so that

$$b^4 = \left(1 + \frac{A_1^2 - B_1^2}{2} + A_2\right)^2 + \left(A_1B_1 + B_2\right)^2$$

and

$$\gamma = \frac{1}{2}\tan^{-1}\frac{A_1B_1 + B_2}{1 + \frac{A_1^2 - B_1^2}{2} + A_2}$$

Then the complex coordinate of F is

$$Z_F = (x + iy)_F$$

$$= me^{i\delta} + \frac{b^2}{e^{\psi_o}}e^{i(2\gamma - \beta)}$$

M_F moment at F, constant for all angles of attack $\left(2\pi\rho b^2V^2 \sin 2(\gamma - \beta)\right)$

C_{M_F} moment coefficient referred to F $\left(C_{M_F} = \frac{M_F}{c^2q} = 4\pi\frac{b^2}{c^2}\sin 2(\gamma - \beta)\right)$

α_I ideal angle of attack $\left(-\frac{\epsilon_N + \epsilon_T}{2}\right.$ where ϵ_N and ϵ_T denote, respectively, the values of ϵ at the nose and tail; that is, at $\theta = 0$ and $\theta = \pi$, respectively$\left.\right)$

Circular-arc sections.- In order to obtain the circular-arc sections, the Kármán-Trefftz transformation is used; that is,

$$\frac{\zeta + na}{\zeta - na} = \left(\frac{z + a}{z - a}\right)^n$$

where ζ is the complex coordinate in the physical plane and z is the complex coordinate in the circle plane.

If the center of the circle is on the y-axis at $a \tan \beta$, two circular arcs are given in the physical plane. For $n = 2$ this transformation reduces to the Joukowsky transformation and the circular arcs are coincident. The y-coordinates for the upper and lower surfaces at the center of the circle are

$$y_U = na \cot \frac{n}{2}\left(\frac{\pi}{2} - \beta\right)$$

$$y_L = -na \cot \frac{n}{2}\left(\frac{\pi}{2} + \beta\right)$$

and the percent thickness is given by

$$t = \frac{y_U - y_L}{2na}$$

$$= \frac{1}{2}\left[\cot \frac{n}{2}\left(\frac{\pi}{2} - \beta\right) + \cot \frac{n}{2}\left(\frac{\pi}{2} + \beta\right)\right]$$

$$= \frac{\sin n\frac{\pi}{2}}{\cos n\beta - \cos n\frac{\pi}{2}}$$

The ideal lift coefficient, the lift coefficient at the ideal angle of attack ($\alpha = 0^\circ$), is given by

$$C_{L_I} = \frac{4\pi}{n} \tan \beta.$$

These two equations may be used to determine both n and β for a specified thickness and lift coefficient.

If

$$\mu^2 \equiv \left[\frac{1 + \cos(\varphi - \beta)}{1 - \cos(\varphi + \beta)}\right]^n$$

then the x-coordinate is

$$x = \frac{na}{M}\left(\mu - \frac{1}{\mu}\right)$$

where

$$M \equiv \mu + \frac{1}{\mu} - 2 \cos n\left(\frac{\pi}{2} - \beta\right)$$

for the upper surface and for the lower surface if $+\beta$ is substituted for $-\beta$. The y-coordinates for the upper and lower surfaces are

$$y_U = \frac{2na}{M} \sin n\left(\frac{\pi}{2} - \beta\right)$$

$$y_L = -\frac{2na}{M} \sin n\left(\frac{\pi}{2} + \beta\right)$$

The velocity is given by

$$\frac{v}{V} = M\left(\frac{\sin \varphi + \sin \beta}{n \cos \beta}\right)^2$$

Synthetic airfoils.- The synthesized airfoils are fully defined by the Ψ-function, or

$$\psi = \psi_o + A_1 \cos (\varphi - \delta_1) + A_2 \cos (2\varphi - \delta_2) + \ldots$$

from which

$$\epsilon = A_1 \sin (\varphi - \delta_1) + A_2 \sin (2\varphi - \delta_2) + \ldots$$

The ordinates are given by

$$x = 2a \cosh \psi \cos \theta$$

$$y = 2a \sinh \psi \sin \theta$$

and the velocity by

$$\frac{v}{V} = \frac{\left[\sin (\alpha + \varphi) + \sin (\alpha + \beta)\right] e^{\psi_o}}{\sqrt{\left(\sinh^2\psi + \sin^2\theta\right)\left[\left(1 - \frac{d\epsilon}{d\varphi}\right)^2 + \left(\frac{d\psi}{d\varphi}\right)^2\right]}}$$

where $\beta = \epsilon_T$. The value of ϵ_T may be obtained by solution of the ϵ-equation if $\varphi = \theta + \epsilon$ and

$$\epsilon = A_1 \sin (\theta + \epsilon - \delta_1) + A_2 \sin (2\theta + 2\epsilon - \delta_2) + \ldots$$

For ϵ_T,

$$\theta = \pi$$

and

$$\epsilon_T = - A_1 \sin (\epsilon_T - \delta_1) + A_2 \sin (2\epsilon_T - \delta_2) - \ldots$$

This transcendental equation must be solved by successive approximations. The method is rapidly convergent.

RESULTS

The results are presented in the form of a composite figure for each airfoil. Each figure contains a sketch of the airfoil contour in the upper left-hand corner and the pressure distribution for a lift coefficient of 0, 0.1, 0.2, 0.3, 0.5, 1.0, 1.5, and the ideal lift. For the circular-arc airfoils the pressure distributions are given for the ideal lift only.

Conventional airfoils.- The first group of airfoils comprises some which are in quite common use but for which no complete set of pressure distributions has been published. It is desirable to have the pressure distributions of these airfoils on record since they form convenient references for use or modification of the airfoils. This group of airfoils consists of the NACA 22-, 44-, and 230-series airfoils (figs. 1 to 5, 6 to 11, and 12 to 18, respectively) and the Clark Y family of airfoils (figs. 19 to 30). Ordinates of the NACA 22- and 44-series airfoils are given in reference 4 and of the NACA 230-series, in reference 5. For the Clark Y family of airfoils, a designation of the form CY(c)-t is adopted, where the letters CY are the identifying initials, the letter within the parentheses represents the percent maximum camber at 40 percent chord, and the last letter represents the percent maximum thickness at 30 percent chord. In the notation used herein the original Clark Y airfoil becomes CY(3.576)-11.7. The basic camber and thickness distributions of the Clark Y family are given in table I. In the generation of all airfoils of the Clark Y family, the camber and thickness are measured perpendicular to the chord line.

There are several subgroups in the Clark Y family. In figures 19 to 25 are airfoils of the standard flat-bottom type obtained by multiplying both the camber and thickness ordinates by the same ratio. The airfoil shown in figure 19 is the original Clark Y airfoil [CY(3.576)-11.7]. Figures 26 and 27 give the Clark Y M airfoil in which the camber remains constant and identical to the original Clark Y airfoil and the thickness is the only variable. In figures 28 and 29 the standard thickness of the Clark Y airfoil is maintained and two values are used for the camber. In figure 30 the camber of the airfoil is the same as that of the airfoil in figure 29 but the thickness was decreased to 6 percent.

Circular-arc sections.- The second group of airfoils, figures 31 to 34, is a collection of circular-arc sections obtained by the Kármán-Trefftz transformation. These airfoils are designed for ideal lift coefficients of 0.1, 0.25, 0.5, and 1.0, and the airfoil shape and pressure distribution are given at 0-, 5-, 10-, and 15-percent thickness.

Synthetic airfoils.- The third group, figures 35 to 49, shows examples of airfoils constructed in accordance with the method of the last section of reference 2. In figures 35 to 41, examples of airfoils are presented containing only one harmonic

$$\psi = \psi_o + A_1 \cos (\varphi - \delta_1)$$

Such examples are interesting in that they demonstrate how far it is practicable to obtain airfoil shapes based on only one harmonic. It appears from the results that for one harmonic a phase angle of 45° gives the best airfoil shape.

In figures 42 to 47, the generating function contains first and second harmonics

$$\psi = \psi_o + A_1 \cos (\varphi - \delta_1) + A_2 \sin 2\varphi$$

For these airfoils the constants are chosen to generate airfoils with zero moment coefficient exclusively.

In figures 48 and 49, two examples are shown with functions containing first, second, and fourth harmonics, with the constants again chosen to give zero moment coefficients. Since the fourth harmonic does not affect the moment coefficient, a certain freedom in rearranging the pressure distribution at will is afforded.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., September 13, 1945

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TABLE I

MEAN-CAMBER AND SEMITHICKNESS ORDINATES FOR FAMILY OF AIRFOILS BASED ON CLARK Y SECTION

[All values are given in percent wing chord]

Station	Mean-camber ordinate	Semithickness
0	0	0
1.25	.0822	.1504
2.5	.1597	.2150
5	.3040	.2979
7.5	.4189	.3513
10	.5185	.3923
15	.6812	.4504
20	.8062	.4842
30	.9457	.5000
40	1.0000	.4872
50	.9732	.4496
60	.8778	.3910
70	.7223	.3141
80	.5207	.2231
90	.2785	.1197
95	.1435	.0637
100	0	0
L.E. radius: $0.009t^2$; T.E. radius: $0.005t$		

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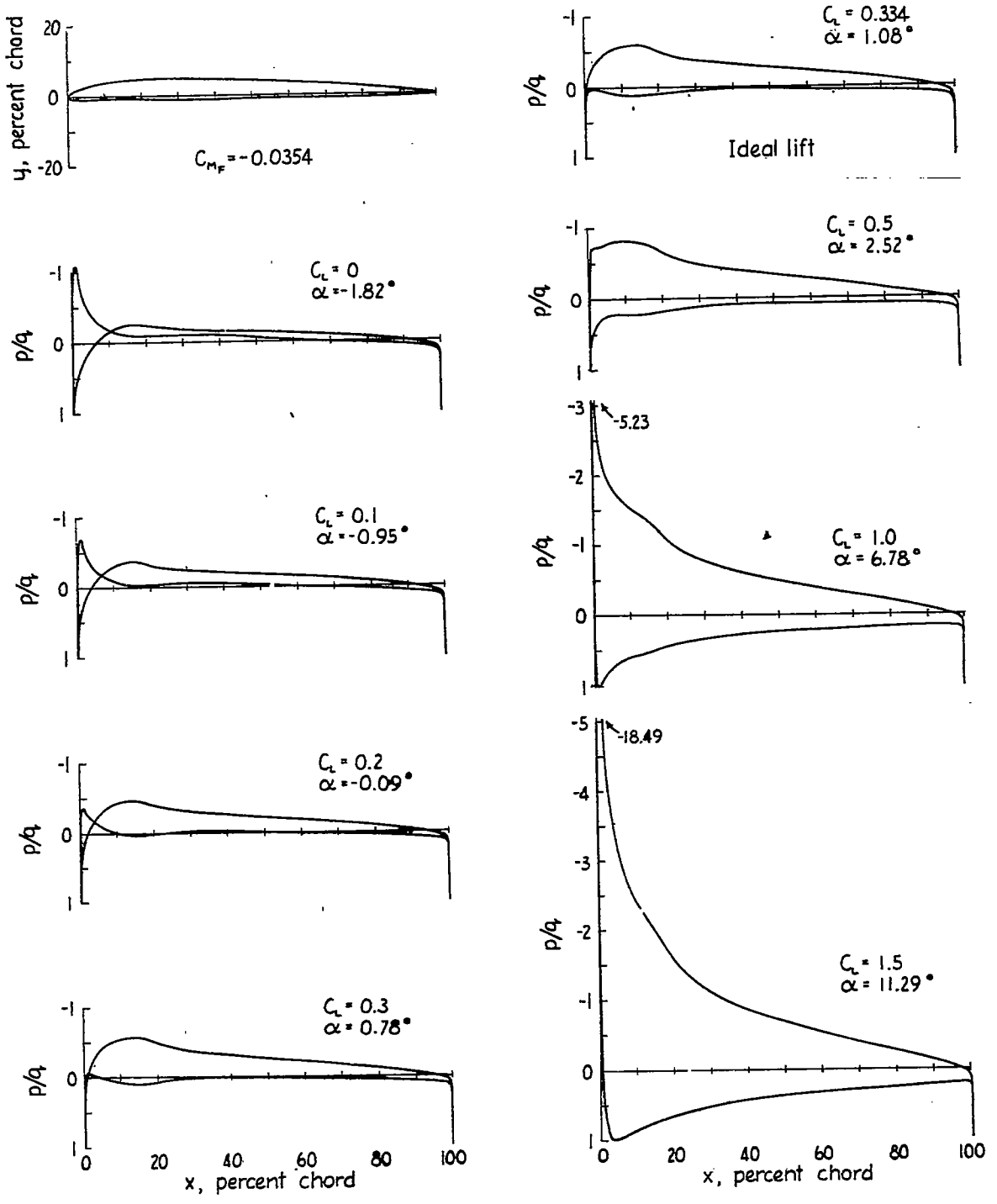

Figure 1.- Shape and pressure distribution for NACA 2206 airfoil.

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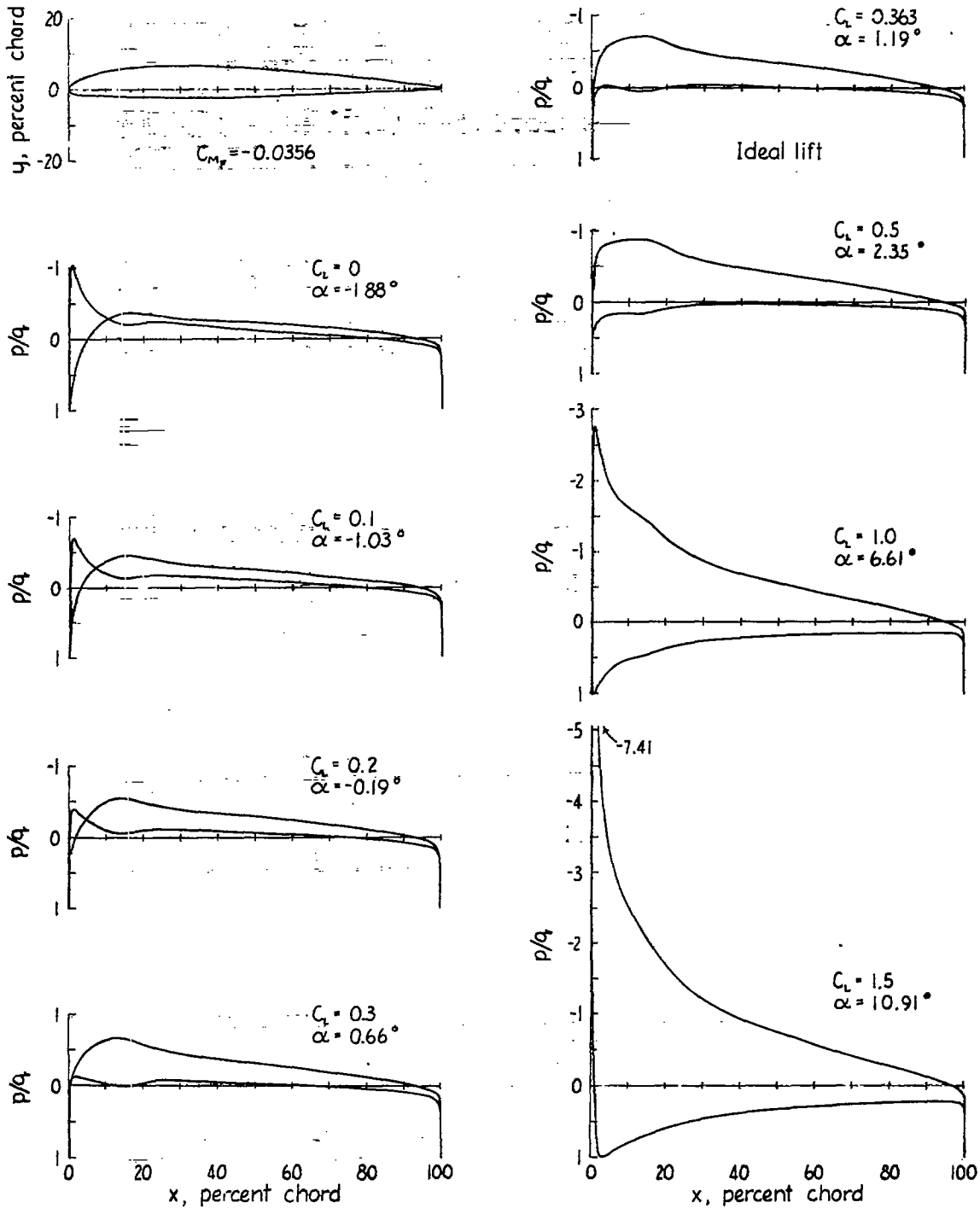

Figure 2.- Shape and pressure distribution for NACA 2209 airfoil.

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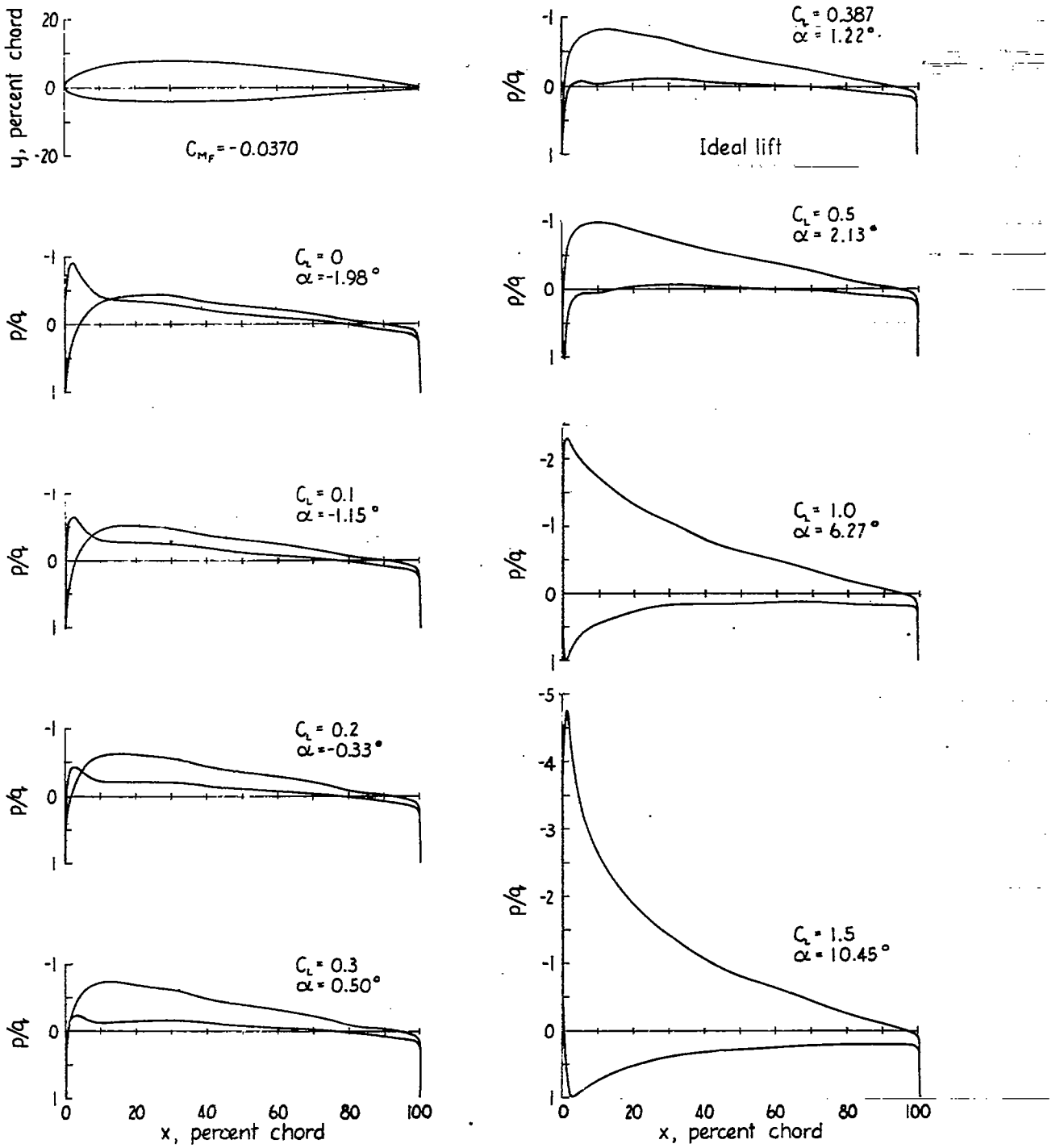

Figure 3.- Shape and pressure distribution for NACA 2212 airfoil.

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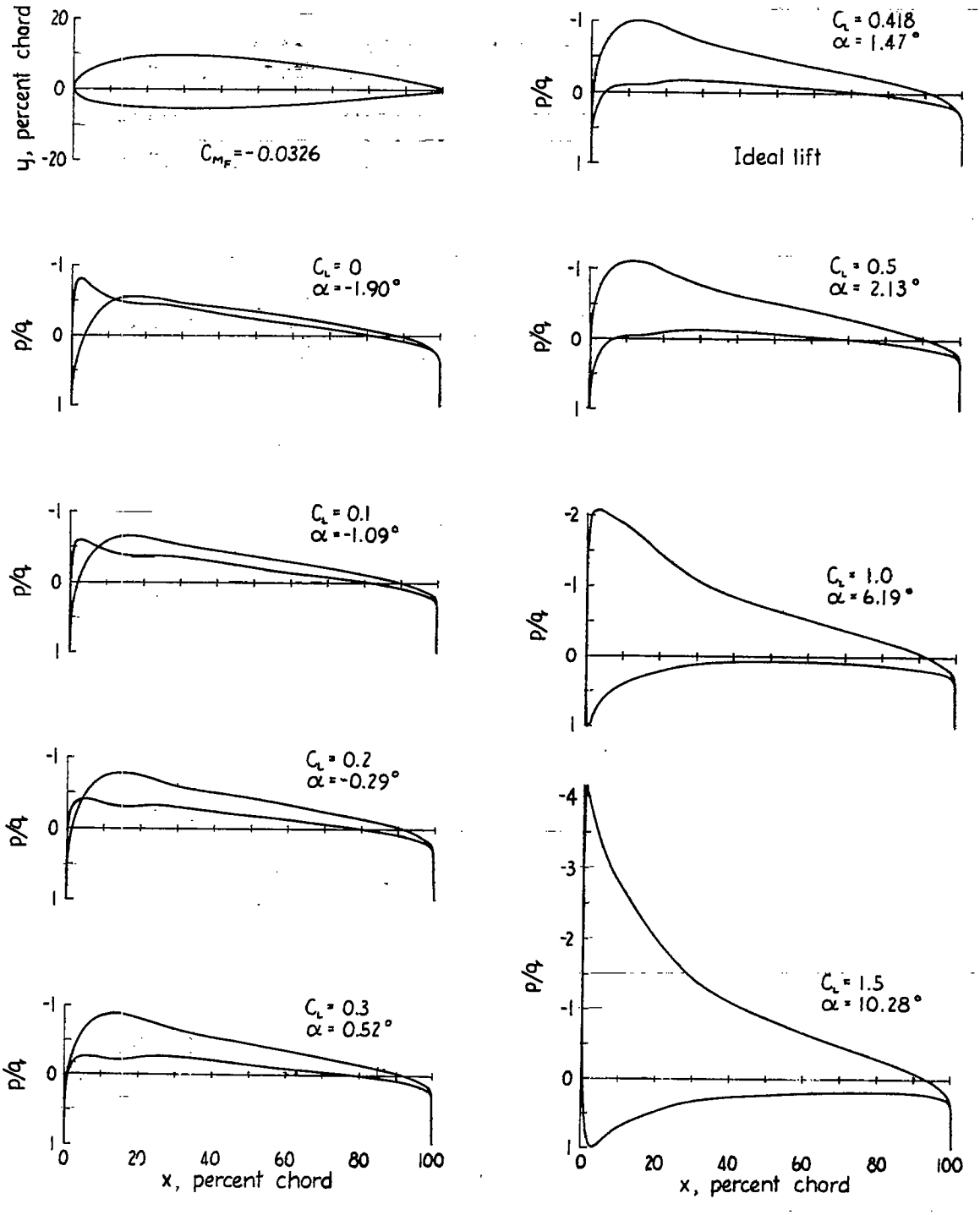

Figure 4.- Shape and pressure distribution for NACA 2215 airfoil.

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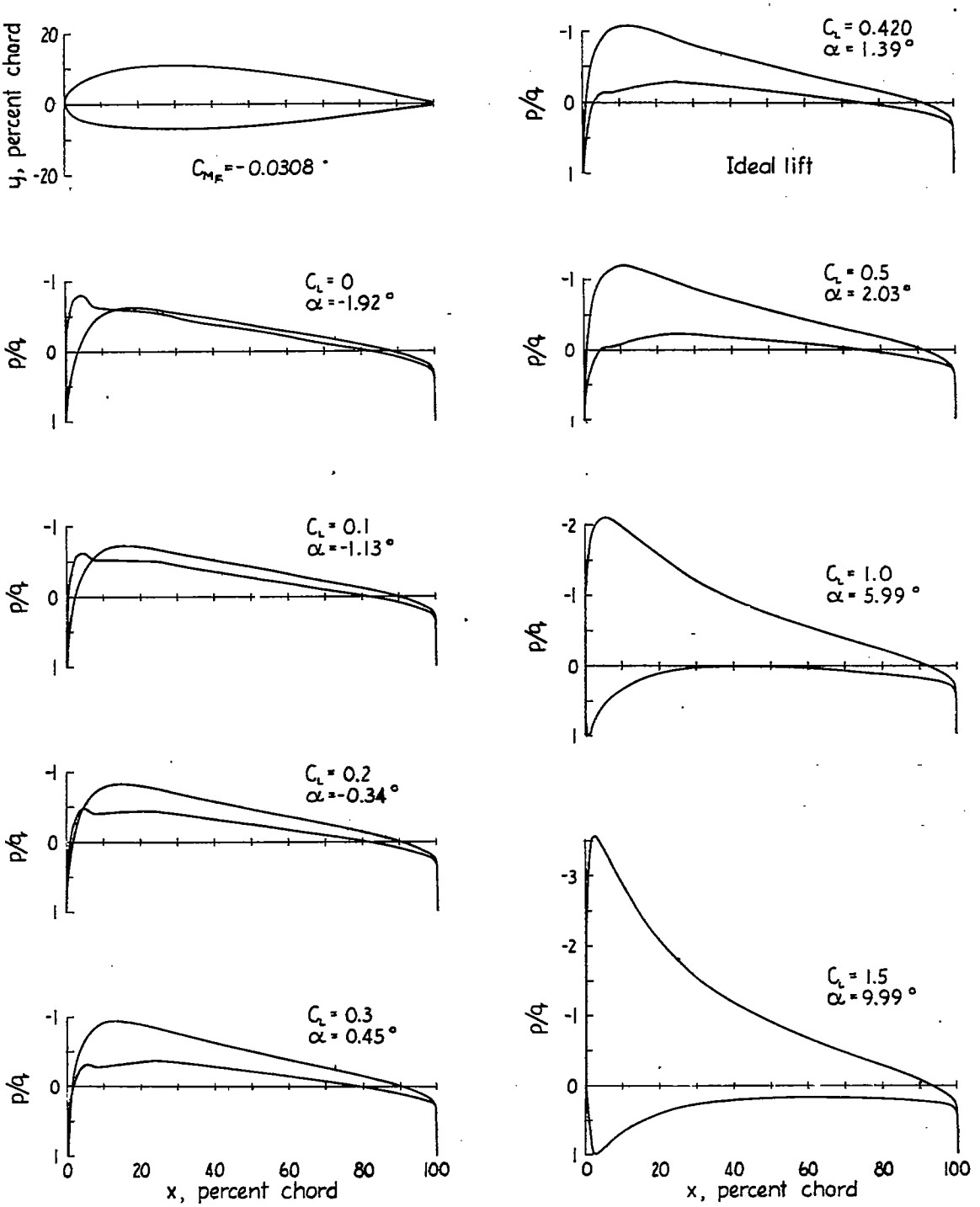

Figure 5.- Shape and pressure distribution for NACA 2218 airfoil.

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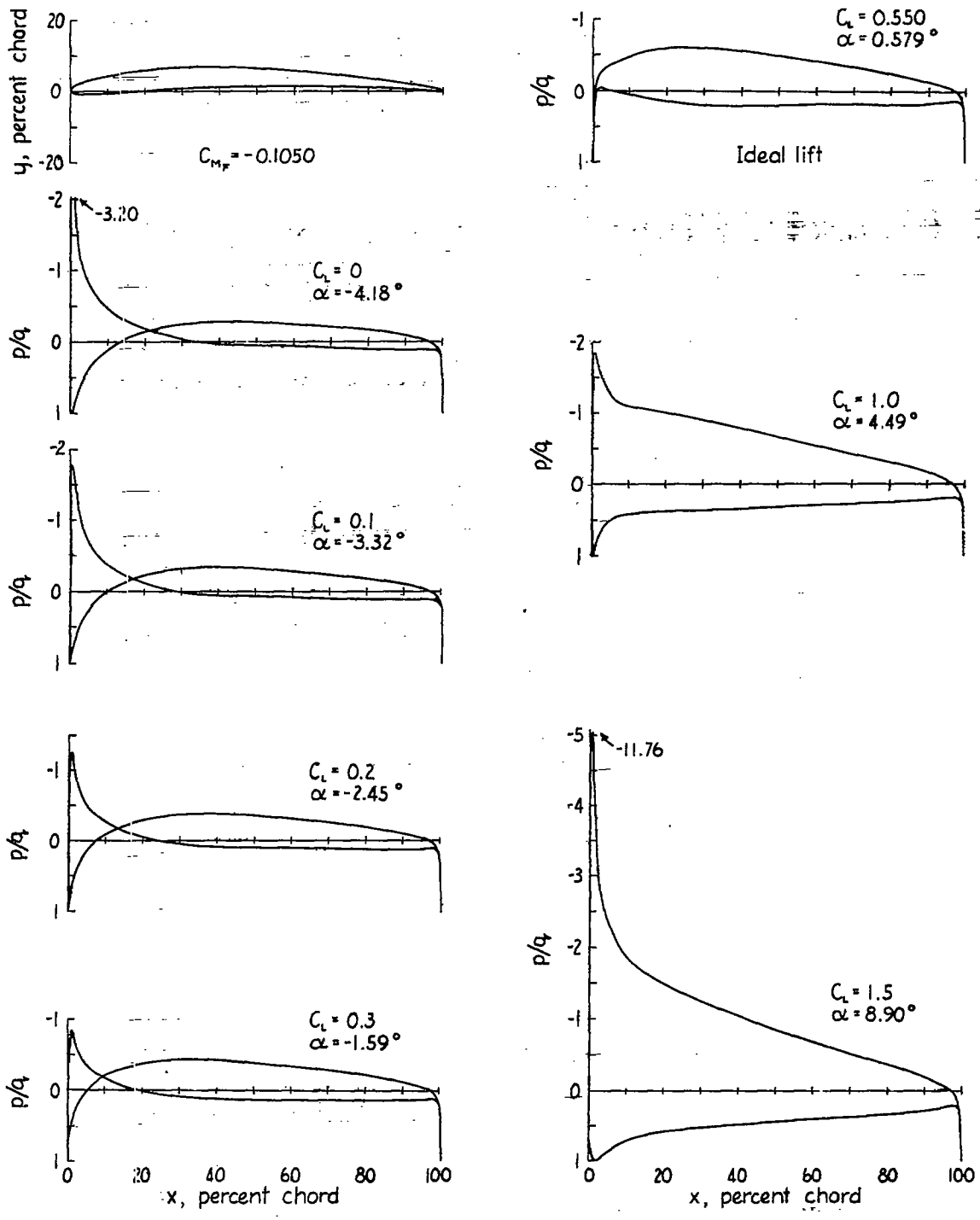

Figure 6.- Shape and pressure distribution for NACA 4406 airfoil.

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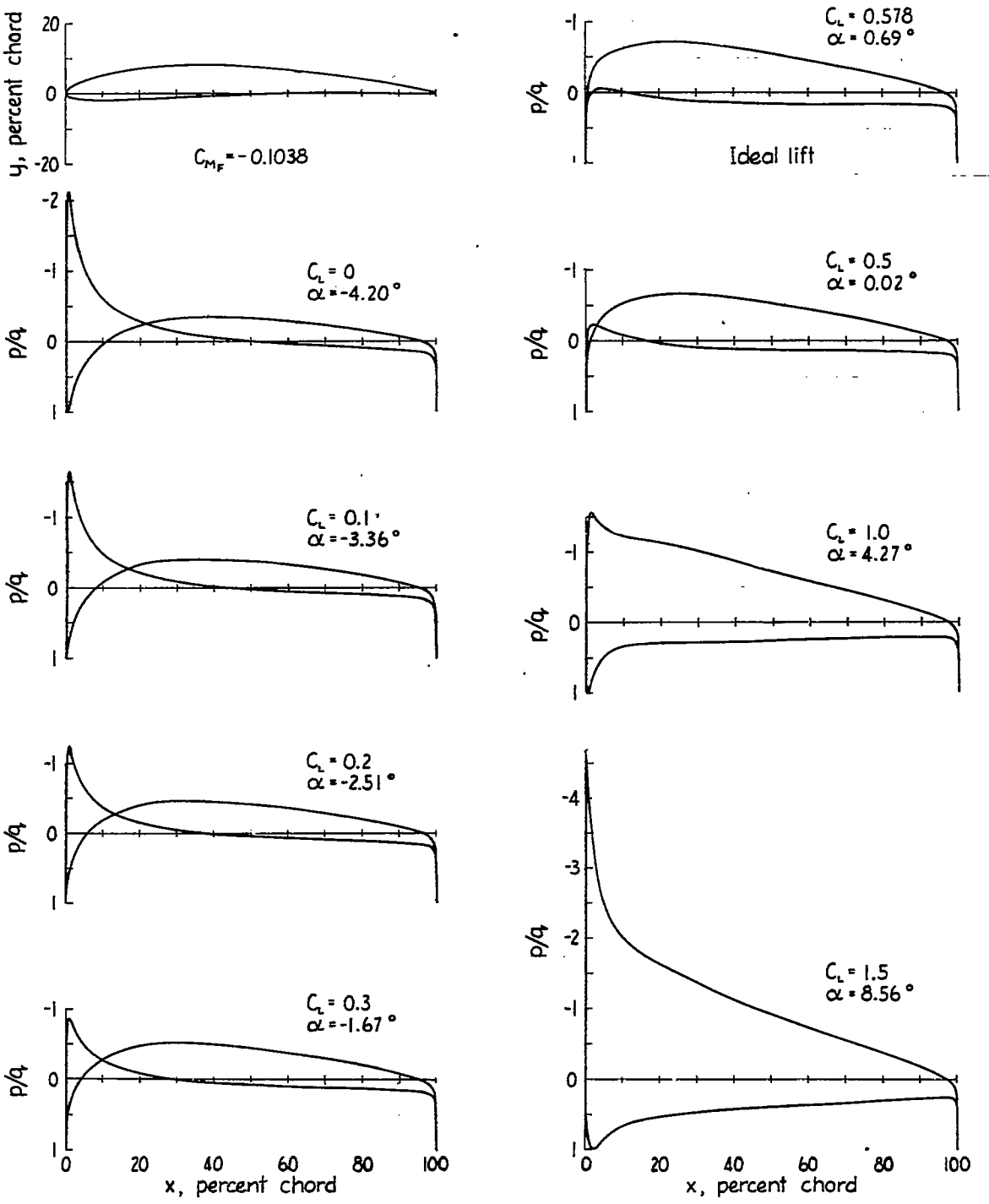

Figure 7.- Shape and pressure distribution for NACA 4409 airfoil.

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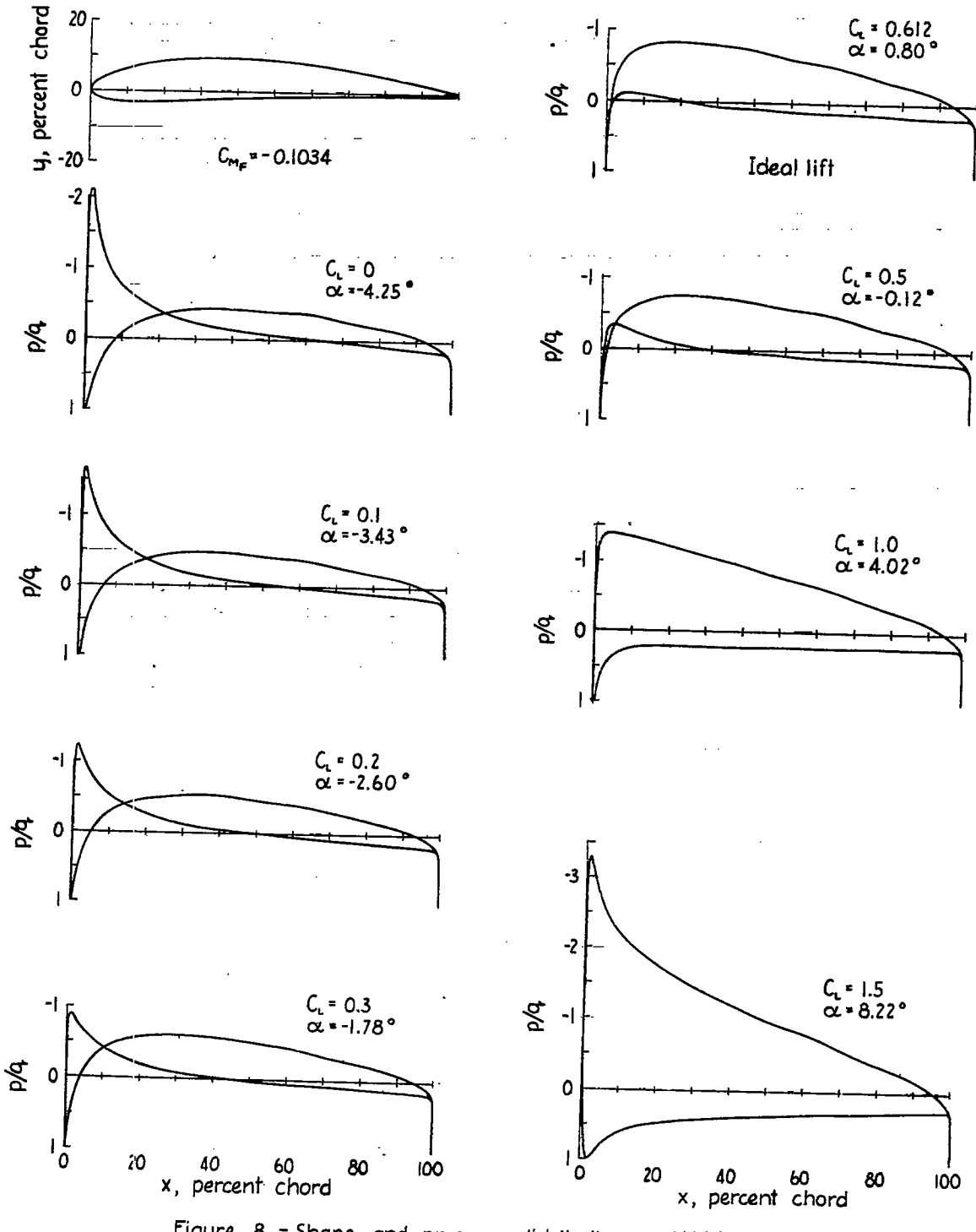

Figure 8.- Shape and pressure distribution for NACA 4412 airfoil

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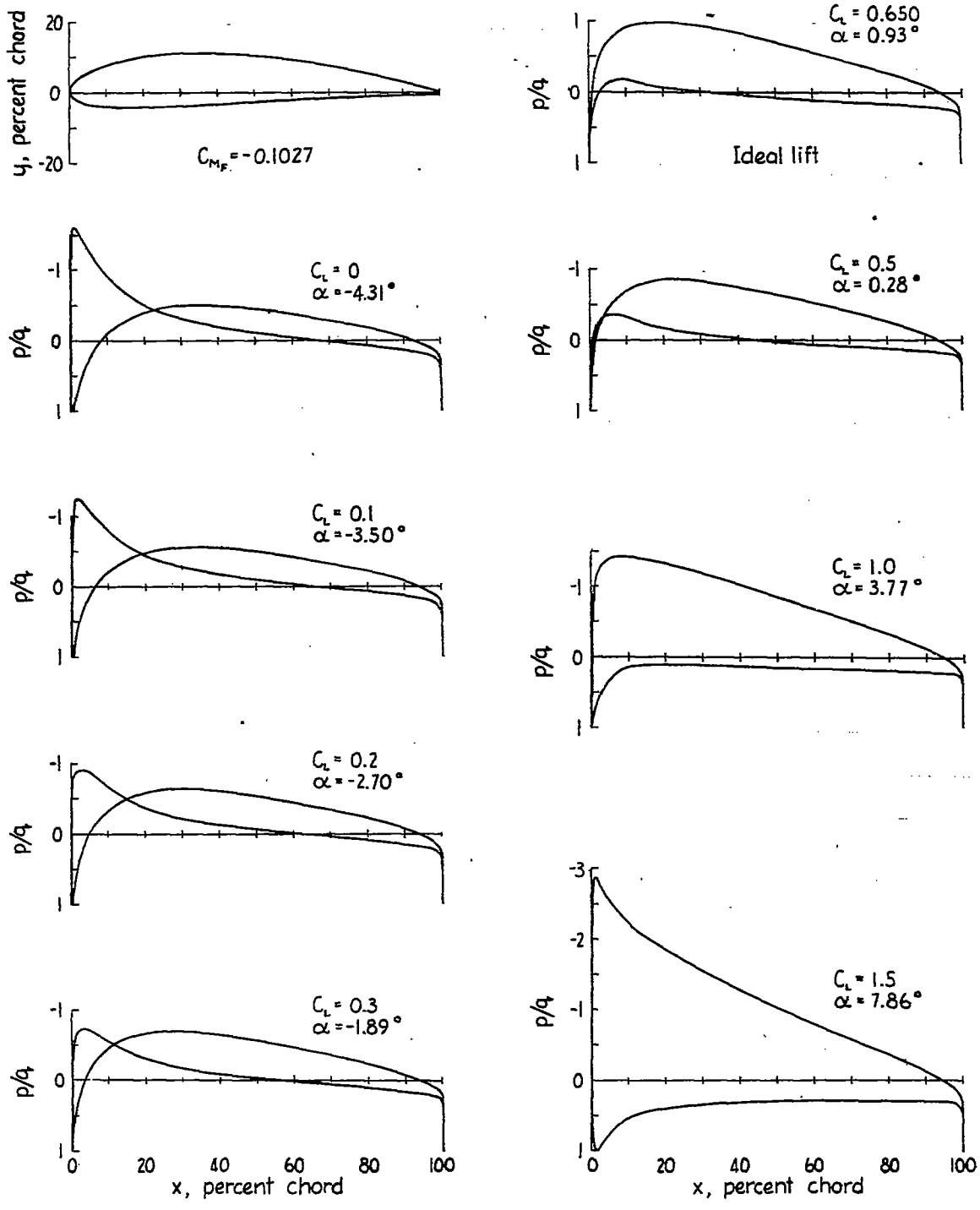

Figure 9.- Shape and pressure distribution for NACA 4415 airfoil.

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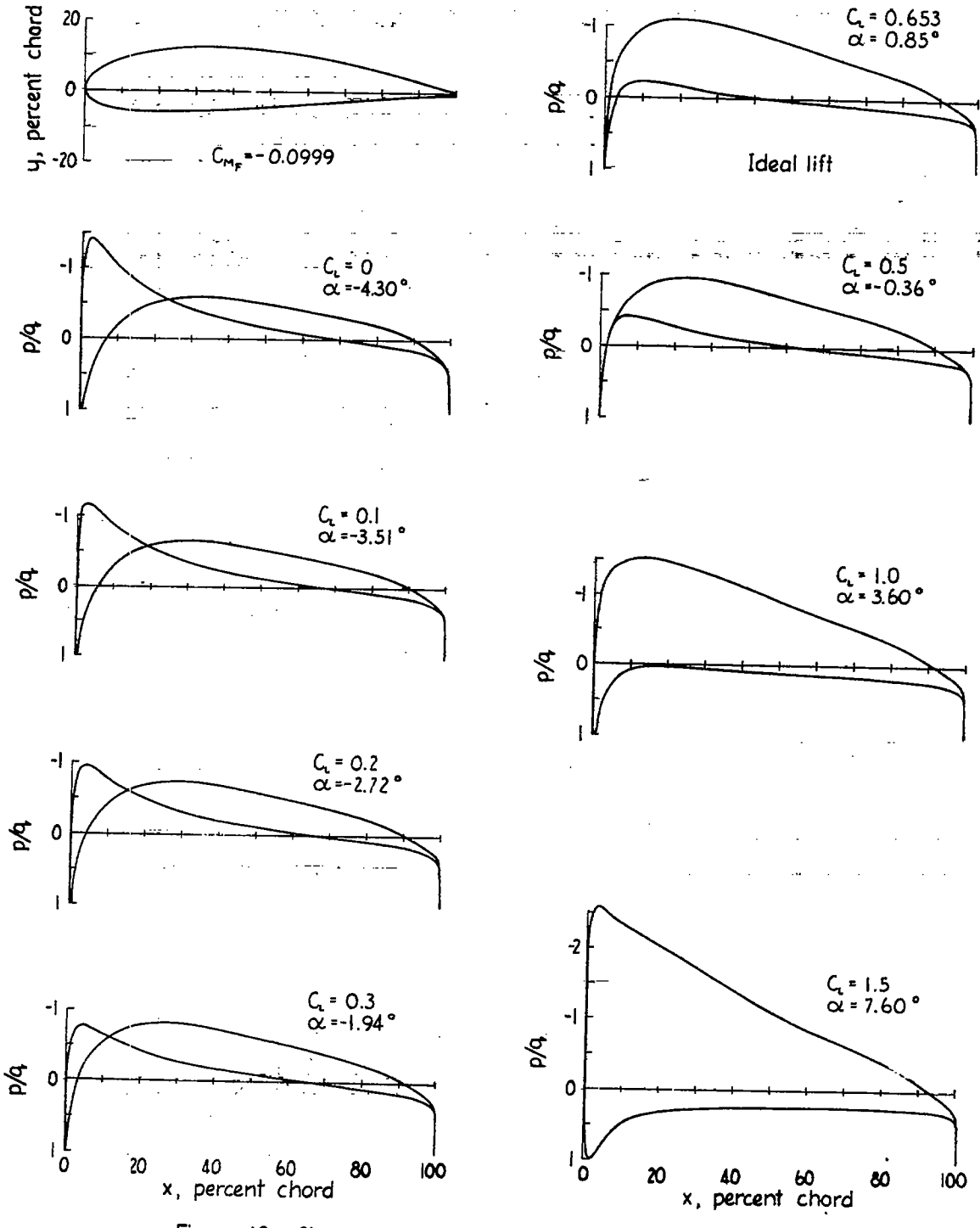

Figure 10.- Shape and pressure distribution for NACA 4418 airfoil.

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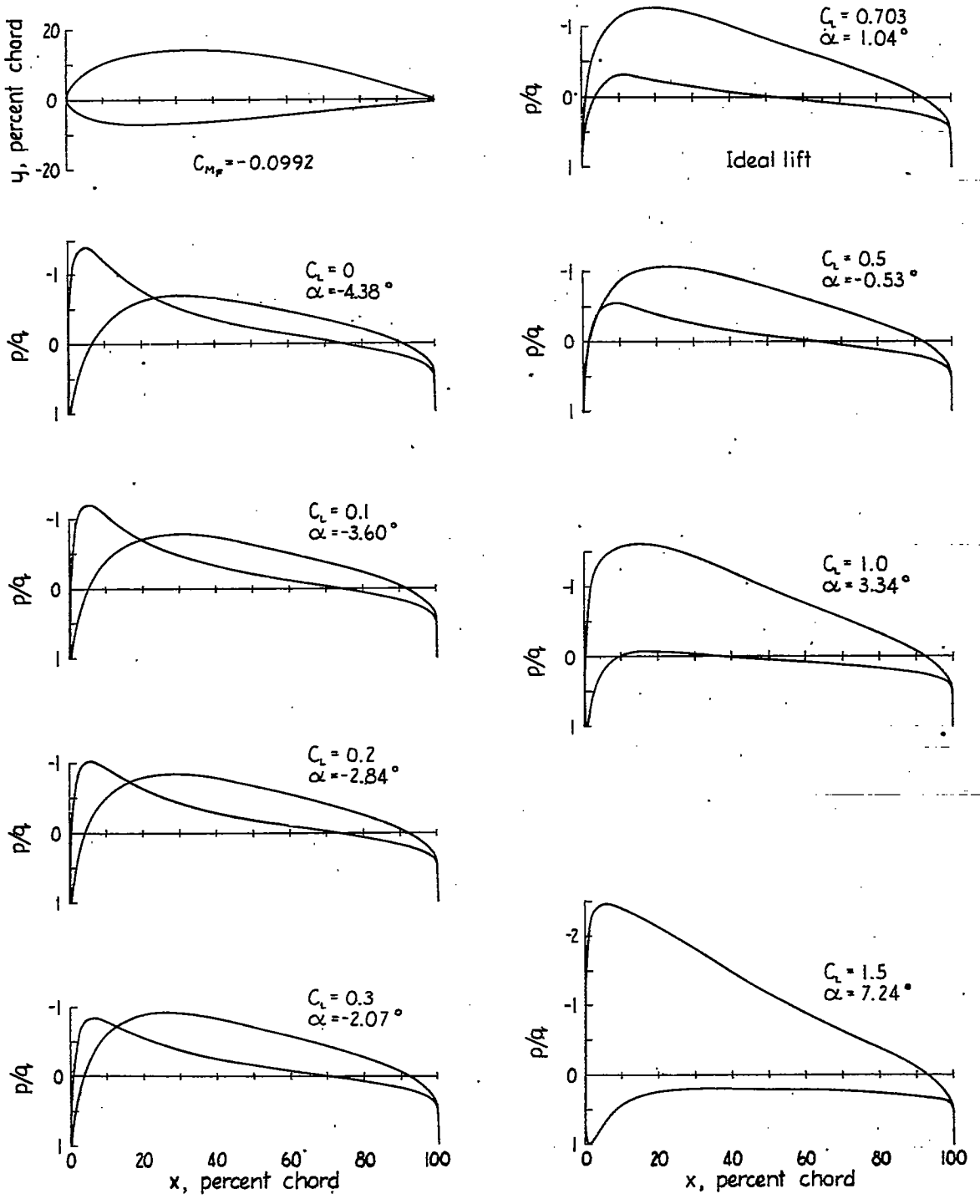

Figure 11.- Shape and pressure distribution for NACA 4421 airfoil.

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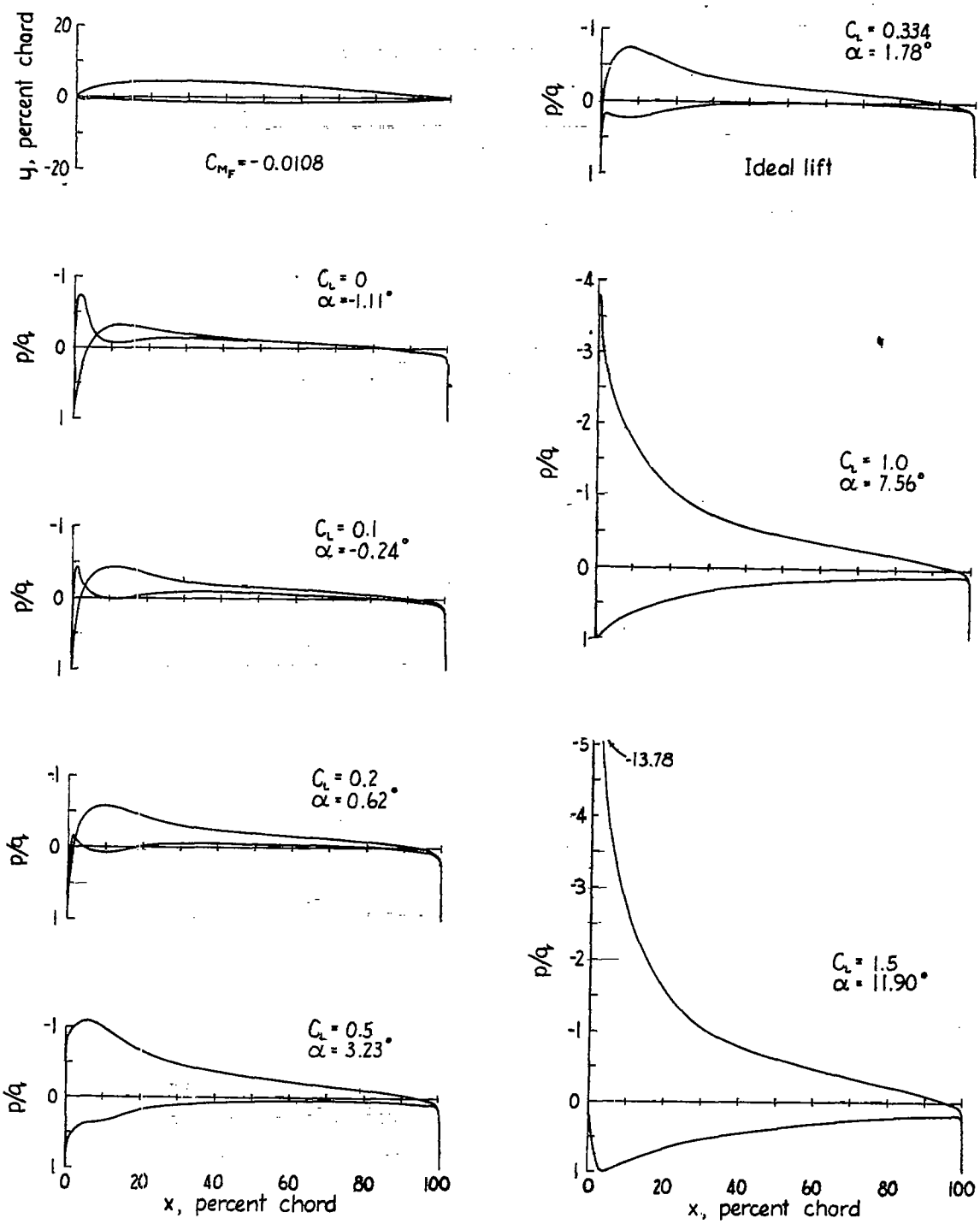

Figure 12.- Shape and pressure distribution for NACA 23006 airfoil.

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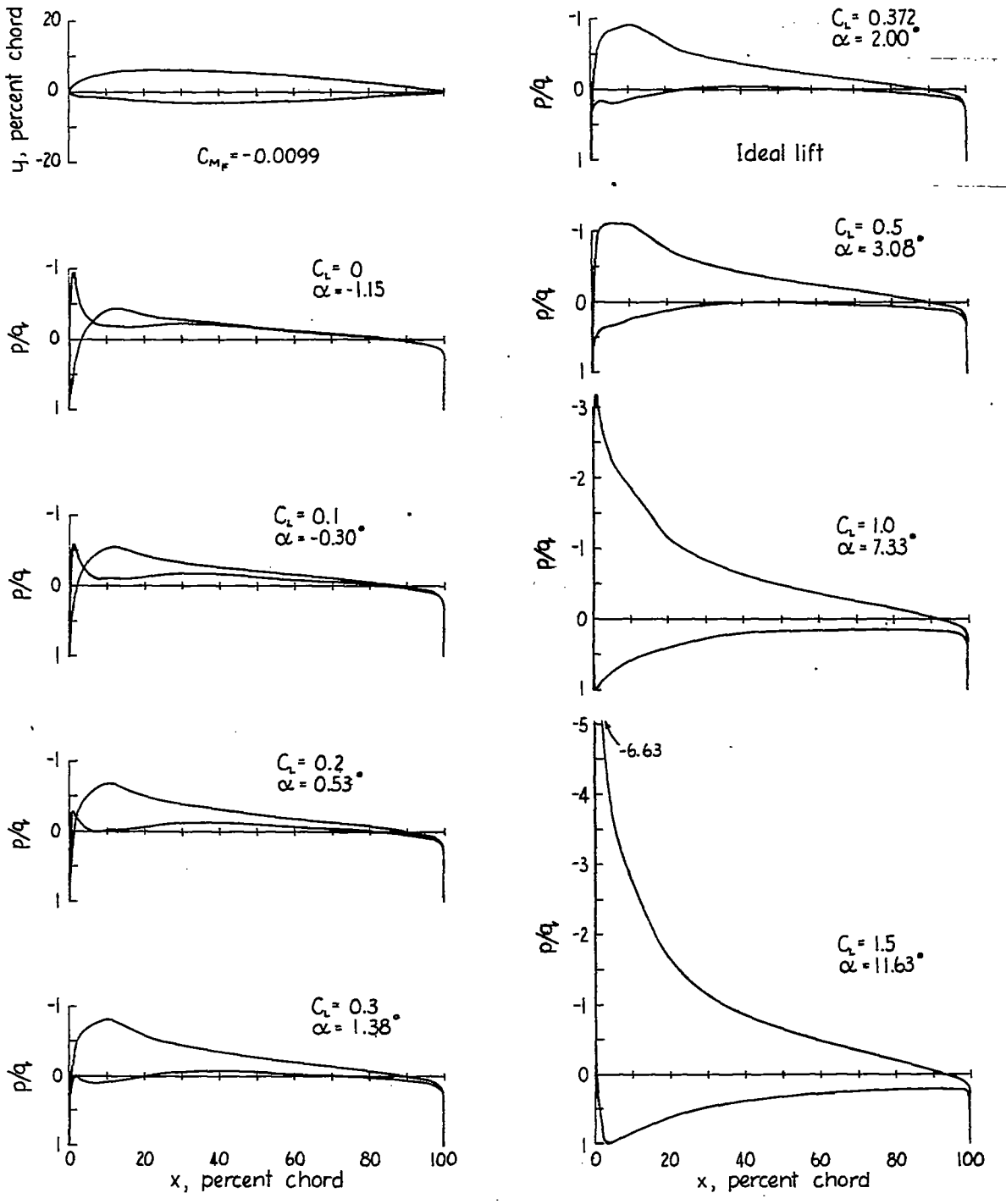

Figure 13.- Shape and pressure distribution for NACA 23009 airfoil.

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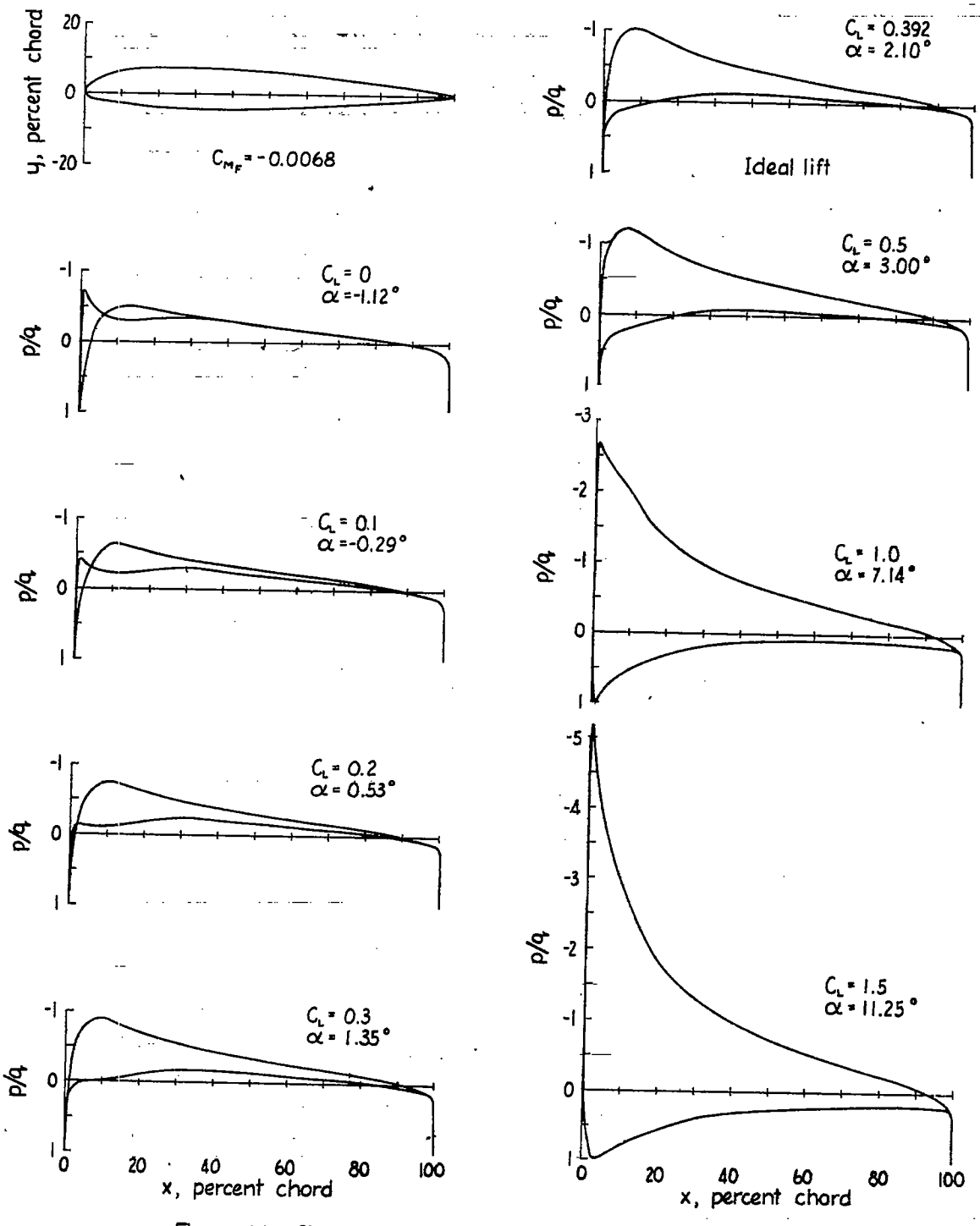

Figure 14.- Shape and pressure distribution for NACA 23012 airfoil.

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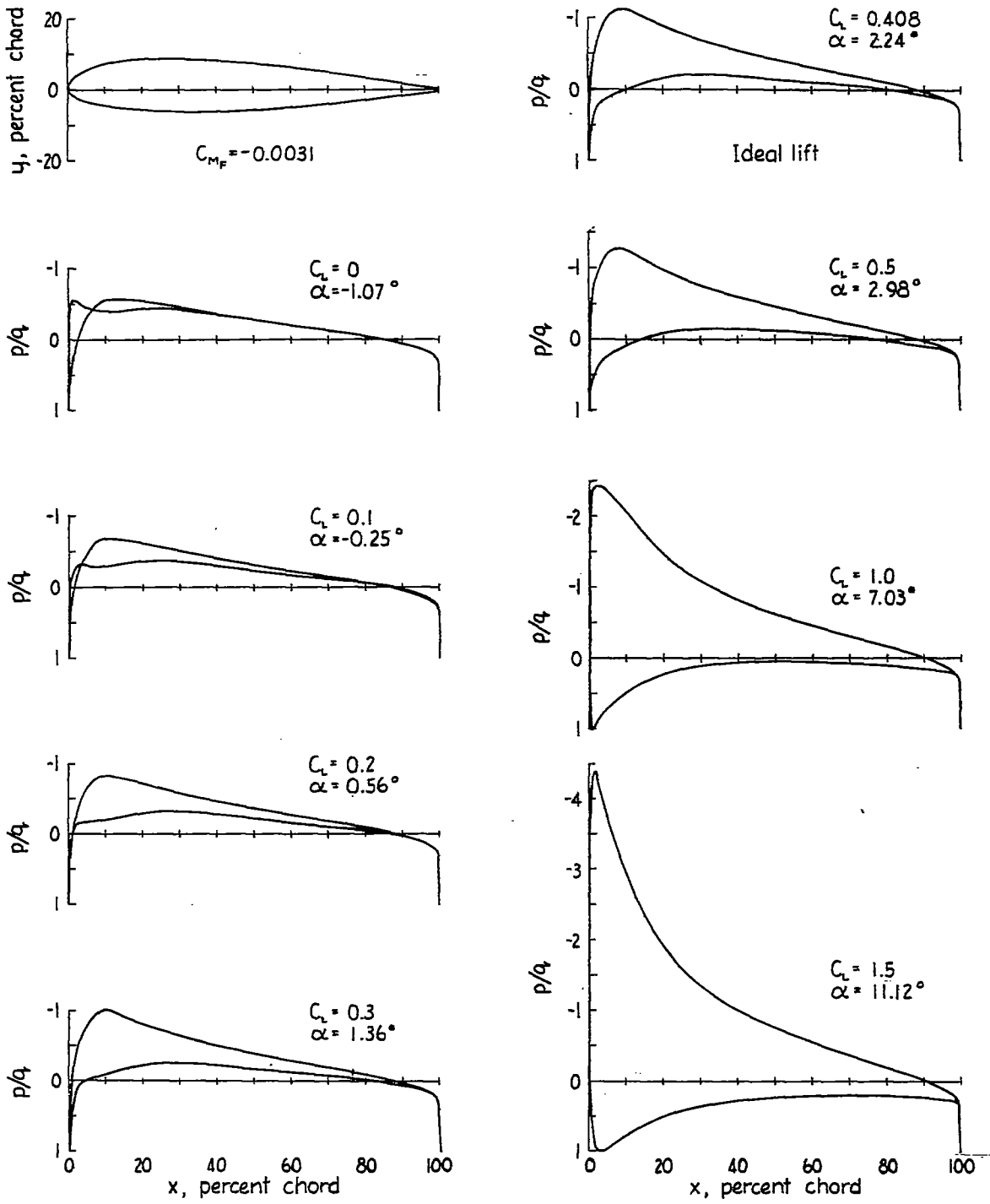

Figure 15.- Shape and pressure distribution for NACA 23015 airfoil.

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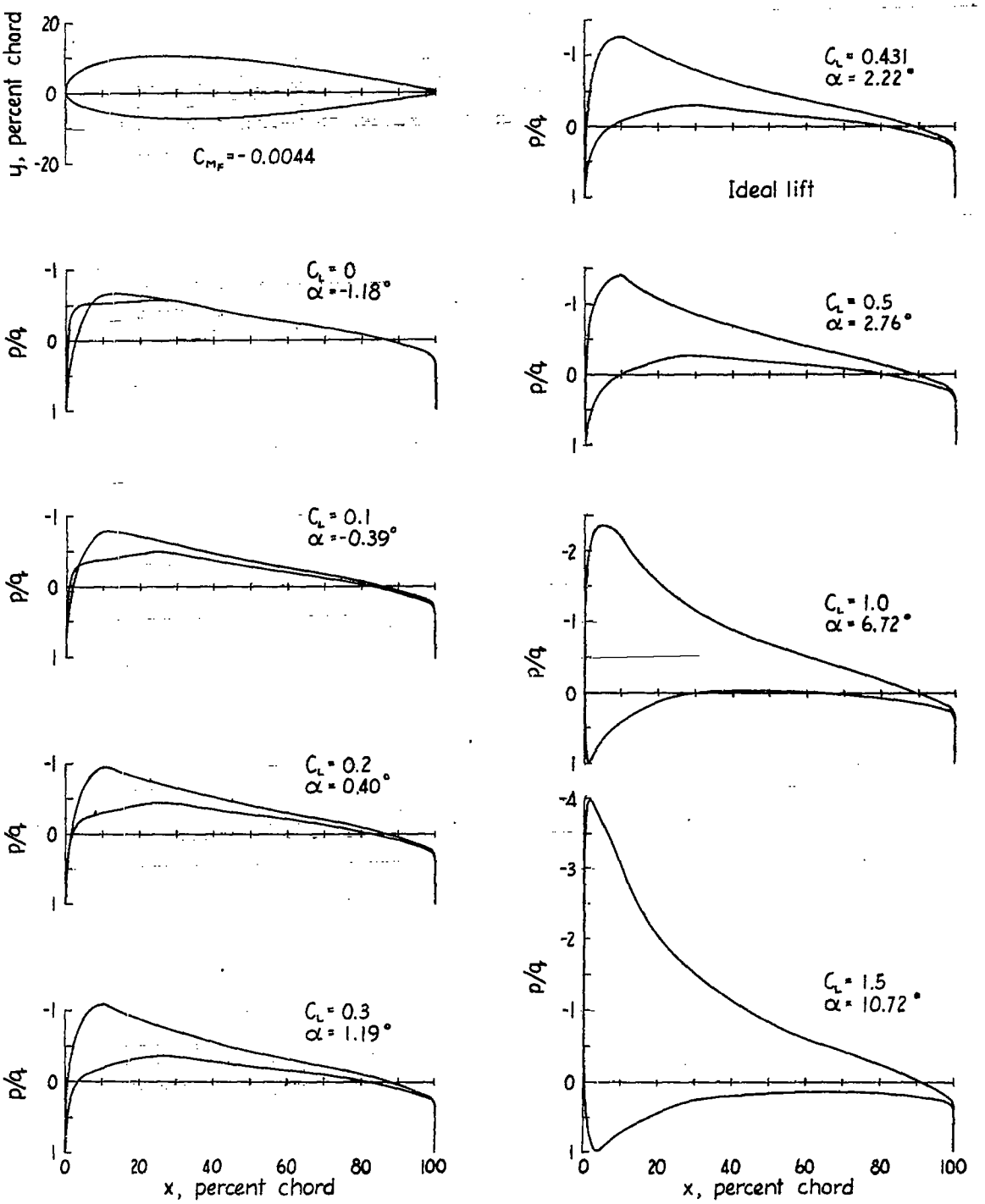

Figure 16.- Shape and pressure distribution for NACA 23018 airfoil.

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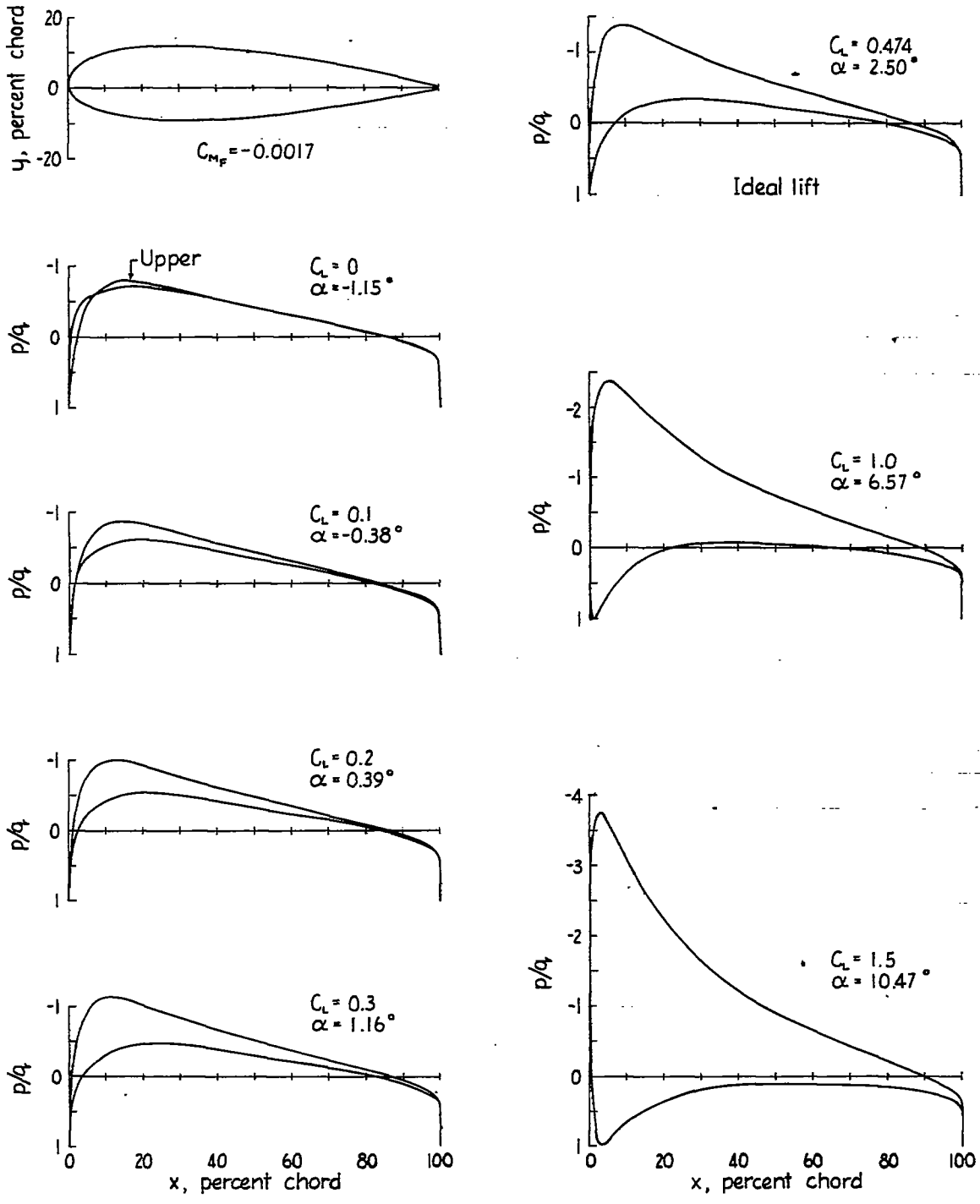

Figure 17.- Shape and pressure distribution for NACA 23021 airfoil.

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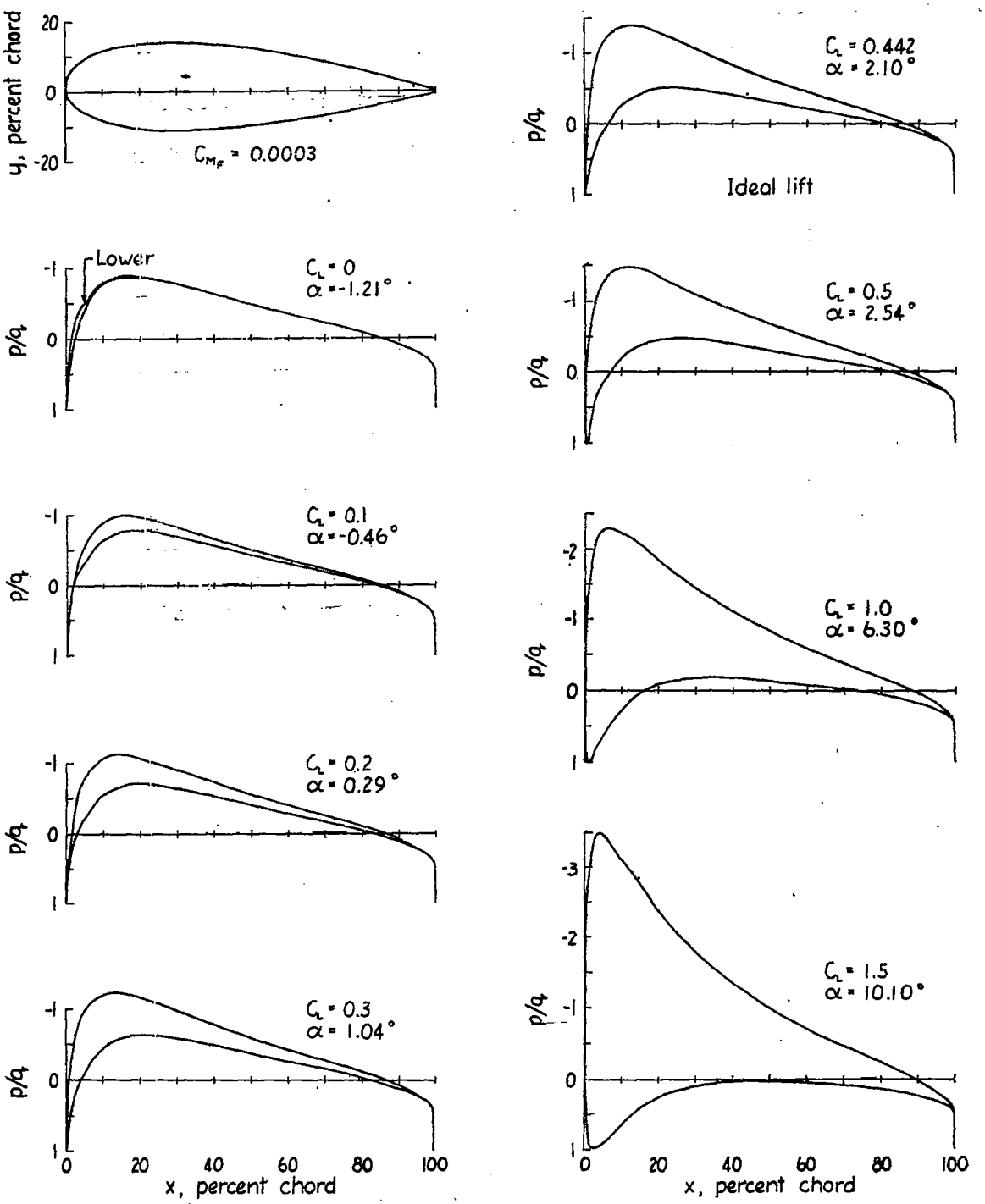

Figure 18.- Shape and pressure distribution for NACA 23025 airfoil.

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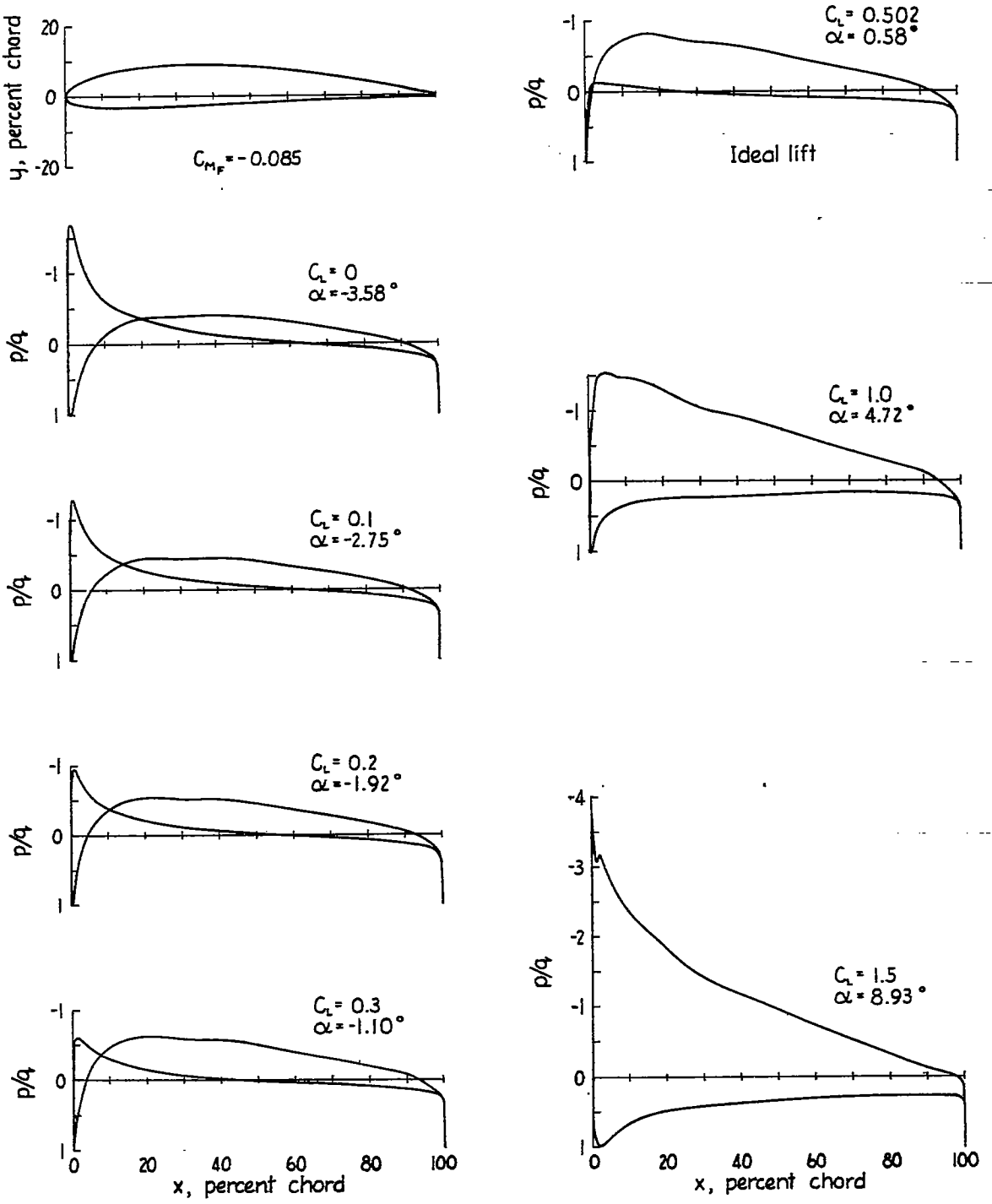

Figure 19.- Shape and pressure distribution for Clark Y airfoil, CY(3.576) - 11.7.

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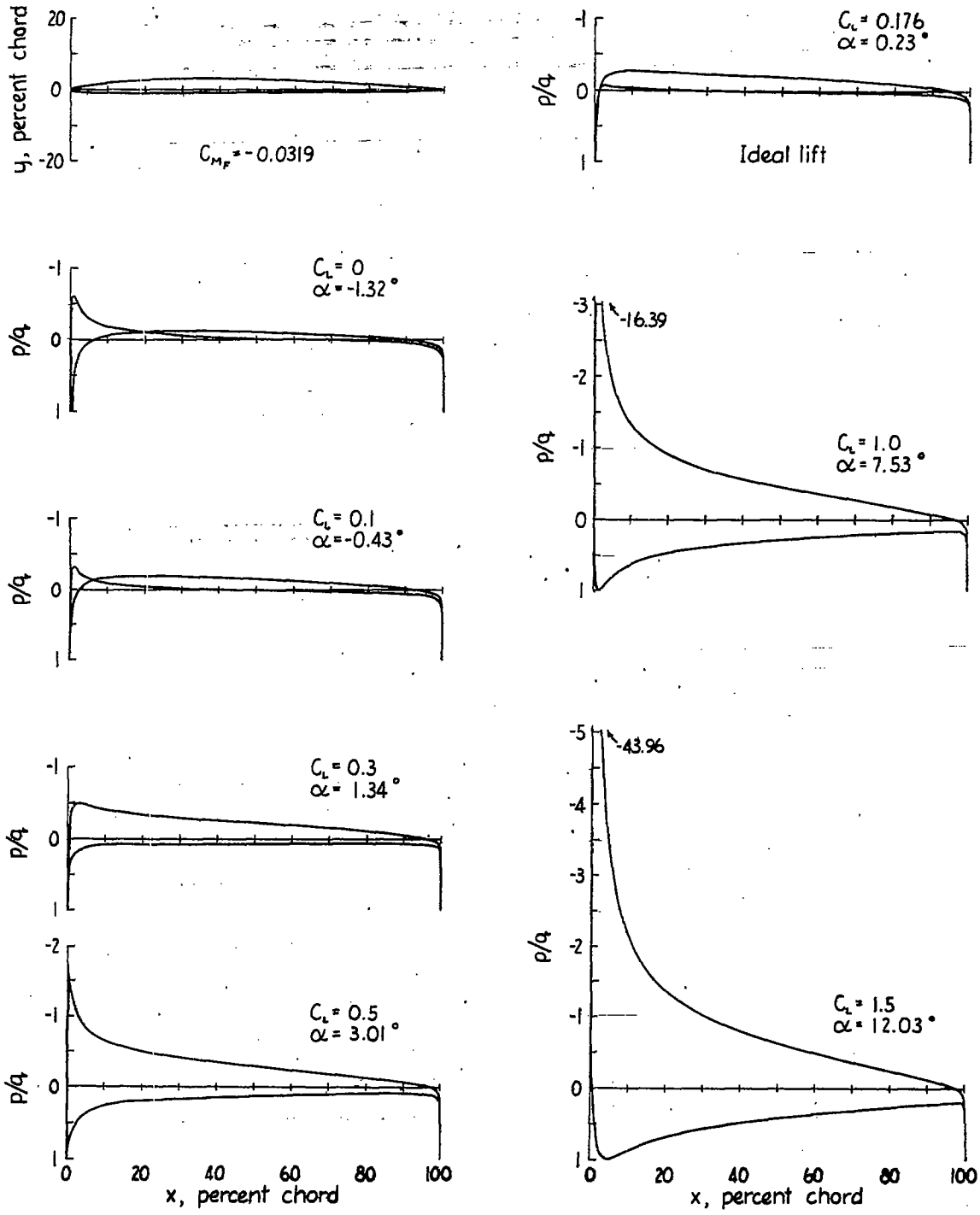

Figure 20.- Shape and pressure distribution for Clark Y-4 airfoil, CY(1.223) - 4.

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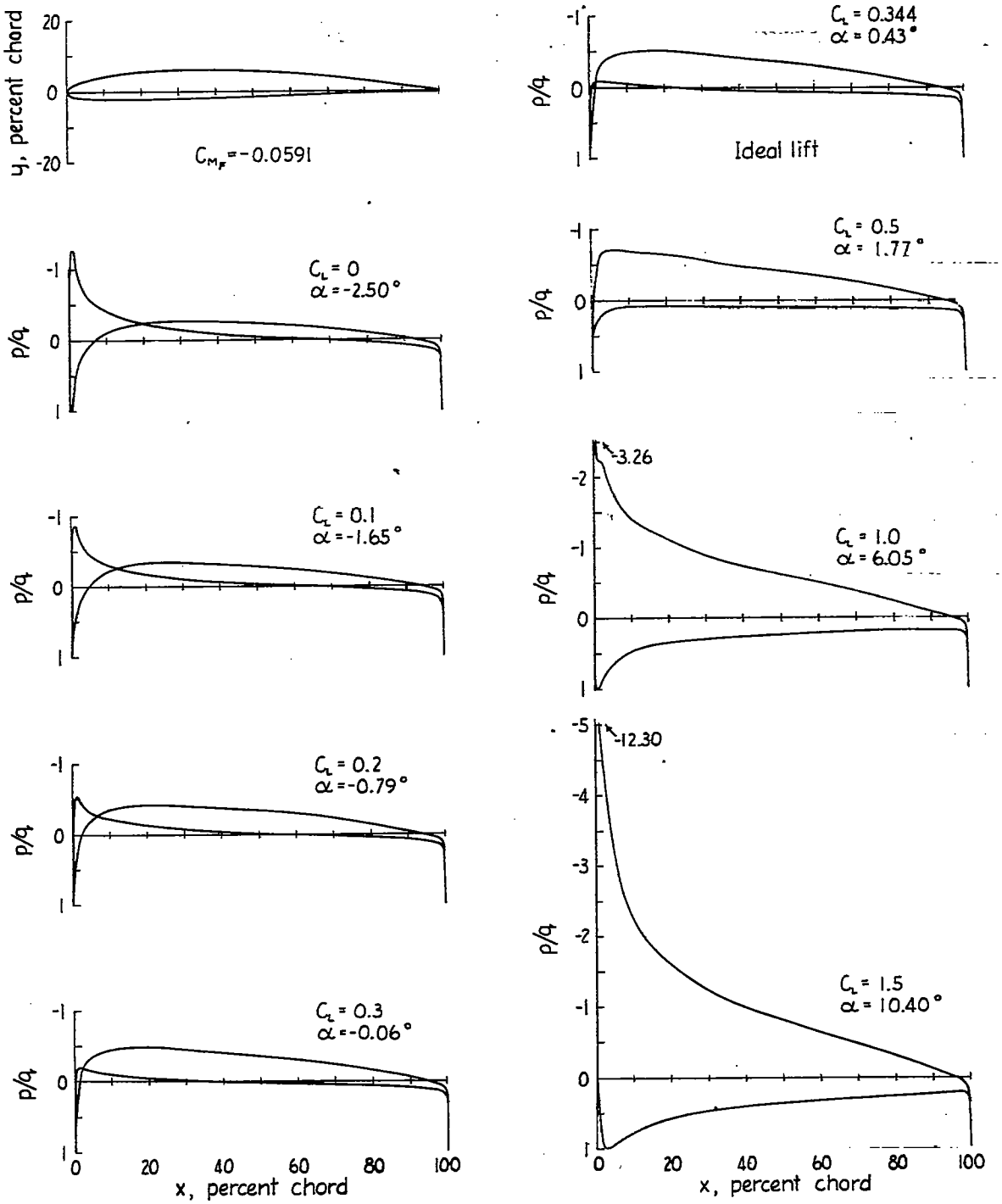

Figure 21.- Shape and pressure distribution for Clark Y-8 airfoil, CY(2.445)-8.

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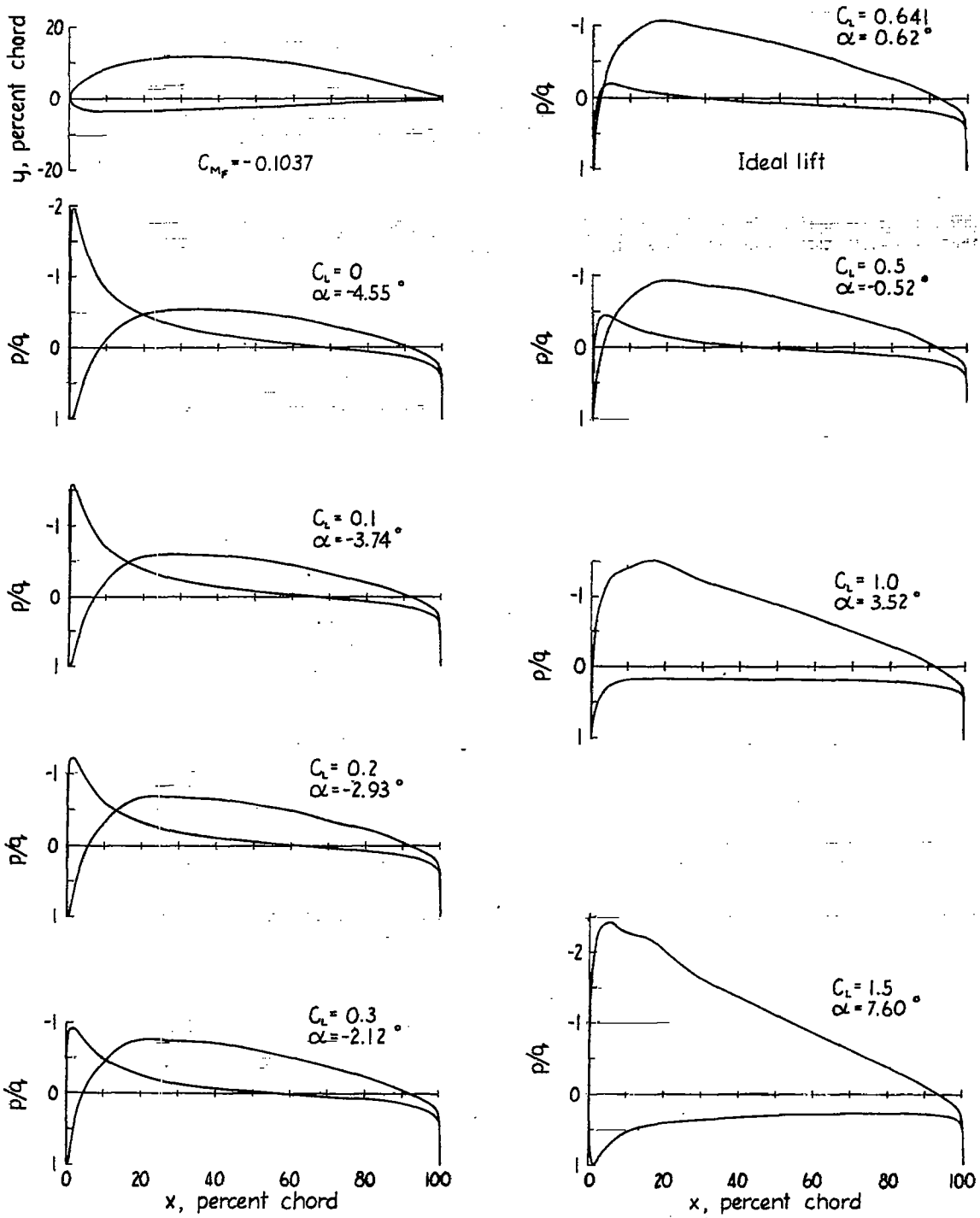

Figure 22.- Shape and pressure distribution for Clark Y-15 airfoil, CY(4.585)-15.

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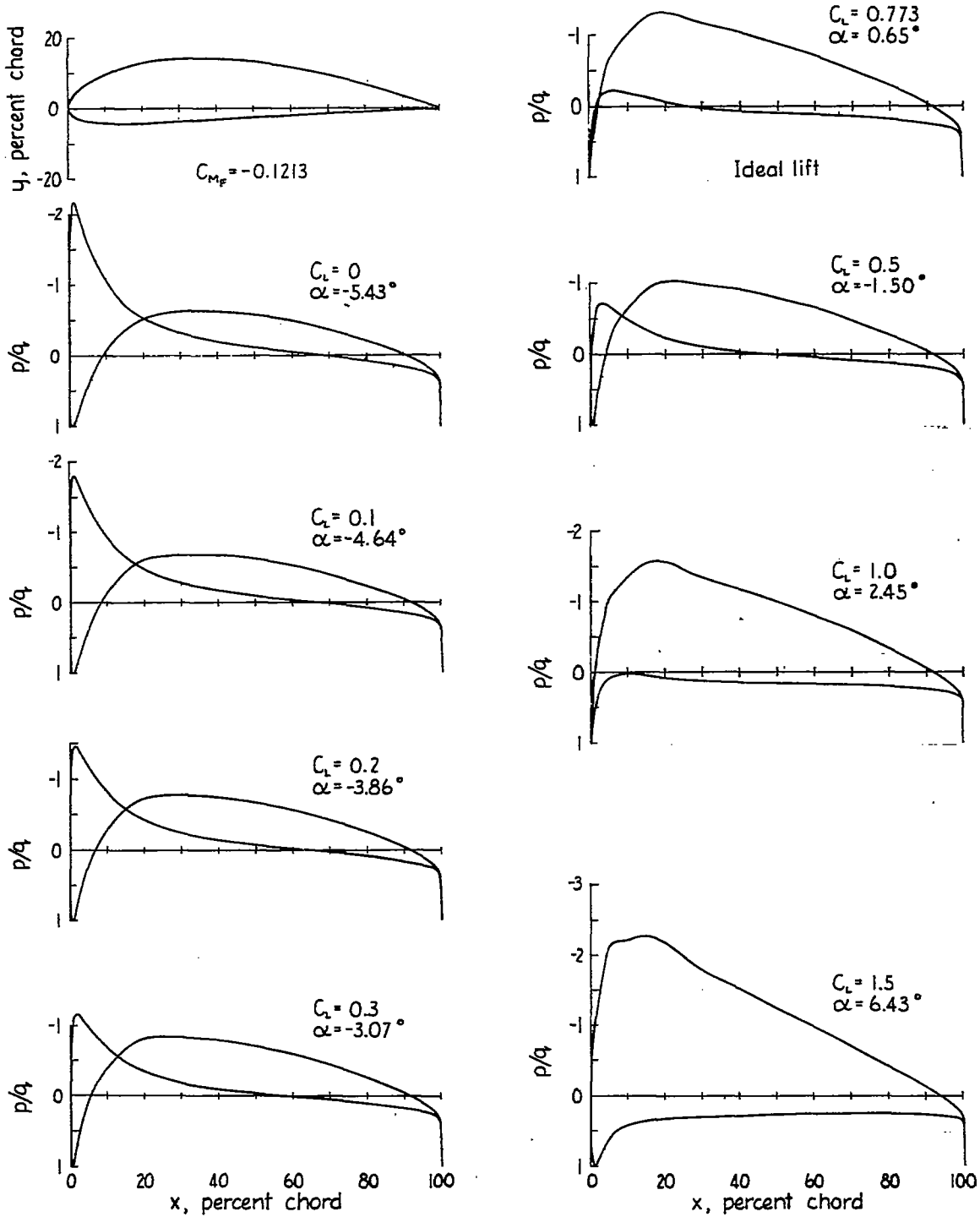

Figure 23.- Shape and pressure distribution for Clark Y-18 airfoil, CY(5.502)-18.

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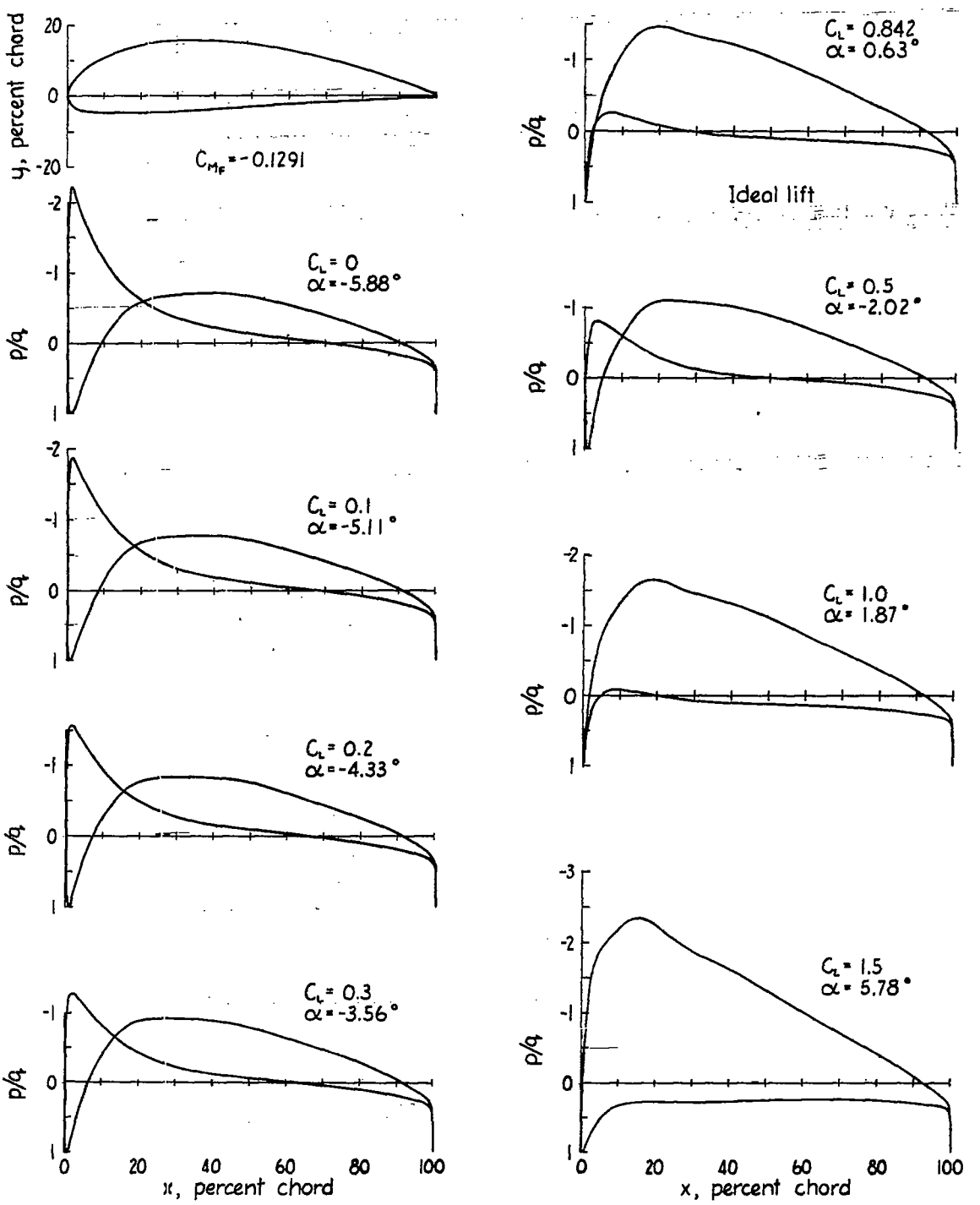

Figure 24.- Shape and pressure distribution for Clark Y-20 airfoil, CY(6.113)-20.

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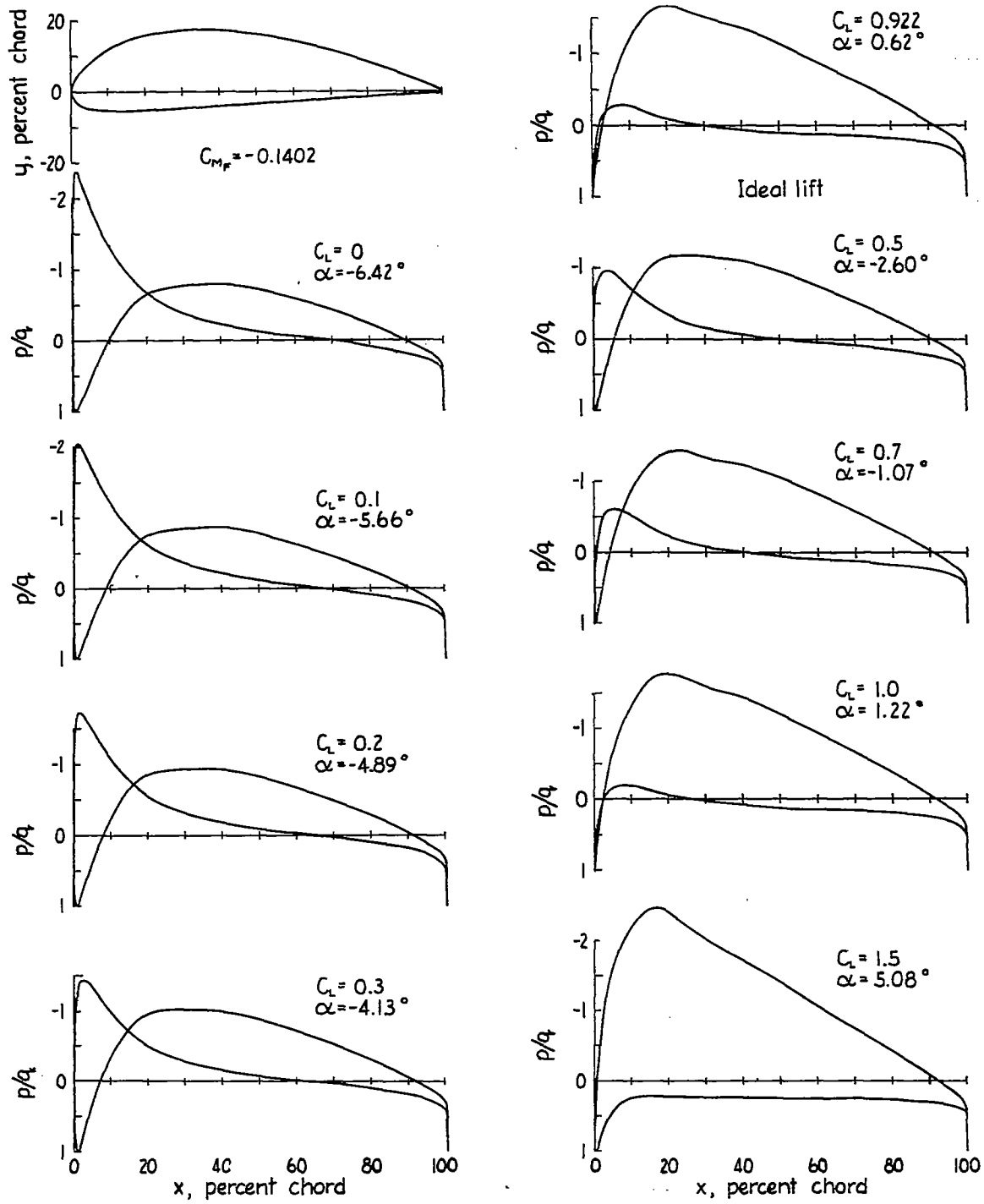

Figure 25.- Shape and pressure distribution for Clark Y-22 airfoil, CY(6.724)-22.

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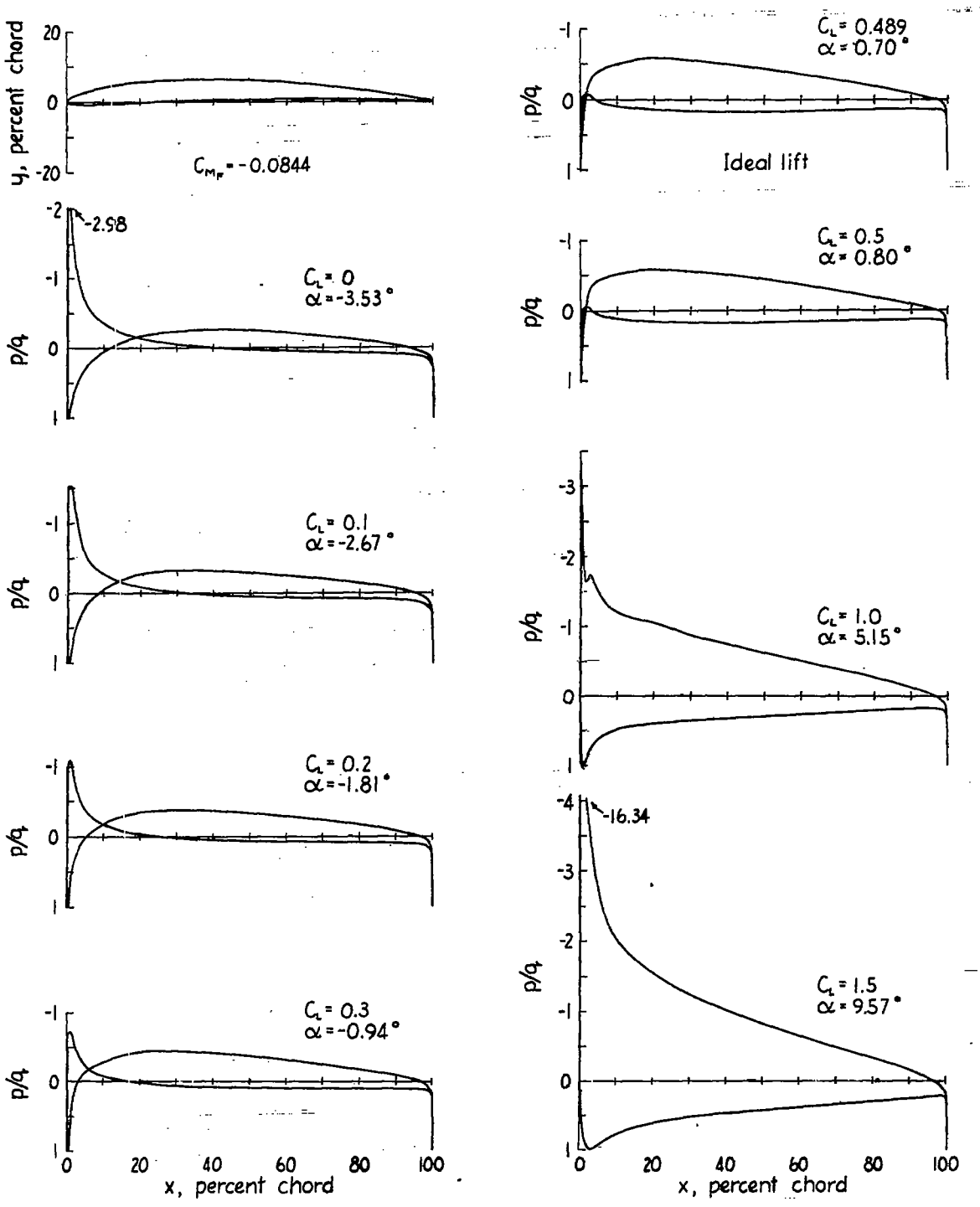

Figure 26.- Shape and pressure distribution for Clark Y M-6 airfoil, CY(3.576)-6.

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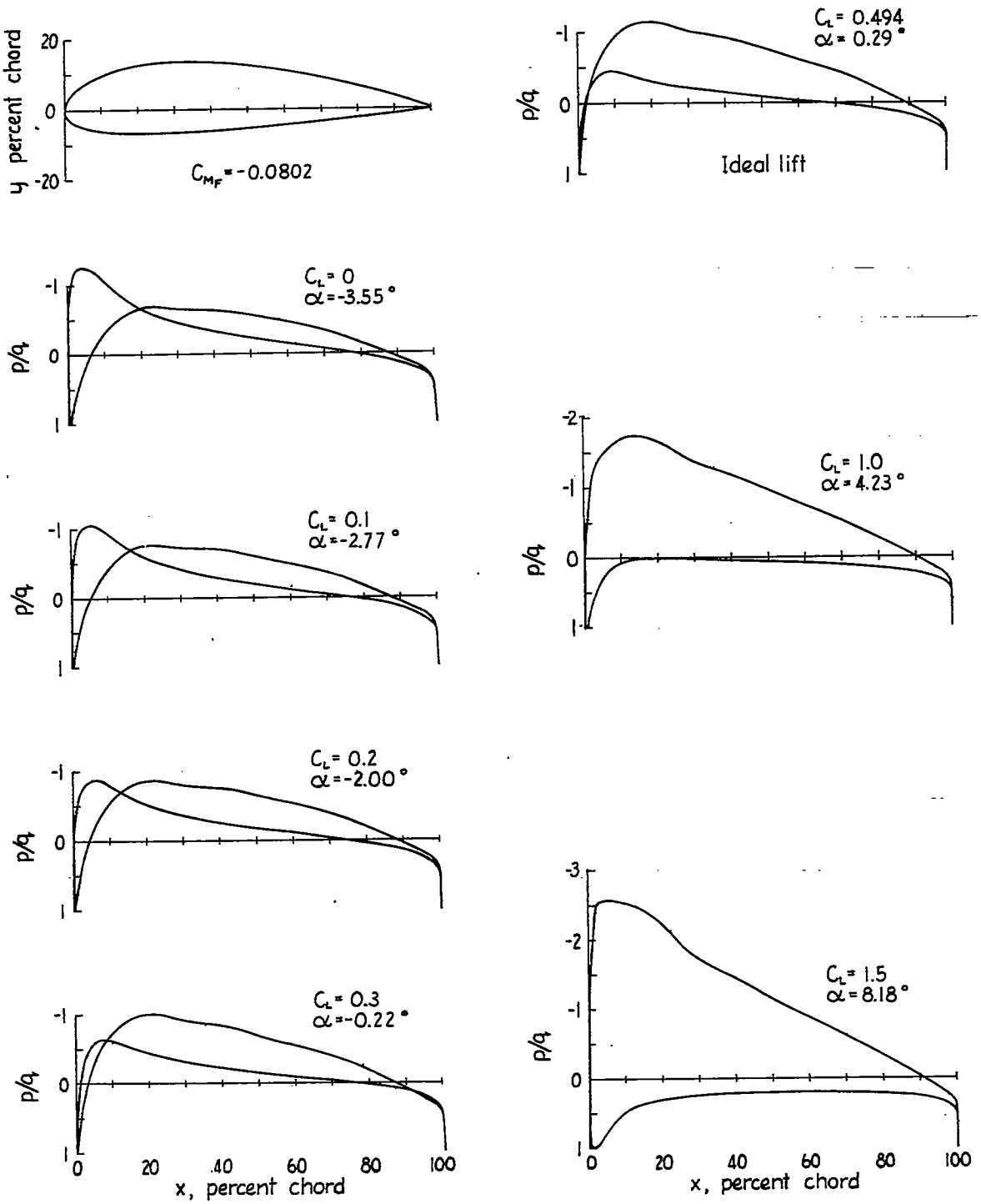

Figure 27.- Shape and pressure distribution for Clark Y M-20 airfoil, CY(3.576) - 20.

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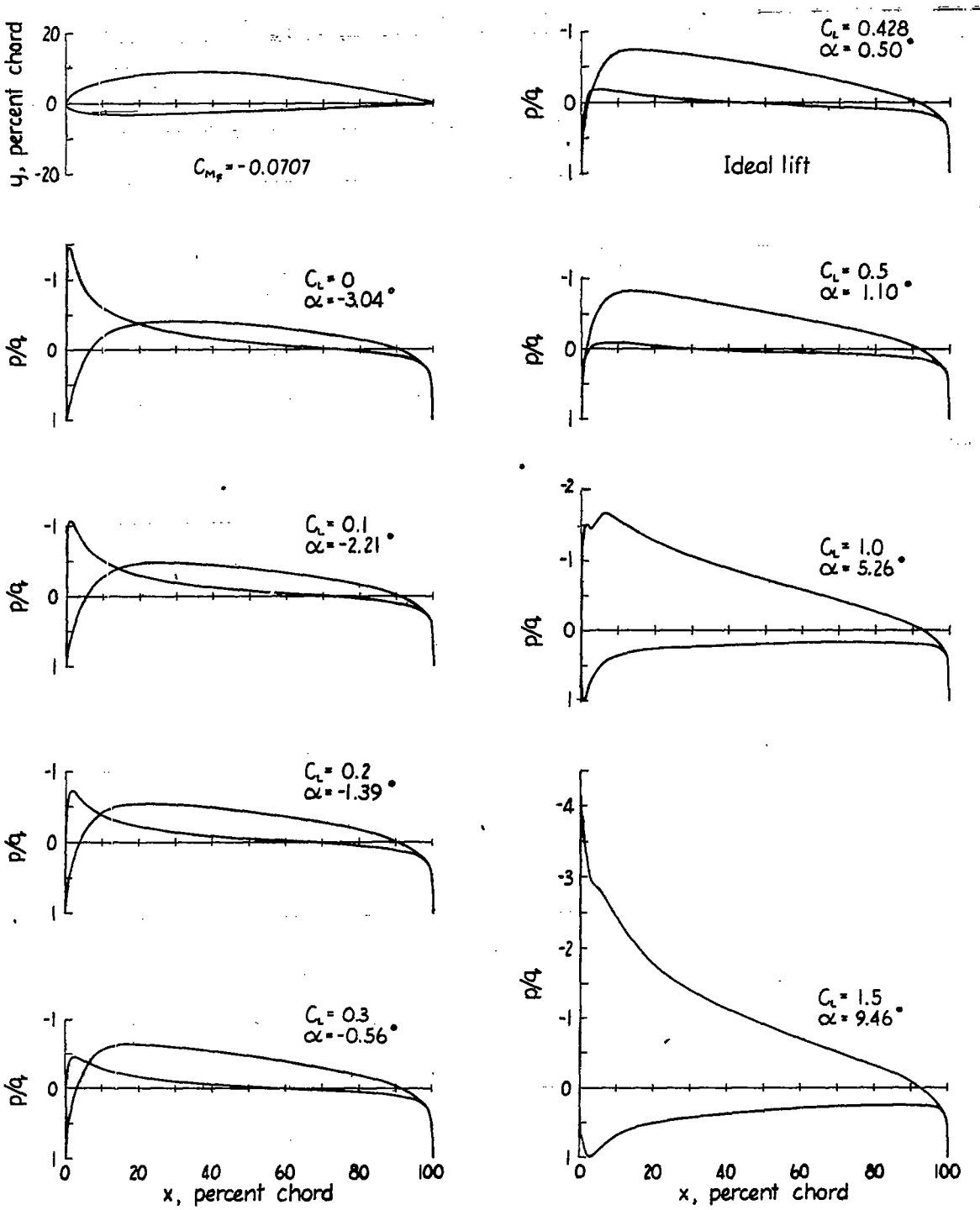

Figure 28.- Shape and pressure distribution for CY(3.056)-11.7 airfoil.

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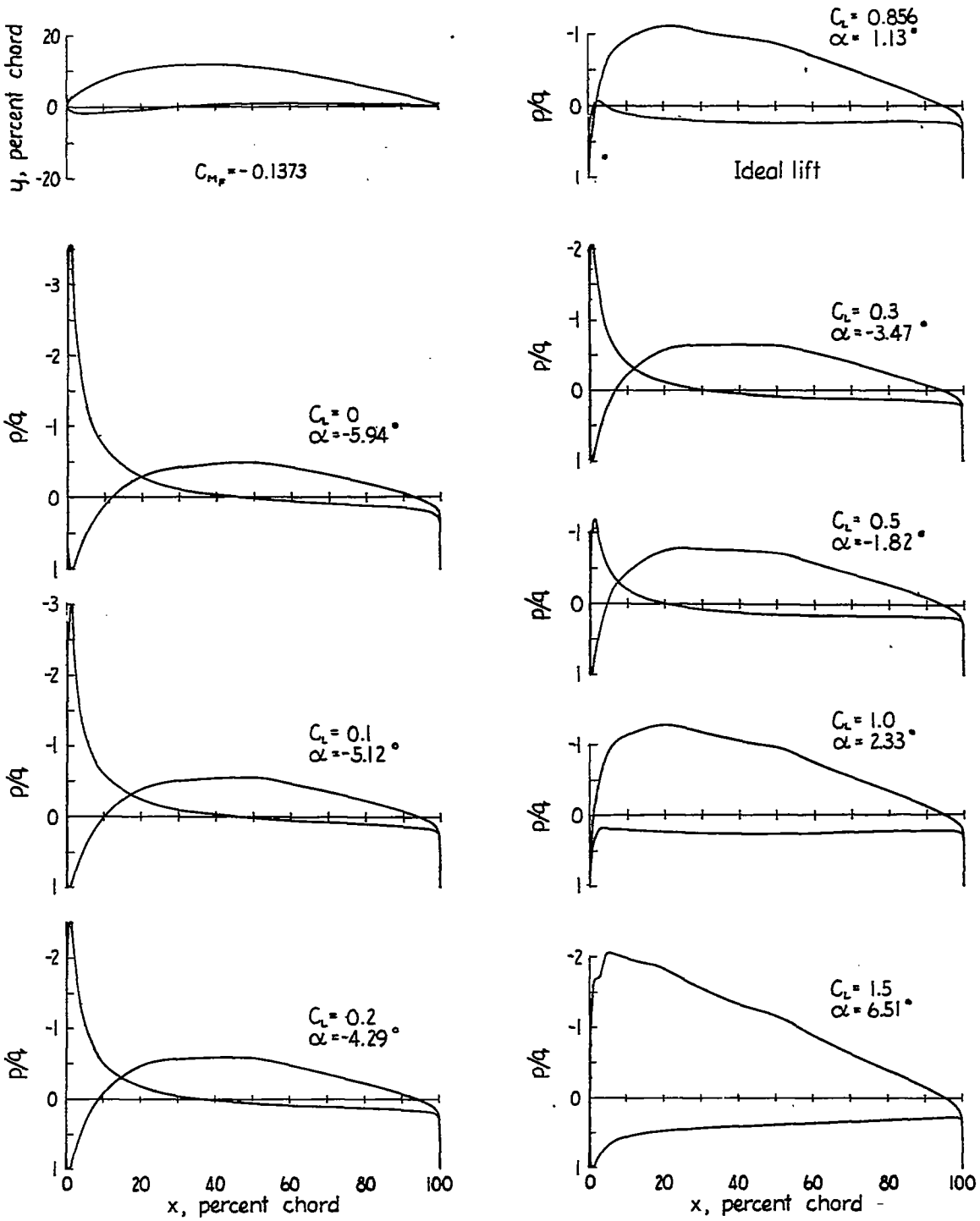

Figure 29.- Shape and pressure distribution for CY(6.113)-11.7 airfoil.

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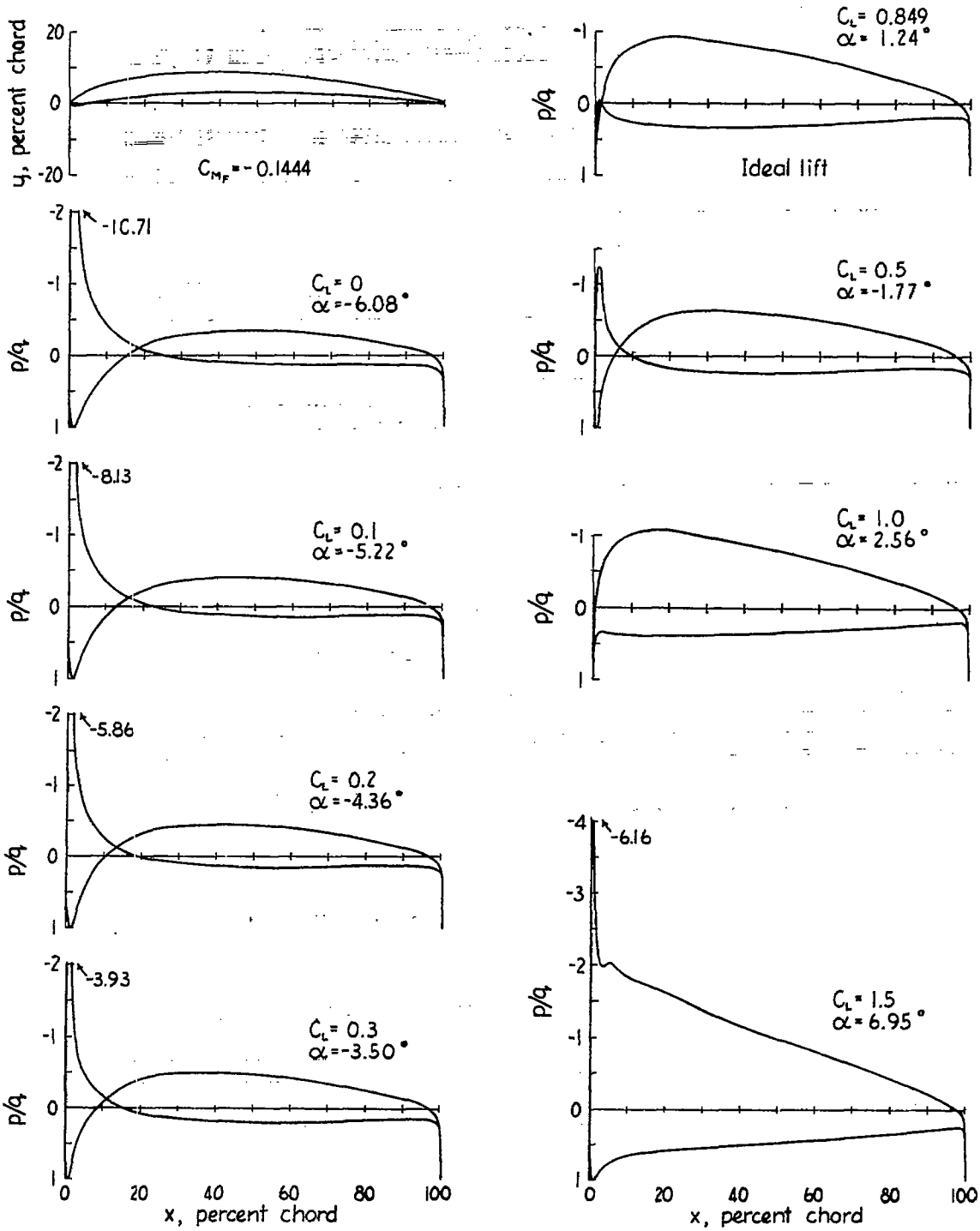

Figure 30.- Shape and pressure distribution for CY(6.113)-6 airfoil.

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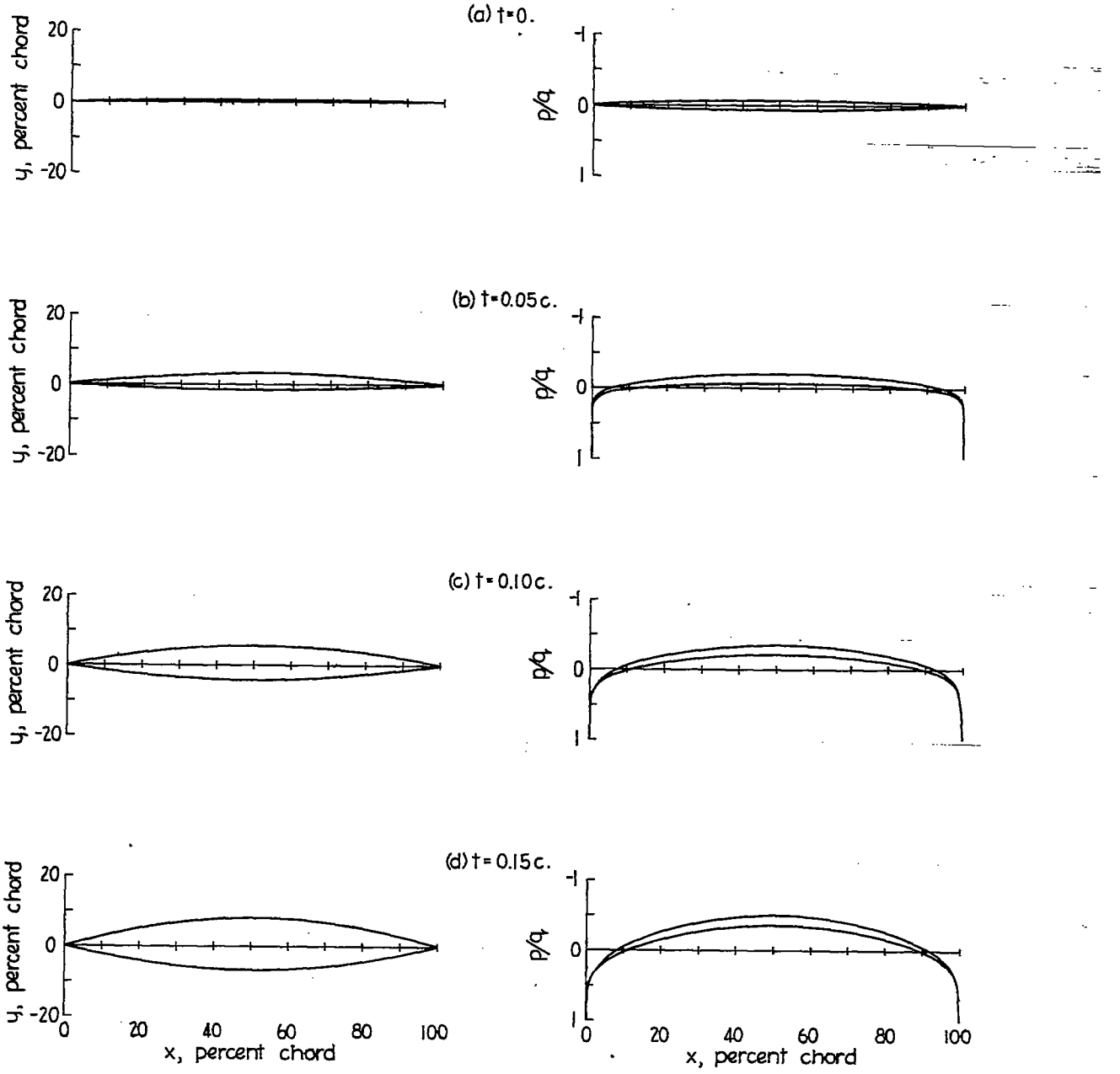

Figure 31.- Shape and pressure distribution for circular-arc airfoils of various thicknesses. $C_L=0.1$.

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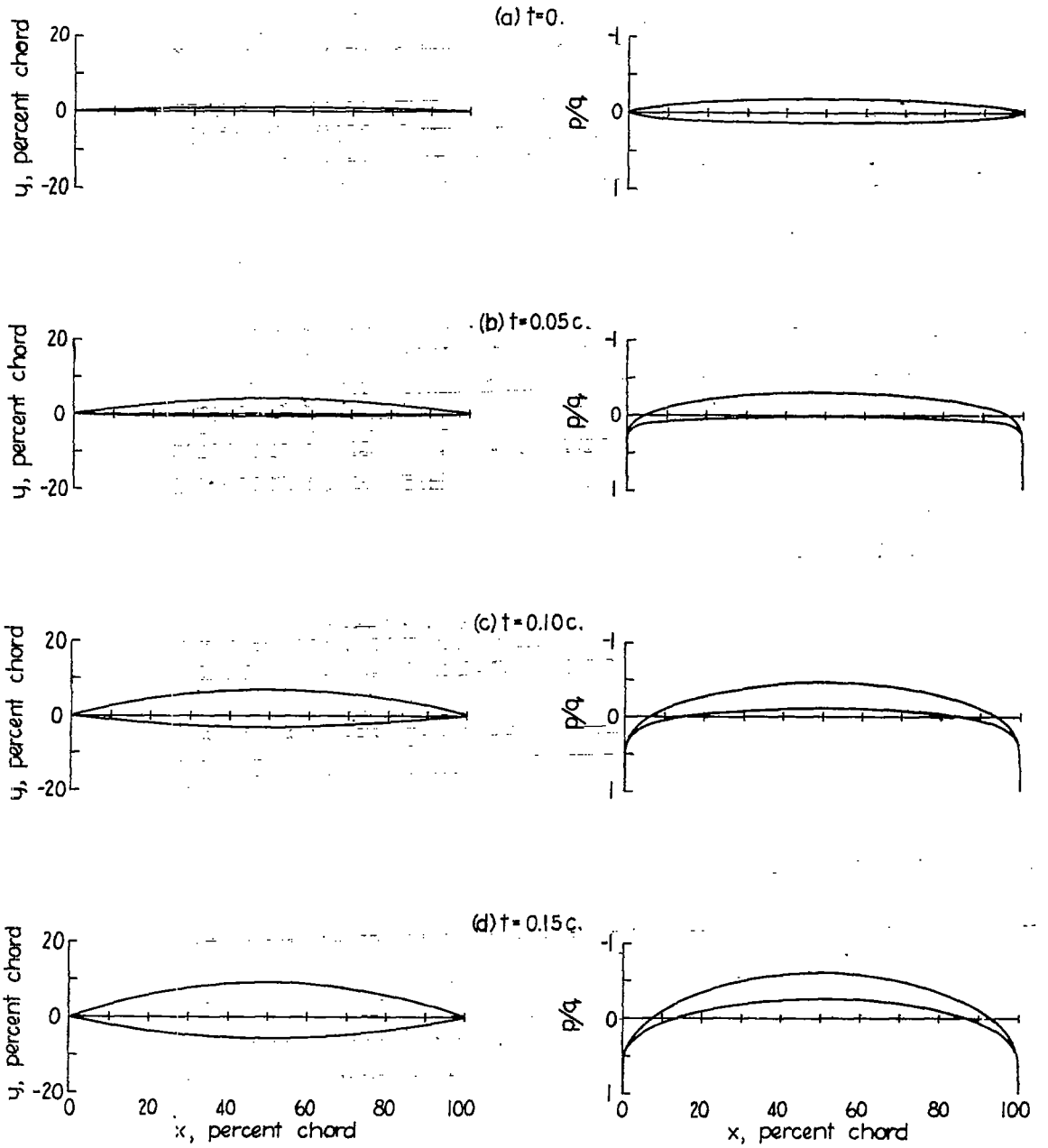

Figure 32.- Shape and pressure distribution for circular-arc airfoils of various thicknesses. $C_L = 0.25$.

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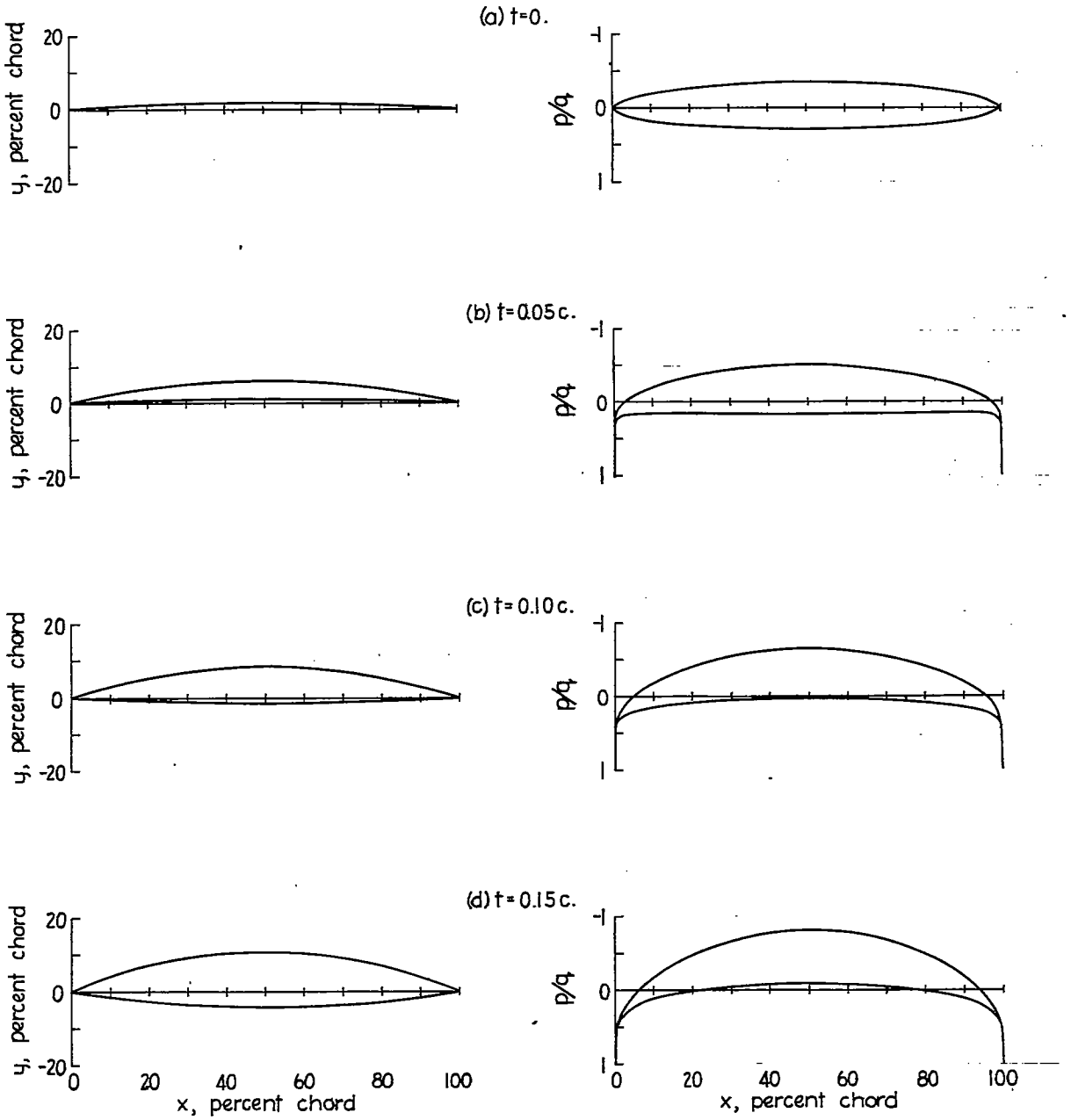

Figure 33.- Shape and pressure distribution for circular-arc airfoils of various thicknesses. $C_L = 0.5$.

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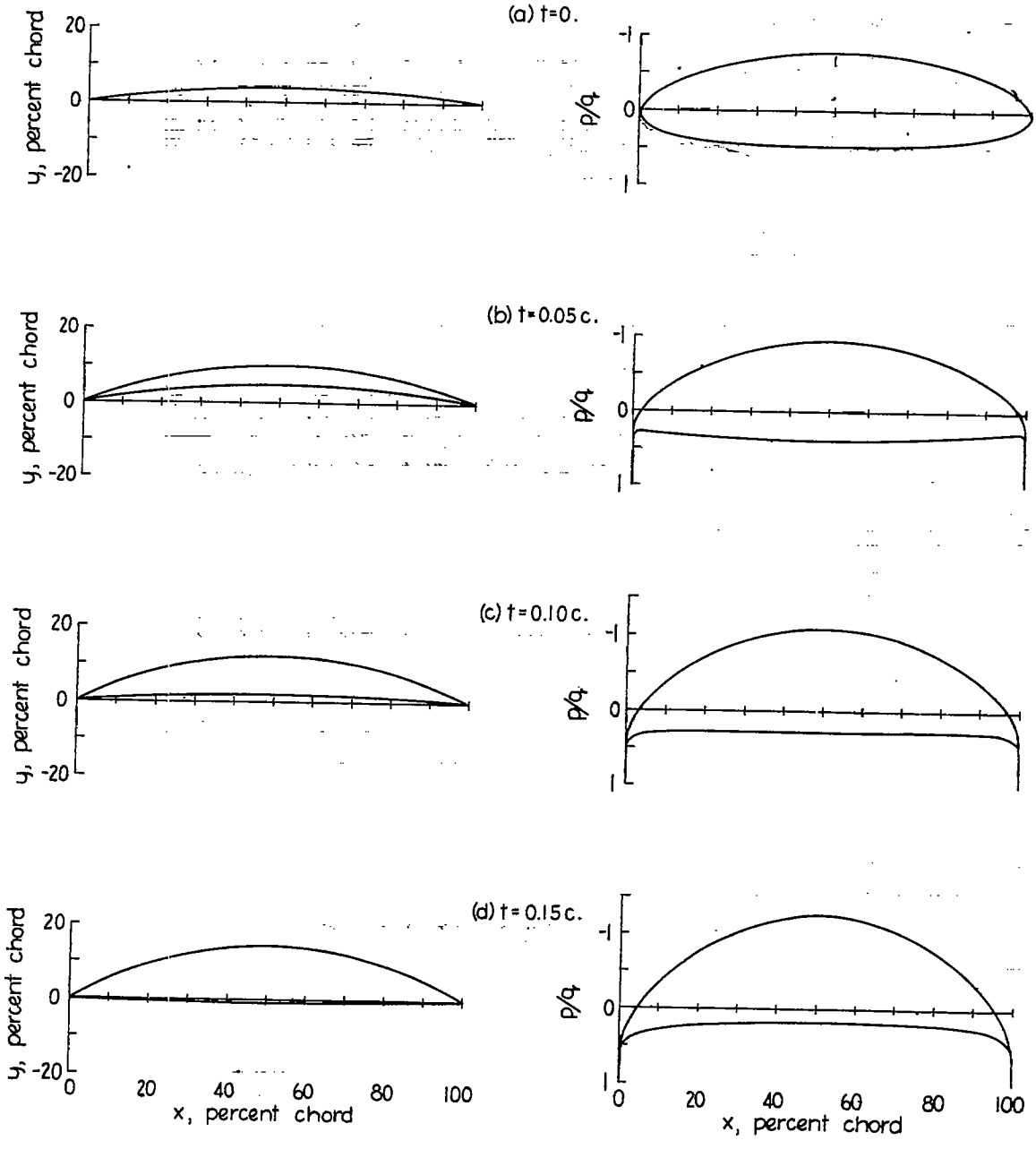

Figure 34.- Shape and pressure distribution for circular-arc airfoils of various thicknesses. $C_L = 1.0$.

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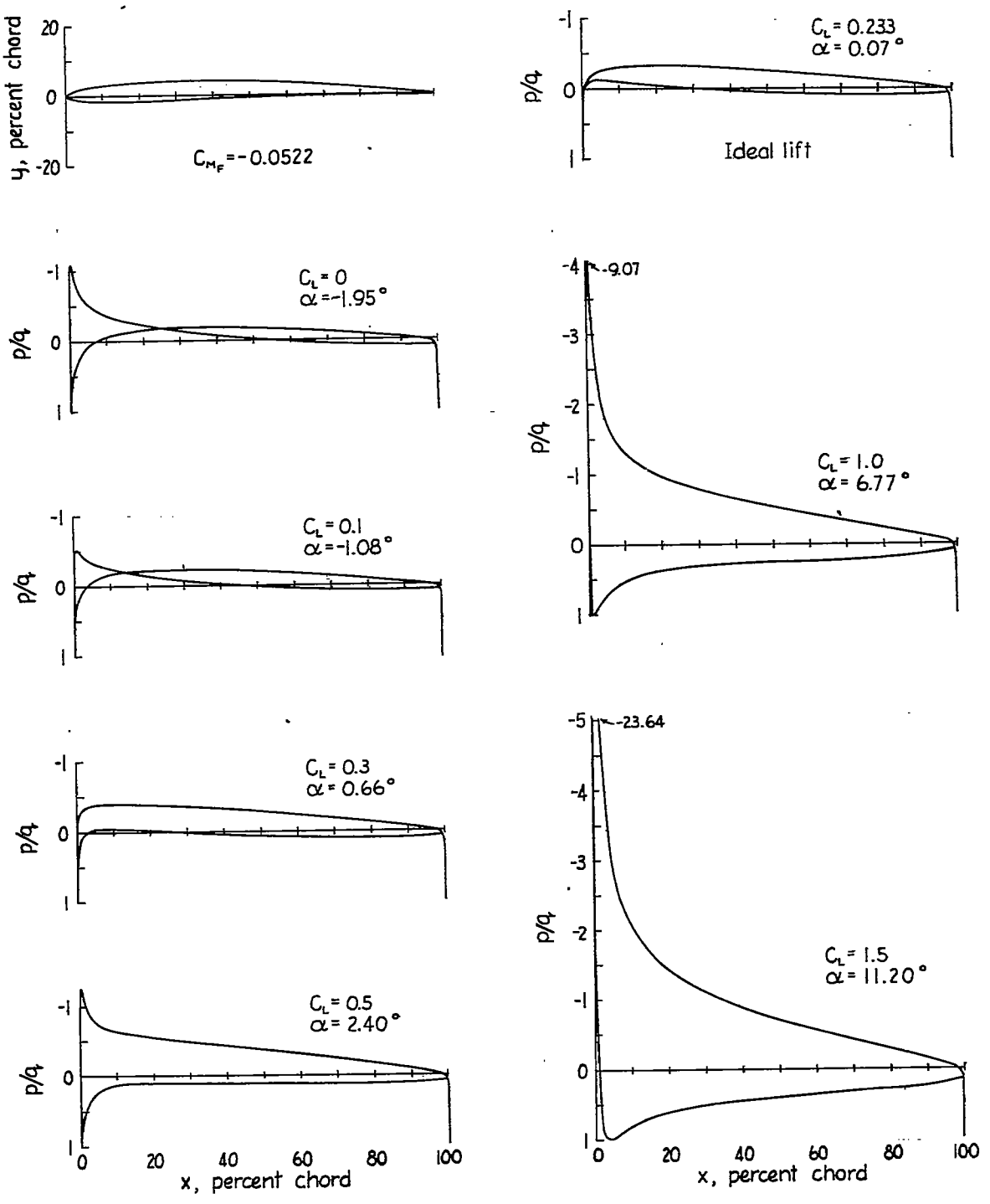

Figure 35.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.05 + 0.05 \cos(\varphi - 45°)$.

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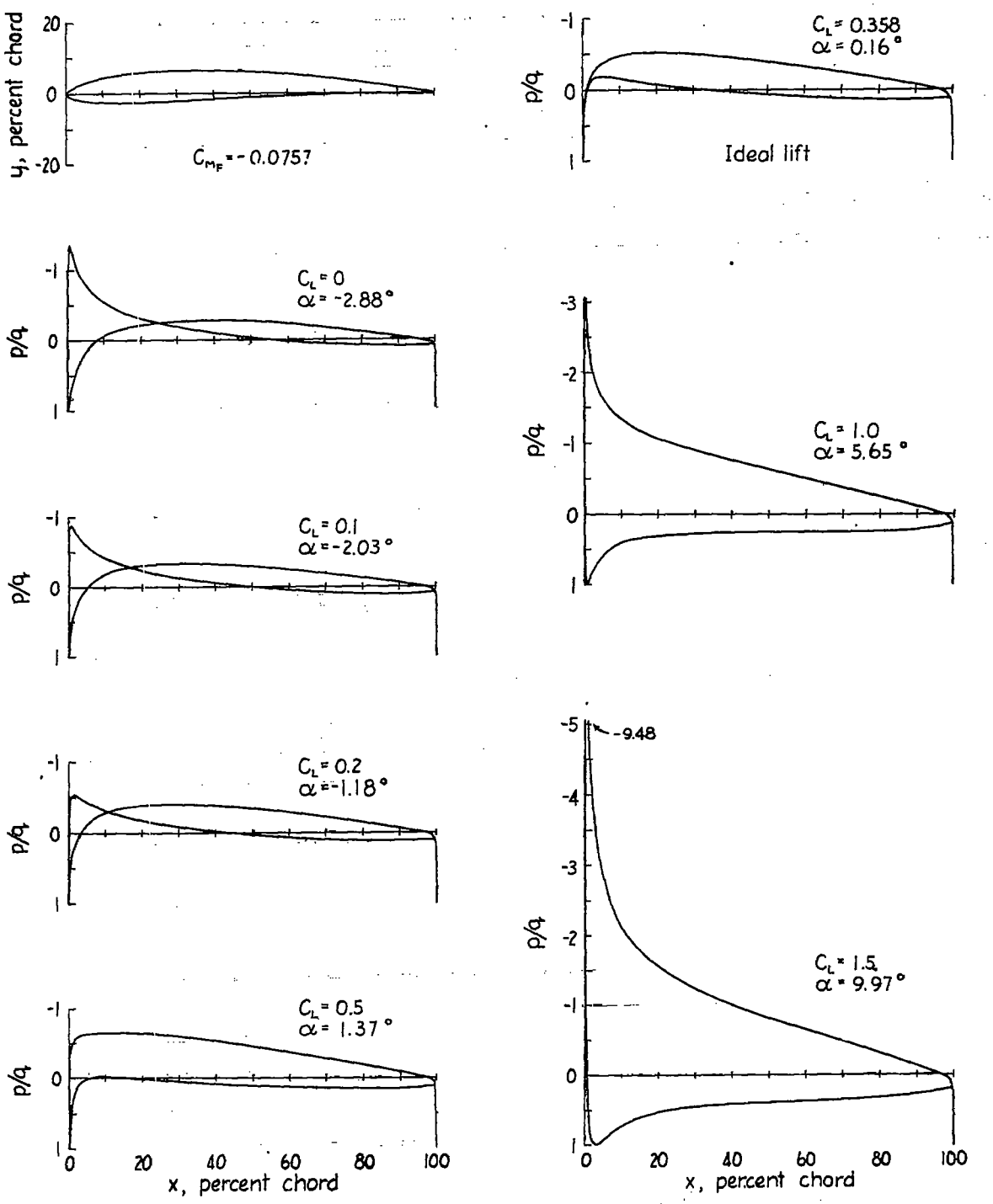

Figure 36.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.075 + 0.075 \cos(\varphi - 45°)$.

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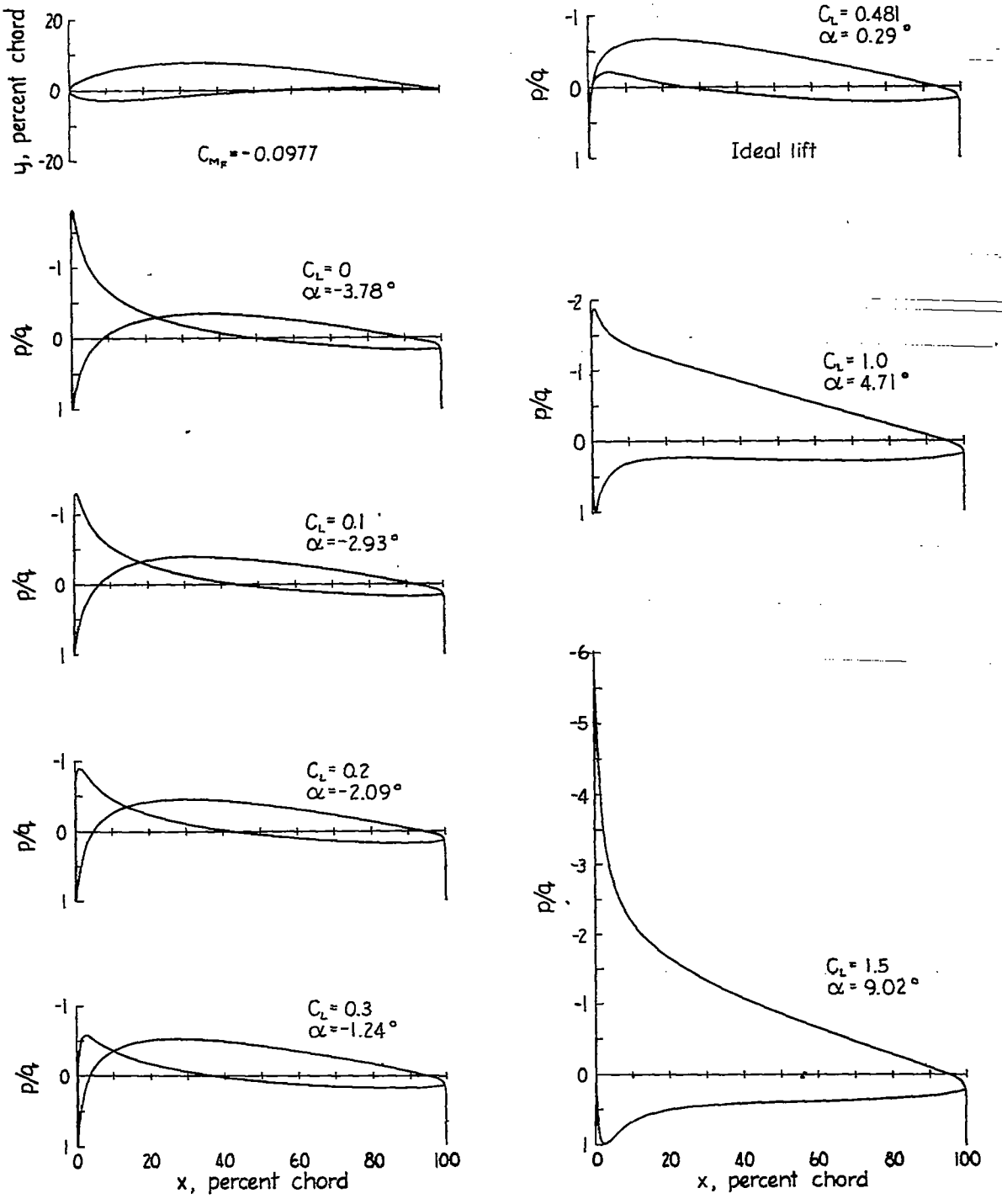

Figure 37.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.08 + 0.1 \cos(\varphi - 45°)$.

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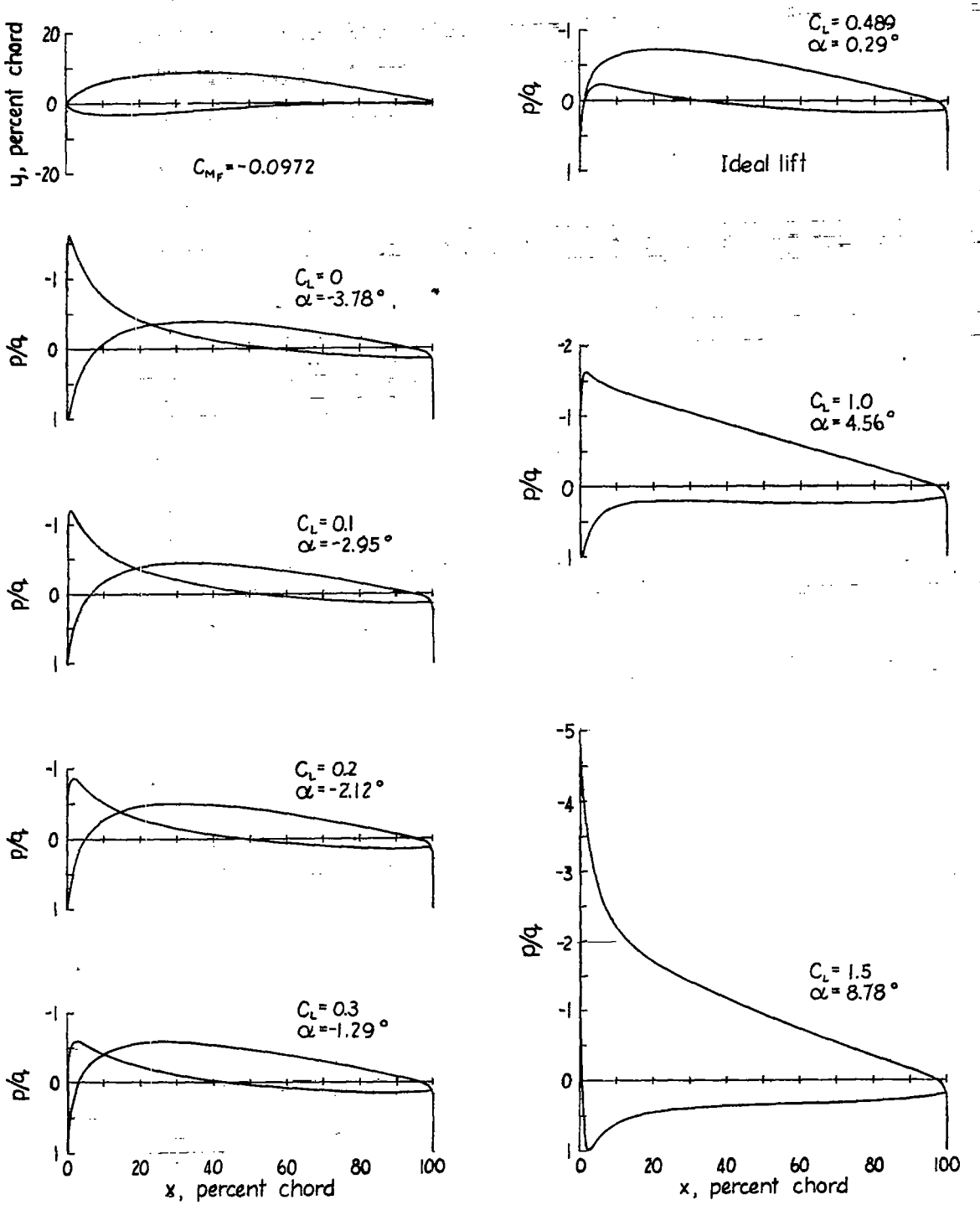

Figure 38.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.1 + 0.1 \cos(\varphi - 45°)$.

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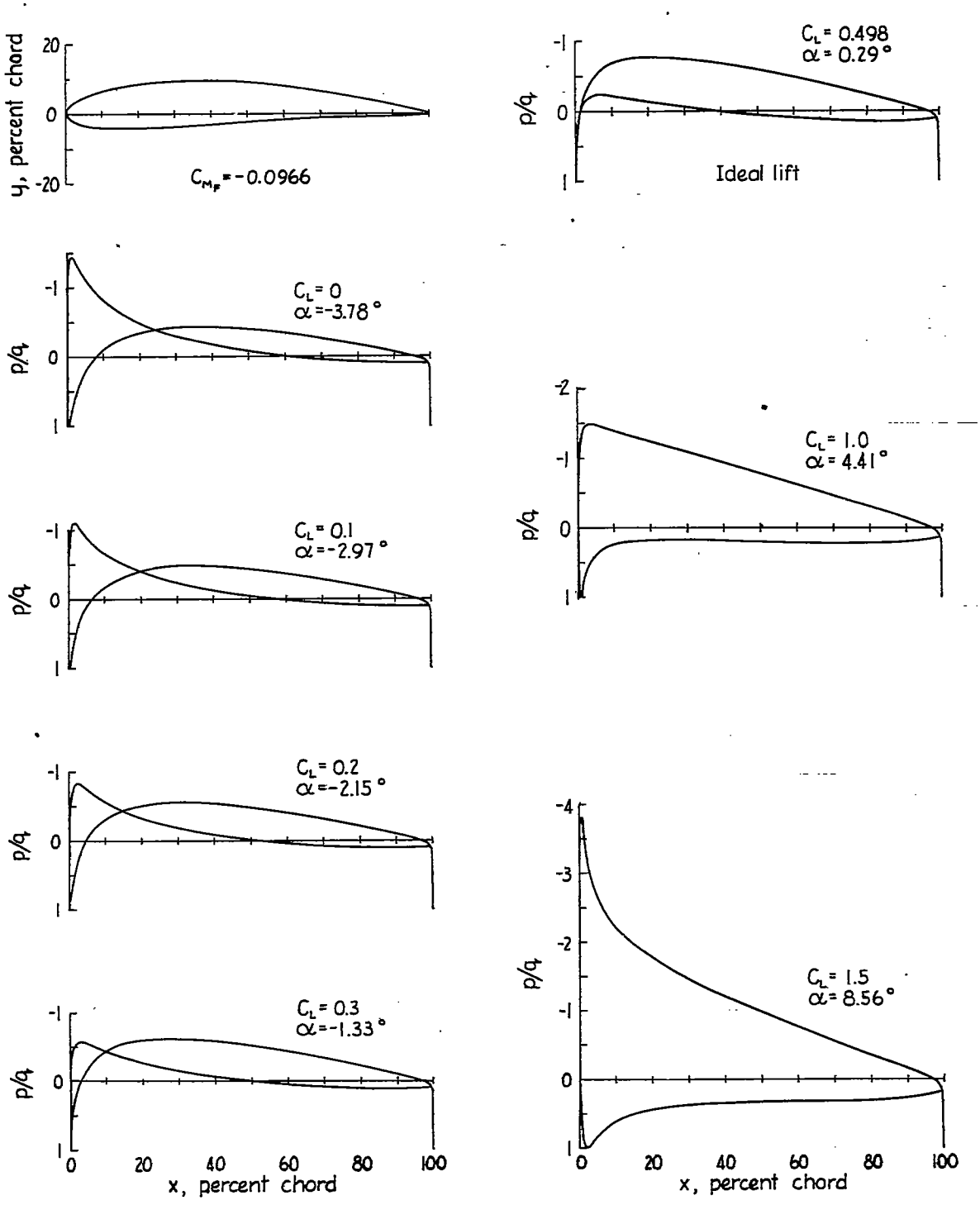

Figure 39.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.12 + 0.1 \cos(\varphi - 45°)$.

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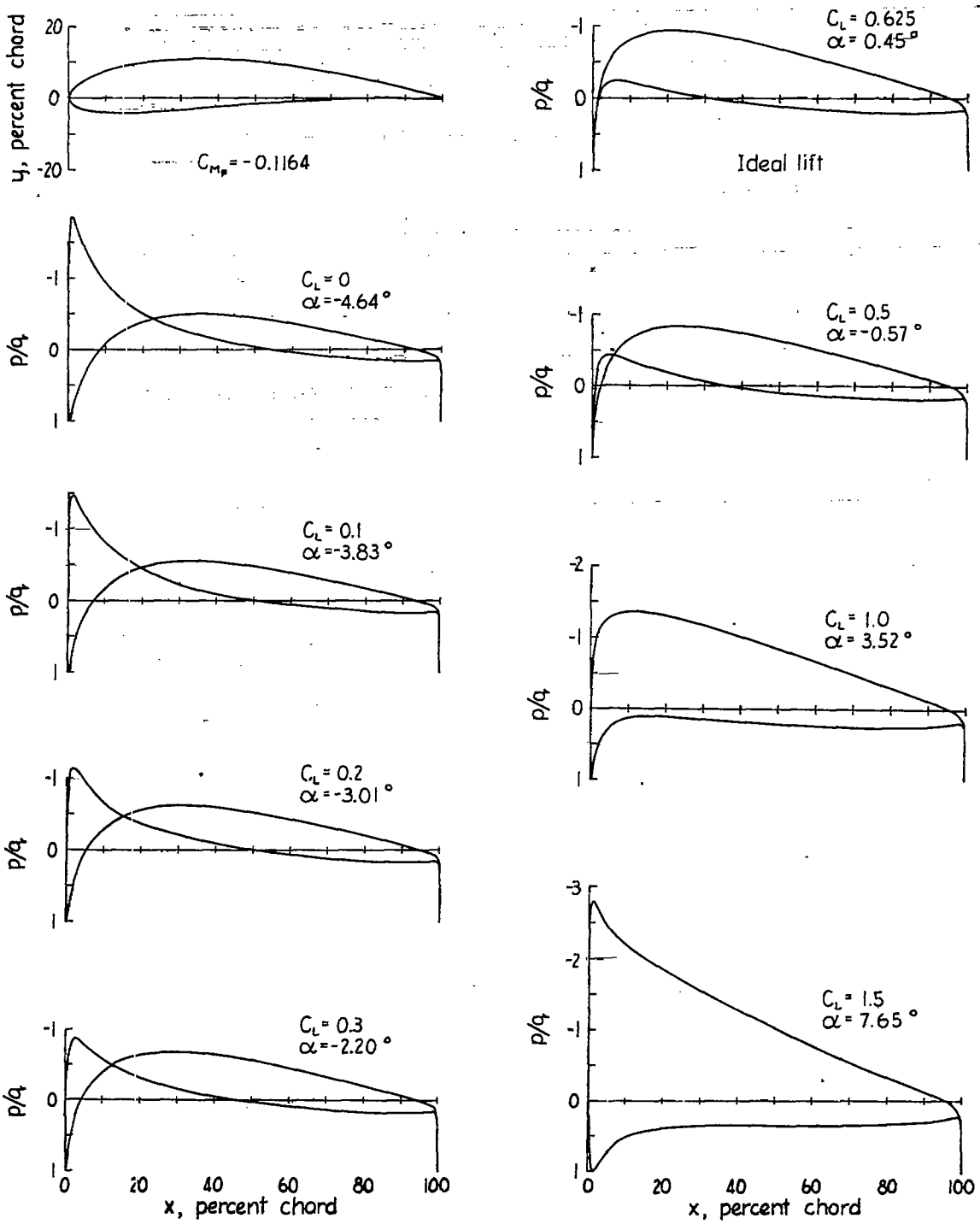

Figure 40.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.125 + 0.125 \cos(\varphi - 45°)$.

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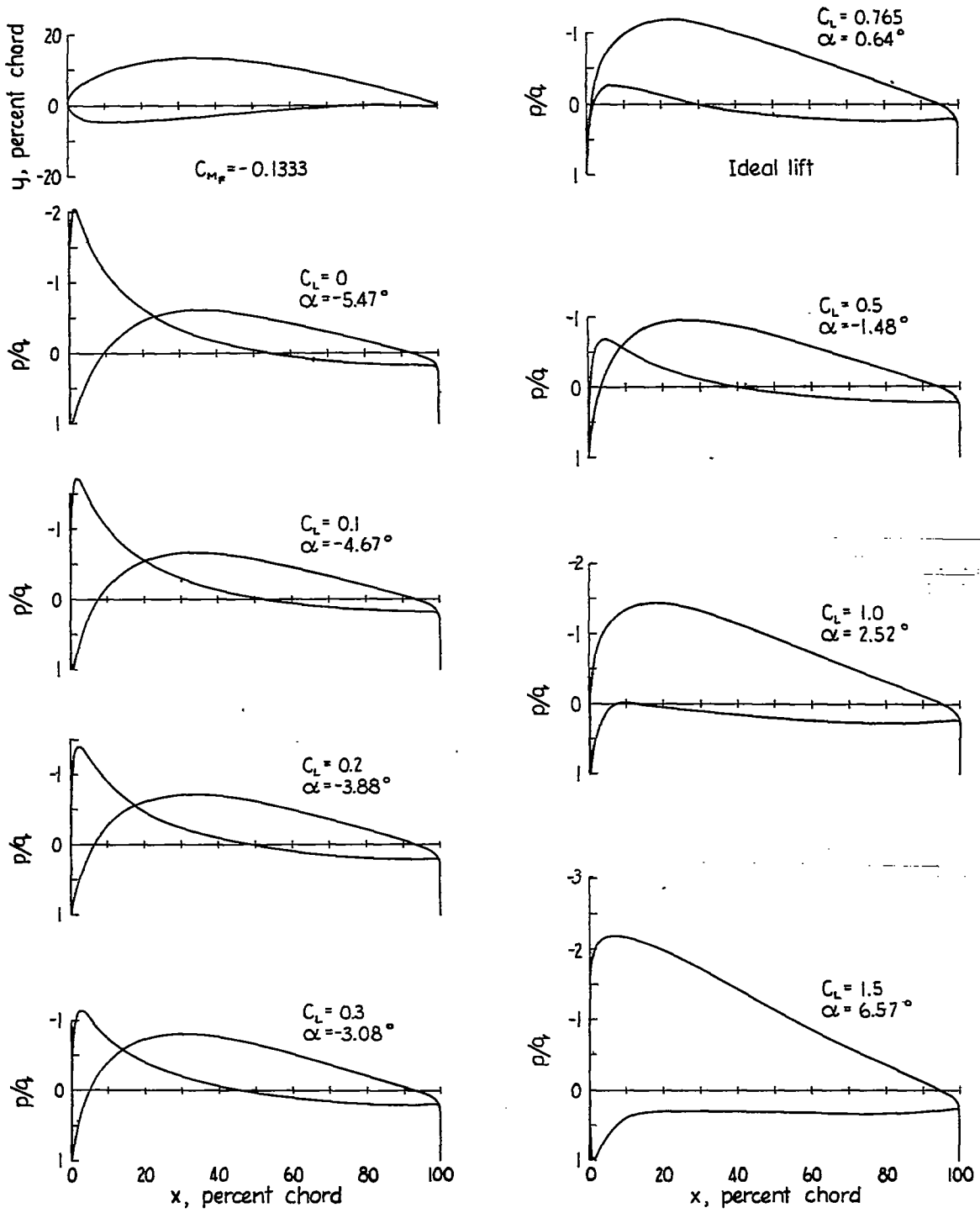

Figure 41.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.15 + 0.15 \cos(\varphi - 45°)$.

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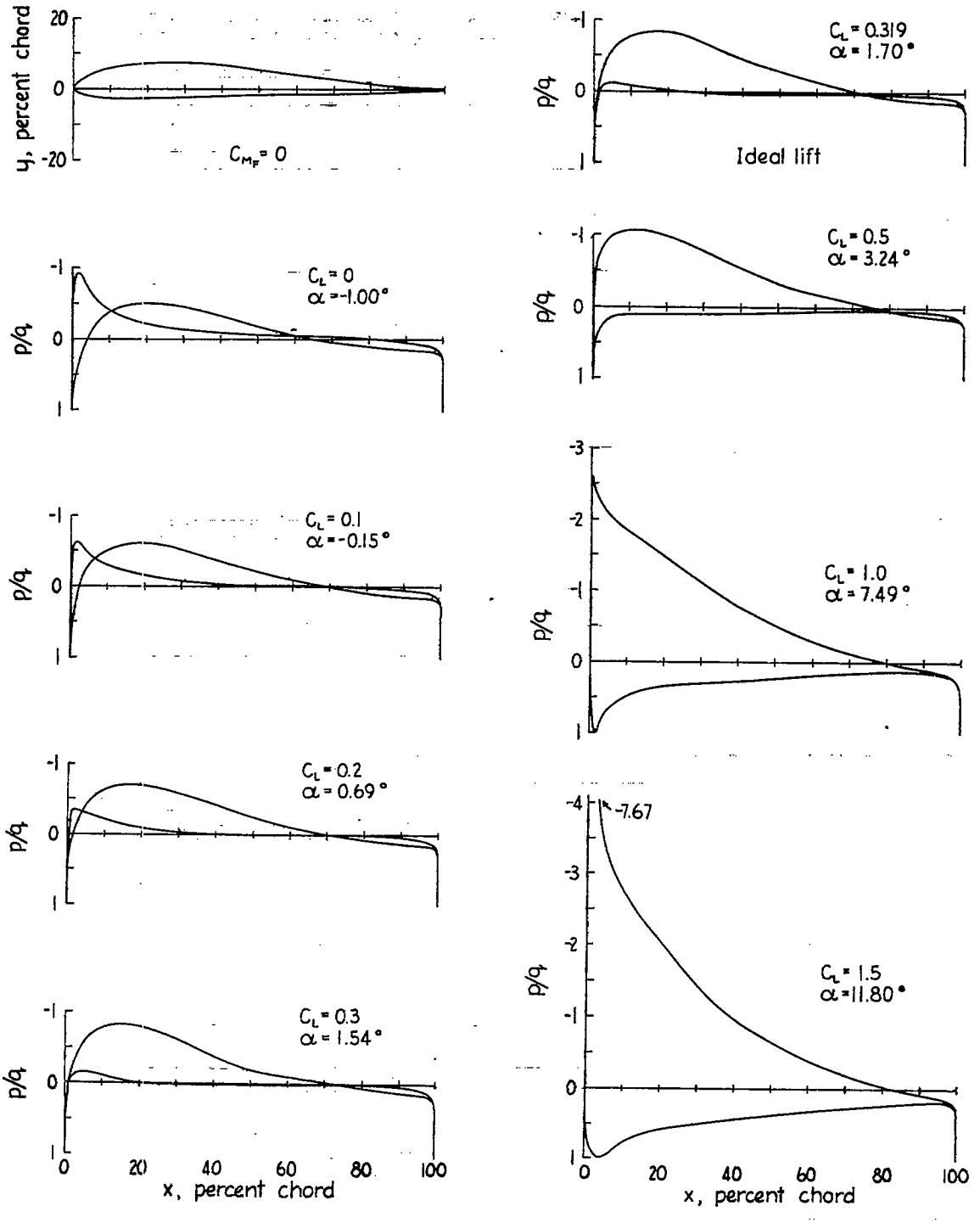

Figure 42.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.0804 + 0.09\cos(\varphi - 30°) + 0.0262\sin 2\varphi$.

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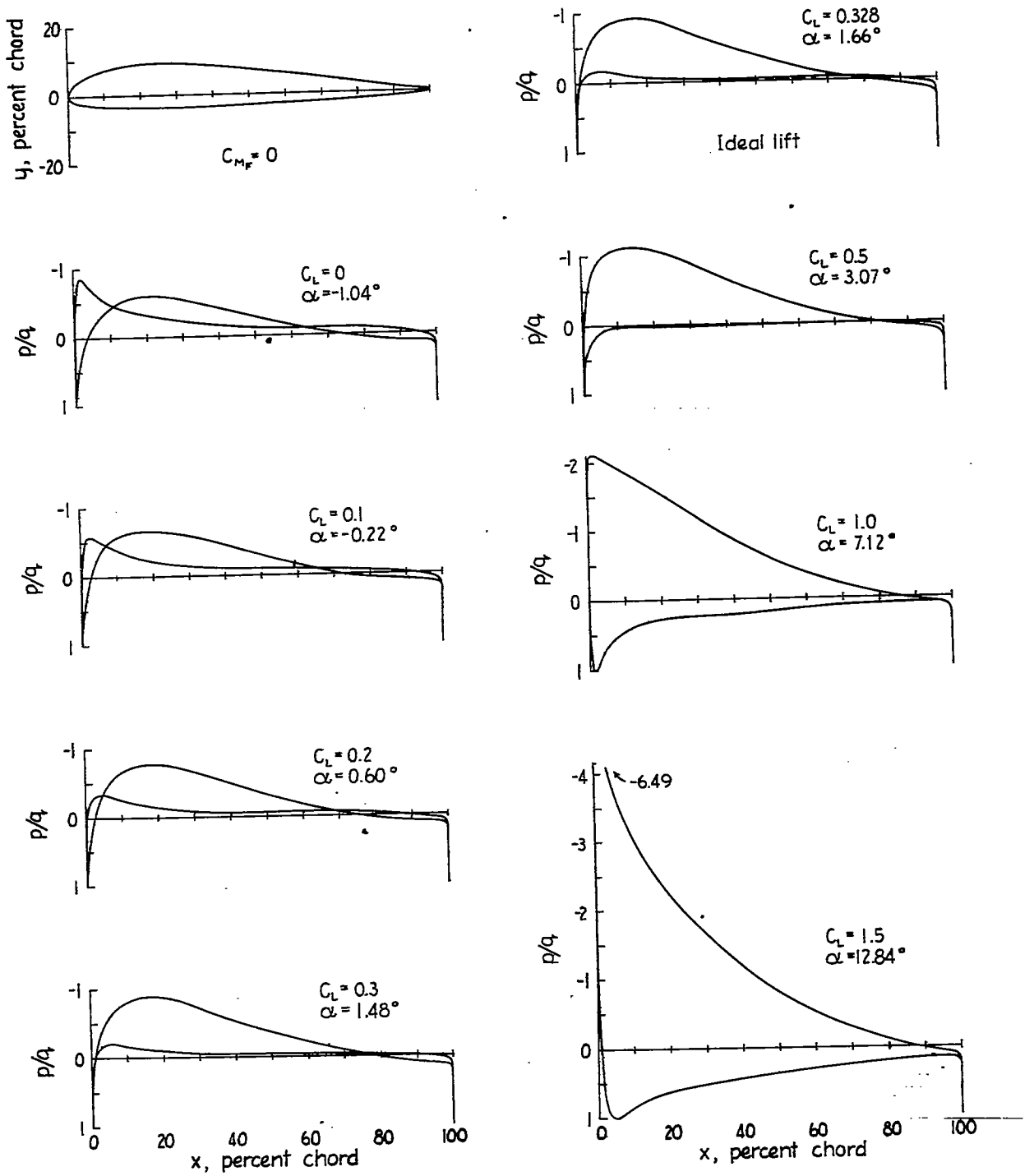

Figure 43.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.1122 + 0.09\cos(\varphi - 30°) + 0.0255\sin 2\varphi$.

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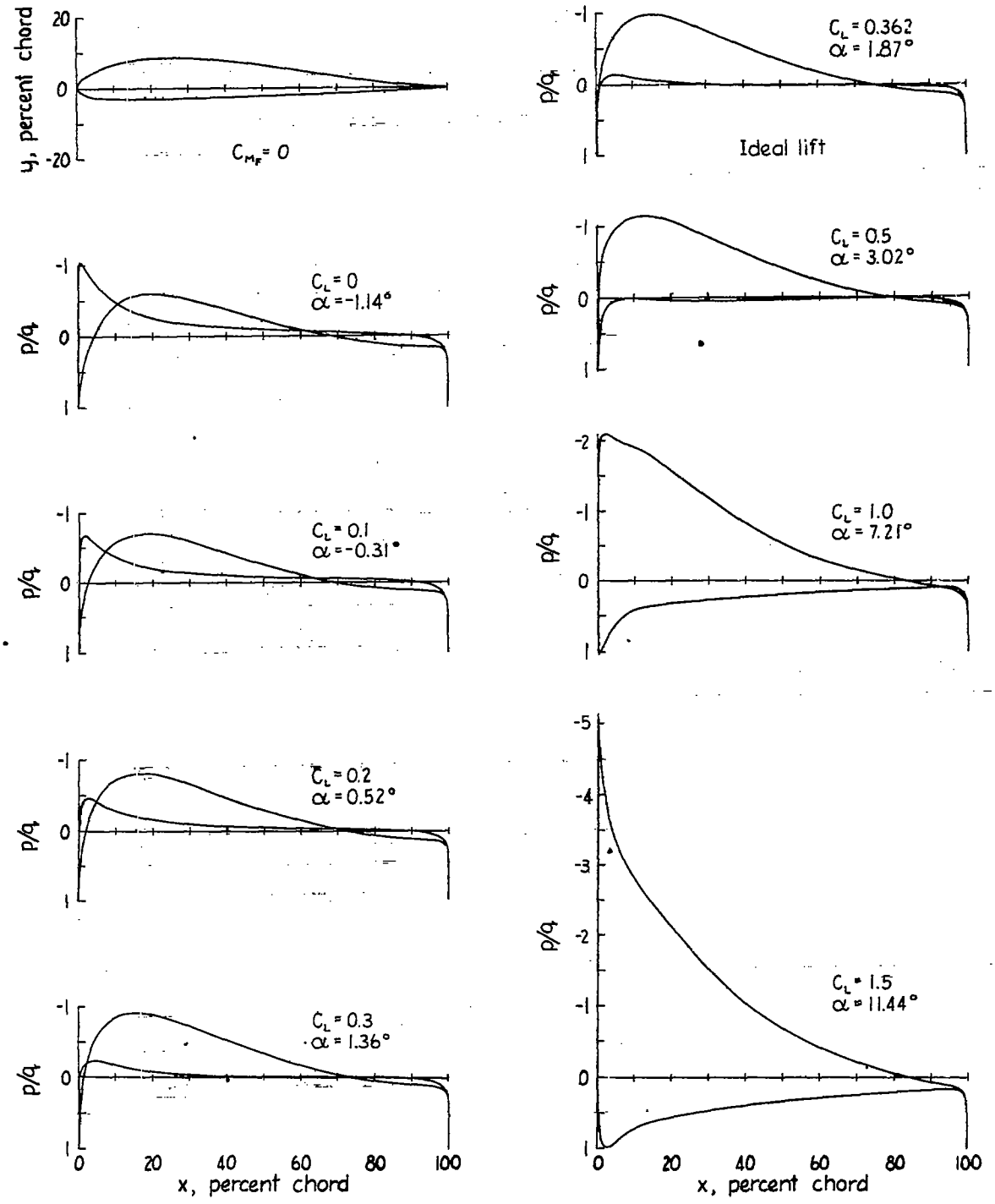

Figure 44.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.0996 + 0.1 \cos(\varphi - 30°) + 0.0284 \sin 2\varphi$.

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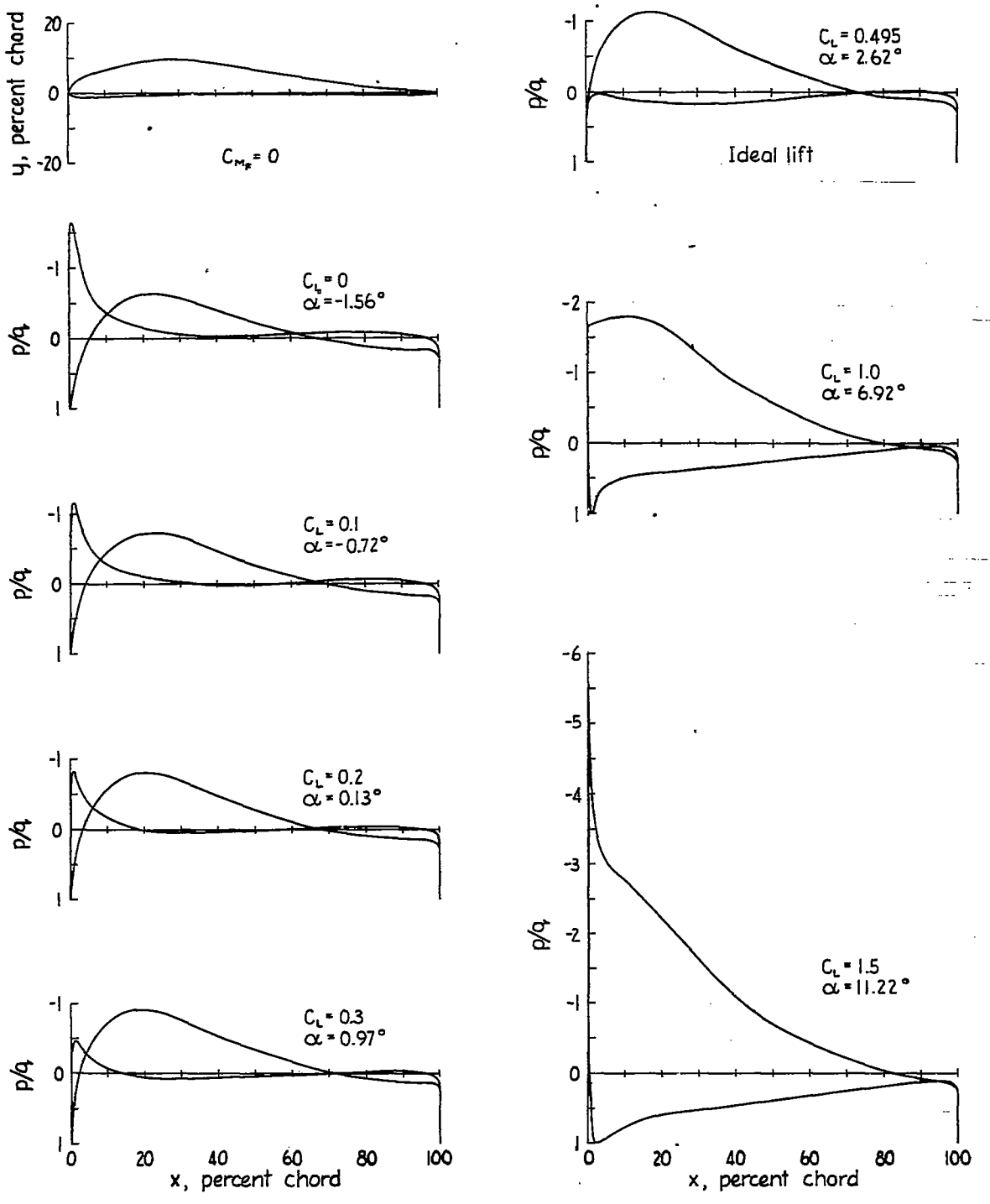

Figure 45.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.0808 + 0.1\cos(\varphi - 45°) + 0.0415\sin 2\varphi$.

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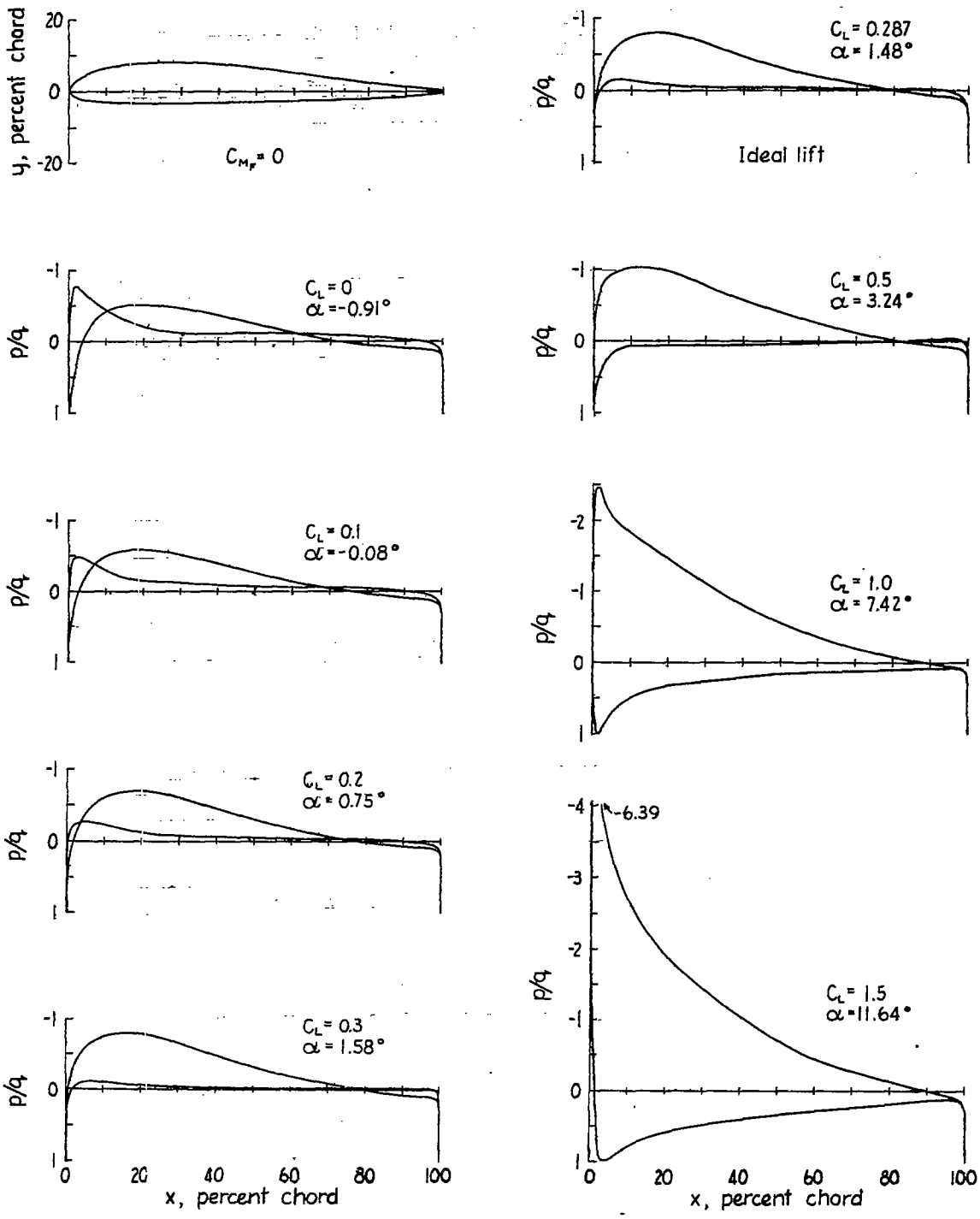

Figure 46.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.1012 + 0.08 \cos(\varphi - 30°) + 0.0231 \sin 2\varphi$.

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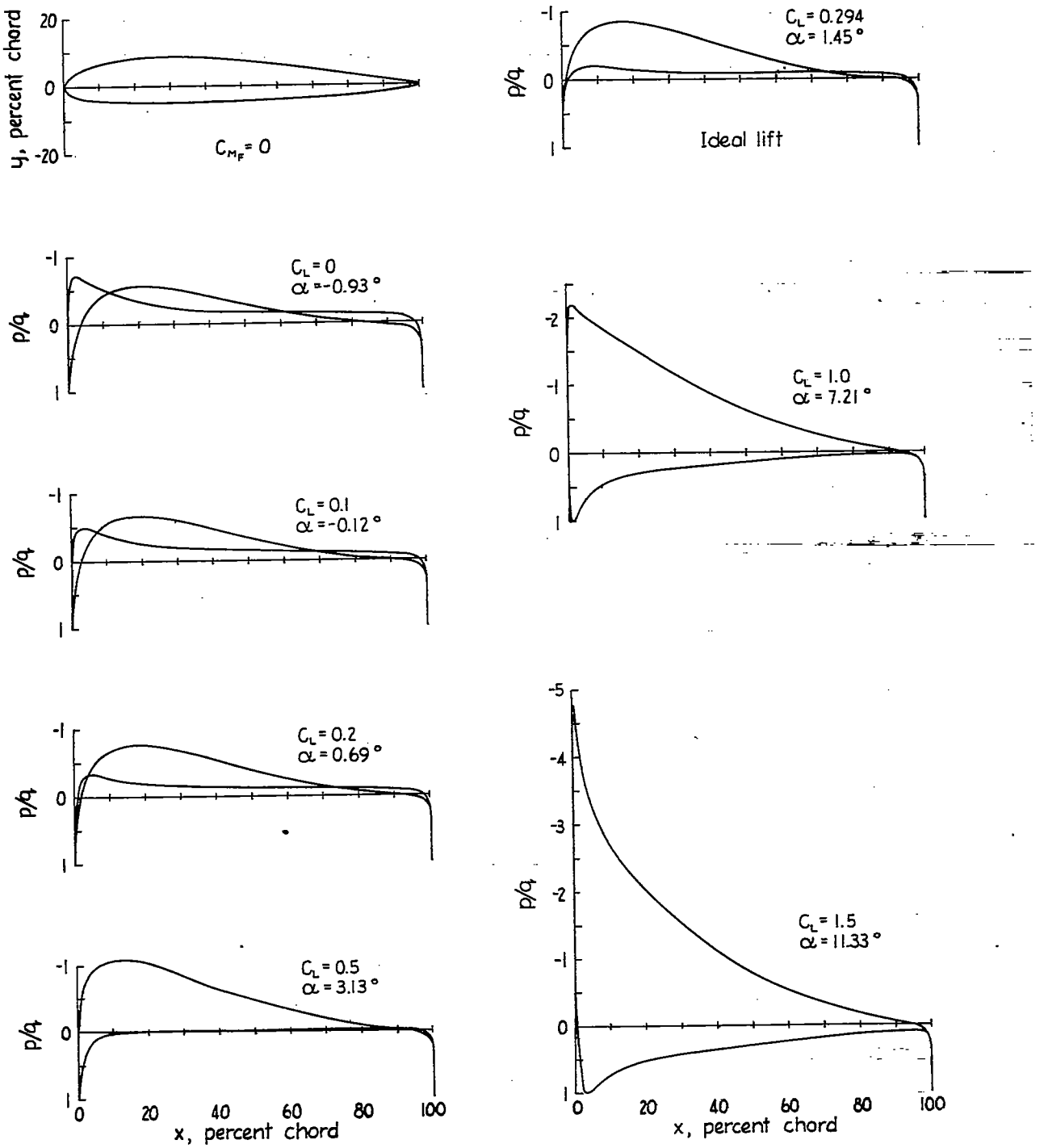

Figure 47.- Shape and pressure distribution for synthetic airfoil;
$\psi = 0.1226 + 0.08 \cos(\varphi - 30°) + 0.0226 \sin 2\varphi$.

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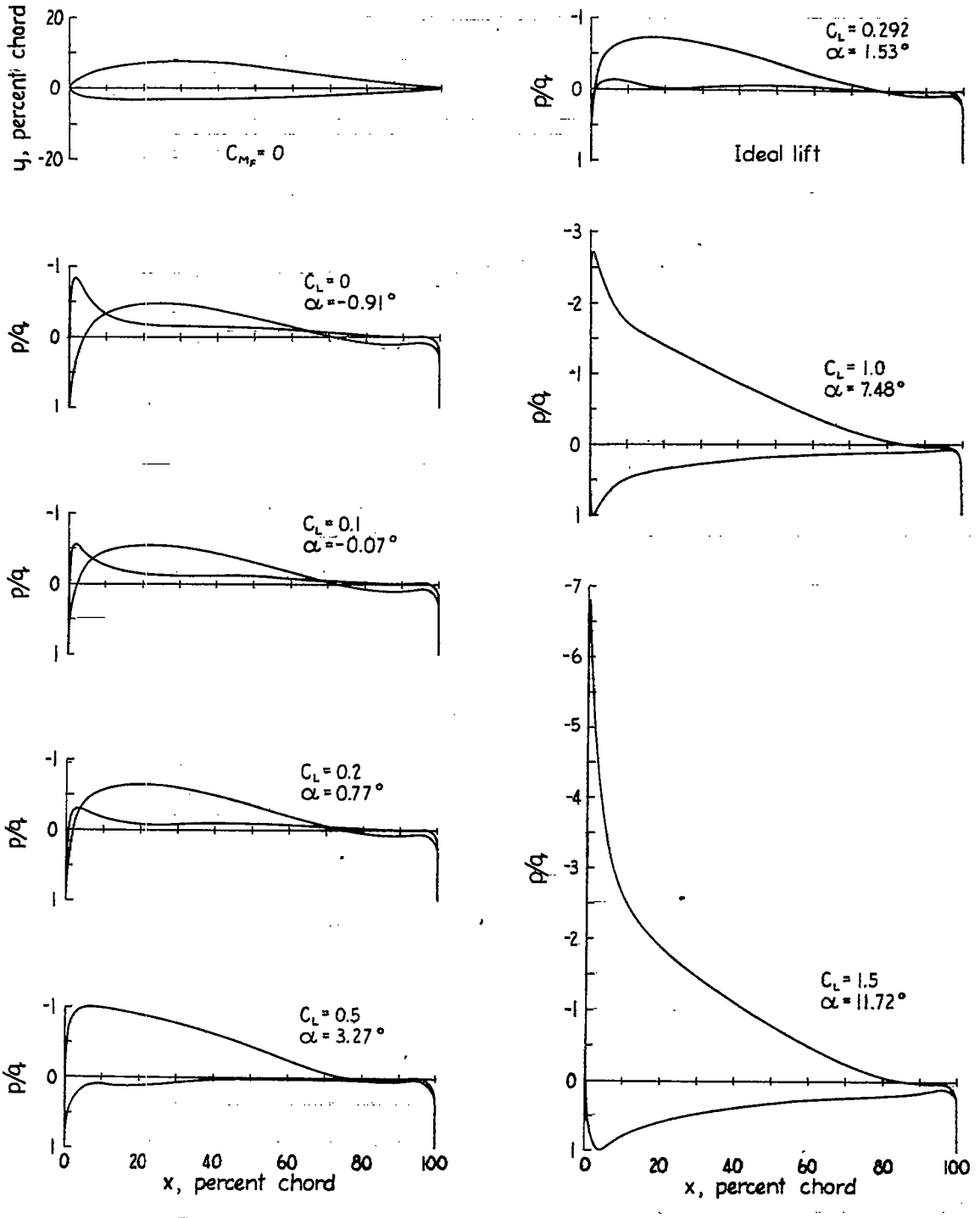

Figure 48.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.0948 + 0.08\cos(\varphi - 30°) + 0.0234\sin 2\varphi + 0.005\cos 4\varphi$.

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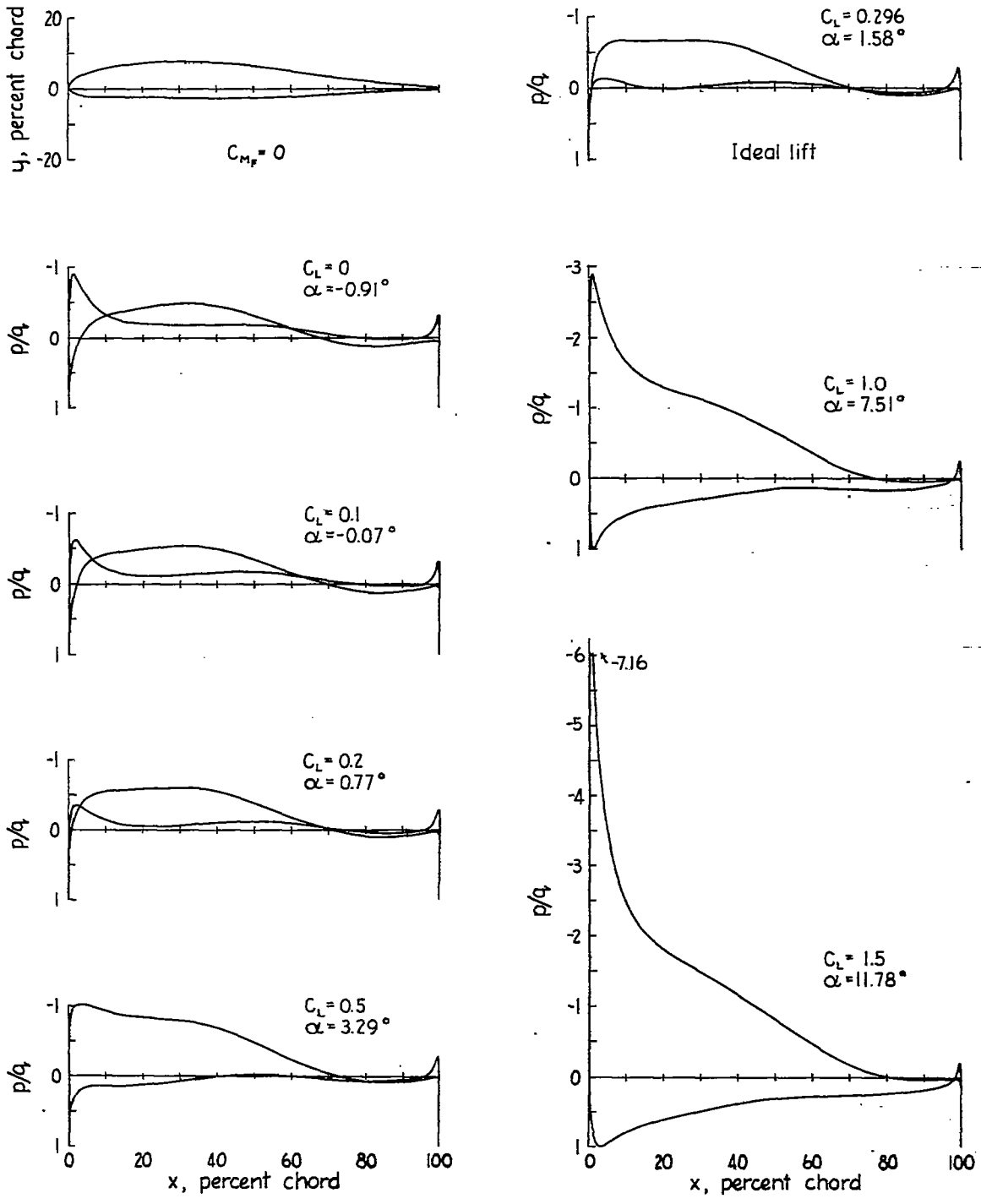

Figure 49.- Shape and pressure distribution for synthetic airfoil; $\psi = 0.0897 + 0.08\cos(\varphi - 30°) + 0.0237\sin 2\varphi + 0.01\cos 4\varphi$.

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