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## QUANTITATIVE TREATMENT OF THE CREEP OF METALS BY DISLOCATION AND RATE-PROCESS THEORIES

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# QUANTITATIVE TREATMENT OF THE CREEP OF METALS BY DISLOCATION AND RATE-PROCESS THEORIES

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## SUMMARY

An equation for the steady-state rate of creep has been derived by applying the theory of dislocations to the creep of pure metals. The form of this equation is in good agreement with empirical equations describing creep rates. The theory was also used to predict the dependence of steady-state rate of creep on physical constants of the material and good agreement was obtained with data in the literature for pure annealed metals. The rate of creep was found to decrease with increasing modulus of rigidity. This fact suggests that one of the requirements for a heat-resisting alloy is that its matrix be a metal that has a high modulus of rigidity and therefore a high modulus of elasticity.

## INTRODUCTION

The development of the gas turbine in combination with the jet as a power plant for military aircraft has focused attention on the need for heat-resisting alloys. One of the criteria used to evaluate heat-resisting alloys is creep resistance, that is, the resistance to plastic deformation over a period of time. Current evaluation of creep resistance is accomplished by creep tests in which creep curves (elongation plotted against time) are obtained at constant stress and temperature. A typical creep curve is shown in figure 1. The initial stage, in which the slope of the curve or rate of creep is rapidly decreasing, is commonly designated the primary or transient stage; secondary or steady-state creep refers to the straight-line portion of the creep curve. After a sufficient length of time, the rate of creep increases in the region designated the tertiary stage. In certain cases, the rate of creep continuously increases and cannot be divided into these three stages (reference 1(a)). The steady-state rate of creep is generally the criterion by which the creep resistance of heat-resisting alloys is expressed. Because this rate rapidly increases with temperature, creep becomes an important factor in limiting safe-operating temperatures.

One of the first attempts to analyze steady-state creep by other than empirical methods was by Kanter (reference 2) in 1938. Further work of a fundamental nature was carried out by Kauzmann (reference 3) who applied Eyring's theory of liquid flow, which is a special application of the theory of rate processes, to the steady-state flow of metals. The problem of creep has also been attacked by the use of the theory of dislocations. A survey of results obtained thus far by this approach is reported by Seitz and Read (reference 4). Although these three theoretical treatments predict equations for the steady-state rate of creep that showed the correct dependence on stress and temperature, quantitative predictions cannot be made from them as to how physical and structural constants of materials affect the creep rate. These treatments therefore are not of great value in leading to the synthesis of new materials that will have better creep properties than those currently used for heat-resisting purposes.

In order to determine the dependence of steady-state creep rate on physical constants of materials, the investigation reported herein, which continues a fundamental approach to the problem of creep, was conducted at the NACA Cleveland laboratory in the early part of 1945. An equation for the steady-state rate of creep as a function of applied stress and temperature is derived by applying Eyring's theory of rate processes and the theory of dislocations to the problem. This investigation is part of a program being conducted at the Cleveland laboratory to evaluate the physical properties of heat-resisting alloys in terms of physical constants, which are either known or easily measurable, in order to minimize the number of tests as well as to make possible the prediction of new compositions and structures for better heat-resisting alloys than those currently used.

## SYMBOLS

The following symbols are used in the theoretical analysis:

$d_1$ distance between atoms in slip direction

$d_2$ interplanar spacing (fig. 4(a))

$E$ modulus of elasticity

$F(T)$ function of temperature and material

$\Delta F_a$ free energy of activation per molecule

$f$ fraction whose value is about 1/2

$G$ modulus of rigidity at any temperature

$G_0$ modulus of rigidity at absolute zero

$h$ Planck's constant, $6.62 \times 10^{-27}$ erg seconds

$\Delta H_a$ heat of activation per molecule

$k$ Boltzmann's constant, $1.38 \times 10^{-16}$ ergs per molecule per $^{\circ}K$

$L$ distance between imperfections in a single crystal (of the order of 1 micron)

$N_d$ number of dislocations per unit area that intersect a plane normal to the slip plane and containing the slip direction

$N_s$ number of sources of dislocations per unit volume

$p$ probability of occurrence of oscillation in crystallographic direction under consideration

$P^+$ ratio of sources of dislocations at which a positive activated complex has formed to total number of right-hand sources of dislocations; that is, probability of formation of a positive activated complex at right-hand source

$P^-$ probability of formation of a negative activated complex at right-hand source

$P' = -k \log_e p^5$

$q$ stress concentration factor

$r$ rate of reaction per unit concentration of reactants

$R_g$ rate of generation (number of dislocations generated/sec for single generating source)

$R_g^+$ rate of generation of positive dislocations at right-hand source

$R_g^-$ rate of generation of negative dislocations at right-hand source

$\Delta S_a$ entropy of activation per molecule

$t$ average time for activated complex to pass into product form

$T$ absolute temperature, $^{\circ}K$

| | |
|---|---|
| $u$ | tensile creep rate at steady-state conditions |
| $u'$ | shear rate |
| $v$ | average velocity of motion of dislocations |
| $V$ | volume associated with one atom |
| $W$ | elastic energy |
| $W_t$ | energy to be attained by thermal motion |
| $x$ | ratio of $d_1$ to $d_2$ |
| $\alpha$ | temperature coefficient of modulus of rigidity |
| $\delta$ | displacement shown in figure 5(a) |
| $\mu$ | Poisson's ratio |
| $\sigma$ | externally applied tensile stress |
| $\tau$ | shear stress |
| $\tau_b$ | back stress |
| $\tau_e$ | externally applied shear stress |
| $\tau_s$ | average shear stress at sources of dislocations |
| $\tau_t$ | shear stress to be attained by thermal motions |
| $\tau_0$ | critical local shear stress for generation of dislocation |

## THEORY

The features of the theory of rate processes and the theory of dislocations required for the development of the creep equation are first reviewed; the derivation of the equation for the steady-state rate of creep then follows.

### Eyring's Theory of Rate Processes

The theory of rate processes (reference 5), as developed by Eyring and others, considers a reaction or any rate process to be the result of the crossing of a potential-energy barrier by molecules

whose energies have exceeded a certain minimum value. The reaction consists of the formation of an "activated complex" capable of crossing the barrier followed by the passage of this complex over the barrier. (See fig. 2.) The most important assumption is that the initial reactants and activated complexes are always in equilibrium. Application of thermodynamic and statistical mechanical considerations shows that the number of activated complexes passing over the barrier per second (or rate of reaction) per unit concentration of reactants is given by

$$r = \frac{kT}{h} e^{-\Delta F_a/kT} \tag{1}$$

The term $kT/h$ can be regarded as the effective frequency at which activated complexes cross over the barrier; the exponential factor represents the probability of formation of an activated complex. In the calculation of this probability term, the contribution due to the translational degree of freedom along the "reaction coordinate" (the most favorable reaction path on the potential-energy surface) has been disregarded because it is included in the factor $kT/h$. The free energy of activation $\Delta F_a$ is interpreted as an ordinary free-energy term and can be expressed by

$$\Delta F_a = \Delta H_a - T\Delta S_a \tag{2}$$

where the heat of activation per molecule $\Delta H_a$ is the height of the potential-energy barrier in figure 2.

## General Features of Dislocation Theory in Relation to Slip

The theory of dislocations arose in relation to the mechanism of slip in single crystals of metal. Slip is the plastic deformation that occurs in an arbitrary short period of time when a metal is subjected to stress. This deformation is characterized by the displacement of one part of the crystal lattice relative to another along a particular crystallographic plane and in a definite crystallographic direction. These slip planes and slip directions are detected by the occurrence of slip bands, step-like discontinuities on the surface of single crystals that have been subjected to stress. The spacings of the bands are generally of the order of 1 micron (reference 1(b)). For any given crystallographic plane, there is a more or less critical value of the component of the applied stress above which the rate of plastic deformation rapidly increases. This value is called the critical shearing stress. Early attempts to explain the mechanism of slip based upon the assumption that the process consists in the sliding of parallel planes over one another lead to theoretically determined values of the critical shearing

stresses, which were of the order of 1000 times too large. It therefore became evident that the true nature of slip involves deviations from a perfect lattice.

Of several mechanisms suggested the one that has been most successful in explaining the slip process is the theory of dislocations. This theory, which was advanced by G. I. Taylor (reference 6), Orowan (reference 7), and Polanyi (reference 8), proposes that local deviations from a perfect lattice called dislocations exist in single crystals and that movement of dislocations through the stressed crystal produces slip. A dislocation consists in a stable arrangement of atoms such that, in a region of a few atomic distances, $n + 1$ atoms in the slip direction face $n$ atoms across the slip plane. The point at which the atoms are one-half an atomic spacing "out of step" is called the center of the dislocation. The extension of the dislocation in the slip plane and normal to the slip direction is called the length of the dislocation.

A simplified model of a dislocation in which two neighboring planes are shown in cross section is presented in figure 3; the plane of the figure is normal to the length of the dislocation. A small shear stress applied as shown in figure 3(a) will cause a displacement of the center of the dislocation by one atomic distance (fig. 3(b)). The dislocation finally passes completely out of the crystal and a perfect lattice is restored, differing from the original lattice in that one plane has now been translated relative to the other by one atomic distance. In the process just described, the entire linear dislocation must move as a unit.

Barrett (reference 9) presents a sketch taken from an earlier work by Taylor (reference 6) showing the two types of dislocation that can exist (fig. 4). If, in the neighborhood of the center of the dislocation, more atoms are above the slip plane, a positive dislocation occurs and, if more atoms are below, a negative dislocation occurs. For the same shear stress, positive and negative dislocations move in opposite directions but the net result of the passage through the crystal is the same, a translation of one part of the lattice with respect to the other by one atomic distance (fig. 4).

Inasmuch as the passing of a dislocation out of the crystal lattice results in the loss of the dislocation, new dislocations must be generated in order for continuous plastic deformation to be possible. A dynamic theory of dislocations is therefore necessary and was developed by Orowan (reference 7) and extended by others, including Seitz and Read (reference 4). Because of the great energy required, the process of generation generally takes place at regions of high stress concentration resulting from imperfections in the

single crystals. The existence of such imperfections as a result of accidents of growth during the formation of single crystals is generally accepted but the nature of the flaws that exist is still a matter of controversy. For example, the theory that single crystals are made up of a mosaic block structure is discussed in reference 4. X-ray evidence indicates that the average spacing of the imperfection is of the order of 1 micron. The general dynamic picture obtained is as follows: Dislocations are generated under shear stress at the regions of high stress concentration, which will be called sources of dislocations. A source of dislocations will be called a right-hand source if it generates dislocations to the right and a left-hand source if it generates dislocations to the left. The dislocations as they are generated are only a few atoms long because of the extremely low probability of generating a full-length dislocation (one whose length is of the order of the spacing between imperfections) (reference 4). As the dislocations move through the crystal under shear stress, they increase in length and move in the slip plane and slip direction until they become stuck at an imperfection or in the vicinity of other dislocations.

Further properties of dislocations and their use to explain the qualitative phenomena of plastic deformation are summarized in a series of four papers by Seitz and Read (reference 4).

## Derivation of an Equation for Creep Rate

In the analysis of the creep process, it will be assumed as in reference 4 that creep as well as slip takes place by the motion of dislocations. The dynamic model previously described will be used. The procedure is as follows: An equation will be derived for the steady-state rate of creep, which will show that the rate of generation of dislocations is the primary factor that determines the variation of creep properties among materials. The rate of generation will then be calculated in terms of rate-process theory as a frequency factor multiplied by the probability that a source of generation will attain the energy required to generate a dislocation.

For a given block of volume $L^3$, if it is assumed that no "annihilation" (union of pairs of dislocations of opposite sign) takes place and that steady-state creep is being dealt with, the number of dislocations generated per second within the block must equal the number leaving the block per second. The number of dislocations leaving the block per second is the number of dislocations in an area $vL$ so

$$R_g N_s L^3 = N_d v L \tag{3}$$

Inasmuch as the displacement produced when a dislocation passes out of the block is $d_1$, the shear rate $u'$ is given by

$$u' = \frac{1}{L}(N_d v L)d_1 = N_d v d_1 \tag{4}$$

The occurrence of slip bands at distances apart of the order of 1 micron indicates an average of one source of dislocations for each imperfection; that is

$$N_s = 1/L^3 \tag{5}$$

When equations (3), (4), and (5) are combined, the expression for shear rate becomes

$$u' = \frac{d_1}{L} R_g \tag{6a}$$

Up to this point, only the creep of single crystals has been considered. In a consideration of polycrystalline metals, it would appear that two types of creep may take place: creep occurring within the individual grains or single crystals, and creep resulting from intergranular motion. As noted in reference 4, present data on the effect of grain size on creep leaves doubt as to the possibility of the occurrence of intergranular creep. For example, the work reported in reference 10 on copper of various grain sizes shows no detectable difference in creep strength other than that resulting from oxidation. Unfortunately, this work was carried out only at a single temperature. Creep in polycrystalline metals is therefore assumed to be predominately the result of deformation occurring within the individual grains by the motion of dislocations. If the orientation of the grains is such that slip occurs in the plane of maximum shear stress, then, as in reference 3, the strain rate $u'$ can be corrected to the tensile creep rate $u$ by an additional factor of about two-thirds. Such a factor would be of no consequence here because $d_1/L$ is known only in order of magnitude; accurate values cannot be given until more is known about the nature of the crystal imperfections. Equation (6a) can therefore be written approximately as

$$u = \frac{d_1}{L} R_g \tag{6b}$$

The rate of generation $R_g$ will now be calculated. A specific model for the mechanism of generation of a dislocation must be set up and certain simplifying assumptions made. The approximations that will be made are similar to those used by Orowan (reference 7) but the treatment presented here is based upon a more definite model than he used. Furthermore, considerations in reference 7 were not applied to steady-state creep.

The generated dislocation will be taken to be 1 atom long. (The length of a dislocation is its extension in the slip plane and perpendicular to the slip direction.) The process of generation will involve displacements of atoms lying along the slip direction by large fractions of an atomic distance. A simple model for the generation of a positive dislocation at a right-hand source is presented in figures 5(a) and 5(b). If atoms 1 and 2 are simultaneously displaced in the directions shown in figure 5(a) by large fractions of the lattice spacing and atoms 3, 4, and 5 are moving in the slip direction as shown, the generation of a dislocation becomes possible. Before a dislocation will form, however, a rearrangement of the neighboring atoms must take place in order to achieve a stable configuration. Figure 5(b) shows the rearranged condition. Comparison of figure 5(b) with the positive dislocation in figure 4(b) shows that the two are essentially the same. The new positions of atoms 5 and 6 after rearrangement were found by using the approximate method used by Taylor (reference 6) in his discussion of the motion of a dislocation under stress. The contributions to the potential field in row B by the neighboring rows A and C are taken to be sinusoidal with periods equal to the spacing of atoms 1 and 3 in row A and atoms 2 and 4 in row C. Figure 5(c) shows that addition of the two sinusoidal potential functions gives a resultant potential function having four minima. Atoms 5, 6, and 7 can be expected to fall into the deepest of these minima if atom 5 had begun by oscillating to the left as in figure 5(a).

The displacement $\delta$ of atoms 1 and 2 will be some fraction $f$ of the atomic separation $d_1$ in the slip direction. A value of $f$ of somewhat less than one-half makes it possible to form a stable configuration with a minimum activation energy.

In the calculation of $R_g$, it will be necessary to use Eyring's general theory of rate processes. A right-hand source, under a shear stress $\tau_s$ having the direction shown in figure 5, will be considered. The "reactant" is the perfect lattice at the region of high stress concentration. The activated complex for a positive dislocation, which for brevity will be called a positive activated complex, can be taken as the configuration shown in figure 5(a) and the product as the dislocation after atomic rearrangement has taken place (fig. 5(b)). If the fundamental assumption of rate-process theory that equilibrium exists between reactant and activated complex is made, the considerations involved in the general theory of rate processes (reference 5) can be applied to the generation of dislocations. From the theory of rate processes, the rate of generation of positive dislocations $R_g^+$ can be expressed as

$$R_g^+ = \frac{1}{t} P^+ \tag{7}$$

The discussion in reference 5 (p. 189) shows that, if the displacement involved in the transition from the activated complex to the final state is on the order of $10^{-8}$ centimeters, the factor $1/t$ is approximately equal to $kT/h$. In the formation of a dislocation, this transition corresponds to passing from the configuration of figure 5(a) to that of figure 5(b), which clearly involves a displacement of this order of magnitude. Hence, $R_g^+$ is given approximately by

$$R_g^+ = \frac{kT}{h} P^+ \tag{8a}$$

Similarly

$$R_g^- = \frac{kT}{h} P^- \tag{8b}$$

The factors $P^+$ and $P^-$ will now be calculated. In order to simplify the calculation, the theory of elasticity is assumed to be applicable on an atomic scale. The generation of a dislocation will be treated as a "local shear," which takes place by means of thermal oscillations. The energy involved in this process will be calculated by the theory of elasticity. Furthermore, Hooke's law will be assumed valid for large strains. The use of these assumptions is necessary because the forces between individual atoms have not been evaluated in terms of physical constants but it should be remembered that the approximations may be serious enough to lead to large errors in the results.

The thermal atomic oscillations in figure 5(a) will be regarded as resulting in a local shearing of atoms 1, 6, and 2 producing a strain of $\delta/d_2$. Inasmuch as the elastic energy per unit volume for a shear stress $\tau$ is $\tau^2/2G$ and the volume of the region of shear is $2V$, the elastic energy involved is

$$W = \frac{V\tau^2}{G} \tag{9}$$

The result will be applied to the generation of a dislocation as follows: Let $\tau_0$ be the local shear stress attained by atoms 1, 6, and 2 in figure 5(a). If the displacement $\delta$ is $fd_1$, application of Hooke's law to the displacement yields

$$\tau_0 = \frac{Gfd_1}{d_2} \equiv Gxf \tag{10}$$

The stress to be produced by thermal motion is

$$\tau_t = \tau_0 - \tau_s$$

if it is assumed that the local shear stress to be attained by atoms 1, 6, and 2 in figure 5(a) must be $\tau_0$, independent of applied stress. The stress $\tau_s$ depends on the externally applied stress and is in the same direction as $\tau_0$. From equation (9) the energy to be attained thermally $W_t$ therefore is

$$W_t = \frac{V}{G}(\tau_0 - \tau_s)^2 \tag{11}$$

Inasmuch as $\tau_s$ can be expected to be only a small fraction of $\tau_0$ for rigid materials, the term in $\tau_s^2$ may be neglected relative to the other terms in the binomial expansion of $(\tau_0 - \tau_s)^2$. The approximation obtained is therefore

$$W_t = VGx^2f^2 - 2Vxf\tau_s \tag{12}$$

where equation (10) has been used.

In order to evaluate $\tau_s$, work-hardening must be considered. A crystal in the steady-state range of creep is generally work-hardened because deformation has taken place. From the nature of a dislocation it is clear that a stress field exists about it. By making use of this stress field, Taylor (reference 6) has explained work-hardening on the basis of a "lattice" of stuck dislocations. Taylor's explanation is a static one and is applied only to determining the form of the stress-strain curve. In terms of a dynamic approach, the existence of a stress field about a dislocation leads to the concept suggested by Kochendörfer (reference 11) that the lattice of stuck dislocations creates an inner stress field whose direction at the point of generation is opposite to the externally applied shear stress $\tau_e$ in the slip plane. The value of the inner stress field at the point of generation will be called the back stress $\tau_b$. The dynamic concept of hardening therefore consists in the formation of a lattice of stuck dislocations, which results in a back stress at the point of generation. This back stress lowers the effective shear stress at these points and as a result the rate of generation, hence the creep rate, decreases. These considerations lead to the result that

$$\tau_s = q(\tau_e - \tau_b) \tag{13}$$

Inasmuch as $\tau_b$ is always less than $\tau_e$, the directions of $\tau_e$ and $\tau_s$ are the same.

In a large assembly of systems, which are in thermal equilibrium with each other, the fraction of them that will have energies greater than some arbitrary value $\epsilon$ is given by $e^{-\epsilon/kT}$. In addition to

the energy requirement, the occurrence of the configuration of figure 5(a) involves a limitation on the direction of atoms 1, 2, 3, 4, and 5 to a particular crystallographic direction within a very small solid angle. If the probability of oscillation in this direction is $p$, the total probability of the configuration of figure 5(a), that is, the probability that a right-hand source will form a positive activated complex is given by

$$P^{+} = p^{5}e^{-W_t/kT} = p^{5}e^{-(VGx^2f^2 - 2Vxf\tau_s)/kT} \tag{14a}$$

Similarly the probability of forming a negative activated complex (the configuration of fig. 5(d)) at a positive source is

$$P^{-} = p^{5}e^{-(VGx^2f^2 + 2Vxf\tau_s)/kT} \tag{14b}$$

inasmuch as $\tau_s$ is opposite to $\tau_0$ in this case. The applied stress $\tau_e$ therefore makes the generation of a positive dislocation more probable than the generation of a negative dislocation at a right-hand source.

If a negative dislocation is generated in the region under discussion, it would move to the left and out of the crystal without producing deformation if it were the only dislocation present. The last positive dislocation generated from this source, however, became stuck after moving through part of the crystal lattice; that is, all forces upon it were balanced. The generation of a negative dislocation at the point being considered will upset this equilibrium and attract the previous positive dislocation. As the two dislocations approach each other, the stress that each exerts on the other increases (from the equation for the stress field about a dislocation, reference 6) and a union takes place resulting in the annihilation of the two dislocations and restoration of a perfect lattice. The net rate of generation $R_g$, which is the rate of generation of positive dislocations that will eventually produce plastic deformation, is $R_g^{+} - R_g^{-}$ or, from equations (8a), (8b), (13), (14a), and (14b)

$$R_g = \frac{2kT}{h} e^{-(VGx^2f^2 + P'T)/kT} \sinh\left[2qVxf(\tau_e - \tau_b)/kT\right] \tag{15}$$

where

$$P' = -k \log_e p^5 \tag{16}$$

The equation for the net rate of generation of negative dislocations at a negative source is clearly identical with equation (15). Equation (15) therefore gives the expression for the factor $R_g$ of equation (6b).

According to this derivation, it is now clear that the generation of a dislocation 2 atoms in length involves the simultaneous oscillation of twice as many atoms with sufficiently great energies. Thus, the probability of such a generation taking place can be neglected as compared with the probability of generating a 1-atom dislocation.

The hyperbolic sine term in equation (15) arose from the lowering of the potential-energy barrier in the direction of the applied stress. The heat of activation per molecule $\Delta H_a$ (the height of the barrier for zero applied stress) and the entropy of activation per molecule $\Delta S_a$ are (by comparison with equations (1) and (2))

$$\Delta H_a = VGx^2f^2 \tag{17a}$$

$$\Delta S_a = - P' \tag{17b}$$

$P'$ is always positive because $p$ is less than 1 (equation (16)), therefore $\Delta S_a$ is negative.

When the general expression for $R_g$ given by equation (15) is combined with the creep equation (6b), the complete equation for the steady-state rate of creep becomes

$$u = \frac{2d_1}{L}\frac{kT}{h} e^{-(VGx^2f^2 + P'T)/kT} \sinh\left[qVxf(\sigma - 2\tau_b)/kT\right] \tag{18}$$

where the applied tensile stress $\sigma$ has been substituted for $2\tau_e$ assuming that slip in the individual grains occurs in the plane of maximum shear stress. For high values of applied tensile stress $\sigma$ the hyperbolic sine function, to a good approximation, can be replaced by an exponential and equation (18) becomes, after taking logarithms of both sides

$$\log_e u = \log_e\left(\frac{d_1}{L}\frac{kT}{h}\right) - \left(\frac{VGx^2f^2 + P'T}{kT}\right) + \frac{qVxf}{kT}(\sigma - 2\tau_b) \tag{19}$$

Equation (18), or its approximation equation (19), is the form of the theoretical equation to be used for the steady-state creep rate of pure polycrystalline materials as a function of stress and temperature.

## COMPARISON OF THEORETICAL AND EXPERIMENTAL CREEP EQUATIONS

The evaluation of the back stress $\tau_b$ as a function of temperature and applied stress is difficult and has not yet been accomplished. A great deal may be learned, however, from a comparison of the theoretical equation (19) with empirical equations. It has been empirically

determined that a hyperbolic sine function represents the dependence of creep rate on applied tensile stress $\sigma$ for many polycrystalline materials (reference 12) in agreement with equation (18). At high values of stress, the hyperbolic sine function reduces to an exponential. In reference 13 creep data are presented, which show that a relation of the form

$$\log_e\left(\frac{u}{T}\right) = -\frac{a}{T} - b + \gamma\sigma \tag{20}$$

represents the experimental data in the range of high stresses, where $a$ and $b$ are constants for a given material and $\gamma$ is a function of temperature but not of stress. If the notation is so changed as to make this equation similar to equation (19), equation (20) can be rewritten

$$\log_e u = \log_e\left(\frac{d_1}{L}\frac{kT}{h}\right) - \frac{A + BT}{kT} + \frac{\beta\sigma}{kT} \tag{21}$$

where

$A = ka$

$B = kb + k \log_e\left(\frac{d_1 k}{Lh}\right)$

$\beta = \gamma kT$

Comparison of the forms of equations (19) and (21) show good agreement. The simultaneous validity of the two equations requires that $\tau_b$ be a linear function of stress. In addition, the condition $\tau_b = 0$ for $\sigma = 0$ leads to the following expression for $\tau_b$

$$\tau_b = F(T)\sigma \tag{22}$$

Equation (21) shows that three factors completely determine the steady-state rate of creep: the constants $A$ and $B$ and the temperature function $\beta$. A comparison of equation (21) with equations (19) and (22) shows that these factors are expressed in terms of physical constants of the material as

$$A = VG_0 f^2x^2 \tag{23}$$

$$B = P' - Vf^2x^2G_0\alpha \tag{24}$$

$$\beta = qVxf(1 - 2F(T)) \tag{25}$$

where the modulus of rigidity $G$ has been taken in the approximate form

$$G = G_0(1 - \alpha T) \tag{26}$$

## COMPUTATIONS AND PRECISION

Creep data were obtained from literature for a number of polycrystalline "pure" metals that had been annealed (references 13 to 20). (Although the references do not generally state the impurity content, the metals will be considered pure as compared with alloys.) The reproducibility of the creep data from the various sources is not known. From these data, curves of log creep rate against stress were plotted and straight lines were obtained. In terms of equation (21), the coefficient $\beta/kT$ divided by 2.3 is the slope of this straight line and the intercept is equal to $\log\left(\frac{d_1}{L}\frac{kT}{h}\right) - \frac{(A + BT)}{2.3\ kT}$.

The values of $\beta$ and $H$ (defined as $A + BT$) were then determined from the slopes and the intercepts, respectively. Finally, $A$ and $B$ were separately determined from a linear plot of $H$ against $T$ where $d_1/L$ was taken as $10^{-4}$ for all materials.

The errors involved in the determination of $A$, $B$, and $\beta$ depend in part on the number of stresses for which data were available at each temperature and on the number of temperatures at which observations were made. The ranges in temperatures and stresses used will also affect the error. Generally, the errors for $A$, $B$, and $\beta$ were estimated to be about $\pm 5 \times 10^{-13}$ ergs, $\pm 8 \times 10^{-16}$ ergs per °K, and ±30 percent, respectively.

For a large number of pure polycrystalline metals, it was found that $\beta$ varied with temperature. The function

$$\beta = C_1 e^{C_2 T}$$

approximates the temperature dependence of $\beta$. This function was determined over comparatively short ranges of temperature (200° to 300° K) and therefore may not apply over a large temperature range.

Values of the modulus of rigidity $G$ were obtained from references 21 and 22. These references showed that variations in the value of $G$ for a given material are about ±5 percent. Room-temperature values were used as approximations of the absolute-zero values of $G$.

## RESULTS AND DISCUSSION

### Factor A

Equation (23) gives the theoretical expression

$$VG_0 f^2 x^2$$

for the factor A of the empirical equation (21). In figure 6, A/V is plotted as a function of $G_0 x^2$. The line drawn is the best straight line through the origin and represents the data within the experimental error of A. From the previous theoretical discussion, the slope of this line is $f^2$ and is equal to 0.25 or less. Because the activation energy predicted on the basis of the validity of Hooke's law will be greater than the actual value, it can be expected that the theoretical prediction will be too high. The experimental slope from the straight line of figure 6 is 0.14.

The agreement between theoretical prediction and experimental results is encouraging when the assumptions made and the variety of sources of data are considered.

## Factor B

When equations (19) and (22) were compared with the empirical equation (21), B was found to be equal to

$$P' - Vx^2f^2G_0\alpha$$

where $\alpha$ is determined from the approximate relation

$$G = G_0(1 - \alpha T)$$

The order of magnitude of the term $Vx^2f^2G_0\alpha$, which is $3 \times 10^{-16}$ ergs per °K, is within the estimated error of B, which is $\pm 8 \times 10^{-16}$ ergs per °K; this term can therefore be neglected in comparing the B values for different materials. The values of B for different metals are presented in the following table:

| Material | B (ergs/°K) | Reference |
|---|---|---|
| Aluminum | $40 \times 10^{-16}$ | 13 |
| Copper | 45 | 14, 15 |
| Iron | 39 | 17 |
| Lead | 54 | 20 |
| Nickel | 33 | 16 |
| Platinum | 42 | 13 |
| Silver | 36 | 13 |
| Tin | 47 | 19 |

A correlation exists between the scatter of data presented in figure 6 and the scatter of B values in this table. The straight line in figure 6 falls between the experimental points for aluminum

and iron. The table shows that aluminum and iron have B values equal to $40 \times 10^{-16}$ and $39 \times 10^{-16}$ ergs per °K, respectively. The points above the line in figure 6 are all related to metals that have B values less than $40 \times 10^{-16}$ ergs per °K and the points below the line correspond to B values greater than $40 \times 10^{-16}$ ergs per °K.

The correlation between the direction of scatter of data for A and B is satisfactorily explained by considering the method used to obtain experimental values of A and B. For each material, values of H (defined as A + BT) were plotted against T and the values of A and B were obtained from the intercept and slope, respectively, of the best straight line drawn through the points. For a given material, the value of the intercept A will increase as the slope B of the line drawn through the experimental points is decreased. In view of this explanation, the scatter of data for A about the theoretical line appears to result from experimental error alone. It can also be concluded that the correct values of B fall about their mean value much more closely than those in the table.

When the term containing $\alpha$ is neglected, B becomes equal to P', which from equation (17b) is the negative of the entropy of activation $\Delta S_a$. The high negative value of the entropy of activation in the creep process has been pointed out in references 3 and 13 but satisfactory quantitative explanations of the magnitude of this factor could not be given.

On the basis of the present treatment, the restriction of direction of 5 atoms participating in the generation of an activated complex accounts for the large negative entropy of activation. A good approximation of the value of the probability of oscillation p of an atom in the slip direction can be obtained by substituting values of B for P' in equation (16). A value of about $3 \times 10^{-3}$ was obtained for p when P' was equal to $40 \times 10^{-16}$ ergs per °K.

## Factor $\beta$

The theoretical expression for the factor $\beta$ is given by

$$\beta = qVxf(1 - 2F(T))$$

The term F(T) in this equation is related to the back stress $\tau_b$ (equation (22)), which resulted from the inner stress field about a dislocation. This stress field is related to the modulus of rigidity G (reference 6). Therefore, $\beta$ at any given temperature was expected to be a function of G. Figure 7 shows that at room temperature $\beta$ varies as $G^{-n}$ where n is approximately 3.

In order to determine whether the temperature dependence of $G$ could alone account for the temperature dependence of $\beta$, the variation of $G$ with temperature was investigated. It can be seen in figure 8 that for iron, $\beta$ and $G^{-9}$ have about the same temperature dependence. Figure 6 shows, however, that $\beta$ is proportional to $G^{-3}$. Therefore, it can be concluded that $F(T)$ contains a temperature-dependent factor in addition to $G$.

## EVALUATION OF RESULTS

The statement is made in reference 9 (p. 344) that "a full interpretation of creep results in terms of dislocation theory would be welcome but has not yet been achieved." In the present report two contributions are made to a fuller understanding of creep in terms of both rate-process theory and dislocation theory. The first contribution is the quantitative dependence of creep on physical constants of the material; the second is an understanding of the origin of the large negative entropy of activation $\Delta S_a$, which was a source of difficulty in previous treatments (references 3 and 13).

Although the previous theories have been successful in showing the dependence of creep on stress and temperature, none has been able to show the quantitative dependence of creep on physical constants of materials. From a practical point of view, equations that relate creep to stress and temperature have useful engineering application in predicting the creep behavior of a given material. These equations can yield little useful information, however, to the metallurgist who must find criteria to use in synthesizing alloys for better creep resistance. A knowledge of the dependence of creep rate on physical constants of material would yield such criteria and therefore have a practical as well as an academic interest.

The empirical equation that gives the dependence of steady-state rate of creep on stress $\sigma$ and absolute temperature $T$ can be taken as

$$\log u = \log\left(\frac{d_1}{L}\frac{kT}{h}\right) - \frac{A + BT}{2.3\ kT} + \frac{\beta\sigma}{2.3\ kT} \tag{21a}$$

The theoretical treatment in this report has shown that the parameters $A$, $B$, and $\beta$ depend on physical constants of the material in the following manner:

$$A = VG_0\ f^2x^2 \tag{23}$$

$$B = P' - Vx^2f^2G_0\alpha = P' - A\alpha \tag{24}$$

$$\beta = qVxf(1 - 2F(T)) \tag{25}$$

Equation (21a) shows that, in order to have a low rate of creep, $A$ and $B$ must be large and $\beta$ must be small. Equation (24) indicates that the effect of any physical constant on $A$ will be in the opposite direction to its effect on $B$. Inasmuch as $\alpha$ in equation (24) is of the order of $10^{-4}$ and $P'$ has been shown to be approximately constant for all pure annealed materials, the physical constant will have a greater effect on $A$ than on $B$ up to a temperature of $10^{40}$ K. The most important physical constant in equation (23) is the modulus of rigidity at absolute zero $G_0$ inasmuch as the other constants do not vary much in order of magnitude among different materials. Empirically $\beta$ was also found to decrease with increasing $G$. Pure metals having high moduli of rigidity will therefore be creep resistant.

The relation between modulus of rigidity $G$ and modulus of elasticity $E$ is given by

$$G = \frac{E}{2(1 + \mu)}$$

Inasmuch as values of $\mu$ usually run from about 0.3 to 0.4, the effect of variations in this factor for different materials will be so small that in general, pure metals will be creep resistant if their modulus of elasticity is high.

The extension of the theory to alloys, in particular heat-resisting alloys, will require modification to include the effects of strains produced by solid soluble atoms and precipitated particles. In view of the need for criteria that will hasten the development of alloys having heat-resisting properties, any indications that can be obtained from the present theory should be considered. The theory suggests the use of materials of high moduli of rigidity, and therefore of high moduli of elasticity, such as tungsten, molybdenum, and cemented tungsten carbide as matrix materials for heat-resisting alloys. An investigation conducted under the auspices of the National Defense Research Committee has shown that chrome-base alloys having high tungsten or molybdenum content generally have better heat-resisting properties than currently used alloys. The use of still higher percentages of these elements therefore merits serious consideration.

Aircraft Engine Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, January 18, 1946.

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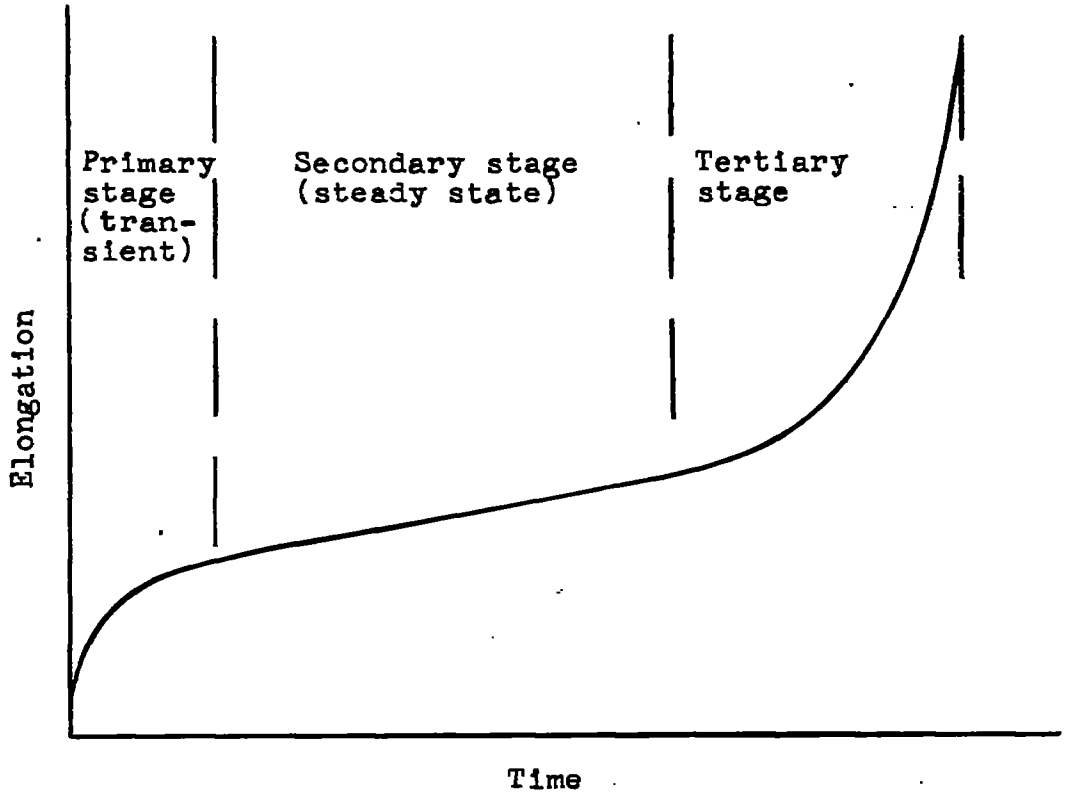


Figure 1.- Typical creep curve showing three stages.

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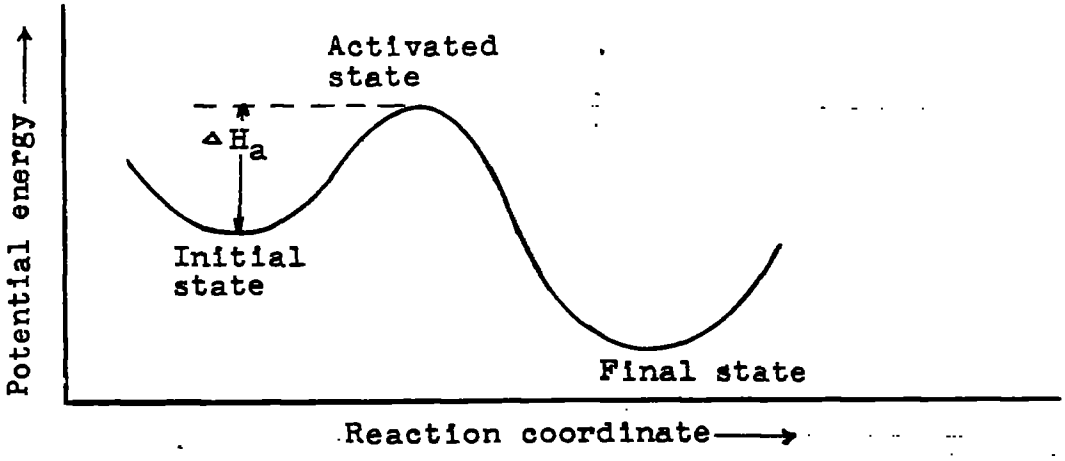


Figure 2.- Schematic representation of potential barrier.

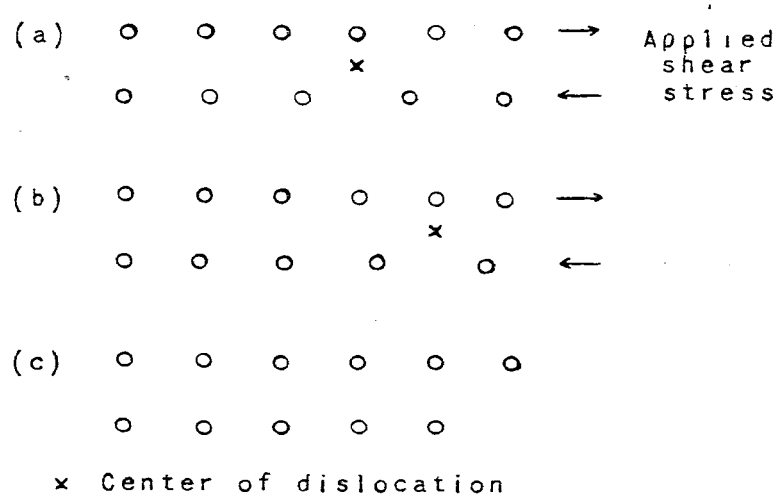


Figure 3. - Schematic representation of displacement of two planes by motion of a dislocation.

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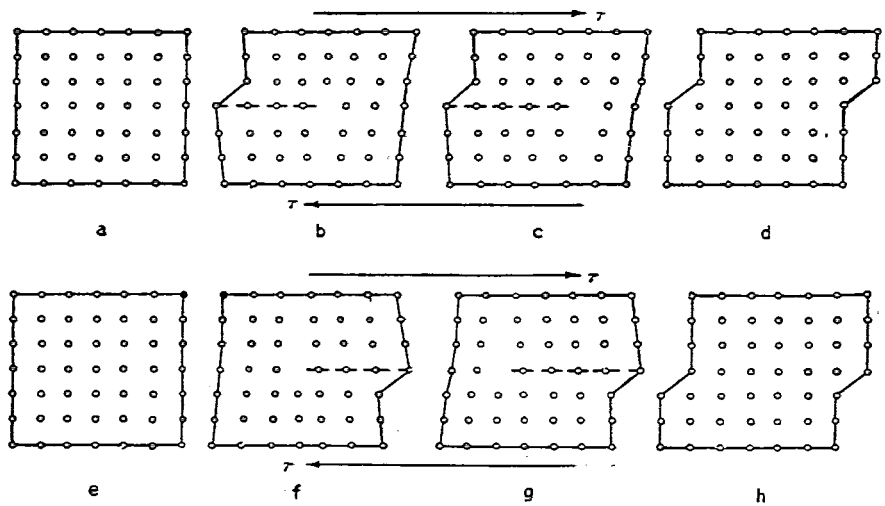
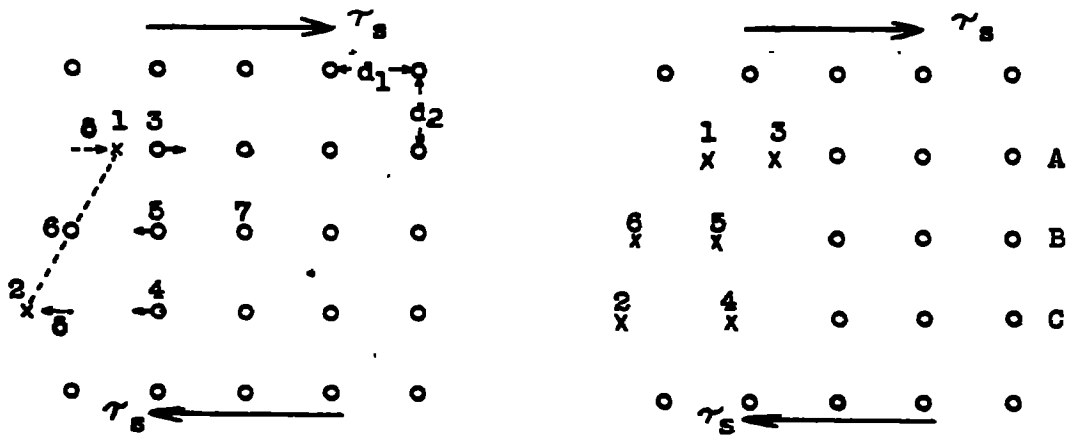


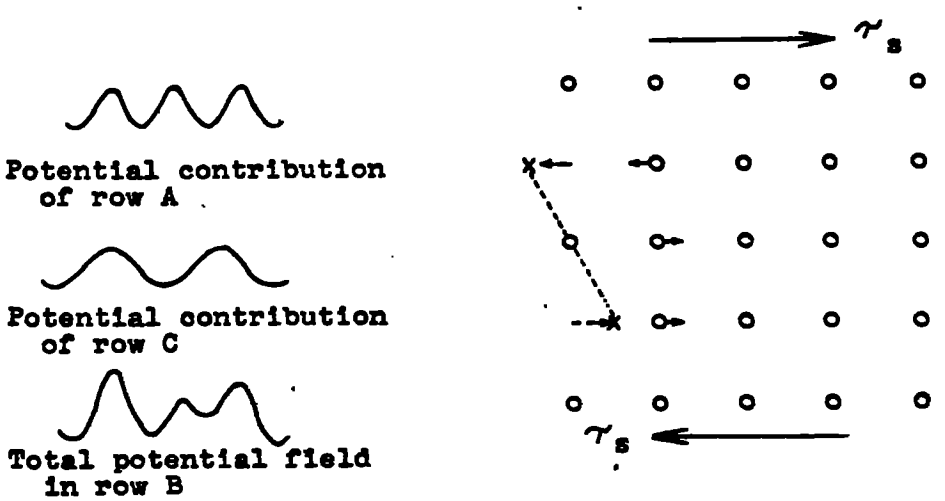

Figure 4. - Generation and movement of a dislocation. In sketches a, b, c, and d a positive dislocation moves to the right; in e, f, g, and h a negative one moves to the left; the resulting deformation is identical. (Taylor.) (Reproduced from fig. 1 of reference 9, p. 335.)




× Atoms that have made critical oscillations
o Atoms that are still unaffected by the oscillations

(a) Activated complex for a positive dislocation.

(b) Dislocation formed by rearrangement of the activated complex.




(c) Determination of potential field in row B.

(d) Activated complex for a negative dislocation

Figure 5.- Generation of a dislocation at a positive source.

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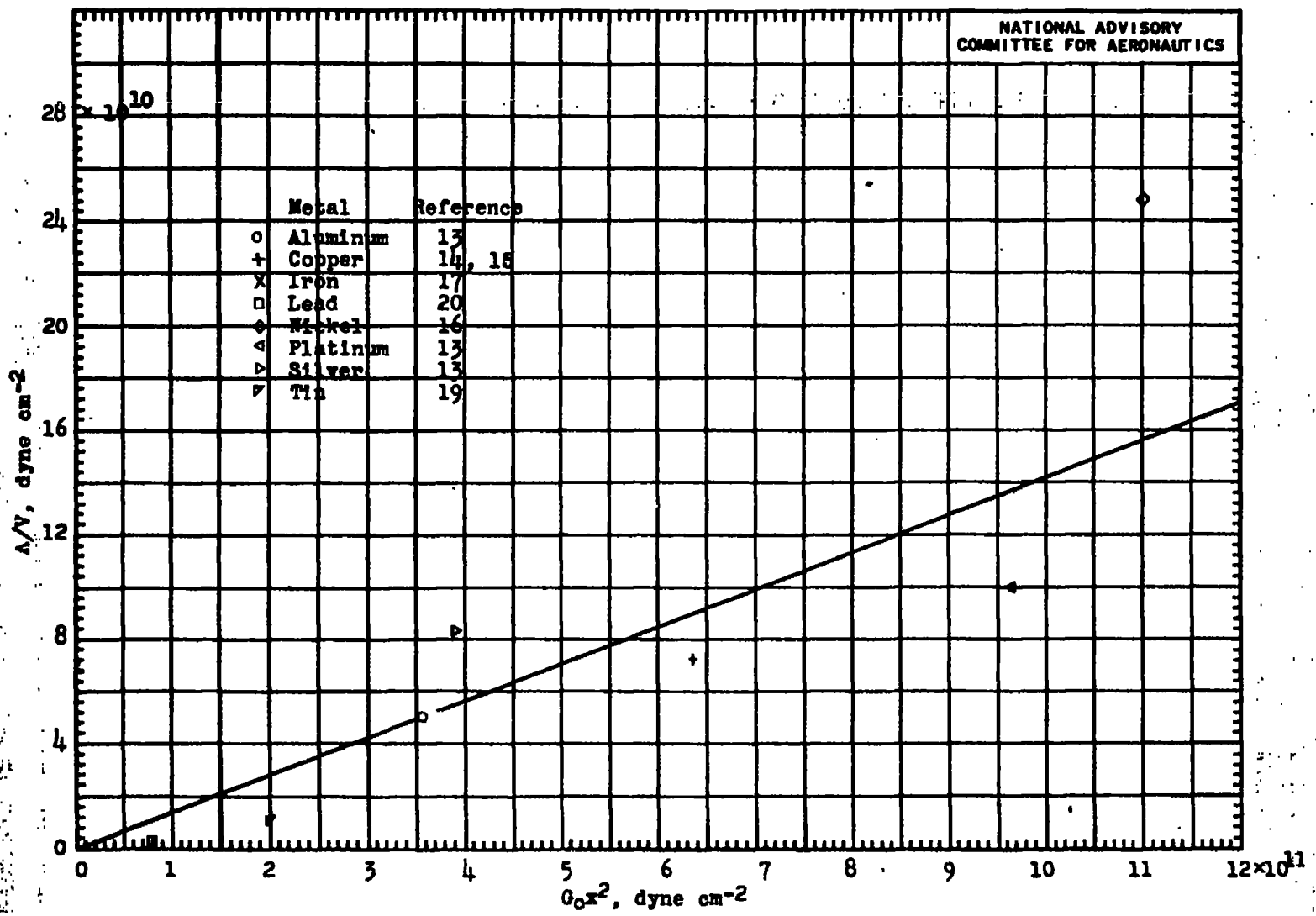


Figure 6. - Comparison of experimental data for factor A of equation (21) with theory.

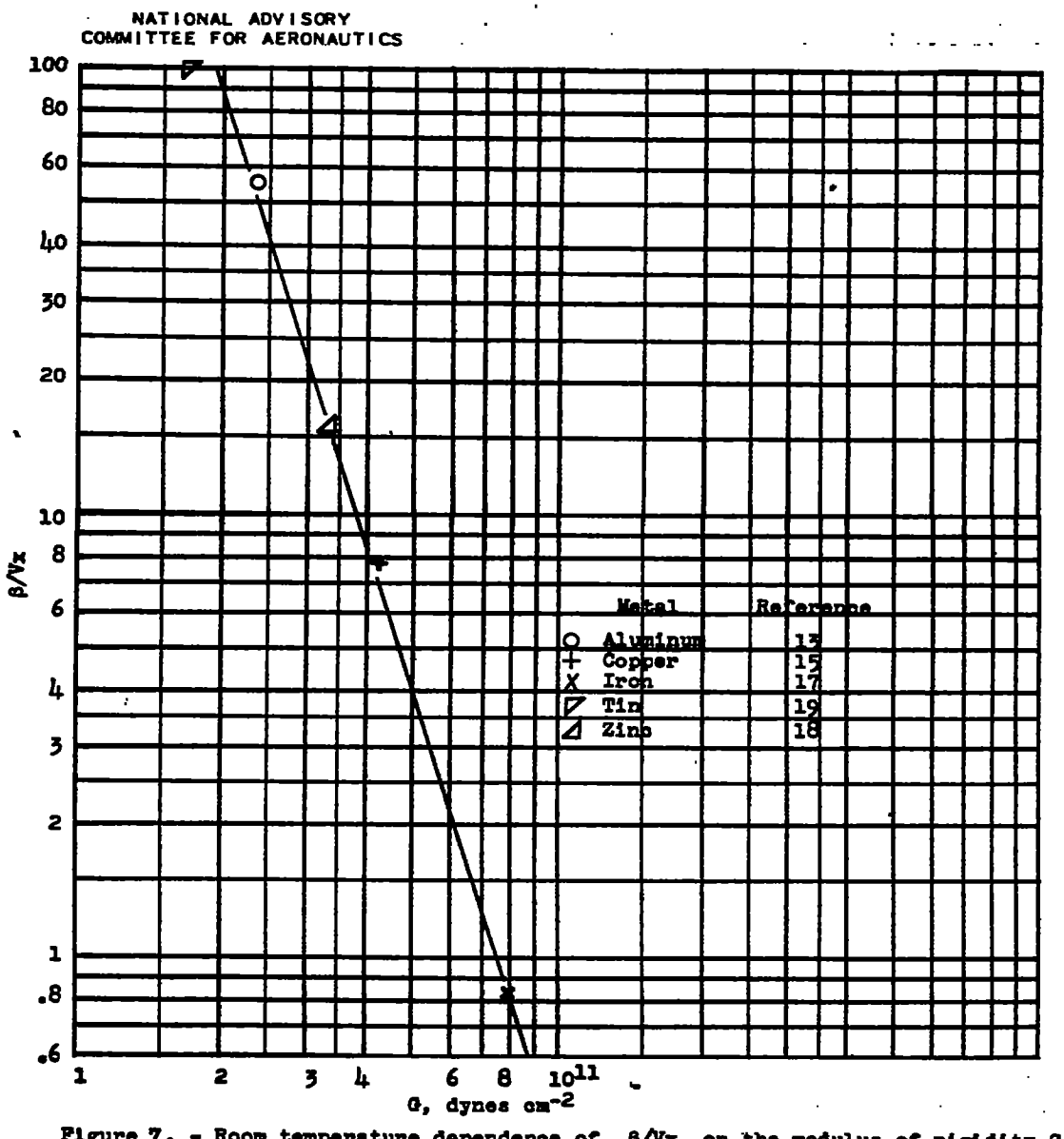


Figure 7. - Room temperature dependence of $\beta/V_x$ on the modulus of rigidity G.

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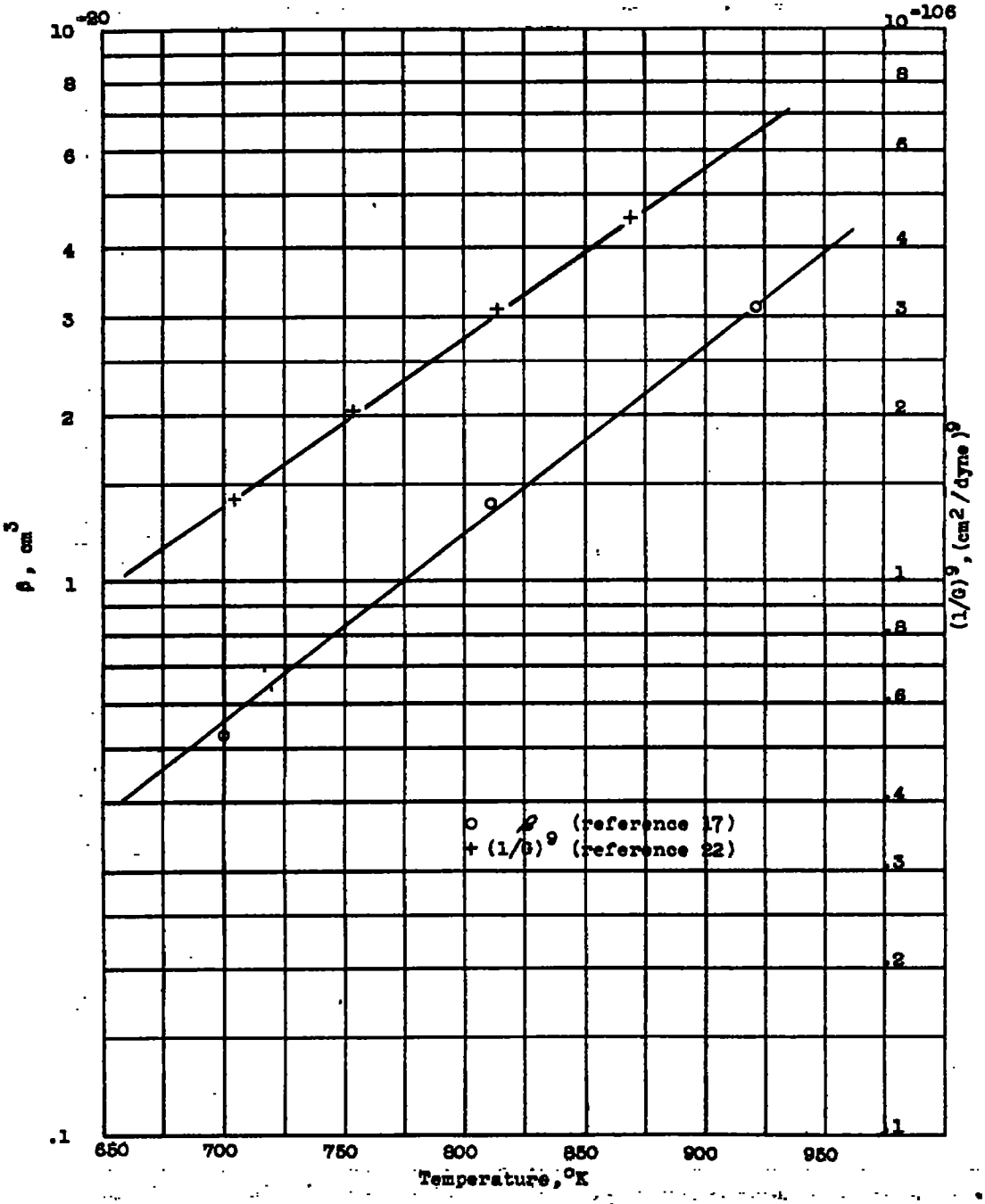


Figure 8.- Temperature dependence of $\beta$ and $1/G$ for iron.